Fees, Reputation, and Rating Quality*

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Abstract

This paper studies how the compensation of a rating agency affects its incentive to collect information before assigning a rating. A sequence of short-lived firms can hire a long-lived monopolistic agency to rate projects of unknown quality. Information acquisition is costly for the agency and unobservable for third parties. The agency might be committed to obtaining an informative signal or fully strategic. I compare the *status quo*, in which the agency is compensated only when a firm publishes a rating, with reformed compensation schemes, in which the agency is paid a share of its fee whenever hired. The scheme that requires the entire fee to be independent of publication ensures the most informative ratings as long as the agency puts sufficient weight on future revenues. If instead the agency is impatient, fees entirely contingent on publication produce the most informative ratings. My results hinge on the assumption that the agency's information acquisition is unobservable. If information acquisition is observable, fees not contingent on publication always result in the most informative ratings. I discuss implications for the policy debate on rating agency regulation.

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This is a major overhaul of the system ... it is a dramatic change.
- Andrew Cuomo on the Cuomo Plan. June, 2008.[1]

This feels cosmetic to me, ... getting paid for just showing up doesn't strike me as a good model or incentive structure.
- Lawrence White on the Cuomo Plan. June, 2008.[1]

1. Introduction

In 2011, the United States Senate issued a report on the causes of the recent financial crisis (US Senate (2011)). The report devoted an entire section to the role of credit rating agencies, and blamed them for overlooking factors that would have induced lower ratings for Residential Mortgage-Backed Securities (RMBS) and Collateralized Debt Obligations. The report argued that a more careful rating process would have increased awareness of the riskiness of mortgage-related securities. The following excerpt is illustrative:

> Despite the increasing number of ratings issued each year and record revenues as a result, neither Moody's nor S&P hired sufficient staff or devoted sufficient resources to ensure that the initial rating process ... produced accurate credit ratings. US Senate (2011), page 304.

The report was one of many voices critical of the performance of rating agencies.[2] Such criticism has spurred a wave of proposals to reform the credit rating process. The way rating agencies are paid for their services has received the most attention.[3] The largest rating agencies receive fees from the issuers of the securities to be rated. Those fees are paid only if the rating is ultimately published by the security issuer.[4] Although the issuer-pay model has been heavily criticized, it is still by far the most common compensation scheme.[5]

[1]http://www.crainsnewyork.com/article/20080605/FREE/323523855/cuomo-reaches-deal-with-ratings-agencies.
[2]See Benmelech and Dlugosz (2009), White (2010) and Haan and Amtenbrink (2011).
[3]The Dodd-Frank Act requires the Securities and Exchange Commission (SEC) to carry out an assessment of "potential mechanisms for determining fees for the nationally recognized statistical rating organizations"; and "appropriate methods for paying fees to the nationally recognized statistical rating organizations"(US Senate (2010), page 514).
[4]As indicated in US SEC (2012), "An NRSRO [nationally recognized statistical rating organization] that operates under an firm-pay model typically is paid only if the credit rating is issued, though sometimes it receives a partial fee for the analytic work undertaken if the credit rating is not issued" (page 12).
[5]As of the end of 2013, more than 99% of ratings issued by rating agencies registered as NRSRO were issued under the issuer-pay model (US SEC (2014)).

As early as 2008, Andrew Cuomo, who then served as Attorney General of New York, proposed an agreement to reform the issuer-pay model.[6] The agreement was known as the Cuomo Plan. The plan required issuers of RMBS to pay an agency whenever they requested a rating, regardless of whether the rating was ultimately published. The goal was to ensure more reliable ratings by eliminating the conflict of interest intrinsic to the issuer-pay model.

I develop a dynamic model to evaluate policies that require a share of the rating fee to be paid whenever a rating agency is hired, regardless of the choice to publish the rating obtained. In every period a new firm has a project of unknown quality. Before taking the project to potential investors, the firm has the option to hire an infinitely-lived rating agency to collect information about the project and rate it. If hired, the agency can incur the cost necessary to learn the true quality of the project and rate it accordingly. The alternative is to shirk and assign a favorable, high rating. The agency has an unobservable type, as in the models pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982). The agency can be *committed* to obtain an informative signal or fully *strategic*.

I compare the informative content of ratings in an unregulated market in which the agency collects its rating fee only if the firm publishes the rating, referred to as contingent fees, with the informative content in regulated markets, in which firms pay a share of the rating fee whenever they hire the agency. When the rating agency is not very patient, an unexpected result holds. Regulations that require a share of the fees to be independent of publication *reduce* the probability that the agency will collect an informative signal, and as a result *reduce* the overall welfare. When instead the rating agency is sufficiently patient, a regulation requiring the entire fee to be independent of publication, referred to as fixed fees, ensures the highest probability that the agency will collect the informative signal. Interestingly, "intermediate" policies, requiring part of the fee to be contingent on publication and the rest to be independent, are never the unique socially optimal policy.

In order to explain the results it is useful to focus on the policy of fixed fees. Fixed fees have a static and a dynamic effect. The static effect works as expected. As long as fees are contingent, honest ratings entail an opportunity cost, as the agency receives no compensation whenever it assigns unfavorable, low ratings. Fixed fees eliminate this opportunity cost and provide stronger static incentives to collect information and rate honestly than contingent fees.

At the same time, fixed fees have a dynamic effect, namely that they reduce the incentive to build a reputation for commitment. The effect can be explained as follows. All else being equal, a firm expects to pay the same amount to the agency regardless of whether the regulation is in place. The expected revenue of the agency in each period increases whenever the reputation for commitment becomes stronger. Under fixed fees, the agency's expected

[6] http://www.ag.ny.gov/press-release/attorney-general-cuomo-announces-landmark-reform-agreements-nations-three-principal.

revenue corresponds to the realized revenue, and a *strategic* or a *committed* agency earn the same revenue. In contrast, under contingent fees, the agency's realized revenue differs from the expected revenue because the fee depends on the rating assigned. If the *strategic* agency shirks, and assigns a high rating, its current revenue is higher than that of a *committed* agency. This option is particularly valuable when the agency has a strong reputation for commitment because in this case a favorable rating is valuable to the firm and the contingent rating fee is high.

These observations reveal that a *strategic* agency earns a higher revenue under contingent fees than under fixed fees and, most importantly, the difference in revenue becomes larger as the reputation for commitment becomes stronger. Fixed fees result in the revenue of a *strategic* agency being less responsive to shifts in reputation. To the extent that fixed fees reduce the value of improving reputation, they weaken the dynamic incentive for a *strategic* agency to collect informative signals.

For small discount factors, a *strategic* agency is more likely to exert effort under contingent fees than under fixed fees, because the dynamic effect is relatively strong and the static effect relatively weak. If the discount factor is small, a *strategic* agency has weak incentives to collect informative signals under either compensation scheme. When the agency is expected to shirk, its rating has little value for a firm, the rating fee is low, and the static effect is weak. At the same time, the dynamic effect is strong. As a *strategic* agency is expected to shirk and assign a high rating, the agency's reputation becomes considerably stronger whenever it assigns a low rating. As discussed above, the prospective of a strong reputation is more valuable to a *strategic* agency under contingent fees than it is under fixed fees.

I argue that policies which prohibit contingent fees can curb the incentive to acquire costly information. This result hinges on the assumption that the agency incurs a cost to obtain an informative signal. If the agency can collect the signal at no cost, fixed fees always eliminate all incentives to shirk and ensure that the agency assigns reliable ratings.

My results show that policies which require the entire rating fee, or part of it, to be independent of publication are counterproductive when the rating agency has a small discount factor and when the cost to collect the signal is not negligible. The discouraging conclusion to draw from these results is that the policies considered are socially counterproductive exactly in the cases in which we should expect the rating process to be less efficient to start with. A more constructive interpretation of the results however, shows that fixed fees unequivocally improve the rating process, as long as they are matched with supervision of the resources that the agency devotes to the rating process. The idea is that while regulators cannot eliminate the cost of obtaining informative ratings, they can scrutinize and publicly disclose the amount of resources that an agency devotes to the rating process. This is particularly relevant given recent efforts by regulators to facilitate the external evaluation of procedures followed by rating agencies.[7] In an extension, I compare fixed and contingent fees while assuming that firms and investors can observe whether the agency shirks or obtains an informative signal. In this setting, fixed fees eliminate all incentives to shirk and ensure higher welfare than contingent fees.

My paper provides the first comparative analysis of rating agency compensation schemes within a dynamic framework. The model follows the steps of Mathis, McAndrews and Rochet (2009). The analysis departs from their work as I introduce a moral hazard component to the rating process and I compare the rating process under different compensation schemes. The results strengthen the main conclusion of Mathis, McAndrews and Rochet (2009): reputation motives are not always sufficient to ensure a reliable rating process.

Previous papers have considered the policy introduced by the Cuomo Plan in a static setting. Bolton, Freixas and Shapiro (2012) show that whenever the quality of the signal obtained by the agency is exogenous, fixed fees eliminate all incentives to lie. This is similar to the result I obtain for costless signals. The authors warn that fixed fees might lead to uninformative ratings if the agency chooses the precision of its signal, effort is observable but not contractible, and there is no uncertainty over the type of the rating agency. Similarly, in my model, whenever firms and investors know the type of the rating agency, a *strategic* agency has no incentive to exert effort. When instead firms and investors are uncertain about the type of the agency, the desire to maintain a reputation for commitment provides incentives to exert effort and assign reliable ratings. In Ozerturk (2013) fixed fees induce the rating agency to provide more or equally informative ratings than the agency would under contingent fees. According to Kovbasyuk (2013), the effect of the Cuomo Plan depends on whether the contract between a firm and the rating agency can be observed by the investors. In my model, contracts are not observable. Kovbasyuk (2013) argues that in this case the Cuomo Plan reduces the incentive to assign high ratings to low-quality projects. Kashyap and Kovrijnykh (2013) study the optimal compensation of a rating agency in a static environment. They consider a broader range of compensation schemes than I do, including investor-pay schemes, but they do not consider the reputation building process.

My paper is also related to models in which rating agencies receive fixed fees. Frenkel (2015) shows that, under fixed fees, an agency has an incentive to assign undeserved high ratings only if firms are long lived and require ratings for more than one project. In Frenkel (2015) the agency obtains its signals at no cost. In a market with short-lived firms and costless signals, my model has the same prediction as Frenkel (2015): under fixed fees, the

[7]Sections 932 and 939E of the Dodd Frank Act, contain, among other things, provisions that require (a) the SEC to monitor rating agencies and sanction those that do not have sufficient resources to perform their task; (b) rating agencies to document their internal controls over procedures and (c) the creation of a professional organization that would set standards for the profession of rating analysts.

agency rates honestly. Bouvard and Levy (2013) also consider a rating agency that receives fixed fees. The rating fee is set exogenously and firms know the quality of their projects. These differences from my model lead to contrasting conclusions about the role of reputation. In my model, (i) the agency monotonically prefers a stronger reputation for commitment and (ii) reputation motives lead to more informative ratings. In Bouvard and Levy (2013) revenues are maximized at intermediate levels of reputation, so that when reputation for commitment is strong, reputation concerns motivate a *strategic* agency to assign undeserved high ratings.

Recent theoretical research on credit ratings has addressed a variety of topics. Farhi, Lerner and Tirole (2013), Skreta and Veldkamp (2009), and Sangiorgi and Spatt (2013) focus on credit shopping, that is, a strategy of cherry-picking the most favorable ratings. Unsolicited ratings are considered in Fulghieri, Strobl and Xia (2013). Pagano, Volpin and Wagner (2010) and Farhi, Lerner and Tirole (2013) study the transparency of rates and Damiano, Li and Suen (2008) consider the role of coordination among raters working for the same credit rating agency. Credit ratings are meant to evaluate the default probability of debtors, but ratings can also influence probability of default. This feedback effect is considered in Manso (2013) and Holden, Natvik and Vigier (2014). White (2010) and Dranove and Jin (2010) provide comprehensive reviews on the subject.

The paper is structured as follows. Section 2 introduces the baseline model with contingent fees. Section 3 presents the equilibrium. Section 4 shows the equilibrium in regulated markets. Section 5 compares the properties of the equilibria under the different compensation schemes. Section 6 extends the analysis. Section 7 concludes. All the proofs are contained in the appendices.

2. Baseline Model

The game has infinite periods and three types of risk neutral agents: a monopolistic rating agency that is present in every period, and a sequence of one firm and $n \geq 2$ investors, active for one period only. In each period t, the active firm owns a project of unobservable quality q_t. Quality can be high ($q_t = H$) or low ($q_t = L$). All the agents know that qualities are independent, identically distributed and

$$Pr\{q_t = H\} = \lambda \in (0,1), \quad \forall t.$$

The firm does not have sufficient resources to start the project, so it looks for an investor to buy the project opportunity. Every project requires an investment of α. A project ensures a gross return equal to 1 if its quality is high and 0 if its quality is low. I assume $\alpha \in (0,\lambda)$, so without additional information it is profitable to finance a project. In Section 6, I consider the case of $\alpha \in (\lambda,1)$.

At the beginning of each period, the agency publicly announces its rating fee ϕ_t. The firm can put the project up for sale immediately or wait for the agency to rate it. If the firm waits, the agency decides privately whether to scrutinize a project, i.e. "exert effort", or shirk. If quality is low, hard information about the nature of the project exists. No hard information exists when the quality is high. The agency observes a signal $s_t \in \{h, l\}$. If the agency exerts effort, the signal is perfectly informative: $s_t = h$ whenever $q_t = H$ and $s_t = l$ whenever $q_t = L$. If the agency shirks, the signal is completely uninformative: $s_t = h$ regardless of the quality. Signal l corresponds to observing the hard information.

The rating process corresponds to the choice to disclose the hard information or not. Without loss of generality, I assume that the agency assigns a rating identical to the signal observed. This is equivalent to assuming that the agency always discloses any unfavorable information. This does not entail a loss of generality as even if the agency had the option to hide the hard information, it would never need to do so. Rather than hiding information, the agency could shirk and observe $s_t = h$.

These signal and rating structures require the agency to provide evidence only when it assigns an unfavorable rating. A similar assumption is present in Bouvard and Levy (2013) and Frenkel (2015). As noted in Frenkel (2015), as the agency is hired by the active firm, it seems reasonable that the burden of the proof is heavier when the agency gives an unfavorable rating.

The agency has a type, determined once and for all at the beginning of the game. The agency is *committed* with probability $\mu_0 \in (0,1)$, and *strategic* otherwise. The agency knows its type, while the other agents only know the prior distribution. A *committed* agency exerts effort, at no cost, whenever the firm waits for a rating. A *strategic* agency chooses whether to exert effort, at a cost $\kappa > 0$, in order to maximize the discounted sum of its instantaneous payoffs.[8] The probability that a *strategic* agency exerts effort is denoted $e_t \in [0,1]$. I assume that the cost κ is low enough that it is socially efficient to obtain a signal and finance only high-quality projects.[9]

Assumption 1. $0 < \kappa < (1-\lambda)\alpha$.

The agency receives the rating fee if and only if the firm decides to publish the signal observed by the agency s_t as a rating. I will refer to this form of compensation as contingent

[8]The assumption that a *committed* agency gets the informative signal at no cost ensures that a *committed* agency always obtains a non-negative payoff from rating. If the initial reputation for commitment is strong enough, namely $\mu_0 \geq \frac{\kappa}{(1-\lambda)(\lambda^2+\kappa)}$, then the equilibrium strategies would not change even if a *committed* agency incurred a cost κ to obtain the informative signal.

[9]In a market without a rating agency, every project is financed and the expected social welfare equals $\lambda - \alpha$ in each period. If instead an informative signal could be observed directly by the market participants, then a project would be financed if and only if its quality is high, and the expected social welfare would be equal to $\lambda(1-\alpha)$.

fees.[10] Investors observe whether the firm sells early, and whether any rating are published. Let $r_t \in \{h,l,\emptyset\}$ denote the rating observed by the investors, where $r_t = \emptyset$ if no rating is published. Investors bid simultaneously and the project is financed only if the largest bid covers the investment cost α. Figure 2.1 shows the sequence of actions taking place in each period.

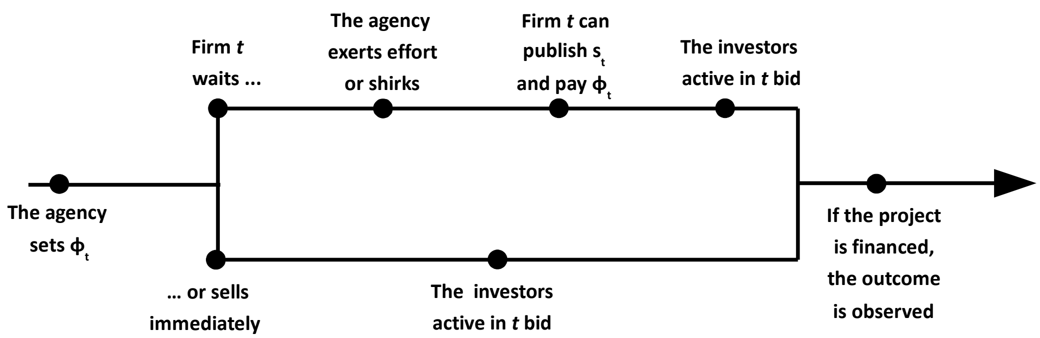

FIGURE 2.1. *Time-line, contingent fees.*

If, and only if, a project is financed, its outcome is publicly observed at the end of the period. Firm and investors active in period t observe the entire history of ratings, outcomes, and fees

$$\mathcal{H}_t = \{(r_1, y_1, \phi_1), .., (r_{t-1}, y_{t-1}, \phi_{t-1})\},$$

where $y_t \in \{0,1,\emptyset\}$ is the outcome of the project in period t, with $y_t = \emptyset$ meaning that the project is not financed. $\mathcal{H}_t$ provides information about the agency's type. As new information becomes available, the rest of the market updates its beliefs. Let μ_t refer to the agency's reputation at the beginning of period t.

I consider Perfect Bayesian Equilibria (PBE) in stationary Markov strategies, where the state variable is reputation μ. A PBE is defined by strategies and beliefs. In equilibrium, strategies are individually rational on and off the equilibrium path and beliefs are based on Bayes' Rule whenever possible. The agency chooses a fee for each level of reputation and, if *strategic*, also chooses whether to exert effort. The firm can sell early or wait; if the firm waits, it must decide whether to publish the rating. The strategy of each investor consists of a bid for any rating, and for any reputation of the agency μ, and a bid in case the firm sells early.

[10] As my goal is to model a rating market in which an agency is paid contingent fees, I do not allow the agency to request to be paid a fee whenever hired. Even if the agency had the option to set such fees, there would exists an equilibrium in which the agency voluntarily sets its fee entirely contingent on the publication of the rating.

Equilibria in stationary Markov strategies require players to choose the same actions following any two histories $\mathcal{H}_t$, $\mathcal{H}'_{t+n}$ whenever μ_t resulting from $\mathcal{H}_t$ is identical to μ_{t+n} resulting from $\mathcal{H}'_{t+n}$. As I consider stationary strategies, from now on I will drop the time subscript wherever possible. My focus on Markov strategies ensures that at the beginning of any period the expected continuation payoff of a *strategic* agency is only a function of its reputation, and can be described with a value function $V(\mu)$.

I impose two restrictions on equilibria. The first, in the spirit of Mathis, McAndrews and Rochet (2009), allows me to focus on equilibria in which a project is rated as long as the rest of the market does not believe with certainty that the agency is *strategic*.

Restriction 1. *Whenever $\mu>0$, a firm with a high-quality project publishes a rating.*

Restriction 1 rules out uninteresting equilibria in which rating does not take place simply because market participants hold arbitrary out-of-equilibrium-path beliefs about the agency's type whenever a rating is assigned. The second restriction ensures that the equilibrium fees are unique.

Restriction 2. *The agency, regardless of its type, sets a rating fee equal to the highest fee that the firm is willing to pay.*

Restriction 2 requires all the surplus generated by the rating process to be earned by the agency. This seems reasonable to the extent that the agency is a monopolist and sets the price. Restriction 2 also rules out equilibria in which an agency signals its type with its choice of fee. Note however that Restriction 1 is sufficient to rule out fully separating equilibria.
In the remainder of the paper, the term "equilibrium" refers to PBE in stationary Markov strategies that satisfy Restrictions 1 and 2.

Let b_t denote the highest bid in period t. If and only if $b_t \geq \alpha$, one of the highest bidders finances the project, and earns $y_t - b_t$. Investors' outside options equal 0. The payoff of the firm active in period t amounts to $\max\{b_t - \alpha, 0\} - 1^r_t \phi_t$, where $1^r_t = 1$ if $r_t \in \{h, l\}$ and $1^r_t = 0$ otherwise. The payoff of a *strategic* agency is the sum of the instantaneous payoffs discounted at a rate $\delta \in (0,1)$: $\Sigma_{t=1}^{\infty} \delta^{t-1}(1^r_t \phi_t - \kappa e_t)$. The payoff of a *committed* agency is $\Sigma_{t=1}^{\infty} \delta^{t-1} 1^r_t \phi_t$.

3. Baseline Equilibrium

In this section I characterize the equilibrium under contingent fees. If the firm sells without waiting, the highest bid is equal to the expected gross return of the project, λ. If the firm waits, investors bid taking into account the rating assigned to the project, or the lack of a rating. Whenever the firm waits, it publishes the rating only if it is favorable ($s=h$).[11] If

[11] In Appendix B, I show that it is without loss of generality to restrict attention to equilibria in which the firm does not publish an l rating.

the agency is believed to be *strategic* with certainty, a favorable rating does not provide any information. The firm is not willing to pay a fee larger than zero for such rating and the agency does not earn any revenue.

Lemma 1. *Suppose* $\mu{=}0$. *Then* $e{=}0$ *and, as a result,* $V(0){=}0$.

As long as the other market participants do not believe the agency to be *strategic* with certainty, the agency sells its rating at a positive fee.

Lemma 2. *Suppose* $\mu{>}0$. *If a strategic agency is expected to exert effort with probability* $e^*{<}1$, *the agency demands the entire fee contingent on publication and sets:*

$$\phi{=}\phi(\mu,e^*)\equiv\frac{(1-\lambda)\alpha[1-(1-\mu)(1-e^*)]}{\lambda+(1-\lambda)(1-\mu)(1-e^*)}. \tag{3.1}$$

Moreover, the project is financed only if $r{=}h$, *and the investor that finances the project pays*

$$b{=}b(\mu,e^*)\equiv\frac{\lambda}{\lambda+(1-\lambda)(1-\mu)(1-e^*)}. \tag{3.2}$$

As both types of agencies set the same fee, the agency's reputation on the equilibrium path depends exclusively on its ratings and the quality of the projects. The agency's reputation evolves as follows:

$$\text{(3.3)}\qquad \text{if the firm waits:}\quad \mu_{t+1}{=}\psi(r_t,y_t|\mu_t,e_t)\equiv\begin{cases}\mu_t & \text{if } y_t{=}1 \text{ and } r_t{=}h \qquad (I)\\ \mu_t^l(e_t) & \text{if } y_t{=}\emptyset \text{ and } r_t{=}\emptyset \qquad (II)\\ 0 & \text{if } y_t{=}0 \text{ and } r_t{=}h \qquad (III)\end{cases}$$

$$\text{otherwise:}\qquad \mu_{t+1}{=}\mu_t \qquad (IV)$$

where $\mu^l(e)\equiv\frac{\mu}{\mu+(1-\mu)e}$. When the project has high quality, the signal does not depend on the choice of effort and the reputation of the agency does not change (I). When instead the market infers that the agency observed $s{=}l$, its reputation weakly improves (II); if the agency fails to collect the hard information, then its type is inferred with certainty (III). If the firm sells the project without waiting for a rating, the reputation of the agency is left unchanged (IV). Reputation is updated according to Bayes' rule whenever possible. For the cases in which Bayes' Rule does not hold, I show in Appendix B that (3.3) imposes only restrictions that hold without loss of generality.

The value function of a *strategic* agency can be represented recursively with a Bellman equation. Lemma 2 allows a term to be dropped from the equation:

$$V(\mu) = \max_{e\in[0,1]} (1-(1-\lambda)e)\phi - e\kappa + \delta\Big[\lambda V(\mu) + (1-\lambda)\Big(eV(\mu^l(e^*)) + (1-e)V(0)\Big)\Big].$$

$$= \max_{e\in[0,1]} (1-(1-\lambda)e)\phi - e\kappa + \delta\Big[\lambda V(\mu) + (1-\lambda)eV(\mu^l(e^*))\Big]. \tag{3.4}$$

The next proposition characterizes the unique equilibrium of the game.

Proposition 3. *A unique equilibrium exists. For positive reputation, $\mu>0$:*
1) if $\delta\in(\overline{\delta},1)$, a strategic agency exerts effort,
2) if $\delta\in[\underline{\delta},\overline{\delta}]$, a strategic agency mixes between effort and shirking if $\mu<\overline{\mu}$; if instead $\mu\geq\overline{\mu}$, then the agency shirks,
3) if $\delta\in(0,\underline{\delta})$, a strategic agency shirks,
where: $\overline{\delta}\equiv\frac{\alpha(1-\lambda)^2+\kappa\lambda}{(1+\lambda)(1-\lambda)^2\alpha+\lambda^2\kappa}$, $\underline{\delta}\equiv\frac{\kappa\lambda}{\kappa\lambda^2+\alpha(1-\lambda)^2}$ and $\overline{\mu}\equiv\frac{\delta(1-\lambda)^2\alpha-\kappa\lambda(1-\delta\lambda)}{(1-\lambda)[(1-\lambda)^2\delta\alpha+\lambda((1-\lambda)\alpha-\kappa)(1-\delta\lambda)]}$.

Figure 3.1 shows the effort choice as a function of discount factor and reputation. For a large enough discount factor, a *strategic* agency exerts effort regardless of its reputation. For intermediate discount factors, a *strategic* agency exerts effort with positive probability as long as its reputation is weak. If the reputation is strong enough, the *strategic* type "cashes in": it shirks and collects positive fees until it rates a project of low quality and loses all its reputation for commitment. If the discount factor is small, a *strategic* agency does not exert effort and always assigns a high rating.

The thresholds $\overline{\delta}$ and $\underline{\delta}$ increase monotonically with the cost κ (see Figure 5.2). The case of a costless signal is the closest to Mathis, McAndrews and Rochet (2009). As in their model, the threshold $\overline{\delta}$ is larger than 0 even if $\kappa=0$: if the agency's discount factor is sufficiently small, a *strategic* agency shirks regardless of the cost of obtaining an informative signal.

4. Regulated Fees

In this section, I consider policies that require firms to pay a share $\gamma\in(0,1]$ of the rating fee when they hire the agency. The remaining share of the fee $(1-\gamma)$ is paid only if a firm decides to publish the rating. The Cuomo Plan corresponds to a policy that sets $\gamma=1$. I refer to this policy as fixed fees. The contingent fees considered in the last sections are equivalent to $\gamma=0$.

In the baseline scenario, the firm did not have to pay any fee to hire the agency, so in order to economize on modeling I assumed that whenever the firm waited, it also hired the agency. In this section, the firm's decision to request a rating is modeled explicitly. If the agency is not hired, it does not assign a rating. Figure 4.1 summarizes the sequence of actions.

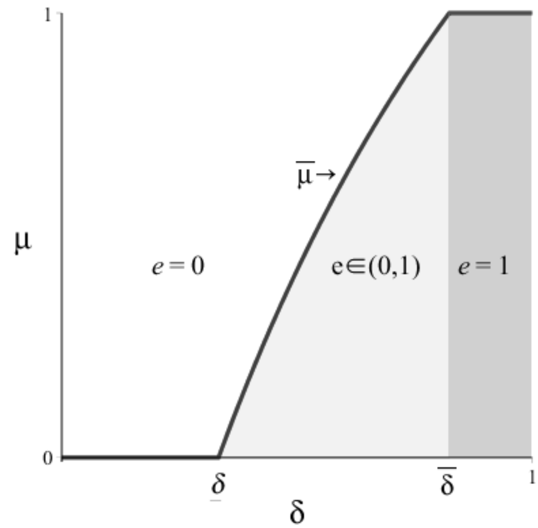

Figure 3.1. Effort of a *strategic* agency under contingent fees.

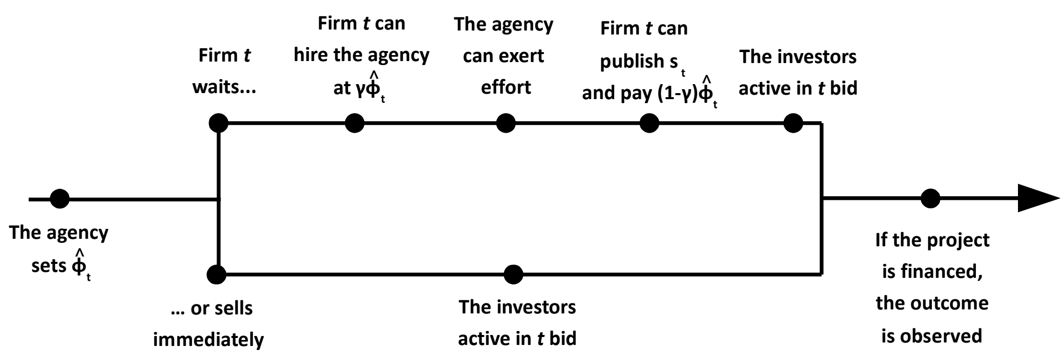

Figure 4.1. *Time-line: regulated fees.*

For a policy γ, $V^\gamma(\mu)$ denotes the agency's value function, $\phi^\gamma(\mu)$ is the rating fee b^γ the highest bid and e^γ the probability of effort. As in the baseline case, if the firm sells the project without waiting, the project is sold at its expected value. I show in Appendix B that if the firm waits it can be assumed without loss of generality that the firm publishes the rating only if $s{=}h$. The first result is the counterpart of Lemma 1.

Lemma 4. *For any $\gamma{\in}(0{,}1]$, if $\mu{=}0$ then $e^\gamma{=}0$, and, as a result, $V^\gamma(0){=}0$.*

Regardless of how the agency is compensated, the agency does not earn any revenue if it is believed to be *strategic* with certainty. The next lemma characterizes the fee paid to obtain a rating and the price at which a project is sold.

Lemma 5. *Suppose $\mu>0$ and a regulation γ is in place. If a strategic agency is expected to exert effort with probability $e^{\gamma*}$, the agency sets*

$$\phi^{\gamma}=\phi^{\gamma}(\mu,e^{\gamma*})\equiv\frac{(1-\lambda)\alpha[1-(1-\mu)(1-e^{\gamma*})]}{\gamma+(1-\gamma)(\lambda+(1-\lambda)(1-\mu)(1-e^{\gamma*}))}.$$

The project is financed if and only if $r=h$. The investor that finances the project pays $b(\mu,e^{\gamma*})$.

The market price of a high-rated project depends exclusively on the probability that an agency observes an informative signal. The expected amount that the firm pays to the agency, given effort and reputation, is identical regardless of whether regulation is in place, that is:

$$\phi^{\gamma}(\mu,e)(\gamma+(1-\gamma)Pr(r=h|\mu,e))=\phi(\mu,e)Pr(r=h|\mu,e)\quad\forall e,\mu,\gamma.$$

The reputation of the agency evolves according to (3.3). The Bellman equation takes the form:

$$V^{\gamma}(\mu)=\max_{e\in[0,1]}[\gamma+(1-\gamma)(\lambda+(1-\lambda)(1-e))]\phi^{\gamma}-e\kappa+\delta\Big\{\lambda V^{\gamma}(\mu)+(1-\lambda)eV^{\gamma}(\mu^{l}(e^{\gamma*}))\Big\}. \tag{4.1}$$

As discussed above, regulating the compensation of the rating agency does not directly affect the value for a firm to obtain a high rating. These regulations indirectly determine the value of a high rating, as they have an effect on a *strategic* agency's choice of effort. The next proposition characterizes the equilibrium effort.

Proposition 6. *Suppose a regulation γ is in place. A unique equilibrium exists. For positive reputation $\mu>0$:*
1) if $\delta\in(\overline{\delta}^{\gamma},1)$, a strategic agency exerts effort,
2) if $\delta\in[\underline{\delta}^{\gamma},\overline{\delta}^{\gamma}]$, a strategic agency mixes between effort and shirking if $\mu<\overline{\mu}^{\gamma}$; if instead $\mu\geq\overline{\mu}^{\gamma}$, then the agency shirks,
3) if $\delta\in(0,\underline{\delta}^{\gamma})$, a strategic agency shirks,
where: $\overline{\delta}^{\gamma}\equiv\frac{\alpha(1-\lambda)-(\alpha(1-\lambda)-\kappa)(\gamma+(1-\gamma)\lambda)}{\alpha(1-\lambda)-\lambda(\alpha(1-\lambda)-\kappa)(\gamma+(1-\gamma)\lambda)}$, $\underline{\delta}^{\gamma}\equiv\frac{(\gamma+(1-\gamma)\lambda)\kappa}{\alpha(1-\lambda)^{2}+\lambda\kappa(\gamma+(1-\gamma)\lambda)}$ *and*
$\overline{\mu}^{\gamma}\equiv\frac{\delta(1-\lambda)^{2}\alpha-\kappa(1-\delta\lambda)(\gamma+(1-\gamma)\lambda)}{(1-\lambda)(1-\gamma)[(1-\lambda)^{2}\delta\alpha+(\gamma+(1-\gamma)\lambda)((1-\lambda)\alpha-\kappa)(1-\delta\lambda)]}$.

Figure 4.2 shows the effort of a *strategic* agency as a result of a policy that requires half of the fee to be independent of publication. The thresholds $\underline{\delta}^{\gamma}$ and $\overline{\delta}^{\gamma}$ are the equivalent of thresholds $\underline{\delta}$ and $\overline{\delta}$ from Section 3. $\overline{\delta}^{\gamma}$and $\underline{\delta}^{\gamma}$ move closer to each other when a larger share of the fee is paid at the time of hiring the agency ($\frac{\partial\underline{\delta}^{\gamma}}{\partial\gamma}>0>\frac{\partial\overline{\delta}^{\gamma}}{\partial\gamma}$). If the fee is fixed, the two thresholds coincide (for later use, I denote this unique threshold as $\hat{\delta}\equiv\underline{\delta}^{1}=\overline{\delta}^{1}$).

Figure 4.3 describes effort under fixed fees. Effort does not take intermediate values: either $e^1=0$ or $e^1=1$, and, as long as $\mu>0$, the choice to exert effort does not depend on the agency's reputation. This is somewhat unusual. An interval of parameter values, for which a *strategic* type becomes less likely to mimic the *commitment* type as the reputation for commitment improves, is commonly found in similar models (e.g. Mathis, McAndrews and Rochet (2009) and Benabou and Laroque (1992)).

In order to understand where this feature of the equilibrium comes from, it is useful to focus on the interaction between reputation for commitment and incentive to exert effort. If fees are contingent, reputation for commitment has two contrasting effects on a *strategic* agency's incentive to shirk. First, a strong reputation for commitment ensures that the agency earns a large fee whenever it assigns a favorable rating. This increases its incentive to shirk. Second, a strong reputation ensures that the agency can earn high revenues in future periods as long as it rates honestly and maintains its reputation. This effect provides an incentive to exert effort. For intermediate values of the discount factor δ, the first effect dominates the second, and a strategic agency becomes less willing to exert effort as its reputation improves.

If the agency earns a fixed fee, the first of the two effects discussed above is absent. Shirking becomes less profitable when the agency has a stronger reputation. For $\delta<\hat{\delta}$, the agency is not willing to incur the cost to observe an informative signal, assign the correct rating and maintain a reputation for commitment even if its reputation is strong. For weaker reputations the incentive to exert effort is even smaller and therefore a *strategic* agency shirks regardless of its reputation. For $\delta>\hat{\delta}$, whenever the agency enjoys a strong reputation, investors and firms expect the agency to exert effort, the rating is valuable, and the agency charges a high fee. As a result, earning a strong reputation is extremely valuable. If the agency was expected to shirk for a weak reputation then the expected future revenues following an honest rating would be very large. As a result, shirking cannot be part of the equilibrium and the agency exerts effort as long as $\mu>0$.

The threshold $\hat{\delta}$ is monotonically increasing in the cost κ (see Figure 5.2 in the next Section). This is intuitive: as the informative signal becomes costlier, it takes a larger discount factor to ensure that the *strategic* agency does not shirk. For a costless signal, fixed fees eliminate every incentive to shirk; regardless of its discount factor, the agency collects the informative signal.

5. Comparing Equilibria

Assumption 1 ensures that the expected social welfare in any given period increases monotonically if the probability of effort increases. Therefore, in order to evaluate the policies considered in the last section, it is sufficient to focus on the *strategic* agency's effort. The next proposition is the main result of the paper.

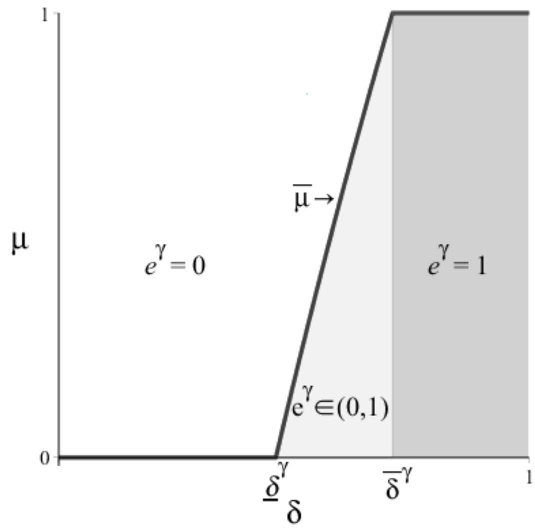

FIGURE 4.2. Effort of a *strategic* agency, γ=0.5.

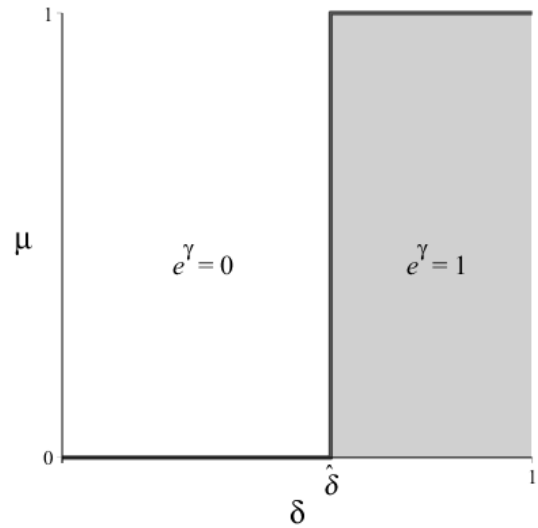

FIGURE 4.3. Effort of a *strategic* agency, γ=1 (fixed fee).

Proposition 7. *If $\delta \geq \hat{\delta}$, a policy of fixed fees ensures that the agency collects the informative signal. Fixed fees ensure larger social welfare than contingent fees or any other policy. If $\delta < \hat{\delta}$, every policy $\gamma > 0$ reduces the probability that the agency collects an informative signal and therefore reduces social welfare.*

The agency collects the informative signal (with probability 1) only if it assigns a sufficiently large weight to future revenues: $\delta > \overline{\delta}$ when fees are contingent (Proposition 3), and $\delta > \overline{\delta}^{\gamma}$ when fees are regulated (Proposition 6). As $\hat{\delta} < \overline{\delta}^{\gamma} < \overline{\delta}$ for any $\gamma \in (0,1)$, whenever costly information acquisition can be sustained with contingent fees or under any policy, then information acquisition can be sustained under fixed fees. The opposite is not true. If instead the agency's discount factor is small enough, contingent fees are the socially optimal policy: for $\delta \in (\underline{\delta}, \hat{\delta})$ a *strategic* agency is more likely to exert effort if fees are contingent than under any other compensation scheme.

In order to provide an intuition for the results of Proposition 7, I focus on the comparison between contingent and fixed fees. Intermediate policies are never the unique optimal policy and in any case can be thought of as a combination of these two extreme regimes. I proceed in two steps. First, I discuss why contingent fees can dominate fixed ones. Then I consider why contingent fees dominate when the discount factor is small.

A policy that prohibits contingent fees has two effects on the incentive to exert effort. The first effect is static. Fixed fees paid regardless of the rating reduce the incentive to shirk. The second effect is dynamic. The agency exerts effort to improve its reputation for commitment, as a stronger reputation ensures larger revenues. The revenues of a *strategic* agency are more elastic to reputation changes under contingent than under fixed fees, as shown in Figure 5.1. Fixed fees make reputation less valuable and reduce the dynamic incentive to exert effort.

The dynamic effect works as follows. As discussed above, a firm expects to pay the same amount under the two compensation schemes, for given reputation and equilibrium effort. While on average the agency earns the same revenue, the two compensation schemes ensure different revenues for a *strategic* and a *committed* agency. Under fixed fees, the agency earns the same revenue regardless of its type. Under contingent fees, however, whenever the *strategic* agency shirks with positive probability, it is more likely to obtain the fee than a *committed* agency. As a result, whenever the reputation of the agency improves, the average revenue of the agency increases at the same rate under the two compensation schemes, but the revenue of a *strategic* agency increases at a faster rate under contingent fees.

Static and dynamic effects work in opposite directions and a policy of fixed fees increases social welfare if and only if the static effect of the policy is stronger than its dynamic one. The dynamic effect is absent whenever the *strategic* agency is expected to always exert effort. As a result, an equilibrium in which the agency always exerts effort can be sustained with a lower discount factor when fees are fixed ($\delta > \hat{\delta}$). For small discount factors ($\delta < \hat{\delta}$) instead, investors conjecture that a *strategic* agency might shirk, the rating fee is relatively small, and the static effect is weak. At the same time, honest ratings considerably improve the reputation of the agency and therefore the dynamic effect is strong. As a result, contingent fees dominate fixed fees for low discount factors.

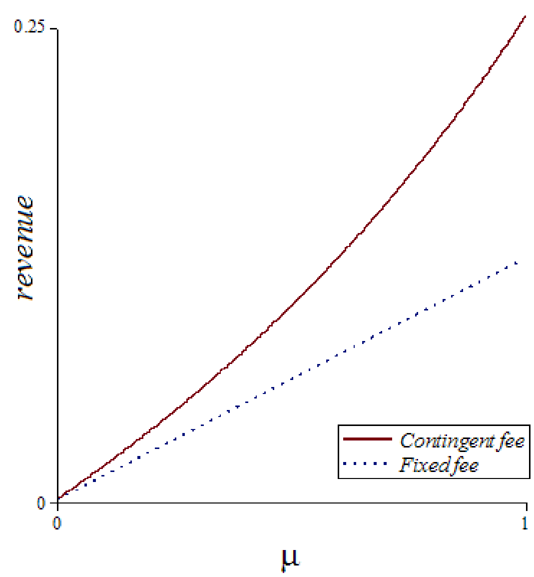

FIGURE 5.1. Revenue of a *strategic* agency (for e=$\hat{e}$=0.5).

Figure 5.2 shows how the thresholds $\overline{\delta}$, $\underline{\delta}$ and $\hat{\delta}$ depend on the cost of the informative signal. When the signal is costless, fixed fees ensure that the agency, regardless of its discount factor, always collects the informative signal ($\hat{\delta}$=0 for κ=0). An agency that collects fixed fees, might instead shirk even if the signal is costless ($\overline{\delta}$>0 for κ=0). As the cost of the signal increases, the interval of discount factors for which contingent fees ensure more effort than fixed fees becomes larger. These comparative statics show how the tradeoff discussed above hinges crucially on the presence of a cost to obtain informative signals.This cost has been singled out as an important, indirect, determinant of the quality of financial ratings. The agencies' lack of investment in information acquisition has been single out in the aftermath of the recent crisis as one of the main causes of the poor performance of ratings of structured financial products (e.g. US Senate (2011)).

Figure 5.3 compares the thresholds discount factors for different shares of high-quality projects. When the fraction of high-quality projects is large, shirking is hard to detect, and all the thresholds are relatively high. In the limit, as $(1-\lambda)\alpha\to\kappa$, (or $\lambda\to 1-\frac{\kappa}{\alpha}$) then $\underline{\delta}\to\hat{\delta}$ and contingent fees dominate fixed ones. If the share λ is large, the firm is not willing to pay much for a favorable rating, the rating fees are small, and the static effect is weak. Fees associated with a strong reputation shrink proportionally less as λ increases, so the dynamic effect is strong and contingent fees generate the strongest incentives to exert effort.

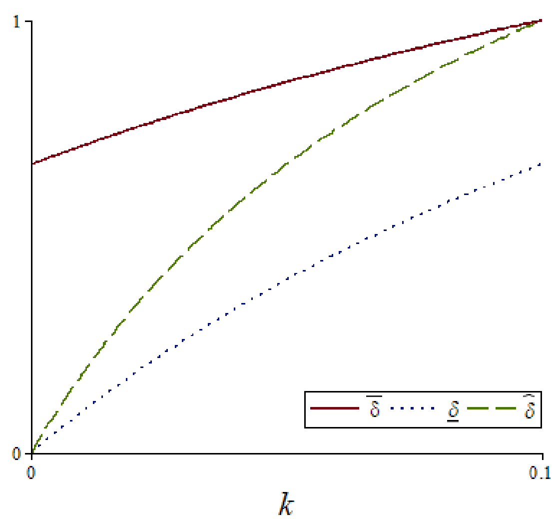

FIGURE 5.2. Thresholds discount factors as a function of κ.

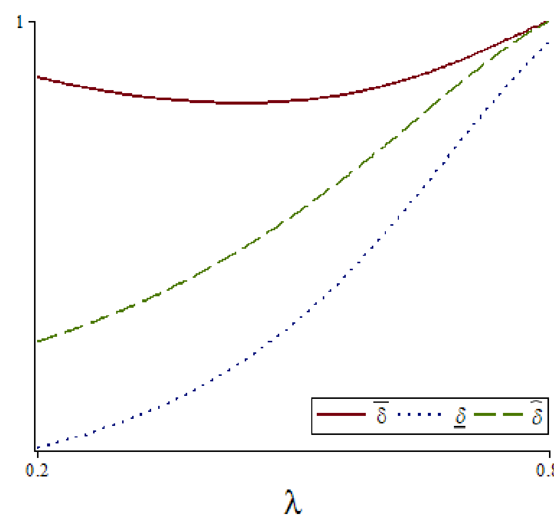

FIGURE 5.3. Threshold discount factors as a function of λ.

6. Extensions

6.1 No Financing Without Rating

In this subsection, I consider a market in which the investment cost is high ($\alpha>\lambda$) and therefore investors finance a project only if it receives a favorable rating. Without informative ratings, the market collapses. I show that as long as the initial reputation of the agency is strong enough, the results of the last section extend to this market.

High investment cost has two implications. First, the firm does not earn any revenue by selling early: a project sold without obtaining a rating would not be financed. As a project is not sold without a rating, Assumption 1 is not sufficient to ensure that honest rating is socially efficient. Informed ratings are socially efficient if collecting a signal and investing in high-rated projects ensures a larger welfare than not investing at all. The following condition is necessary and sufficient to ensure that an honest rating is indeed socially desirable:

Assumption 2. $\kappa\leq(1-\alpha)\lambda$.

A high investment cost has a second implication. Whenever the agency is expected to assign favorable ratings "too often", a favorable rating is insufficient to induce investors to finance the project. The following assumption ensures that a favorable rating from an agency with a positive reputation always results in investment.

Assumption 3. $\mu_0>\underline{\mu}$ *where* $\underline{\mu}$ *satisfies* $b(\underline{\mu},0)=\alpha$.

The next lemma compares the effort choice of a *strategic* agency under fixed and contingent fees. Assumption 2 ensures that effort and total welfare are positively correlated. As a result, whichever compensation scheme ensures effort with a higher probability results in a larger overall welfare.

Proposition 8. *Let Assumptions 2 and 3 hold. Under contingent as well as fixed fees the equilibrium is unique. Suppose* $\mu>0$*, then*
(I) for $\delta\geq\bar{\bar{\delta}}$ *, a strategic agency always exerts effort, regardless of the compensation scheme:* $e=e^1=1$,
(II) for $\bar{\bar{\delta}}>\delta>\hat{\hat{\delta}}$ *, a strategic agency is more likely to exert effort under fixed fees than under contingent ones:* $e^1=1>e$,
(III) for $\hat{\hat{\delta}}>\delta>\underline{\underline{\delta}}$*, a strategic agency exerts effort only if the fees are contingent:* $e\geq e^1=0$,
(IV) for $\delta\leq\underline{\underline{\delta}}$ *, a strategic agency shirks, regardless of the compensation scheme:* $e=e^1=0$,
where $\bar{\bar{\delta}}=\frac{\kappa+(1-\lambda)(1-\alpha)}{\lambda\kappa+(1-\lambda^2)(1-\alpha)}$, $\underline{\underline{\delta}}=\frac{\kappa}{\lambda\kappa+(1-\lambda)(1-\alpha)}$ *and* $\hat{\hat{\delta}}=\frac{\kappa}{\lambda\kappa+(1-\lambda)\lambda(1-\alpha)}$.

The main result for the case $\alpha<\lambda$ (Proposition 7) also holds substantially unchanged for the case of $\alpha>\lambda$. For very high or very low discount factors, the ratings have the same informative content whether the fees are fixed or contingent. For relatively large discount factors, fixed fees eliminate any incentive to shirk (II), while for relatively low discount factors, a *strategic* agency exerts effort only if the fees are contingent (III).

6.2 Observable Effort

The Dodd Frank Act introduced new regulation for rating agencies (US Senate (2010)). The new provisions intend, among other things, to subject the rating process to external evaluation. The Act allows the Securities and Exchange Commission (SEC) to:

> .. temporarily suspend or permanently revoke the registration of a nationally recognized statistical rating organization with respect to a particular class or subclass of securities, if the Commission (SEC) finds ... that the nationally recognized statistical rating organization does not have adequate financial and managerial resources to consistently produce credit ratings with integrity. Us Senate (2010), page 499.

These policies have two potential effects. They can make the agency's choice of effort observable for the other market participants, and they can enforce a desired level of investment in the production of credit ratings. I focus exclusively on the first effect. I consider how the two compensation schemes fare if the agency's choice of effort can be observed by firms and investors. As effort is observable, it is not without loss of generality to assume that the agency always assigns a rating identical to the signal observed. In this setting, I assume that a *committed* agency always discloses the unfavorable hard information and a *strategic* agency decides whether to disclose in order to maximize its payoff. The rating assigned by the agency is denoted $\tilde{r}$, while r still refers to the rating published by the firm.

I assume that effort is contractible: rating fees can depend on the observed level of effort. If fees are contingent on publication of the rating, the rating fee equals the firm's willingness to pay for a high rating. The willingness to pay, in turn, depends on the anticipated bids. As investors bid after observing the agency's effort, a high rating is valuable to the firm only if the agency exerted effort. If the agency shirks, its rating has no value and therefore the agency does not earn any revenue. A *strategic* agency exerts effort and discloses the information observed only if the discount factor is sufficiently large, as stated in the next lemma.

Proposition 9. *Suppose the rating fees are contingent. An equilibrium in which the agency exerts effort and rates honestly ($\tilde{r}=s\ \forall s$) exists if and only if $\delta \geq \tilde{\delta} \equiv \frac{(1-\lambda)\alpha}{(1-\lambda^2)\alpha-\kappa\lambda}$.*

If the discount factor is small, the agency exerts effort but does not disclose the information obtained: $\tilde{r}=h$ regardless of s. This is even less efficient than shirking, as the agency incurs the cost to obtain an informative signal, but the information obtained is not used.

Under fixed fees, effort and honest reporting can be sustained for any discount factor: it is enough to set ϕ^1=0 whenever the agency shirks to ensure the most efficient outcome.

Proposition 10. *Suppose the rating fees are fixed. For any δ>0 an equilibrium in which the agency exerts effort and rates honestly, that is $\tilde{r}$=s, $\forall s$ exists.*

As the fee depends on effort, but not on the rating assigned, a *strategic* agency has no incentive to misreport the signal. When effort is observable and contractible, fixed fees dominate contingent ones.

Bolton, Freixas and Shapiro (2012) compare fixed and contingent fees when investment in information acquisition is observable but not contractible. They conclude that under fixed fees the agency does not invests in information acquisition unless the investment in information acquisition takes place before the agency is paid. This latter case is in fact the closest to a setting with contractible effort choice. Unlike Bolton, Freixas and Shapiro (2012), in my setting the agency would invest in information acquisition and rate honestly under fixed fees even if effort is not contractible.

7. Conclusion

I model the market for ratings as a game in which short-lived firms can hire a long-lived monopolistic rating agency to rate their projects. The agency takes the unobservable decision to acquire costly information in order to assign informative ratings. I use the model to study the effect of different ways to compensate a rating agency. I compare an unregulated market, in which a firm pays only if it decides to publish a rating, with regulated markets, in which a firm pays a share of the rating fee whenever it hires the agency. I show that regulation does not necessarily lead to more informative ratings. In particular, the rating agency is less likely to exert effort as a result of regulation whenever the agency's discount factor is sufficiently small.

Policies that prohibit contingent fees are counterproductive in exactly the markets in which reputation incentives are weakest. On a positive note, when a policy of fixed fees (that is, a policy that requires the entire fee to be paid regardless of whether the rating is published) is matched with a regulation that makes the agency's investment in information acquisition observable, then the policy ensures that the agency always collects the informative signal.

The model could be extended to consider the effect of competition among rating agencies and to allow for repeated interaction between firms and rating agency. The previous work on the effect of competition among rating agencies include, among others, Bolton, Freixas and Shapiro (2012), Camanho and Deb (2012), Doherty, Kartasheva and Phillips (2012), Bouvard and Levy (2013), Hirth (2014) and Bizzotto (2014). This literature shows how specific features of the market for ratings determine whether competition is feasible and desirable. Along these lines, it would be interesting to evaluate the effect of the entry of new raters when different compensation schemes are in place.

In Frenkel (2015), whenever firms hire the same agency multiple times, an agency has an incentive to inflate ratings. My framework could be extended to study how compensation schemes compare in markets in which firms repeatedly hire the same rating agency.

Appendix A

Proof of Lemma 1. Let μ=0. Assume there is an equilibrium in which the firm waits for a rating and the agency is expected to exert effort with positive probability. As a result, the firm is willing to pay a fee larger than 0 to publish s=h. In such equilibrium, μ_{t+1}=0 regardless of r_t, therefore a *strategic* agency maximizes its payoff by setting the fee equal to the firm's largest willingness to pay and setting e=0, which is a contradiction. Therefore in any equilibrium the firm waits for a rating only if ϕ=0. As a result, the agency does not earn any revenue and $V(0)$=0 □

Proof of Lemma 2. Let μ>0. Consider a candidate equilibrium that satisfies Restriction 1: the firm waits and publishes if s=h. $b(\mu,e^*)$ is the expected value of the project for the investors conditional on observing r=h. Therefore the highest bid satisfies b=$b(\mu,e^*)$. If $r\in\{\emptyset,l\}$, investors infer that s=l and sequential rationality requires that the project is not financed. Fee ϕ=$\phi(\mu,e^*)$ is the unique fee that satisfies Restriction 2. Therefore in any equilibrium ϕ=$\phi(\mu,e^*)$ □

Proof of Proposition 3.[12] Suppose μ>0. The one-stage deviation principle ensures that there are no profitable deviations from the choice of effort e if and only if for any deviation e':

$$(e'-e)\Big(\phi(\mu,e)+\frac{\kappa}{1-\lambda}\Big)\geq\delta(e'-e)\Big(V(\mu^l(e))\Big). \tag{7.1}$$

Suppose $\delta>\overline{\delta}$. Existence: Let ϕ=$\phi(\mu,e(\mu))$ as defined in Lemma 2, for some $e(\mu)$. If $e(\mu)$=1 $\forall\mu$, then the value function is $V(\mu)=v\equiv\frac{\alpha(1-\lambda)-\kappa}{1-\delta}$, (see 3.4) and the no-deviation condition (7.1) becomes $\phi(\mu,1)\leq\delta V(\mu)-\frac{\kappa}{1-\lambda}\leftrightarrow\delta\geq\overline{\delta}$. Assumption 1 implies $\overline{\delta}<1$ and $v\geq0$.
Uniqueness: *Property 1:* $\exists\tilde{\mu}\in(0,1)$ *such that,* $\forall\mu>\tilde{\mu}$, $e(\mu)$=1. Let $\tilde{\mu}$ satisfy $g(\tilde{\mu})\equiv\frac{(\lambda\phi(\tilde{\mu},0)-\kappa)}{1-\delta}(1-\lambda)\delta-\kappa-\phi(1,1)(1-\lambda)=0$. As $g(\mu)$ is continuous, $g'>0$, and $g(0)<0<g(1)$, then $\tilde{\mu}$ exists, is unique and satisfies $\tilde{\mu}\in(0,1)$. For any $\mu>\tilde{\mu}$, in any equilibrium $V(\mu^l)(1-\lambda)\delta-\kappa-\phi(\mu,e(\mu))(1-\lambda)>g(\mu)>0$ and therefore $e(\mu)$=1.
Property 2: $\nexists m>0$ *s.t. if* $\mu<m$ *then* $e(\mu)<1$ *and if* $\mu>m$ *then* $e(\mu)=1$. Suppose such m existed. This would imply, that $\forall\mu<m$:

$$\kappa+\phi(\mu,e(\mu))\geq(1-\lambda)\delta V(\mu^l), \tag{7.2}$$

which in turn implies: $\mu^l\leq m$, which is equivalent to $e(\mu)>\underline{e}(\mu)\equiv\frac{\mu(1-m)}{m(1-\mu)}$ (otherwise $e(\mu^l)$=1, and $\kappa+\phi(\mu,e(\mu))<(1-\lambda)\delta V(\mu^l)$, which contradicts (7.2)). $\underline{e}$ is continuous, increasing, and $\underline{e}(m)$=1. This in turn implies that for some $\mu<m$ close enough to m, $e(\mu')>\overline{e}(\mu)$ $\forall\mu'>\mu$ where $\overline{e}(\mu)$ s.t. $\kappa+\phi(\mu,\overline{e}(\mu))=(1-\lambda)\delta\frac{\phi(\mu,\overline{e}(\mu))}{1-\delta\lambda}$ (for $\delta>\overline{\delta}$, $\overline{e}\in(0,1)$ exists and is unique). This implies

[12]The proof follows the steps of the proof of Proposition 2 in Mathis *et al.* (2009).

that $e(\mu)<1$ violates the no-deviation condition and therefore $e(\mu)=1$. Property 1 and 2 imply uniqueness.
Let $\delta<\overline{\delta}$. In any equilibrium $e(1)=0$ and $V(1)=\frac{\alpha(1-\lambda)}{1-\delta\lambda}$. Suppose $e(\mu)=1$ for some μ, then $\phi(\mu)=\alpha(1-\lambda)$, $\mu^l=\mu$, $V(\mu^l)=\frac{\lambda\phi(\mu)-\kappa}{1-\delta}$. As a result, $e(\mu)=1$ violates (7.1). Therefore the set of candidate equilibrium value functions (e.v.f.) is

$$W\equiv\{w:(0,1]\rightarrow\mathbb{R}|w(\mu)=v(\mu,e(\mu)),e(\mu)<1\forall\mu \text{ and } w(1)=v(1,0)\},$$

where $v(\mu,e)\equiv\frac{\phi(\mu,e)}{1-\delta\lambda}$. If $\kappa\geq(1-\lambda)\delta v(1,0)$, (or, equivalently $\delta\leq\underline{\delta}$), then $e(\mu)=0$ $\forall\mu$. If instead $\kappa<(1-\lambda)\delta v(1,0)$, then $\overline{\mu}>0$. Let $m_1=\overline{\mu}$, which implies $\phi(m_1,0)=\delta v(1,0)-\frac{\kappa}{1-\lambda}$, and $\forall w\in W$ and $\forall e\in[0,1]$;

$$\phi(\mu,e)+\frac{\kappa}{1-\lambda}\geq\delta w(\mu^l(e)) \quad \forall\mu\geq m_1. \tag{7.3}$$

Therefore for $\mu\geq m_1$ (7.1) is satisfied if and only if $e(\mu)=0$ and any candidate e.v.f. belongs to the set $W^1\equiv\{w^1\in W|w^1(\mu)=v(\mu,0) \text{ if } \mu\in[m_1,1]\}$.
Moreover, for $\mu<m_1$ (7.1) implies $e(\mu)\in(0,1)$ (I) and, as $\phi(\mu,e)=\delta V(\mu^l(e))-\frac{\kappa}{1-\lambda}<\delta V(1)-\frac{\kappa}{1-\lambda}$ $\forall e\in(0,1)$, then $w^1(\mu)<w^1(m_1)$ $\forall w^1$ (II).
Let $e_i(\mu)\equiv\frac{\mu(1-m_i)}{m_i(1-\mu)}$. As $e_1(\mu)$ is continuous and increasing for $\mu\in(0,1]$, and $\frac{d\phi(x,e_1(x))}{dx}>0$, at most one $m_2\geq0$ exists that satisfies

$$\phi(m_2,e_1(m_2))+\frac{\kappa}{1-\lambda}=\delta V(m_1). \tag{7.4}$$

(II), (7.4) and $V\in W^1$ ensure that (7.1) is satisfied for $\mu=m_2$ if and only if $e(m_2)=e_1(m_2)$. For all $\mu\in(m_2,m_1)$, $e(\mu)\geq e_1(\mu)$ violates (7.1) and a unique $e(\mu)\in(0,e_1(\mu))$ that satisfies (7.1) exists. As a result, $e(\mu)$ and $V(\mu)$ are uniquely defined and continuous for $\mu\geq m_2$. Moreover let $\tilde{\mu},\hat{\mu}\in[m_2,m_1]$ and $\tilde{\mu}>\hat{\mu}$. Let $\hat{\mu}$ be defined by: $\phi(\mu,\hat{e})=\phi(\tilde{\mu},e(\tilde{\mu}))$, then, $\hat{\mu}^l(\hat{e})<\tilde{\mu}^l(e(\tilde{\mu}))$ as V is (strictly) increasing in $[m_1,1]$, and therefore $\phi(\hat{\mu},\hat{e})+\frac{\kappa}{1-\lambda}>V(\hat{\mu}^l(\hat{e}))$. Therefore $e(\hat{\mu})<\hat{e}$, which implies $V(\hat{\mu})<V(\tilde{\mu})$: V is (strictly) increasing for $\mu\geq m_2$.
Define a sequence: m_i: $\phi(m_i,e_{i-1}(m_i))+\frac{\kappa}{1-\lambda}=\delta v(m_{i-1},e_{i-2}(m_{i-1}))$, for $i=3,4,...$ Following the argument used above for $e(m_2)=e_1(m_2)$, in equilibrium $e(m_i)=e_{i-1}(m_i)$ $\forall m_i>0$. If $\mu\in[m_i,m_{i-1}]$, $e(\mu)$ is uniquely defined as long as the value function is unique, continuous and strictly increasing in the interval $[m_{i-1},1]$. $V(\mu)$ for $\mu\in[m_i,m_{i-1}]$ is defined continuous and increasing as long as $V(\mu)$ is unique, continuous, and strictly increasing in $[m_{i-1},1]$. As there is a unique and strictly increasing value function in the interval $[m_2,1]$, a unique value function can be defined recursively for any $\mu\geq m_i$ $\forall i$. $\delta<\overline{\delta}$ implies $\exists\epsilon>0$ s.t. $e(m_i)<1-\epsilon\forall i$, therefore $lim_{i\rightarrow\infty}m_i=0$, so a unique value function is defined $\forall\mu>0$ $\square$

Proof of Lemma 4. In any equilibrium in which the agency is hired for μ=0, e^γ=0. This is the case as μ_t=0 implies μ_{t+1}=0 regardless of r_t (see (3.3)). If the agency is expected to shirk, the firm hires the agency only if ϕ^γ=0. Therefore, in any equilibrium that satisfies Restriction 1, whenever μ=0 then ϕ^γ=0 and $V^\gamma(0)$=0 □

Proof of Lemma 5. Let $\mu>0$. Consider a candidate equilibrium that satisfies Restriction 1: the firm waits and publishes if s=h. $b(\mu,e^{\gamma *})$ is the expected value of the project for the investors conditional on observing r=h, and therefore b^γ=$b(\mu,e^{\gamma *})$. If instead a rating $r\in\{l,\emptyset\}$ is observed, the bids are not high enough to cover the cost of the project. Moreover, in any equilibrium in which the firm is expected to hire the agency, the firm strictly prefers to sell early rather than waiting without hiring the agency. Whenever buyers bid $b(\mu,e^{\gamma *})$, the unique fee that satisfies Restriction 2 is ϕ^γ=$\phi^\gamma(\mu,e^{\gamma *})$ □

Proof of Proposition 6. The one-stage deviation principle ensures that there are no profitable deviations from the choice of effort e if and only if for any deviation $e'\in[0,1]$:

$$(e'-e)\Big((1-\gamma)\phi^\gamma(\mu,e)+\frac{\kappa}{1-\lambda}\Big)\geq\delta(e'-e)V(\mu_t^l(e)). \tag{7.5}$$

Suppose $\delta>\overline{\delta}^\gamma$. Existence: Let ϕ^γ=$\phi^\gamma(\mu,e^\gamma(\mu))$ as defined in Lemma 5, for some $e^\gamma(\mu)$. If $e^\gamma(\mu)$=1 $\forall\mu$, then $V^\gamma(\mu)$=$v^\gamma\equiv\frac{1}{1-\delta}(\frac{\lambda(1-\lambda)\alpha}{\gamma+(1-\gamma)\lambda}-\kappa)$, (see (4.1)). Assumption 1 $v^\gamma\geq0$ and therefore the agency does not gain from deviating to $\phi^\gamma>\phi^\gamma(\mu,e^\gamma(\mu))$. Condition (7.5) becomes $(1-\gamma)\phi(\mu,1)\leq\delta V(\mu)-\frac{\kappa}{1-\lambda}\leftrightarrow\delta\geq\overline{\delta}^\gamma$, so for $\delta\geq\overline{\delta}^\gamma$ the agency does not gain from deviating to $e^\gamma(\mu)<1$.
Uniqueness: *Property 1*: $\exists\tilde{\mu}\in(0,1)$ *such that,* $\forall\mu>\tilde{\mu}$, $e^\gamma(\mu)$=1. Let $\tilde{\mu}$ satisfy $g(\tilde{\mu})$=0 where $g(\tilde{\mu})\equiv\frac{((\gamma+(1-\gamma)\lambda)\phi(\tilde{\mu},0)-\kappa)}{1-\delta}(1-\lambda)\delta-\kappa-\phi(1,1)(1-\gamma)(1-\lambda)$. As $g(\mu)$ is continuous, $g'>0$, and $g(0)<0<g(1)$, then $\tilde{\mu}$ exists and is unique. For any $\mu>\tilde{\mu}$, in any equilibrium $V(\mu^l)(1-\lambda)\delta-\kappa-\phi^\gamma(\mu,e^\gamma(\mu))(1-\gamma)(1-\lambda)>g(\mu)>0$ and therefore $e^\gamma(\mu)$=1.
Property 2: $\nexists m>0$ *s.t. if* $\mu<m$ *then* $e^\gamma(\mu)<1$ *and if* $\mu>m$ *then* $e^\gamma(\mu)$=1. Suppose such m existed. This would imply, that $\forall\mu<m$:

$$\kappa+(1-\gamma)\phi^\gamma(\mu,e(\mu))\geq(1-\lambda)\delta V^\gamma(\mu^l), \tag{7.6}$$

which in turn implies: $\mu^l\leq m$, which is equivalent to $e^\gamma(\mu)>\underline{e}(\mu)\equiv\frac{\mu(1-m)}{m(1-\mu)}$ (otherwise $e(\mu^l)$=1, and $\kappa+\phi(\mu,e(\mu))(1-\gamma)<(1-\lambda)\delta V(\mu^l)$, which contradicts (7.6)). $\underline{e}$ is continuous, increasing and $\underline{e}(m)$=1. This implies that for some $\mu<m$ close enough to m, $\kappa+(1-\gamma)\phi(\mu,1)=(1-\lambda)\delta\frac{\phi(\mu,\underline{e}(\mu))}{1-\delta\lambda}$ (for $\delta>\overline{\delta}$, $\mu\in(0,1)$ exists and is unique). This implies that $\kappa+(1-\gamma)\phi(\mu,e^\gamma(\mu))<(1-\lambda)\delta\frac{\phi(\mu^l,e^\gamma(\mu^\lambda))}{1-\delta\lambda}$ $\forall e^\gamma(\mu)>\underline{e}(\mu)$ and therefore $e(\mu)$=1, which is a contradiction.
Properties 1 and 2 imply uniqueness.

Let $\delta<\overline{\delta}^{\gamma}$. In any equilibrium $e^{\gamma}(1)=0$ and $V^{\gamma}(1)=\frac{\alpha(1-\lambda)}{1-\delta\lambda}$. Suppose $e^{\gamma}(\mu)=1$, then $\phi^{\gamma}(\mu)=\frac{\alpha(1-\lambda)}{\gamma+(1-\gamma)\lambda}$, $\mu^{l}=\mu$, $V^{\gamma}(\mu^{l})=\frac{\alpha(1-\lambda)-\kappa}{1-\delta}$. As a result, $e^{\gamma}(\mu)=1$ violates (7.1). The set of candidate equilibrium value functions (e.v.f.) is

$$W\equiv\{w:(0,1]\to\mathbb{R}|w(\mu)=v(\mu,e^{\gamma}(\mu)),e^{\gamma}(\mu)<1\forall\mu \text{ and } w(1)=v(1,0)\},$$

where $v^{\gamma}(\mu,e)\equiv\frac{\phi^{\gamma}(\mu,e)}{1-\delta\lambda}$. If $\kappa\geq(1-\lambda)\delta v^{\gamma}(1,0)$, (or, equivalently $\delta\leq\underline{\delta}^{\gamma}$), then $e^{\gamma}(\mu)=0$ $\forall\mu$. If instead $\kappa<(1-\lambda)\delta v^{\gamma}(1,0)$, then $\overline{\mu}^{\gamma}>0$. Let $m_1=\overline{\mu}^{\gamma}$, which implies $\phi^{\gamma}(m_1,0)=\delta v^{\gamma}(1,0)-\frac{\kappa}{1-\lambda}$, and $\forall w\in W$ and $\forall e\in[0,1]$;

$$\phi^{\gamma}(\mu,e)+\frac{\kappa}{1-\lambda}\geq\delta w(\mu^{l}(e)) \quad \forall\mu\geq m_1. \tag{7.7}$$

Therefore for $\mu\geq m_1$ (7.1) is satisfied if and only if $e(\mu)=0$ and any candidate e.v.f. belongs to the set $W^1\equiv\{w^1\in W|w^1(\mu)=v^{\gamma}(\mu,0) \text{ if } \mu\in[m_1,1]\}$.
Moreover, for $\mu<m_1$ (7.1) implies $e^{\gamma}(\mu)\in(0,1)$ (I) and, as $(1-\gamma)\phi^{\gamma}(\mu,e)=\delta V^{\gamma}(\mu^{l}(e))-\frac{\kappa}{1-\lambda}<\delta V^{\gamma}(1)-\frac{\kappa}{1-\lambda}$ $\forall e\in(0,1)$, then $w^1(\mu)<w^1(m_1)$ $\forall w^1$ (II).
Let $e_i(\mu)\equiv\frac{\mu(1-m_i)}{m_i(1-\mu)}$. As $e_1(\mu)$ is continuous and increasing for $\mu\in(0,1]$, and $\frac{d\phi^{\gamma}(x,e_1(x))}{dx}>0$, there exists at most one $m_2\geq0$ that satisfies

$$(1-\gamma)\phi^{\gamma}(m_2,e_1(m_2))+\frac{\kappa}{1-\lambda}=\delta V^{\gamma}(m_1). \tag{7.8}$$

(II), (7.8) and $V^{\gamma}\in W^1$ ensure that (7.1) is satisfied for $\mu=m_2$ if and only if $e^{\gamma}(m_2)=e_1(m_2)$. For all $\mu\in(m_2,m_1)$, $e^{\gamma}(\mu)\geq e_1(\mu)$ violates (7.1) and there exists a unique $e^{\gamma}(\mu)\in(0,e_1(\mu))$ that satisfies (7.1). $e^{\gamma}(\mu)$ is continuous and, as a result, $V^{\gamma}(\mu)$ is uniquely defined and continuous for $\mu\geq m_2$. Moreover let $\tilde{\mu},\hat{\mu}\in[m_2,m_1]$ and $\tilde{\mu}>\hat{\mu}$. Let $\hat{\mu}$ be defined by: $\phi^{\gamma}(\mu,\hat{e})=\phi^{\gamma}(\tilde{\mu},e^{\gamma}(\tilde{\mu}))$, then, $\hat{\mu}^{l}(\hat{e})<\tilde{\mu}^{l}(e^{\gamma}(\tilde{\mu}))$ as V^{γ} is (strictly) increasing in $[m_1,1]$, and therefore $(1-\gamma)\phi^{\gamma}(\hat{\mu},\hat{e})+\frac{\kappa}{1-\lambda}>V^{\gamma}(\hat{\mu}^{l}(\hat{e}))$. So $e^{\gamma}(\hat{\mu})<\hat{e}$, which implies $V^{\gamma}(\hat{\mu})<V^{\gamma}(\tilde{\mu})$: V^{γ} is (strictly) increasing for $\mu\geq m_2$.
Define a sequence: m_i: $(1-\gamma)\phi^{\gamma}(m_i,e_{i-1}(m_i))+\frac{\kappa}{1-\lambda}=\delta v^{\gamma}(m_{i-1},e_{i-2}(m_{i-1}))$, for $i=3,4,...$ Following the argument used above for $e^{\gamma}(m_2)=e_1(m_2)$, in equilibrium $e^{\gamma}(m_i)=e_{i-1}(m_i)$ $\forall m_i>0$. If $\mu\in[m_i,m_{i-1}]$, $e^{\gamma}(\mu)$ is uniquely defined as long as the value function is unique, continuous and strictly increasing in the interval $[m_{i-1},1]$. And $V^{\gamma}(\mu)$ for $\mu\in[m_i,m_{i-1}]$ is defined continuous and increasing as long as $V^{\gamma}(\mu)$ is unique, continuous and strictly increasing in $[m_{i-1},1]$. As there is a unique and strictly increasing value function in the interval $[m_2,1]$, a unique value function can be defined recursively for any $\mu\geq m_i$ $\forall i$. $\delta<\overline{\delta}$ implies $\exists\epsilon>0$ s.t. $e^{\gamma}(m_i)<1-\epsilon\forall i$, therefore $lim_{i\to\infty}m_i=0$, so a unique value function is defined $\forall\mu>0$ □

Proof of Proposition 7. Let $\delta<\hat{\delta}$.

Property 1. Let $\mu^{l\gamma}\equiv\mu^l(e^\gamma(\mu))$. For any $\gamma>0$, $e(\mu^{l\gamma})=e^\gamma(\mu^{l\gamma})$, implies $e(\mu)>e^\gamma(\mu)$ $\forall\mu$. Suppose this is not the case, then for some μ it is the case that $e(\mu^{l\gamma})=e^\gamma(\mu^{l\gamma})$ and $e(\mu)\leq e^\gamma(\mu)$, which is equivalent to

$$\frac{\delta\alpha(1-\lambda)^2(1-(1-\mu^l)(1-e^{l\gamma}))}{(1-\delta\lambda)(\gamma+(1-\gamma)d^l)}=\kappa+\frac{\alpha(1-\lambda)^2(1-\gamma)(1-(1-\mu)(1-e^\gamma))}{\gamma+(1-\gamma)d}, \tag{7.9}$$

and

$$\frac{\delta\alpha(1-\lambda)^2(1-(1-\mu^l)(1-e^{l\gamma}))}{(1-\delta\lambda)d^l}\leq\kappa+\frac{\alpha(1-\lambda)^2(1-(1-\mu)(1-e^\gamma))}{d} \tag{7.10}$$

where $e^{l\gamma}\equiv e(\mu^{\gamma l})=e^\gamma(\mu^{\gamma l})$, $e^\gamma\equiv e^\gamma(\mu)$, $d\equiv\lambda+(1-\lambda)(1-\mu)(1-e^\gamma)$ and $d^l\equiv\lambda+(1-\lambda)(1-\mu^l)(1-e^{l\gamma})$. Using (7.9) to substitute for $\alpha(1-\lambda)^2(1-(1-\mu)(1-e^\gamma))$ in (7.10), one obtains:

$$\gamma\kappa\frac{(1-\delta\lambda)}{\delta\alpha(1-\lambda)^2}\leq(1-(1-\mu^l)(1-e^l))\gamma\left(\frac{d^l+(1-\gamma)dd^l-(1-\gamma)d}{(\gamma+(1-\gamma)d^l)d^l}\right) \tag{7.11}$$

The right side of (7.11) is smaller than 1, so (7.11) holds only if $\gamma\kappa\frac{(1-\delta\lambda)}{\delta\alpha(1-\lambda)^2}\leq1$, which is equivalent to $\delta\geq\hat{\delta}$.

Property 1 implies that $\overline{\mu}>\mu(\gamma)$ $\forall\gamma>0$ and for any $\mu\leq\overline{\mu}(\gamma)$ $e(\mu)>e^\gamma(\mu)$, which proves the first half of the proposition. The second half of the proposition is immediate, as for $\delta\geq\hat{\delta}$ $e^1(\mu)=1$ $\forall\mu>0$ $\square$

Proof of Proposition 8. Let fees be contingent. Following the same steps of the proof of Lemma 1, it can be shown that $e(0)=V(0)=0$. For $\alpha>\lambda$, the firm is willing to pay up to $\tilde{\phi}(\mu,e^*)\equiv b(\mu,e^*)-\alpha$ for a high rating, so in equilibrium if the agency has reputation μ and a *strategic* agency is expected to exert effort with probability e^*, the agency regardless of its type sets $\phi=\tilde{\phi}(\mu,e^*)$. If $\phi(\mu,e^*)$ is replaced by $\tilde{\phi}(\mu,e^*)$ Lemma 2 applies to the case of $\alpha>\lambda$. Similarly, if $\overline{\delta}$ is substituted by $\overline{\overline{\delta}}$ and $\underline{\delta}$ is substituted by $\underline{\underline{\delta}}$, the same steps of the proof of Proposition 3 can be followed to show that for $\delta\geq\overline{\overline{\delta}}$, $e=1$, for $\overline{\overline{\delta}}>\delta>\underline{\underline{\delta}}$, $e=0$ if $\mu\geq\overline{\overline{\mu}}$ and $e\in(0,1)$ if $\mu<\overline{\overline{\mu}}$, where $\overline{\overline{\mu}}\equiv\frac{1}{1-\lambda}\left(1-\frac{1}{\frac{\delta(1-\alpha)}{1-\delta\lambda}-\frac{\kappa}{1-\lambda}+\alpha}\right)$ while for $\delta\leq\underline{\underline{\delta}}$, $e=0$.

Let fees be fixed. Following the same steps of the proof of Lemma 4, it can be shown that $e^1(0)=V^1(0)=0$. For $\alpha>\lambda$, the firm is willing to pay up to $\tilde{\phi}^1(\mu,e^{1*})\equiv(\lambda+(1-\lambda)(1-\mu)(1-e^{1*}))(b(\mu,e^{1*})-\alpha)$ for a high rating, so in equilibrium if the agency has reputation μ and a *strategic* agency is expected to exert effort with probability e^{1*}, the agency regardless of its type sets $\phi^1=\tilde{\phi}^1(\mu,e^*)$. If $\phi^1(\mu,e^*)$ is replaced $\tilde{\phi}^1(\mu,e^*)$ Lemma 5 applies to the case of $\alpha>\lambda$. Similarly, if $\hat{\delta}$ is substituted by $\hat{\hat{\delta}}$ the same steps of the proof of Proposition 6 can be followed to show that for $\delta>\hat{\hat{\delta}}$, $e^1=1$, while for $\delta<\hat{\hat{\delta}}$, $e^1=0$ $\square$

Proof of Proposition 9. Let 1_e=1 if effort is exerted, and 1_e=0 if the agency shirks. Let a=$a(\mu)$ denote the probability that a *strategic* agency assigns a rating r=l upon observing s=l. Consider the following equilibrium: every period $\phi(1_e)=\frac{(1-\lambda)\alpha}{\lambda}$ if 1_e=1 while $\phi(1_e)$=0 if 1_e=0, $e(\mu)$=$a(\mu)$=1 $\forall\mu>0$, and $e(0)$=$a(0)$=0. These strategies ensure an expected payoff strictly larger than 0 for the agency in each period if e=1 and a payoff equal to zero if e=0. Moreover $a(\mu)<1$ is never a profitable deviation if and only if $\delta\geq\tilde{\delta}$. Therefore $\delta\geq\tilde{\delta}$ is a sufficient condition for the existence of an equilibrium in which the rating agency always rates honestly. The condition is also necessary, as in any equilibrium in which on the equilibrium path e=a=1 then μ=μ_0 in each period, which in turn implies that the same fee ϕ is paid in each period. The fee must satisfy $\phi\leq1-\lambda$, otherwise it would be strictly larger than the firm's willingness to pay for a rating. Moreover a=0 is not a profitable deviation if and only if $\delta\geq g(\phi)=\frac{\phi}{\phi(1+\lambda)-\kappa}$. As $g'(\phi)<0$ $\forall\phi$ then an equilibrium with honest ratings exists only if $\delta\geq g(1-\lambda)=\tilde{\delta}$ □

Proof of Proposition 10. Consider the following equilibrium: every period, the fee satisfies $\hat{\phi}(1_e)=(1-\lambda)$ if 1_e=1 while $\hat{\phi}(1_e)$=0 if 1_e=0, where 1_e if effort is exerted, and 1_e=0 if the agency shirks.. The *strategic* type chooses $e^1(\mu)$=$a^1(\mu)$=1 $\forall\mu>0$, $e^1(0)$=0 and $a^1(0)$=0. These strategies ensure an expected positive payoff for the agency in each period if e^1=1 and a payoff equal to zero if e^1=0. Moreover $a^1<1$ is never a profitable deviation, as the rating fee does not depend on the rating □

Appendix B

Lemma 1b. *Regardless of whether the fees are entirely contingent on the publication of the rating or a share* $\gamma\in(0,1]$ *of the fee is fixed, it is without loss of generality to consider only equilibria in which no firm ever publishes a rating if* s=l.

Proof. In any PBE, the bids following r=l are lower or equal to α. So a firm publishes a rating following s=l only if the buyers bids are lower or equal to α if no rating is published and ϕ=0 (in case of contingent fees) or either γ=1 or $\hat{\phi}$=0 (in case of fixed fees). So the probability that the project is financed and the payoffs of all agents are the same as in an equilibrium in which all the players follow the same strategies with the exception that no firm ever publishes a rating if s=l. Moreover, as Restriction 1 ensures that in equilibrium the agency is hired in any period, also the reputation of the agency evolves in the same way regardless of whether a firm publishes a rating following s=l or not □

(3.3) imposes three restrictions on out-of-equilibrium-path beliefs:
(a) $\psi(h,0|1,e_t)$=0
(b) $\psi(\emptyset,0|0,e_t)$=0
(c) if the firm does not wait, μ_{t+1}=μ_t

Lemma 2b. *Regardless of whether fees are contingent or not, it is without loss of generality to focus on equilibria in which beliefs satisfy restrictions (a), (b) and (c).*

Proof. (a) Let fees be contingent. Suppose there exists an equilibrium in which $\psi(h,0|1,e_t)>0$ for some e_t. Let $V(1)$ be the value function in this equilibrium for $\mu_t=1$. Let $V^0(1)$ be the value function for $\mu_t=1$ in an equilibrium in which $\psi(h,0|1,e_t)=0$ $\forall e_t$. $V^0(1)$ is uniquely defined. As $V(0)=0$, it must be the case that $V(1)\geq V^0(1)$. If $V(1)=V^0(1)$, then as $\mu_t=1$ is out the equilibrium path if the agency is *strategic*, there exists another equilibrium identical to the original one with the exception that $\psi(h,0|1,e_t)=0$ $\forall e_t$. If $V(1)>V^0(1)$, 3 alternatives can hold:

(I) $e(\mu_t)=0$ $\forall \mu_t<1$: then as the agency does not exert effort for $\mu_t^l=1$ and $V(1)>V^0(1)$, it also does not exert effort if $V(1)=V^0(1)$, so there exists another equilibrium characterized by the same equilibrium strategies, in which $\psi(h,0|1,e_t)=0$ $\forall e_t$.

(II) $e(\mu_t)>0$ $\forall \mu_t<1$: then the equilibrium strategies do not depend on $V(1)$, so there exists another equilibrium characterized by the same equilibrium play as the original one, in which $\psi(h,L|1,e_t)=0$ $\forall e_t$.

(III) $\exists m\in(0,1)$: $e(m)>0$ and $\exists n\in(0,1)$: $e(n)=0$. Then $\exists \mu_t'$ such that $\phi(\mu_t',0)+\frac{\kappa}{1-\lambda}=\delta V(1)$ (if $\phi(\mu_t,0)+\frac{\kappa}{1-\lambda}>\delta V(1)$ $\forall \mu_t$ case I holds, if $\phi(\mu_t,0)+\frac{\kappa}{1-\lambda}<\delta V(1)$, $\forall \mu_t$, case II holds; by continuity of $\phi(\mu_t,0)$ in μ_t, μ_t' exists). For any $\mu_t=\mu_t'-\epsilon$ for $\epsilon>0$ it is the case that $\phi(\mu_t,0)+\frac{\kappa}{1-\lambda}<\delta V(1)$, so in equilibrium $e(\mu_t)>0$. Moreover, $V(1)>\lim_{\mu\to 1}V(\mu)$. As by Prop.1 $\lim_{\mu\to 1}e(\mu_t)=0$ which implies $\lim_{\mu\to 1}V(\mu)=V^0(1)$. But $\lim_{\epsilon\to 0}\phi(\mu_t,0)=\delta V(1)-\frac{\kappa}{1-\lambda}>\delta V^0(1)-\frac{\kappa}{1-\lambda}$. As in any equilibrium $V(\mu)\leq V^0(1)$, for ϵ small enough $\phi(\mu_t'-\epsilon,e(\mu_t'-\epsilon))>\delta V^0(1)-\frac{\kappa}{1-\lambda}$ implies that in equilibrium $e(\mu_t'-\epsilon)=0$, which is a contradiction. So if $\exists m\in(0,1)$: $e(m)>0$ and $\exists n\in(0,1)$: $e(n)=0$ it must be the case that $\psi(h,0|1,e_t)=0$ $\forall e_t$.

Let fees be fixed. The proof for the case $\hat{V}(1)=\hat{V}^0(1)$ and cases (I) and (II) are identical to the case of contingent fees. Case (III) follows the same steps with the exception that condition (7.5) holds instead of (7.1).

(b) As stated in Lemma 1 (Lemma 4), $e(0)>0$, ($\hat{e}(0)>0$) is not part of an equilibrium. So it is without loss of generality to assume $\psi(\emptyset,0|0,e_t)=0$ $\forall e_t$.

(c) Restriction 1 ensures that the firm always waits for a rating, and restriction 2 requires that the firm holds belief equal to the belief at the beginning of the period is the agency sets an out of equilibrium fee. Consistency of beliefs requires $\mu_{t+1}=\mu_t$ whenever the firm does not wait for a rating

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