High Frequency Traders: Taking Advantage of Speed

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Joint work with Yacine Aït-Sahalia (Princeton).

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Stylized Facts and Motivation

- Increase market quality [Hendershott et. al., (2011), Jovanovic and Menkveld (2012)].
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- Cancel orders with high probability [Hasbrouck and Saar, 2009].
  Question: What could be the driver of this behavior?

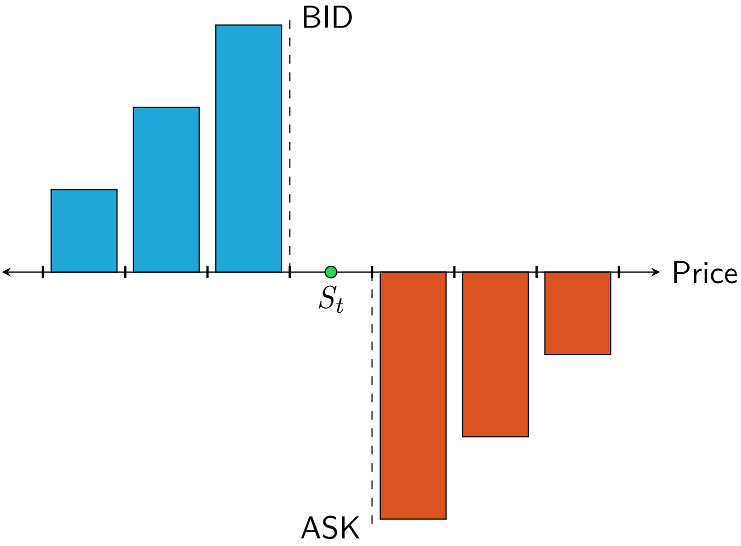
Our Contributions

- We derive the HFT's optimal liquidity provision in a dynamic model as a function of his speed, asset volatility in monopolistic and duopolistic markets.

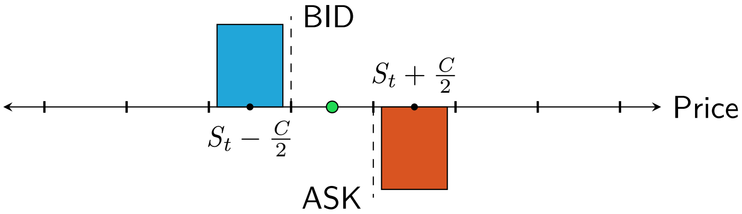
- Our model reproduces endogenous cancellation of limit orders.

- We evaluate various recent proposals to regulate high-frequency trading.

Limit Order Book



Simplified Limit Order Book



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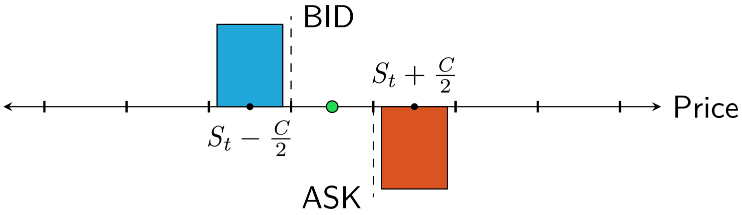

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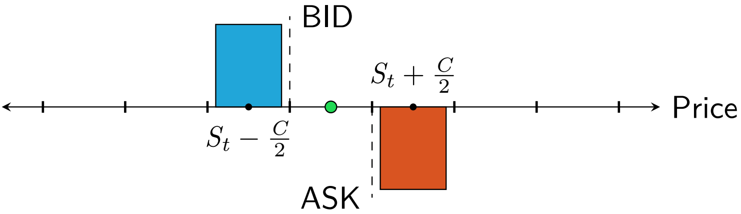

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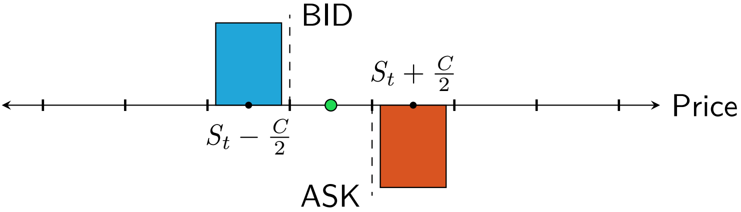

- The fundamental price for the asset is S_t.
- Bid-offer spread is given by C.
- The LFTs submit market orders which arrive at random times according to a Poisson process with parameter λ.

Trading Technology of HFTs

- HFT receives a signal s with rate μ which is informative about the sign of the incoming market order.
  $\Rightarrow$ Signals can be generated from hard information
  - order book imbalance
  - index arbitrage
  - new trades on a correlated asset

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- The signal is an iid Bernoulli random variable, $s \in \{\text{sell,buy}\}$ with each being equally likely.

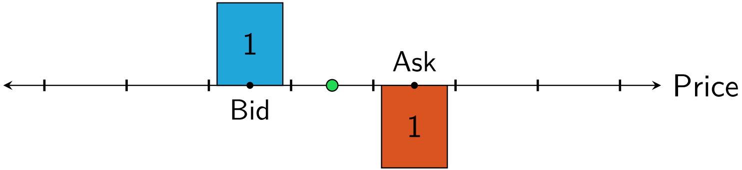
- Conditional on buy (sell) signal the next market order will be a sell (buy) order with probability p and buy (sell) with probability $1 - p$.

HFT's Quoting Mechanism

- The HFT makes quoting decisions immediately after observing a signal or market order.

- The HFT can post limit orders at the best bid ($\ell^b = 1$) and/or the best ask price ($\ell^a = 1$).

- The quantity is fixed at 1 lot: no optimization over quantity. The number of possible actions is 4:



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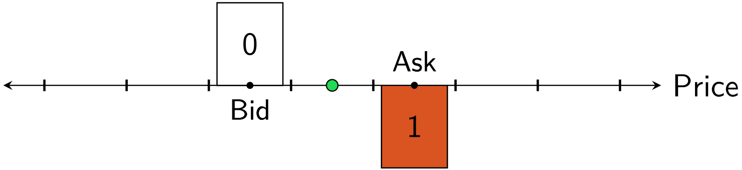

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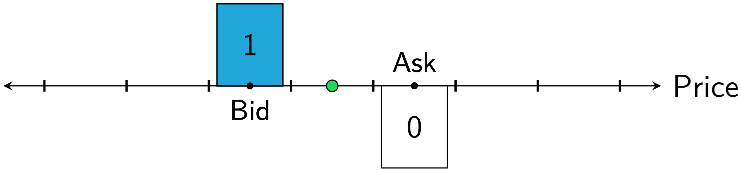

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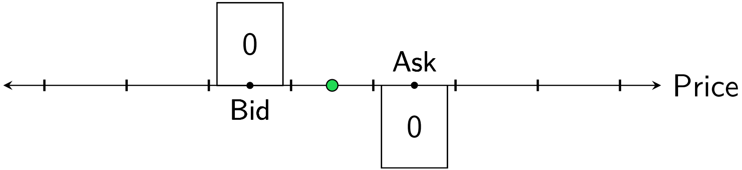

HFT's Objective Function

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- With T_i^s the ith market sell order and T_j^b the jth market buy order, the HFT maximizes over any feasible π that chooses ℓ^b and ℓ^a at decision times:

$$\max_{\pi} \mathbb{E}^{\pi} \left[\frac{C}{2} \sum_{i=1}^{\infty} e^{-DT_i^{\text{sell}}} \mathbb{1} \left(\ell^b_{T_i^{\text{sell}}} = 1 \right) + \frac{C}{2} \sum_{j=1}^{\infty} e^{-DT_j^{\text{buy}}} \mathbb{1} \left(\ell^a_{T_j^{\text{buy}}} = 1 \right) - \Gamma \int_0^{\infty} e^{-Dt} |x_t| dt \right]$$

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- The corresponding action at each state is whether to quote a limit order or not at the best bid and best ask, i.e., $\ell^b \in \{0, 1\}$ and $\ell^a \in \{0, 1\}$.

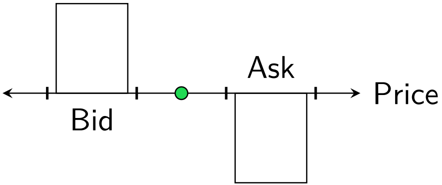
Optimal Market Making Policy

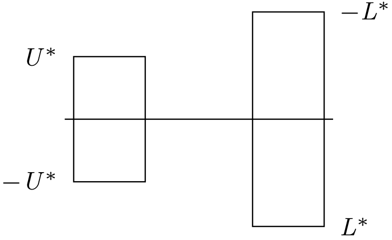
- The optimal quoting policy of the HFT, π^*, consists in quoting at the best bid and the ask according to a threshold policy.
- We prove that there exists L^* and U^* with $-L^* \geq U^*$ such that

$$\ell^{b*}(x,1) = \begin{cases} 1 & \text{if } x < U^* \\ 0 & \text{if } x \geq U^* \end{cases} \qquad \ell^{a*}(x,1) = \begin{cases} 1 & \text{if } x > L^* \\ 0 & \text{if } x \leq L^* \end{cases}$$

$$\ell^{b*}(x,-1) = \begin{cases} 1 & \text{if } x < -L^* \\ 0 & \text{if } x \geq -L^* \end{cases} \qquad \ell^{a*}(x,-1) = \begin{cases} 1 & \text{if } x > -U^* \\ 0 & \text{if } x \leq -U^* \end{cases}$$

Interpretation





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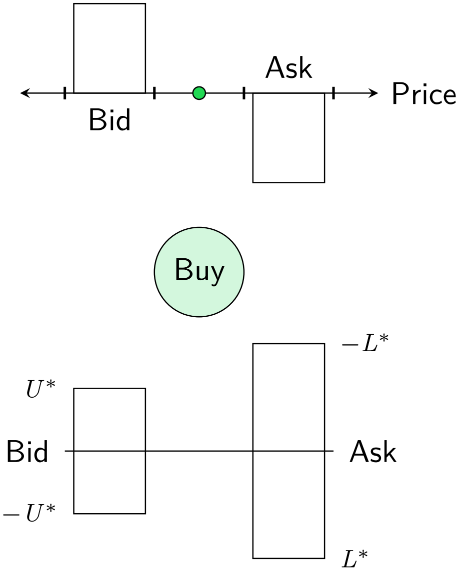

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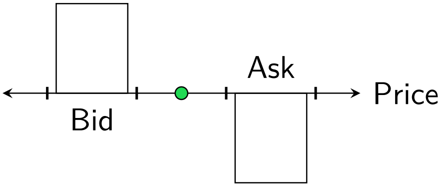

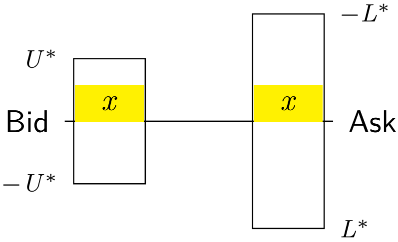

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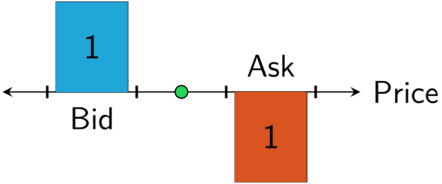

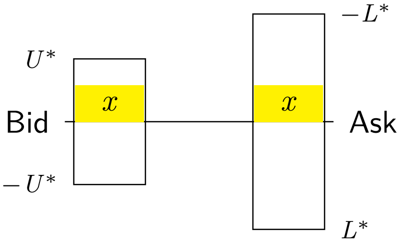

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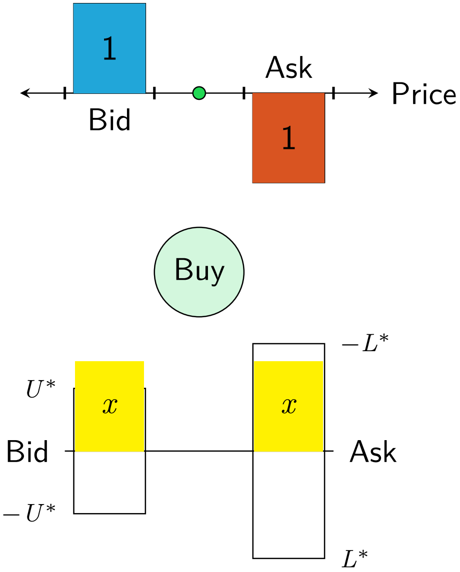

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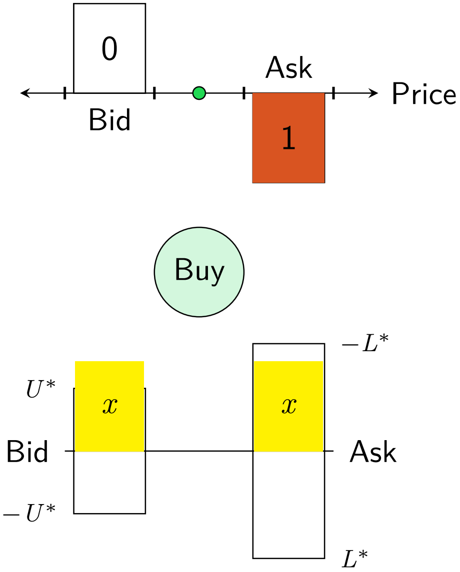

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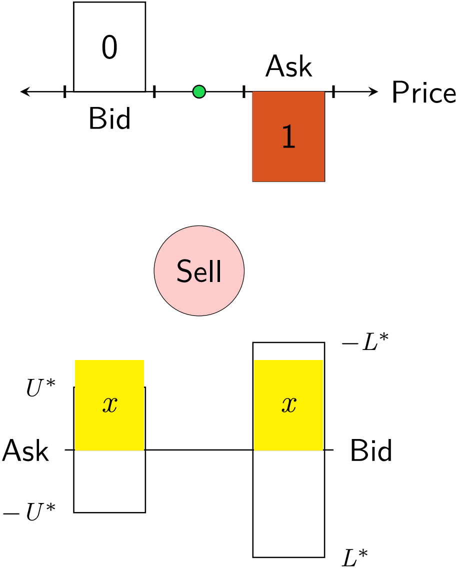

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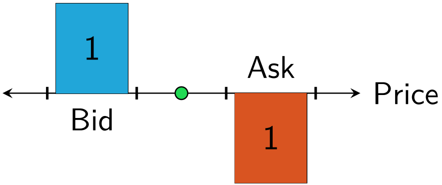

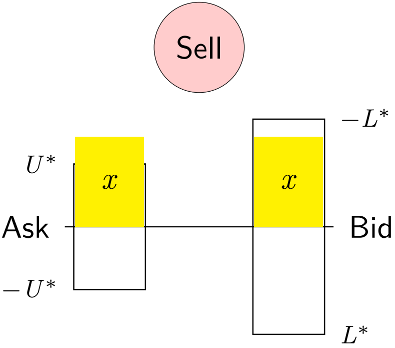

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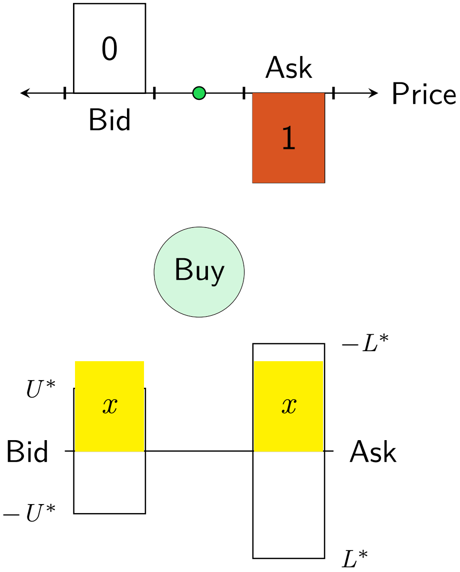
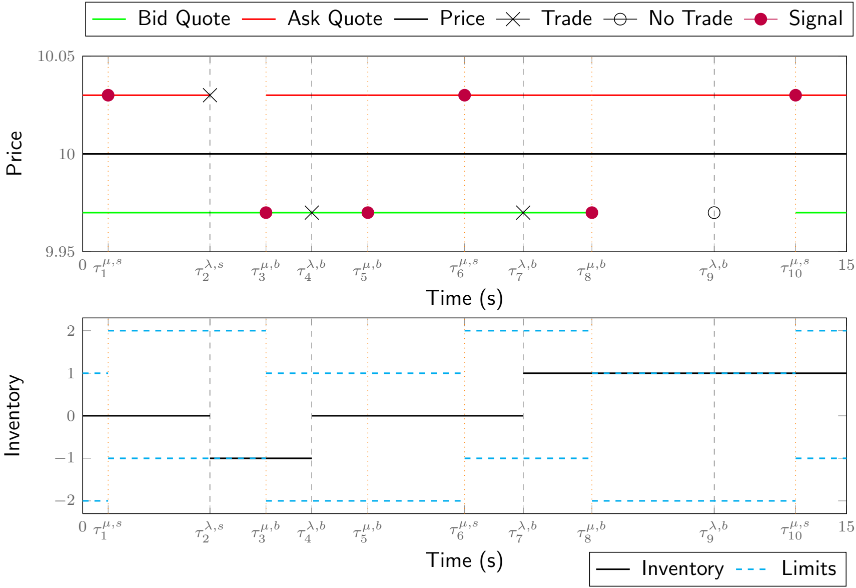
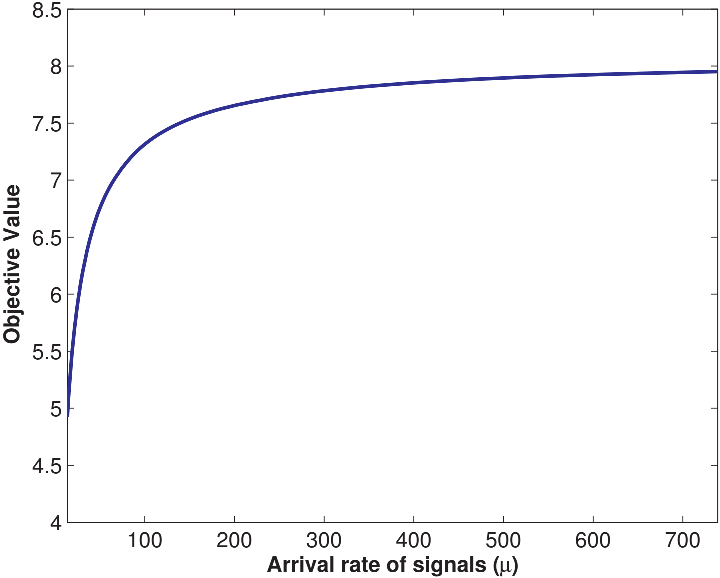


Illustration: A Simulated Path



Comparative Statics

Arrival Rate of the Signal: μ



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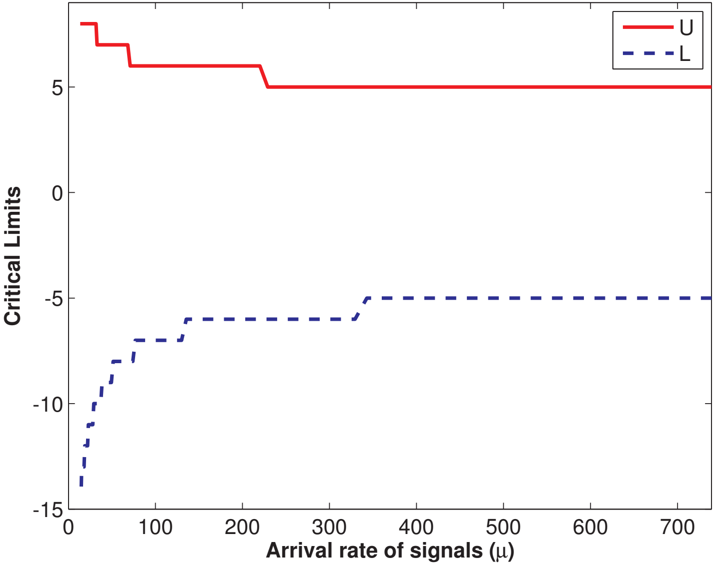
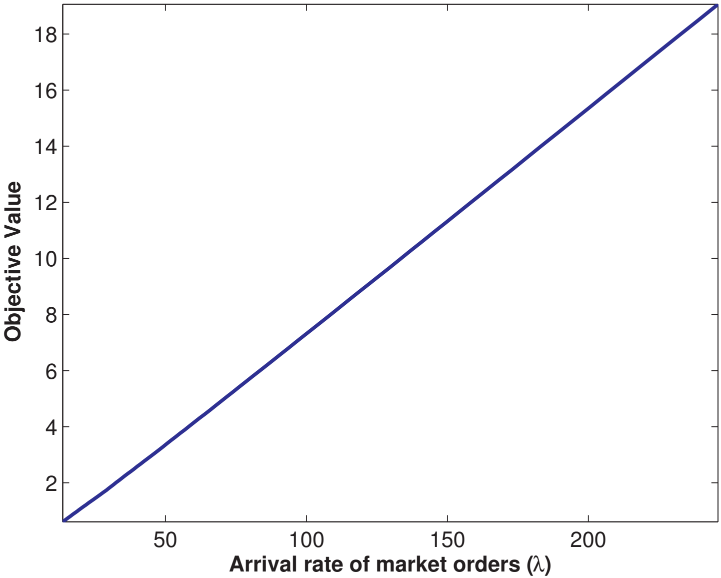


Arrival Rate of the LFTs: λ



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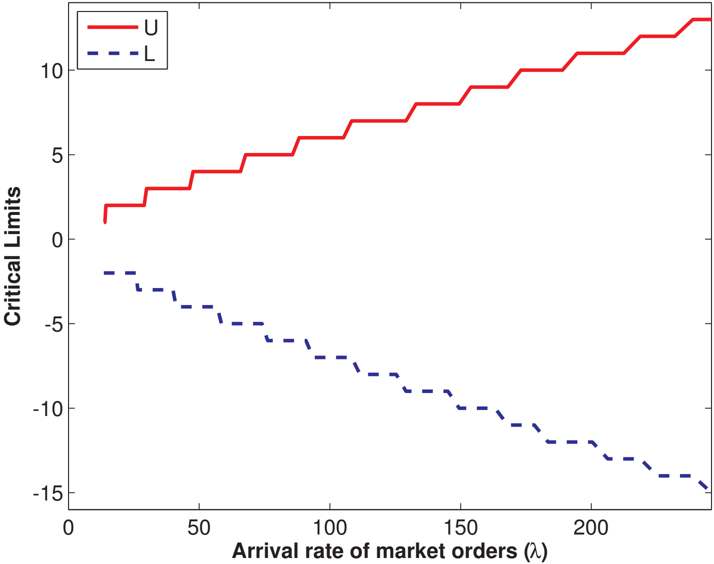


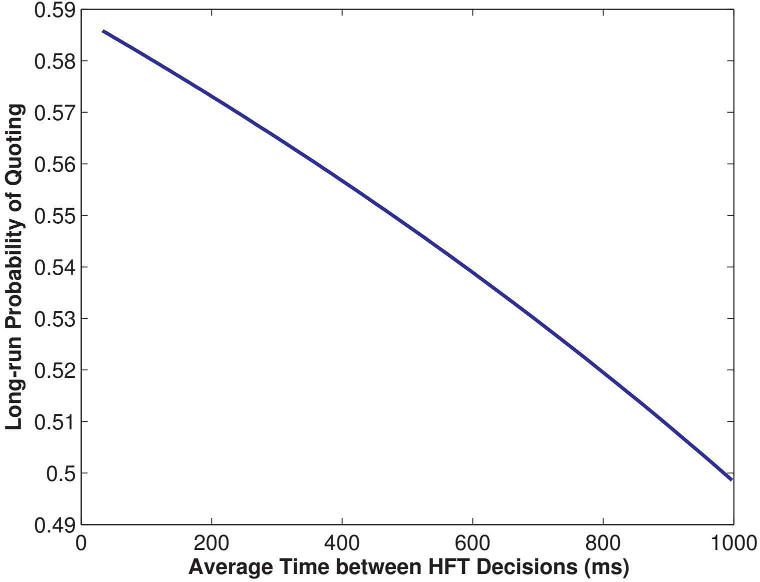
Implications for the Market Structure

Implications from stationary probabilities

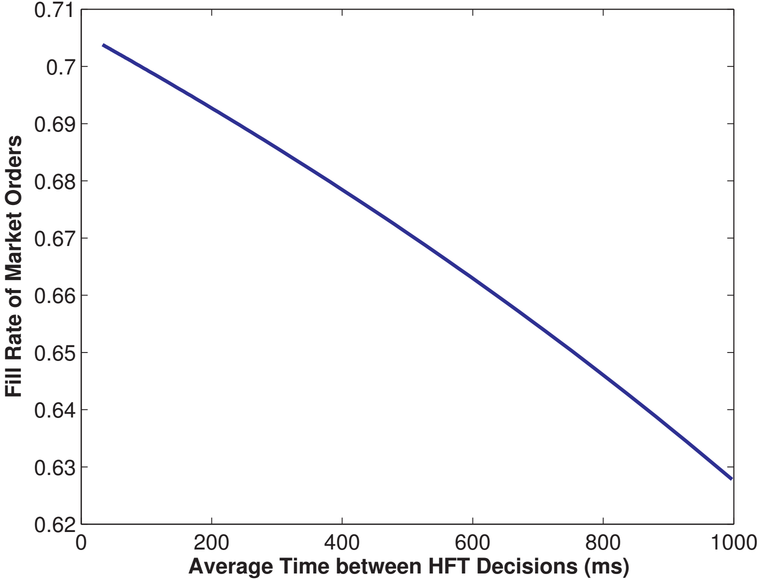
- Inventory fluctuates between $[L, -L]$.
- Signals can take values from $\{-1, 1\}$.
- Under optimal policy, we have a finite state irreducible Markov Chain.
- Long-run stationary probabilities, $\pi(x, s)$, exist.
- Long-run probability of quoting at both sides of the market can be found by

$$q_{\text{quote}} = \sum_{x \in (L,U)} \pi(x, 1) + \sum_{x \in (-U,-L)} \pi(x, -1).$$

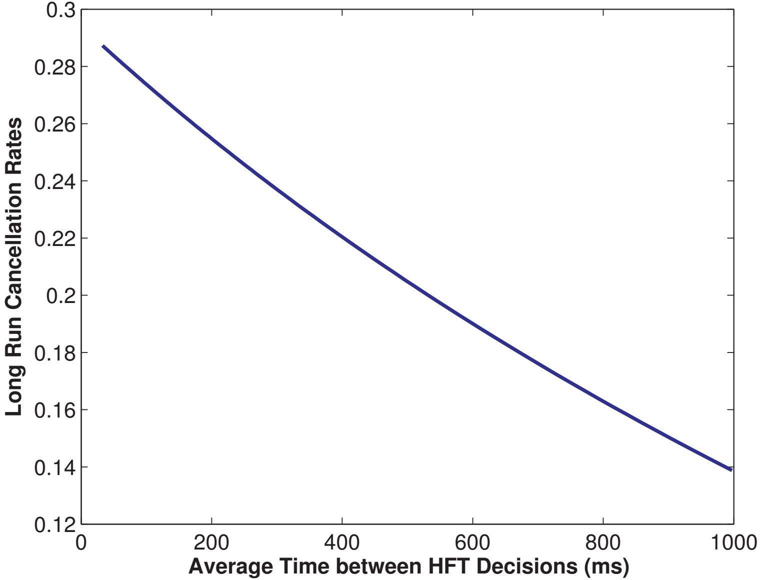
HFT's Liquidity Provision



Welfare of the LFTs: Fill Rate



Cancellation Rates



Price Volatility

- Let the fundamental price of the security, S_t, be specified by a pure jump process

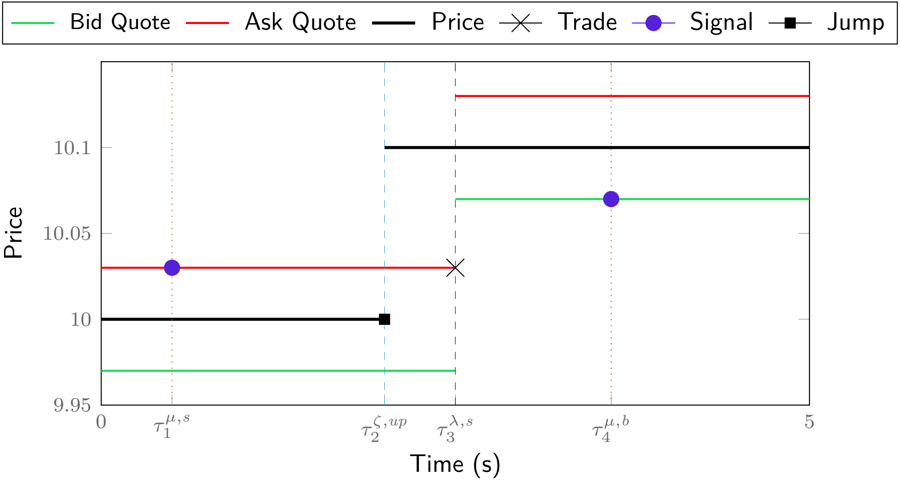
$$S_t = S_0 + \sum_{i=1}^{N_t} Y_i,$$

- N_t is a Poisson process with arrival rate ζ counting the number of tick movements up to time t and Y_i is the jump size with

$$Y_i \sim \begin{cases} J & \text{with probability } \frac{1}{2}, \\ -J & \text{with probability } \frac{1}{2}. \end{cases}$$

- When the price jump occurs, an LFT may arrive and possibly trade with a stale HFT quote, effectively imposing adverse selection on the HFT.

Illustration: Volatility Model



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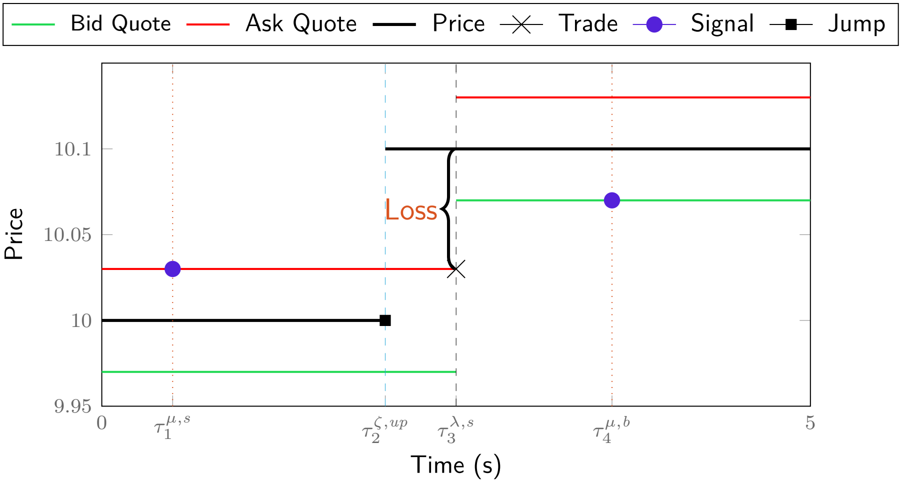
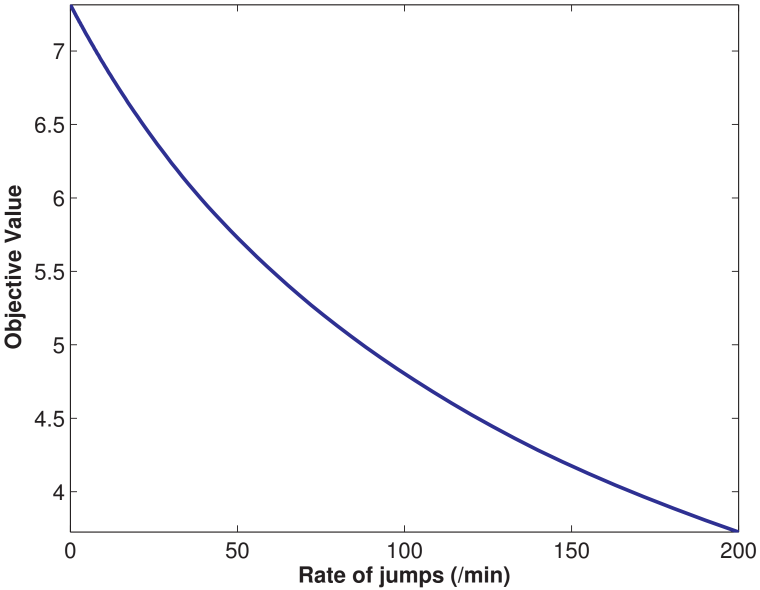
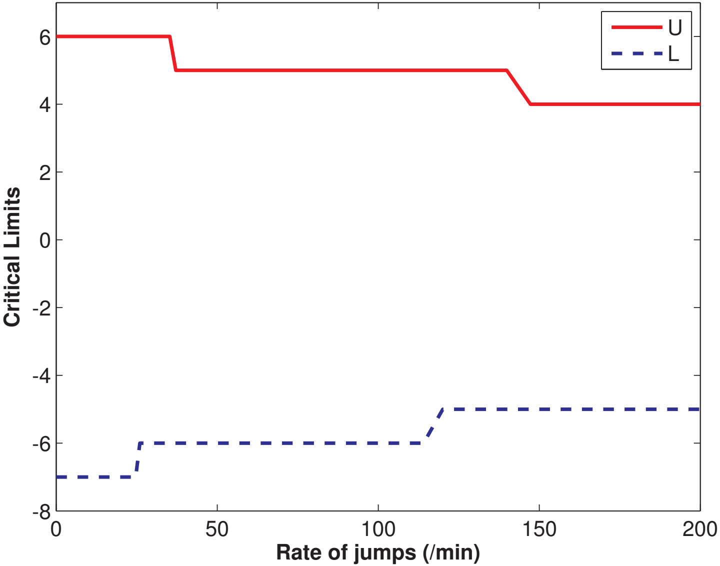


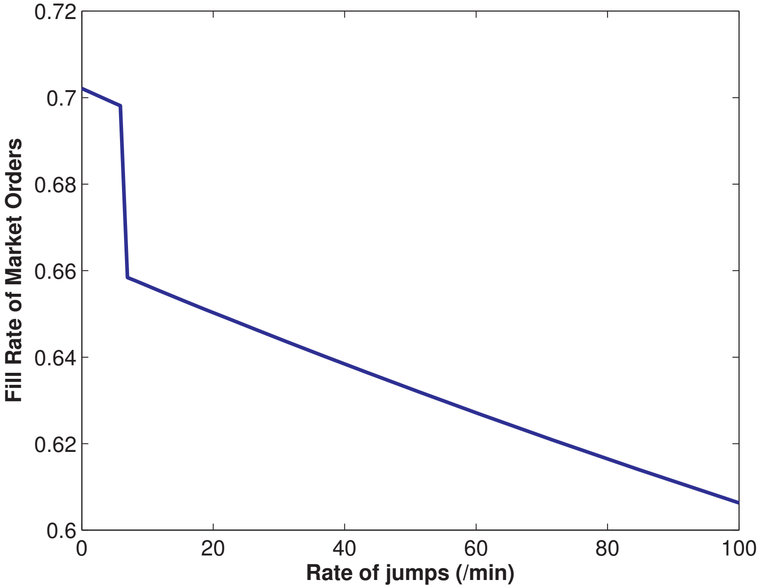
Impact of Volatility on HFT's Profits



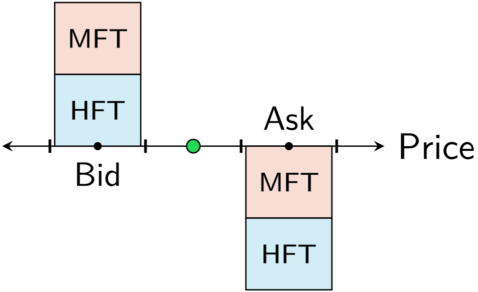
Impact of Volatility on HFT's Quoting Policy



Impact of Volatility on Welfare of the LFTs



Duopoly Model - Priority Issues



- Medium-frequency trader (MFT) competes with HFT.

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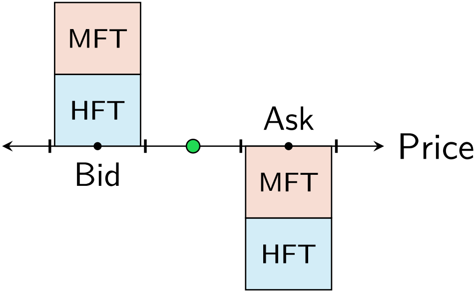

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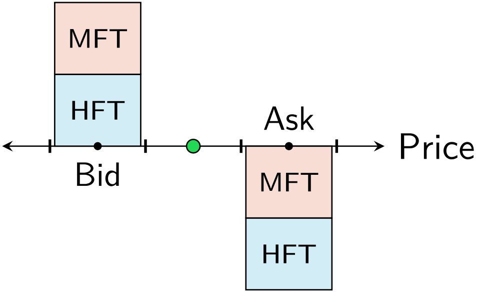
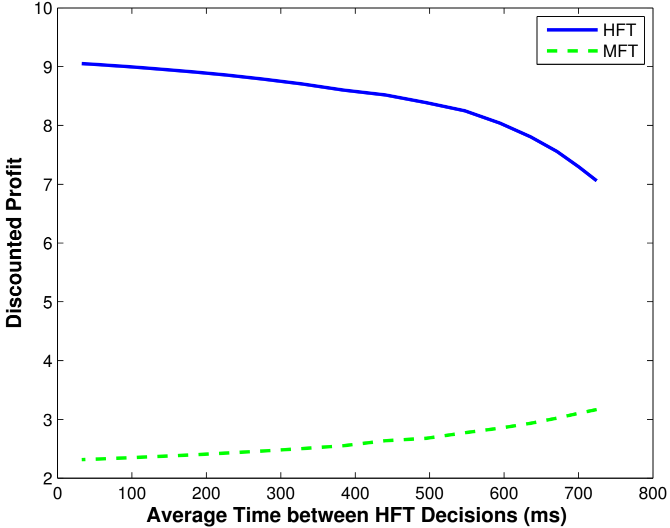
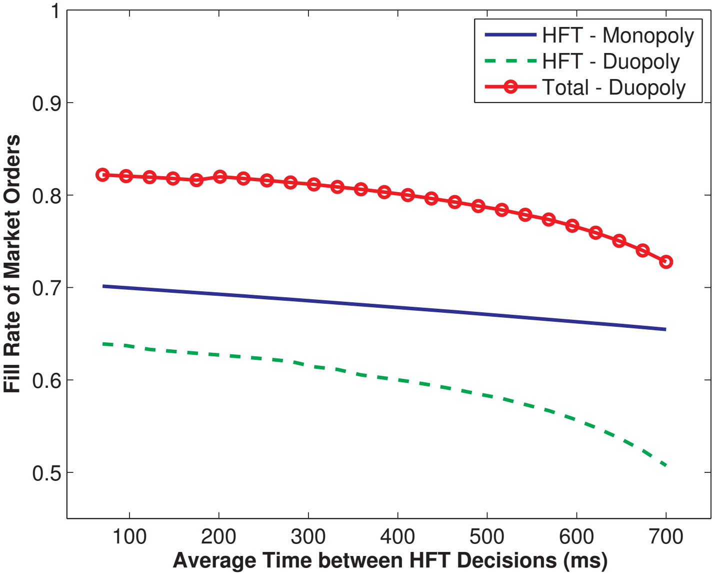


- Medium-frequency trader (MFT) competes with HFT.
- MFT submits and cancels orders at an exogenous rate of β.
- Our model can accommodate priority issues with additional states.

Impact of Competition on HFT's Profits



Impact of Competition on Liquidity



Policy Implications

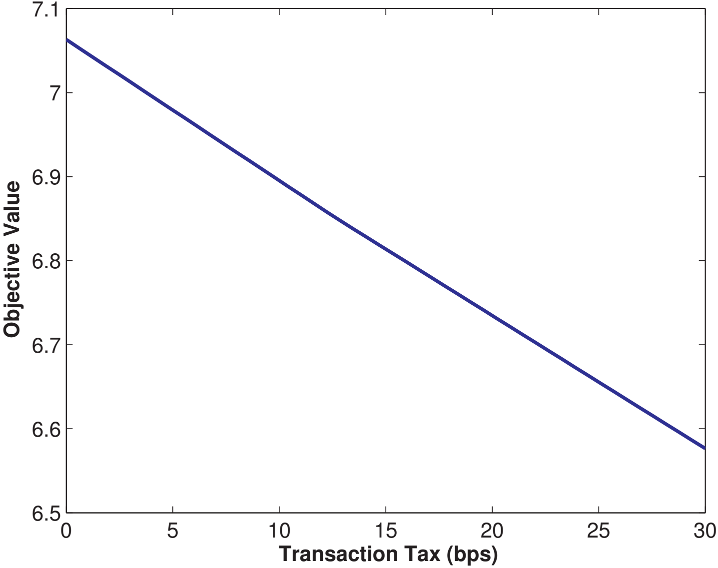
Discussion of Highly Cited Policies

- Tobin Tax: Suppose that HFT pays $\frac{\kappa}{2}$ dollars each time for every trade.
  - Equivalent to changing the spread in our model. Define the tax-adjusted bid-offer spread as $\tilde{C} \equiv C - \kappa$.

- Speed Bumps for HFTs: We can impose (random) minimum time before a quote can be cancelled.
  - Random minumum time limits can be modeled using another Poisson clock with rate θ. Lower θ imposes larger order resting times.

- Cancellation Taxes: We can tax the HFT by ε dolars whenever he cancels an existing quote.
  - This extension can be accommodated via additional states that keeps track of the previous quotes.

Tobin Tax decreases HFT's Objective Value



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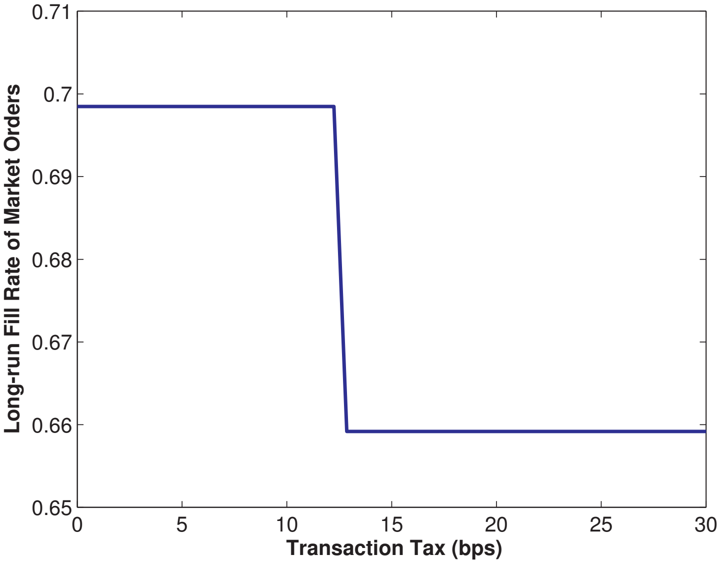

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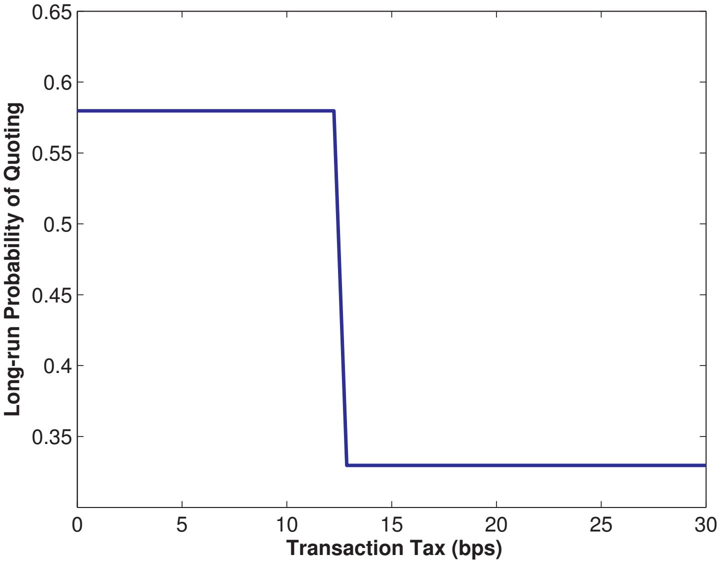
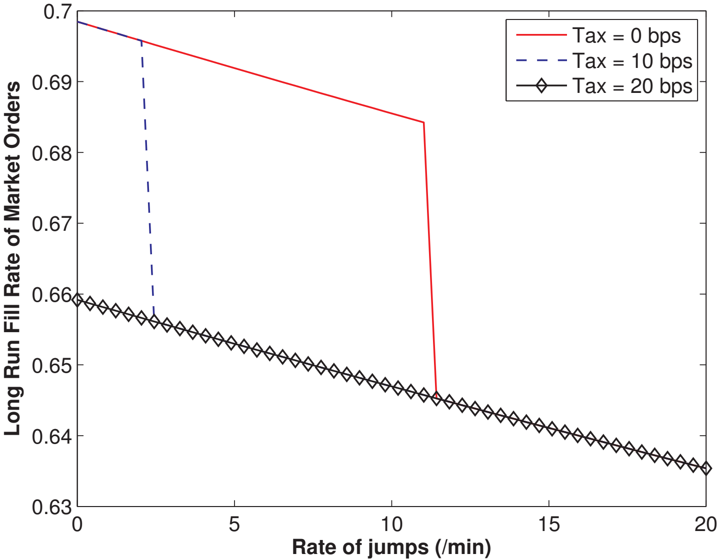
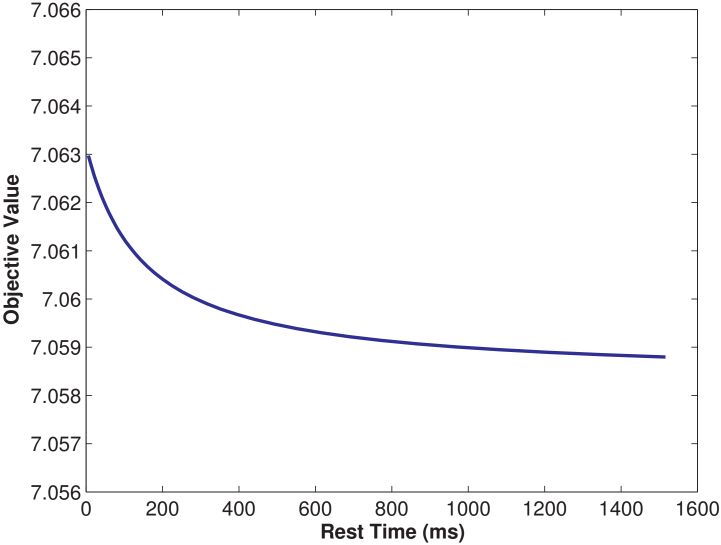


Sensitivity to Volatility



Resting Time decreases HFT's Objective Value



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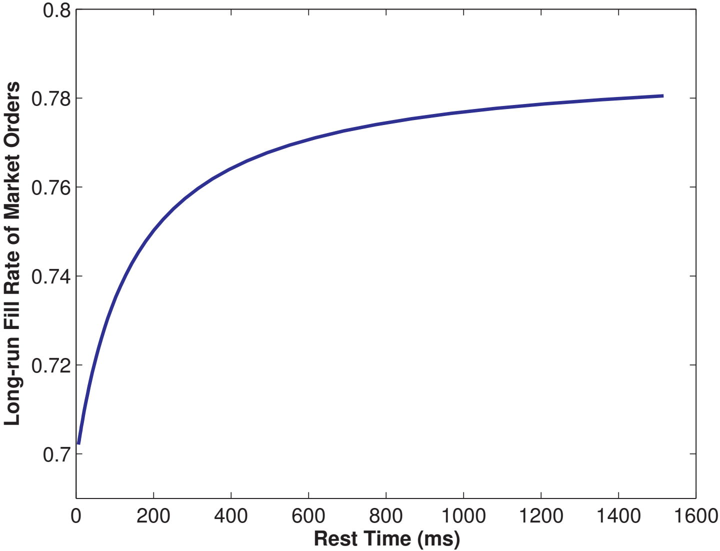

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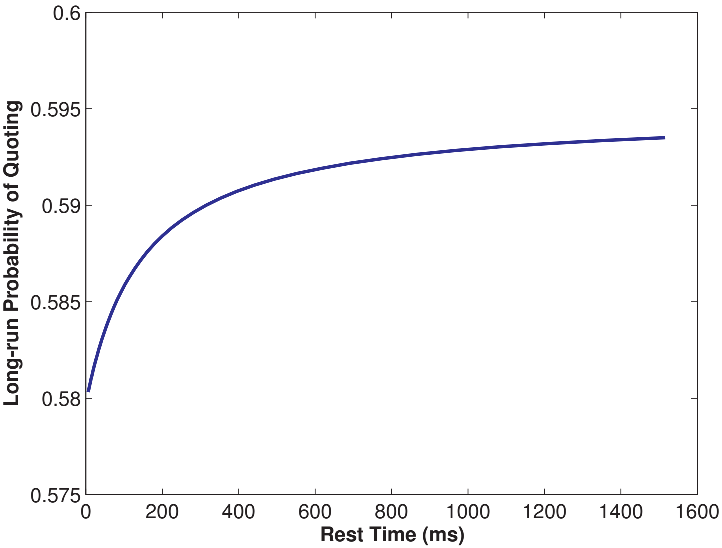
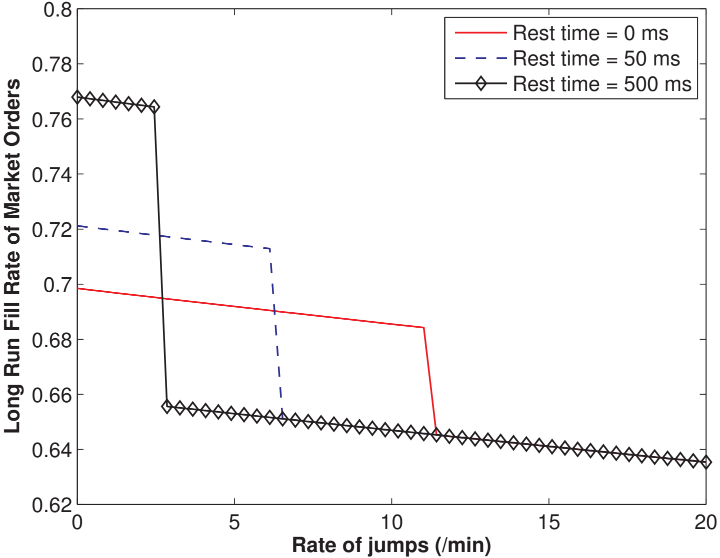
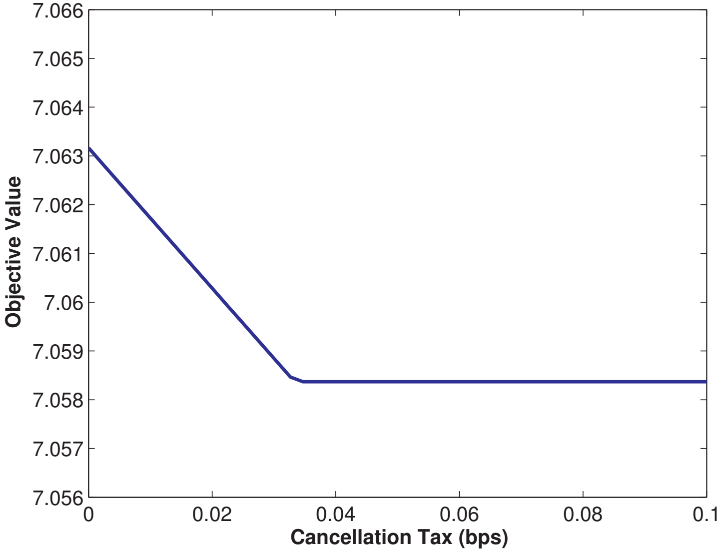


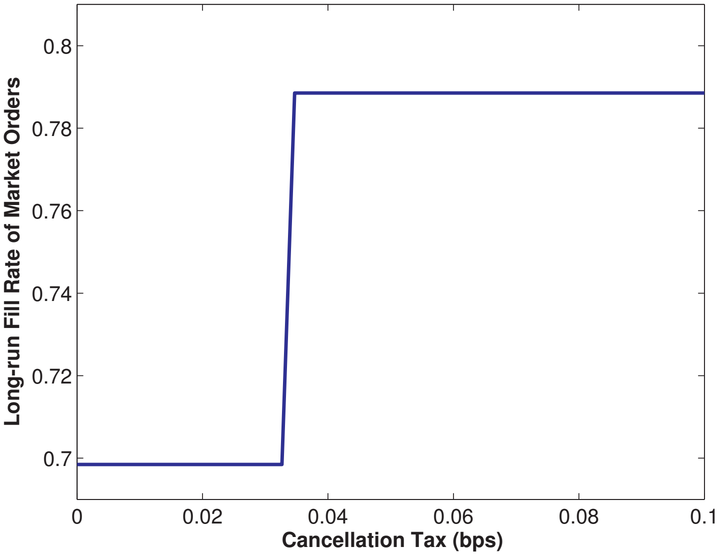
Downside: Countercyclical with Volatility



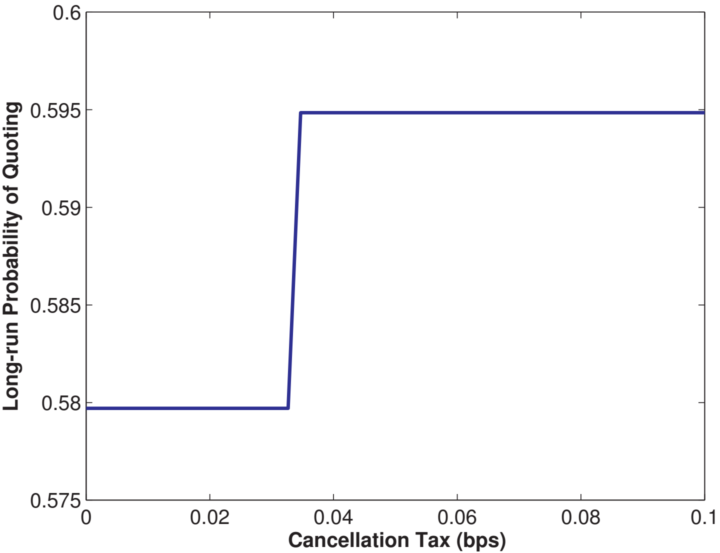
Cancellation Tax decreases HFT's Objective Value



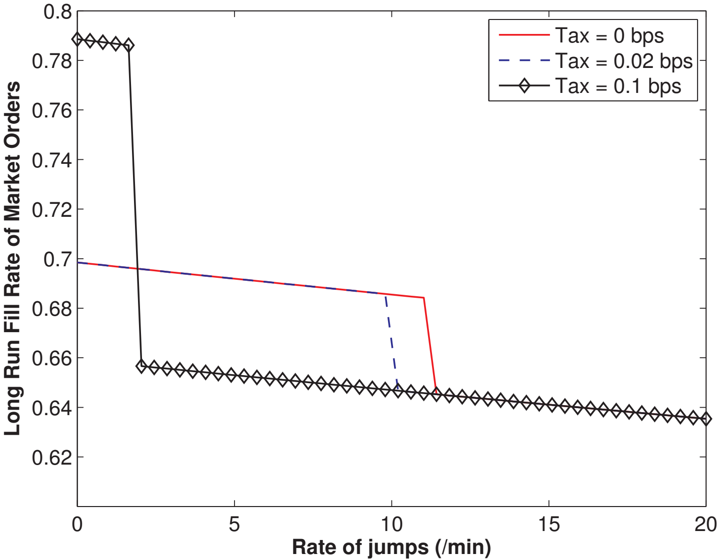
Cancellation Tax improves Liquidity



Cancellation Tax improves Liquidity



Downside: Countercyclical with Volatility



Conclusion

- We develop a fully dynamic trading model in which we study HFT's optimal quoting policy
- Model is very tractable and allows multiple extensions with interesting research questions.

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Key implications:

- HFTs improve market liquidity but they shy away providing liquidity in high volatility regimes.

- Tobin tax is a bad policy for the market but minimum time limits and cancellation taxes can improve liquidity.