Perks as Second Best Optimal Compensations

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Abstract

The finance literature views perks either as productivity enhancing expenditures or as a result of poor managerial control by shareholders. Using a corporate jet to attend a business meeting may be justified because of the returns generated for the firm; but flying on the same jet to reach a vacation resort reflects a misappropriation of the firm's resources by the manager. Our paper challenges this view. We argue that complementarity between leisure and wages creates difficult incentive problems, because the bonuses or stock options that reward success increase the marginal disutility of effort. In such a context, we show that whenever there exist commodities ('perks') that are substitute to leisure (or even less complementary to leisure than money), the optimal incentive scheme involves overprovision of such commodities, in the sense that the agent should consume more of them than she would elect to should she given a choice between money and perks at the current market prices. This conclusion is valid even when perks must be provided independently of success, i.e. cannot be used as an incentive device. Finally, we discuss the role of governance by introducing manipulations a la Peng and Röell (2006), and show that, in contrast with standard intuition, perks are used even when governance is perfect, and poorer governance may result in *less* perks being offered to the agent.

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Perquisites, or 'perks', are non monetary compensations that firms offer to select employees. They may include the personal use of chauffeur-driven limos or executive jets, membership in select clubs, financial counseling, tax preparation and estate planning, retirement packages, etc. In many cases, perks represent significant amounts. A recent survey of the compensations received in 2006 by several Silicon Valley's chief executives[1] indicates that Larry Ellison, Oracle's CEO, received almost two millions in 'perks and other compensations'; Meg Whitman (eBay) landed more than one million, and several other CEOs received $500,000 or more. Moreover, perks are not exclusive to CEOs; the same survey indicates that in several firms, the perks received by other top executives (CFO, COO, SVP) are close to, and sometimes larger than those of the CEO.[2] Nor is the perk phenomenon specific of large, private corporations. Several universities provide faculty housing below market prices; and a recent newspaper article reported that the Chancellor of a top university charged its institution $11,000 in season tickets to a theater. Finally, while some perks may seem largely related to business and professional activities, others are not; some do not actually benefit the executive herself, but rather her family - examples include kindergarten services or access to selective private schools, airplane tickets or use of company jet for spouse or children, and others.[3]

While the amounts spent on perks often represent only a small fraction of the total compensation received by the beneficiaries, their mere existence raises a simple question - namely, why not pay the corresponding amounts in cash and let the employees free to purchase these products (or any alternative they may fancy) by themselves? Particularly intriguing is the fact that most of the time, the corresponding products or services are unlikely to be purchased by the agents (or at least not in the same amount), should they receive the cash equivalent. This suggests that the utility derived from this particular form of compensation may be quite small in regard of its cost - therefore that perks are, at least in a first best world, a particularly inefficient (and distortional) way of paying compensations.

In the financial literature, the standard explanation of perks relies on some

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[1] Equilar, Mercury News research, 2007

[2] For instance, Stephen McGowan, CFO and EVP of Sun Microsystems, received perks for an amount of $922,830, more than the CEO.

[3] As an example, consider the following excerpt from an article published in the New York Times:

> "Some high school students will be making their way back to school this week on a bus or, if they are lucky, in their own car. But the stepdaughter of Edward Mueller, the new chief executive of Qwest Communications, has a much fancier option....A regulatory filing made Friday, on the eve of the holiday weekend, disclosed that Qwest has authorized Mr. Mueller's wife and her daughter to use Qwest Corporate jet to travel between Denver, where the telecommunications company is based, and California, where Mr. Mueller's stepdaughter is finishing high school. [...] Asked about the filing by the Rocky Mountain News, a Qwest spokesman said the agreement reflects an appreciation for his family situation as his daughter wraps up her schooling in California". New York Times, September 4, 2007

form of agency problem; namely, perks are used by managers to (mis)appropriate some of the surplus generated by the firm, in a way that is neither approved nor even acknowledged by shareholders. Cash transfers, while more efficient, would be more visible, hence less useful in terms of surplus extraction. In short, perks are the consequence of poor monitoring of the managers by the shareholders - a view that lies at the core of a host of theoretical papers in corporate finance.[4] This interpretation, however, has recently be challenged by Raghuram Rajan and Julie Wulf (2006). In their paper, Rajan and Wulf use a large database on executive compensation to carefully examine the empirical relevance of several predictions generated by the standard agency model used in most of the corporate finance literature. Overall, they find little support for it. For instance, they find no direct relation between governance and perks, nor any impact of exogenous changes in governance on perk consumption; and they show that standard indicators of external monitoring, such as board size, fraction of outside directors or institutional investor ownership, are unrelated to perks used for personal purposes. They tellingly conclude that despite 'occasional aberrations, [...] treating perks purely as managerial excess is incorrect'.

Alternative justifications for perks have been suggested. For instance perks may in some case allow to exploit tax loopholes. This argument helps explaining some but not all perks, in particular because the tax advantage has to be traded off with the utility loss linked with the in kind provision of less desired commodities.[5] Rajan and Wulf, on the basis of their empirical analysis, suggest still another explanation: firms offer as perks goods that increase workers' productivity. Such a direct provision is efficient because managers' private incentives to consume perks fall short of their 'social' incentives whenever firm can appropriate part of these productivity gains[6]. This intuition has been formalized in a recent paper by Marino and Zabojnık (2008). In their setting, perks serve as a non-labor input that increases the agent's productivity. They show that perks are generally offered even when their direct consumption benefits are offset by their costs, and derive comparative static results.

The Rajan/Wulf/Marino/Zabojnık argument can justify some of the perks actually observed, from the provision of a laptop to the availability of the company jet for business-related travel. However, it applies only insofar as the perks actually increase the agent's productivity; it precludes any purely private (non business-related) consumption of perks. Advantages like financial counseling, retirement packages, or private use of the corporate jet - let alone kindergarten, private schools or family use of the corporate jet - can hardly be justified by their impact on the agent's productivity. Therefore, in the traditional view, their existence can only be the by-product of some kind of managerial misconduct.

[4] See for instance Grossman and Hart (1980), Jensen and Meckling (1976), Jensen (1986) and Aghion and Bolton (1992).

[5] Rajan and Wulf find that state marginal tax rates have a statisically significant impact on the use of company car, country club membership and financial counseling. However, the use of chauffeur and corporate jets is not significantly linked to state taxes.

[6] Noteworthy, this explanation seems to implicitly refer to a moral hazard problem : were it possible to pay a manager conditional on its true effort, a worker would have appropriate incentives to consume productivity enhancing commodities

In this paper, we propose an alternative and complementary justification for perks that challenges traditional wisdom. Specifically, we argue that in a context of asymmetric information, perks are typically part of the (second-best) optimal incentive scheme *even when they have no impact on the agent's productivity*. Firms may rationally want to pay perks that are not directly productive, because such a compensation reduces the cost of providing adequate incentives to the employee. The basic idea is that, in general, an agent's utility depends on effort (or leisure) and consumptions in a *non separable* way. Non separability of leisure and consumption is a standard finding of the empirical literature on labor supply (see for instance Browning and Meghir (1997)). It reflects the very natural intuition that, in general, the marginal utility of leisure increases with wealth, if only because number of consumable goods (travel, services,...) are complements of leisure; for instance, a free week-end is both more expensive and more enjoyable when spent skiing down Colorado slopes or sailing along the coast of some Caribbean island rather than idling in Brooklyn.[7] In the moral hazard setting we consider, complementarity of leisure and consumption has a crucial consequence[8]; namely, direct compensations, in terms of cash payments, tend to increase the disutility of effort, which exacerbates moral hazard issues. At the very high income level reached by many top executives, the (marginal) value of leisure is so high (and the marginal utility of money so low) that providing adequate incentives becomes extremely costly; moreover, such incentives, by further increasing the subjective cost of activity, may even be counterproductive, a case illustrated by Bennardo and Chiappori (2003).

We investigate the consequences of this remark from a second best perspective, in a multicommodity setting. A standard second best analysis concludes that the optimal contract typically involves consumptions patterns that are distorted vis a vis both the first best and the agent's unconstrained choices, in order to fully exploit the different complementarity/substitutability properties of each commodity with respect to leisure. Specifically, we show, in a simple model in which the agent can consume leisure, a numeraire good ('money') and a third commodity, the 'perk', that if the perk good is a *substitute to leisure*, in the (usual) sense that consumption of the former decreases the marginal utility of the latter, then the second best optimum entails a *larger* consumption of perks than what the agent would freely select. Moreover, the conclusion holds under

[7] A Beckerian justification, used for instance by Bennardo and Chiappori (2003), relies upon the existence of a domestic production function that produce some agent-specific commodity, using time and the consumption good as complementary inputs. In this case, the marginal utility of consumption typically increases with leisure. To see why, assume that well-being is proportional to the consumption of a single household good ξ, produced from some constant return to scale technology :

$$\xi = f\left(c, 1-e\right) = \left(1-e\right)\phi\left(\frac{c}{1-e}\right)$$

where ϕ is increasing concave. Then $\frac{\partial^2 v}{\partial c \partial e} = \frac{\partial^2 f}{\partial c \partial e}$ is always negative.

[8] The implications of non separable preferences for moral hazard problems were initially recognized by Grossman and Hart (1983). For a recent and more thorough investigation, see Bennardo and Chiappori (2003)

weaker assumptions; actually, one only needs perks to be 'less complement to leisure' (in a sense we precisely define) than the numeraire.

One can easily accept that giving a top executive access to a corporate jet to facilitate her professional travels could be efficient. We argue, however, that the conclusion may extend to *private* use of the jet, insofar as it can be a substitute for alternative uses of the agent's time. When she wants to reach Aspen for a ski week-end, a top executive faces a choice between leaving her office early on Friday afternoon in prevision of the long hours required for the connections between regular flights, or attending this crucial but late meeting on Friday evening and taking next a direct flight on the company jet. Here, access to the private flight on Friday night reduces the marginal utility of leisure; in a second best context, this property should be exploited to provide incentives optimally. By the same token, saving an employee's time by providing her with adequate assistance in tax preparation, estate planning or financial counseling will generally be efficient, even if the consumption of these services is purely private and does not directly increase her productivity in the office. On a less fancy tone, availability of subsidized housing located near the campus reduces the time academics spend commuting; it is thus second best efficient in general.

A second conclusion is that if, as argued above, leisure and money are complement, then the optimal compensation plan will entail overconsumption of the perk good whenever the latter is 'less complementary to leisure' than money, in a sense we precisely define. In particular, even if the agent's utility is separable in leisure and perks, so that consumption of perks has no impact on the marginal disutility of effort, one still expect overprovision of perk at the optimum whenever leisure and money are complement, precisely because perks alleviate the negative impact of high wages on the cost of providing adequate incentives. In all these cases, the optimal incentive scheme requires that a fraction of the compensation be paid as perks. A regulation reducing or prohibiting the use of perks would actually directly harm shareholders and result in social losses.

Our basic model can be extended in several directions. The second best efficient allocation of perks can, alternatively, be implemented by the introduction of subsidies over perk goods. We also consider the coexistence of several perk goods, and show that the basic intuition can be generalized: if a commodity, say i, is less complement to leisure than some other commodity, say k, then commodity i is 'more overconsumed' than commodity k at the optimum, in the sense that given the total expenditures on goods k and i, the agent would like to consume more good k and less good i than provided at the second best. Our basic intuition is also remains valid whether the provision of perk can depend on the state of the world (so that incentives cannot be provided by additional perks expenditures rewarding good performance) or not. Finally, we discuss the links between our second-best explanation of perks and governance issues. While in our story perks are not directly caused by lack or governance, it is nevertheless the case that the provision of perks may, in the same second best spirit, be used to alleviate governance problems. We illustrate this general idea in a simple extension of our model, the main features of which are borrowed from Peng and Röell (2006). We show that, indeed, severe governance problems may impact the optimal allocation of perks. The conclusions, however, are quite different from the standard insights. Not only are perks present even when governance is perfect, but one can find robust examples in which more serious governance problems may result in *less* perks being provided at the optimum.

The related literature includes the seminal contribution of Grossman and Hart (1983), which was the first, to the best of our knowledge, to consider the effects of non separability of preferences on second best rewards schemes. Bennardo and Chiappori (2003) show that when the agent's preferences are non separable in effort and consumption, Bertrand competition may result in positive equilibrium profit for perfectly competitive principals. Our paper is an extension of theirs to a multi-commodity setting, but in a partial equilibrium environment. Peng and Röell (2006) assume non separable preferences to investigate the effects of managerial manipulation of performance measurement and characterize second best contracts in a single good environment. Jensen (1986), a representative of the conventional corporate finance literature on perks, argues managers working for firms generating larger cash-flow get larger perks, while firms with better external governance pay less perks in the spirit of Jensen and Meckling (1976). Yermack (2006) empirically investigates the link between perks and external governance. He finds that the disclosure of a CEO's personal use of a company plane leads to underperforming average shareholder returns.

Section 1 describes basic setting. Section 2 provides basic results with comparative statics. Section 3 discuss extensions of subsidized perks, several perks and state independent perks. Section 4 analyzes the effect of governance structure on perks. Section 5 concludes.

1 The Model

We consider a simple, principal-agent model. As it is standard in moral hazard models, we assume that the principal (the firm or its shareholders) is risk neutral and maximizes expected profit while the agent (the top executive) is risk averse. When employed by a principal, an agent produces an output that can take two values Y and y, with $Y > y$. The probability of achieving the high output Y depends on some unobservable effort level e. We assume for simplicity that effort can take only two values, e_L and e_H, with $e_H > e_L$; the 'good' outcome Y obtains with probability $P(e)$, where $P(e_H) = P$ and $P(e_L) = p < P$.

There are three commodities in this economy: leisure l, a numeraire good ('money') c and a third commodity, q, which we call a perk; we define 'effort' $e = 1 - l$. We ignore price variations, and normalize both prices to one. Note than, in this simplified setting, any compensation received by the agent that does not take the form of a perk is used to consume the alternative commodity. In other words, the (state-dependent) consumption of the numeraire good can be seen as a reduced form for any 'standard' type of outcome-related payment; these includes wage and bonuses, but also stock options or any sophisticated compensation.

In our analysis, the complementarity or substitutability between leisure on the one hand and consumption goods on the other hand plays a crucial role. To emphasize these aspects, we assume that an agent's VNM utility function has the form

$$u(c,q,l) = v(c,l) + w(q,l) \tag{1}$$

so that we can ignore complementarity or substitutability between money and perk. The crucial aspect, which directly generalizes Bennardo and Chiappori (2003), is that effort is not separable from consumption of the numeraire good; i.e., the marginal utility of leisure (or equivalently the marginal disutility of effort) is

$$\frac{\partial u(c,q,l)}{\partial l} = \frac{\partial v(c,l)}{\partial l} + \frac{\partial w(q,l)}{\partial l}$$

which depends on both consumptions.

To keep the discussion more intuitive, we first assume that leisure and consumption of the numeraire are complements while leisure and perks are substitute (equivalently, that effort and money are substitutes while efforts and perks are complements), implying that:

$$\frac{\partial^2 v(c,l)}{\partial c \partial l} \geq 0 \quad \text{and} \quad \frac{\partial^2 w(q,l)}{\partial c \partial l} < 0$$

or equivalently, using effort:

$$\frac{\partial^2 v(c,1-e)}{\partial c \partial e} \leq 0 \quad \text{and} \quad \frac{\partial^2 w(q,1-e)}{\partial c \partial e} > 0$$

In particular, for the two effort level $e_L < e_H$, complementarity implies that

$$\frac{\partial v(c,1-e_L)}{\partial c} \geq \frac{\partial v(c,1-e_H)}{\partial c} \quad \text{for all } c.$$

while

$$\frac{\partial w(q,1-e_L)}{\partial q} < \frac{\partial w(q,1-e_H)}{\partial q} \quad \text{for all } q.$$

It must however be stressed that these assumptions, while natural, are significantly stronger than what we need. Our results simply require that perks be 'less' complementary to leisure than the numeraire is, in the following, technical sense:

$$\frac{\partial v(c,1-e_L)}{\partial c} - \frac{\partial v(c,1-e_H)}{\partial c} > \frac{\partial w(q,1-e_L)}{\partial q} - \frac{\partial w(q,1-e_H)}{\partial q} \tag{2}$$

for all (c,q). When leisure and consumption of the numeraire are complements while leisure and perks are substitute, we have indeed that:

$$\frac{\partial v(c,1-e_L)}{\partial c} - \frac{\partial v(c,1-e_H)}{\partial c} > 0 > \frac{\partial w(q,1-e_L)}{\partial q} - \frac{\partial w(q,1-e_H)}{\partial q},$$

and equation (2) is satisfied; but, obviously, this is a special case.

2 Optimal incentive-compatible contract

2.1 The main result

In what follows, we consider only deterministic contracts; i.e. we exclude randomization both ex ante (whereby agents face a lottery of possible contracts) and ex post (whereby each agent, contingent on his outcome realization, receives a lottery of possible payments). The interested reader is referred to Bennardo and Chiappori for a detailed discussion of these issues. Let (C, Q) (resp. (c, q)) denote the agent's consumption vector when the high (low) production level is achieved. One can for instance think of c as the agent's basic wage, and of $C - c$ as the bonus paid in case of success. Alternatively, $C - c$ can be the value of the stock options received by the agent (assuming that the strike is such that they are exercised only in the good state of the world), or the capital gain made by the agent on the stocks she owns. Throughout the paper, we assume that the technology is such that the second best optimum entails provision of the high effort by the agent: incentives are worth being used.

The optimal incentive-compatible contract maximizes the principal's expected profit, subject to a participation constraint for the agent and the incentive compatibility constraint. The program is thus (assuming the high effort level is implemented):

$$\max PY + (1-P)\,y - (P\,(C+Q) + (1-P)\,(c+q))$$

under the constraints

$$P\,(v\,(C, 1-e_H) + w\,(Q, 1-e_H)) + (1-P)\,(v\,(c, 1-e_H) + w\,(q, 1-e_H)) \geq \bar{U}$$

and

$$\begin{aligned} & P\,(v\,(C, 1-e_H) + w\,(Q, 1-e_H)) + (1-P)\,(v\,(c, 1-e_H) + w\,(q, 1-e_H)) \\ \geq\ & p\,(v\,(C, 1-e_L) + w\,(Q, 1-e_L)) + (1-p)\,(v\,(c, 1-e_L) + w\,(q, 1-e_L)) \end{aligned}$$

where $\bar{U}$ is the agent's reservation utility.

Let us first consider the first best contract (i.e., the optimal contract if effort was contractible). It entails full insurance for the agent; i.e., the compensation package (c, q) does not depend on the output realization. Regarding the allocation between money and perks, the first order conditions imply that:

$$\frac{\partial v\,(C, 1-e_H)}{\partial C} = \frac{\partial w\,(Q, 1-e_H)}{\partial Q} \text{ and } \frac{\partial v\,(c, 1-e_L)}{\partial c} = \frac{\partial w\,(q, 1-e_L)}{\partial q}$$

In words, the individual's MRS between the two types of consumptions equals their relative price. Therefore the firm does not need to directly cover expenditures on perks; it may as well pay a global wage equal to $C + Q$ in the high output case and $c + q$ otherwise, and let the agent freely choose her consumptions on the spot market. We conclude that in the absence of moral hazard, perks should not be part of the compensation package.

We now analyze the moral hazard situation. The first order conditions are:

$$\frac{1}{\partial v\left(C,1-e_H\right)/\partial C} = \lambda+\mu\left(1-\frac{p}{P}\frac{\partial v\left(C,1-e_L\right)/\partial C}{\partial v\left(C,1-e_H\right)/\partial C}\right) \tag{C1}$$

$$\frac{1}{\partial w\left(Q,1-e_H\right)/\partial Q} = \lambda+\mu\left(1-\frac{p}{P}\frac{\partial w\left(Q,1-e_L\right)/\partial Q}{\partial w\left(Q,1-e_H\right)/\partial Q}\right) \tag{C2}$$

$$\frac{1}{\partial v\left(c,1-e_H\right)/\partial c} = \lambda+\mu\left(1-\frac{1-p}{1-P}\frac{\partial v\left(c,1-e_L\right)/\partial c}{\partial v\left(c,1-e_H\right)/\partial c}\right) \tag{C3}$$

$$\frac{1}{\partial w\left(q,1-e_H\right)/\partial q} = \lambda+\mu\left(1-\frac{1-p}{1-P}\frac{\partial w\left(q,1-e_L\right)/\partial q}{\partial w\left(q,1-e_H\right)/\partial q}\right) \tag{C4}$$

where λ and μ are the respective Lagrange multipliers of the participation and the incentive compatibility constraint. Constraints (C1) and (C2) imply the following (the derivation is in Appendix A.1).

$$MRS(C,Q) := \frac{\frac{\partial v(C,1-e_H)}{\partial C}}{\frac{\partial w(Q,1-e_H)}{\partial Q}} = \frac{1+\mu\frac{p}{P}\left(\frac{\partial v(C,1-e_L)}{\partial c}-\frac{\partial v(C,1-e_H)}{\partial c}\right)}{1+\mu\frac{p}{P}\left(\frac{\partial w(Q,1-e_L)}{\partial q}-\frac{\partial w(Q,1-e_H)}{\partial q}\right)} \tag{3}$$

From property (2) above, the right hand side of (3) is positive. Therefore

$$\frac{\partial v\left(C,1-e_H\right)/\partial C}{\partial w\left(Q,1-e_H\right)/\partial Q} > 1$$

and by the same token

$$\frac{\partial v\left(c,1-e_H\right)/\partial c}{\partial w\left(q,1-e_H\right)/\partial q} > 1$$

We can therefore state the following result:

Proposition 1 *If condition (2) is satisfied, then the optimal second best contract is such that the perk good is overconsumed, in the sense that the marginal rate of substitution between money and the perk good is larger than the corresponding price ratio.*

In words, the optimal contract is such that the marginal utility of money is strictly larger than that of perks: the agent would, from an ex post perspective, prefer to consume less perks and more of the numeraire good. In particular, should she receive her entire compensation as a monetary wage, the quantity of perk she would buy would be smaller than the second best quantity. Therefore, *efficiency requires perks to be provided in kind, in excess of the quantity the agent would freely purchase on the market.* Not surprisingly, perks are usually luxury goods, which even wealthy executive would not buy in large quantities by themselves.

In practice, our framework encompasses a number of special cases. One is the company plane example described above. Here, the perk (access to a company plane) is a direct substitute to leisure. Even if the agent's utility is separable in leisure and money (i.e., $v(C, 1-e) = \bar{v}(C) + \bar{u}1 - e)$) - so that a higher wage does *not* increase the marginal disutility of effort - the optimal contract still entails overprovision of perks because perks, by reducing the agent's disutility of effort, lower the cost of providing adequate incentives.

Alternatively, if money and leisure are indeed complements in the agent's utility function, then any commodity that can be consumed without (too much) increasing the disutility of effort can be used as a perk, and the second best optimum entails overprovision of it. It is tempting to think of symbolic gratifications, status or positional goods in these terms. A larger office, the availability of a private chauffeur, membership of an exclusive club are signals that convey important information about the person's status within the organization.[9] To the extent that they do not increase disutility of effort (and one could actually argue that, if anything, they reduce it), they should be part of the optimal compensation package.[10]

2.2 Comparative statics

In general, moral hazard models with non separable preferences do not generate clear-cut comparative statics properties. However, two specific features of our context turn out to be very helpful. One is that perks are typically luxury goods, in the sense that the fraction of total expenditures devoted to their consumption increases with income. Secondly, perks almost always represent a small fraction of an agent's total compensation. This suggests that the complementary/substitutability effect between consumption goods and effort is probably small in magnitude; therefore, we can perform our comparative static analysis 'in the neighborhood of separability'. Technically, denoting $\Delta v(c)$ the difference $\partial v(c, 1-e_L)/\partial c - \partial v(c, 1-e_H)/\partial c$ considered in subsection 1.1, we shall assume in what follows that the absolute value of both the function $\Delta v(c)$ and its derivative $\partial \Delta v(c)/\partial c$ are small with respect to unity. Finally, we disregard the impact of consumption and leisure on risk aversion by assuming that the agent's index of relative risk aversion is constant.

In this context, we can prove two results.[11] First, when the agent's reservation utility $\bar{U}$ increases, then both his monetary wage and the amount of perks she receives increase; moreover, the fraction of total compensation paid as perks increases as well. In short, better paid agents receive proportionally more perks - a finding that is consistent with the findings of Rajan and Wulf. Secondly, perks should be larger for more productive managers, and also for managers supplying their services in more competitive markets. Thirdly, if the

[9] As noted by Rajan and Wulf, the signal is all the more credible that the total supply of such signals is limited. 'There are only so many corner offices or so many places on the corporate jet, and who gets them can signal the recipient's place in the pecking order better than cash compensation can' (Rajan and Wulf, 2006, p. 6).

[10] This intuition is well expressed by Rajan and Wulf: 'If relative standing within the firm is an important element of the utility derived from compensation (see Frank, 1985a,b), then perks can motivate far more cost-effectively than equivalent amounts of cash.' (2006, p.6).

[11] See the Appendix for a formal derivation.

severity of the moral hazard problem increases, in the sense that the probability p of a low effort being undetected is larger, then the amount Q of perks paid if the outcome is high must increase; however, the amount q paid in the alternative situation may either increase or decrease, so that the overall impact is indeterminate. The (somewhat counterintuitive) conclusion is that although perks, in our model, are used to alleviate moral hazard problems, more severe moral hazard may not result in more perks being offered on average. Our model generates a more subtle prediction - namely that when moral hazard issues are more stringent, the level of perks *rewarding good performance* should be higher.

3 Extensions

3.1 Subsidized perks

The model can readily be extended in several directions. First, the mere notion of perks implicitly relies on an exclusivity assumption, in the contract theory sense; i.e., it must be the case that the principal can monitor the agent's consumption, and in particular impose a consumption of perks larger than the amount the agent would have freely chosen. While this assumption makes sense in our specific context, it is not indispensable. In the absence of exclusivity, the optimum could still be implemented using subsidies; it would then require a lower monetary wage compensated by a subsidized access to perk goods. To see how, just note that the previous program characterizes the marginal rate of substitution between perks and the numeraire for each outcome realization. This MRS is larger than one, implying that the agent, if facing the market prices, would voluntarily purchase less perks than the second best amount. If, on the other hand, perks are subsidized so as to equate the price ratio to the second best MRS, then the agent's compensation may be paid in numeraire - she will spend the optimal amount on perk goods. Note, however, that the second best outcome requires a subsidy that varies with the outcome; in other words, the agent's bonus in case of success is partly paid by giving her access to more subsidized perks. In that sense, providing agents with subsidized meals at the firm's cafeteria can be an efficient perk.

3.2 Several perks

A second extension is related to the case when several perks goods coexist. Assume that there exist n commodities that can be used as perks, and let us disregard issues linked to complementarity/substitutability between perks by assuming the following utility function, a direct generalization of (1):

$$u\left(c,q,l\right)=v\left(c,l\right)+\sum_{i}w_{i}\left(q_{i},l\right) \tag{4}$$

If the marginal utility $\partial w_i (q_i, l) / \partial q_i$ is large enough when q_i goes to zero, the program leads to the following:

$$\frac{\partial v (C, 1 - e_H) / \partial C}{\partial w_i (Q_i, 1 - e_H) / \partial Q_i} = \frac{1 + \mu \frac{p}{P} \left(\frac{\partial v(C,1-e_L)}{\partial C} - \frac{\partial v(C,1-e_H)}{\partial C} \right)}{1 + \mu \frac{p}{P} \left(\frac{\partial w_i(Q_i,1-e_L)}{\partial Q_i} - \frac{\partial w_i(Q_i,1-e_H)}{\partial Q_i} \right)} \tag{5}$$

We conclude, again, that all perk that are substitute to leisure (or less complement to leisure than money, in the sense defined above) are overprovided at the optimum. An interesting consequence is that for any two perk commodities (i, k), we have that:

$$\frac{\partial w_k (Q_k, 1 - e_H) / \partial Q_k}{\partial w_i (Q_i, 1 - e_H) / \partial Q_i} = \frac{1 + \mu \frac{p}{P} \left(\frac{\partial w_k(Q_k,1-e_L)}{\partial Q_k} - \frac{\partial w_k(Q_k,1-e_H)}{\partial Q_k} \right)}{1 + \mu \frac{p}{P} \left(\frac{\partial w_i(Q_i,1-e_L)}{\partial Q_i} - \frac{\partial w_i(Q_i,1-e_H)}{\partial Q_i} \right)} \tag{6}$$

If commodity i is less complement to leisure than commodity k, in the sense that

$$\frac{\partial w_k (Q_k, 1 - e_L)}{\partial Q_k} - \frac{\partial w_k (Q_k, 1 - e_H)}{\partial Q_k} \geq \frac{\partial w_i (Q_i, 1 - e_L)}{\partial Q_i} - \frac{\partial w_i (Q_i, 1 - e_H)}{\partial Q_i}$$

then

$$\frac{\partial w_k (Q_k, 1 - e_L) / \partial Q_k}{\partial w_i (Q_i, 1 - e_H) / \partial Q_i} > 1$$

and commodity i is 'more overconsumed' than commodity k at the optimum, in the sense that given the total expenditures on goods k and i, the agent would like to consume more good k and less good i than provided at the second best.

3.3 State independent perks

In the previous analysis, perks are used, together with money, to reward effort. As a consequence, the level of perks depends on (and actually increases with) the outcome; the implicit assumption being that it is indeed possible to vary the level of perks in response to the agent's observed performance. A more complex but sometimes more realistic situation occurs when the level of perks is either not flexible or has to be decided ex ante, i.e. before the outcome can be observed (say, because it affects the marginal disutility of effort only if consumed when the effort is actually performed). Then the agent's consumption of perks must be the same in all states of the world; in particular, the principal cannot use variations in the amounts of perks provided to create additional incentives. However, the previous conclusions are still valid (in average) under a condition similar to inequality (2): the optimal contract still involves overprovision of perks in average, even though they cannot be used as rewards.

With state-independent provision of perks, the principal's program is:

$$\begin{aligned}
\max \quad & PY + (1-P)\,y - PC - (1-P)\,c - Q \\
s.t. \quad & Pv\,(C, 1-e_H) + (1-P)\,v\,(c, 1-e_H) + w\,(Q, 1-e_H) \\
& \quad \geq pv\,(C, 1-e_L) + (1-p)\,v\,(c, 1-e_L) + w\,(Q, 1-e_L)\,, \\
& Pv\,(C, 1-e_H) + (1-P)\,v\,(c, 1-e_H) + w\,(Q, 1-e_H) \geq \bar{U}.
\end{aligned}$$

where Q denotes the (state-independent) level of perks.

By the similar way we derived equation (3), we get

$$\begin{aligned}
MRS(C,Q) &= \frac{1+\mu[\frac{p}{P}v_c(C,1-e_L) - v_c(C,1-e_H)]}{1+\mu[w_q(Q,1-e_L) - w_q(Q,1-e_H)]}, \\
MRS(c,Q) &= \frac{1+\mu[\frac{1-p}{1-P}v_c(c,1-e_L) - v_c(c,1-e_H)]}{1+\mu[w_q(Q,1-e_L) - w_q(Q,1-e_H)]}.
\end{aligned}$$

Under the assumption that leisure and consumption of the numeraire good are complementary, but leisure and perks good are substitute (inequality (2)), the second equation above implies $MRS(c,Q) > 1$ since $\frac{1-p}{1-P} > 1$. However, it is ambiguous whether $MRS(C,Q)$ is larger or smaller than unity under the assumption in (2) since $\frac{p}{P} < 1$. In other words, the perks are over-provided when the outcome is unsuccessful, but not necessarily so when it is successful.

However, notice that

$$\begin{aligned}
& P \cdot MRS(C,Q) + (1-P) \cdot MRS(c,Q) \\
&= \frac{1+\mu([pv_c(C,1-e_L) + (1-p)v_c(c,1-e_L)] - [Pv_c(C,1-e_H) + (1-P)v_c(c,1-e_H)])}{1+\mu[w_q(Q,1-e_L) - w_q(Q,1-e_H)].}.
\end{aligned}$$

Suppose

$$\begin{aligned}
& pv_c(C,1-e_L) + (1-p)v_c(c,1-e_L)] - [Pv_c(C,1-e_H) + (1-P)v_c(c,1-e_H)] \\
& > w_q(Q,1-e_L) - w_q(Q,1-e_H),
\end{aligned}$$

i.e., perks are "in average" less complementary to leisure than the numeraire good is. Then the expected marginal rate of substitution is larger than unity. Thus we can conclude that the optimal contract still involves over-provision of perks "in average." Again, the intuition is that perks, by decreasing the marginal disutility of effort, alleviate the incentives problem.

4 Perks and Governance

In our model, lack of governance is not the main culprit for the existence of perks - moral hazard is. It does not follow, however, that perks are irrelevant for governance issues. To the extent that governance problems involve moral hazard - as they usually do - the provision of perks can, and will at the optimum, be used to alleviate these problems. We shall illustrate this claim using a simple extension of our model that closely follows the technology introduced by Peng and Röell (2006). We therefore assume that, in addition to her productive effort, the agent can influence the principal's evaluation of her performance by undertaking a set of activities, ranging from devoting time and effort to develop a network of relations to any kind of creative accounting affecting her division's books. The crucial idea is that these activities are not beneficial to the principal (for instance, they do not increase the long term value of the stock) but may increase the agent's compensation (for instance because it is based on short term performance). One may think, for instance, that the agent's reward is based on a signal (say, end of year value of the stock) which is available before the realization of the true outcome (the long term value), and is only imperfectly correlated with it. The explanation of this divergence between the agent's payoff and the firm's (long term) interest is a standard theme of the financial literature (see for instance Bolton, Scheinkman, Xiong 2006), that we do not address here.[12]

We therefore assume, following Peng and Röell, that the agent chooses two types of effort. One, denoted e as above, is productive and has a direct impact on the probability of reaching the good outcome. The other, denoted a, is a manipulation that has no impact on the long term output but may affect the interim signal on which the agent's reward is based. Formally, P, which is now interpreted as the probability of receiving the positive *signal*, is a function $P(e,a)$ of two variables; and utilities depend on both efforts, i.e. $v(C, 1-f(e,a))$ and $w(Q, 1-f(e,a))$ where $f(e,a)$, the cost of choosing the pair (e,a), is increasing in its two arguments. Note that now the principal, while still willing to promote the productive effort e, would however like to discourage manipulation, which increases expected costs without benefits. Finally, we maintain the assumption that e can take only two values, $e_H > e_L$, and we similarly assume that $a \in \{a_L, a_H\}$ with $a_H > a_L$; and we simplify the notations by posing

$$P = P(e_H, a_L), P' = P(e_H, a_H), p = P(e_L, a_L), p' = P(e_L, a_H)$$

As before, $p < P$ and $p' < P'$; and $P' \geq P$ and $p' \geq p$, expressing the fact that manipulation works.

In this setting, the quality of governance is inversely related to the differences $P' - P$ and $p' - p$. If governance is perfect, both numbers are zero, reflecting the impossibility of successful manipulations; and an increase in these differences can be interpreted as a worsening of governance.

A complete characterization of the relationship between governance and optimal level of perks is quite complex, and the conclusions generally depend on the parameters of the model. We shall simply emphasize two points, both of which go against the standard intuition that poor governance results in higher perks. First, if governance is 'good enough', manipulation is not a problem.

[12] A more complex but probably more interesting setting would involve three players - the agent, the principal and the market - and allow for richer interactions between them - for instance, the principal may sometimes collude with the agent, whose manipulations may deceive the market. This is the topic of ongoing research.

The optimal contract is then the same as before, and involves perks. In other words, even under perfect governance, we expect the optimal contract to entail perks. The second point is more surprising. Start from a context of perfect governance (in which $P' = P$ and $p' = p$), and gradually increase the severity of the manipulation problem up to a point at which the initial contract is no longer incentive compatible. Then the second best contract has to be adapted to deter incentives to manipulate. In such a case, the optimal response may consist in *reducing* the perks. In other words, not only are perks compatible with good governance, but a deterioration of governance may optimally reduce the level of perks offered by the contract.

For the sake of simplicity, we consider the case of state-independent perks. To get the intuition of the first result, note that the optimal contract now involves three incentives constraints, namely:

$$\begin{aligned} & P\left(v\left(C, 1-f\left(e_H, a_L\right)\right)+w\left(Q, 1-f\left(e_H, a_L\right)\right)\right) \\ & \quad +(1-P)\left(v\left(c, 1-f\left(e_H, a_L\right)\right)+w\left(q, 1-f\left(e_H, a_L\right)\right)\right) \\ & \geq p\left(v\left(C, 1-f\left(e_L, a_L\right)\right)+w\left(Q, 1-f\left(e_L, a_L\right)\right)\right) \\ & \quad +(1-p)\left(v\left(c, 1-f\left(e_L, a_L\right)\right)+w\left(q, 1-f\left(e_L, a_L\right)\right)\right) \end{aligned} \tag{7}$$

$$\begin{aligned} & P\left(v\left(C, 1-f\left(e_H, a_L\right)\right)+w\left(Q, 1-f\left(e_H, a_L\right)\right)\right) \\ & +(1-P)\left(v\left(c, 1-f\left(e_H, a_L\right)\right)+w\left(q, 1-f\left(e_H, a_L\right)\right)\right) \\ \geq \quad & P'\left(v\left(C, 1-f\left(e_H, a_H\right)\right)+w\left(Q, 1-f\left(e_H, a_H\right)\right)\right) \\ & +(1-P')\left(v\left(c, 1-f\left(e_H, a_H\right)\right)+w\left(q, 1-f\left(e_H, a_H\right)\right)\right) \end{aligned} \tag{8}$$

$$\begin{aligned} & P\left(v\left(C, 1-f\left(e_H, a_L\right)\right)+w\left(Q, 1-f\left(e_H, a_L\right)\right)\right) \\ & \quad +(1-P)\left(v\left(c, 1-f\left(e_H, a_L\right)\right)+w\left(q, 1-f\left(e_H, a_L\right)\right)\right) \\ & \geq p'\left(v\left(C, 1-f\left(e_L, a_H\right)\right)+w\left(Q, 1-f\left(e_L, a_H\right)\right)\right) \\ & \quad +(1-p')\left(v\left(c, 1-f\left(e_L, a_H\right)\right)+w\left(q, 1-f\left(e_L, a_H\right)\right)\right) \end{aligned} \tag{9}$$

The first constraint expresses the fact that, in the absence of manipulation, the agent prefers taking the high level of productive effort. The second implies, conversely, that when choosing the high productive effort, the agent does not try to manipulate. Finally, the last constraint states that the combination high productive effort - no manipulation is preferred over low productive effort with manipulation. Now, if $P' = P$ and $p' = p$, manipulation has a cost (for the agent) but no benefit; the agent will therefore never choose a_H. In practice, (7) implies (9) and (8) is always satisfied. By continuity, the same conclusion holds if (P', p') is 'close to' (P, p).

Regarding the second point, for the sake of brevity we simply provide an intuitive argument; a complete example is available upon request. Start from a situation in which (P', p') is 'close to' (P, p), so that the second best contract can be implemented without manipulation risk, and increase (P', p') up to the point where the second best contract is no longer implementable, because it would induce some manipulation from the agent. The contract must therefore be modified so as to reduce the incentives to manipulate, but without killing the incentives to choose the productive effort. Depending on the parameters, this may require a substitution between perks and the numeraire bonus, whereby perks are reduced while the bonus is increased.

5 Conclusion

The finance literature views perks either as productivity enhancing expenditures or as a result of poor managerial control by shareholders. Using a corporate jet to attend a business meeting may be justified because of the returns generated for the firm; but flying on the same jet to reach a vacation resort reflects a misappropriation of the firm's resources by the manager. Our paper challenges this view. We argue that complementarity between leisure and wages creates difficult incentive problems, because the bonuses or stock options that reward success increase the marginal disutility of effort. In such a context, we show that whenever there exist commodities ('perks') that are substitute to leisure (or even less complementary to leisure than money), the optimal incentive scheme involves overprovision of such commodities, in the sense that the agent should consume more of them that she would elect to, should she given a choice between money and perks at the current market prices. Such perks can profitably be used for pure incentive purposes even when they generate no productivity gains.

Clearly, our story complements other explanations. There is little doubt that, in some situations, aberrant perks may signal managerial excess and surplus misappropriation. In other cases, perks directly increase the employee's productivity,[13] or can be simply explained by a desire to exploit tax loopholes. What out results suggests, however, is that perks may deserve a more careful investigation. A crucial aspect is the impact of the corresponding consumptions on the marginal disutility of effort. Obviously, assessing complementarity or substitutability between any given perk and managerial effort is a challenging task. We believe, nevertheless, that further work is needed in this direction.

Finally, we may expect the coming years to provide some natural tests of the various explanations at stake. In a 2006 report, the Security and Exchange Commission, while refusing to define the term "perk" because of its elusiveness, recommended more restrictive disclosure rules; in particular, all perks worth more that $10,000 should be *publicly declared* by firms. The agnostic position of the regulatory agency seems quite appropriate in light of our results. More interestingly, if the traditional explanation of perks by the corporate governance literature - private appropriation is less visible, hence easier through perks than through wages, bonuses or stock options - is correct, then perks should all

[13] It is interesting to note, in particular, that most of the empirical findings of Rajan and Wulf support both our explanation and the productivity enhancement story. For instance, they find that executives are more likely to be granted access to a corporate plane is when local airports are small and poorly connected; obviously, these features increase the value of the plane both in terms of productivity gains and of substitute to leisure.

but disappear once they have to be publicly declared. If, on the contrary, perks are indeed an efficient productivity-enhancing or incentive device, then we should expect that they will mostly be maintained in the long run. From this perspective, the coming years should be quite informative.

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A Appendix

A.1 Computation note for section 2.1

Constraints (C1) and (C2) imply the following.

$$\frac{1}{\partial w\left(Q,1-e_H\right)/\partial Q}-\frac{1}{\partial v\left(C,1-e_H\right)/\partial C}=\mu\frac{p}{P}\left(\frac{\partial v\left(C,1-e_L\right)/\partial C}{\partial v\left(C,1-e_H\right)/\partial C}-\frac{\partial w\left(Q,1-e_L\right)/\partial Q}{\partial w\left(Q,1-e_H\right)/\partial Q}\right)$$

Multiplying both sides by $\partial v\left(C,1-e_H\right)/\partial C$, we derive

$$MRS(C,Q)-1=\mu\frac{p}{P}\left(\partial v\left(C,1-e_L\right)/\partial C-\frac{\partial w\left(Q,1-e_L\right)/\partial Q}{\partial w\left(Q,1-e_H\right)/\partial Q}\partial v\left(C,1-e_H\right)/\partial C\right)$$

$$=\mu\frac{p}{P}\left(\frac{\partial v\left(C,1-e_L\right)}{\partial C}-\frac{\partial v\left(C,1-e_H\right)}{\partial C}+\frac{\partial v\left(C,1-e_H\right)}{\partial C}-\frac{\partial w\left(Q,1-e_L\right)/\partial Q}{\partial w\left(Q,1-e_H\right)/\partial Q}\frac{\partial v\left(C,1-e_H\right)}{\partial C}\right)$$

$$=\mu\frac{p}{P}\left(\frac{\partial v\left(C,1-e_L\right)}{\partial C}-\frac{\partial v\left(C,1-e_H\right)}{\partial C}+MRS(C,Q)\left(\frac{\partial w\left(Q,1-e_H\right)}{\partial Q}-\frac{\partial w\left(Q,1-e_L\right)}{\partial Q}\right)\right).$$

Rearranging $MRS(C,Q)$ to the left-hand side, one obtains equation (3).

A.2 Computation note for section 2.2

Let $\Delta u_x(c,q)=u_x(c,q,e_L)-u_x(c,q,e_H)$ denote the difference between the marginal utility of good $x\in\{c,q\}$ under low and high effort, respectively. Throughout this section, we impose the following assumption.

A.1 $\left|\Delta u_x(c,q)\right|<\delta$, $\partial\left|\Delta u_x(c,q)\right|/\partial x<\delta$ for $x\in\{c,q\}$ and for all (c,q), with δ positive and sufficiently small.

This restriction is meant to capture the fact that perks represent a relatively small fraction of employees' entire compensation.

Our main comparative statics results shall rely on **A.1** only.

To get a sharper characterization, however, we shall also consider the case where the following restriction is satisfied:

A.2 $MRS(\alpha c,\alpha q)-MRS(c,q)\leq 0$ for all $\alpha>1$.

A direct implication of this assumption is that an agent who "freely" chooses his consumption of money and perks subject only to his budget constraint spends a larger fraction of his wealth in commodity q (in perks) as his wealth increases. This is perfectly consistent with the observation that goods provided as perks are usually luxurious or conspicuous goods, and very rarely inferior goods.

Lemma 2 *Assume **A.1** holds. The second best contract is unique..*

Proof. Consider an environment that differs from that studied in the previous sections only because of the presence of exogenous, purely extrinsic, uncertainty. Formally, this environment features two equilikely aggregate states: a sunny states $s = 1$ and a raining state $s = 2$. The uncertainty is resolved after the agent has exerted effort and does not affect neither preferences nor productivity so that $P(Y \mid e_H, s) = P$ and $p(Y \mid e_L, s) = p$, for $s = 1, 2$, while the agent's utility function is state independent. In this *sunspot* environment, a contract is defined as a vector $\chi = (\chi_1, \chi_2)$ with $\chi_s = (C_s, Q_s, c_s, q_s)$, so as to take into account the possibility that the agent's payment are contingent on s. Let $\pi(\chi) = (1/2)(\pi(\chi_1) + \pi(\chi_2)$ be the expected profit of $\chi = (\chi_1, \chi_2)$, when the agent exerts high effort. Let $u(\chi_s, e) = p(e)u(C_s, Q_s, e) + (1 - p(e))u(c_s, q_s, e)$ be the expected utility that an agent signing χ and exerting e obtains in state s. The contract offered by the principal in the *sunspot* environment just described maximizes $\pi(\chi) = (1/2)(\pi(\chi_1) + \pi(\chi_2))$ subject to $Eu(\chi_s, e_H) = (1/2)(Eu(\chi_1, e_H) + Eu(\chi_2, e_H)) \geq \bar{U}$ and $Eu(\chi_s, e_H) \geq Eu(\chi_s, e_L)$.

We shall now characterize first the principal's offer in the sunspot environment and then use this characterization to show that the solution of the second best program is unique. The first order condition with respect to C_s of the principal's program in the sunspot environment can be written as

$$1/\nu_{C_s}(C_s, e_H) - \mu^*[1 - (p/P)(\nu_{C_s}(C_s, e_L)/\nu_{C_s}(C_s, e_H)] - \lambda^* = 0 \text{ for } s = 1, 2 \tag{10}$$

where μ^* and λ^* are the value of the lagrangean multipliers calculated at a solution of the principal program.

Since the function appearing in the left-hand-side of this equality is increasing in C_s for all $C_s \geq 0$, any solution of (10) entails $C_1 = C_2$. By using the same argument, one verifies that $\chi_s = \bar{\chi}$ for $s = 1, 2$ for any $\chi = (\chi_1, \chi_2)$ solving the principal program. This in turn implies that $\chi = (\chi_1 = \bar{\chi}, \chi_2 = \bar{\chi})$ solves the principal program in the sunspot environment if and only if $\bar{\chi}$ solves the second best program in the environment without sunspot. This is true just because the two programs coincide under the additional restrictions $\chi_1 = \bar{\chi}$ and $\chi_2 = \bar{\chi}$. Now suppose that the second best program in the environment without sunspots has two solutions, $\tilde{\chi}'$ and $\tilde{\chi}''$. We shall prove that this is impossible by contraddiction. The result derived above implies that $\chi' = (\chi_1 = \tilde{\chi}', \chi_2 = \tilde{\chi}')$ and $\chi'' = (\chi_1 = \tilde{\chi}'', \chi_2 = \tilde{\chi}'')$ are both solutions of the principal's program in the sunspot environment. But then $\chi''' = (\chi_1 = \tilde{\chi}', \chi_2 = \tilde{\chi}'')$ must also be a solution of that program because $\pi(\chi_1 = \tilde{\chi}') = \pi(\chi_2 = \tilde{\chi}'')$ and $\chi = (\chi_1 = \tilde{\chi}', \chi_2 = \tilde{\chi}'')$ satisfies the reservation constraint and the incentive constraint of the principal program in the sunspot environment. This is because $Eu(\chi_s, e_H) = (1/2)[Eu(\chi_1, e_H) + Eu(\chi_2, e_H)] = (1/2)[\bar{U} + \bar{U}]$ and $Eu(\chi_s, e_H) \geq Eu(\chi_s, e_L)$ since $\mathrm{Eu}(\tilde{\chi}', e_H) \geq Eu(\tilde{\chi}', e_L)$ and $\mathrm{Eu}(\tilde{\chi}'', e_H) \geq Eu(\tilde{\chi}'', e_L)$. This however contraddicts the fact that one necessarily has $\chi_1 = \chi_2$ in the solution of the principal program in the sunspot environment. ■

The following lemmas allow to simplify our comparative statics analysis by taking advantage of **A.1**.

Lemma 3 *Assume* ***A.1*** *holds. For any* $z > 0$ *there is a continuous function* $f : \Re \to \Re$, $e \to f(e)$, *with* $f'(e) > 0$, *and a sufficiently small number* ε, *such that* $|u_{xe}(c, q, e)| < \varepsilon$ *for any possible* (c, q, e) *implies* $|u(c, q, e) - u(c, q, e_L) - f(e)| < z$ *for any possible* (c, q, e).

Proof. We shall show that whenever $|u_{xe}(c, q, e)| \leq \delta$, for $x \in \{c, q\}$ and δ sufficiently small, there exists an increasing function $f(\cdot)$ and two real numbers, k_c and k_q, such that $u(c, q, e) \approx u(c, q, e = 0) - f(e) + \sum_{x \in \{c,q\}} k_x u_{xe}(c, q, e)$ for all e. Clearly this relation holds for $e = 0$. Hence, **A.1** implies:

$$\begin{aligned} u(c, q, \hat{e}) &\approx u(c, q, e = 0) + u_e(c, q, e = 0)\hat{e} \\ &= u(c, q, e = 0) - f(e = 0) + \left(\frac{\partial \sum_x k_x u_{xe}(c, e = 0)}{\partial e} - f'(e = 0) \right) \hat{e} \end{aligned}$$

for all $\hat{e} \in (0, e_H]$.Since $\frac{\partial \sum_x k_x u_{xe}(c, e_L)}{\partial e} \to 0$ as $|u_{xe}(c, q, e)| \to 0$, and $f(\hat{e}) \approx f(e = 0) + v'(e = 0)\hat{e}$, we have $u(c, q, \hat{e}) \approx u(c, q, 0) - f(\hat{e})$ for sufficiently small values of $|u_{xe}(c, q, e)|$. ■

Denote $\chi = (c, q, C, Q)$ a generic contract, and

$$\chi(p, \bar{U}) \equiv (c(p, \bar{U}), q(p, \bar{U}), C(p, \bar{U}), Q(p, \bar{U}))$$

a (generic) solution of the second best program. Moreover, let $\hat{f}()$ satisfy $\left| u(c, q, e) - u(c, q, e_L) - \hat{f}(e) \right| \leq \hat{z}$, and let

$$\hat{\chi}(p, \bar{U}) \equiv (\hat{c}(p, \bar{U}), \hat{q}(p, \bar{U}), \hat{C}(p, \bar{U}), \hat{Q}(p, \bar{U}))$$

be a solution of the auxiliary program:

$$\max_{(c,q,C,Q)} P(Y - C - Q) + (1 - P)(y - c - q)$$

subjecto to

$$Pu(C, Q, e_L) + (1 - P)u(c, q, e_L) - \hat{f}(e_H) \geq pu(C, Q, e_L) + (1 - p)u(c, q, e_L) - \hat{f}(e_L)$$

and

$$Pu(C, Q, e_L) + (1 - P)u(c, q, e_L) - \hat{f}(e_H) \geq \bar{U}$$

Denote $\Pi(\chi)$ the expected profit of χ and let $U(\chi) = Pu(C, Q, e_H) + (1 - P)u(C, Q, e_H)$ and $\hat{U}(\chi) = Pu(C, Q, e_L) + (1 - P)u(C, Q, e_l) - \hat{f}(e_H)$

Assume **A.1** holds. For any $z > 0$, there is a sufficiently small number ε such that $|u_{xe}(c, q, e)| < \varepsilon$ implies that the solutions of the second best and the auxiliary program are continuous and differentiable feature the following properties:

$$\left| \chi(p, \bar{U}) - \hat{\chi}(p, \bar{U}) \right| \leq z, \quad \left| \frac{d\chi(p, \bar{U})}{d\bar{U}} d\bar{U} - \frac{d\hat{\chi}(p, \bar{U})}{d\bar{U}} d\bar{U} \right| \leq z, \quad \left| \frac{d\chi(p, \bar{U})}{dp} dp - \frac{d\hat{\chi}(p, \bar{U})}{dp} dp \right| \leq z.$$

Proof. The proof is developed in several steps.

Step 1 For any number $\bar{\delta} > 0$, and any pair of incentive compatible contracts, $\chi_1 \equiv (c_1, q_1, C_1, Q_1)$ and $\chi_2 \equiv (c_2, q_2, C_2, Q_2)$ such that $|\chi_1 - \chi_2| \geq \bar{\delta}$, there are a sufficiently small number ε and another incentive compatible contract $\tilde{\chi}$ such that

$$\Pi(\tilde{\chi}) > \max\{\Pi(\chi_1), \Pi(\chi_2)\} \text{ and } U(\tilde{\chi}) \geq \max\{U(\chi_1), U(\chi_2)\}$$

whenever $|U(\chi_1) - U(\chi_2)| < \varepsilon$, and $|\Pi(\chi_1) - \Pi(\chi_2)| < \varepsilon$.

proof Take $\tilde{\chi} \equiv (\tilde{c}, \tilde{q}, \tilde{C}, \tilde{Q})$ such that:

$$u(\tilde{c}, \tilde{q}, e_H) = (1/2)u(c_1, q_1, e_H) + (1/2)u(c_2, q_2, e_H) \tag{11}$$

$$\tilde{c} \leq (c_1 + c_2)/2; \;\; \tilde{q} \leq (q_1 + q_2)/2; \tag{12}$$

$$\tilde{C} = (C_1 + C_2)/2 + [(c_1 + c_2)/2 - \tilde{c}](1 - P)/P - t \tag{13}$$

$$\tilde{Q} = (Q_1 + Q_2)/2 + [(q_1 + q_2)/2 - \tilde{q}](1 - P)/P - t \tag{14}$$

Note that a vector $(\tilde{c}, \tilde{q})$ satisfying (**??** and **??**) exists because u is continuous and strictly concave. Moreover, $|\chi_1 - \chi_2| \geq \bar{\delta} > 0$, together with the strict concavity of u, imply that, for any $\delta \geq \bar{\delta}$, there are a positive number $t > \bar{t}(\bar{\delta}) > 0$ and a vector $(\tilde{C}, \tilde{Q})$ satisfying (**??**),(**??**) and

$$u(\tilde{C}, \tilde{Q}, e_H) - (1/2)u(C_1, Q_1, e_H) - (1/2)u(C_2, Q_2, e_H) > D > 0$$

Thus, (**??**) implies:

$$Pu(\tilde{C}, \tilde{Q}, e_H) + (1 - P)u(\tilde{c}, \tilde{q}, e_H) > \max\{U(\chi_1), U(\chi_2)\}$$

for $|U(\chi_1) - U(\chi_2)|$ sufficiently small. In addition, $\tilde{\chi}$ is incentive compatible by construction under **A.1** and satisfies $\Pi(\tilde{\chi}) > (1/2)\Pi(\chi_1) + (1/2)\Pi(\chi_2) + P\bar{t}$. For $|\Pi(\chi_1) - \Pi(\chi_2)| < \varepsilon$, with ε sufficiently small, the latter inequality implies: $\Pi(\tilde{\chi}) > \max\{\Pi(\chi_1), \Pi(\chi_2)\}$.

step 2 *For any $z > 0$, there is a sufficiently small number ε such that $|u_{xe}(c, q, e)| < \varepsilon$ implies $\left|\chi(p, \bar{U}) - \hat{\chi}(p, \bar{U})\right| \leq z$.*

proof The proof is by contraddiction. Assume that for any positive ε there exists $\bar{z} > 0$ such that $\left|\chi(p, \bar{U}) - \hat{\chi}(p, \bar{U})\right| > \bar{z}$, for any ε such that $|u_{xe}(c, q, e)| < \bar{\varepsilon}$.

The continuity of the utility functions appearing in the constraints of the second best and the auxiliary programs, together with the assumption that $|u_{xe}(c, q, e)| < \varepsilon$, have the following implication. For any $z' > 0$, there is ε small enough such that one can find two contracts $\hat{\chi}'$ and χ' with these properties: $\hat{\chi}'$ satisfies the incentive and the participation constraint of the second best program as well as the inequality $\left|\hat{\chi}' - \hat{\chi}(p, \bar{U})\right| < z'$; while χ' satisfies the incentive and the participation constraint of the auxiliary program as well as the inequality $\left|\chi' - \chi(p, \bar{U})\right| < z'$. This implies $\Pi(\hat{\chi}(p, \bar{U})) \geq \Pi(\chi(p, \bar{U})) + \delta$, hence $\Pi(\hat{\chi}') \geq \Pi(\chi(p, \bar{U})) + \delta$, with $\delta \to 0$, for $|u_{xe}(c, q, e)| \to 0$. Moreover, one has

$U(\hat{\chi}') = U(\chi(p,\bar{U})) + \delta'$ with $\delta' \to 0$ for $|u_{xe}(c,q,e)| \to 0$ because $\hat{U}(\hat{\chi}(p,\bar{U})) = U(\chi(p,\bar{U})) = \bar{U}$ and $\left|\hat{\chi}' - \hat{\chi}(p,\bar{U})\right| < z'$. Finally, as $\left|\chi(p,\bar{U}) - \hat{\chi}(p,\bar{U})\right| > \bar{z}$ one also has $\left|\chi(p,\bar{U}) - \hat{\chi}'\right| > z' > 0$. But this inequality contraddicts the statement proved in **step 1**.

step 3 *the solutions of the second best and the auxiliary program are continuous and differentiable and satisfy:* $\left|\frac{d\chi(p,\bar{U})}{d\bar{U}}d\bar{U} - \frac{d\hat{\chi}(p,\bar{U})}{d\bar{U}}d\bar{U}\right| \leq z$, $\left|\frac{d\chi(p,\bar{U})}{dp}dp - \frac{d\hat{\chi}(p,\bar{U})}{dp}dp\right| \leq z$.

proof The continuity and the differentiability of $\chi(p,\bar{U})$ and $\hat{\chi}(p,\bar{U})$ can be easily verified by using the implicit function theorem. Thus, one has: $\chi(p,\bar{U}+\Delta U) - \chi(p,\bar{U}+\Delta U) \approx \frac{d\chi(p,\bar{U})}{d\bar{U}}\Delta U$ and $\hat{\chi}(p,\bar{U}+\Delta U) - \hat{\chi}(p,\bar{U}+\Delta U) \approx \frac{d\hat{\chi}(p,\bar{U})}{d\bar{U}}\Delta U$

so that by applying the result in step 2 one obtains $\left|\frac{d\chi(p,\bar{U})}{d\bar{U}}d\bar{U} - \frac{d\hat{\chi}(p,\bar{U})}{d\bar{U}}d\bar{U}\right| \leq z$. The same argument proves $\left|\frac{d\chi(p,\bar{U})}{dp}dp - \frac{d\hat{\chi}(p,\bar{U})}{dp}dp\right| \leq z$. ■

The auxiliary program can be solved by steps. In the first step, $(c(I_y), q(I_y))$ and $(C(I_Y), Q(I_Y))$ are obtained by maximizing $u(c,q,e_L)$ subject $c+q = I$, for $I = I_y$ and $I = I_Y$, respectively. Finally, $\mathbf{I} = (I_Y, I_y)$ is obtained in a second step by solving:

$$\max_{I_Y, I_y} \quad PY + (1-P)y - (PI_Y + (1-P)I_y)$$

$$s.t. \quad P\hat{V}(I_Y) + (1-P)\hat{V}(I_y) - f(e_H) = p\hat{V}(I_Y) + (1-p)\hat{V}(I_y) - f(e_L) \tag{15}$$

$$P\hat{V}(I_Y) + (1-P)\hat{V}(I_y) - f(e_H) = \bar{U} \tag{16}$$

where $\hat{V}(I) = \max_{c,q} u(c,q)$ subject to $c+q = I$ is the agent's indirect utility function.

The two constraints of this program yields:

$$\hat{I}_Y = \hat{V}^{-1}(\bar{U} - PK) \text{ and } \hat{I}_y = \hat{V}^{-1}(\bar{U} + (1-P)K)$$

with $K = (f(e_H) - f(e_L))/(P-p)$.

An immediate corollary of Lemma is that for $|\Delta u_x(c,q)|$ sufficiently small, with $x = c, q$, $\chi(p,\bar{U})$ satisfies:

$$C(\bar{U},K) + Q(\bar{U},K) = I_Y(\bar{U},K) \approx \hat{I}_Y; \;\; c(\bar{U},K) + q((\bar{U},K) = I_y(\bar{U},K) \approx \hat{I}_y$$

A.2.1 Perks and managers' outside options

Next proposition show that better paid managers receive either larger amount of perks, or a larger fraction of their compensation in the form of perks. Let $(C(\bar{U}), Q(\bar{U}), c(\bar{U}), q(\bar{U}))$ be the second best contract parametrized by $\bar{U}$.

Proposition 4 *Assume **A.1** holds.$Q(\bar{U})$ is strictly increasing in $\bar{U}$. Moreover, under **A2** both $Q(\bar{U})/(C(\bar{U}) + Q(\bar{U}))$ and $q(\bar{U})/(c(\bar{U}) + q(\bar{U}))$ increase in $\bar{U}$*

Proof. By differentiating the optimality condition

$$MRS(C(\bar{U}), Q(\bar{U}), e_H) = \frac{v_C(C(\bar{U}), e_H)}{w_Q(Q(\bar{U}), e_H)} = \frac{1 + \mu \frac{p}{P} \Delta v_C(C(\bar{U}))}{1 + \mu \frac{p}{P} \Delta w_Q(Q(\bar{U}))}$$

in the point $(C(\bar{U}), Q(\bar{U}), c(\bar{U}), q(\bar{U}))$, and using **A.1,** one obtains:

$$\frac{dMRS(C(\bar{U}), Q(\bar{U}), e_H)}{d\bar{U}} d\bar{U} \approx$$

$$\frac{v_{CC}(C(\bar{U}))w_Q(Q(\bar{U}), e_H)\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}} - w_{QQ}(Q(\bar{U}), e_H)v_C(C(\bar{U}), e_H)\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}}{[w_Q(Q(\bar{U}, e_H)]^2} d\bar{U} \approx$$

$$\frac{[\frac{d\mu}{d\bar{U}}\frac{p}{P}\Delta v_C(C(\bar{U})) + \mu\frac{p}{P}\Delta v_{CC}(C(\bar{U}))\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}][1 + \mu\frac{p}{P}\Delta w_Q(Q(\bar{U}))] +}{[1 + \mu\frac{p}{P}\Delta w_Q(Q(\bar{U}))]^2} d\bar{U}$$
$$-\frac{[\frac{d\mu}{d\bar{U}}\frac{p}{P}\Delta w_Q(Q(\bar{U})) + \mu\frac{p}{P}\Delta w_{QQ}(Q(\bar{U}))\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}][1 + \mu\frac{p}{P}\Delta v_c(C(\bar{U}))]}{[1 + \mu\frac{p}{P}\Delta w_Q(Q(\bar{U}))]^2} d\bar{U}$$

For $\left|\Delta v_C(C(\bar{U})), \Delta w_Q(Q(\bar{U}))\right| \to 0$ and $\left|\Delta v_{CC}(C(\bar{U})), \Delta w_{QQ}(Q(\bar{U}))\right| \to 0$, the right-hand-side of this equality goes to zero. Since the utility function as well as its derivatives are continuous, there exists a continuous function $g(\varepsilon)$, with $g(0) = 0$, such that $dMRS(C(\bar{U}), Q(\bar{U}), e_H)/d\bar{U} < g(\varepsilon)$ for $\left|\Delta v_C(C(\bar{U})), \Delta w_Q(Q(\bar{U}))\right| < \varepsilon$ and $\left|\Delta v_{CC}(C(\bar{U})), \Delta w_{QQ}(Q(\bar{U}))\right| < \varepsilon$.In turn, $dMRS(C(\bar{U}), Q(\bar{U}), e_H)/d\bar{U} < g(\varepsilon)$ for ε sufficiently small, implies:

$$\left(\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}\right) / \left(\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}\right) \approx \frac{w_{QQ}(Q(\bar{U}), e_H)v_C(C(\bar{U}), e_H)}{v_{CC}(C(\bar{U}), e_H)w_Q(Q(\bar{U}), e_H)} \tag{17}$$

Since $C(\bar{U}) + Q(\bar{U}) \approx I_Y(\bar{U}) = \hat{V}^{-1}(\bar{U} - PK)$ is increasing in $\bar{U}$, either $dC(\bar{U})/d\bar{U}$ or $dQ(\bar{U})/d\bar{U}$, or both of them, must be strictly positive. As the right-hand side of (17) is positive, $dC(\bar{U})/d\bar{U}$ and $dQ(\bar{U})/d\bar{U}$ have the same sign; hence both of them are strictly positive, which implies that $Q(\bar{U})$ increases in $\bar{U}$.Moreover, **A.2** implies

$$\left.\frac{\partial\left(\frac{v_C(\alpha C)}{w_Q(\alpha Q)}\right)}{\partial\alpha}\right|_{\alpha=1} = \frac{v_{CC}(C(\bar{U}), e_H)w_Q(Q(\bar{U}), e_H))C(\bar{U}) - w_{QQ}(Q(\bar{U}), e_H)v_C(C(\bar{U}), e_H)Q(\bar{U})}{[w_Q(Q(\bar{U}), e_H)]^2} < 0.$$

Using (17) this inequality rewrites as:

$$v_{cc}(\alpha C(\bar{U}), e_H)w_q(\alpha Q(\bar{U}), e_H))[C(\bar{U}) - Q(\bar{U})\left(\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}\right) / \left(\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{d\bar{U}}\right)] < 0.$$

Hence,

$$\frac{dQ}{d\widehat{I}_Y}\left(\frac{\widehat{I}_Y(\bar{U})}{Q(\bar{U})}\right) > \frac{dC}{d\widehat{I}_Y}\left(\frac{\widehat{I}_Y(\bar{U})}{C(\bar{U})}\right),$$

so that $[Q(\bar{U})+dQ(\bar{U})]/(\widehat{I}_Y(\bar{U})+d\widehat{I}_Y) > Q(\bar{U})/\widehat{I}_Y(\bar{U})$.Finally, exactly the same logic developed in the first part of this proof implies $dq(\bar{U})/d\bar{U} > 0$, and $[q(\bar{U})+dq(\bar{U})]/(\widehat{I}_y(\bar{U})+d\widehat{I}_y) > q(\bar{U})/\widehat{I}_y(\bar{U})$ for $u_c(\alpha c,\alpha q)/u_q(\alpha c,\alpha q) < u_c(c,q)/u_q(c,q)$. ■

A.2.2 Perks and the intensity of moral hazard problem

Let $(C(p),Q(p),c(p),q(p))$ be the second best contract parametrized by p.

Proposition 5 *The second best contract satisfies the following properties:*
*(i) $Q(p)$ is strictly increasing in p; under **A.2** $Q(p)/(C(p)+Q(p))$ also increases in p.*
*(ii) $q(p)$ is strictly decreasing in p; under **A.2** $q(p)/(c(p)+q(p))$ also decreases in p.*

Proof. The first order conditions of the optimal program imply:

$$MRS(C(p),Q(p),e_H)=\frac{v_C(C(p),e_H)}{w_Q(Q(p),e_H)}=\frac{1+\mu\frac{p}{P}\Delta v_c(C(p))}{1+\mu\frac{p}{P}\Delta w_q(Q(p))}$$

Its differentiation in the point $(C(p),Q(p),c(p),q(p))$ yields:

$$\frac{\partial MRS(C(p),Q(p),e_H)}{\partial p}\approx$$

$$\frac{v_{CC}(C(p))w_Q(Q(p),e_H)\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}-w_{QQ}(Q(p),e_H)v_C(C(p),e_H)\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}}{[w_Q(Q(p),e_H)]^2}dp\approx$$

$$\frac{\left(\frac{1}{P}\right)\left[\frac{d\mu}{dp}p\Delta v_c(C(p))+\Delta u_c(C(p))\mu+\mu p\Delta u_{cc}(C(p))\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\right]\left[1+\mu\left(\frac{p}{P}\right)\Delta w_q(Q(p))\right]}{\left[1+\mu\left(\frac{p}{P}\right)\Delta w_q(Q(p))\right]^2}dp$$
$$-\frac{\left(\frac{1}{P}\right)\left[\frac{d\mu}{dp}p\Delta u_q(Q(p))+\Delta w_q(Q(p))\mu+\mu p\Delta w_{qq}(Q(p))\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\right]\left[1+\mu\left(\frac{p}{P}\right)\Delta v_c(C(p))\right]}{\left[1+\mu\left(\frac{p}{P}\right)\Delta w_q(Q(p))\right]^2}dp$$

Moreover for $|\Delta v_c(C(p)),\Delta w_q(Q(p)),|\to 0$ and $|\Delta v_{cc}(c(p)),\Delta w_{qq}(q(p))|\to 0$ the right-hand-side of this equality goes to zero. Hence, as in the previous Proposition, there exists a continuous function $g(\varepsilon)$ with $g(0)=0$ such that $dMRS(C(p),Q(p),e_H)/dp < g(\varepsilon)$ for $|\Delta v_c(C(p)),\Delta w_q(Q(p))| < \varepsilon$ and $|\Delta v_{cc}(c(p)),\Delta w_{qq}(q(p))| < \varepsilon$. Thus, for ε small enough, we have:

$$\frac{dC(p)/dp}{dQ(p)/dp}\approx\left(\frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\right)/\left(\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\right)\approx\frac{w_{qq}(Q(p),e_H)v_c(C(p),e_H)}{v_{cc}(C(p),e_H)w_q(Q(p),e_H)}\quad(18)$$

In addition, $C(p)+Q(p)=I_Y(p)\approx\hat{V}^{-1}(\bar{U}-PK(p))$ so that

$$\frac{d\widehat{I}_Y}{dp}=\frac{1}{\hat{V}_I(\hat{I}_Y)}P\frac{dK}{dp}=\frac{1}{\hat{V}_I(\hat{I}_Y)}P\left[\frac{f(e_H)-f(e_L)}{(P-p)^2}\right]>0,$$

As a consequence, $dC(p)/dp$ and/or $dQ(p)/dp$ must be strictly positive. But $dC(p)/dp$ and $dQ(p)/dp$ have the same sign as the right-hand side of (18) is positive; hence they are both strictly positive, thereby $Q(p)$ increases in p. Finally, **A.2** implies

$$\left.\frac{\partial\left(\frac{v_c(\alpha c)}{w_q(\alpha q)}\right)}{\partial\alpha}\right|_{\alpha=1} = \frac{v_{cc}(C(p),e_H)w_q(Q(p),e_H))C(p) - w_{qq}(Q(p),e_H)v_c(C(p),e_H)Q(p)}{[w_{qq}(Q(p),e_H)v_c(C(p),e_H)]^2} < 0$$

and using this equality together with (18) one obtains:

$$\frac{dQ}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\left(\frac{\widehat{I}_Y}{Q}\right)dp > \frac{dC}{d\widehat{I}_Y}\frac{d\widehat{I}_Y}{dp}\left(\frac{\widehat{I}_Y}{C}\right)dp$$

which in turn yields $[Q(p) + dQ(p)]/(I(p) + (dI/dp)dp) > Q(p)/I(p)$ since $d\widehat{I}_Y/dp > 0$.The proof of part (ii) relies on the fact that $d\widehat{I}_Y/dp < 0$, and can be developed along the same lines as that of part (i). For brevity, it is left to the reader. ■