Market Maker Pricing and Information about Prospective Order Flow

EIEF

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Goal

Use a risk averse market making model to investigate ….

- [Microstructural determinants of volatility, liquidity and serial correlation of returns]
- Impact and effectiveness of **simple** dynamic order placement strategies
- Impact of "high frequency traders"
  - Modelled as informed about future order flow
  - Potential asymmetries of information about this
- Other market design / microstructure issues

Model with Inventory Risk

Two sources of risk:

- **Fundamental risk**: innovations in underlying value of the security
  - With or without inside information
- **Order flow risk**: noisy demand impacts the ability to unwind the position effectively

Model: main features

- Repeated (OLG) model
- risk averse market makers
- extended deviations of price from fundamental value - due to **limited market participation**

Simplifying features

- **No** asymmetric information about fundamentals
- not a GE asset pricing model:
  - fundamental value is exogenous (wlog, constant),
  - no true "price discovery" role for market makers
- CARA normal market makers
  - preferences defined over end-of-period wealth marked to market
  - competitive (price takers)
- exogenous behavior of general universe of investors

Related Literature

- Noise trading, inventory and returns
  - Dennert dissertation (LSE, ca. 1992)
  - Campbell and Kyle (1993)
  - Weill (2007) matching model
  - Hendershott and Seasholes (2007)
  - Hendershott and Menkveld (2011)
  - Duffie (2010)
  - Comerton-Forde, Hendershott, Jones, Moulton and Seasholes (2011)
  - **Suominen and Rinne (2011)**
- HFT models and evidence
  - Foucault, Moinas and Biais (2011): HFT informed, adverse selection
  - Brogaard (2010, 2012): mixed evidence of anticipatory trading; HFT decreases intraday volatility
  - Kirilenko, Kyle, Samadi and Tuzun (2011): HFT exacerbated but did not cause flash crash; short holding periods of HFT
  - Agency broker complaints: HFTs reverse-engineer the data feed to identify order placers and hidden orders

The Model: Fundamentals

- Simple Lucas tree. At time t, the fundamental value v_t is the discounted NPV of expected future dividends $\{d_{t+s}: s=1,\dots,\infty\}$, discounted at rate r.
- v_t is common knowledge at time of trade t for all market participants. At time t all future dividends have an expected value of d and a variance of Σ
  - w.l.o.g. dividends are assumed to be paid out each period before trading
- This means that: $v_t = d/r \equiv v$ for all t.

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Public Order Flow

Two components in baseline model

- order flow noise that is i.i.d. normally distributed $\varepsilon_t \sim N(0,\Psi)$, ε_t and ε_s independent for all $t \neq s$,
- Responsiveness to market maker inventory so that total demand for the stock is: $u_t = \gamma z_t + \varepsilon_t$ where $\gamma \in (0,1]$.
  - z_t is market makers' inventory
  - Ideally γ should be endogenous, so response is proportional to the distance of the current market price from fundamental value – e.g. $\gamma = k\delta$ for some exogenous k
- Think of public as risk neutral long-term investors overall, but only a small fraction visits the market in any period

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Market Makers

- Market makers are myopic in the sense that they trade to maximise mean-variance utility, with collective CARA coefficient ϕ, defined over their wealth at the end of the upcoming period
  - But rational and fully aware of how the market works
  - Competitive, that is, price takers.
  - End-of-period wealth computed by marking inventory to market at the end-of-period equilibrium price.

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Solve for equilibrium

- Conjecture that the equilibrium market price can be written as: $p_t = v - \delta z_t$ for some parameter value $\delta > 0$ to be determined endogenously
  - Look for stationary solution
- Derive associated distribution of p_{t+1}
- Supply = demand at time t

$$p_t \;=\; \overline{v} - \frac{\phi\left(\Sigma+\sigma_p^2\right)}{1+r} z_t + \frac{1}{1+r}\left(\mathbf{E}_t\left[\widetilde{p}_{t+1}\right] - \overline{v}\right)$$

- Equate coefficients

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Solution: Base Case

$p_t = v - \delta z_t$ where

$$\delta \;=\; (1+\gamma)\frac{(r+\gamma+r\gamma)-\sqrt{(r+\gamma+r\gamma)^2-4\phi^2\Psi\Sigma}}{2\phi\Psi}$$

$$\sigma_p^2 \;=\; \left(\frac{\delta}{1+\gamma}\right)^2 \Psi$$

- Focusing on smallest root (most liquid market solution)

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Public order flow: alternative models

1. This talk (2,2)

   $u_t = \gamma z_t + \varepsilon_t$

   where $\gamma \in (0,1]$ and z_t is market makers' inventory
2. Better model - harder to analyse (3,5)

   $u_t = \gamma(v_t - p_t) + \varepsilon_t$

   where $\gamma > 0$

   Can talk about potential "sunshine trading" advantages of anticipatory trading
3. OK model - intermediate (2,4)

   $u_t = \gamma(v_{t-1} - p_{t-1}) + \varepsilon_t$

Last two models simplify drastically in limit towards continuous time (trading interval → 0) but then two-period order placement strategy becomes meaningless

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Multiperiod Order Placement

- Now we will split up the noise trading demand into two parts,
  - one of which (ε_t) is one-shot,
  - the other (η_t) is carried out over multiple periods: order flow shock $\pi\eta_t$ at time t followed by long-run order placement strategy:

$$\pi (1-\pi)^{t+s} \eta_t \quad \text{in period } t+s,\ s=0,1, \ldots\infty$$

- That is, each period the order flow is:

$$u_t = \gamma z_t + \varepsilon_t + \pi \textstyle\sum_{s\in(0,\infty)} (1-\pi)^{t-s} \eta_{t-s}$$

with $\varepsilon_t \sim N(0,\Psi)$ and $\eta_t \sim N(0,\Omega)$.

- Market makers know that $\pi(1-\pi)\eta_t$ is still to come the next period, $\pi(1-\pi)^2\eta_t$ the period after that, etc.

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Simpler Alternative Assumption: Two-Period Order Flow

- Order flow shock η_t is carried out over two consecutive periods t and t+1 in proportions $\pi : 1-\pi$ for some $\pi\in[0,1]$.
- That is, each period the order flow is:

$$u_t = \gamma z_t + \varepsilon_t + \pi\eta_t + (1-\pi)\eta_{t-1}$$

with $\varepsilon_t \sim N(0,\Psi)$ and $\eta_t \sim N(0,\Omega)$.

- The next period, market makers know that $(1-\pi)\eta_t$ is still to come.

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Possible Regimes

I. Market makers can distinguish one-shot and two-period order flow shocks
II. Market makers cannot distinguish the current shocks but learn about the past
    Past is perfectly revealed or not?
III. Mixed: some market makers ("HFT") can, and others cannot

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I. Omniscient Market Makers

….. At time t they know: ε_t, η_t and η_{t-1}

Price is of the form:

$$p_t = v - \delta z_t + \beta\eta_t$$

Where

$$\delta = (1+\gamma)\frac{r+\gamma+r\gamma \pm \sqrt{(r+\gamma+r\gamma)^2 - 4\Sigma\phi^2\left\{\Psi + \left(\frac{1+\pi r}{1+r}\right)^2\Omega\right\}}}{2\phi\left\{\Psi + \left(\frac{1+\pi r}{1+r}\right)^2\Omega\right\}}$$

$$\beta = \frac{1-\pi}{(1+\gamma)(1+r)}\delta$$

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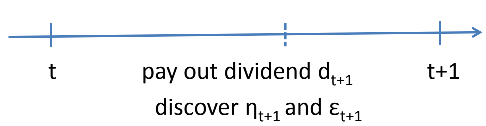
Time Line



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II. Market makers cannot distinguish different types of order flow

….. At time t they cannot distinguish ε_t from η_t (but they do know η_{t-1} for simplicity)

They observe $s_t = \varepsilon_t + \pi\eta_t$

And we conjecture $p_t = v - \delta z_t + \beta s_t$

Inventory impact δ solves:

$$\delta(1+\gamma)(r+\gamma+r\gamma) = \phi\left[\Sigma(1+\gamma)^2 + \delta^2\left[\frac{\left(\Psi + \Omega\pi\left(\frac{1+\pi r}{1+r}\right)\right)^2 + (1-\pi)^2\Omega\Psi}{\pi^2\Omega+\Psi}\right]\right]$$

which is higher than in previous regime. And

$$\lambda = \frac{\delta}{(1+\gamma)(1+r)}\frac{(1-\pi)\pi\Omega}{\pi^2\Omega+\Psi}$$

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Comparison of Regimes I and II

- Inventory has stronger impact on price in regime II
- One-shot traders are better off if market makers are informed
- Two-shot traders worse off in equilibrium?

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Questions to be addressed

- Nail down precise impact of anticipatory trading on liquidity, volatility, welfare of participants
- Model with both informed and uninformed market makers (regime III)
  - More price volatility
  - "informed" market makers (HFT) have shorter holding periods
- Examine optimal order placement
- Continuous time model more tractable
  - Requires long-run order placement model rather than two-period version

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Conclusion

- Explicit model of multi-period order flow placement
- Permits analysis of
  - anticipatory trading by market makers
  - pros and cons of slow order placement strategies
- Market makers' information about future order flow affects price formation
- Many issues still to be examined

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Abstract

Market maker pricing and information about prospective order flow

The paper considers the impact of order placement strategies on market pricing. In the model, some orders are submitted for immediate execution in their entirety, while others are part of an ongoing strategy to trickle in orders slowly so as to reduce price pressure by taking advantage of the resiliency of the market. If market makers can perfectly distinguish the two types of order flow, their prices are different than if they can only guess at the type of orders they face. A common complaint is that exchanges sell such detailed order flow information to high frequency traders that these are able to identify orders that are the tip of an iceberg, generating adverse price pressure. Gainers and losers from exchange policies to sell such information are considered, in a model of risk-averse market making in which order flow exerts price pressure despite the absence of asymmetries of information about fundamental risks.

1 Introduction: Main Features of Model:

- market making with inventory
- risk averse market makers
- temporary deviations of price from fundamental value due to limited market participation
- distinguish *fundamental risk* and *order flow risk*
- no asymmetries of information about fundamentals

Simplifying features

- not a GE asset pricing model: fundamental value is exogenous (wlog, constant), no " price discovery"
- market makers:

  myopic preferences

  competitive (price takers)
- exogenous behavior of general universe of investors
- CARA-normal

Use model to investigate

- determinants of volatility, liquidity and serial correlation of returns
- speed of inventory adjustments

- front running or anticipatory trading
- impact of potential asymmetries of information about future order flow shocks (high frequency traders)
- optimal order placement strategies

2 Baseline Model

Repeated OLG or "relay race" model (no analysis of intertemporal consumption choices)

Fundamentals (simple Lucas tree). At time t, the fundamental value v_t is the discounted NPV of expected future dividends $\{d_{t+s} : s = 1, ..., \infty\}$, discounted at rate $\beta = \frac{1}{1+r}$.

$$v_t = \mathrm{E}_t \left[\sum_{s=1}^{\infty} \beta^s d_{t+s} \right]$$

there is no asymmetric information about fundamental valuation, and so v_t is common knowledge at time of trade t for all market participants.

At time t all future dividends have an expected value of $\overline{d}$ and a variance of Σ; for simplicity dividends are assumed to be paid out each period before trading, though as periods are short (this is a microstructure model!) the underlying idea is that d_t is actually a series of time-t before-interest earnings which, if retained, earn interest r each period until they are paid out.[1] This means that:

$$v_t = \frac{\overline{d}}{r} \equiv \overline{v} \quad \text{for all } t.$$

Order flow from the general public. There is order flow noise that is i.i.d. normally distributed $\varepsilon_t \sim N(0, \Psi)$, ε_t and ε_s independent for all $t \neq s$, as well as a public order flow response proportional to the distance of the current market price fundamental value, so that total demand for the stock is:

$$u_t = \gamma z_t + \varepsilon_t \quad \text{where } \gamma \in (0, 1].$$

The underlying idea is that if all the world were present on the stock market, then $\gamma \to 1$ and the stock price would equal fundamental value at all times. But only a very limited set of risk averse members of the general public are present at any time t; and they trade in proportion to the amount by which the price deviates from fundamental value and then leave the market, to be replaced by a fresh set of public traders in the next period.

Market makers. Market makers are rational and fully aware of how the market works, but myopic in the sense that they trade to maximise mean-variance utility, with collective CARA coefficient ϕ, defined over their wealth

[1] The analysis would be identical except for notation, as one would have to distinguish cum-dividend and ex-dividendend valuations and prices.

at the end of the upcoming period. They are assumed to be competitive, that is, price takers. To compute their end-of-period wealth, they mark their stock to market using the end-of-period equilibrium price. That is, a market maker contemplating supplying an amount y_t to the market at time t who starts out with cash c_t and stock inventory z_{t-1} computes his end-of-period wealth as:

$$
\begin{aligned}
\widetilde{w}_{t+1} &= \widetilde{c}_{t+1} + \widetilde{p}_{t+1} z_t \\
&\text{where} \\
\widetilde{c}_{t+1} &= (1+r)(c_t + p_t y_t) + \widetilde{d}_{t+1} z_t \\
z_t &= z_{t-1} - y_t \\
&\text{i.e.} \\
\widetilde{w}_{t+1} &= (1+r)(c_t + p_t y_t) + \left(\widetilde{d}_{t+1} + \widetilde{p}_{t+1}\right)(z_{t-1} - y_t)
\end{aligned}
$$

where quantities that are uncertain at time of trading at time t are decorated with a $\sim$ superscript. Thus the market makers 'expected utility is

$$
\mathrm{E}_t U_{t+1} = (1+r) c_t + \left(\overline{d} + \mathrm{E}_t[\widetilde{p}_{t+1}] - (1+r) p_t\right)(z_{t-1} - y_t) - \frac{\phi}{2}(z_{t-1} - y_t)^2 \left(\Sigma + \sigma_p^2\right) \tag{1}
$$

where two conjuctures, to be verified in equilibrium, have been made regarding the uncertainty about the end-of-period price: (i) it is independent of the earnings surprise and (ii) it is stationary, that is, the per-period price volatility σ_p^2 is constant over time. This price volatility is endogenous, an unknown to be determined by solving the model.

To solve this model we first write down the solution to the market makers' supply choice problem, then impose equilibrium (demand equal to supply) and then look for a stationary solution. The market makers' FOC, choosing supply y_t to maximize their expected utility (expression 1), is:

$$
\begin{aligned}
\overline{d} + \mathrm{E}_t[\widetilde{p}_{t+1}] - (1+r) p_t &= \phi (z_{t-1} - y_t)\left(\Sigma + \sigma_p^2\right) \\
y_t &= z_{t-1} + \frac{(1+r) p_t - \left(\overline{d} + \mathrm{E}_t[\widetilde{p}_{t+1}]\right)}{\phi\left(\Sigma + \sigma_p^2\right)}
\end{aligned}
$$

The market clears when:

$$
\begin{aligned}
y_t &= u_t \quad \text{, that is} \\
z_{t-1} + \frac{(1+r) p_t - \left(\overline{d} + \mathrm{E}_t[\widetilde{p}_{t+1}]\right)}{\phi\left(\Sigma + \sigma_p^2\right)} &= u_t
\end{aligned}
$$

so that the market clearing price satisfies:

$$\frac{1+r}{\phi\left(\Sigma+\sigma_p^2\right)}\left(p_t-\overline{v}\right) = \frac{1}{\phi\left(\Sigma+\sigma_p^2\right)}\left(\mathrm{E}_t\left[\widetilde{p}_{t+1}\right]-\overline{v}\right)-\left(z_{t-1}-u_t\right) \tag{2}$$

$$p_t = \overline{v}-\frac{\phi\left(\Sigma+\sigma_p^2\right)}{1+r}z_t+\frac{1}{1+r}\left(\mathrm{E}_t\left[\widetilde{p}_{t+1}\right]-\overline{v}\right) \tag{3}$$

We will conjecture that the equilibrium market price can be written as:

$$p_t=\overline{v}-\delta z_t \tag{4}$$

for some parameter value $\delta>0$ to be determined endogenously. Then

$$\widetilde{p}_{t+1}=\overline{v}-\delta\widetilde{z}_{t+1}$$

where

$$\begin{aligned}\widetilde{z}_{t+1} &= z_t-\widetilde{u}_{t+1}=z_t-\gamma z_{t+1}-\widetilde{\varepsilon}_{t+1}\\ \widetilde{z}_{t+1} &= \frac{1}{1+\gamma}\left(z_t-\widetilde{\varepsilon}_{t+1}\right)\end{aligned}$$

so that

$$\widetilde{p}_{t+1}=\overline{v}-\frac{\delta}{1+\gamma}\left(z_t-\widetilde{\varepsilon}_{t+1}\right)$$

Thus expectations about the next market price satisfy, using (4) to substitute out for z_t:

$$\mathrm{E}_t\left[\widetilde{p}_{t+1}\right] = \overline{v}-\frac{\delta}{1+\gamma}z_t \tag{5}$$

$$\sigma_p^2 = \left(\frac{\delta}{1+\gamma}\right)^2\Psi \tag{6}$$

Inserting equations (5) and (6) into the expession for the market clearing price (3),

$$\begin{aligned}p_t &= \overline{v}-\frac{\phi\left(\Sigma+\sigma_p^2\right)}{1+r}z_t+\frac{1}{1+r}\left(\mathrm{E}_t\left[\widetilde{p}_{t+1}\right]-\overline{v}\right)\\ &= \overline{v}-\frac{\phi\left(\Sigma+\delta^2\Psi\right)}{1+r}z_t-\frac{\delta}{(1+r)(1+\gamma)}z_t\end{aligned}$$

$$p_t = \overline{v}-\frac{1}{1+r}\left[\phi\left(\Sigma+\delta^2\Psi\right)+\frac{\delta}{1+\gamma}\right]z_t \tag{7}$$

The two undetermined endogenous parameters, δ and σ_p^2, can now be derived. Equating the coefficient in equation (7) with that of the conjectured price equation (4), and substituting out for σ_p^2 using equation(6),

$$\delta = \frac{1}{1+r}\left[\phi\left(\Sigma + \left(\frac{\delta}{1+\gamma}\right)^2\Psi\right) + \frac{\delta}{1+\gamma}\right] \tag{8}$$

which yields:

$$\begin{aligned}
0 &= \phi\Psi\delta^2 - (1+\gamma)(r+\gamma+r\gamma)\delta + (1+\gamma)^2\phi\Sigma \\
\delta &= (1+\gamma)\frac{(r+\gamma+r\gamma) - \sqrt{(r+\gamma+r\gamma)^2 - 4\phi^2\Psi\Sigma}}{2\phi\Psi} \\
\sigma_p^2 &= \left(\frac{\delta}{1+\gamma}\right)^2\Psi = \frac{(r+\gamma+r\gamma)^2 - 2\phi^2\Psi\Sigma - (r+\gamma+r\gamma)\sqrt{(r+\gamma+r\gamma)^2 - 4\phi^2\Psi\Sigma}}{2\phi^2\Psi} \\
&= \frac{(r+\gamma+r\gamma)^2}{2\phi^2\Psi} - \Sigma - \frac{r+\gamma+r\gamma}{\sqrt{2\phi^2\Psi}}\sqrt{\frac{(r+\gamma+r\gamma)^2}{2\phi^2\Psi} - 2\Sigma} \text{ check this!}
\end{aligned}$$

Note:

- There are two solutions. Argue that the lower solution for δ makes sense, as limit of finite-horizon problem
- For a solution to exist, it must be that $r+\gamma+r\gamma \geq 2\phi\sqrt{\Psi\Sigma}$ if not, there is no steady state.
- As the time interval between trading periods shrinks, if we take r, γ, Σ, Ψ proportional to interval size then in the limit

$$\delta \approx \frac{r+\gamma - \sqrt{(r+\gamma)^2 - 4\phi^2\Psi\Sigma}}{2\phi\Psi}$$