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Monetary Policy, Doubts and Asset Prices

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Abstract

Asset prices and the equity premium might reflect doubts and pessimism. Introducing these features in an otherwise standard New-Keynesian model changes in a quite substantial way its normative conclusions. First, following productivity shocks, optimal policy should be very accommodative even to the point to inflate the equity premium. Second, asset-price movements improve the inflation-output trade-off so that average output can rise without increasing much average inflation. Finally, a strict inflation-targeting policy is dominated by more flexible inflation-targeting policies which increase the comovements between inflation, asset prices and output growth.

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1 Introduction

The theme of monetary policy and asset prices has been widely debated in the literature, especially after the recent financial crisis. Several authors have argued that monetary policy in the last decade was too expansionary when compared to the previous twenty years, and that a policy more aggressive toward inflation would have been beneficial to avoid the spur of the asset price bubble.[1]

In this paper, we revisit the theme of monetary policy and asset prices in a standard New-Keynesian monetary model. An important shortcoming of current models is to have counterfactual implications for the equity premium and other financial relationships. We address this issue by introducing distortions in agents' beliefs– doubts and ambiguity aversion– which enable the model to reproduce realistic values for the equity premium and the market price of risk.[2] The policy conclusions of the benchmark model change in a substantial and interesting way. The benchmark model would predict that following productivity shocks prices should be kept stable and that average output cannot rise because it is too costly to increase average inflation.[3] Our framework instead would suggest that inflation should rise following positive productivity shocks and that monetary policy should be very accommodative. The inflation-output trade-off becomes less severe, because of the interaction between asset prices and firms' price-setting behavior. The equity premium is higher under optimal policy than under a price-stability policy because equity returns are more procyclical.

Indeed, average output can rise without much increase in average inflation if the firms' discounted value of current and future costs does not move much. This can happen in our model if marginal costs are negatively related to the firms' evaluation of future payoffs through the stochastic discount factor. In fact, the impact of an increase in marginal cost on the price setting decision can be at least partially offset by a decrease in the stochastic discount factor associated with that contingency. In our framework, doubts and ambiguity aversion distort the stochastic discount factor creating an inverse relationship with long-run productivity – the more the higher the degree of ambiguity. An expansionary monetary policy leads to procyclical marginal costs and therefore can create a negative comovement between the stochastic discount factor and marginal cost. This

[1] See for instance the discussions of Poole (2007) and Taylor (2007); see section 5 in Greenspan (2010). See also *The Economist* May 18th 2010.

[2] Doubts and aversion to ambiguity are introduced using the framework of Hansen and Sargent (2005, 2007). See Barillas et al. (2009) for the ability of this framework to reproduce realistic values for the equity premium and the price of risk.

[3] For an overview of the main results of the literature see Benigno and Woodford (2005), Khan et al. (2003) and the recent review of Woodford (2009b).

more expansionary policy is optimal because it can correct for the inefficiencies due to monopolistic competition by raising average output while keeping average inflation low, thanks to the flattening of the trade-off between average inflation and average output. In fact, in the standard New-Keynesian monetary model this trade-off is too step leaving no room for improving average output.

We further show that an interest rate rule calibrated to match monetary policy under Greenspan's tenure as a chairman of the Federal Reserve achieves equilibrium allocations that resemble the ones prescribed by optimal policy in our framework. In addition, we show that Greenspan's policy is closer to optimal policy than the traditional Taylor rule. In fact, in our model, exploiting the less severe output-inflation trade-off requires a relatively more procyclical policy. However, we also find that the estimated Greenspan's policy is too accommodative even from the perspective of our model, even if less so than from the perspective of standard New-Keynesian models.

The closest paper to our work is Karantounias (2009) which analyzes a Ramsey problem but in the optimal taxation literature where, like in our model, the private sector distrusts the probability distribution of the model while the government fully trusts it. Beside the different focus of the two economic applications, the other subtle difference is in the approximation method. Whereas Karantounias (2009) approximates around the stochastic no-distrust case for small deviations of the parameter identifying the dimension of the set of nearby model, we approximate around a deterministic steady state allowing for even large deviations of the same parameter while bounding the maximum amplitude of the shocks.

Woodford (2009a) studies an optimal monetary policy problem in which the monetary policymaker trusts its own model but not its knowledge of the private agents' beliefs. In our context, it is just the private sector which has doubts on the true model whereas the policymaker is fully knowledgeable also with respect to the doubts of the private sector. Moreover, Woodford (2009a) uses a New-Keynesian model where distorted beliefs are introduced in an already approximated linear-quadratic environment with the consequence that his model cannot be considered as an approximation to a general equilibrium model of optimal monetary policy under distorted beliefs.[4] Both issues explain why in his context, in contrast to our results, the optimal stabilization policy following productivity shocks is to keep prices stable no matter what is the degree of distrust that the agents might have. Dupor (2005) analyzes optimal monetary policy in a New-Keynesian model in which only

[4]Indeed, in his framework distorted beliefs should not appear in a first-order approximation of the AS equation–as it is instead assumed. Moreover, beliefs will affect second-order terms and therefore the construction of the micro-founded quadratic loss function unless the approximation is taken around a non-distorted steady state.

the investment decisions are distorted by an *ad hoc* irrational expectational shocks. In our framework, the distortions in the beliefs are instead the result of the aversion to model mis-specification on the side of households, which also affects in an important way the intertemporal pricing decisions of the firms on top of the investment decisions.

There are several other papers that have formulated optimal monetary policy in *ad hoc* linear-quadratic framework where the other main difference with respect to our work is that the monetary policymaker distrusts the true probability distribution and the private-sector expectations are aligned with that distrust.[5] We, instead, take a normative perspective from the point of view of a fully knowledgeable policymaker who knows the true probability distribution and understands that the private sector distrusts it.

Another related strand of literature is that on the interaction between monetary policy rules and asset prices. Bernanke and Gertler (2001) have studied whether monetary policy should react to asset prices in order to stabilize fluctuations driven both by fundamental or speculative reasons. A distinctive feature of their model is the role of firms' balance sheets in the transmission mechanism of asset-price movements to the aggregate variables. They show that price stability is optimal in response to both productivity and non-fundamental shocks. An interest-rate rule aggressive with respect to inflation can approximate well the optimal policy. Instead, Cecchetti et al. (2000) finds that conditional on the non-fundamental shock, the interest rate rule should also react to asset prices. Within the class of flexible-inflation targeting rules, we find that the optimal simple rule should move from a strict inflation-targeting policy, when there are no doubts, to a more flexible inflation targeting policy, which also includes output growth and asset-price inflation, when doubts rise. However, similar to Bernanke and Gertler (2001), we find that including asset-price inflation as target does not improve much average welfare.

The structure of the paper is the following. Section 2 discusses model uncertainty. Section 3 presents the model. Section 4 characterizes the optimal policy. Section 5 studies the implementation of the optimal policy through interest-rate rules. Section 6 presents some special cases.

2 Model Uncertainty

We characterize model uncertainty as an environment in which agents are endowed with some probability distribution –the "reference" probability distribution– but they are not sure that it is in fact the true data-generating one, and might instead act using a nearby

[5] See the papers cited in Ellison and Sargent (2009) and among others Dennis et al. (2009), Giannoni (2002), Leitemo and Soderstrom (2008), Rudebusch (2001).

distorted "subjective" probability distribution.

Consider a generic state of nature s_t at time t and define s^t as the history $s^t \equiv [s_t, s_{t-1}, ..., s_0]$. Let $\pi(s^t)$ be the "approximating" or "reference" probability measure on histories s^t, that the agents are endowed with. Decision-makers may seek a different probability measure, a "subjective" one, denoted by $\tilde{\pi}(s^t)$ which is absolutely continuous with respect to the "approximating" measure. Absolute continuity is obtained by using the Radon-Nykodym derivative, which converts the reference measure into the subjective one.[6] First, the two probability measures agree on which events have zero probability. Second, there exists a non-negative martingale $G(s^t)$ with the property

$$E(G_t) \equiv \sum_{s^t} G(s^t)\pi(s^t) = 1 \tag{1}$$

such that, for a generic random variable $X(s^t)$,

$$\tilde{E}(X_t) \equiv \sum_{s^t} \tilde{\pi}(s^t)X(s^t) = \sum_{s^t} G(s^t)\pi(s^t)X(s^t) \equiv E(G_t X_t) \tag{2}$$

in which we have defined $E(\cdot)$ and $\tilde{E}(\cdot)$ the expectation operators under the "reference" and "subjective" probability measures, respectively. Specifically, $G(s^t)$ is a probability measure, equivalent to the ratio $\tilde{\pi}(s^t)/\pi(s^t)$, that allows a change of measure from the "reference" to the "subjective" measure.

Moreover, the martingale-assumption on G_t implies

$$E(G_{t+1}X_t) = E(G_t X_t).$$

with its increment $g(s_{t+1}|s^t)$ given by

$$g(s_{t+1}|s^t) \equiv \frac{G(s^{t+1})}{G(s^t)},$$

with the property $E_t g_{t+1} = 1$. It follows that $g(s_{t+1}|s^t)$ is equivalent to the likelihood ratio $\tilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$, and acts as a change of measure in conditional probabilities. High values of $g(s_{t+1}|s^t)$ imply that the decision-makers assign a higher subjective probability to state s_{t+1} conditional on history s^t.

For each random variable X_{t+1}, the martingale increment g_{t+1} defines a mapping

[6] This way of constructing subjective probability measures is borrowed from Hansen and Sargent (2005, 2007).

between the conditional expectations under the two measures:

$$\tilde{E}_t(X_{t+1}) = E_t(g_{t+1}X_{t+1}), \tag{3}$$

in which $E_t(\cdot)$ and $\tilde{E}_t(\cdot)$ denote the conditional-expectation operators.

As in Hansen and Sargent (2005), we use conditional relative entropy as a measure of the divergence between the "reference" and "subjective" probabilities,

$$E_t(g_{t+1} \ln g_{t+1}),$$

which approximately measures the variance of the distortions in the beliefs. When there are in fact no distortions this measure is zero: in this case, indeed, $g(s_{t+1}|s^t) = 1$ for each s_{t+1}. In particular, since we are going to work with a dynamic model, in what follows, it is more appropriate to exploit the discounted version of conditional relative entropy discussed in Hansen and Sargent (2005)

$$\eta_{t_0} \equiv E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t E_t(g_{t+1} \ln g_{t+1}) \right\}, \tag{4}$$

where $0 < \beta < 1$. A high value of entropy can be interpreted as a very large divergence between the "subjective" and the "reference" beliefs. On the contrary a low value of entropy implies beliefs that are not too distorted or different from the reference model.

3 Model

3.1 Households

We consider a closed-economy model with a continuum of firms and households. Households have doubts about the true probability distribution. As discussed in the previous section, we assume that households are endowed with a "reference" probability distribution but act using a distorted nearby subjective probability distribution. Therefore, the representative household has a subjective expected lifetime utility given by

$$\tilde{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(C_t, L_t) \tag{5}$$

which can be equivalently written in terms of the "reference" expectation operator

$$E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t U(C_t, L_t)$$

with $G_{t_0} = 1$ and where β is the intertemporal discount factor, with $0 < \beta < 1$; C_t is a Dixit-Stiglitz aggregator of the continuum of consumption goods produced in the economy

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\theta}{\theta-1}} dj \right]^{\frac{\theta-1}{\theta}}$$

where θ, with $\theta > 0$, is the elasticity of substitution across the consumption goods and $c_t(j)$ is the consumption of the individual good j; L_t is leisure.

Households are subject to a flow budget constraint of the form

$$W_t N_t + P_t^k K_t + x_{t-1} (Q_t + D_t) = x_t Q_t + P_t (C_t + I_t) + T_t, \tag{6}$$

where W_t denotes the nominal wage received in a common labor market; N_t is labor (notice that $N_t + L_t = 1$); P_t^k represents the nominal rental rate of capital, K_t, which is rented in a common market to all firms operating in the economy; x_t is a vector of financial assets held at time t, Q_t the vector of prices while D_t the vector of dividends; P_t is the consumption-based price index given by

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

where $P_t(j)$ is the price of the individual good j. Finally T_t represents government's lump-sum taxes, and I_t the real resource needed, in terms of units of the consumption good, to increase the household's holdings of capital stock. Given K_t and I_t, next-period capital stock follows from

$$K_{t+1} = \left(1 - \delta - \phi \left(\frac{I_t}{K_t} \right) \right) K_t + I_t, \tag{7}$$

where δ, with $0 < \delta < 1$, represents the depreciation rate and $\phi(\cdot)$ is a convex function of the investment-to-capital ratio. The convexity of the adjustment-cost function captures the idea that is less costly to change the capital stock slowly. It implies that the value of installed capital in terms of consumption varies over the business cycle, therefore the model implies a non-trivial dynamic for the Tobin's q.

Households maximize expected utility (5) by choosing the sequences of consumption, capital, leisure and portfolio holdings under the flow budget constraint (6), the law of accumulation of capital (7) and an appropriate transversality condition. Standard optimality conditions imply the equalization of the marginal rate of substitution between consumption and labor to the real wage

$$\frac{U_l(C_t, L_t)}{U_c(C_t, L_t)} = \frac{W_t}{P_t}. \tag{8}$$

The first-order conditions with respect to asset holdings imply the standard orthogonality condition between the stochastic discount factor and the asset return

$$\tilde{E}_t\{M_{t,t+1}R^j_{t+1}\} = 1 \tag{9}$$

where $M_{t,t+1}$ is the nominal stochastic discount factor between period t and $t+1$ defined by

$$M_{t,t+1} \equiv \frac{U_c(C_{t+1}, L_{t+1})}{U_c(C_t, L_t)} \frac{P_t}{P_{t+1}} \tag{10}$$

and R^j_{t+1} is the one-period nominal return on a generic asset j given by $R^j_{t+1} \equiv (Q^j_{t+1} + D^j_{t+1})/Q^j_t$. Notice that (9) can be written in terms of the "reference" expectation operator

$$\tilde{E}_t\{M_{t,t+1}R^j_{t+1}\} = E_t\{g_{t+1}M_{t,t+1}R^j_{t+1}\} = 1. \tag{11}$$

Moreover, by defining with $m_{t,t+1}$ the real stochastic discount factor as $m_{t,t+1} = M_{t,t+1}P_{t+1}/P_t$, the above condition can be written equivalently as

$$\tilde{E}_t\{m_{t,t+1}r^j_{t+1}\} = E_t\{g_{t+1}m_{t,t+1}r^j_{t+1}\} = 1,$$

where r^j_{t+1} is the real return on the generic asset j, defined by $r^j_{t+1} = R^j_{t+1}P_t/P_{t+1}$.

The optimality condition with respect to capital can be also written in terms of an orthogonality condition of the form

$$\tilde{E}_t\{m_{t,t+1}r^K_{t+1}\} = 1 \tag{12}$$

where the real return on capital is defined by

$$r^K_{t+1} \equiv \frac{1}{q_t}\frac{P^k_{t+1}}{P_{t+1}} + \left[1 - \delta - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right]\frac{q_{t+1}}{q_t} \tag{13}$$

and in particular q_t denotes the model Tobin's q given by

$$q_t = \frac{1}{1 - \phi'\left(\frac{I_t}{K_t}\right)}. \tag{14}$$

Tobin's q measures the consumption cost of a marginal unit of capital and is increasing with the investment-to-capital ratio. The return on capital, described in (13), is given by two components: the first one captures the return on renting capital to firms in the next period, while the second component captures the benefits of additional units of capital in building up capital stocks for the future rental markets.

3.2 Distorted beliefs

Households doubt the reference probability distribution and surround it with a set of nearby distorted beliefs. In this set, they choose the worst-case probability distribution to guide their choices. Following the robust-control literature of Hansen and Sargent (2005), the worst-case distribution is chosen by the decision-maker in the same way as if there is an "evil" agent which seeks to minimize the utility of the decision-maker under the entropy constraint (4). The latter defines the size of the set of alternative models, and imposes a bound on the allowed divergence between the distorted and the approximating measures. In a more formal way, the beliefs' distortion $\{g_t\}$ is chosen to minimize

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t U(C_t, L_t)\right\},$$

under the entropy constraint

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \beta E_t(g_{t+1} \ln g_{t+1})\right\} \leq \Phi,$$

and the restrictions given by the martingale assumption on G_t:

$$G_{t+1} = g_{t+1} G_t \tag{15}$$

$$E_t g_{t+1} = 1. \tag{16}$$

The parameter Φ in the entropy constraint imposes an upper-bound on the divergence between the distorted and the "reference" beliefs. The higher Φ, the more afraid of mis-specification the agent is, because a higher Φ allows the "evil" agent to choose larger distortions.

Hansen and Sargent (2005) propose an alternative formulation of this problem in which the entropy constraint is added to the utility of the agent to form a modified objective function

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_tU(C_t,L_t)\right\}+\kappa E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\beta E_t(g_{t+1}\ln g_{t+1})\right\}, \tag{17}$$

where $\kappa > 0$ is a penalty parameter on discounted entropy.

The problem of the "evil" agent, therefore, becomes that of choosing the path $\{g_t\}$ to minimize (17) under the constraints (15) and (16). Higher values of κ imply less fear of model mis-specification, because the "evil" agent is penalized more by raising entropy when it minimizes the utility of the decision-maker. When κ goes to infinity, the optimal choice of the "evil" agent is to set $g_{t+1} = 1$ at all times, meaning that the optimal distortion is zero: the rational-expectation equilibrium is nested as a special case.

We assume that the utility function is log in both arguments and given by $U(C_t, L_t) = \ln C_t + \eta \ln L_t$ where η is a parameter such that $\eta > 0$. As discussed in the literature, among others by Barillas et al (2009) and Karantounias (2009), the solution of the above minimization problem implies a transformation of the original utility function (17) into a non-expected recursive utility function of the form

$$V_t = (C_tL_t^{\eta})^{1-\beta}[E_t(V_{t+1})^{1-\psi}]^{\frac{\beta}{1-\psi}}, \tag{18}$$

where the coefficient ψ is related to κ through the following equation

$$\psi = 1 + \frac{1}{\kappa(1-\beta)}$$

showing that $\psi \geq 1$. In particular, $\psi = 1$ corresponds to the case of no model uncertainty.[7]

A further implication of the above minimization problem is that the martingale increment g_{t+1} can be written in terms of the non-expected recursive utility as

$$g_{t+1} = \frac{V_{t+1}^{1-\psi}}{E_tV_{t+1}{}^{1-\psi}}. \tag{19}$$

[7] This risk-adjusted utility function coincides with that of the preferences described in Kreps and Porteus (1978) and Epstein and Zin (1989). In that context, ψ represents the risk-aversion coefficient, while in our framework ψ is a measure of the degree of ambiguity aversion. As it will be clear in the next sections, the two environments imply a completely different characterization of the optimal policy.

3.3 Firms

There is a continuum of firms of measure one producing the respective consumption goods using a constant-return-to-scale technology given by

$$Y_t(j) = (K_t^j)^\alpha (A_t N_t^j)^{1-\alpha} \tag{20}$$

for each generic firm j where A_t represents a common labor-productivity shifter and α, with $0 < \alpha < 1$, is the capital share. Given the Dixit-Stiglitz aggregator, a generic firm j faces the following demand

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$$

where total output, Y_t, is equal to consumption and investment

$$Y_t = C_t + I_t. \tag{21}$$

Households own firms which distribute profits in the forms of dividends. Given (11), the value of a generic firm j is given by

$$Q_t^j = \tilde{E}_t\{M_{t,t+1}(D_{t+1}^j + Q_{t+1}^j)\} \tag{22}$$

where nominal dividends are defined as

$$D_t^j = P_t(j)Y_t(j) - W_t N_t^j - P_t^k K_t^j. \tag{23}$$

Given (22) and (23), the nominal value of a generic firm j *cum* current dividend is given by

$$Q_t^j + D_t^j = \tilde{E}_t\left\{\sum_{T=t}^{\infty} M_{t,T}[P_T(j)Y_T(j) - W_T N_T^j - P_T^k K_T^j\right\}$$

where $M_{t,t} = 1$.

We assume that firms choose prices, capital and labor to maximize the firm's value *cum* current dividend. In particular, cost minimization under the production function (20) implies that total costs are linear in current output

$$W_t N_t^j + P_t^k K_t^j = \left(\frac{W_t}{A_t(1-\alpha)}\right)^{1-\alpha} \left(\frac{P_t^k}{\alpha}\right)^{\alpha} Y_t(j)$$

and that the capital-to-labor ratio is not firm specific

$$\frac{K_t^j}{N_t^j} = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t^k}. \tag{24}$$

We assume that firms are subject to price rigidities as in the Calvo's mechanism. In particular, at each point in time, firms face a constant probability $(1-\gamma)$, with $0 < \gamma < 1$, of adjusting their price which is independent of the last time prices were re-set. Firms that can adjust their price set them by maximizing the present-discounted value of the firm *cum* current dividend considering that prices set at time t will last until a future time T with probability γ^{T-t}. The objective function can be written as

$$\tilde{E}_t \left\{ \sum_{T=t}^{\infty} \gamma^{T-t} M_{t,T} \left[P_t(j) Y_{t,T}(j) - \left(\frac{W_T}{A_T(1-\alpha)} \right)^{1-\alpha} \left(\frac{P_T^k}{\alpha} \right)^{\alpha} Y_{t,T}(j) \right] \right\} \tag{25}$$

where we have defined

$$Y_{t,T}(j) = \left(\frac{P_t(j)}{P_T} \right)^{-\theta} Y_T.$$

Notice that firms's pricing decisions, for their forward-looking nature, are influenced by the distorted beliefs through the distorted expectation operator. The optimal price decision, P_t^*, which is common to all firms that can adjust their price, implies that

$$\frac{P_t^*}{P_t} = \frac{Z_t}{F_t} \tag{26}$$

where Z_t is given by the following expression

$$Z_t \equiv \mu \tilde{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\gamma)^{T-t} U_c(C_T, L_T) \left(\frac{W_T}{A_T(1-\alpha)} \right)^{1-\alpha} \left(\frac{P_T^k}{\alpha} \right)^{\alpha} Y_T \right\} \tag{27}$$

in which we have defined the mark-up as $\mu \equiv \theta/(1-\theta)$.

Moreover F_t, in equation (26), is given by the following expression

$$F_t \equiv \tilde{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\gamma)^{T-t} U_c(C_T, L_T) Y_T \right\}. \tag{28}$$

Finally, a further implication of the Calvo's mechanism is the following law of motion for the aggregate price level as a function of the past aggregate price level and the optimal price P_t^* chosen by the firms that can reset their price

$$P_t^{1-\theta} = (1-\gamma)P_t^{*1-\theta} + \gamma P_{t-1}^{1-\theta}.$$

We can use the above law of motion to write (26) as

$$\frac{1-\gamma\pi_t^{\theta-1}}{1-\gamma} = \left(\frac{F_t}{Z_t}\right)^{\theta-1} \tag{29}$$

in which the gross inflation rates is given by $\pi_t = P_t/P_{t-1}$.

3.4 Equilibrium

In equilibrium aggregate output is used for consumption and investment as in (21). Financial market equilibrium requires that households hold all the outstanding equity shares and that all the other assets are in zero net supply.

Capital and labor markets are also in equilibrium

$$K_t = \int_0^1 K_t^j dj$$

$$N_t = \int_0^1 N_t^j dj.$$

In particular, equilibrium in the labor market implies

$$N_t = \int_0^1 N_t^j dj = \frac{1}{A_t^{1-\alpha}}\left(\frac{N_t}{K_t}\right)^{\alpha} Y_t\Delta_t \tag{30}$$

where Δ_t is a measure of price dispersion defined by

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta} di,$$

which follows the law of motion

$$\Delta_t = \gamma\pi_t^{\theta}\Delta_{t-1} + (1-\gamma)\left(\frac{1-\gamma\pi_t^{\theta-1}}{1-\gamma}\right)^{\frac{\theta}{\theta-1}}. \tag{31}$$

Finally, lump-sum taxes are adjusted to balance revenues and costs for the government in each period.

Given the processes for the stochastic disturbances $\{A_t\}$, initial conditions (Δ_{t_0-1},

K_{t_0-1}) and a monetary policy rule, an equilibrium is an allocation of quantities and prices $\{C_t, Y_t. K_t, N_t, I_t, F_t, Z_t, P_t, P_t^k, W_t, q_t, \Delta_t, g_t, V_t\}$ such that equations (7), (8), (12), (14), (18), (19), (21), (24), (27), (28), (29), (30), (31) hold, considering the definitions of the following variables $M_{t,t+1}$, R_t^k, L_t, which are given in the text, and considering that the distorted expectation operator is related to the reference expectation operator through (3).

4 Optimal policy

In this section, we address the analysis of optimal policy from a normative perspective. Indeed, the optimality criterion is taken from the view of a policymaker who understands that the "reference" model is the true model and trusts it. The policymaker can recognize the distortions in the beliefs of the private sector through the Arrow-Debreu prices and manipulate them in order to achieve a higher welfare. Interestingly, even a knowledgeable and "intelligent" policymaker, who is sure about the reference probability distribution, might not necessarily desire to reduce the doubts of the private sector and instead might exploit them, and perhaps amplify them, in order to correct other distortions.

We assume that the policymaker of our normative exercise maximizes

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(C_t, L_t) \tag{32}$$

where expectations are taken under the non-distorted probability distribution. In particular, we are interested in characterizing the optimal policy under commitment. The policymaker seeks to maximize (32) by choosing the sequences $\{C_t, Y_t. K_t, N_t, I_t, F_t, Z_t, P_t, P_t^k, W_t, q_t, \Delta_t, g_t, V_t\}$ under the constraints (7), (8), (12), (14), (18), (19), (21), (24), (27), (28), (29), (30), (31) given the processes for the stochastic disturbance $\{A_t\}$ and initial conditions (Δ_{t_0-1}, K_{t_0-1}), considering the definitions of the variables $M_{t,t+1}$ and R_t^k in (10) and (13), given the relationship between leisure and labor $L_t = 1 - N_t$ and considering that the distorted expectation operator is related to the "reference" expectation operator through (3).[8] This Ramsey optimal policy problem is clearly time-inconsistent

[8]Notice that even though the households' and firms' reaction functions are equivalent to those that can be obtained by assuming that the preferences of the households are of the Epstein-Zin-Kreps-Porteus form, the optimal policy problem is not equivalent to an optimal policy problem in which preferences of households are of the Epstein-Zin-Kreps-Porteus form. Indeed, in the latter case, the objective of a Ramsey policymaker would coincide with the preference of the households and therefore with the Epstein-Zin-Kreps-Porteus preferences and not with (32). See Levin et al. (2008) for an analysis of optimal policy with Epstein-Zin-Kreps-Porteus preferences.

because of the presence of forward-looking constraints (12), (18), (27), (28), therefore it cannot be written in a recursive form. This is not really an issue if we could solve the optimization problem in a non-linear way. Instead we proceed with approximation methods which apply to stationary optimization problems. To this end, we analyze a commitment from a timeless perspective where additional constraints are added at time t_0. These commitments on the variables F_{t_0}, Z_{t_0}, V_{t_0} and $H_{t_0} \equiv M_{t_0-1,t_0} R^k_{t_0}$ are of the same forms as the future constraints to which the Ramsey policymaker is already committing to and are such to allow the problem to be written in a recursive way, as discussed among others by Benigno and Woodford (2007).

Our solution method is to consider the first-order conditions of the optimal policy problem under this stronger form of commitment. First, we compute the optimal policy in absence of uncertainty and in particular under no model uncertainty. In this deterministic steady state, the optimal policy implies zero inflation, which is a result in line with other analyses of optimal monetary policy under timeless perspective, discussed in Benigno and Woodford (2005). Our framework generalizes those results to a model with capital accumulation. As a second step, we take a first-order approximation of the non-linear stochastic first-order conditions of the optimal policy problem (discussed above) around the deterministic steady state by considering small perturbations of the shocks around their deterministic path. We solve the resulting system of linear stochastic difference equations using standard methods in order to characterize the optimal policy for the endogenous variables as a linear function of the state variables.

We calibrate the structural parameters of the model consistently with existing results in the macroeconomic literature. In particular, following Christiano, Eichenbaum and Evans (2005), we set $\alpha = 0.36$ which corresponds to a steady state share of capital income equal to roughly 36 percent. We set $\delta = 0.025$, which implies a rate of capital depreciation equal to 10 percent at annual rates. This value of δ is roughly equal to the estimates reported in Christiano and Eichenbaum (1992). In addition, we set the coefficient determining demand elasticity with respect to prices, θ, equal to 6, implying a steady state price mark-up of 20 percent.[9] We choose $\eta = 0.45$ to match a steady state Frisch elasticity of labor supply of 1.8, as estimated by Smets and Wouters (2007) on U.S. data. We set $\gamma = 0.6$ to match the frequency of price adjustment estimated by Klenow and Kryvtsov (2008) and Christiano, Eichenbaum and Evans (2005). The discount factor is set equal to $\beta = 0.99$, implying an average real interest rate of about one percent at values of ψ consistent with observed equity premium.[10] Following Jermann (1998), we

[9] Similar values are obtained in Smets and Wouters (2007).

[10] These values are computed under the assumption that monetary policy follows a Taylor rule, rep-

set the second-derivative of the adjustment-cost function $\phi(\cdot)$ evaluated at the steady state in such a way that $1/\bar{\phi}'' = 0.25$, which corresponds to the steady-state elasticity of the investment-to-capital ratio with respect to Tobin's q. We assume the following random-walk process for productivity

$$\log(A_{t+1}) = \zeta + \log(A_t) + \varepsilon_{t+1},$$

where ε_{t+1} has zero mean and standard deviation σ, and ζ is a drift in technology. We assume $\sigma = 0.012$ and $\zeta = 0.004$ to match respectively the volatility and the mean of U.S. quarterly total factor productivity estimated by Fernald (2008). The model is consistent with a balanced-growth path and therefore we can obtain a stationary representation by re-scaling the appropriate variables through the level of productivity. We study optimal policy for different values of the parameter $\psi \in \{1, 25, 50, 100, 150\}$.[11] In particular, $\psi = 1$ represents the benchmark model of rational expectations, while $\psi = 150$ is the degree of model uncertainty at which our model matches the average U.S. equity premium of 5.5% per year, as estimated by Fama and French (2002).[12]

We study optimal policy under different degrees of model uncertainty, which corresponds to different values of the parameter ψ. In particular, Figures 1 and 2 show the impulse responses of selected variables to a unitary shock to technology.

The case $\psi = 1$ corresponds to the benchmark model of rational expectations. As it is well known, price stability, and therefore the flexible-price allocation, is the optimal policy.[13] Consumption and output steadily increase towards their new higher steady-state levels. The real and nominal interest rates rise on impact and decline steadily to sustain the increase over time in consumption. The return on capital and the Tobin's q increase on impact and therefore investment.

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resentative of actual U.S. policy: $i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_x x_t\right]$, where x_t is the output gap. All variables are expressed in log-deviations from the steady state, and parameters are set to standard levels: $\rho_i = 0.8$, $\phi_\pi = 1.5$ and $\phi_x = 0.5$.

[11] A value of ψ equal to 150 is not necessarily too high or too low value as explained in Barillas et al. (2009), unless is related to the detection error probability. The detection error probability represents a weighed probability that a likelihood-ratio test between the "reference" and the "distorted" model will select the wrong model. Low values corresponds to alternative models that are "easy to detect". Barillas et al. (2006) consider that detection error probabilities around 0.1 still correspond to alternative models which are "difficult to detect". In our case the detection error probability associated with the "reference" and the "distorted" models is 0.45 when $\psi = 150$ and the monetary policy follows the Taylor rule. Therefore, we are confident that at this value of ψ the two models are difficult to distinguish and therefore doubts remain.

[12] The 5.5% equity premium is obtained under a standard Taylor-rule which requires the nominal interest rate to evolve according $i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_x x_t\right]$, where x_t is the (log) output-gap and π_t inflation. Parameters are set to standard levels: $\rho_i = 0.8$, $\phi_\pi = 1.5$ and $\phi_x = 0.5$.

[13] See Woodford (2009b) for a discussion of optimal policy in this case.

With model uncertainty and under optimal policy, the impulse responses change quite substantially and the more the higher the degree of model uncertainty. The optimal policy becomes very accommodative. Inflation increases on impact and steadily declines toward zero. The increase is higher the higher is the degree of model uncertainty. Nominal interest rates become more volatile: first, they decrease and afterward they rise. In the short run, the real rate now falls; consumption and output increase on impact even to overshoot their long-run levels. The Tobin's q jumps at higher levels leading to a larger change in investment. As ψ increases, optimal policy becomes more and more accommodative to the technology shock. Moreover, the higher ψ is, the higher is the volatility of the return on equity and capital and the price of equity and capital. For instance, after a 1% increase in total factor productivity, equity return and Tobin's q increase on impact by 0.6% and 0.12 %, respectively, if $\psi = 1$, while they jump to 1.05% and 0.6% if $\psi = 150$.

To sum up, optimal policy implies a more pro-cyclical response of inflation which "over - accommodates" the technology shock. Such an increase in inflation is accompanied by an increase in the volatility of quantity variables, such as output, investment and consumption, as well as in the volatility of asset prices, as the Tobin's q, equity and capital returns, nominal and real rates. The larger the degree of distortion in beliefs, the larger the departure of optimal policy from price stability.

4.1 Why does model uncertainty matter for optimal policy?

Why is it optimal to "over-accommodate" the technology shock? The benchmark model with no model uncertainty features two distortions: on the one side sticky and staggered prices and on the other side the monopolistic competition in the goods market. In particular, the frictions in the price adjustment can produce real losses because inflation generates price dispersion and therefore an inefficient allocation of resources among goods that are produced according to the same technology. This can be seen by inspecting equations (30) and (31): everything else being equal, higher inflation requires more labor to produce the same amount of output. To counteract this distortion, the policymaker should reduce the variability of prices and at the best stabilize the price level. On the other side, the presence of a monopolistic-competitive goods market produces an inefficient wedge between the marginal rate of transformation and the marginal rate of substitution between consumption and labor, leading to a too-high price mark-up and a too-low level of production. To counteract this distortion, the policymaker should increase average inflation and therefore create some price dispersion in order to lower the average markup. However, at the margin, the costs of creating some price dispersion overwhelm the benefits in terms of a reduction in the average mark-up. As it is well known in the literature (see Khan et al., 2003, Woodford, 2009b), in this benchmark case, price stability turns out to be the optimal policy.

Our framework adds two additional distortions to the benchmark model which both depend on the distorted beliefs originating from the doubts that the private sector has regarding the "reference" probability distribution. In particular, households, who fear model mis-specification, attach higher probability to the states of nature in which there are bad news regarding the long-run productivity level. To see this, we take a first-order approximation of equations (18) and (19) obtaining

$$\begin{array}{c} \ln g_{t+1} = -(\psi - 1)(1-\beta)\left[\sum_{j=0}^{\infty} \beta^j (E_{t+1}\hat{C}_{t+j+1} - E_t\hat{C}_{t+j+1}) + \right. \\ \left. +\eta \sum_{j=0}^{\infty} \beta^j (E_{t+1} \ln \hat{L}_{t+j+1} - E_t \ln \hat{L}_{t+j+1})\right] \end{array},$$

where hats denote deviations of the variable with respect to the steady state. It is shown that the distortion in the beliefs g_{t+1} depends on the revision in the expected path of consumption and labor effort. However, when β is close to 1 the above expression can be approximated by

$$\ln g_{t+1} \simeq -(\psi - 1)[(E_{t+1}\hat{C}_{\infty} - E_t\hat{C}_{\infty}) + \eta(E_{t+1}\hat{L}_{\infty} - E_t\hat{L}_{\infty})]$$

and therefore by the revisions in the expectations of long-run consumption and labor. Since the long-run level of labor does not vary following a permanent productivity shock, a high level of g_{t+1} mainly reflects bad news with respect to long-run consumption and therefore bad-news with respect to productivity. The policymaker has almost negligible impact on these distorted beliefs since it is optimal to keep long-run inflation to zero and, therefore, not to affect long-run consumption.

However, there is still room for the policymaker to exploit the doubts of the private sector in order to improve the welfare. First, everything else being equal, distorted beliefs cause an inefficient accumulation of capital. To this end, consider the arbitrage conditions pricing the real return on capital, r_{t+1}^k, and the risk-free real rate, r_t^f,

$$E_t\{g_{t+1}m_{t,t+1}r_{t+1}^K\} = 1,$$

$$E_t\{g_{t+1}m_{t,t+1}\}r_t^f = 1.$$

By taking a second-order approximation of the above conditions we derive the excess return on capital with respect to the risk-free rate adjusted for the Jensen's inequality as

$$E_t\hat{r}^K_{t+1} - r^f_t + \frac{1}{2}Var_t\hat{r}^K_{t+1} = -cov_t(\hat{m}_{t,t+1}, \hat{r}^K_{t+1}) - cov_t(g_{t+1}, \hat{r}^K_{t+1}).$$

The distortions in the beliefs add an additional term to the premium on the capital return which now depends on the covariance between the return on capital and the distortions in the beliefs g_{t+1}. This additional term leads to an inefficient accumulation of capital, under a policy of price stability. Indeed, in this case, the return on capital is positively correlated with the current and long-run level of technology and therefore negatively correlated with g_{t+1}. The premium on the return on capital becomes larger. Although monetary policy has no leverage in influencing g_{t+1}, it can instead correct the distortions in the capital accumulation by acting on the return $\hat{r}^K_{t+1}$. The policymaker should in fact aim at making the return on capital less cyclical thereby reducing the premium on capital and increasing the average level of capital.

The second dimension along which distorted beliefs affect the equilibrium allocation is related to the pricing decisions of firms. To get the intuition, let us consider the aggregate-supply equation under the assumption that the cost of adjusting capital is infinite and steady-state investment is equal to zero, $Y_t = C_t$. Under this assumption and log utility, F_t, in equation (29), is constant and equal to $1/(1-\beta\gamma)$ while Z_t collapses to

$$Z_t \equiv \frac{\mu}{G_t} E_t\left\{\sum_{T=t}^{\infty}(\beta\gamma)^{T-t}G_T mc_T\right\}$$

which can be written as

$$Z_t \equiv \mu E_t\left\{\sum_{T=t}^{\infty}(\beta\gamma)^{T-t}mc_T\right\} + \frac{\mu}{G_t}\left\{\sum_{T=t+1}^{\infty}(\beta\gamma)^{T-t}cov_t(mc_T, G_T)\right\}, \qquad (33)$$

where mc denotes the real marginal costs.

Furthermore, we can write (29) as

$$\frac{1-\gamma\pi_t^{\theta-1}}{1-\gamma} = \left[\frac{1}{(1-\beta\gamma)Z_t}\right]^{\theta-1}$$

where Z_t is given by (33). The above equation makes clear that there exists a positive relationship between inflation and the present discounted value of the real marginal costs. In the benchmark model with no model uncertainty, the second term on the right-hand side of (33) is not present. Given the monopolistic-competition distortion, output and the average real marginal costs are too low. The policymaker can increase the average real marginal costs, reduce the average mark-up by raising average inflation. This is too costly, as we have already discussed. Under model uncertainty, the decision maker can instead raise real marginal costs without increasing much average inflation provided real marginal costs covary negatively with G_t and therefore with g_t. In fact, by making the stochastic discount factor negatively related with the future real marginal costs, the present discounted value of the firms' costs does not rise much even when average marginal costs increase. Therefore firms do not have much incentive to increase their prices. It is important to notice that the distortion in the beliefs now interacts with the monopolistic-competition distortion and the decision maker can exploit this interaction by increasing the procyclicality of real marginal costs following a productivity shock. This more procyclical response increases the variability of output and of real marginal cost, making the covariance between the latter and the discount factor more negative.

To sum up, the two dimensions along which distorted beliefs affect allocations, i.e. the capital accumulation decision and the price setting decision have opposite implications for optimal policy. The former calls for a less procyclical policy, while the latter calls for a more procyclical policy. It turns out that the latter dimension always dominates for all parameter values because reducing the inefficiencies due to monopolistic competition is of first order importance in this class of models. In general in New Keynesian models the trade-off between inflation and output is too steep to correct for the distortions due to monopolistic competition. In our model, the comovement between asset prices and marginal cost reduces the severity of this trade-off. Indeed, as it is shown in Figure 1, under the optimal policy real marginal costs become strongly procyclical and the more the higher is the degree of distortions in the beliefs. The return on capital and on equity become also more pro-cyclical and volatile worsening the equity and capital premia.

4.1.1 Welfare and degree of distortion in beliefs

Table 1 reports the unconditional expectations of several variables in comparison with the steady state, computed through a second order linear expansion of the first order conditions around the non-stochastic steady state, for different values of ψ, and evaluated at optimal policy. As it shown in the Table, following the reduction in the average mark-up, the average investment increases together with average consumption and output as ψ increases, despite the inefficient capital accumulation due to higher distorted beliefs. Moreover, the reduction in the average mark-up and the increase in the average output come with negligible costs in term of average inflation. Indeed average inflation is approximately zero at all values of ψ. Therefore, the higher the distortion in beliefs, the

	s.s	$\psi = 1$	$\psi = 25$	$\psi = 50$	$\psi = 100$	$\psi = 150$
Welfare	0.043	0.044	0.055	0.066	0.091	0.120
Output	2.009	2.010	2.053	2.103	2.218	2.350
Consumption	1.580	1.581	1.602	1.627	1.688	1.764
Investment	0.429	0.429	0.451	0.476	0.531	0.592
Hours	0.601	0.601	0.603	0.608	0.619	0.638
Inflation*	0.000	0.000	0.000	0.000	0.000	0.000
Real Marginal Cost	0.833	0.833	0.834	0.839	0.857	0.888
Premium on Capital*	0.000	0.005	0.320	0.910	2.894	5.963
Equity Premium*	0.000	0.017	0.820	1.830	4.402	7.711

Table 1: Means of selected variables; s.s.= steady state, *= in % and at annual rates, **= in %.

higher welfare, as the monetary authority is able to reduce more the inefficiencies due to monopolistic competition.

We are also interested in understanding the implications of the model for the price and the return on equity. To this purpose, we note that the stock-market index is given by

$$Q_t = \int_0^1 Q_t^j dj$$

which implies using (22) that

$$Q_t = \tilde{E}_t\{M_{t,t+1}(D_{t+1} + Q_{t+1})\}$$

where aggregate dividends simplify to

$$D_{t+1} = P_t Y_t - MC_t \cdot Y_t$$

and MC_t are the nominal marginal costs. The stock-market value, in real terms, can be written as

$$q_t^e = \frac{1}{G_t} E_t \left\{ \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{U_c(C_T, L_T)}{U_c(C_t, L_t)} G_T Y_T (1 - mc_T) \right\}$$

where $q_t^e \equiv Q_t/P_t$ and we have used the non-distorted expectation operator. Moreover, to get further insights, we assume that the cost of adjusting capital is infinite and that steady-state investment is equal to zero. In this case, under the preference specification used in the text, we can write

$$q_t^e = C_t E_t \left\{ \sum_{T=t+1}^{\infty} \beta^{T-t} \left[1 - \frac{G_T}{G_t} mc_T \right] \right\}$$

which implies that the mean of aggregate consumption is a good proxy for the average stock-market value in real terms. To see this, notice that under optimal policy the terms in the curly bracket does not move much (unconditionally) since it is strictly related to Z_t and to the inflation rate. Indeed, as it is shown in Figure 1, the impulse response of equity behaves similar to that of consumption. Table 1 shows that, as ψ increases, the equity premium increases under optimal policy, while in the benchmark model with rational expectations it is unrealistically small.

4.2 How does Greenspan's policy compare to optimal policy?

In this section we evaluate Greenspan's policy from the perspective of our benchmark model. We model Greenspan's policy through an interest rate rule,

$$R_t = r + \rho_r R_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_x x_t) + u_t, \tag{34}$$

on the sample period corresponding to Greenspan as chairman of the Federal Reserve, 1987:3-2006:1, where R_t is the quarterly average Federal Funds rate, π_t is the quarterly inflation rate, x_t is a measure of output gap obtained by hp-filtering real GDP and r is defined as $r \equiv \exp(\zeta)/\beta$.[14] We obtain $\rho_r = 0.9$, $\phi_\pi = 0.99$ and $\phi_x = 0.75$. We then solve our model under the estimated policy rule (34) at a degree of model uncertainty $\psi = 150$, where R_t corresponds to the quarterly risk-free nominal interest rate in our model.

In Figure 3 we plot impulse responses of selected variables under Greenspan's policy against the responses obtained under optimal policy. As benchmark of comparison, we also plot the impulse responses under two alternative policy rules: perfect inflation targeting, i.e. $\pi_t = 0$, and the classic Taylor Rule, i.e. the interest rate rule (34) evaluated at $\rho_r = 0$, $\phi_\pi = 1.5$ and $\phi_x = 0.5$. As Figure 3 illustrates, impulse responses of output, consumption, investment, Tobin's q, real risk-free rate and inflation to a productivity shock under Greenspan's policy are relatively close to optimal policy, substantially closer than inflation targeting or the Taylor Rule. However, our exercise also suggests that

[14]The rule (34) has been estimated with the method of instrumental variables suggested by Clarida et al. (2000). Instruments are the four lags of inflation, output gap, M2 growth rate (FM2), commodity price inflation (PSCCOM) and the spread between the long-term bond rate (FYGL) and the three-month Treasury Bill rate (FYGM3).

Greenspan was perhaps too accommodative with respect to productivity shocks. For instance, output under Greenspan's policy rises on impact by about 25% more than it should when compared to the optimal policy. In contrast, output under strict inflation targeting or the Taylor Rule increases on impact by only about 1/3 of what it should at optimal policy.

Remember from previous discussion that strict inflation targeting would roughly approximate optimal policy response to a productivity shock in absence of any model uncertainty, i.e. $\psi = 1$. Therefore, while Greenspan's policy would seem too expansionary from the perspective of a standard New-Keynesian model, it appears to be much closer to optimal policy when evaluated from the perspective of our New-Keynesian model with model uncertainty.

5 Flexible inflation targeting and asset prices

In this section we study whether it is possible to implement optimal policy with simple monetary policy rules. In the model with no doubts, when $\psi = 1$, there is a simple solution. A policy of price stability, or zero inflation, would implement the optimal policy. This is a standard result for New-Keynesian models with only productivity shocks. Given this observation, we focus our attention to a class of flexible inflation-targeting policies which encompasses the strict inflation-targeting policy of zero inflation as a special case.[15] In particular, we specialize to the following rules

$$\ln(P_t/P_{t-1}) + \varphi_y \ln(Y_t/Y_{t-1}) + \varphi_q \ln(q_t/q_{t-1}) = 0, \quad (35)$$

where φ_y and φ_q are parameters. In (35) we include a strict zero inflation target (the first term), a target for the growth of output (the second term), and a target for asset-price changes in terms of Tobin's q.

In the left panel of Figure 4, we set $\varphi_q = 0$ and explore how the optimal choice of φ_y varies when ψ increases. The policy rule (35), for given parameter φ_y, together with the structural equilibrium conditions can be solved to determine an equilibrium allocation. Among the equilibria indexed by φ_y, the optimal choice of φ_y corresponds to the equilibrium allocation that maximizes (32). Clearly, when $\psi = 1$, we obtain that $\varphi_y = 0$ and therefore we get that a strict inflation targeting is optimal. When ψ increases, φ_y decreases monotonically. For instance, when $\psi = 150$, we obtain that

[15] Interest-rate rules with reaction to inflation and other variables would be another class of policy rules to consider. However, we find this choice less appropriate since when $\psi = 1$, the optimal simple interest-rate rule would require an infinite reaction to the inflation rate.

$\varphi_y = -0.22$, implying that a one percentage increase in output is accompanied by 22 basis points of a positive inflation rate. This is not surprising since optimal policy requires an accommodative policy following productivity shocks, and therefore our flexible-inflation targeting policy requires positive comovements between output growth and inflation. A similar picture emerges when we set $\varphi_y = 0$ and analyze the optimal choice of φ_q, the response to asset-price movements. As the right panel of Figure 4 shows, policy should also be accommodative in this direction and create positive comovements between inflation and asset-price movements. We find that φ_q should decrease as ψ rises. For instance, when $\psi = 150$, we obtain that $\varphi_q = -0.69$, implying that a one percentage increase in asset prices is accompanied by 69 basis points increase in CPI inflation.

Finally we investigate the more general form of inflation-targeting policy (35) in which we allow for a simultaneous reaction to output-gap growth and asset prices. Figure 5 shows that the optimal combination of φ_q and φ_y is such that the optimal φ_y has a similar pattern, although amplified, to the case in which we restrict φ_q to zero. However, now, the optimal φ_q becomes positive for all values of ψ between 1 and 150. This is the case because it is optimal to generate positive comovements between output growth and inflation, and at the same time between output growth and asset-price changes.

To sum up, we find that the optimal simple rule should move from a strict inflation-targeting policy, when there are no doubts, to a more flexible inflation targeting policy, which also includes output growth and asset-price inflation, when doubts rise. However, similar to Bernanke and Gertler (2001), we also find that including asset-price inflation as target does not improve much average welfare, i.e. an inflation-targeting rule employing only output would imply very similar allocations.

6 Special cases

In this section we report results obtained under two special cases: i) when only the price setting decision is affected by distorted beliefs; ii) when only the capital accumulation decision is affected by distorted beliefs. Notice that both these cases are hard to interpret theoretically within the standard New-Keynesian framework as firms are owned by households, and should therefore maximize the expected discounted sum of profits using the same discount factor of households. Nevertheless, we think that reporting these results may help to gain more intuition on the factors driving optimal policy in the benchmark specification. In addition, one may think of a different model where firms are not directly owned by households but, for instance, by entrepreneurs and have a theory on why these two classes of agents should have different degrees of distortion in beliefs. While interesting, such an extension is beyond the scope of this paper and left for future research.

6.1 The special case of no distortions in price-setting decision

Results about optimal policy are clearly driven by the interaction between the distortion in the beliefs, the monopolistic distortion and the forward-looking pricing behavior of firms under the Calvo's model. To make this clear, we consider an environment in which the stochastic discount factor through which firms evaluate future profits is not distorted, i.e. $\tilde{E}$ in (25) coincides with E. Figure 6 plots the optimal policy responses of selected variables following a permanent productivity shock under this assumption, everything else being equal to the benchmark specification. After a positive technology shock, monetary policy becomes now less accommodating. In particular output, consumption, investment, inflation and Tobin's q are lower than under the benchmark case of $\psi = 1$ and in particular the Tobin's q and inflation on impact decreases. In this way, monetary policy reduces the pro-cyclicality of equity returns (causing counter-cyclical returns on equity and capital for high enough ψ) and, therefore, reduces the equity premium to allow for a relatively high level of physical capital accumulation.[16] The real rate rises to sustain a steadily increase in consumption. Since, under this experiment, beliefs of price setters are not distorted, the monetary authority does not have the room to reduce the average markup as in the previous case. Therefore, optimal policy works to minimize the distortions in the valuation of the return on capital. However, it is worth noticing that, even in this case, monetary policy deviates in an important way from a price-stability policy and the more the higher the degree of ambiguity.

6.2 The special case of no distortions in the capital accumulation decision

Finally we derive optimal policy responses to a permanent productivity shock under the assumption that the stochastic discount factor through which households evaluate future returns on capital is not distorted, everything else being equal to the benchmark specification. Figure 7 plots the optimal policy responses of selected variables. Not surprisingly, the responses of all variables to the productivity shock are qualitatively very similar to the ones obtained under the benchmark specification. However, for given parameter values, responses at optimal policy are more procyclical than under the benchmark specification.

[16] Notice that under the previous case of distortions in the beliefs of price-setters, the valuation of the return on capital is distorted and the equity premium increases. However, average investment is pushed up by the reduction in the average mark-up.

In fact, absent the distortion in the capital accumulation decision, the monetary authority can reduce the inefficiency due to the monopolistic distortion without causing more inefficient capital accumulation.

7 Conclusion

In this paper, we departed from the standard New-Keynesian monetary model by introducing doubts. In our model, households express distrust regarding the true probability distribution. These doubts are reflected in asset prices and might generate, together with ambiguity aversion, equity premia of similar size as those found in the data. This is an important feature of our framework with respect to the benchmark model which, on the contrary, is unable to match asset-price data. In this environment we study how a policymaker, who instead trusts the model, would set optimal policy.

Results change in a substantial way with respect to the benchmark model. A standard result in the literature is the optimality of a policy of price stability following productivity shocks. With doubts, we find that policy should become more accommodative with respect to productivity shocks and work to increase the equity premium. The departure is larger, the higher is the degree of distrust that agents have. Most important flexible-inflation targeting policy might include a reaction to asset-price inflation in the direction to create positive comovements between inflation and asset-price changes or between output growth and asset-price changes.

There are several limitations of our modeling strategy. First, we assume that households and firms share the same degree of doubts. Households' doubts are reflected in Arrow-Debreu prices and those are used to evaluate both asset prices and the future profits of the firms. We show that if doubts are just reflected in asset prices and do not instead distort the evaluation of future firms' profits, then policy should be countercyclical and in this case should work to reduce the equity premium. Second, we assume that the only disturbance affecting the economy is a productivity shock. Results would not change if we were allowing for mark-up shocks modeled using a stationary process. Indeed, doubts and ambiguity aversion are reflected in fears of bad news regarding long-run consumption. Persistent productivity shocks, in contrast to transitory mark-up shocks, can indeed have an influence on long-run consumption. Third, we are conducting a pure normative exercise under the assumption that the true probability distribution coincides with the reference probability distribution distrusted by the agents. Results would change when reference and true probability distributions do not coincide. Most interesting it is the case in which the policymaker also distrusts the reference probability distribution.

We leave these analyses for future work. Finally, we have abstracted from credit frictions and asset-market segmentation which can be important features to add to properly model asset prices and the transmission mechanism of shocks. This is also material for future works. Here, we have kept the analysis the closest as possible to the benchmark New-Keynesian model to show how a small departure from that model delivers important differences in the policy conclusions and how this departure can rationalize a too accommodative monetary policy as an optimal policy following productivity shocks.

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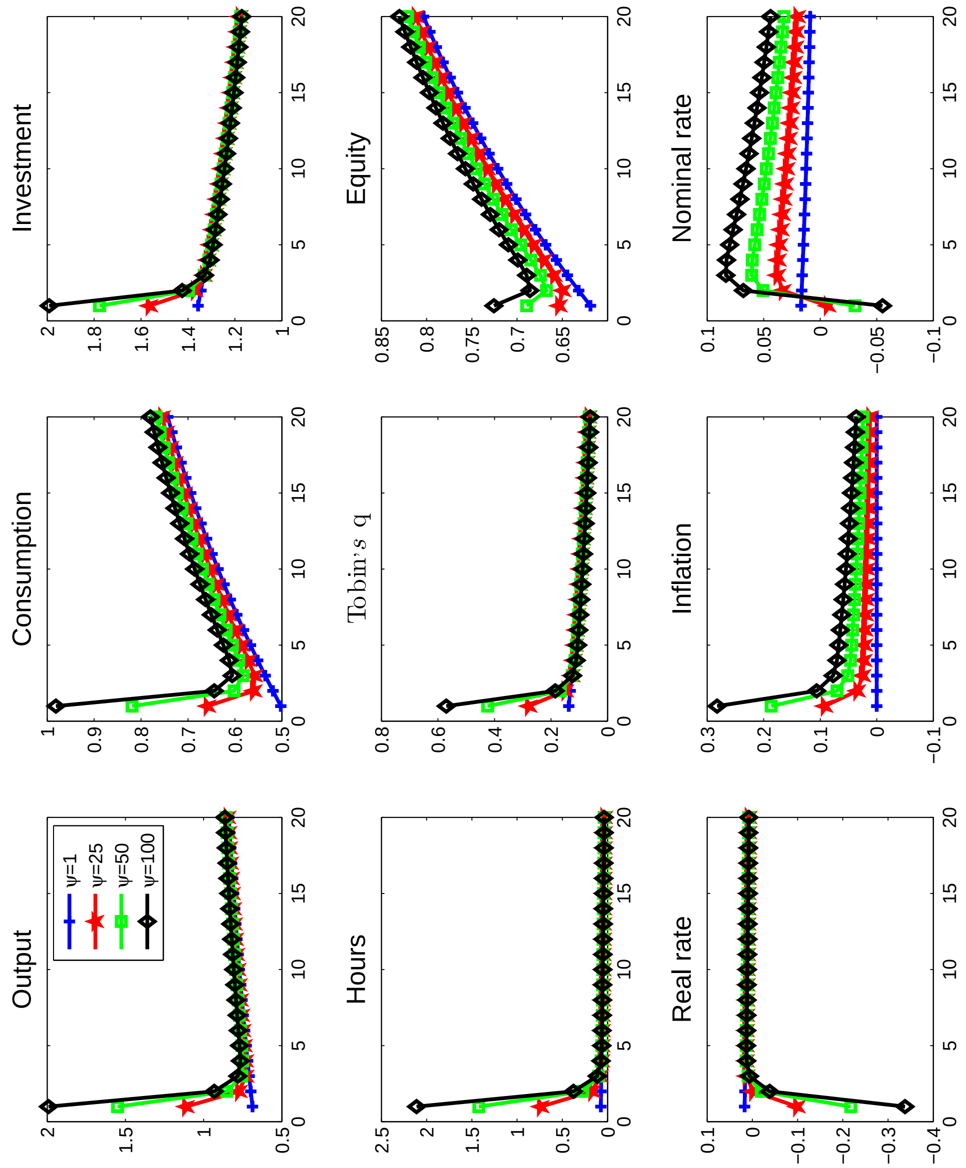

Figure 1: Impulse response of selected variables to a unitary permanent positive productivity shock under optimal policy for different values of the degree of ambiguity aversion ψ.

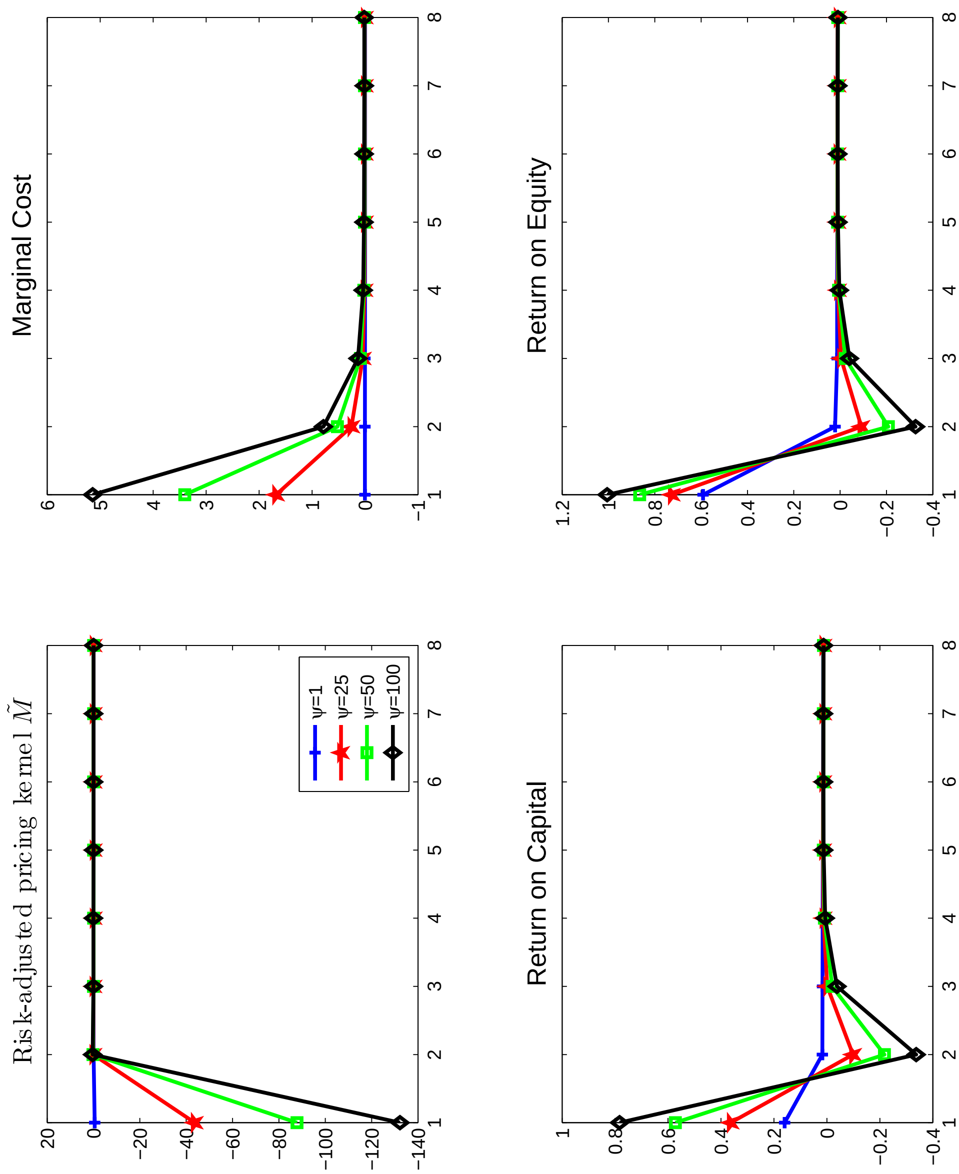

Figure 2: Impulse response of selected variables to a unitary permanent positive productivity shock under optimal policy for different values of the degree of ambiguity aversion ψ.

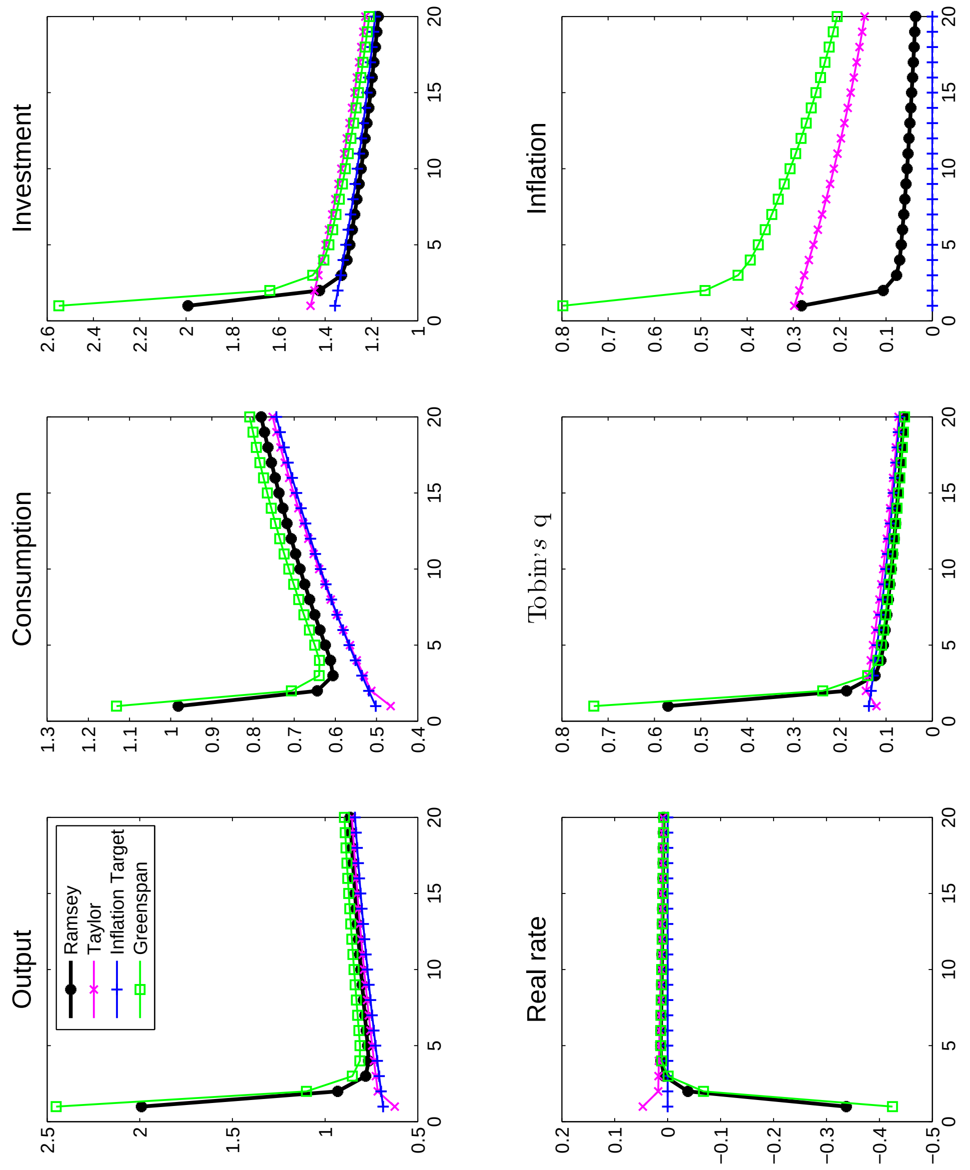

Figure 3: Impulse response of selected variables to a unitary permanent positive productivity shock under optimal policy and under different monetary policy rules for $\psi = 150$.

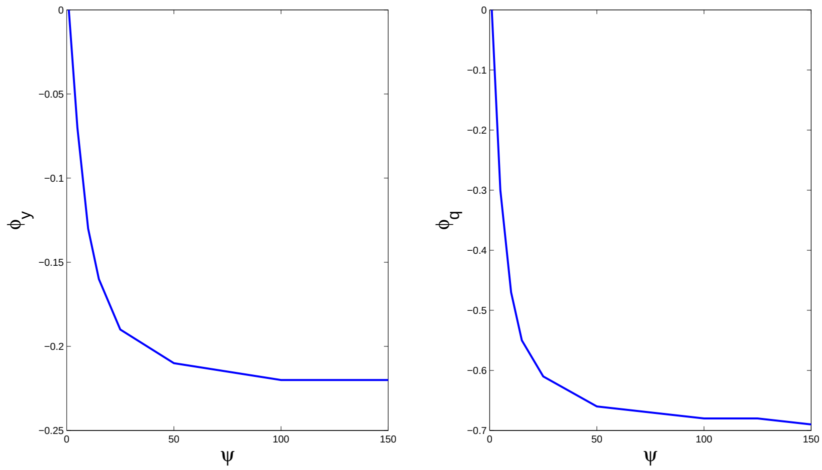

Figure 4: Optimal inflation-targeting policies in the class (35) by varying the degree of model uncertainty, ψ. Left panel: optimal φ_y when $\varphi_q = 0$. Right panel: optimal φ_q when $\varphi_y = 0$.

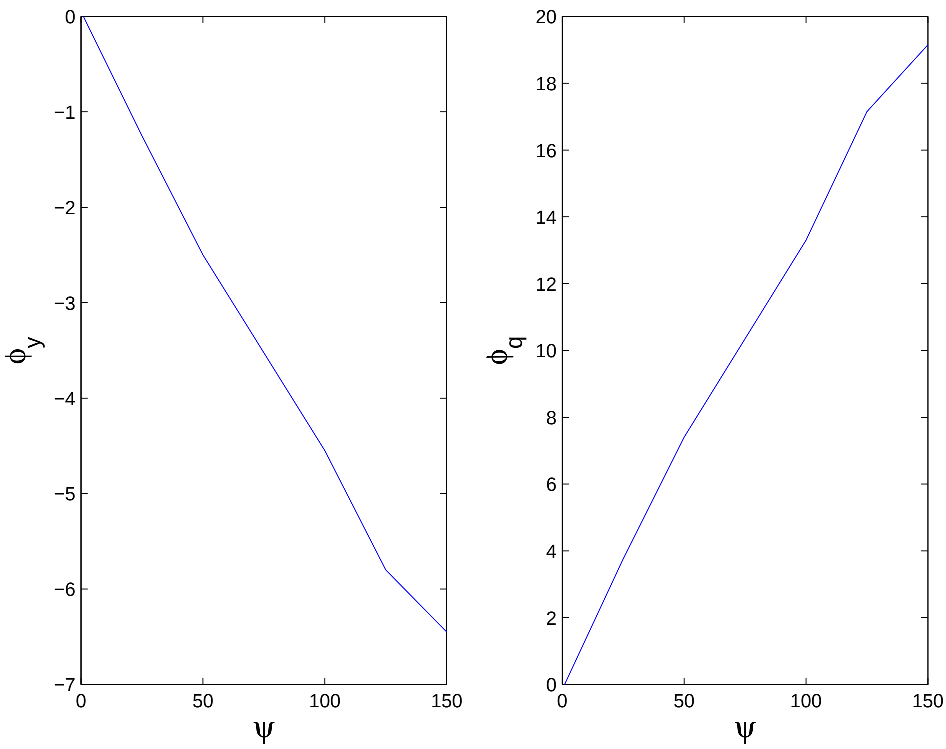

Figure 5: Optimal inflation-targeting policies in the class (35) by varying the degree of model uncertainty, ψ. Left panel: optimal φ_y. Right panel: optimal φ_q.

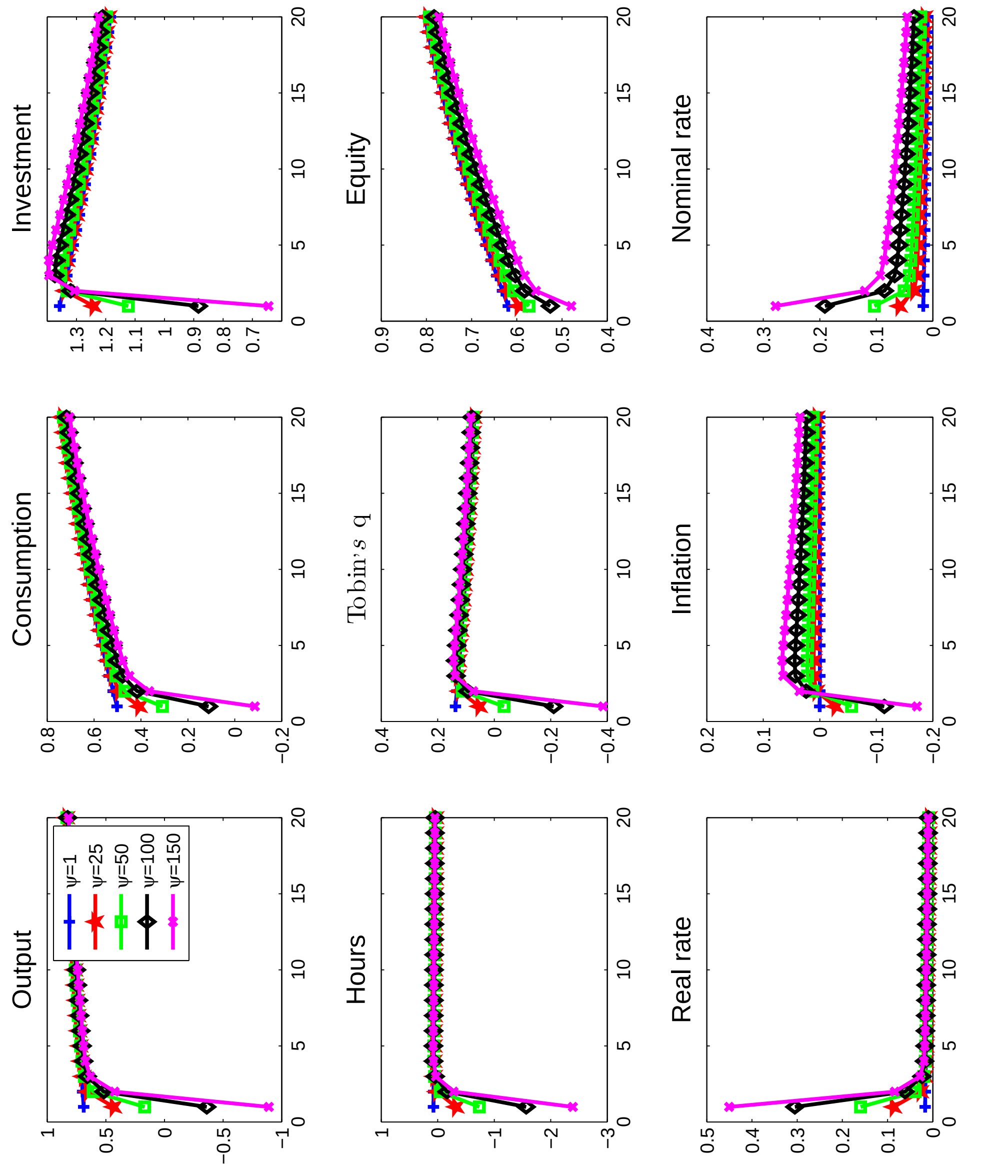

Figure 6: Impulse response of selected variables to a unitary permanent positive productivity shock under optimal policy for different values of the degree of ambiguity aversion ψ under the case in which only investment decisions are distorted.

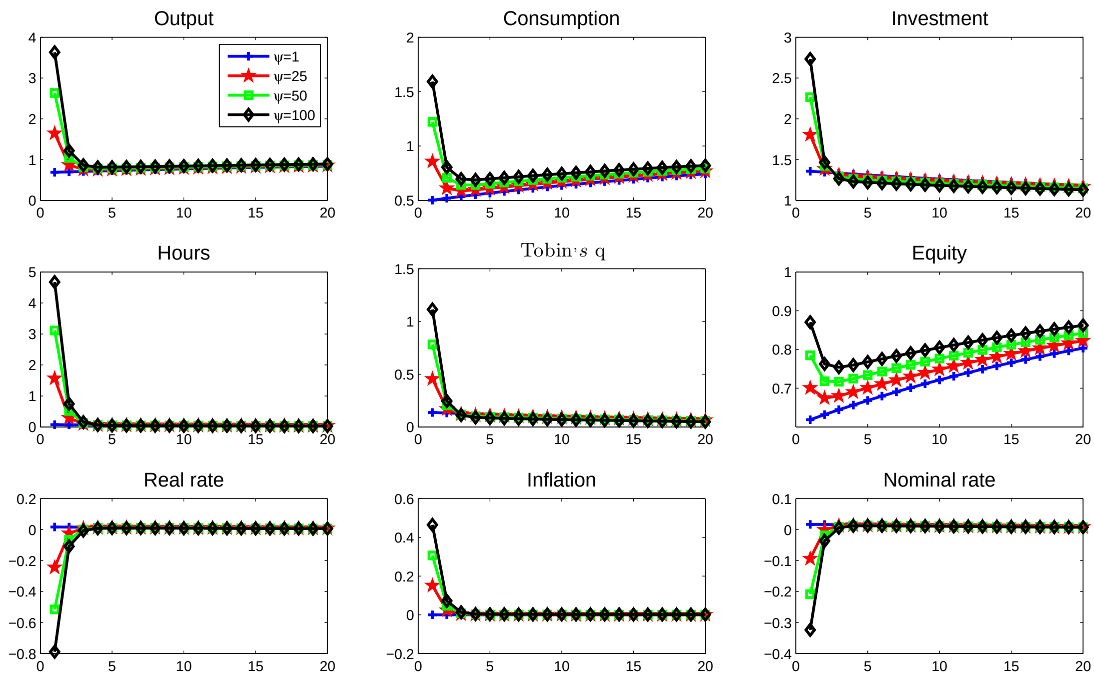

Figure 7: Impulse response of selected variables to a unitary permanent positive productivity shock under optimal policy for different values of the degree of ambiguity aversion ψ under the case in which only price-setting decisions are distorted.