Capital is Heterogeneous: tax Land rather than Wealth and Investment*

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*March 06, 2016. This work is a development of the May 14, 2014 discussion paper 2014-07 of the Economics Department in Sciences Po (http://ideas.repec.org/p/spo/wpecon/infohdl2441-30nstiku669glbr66l6n7mc2oq.html), itself a translation of a first note in French (LIEPP's discussion paper 25, April 2014). We would like to thank, among many others, Philippe Aghion, Yann Algan, Robert Gary-Bobo, Zvi Hercowitz, Ricardo Lagos, Eric Rasmusen, Peter Rupert, Philippe Weil, Mike Woodford and Cornelia Woll for discussions, useful suggestions and comments. Etienne Wasmer acknowledges support from ANR-11-LABX-0091 (LIEPP) and ANR-11-IDEX-0005-02

Abstract

Thomas Piketty's early thesis in *Capital in the 21st Century* is that capital has been accumulating faster than income in advanced countries over the last decades, and this trend would be due to high returns on capital relative to growth rates. We challenge this view in this article.

First, capital is heterogeneous; second, capital (return value) should be distinguished from wealth (market value). Second, the trend arises from only one of the components of capital, namely housing capital, and, as we show, the non-productive component of housing capital, that is land. Third, if the value of housing capital was measured with rental prices, the capital/income ratio would actually remain fairly stable over the last four decades in most countries.

We then examine the consequences of these findings for optimal capital taxation. Instead of a uniform world tax on wealth, we claim that taxation should account for the fact that capital is heterogenous, extending Judd (1985) to housing and land. A positive tax rate of land restores the first best, and if a property tax is not available, a positive tax rate on housing rents leads to a second-best. More generally, taxation should differ depending on the types of capital: land should be taxed rather than productive capital: land taxation avoids the under-accumulation of physical capital.

1 Introduction

The issue of capital has always led to the greatest economic controversies, the most famous of which was between the two Cambridge, in the 1960s. The debate was about possible inconsistencies and tautologies in the measurement of the capital stock and its earnings. *Capital in the 21st Century* has similarly generated its fair amount of controversies. The main argument under scrutiny here is the idea that capital accumulation brings about a self-sustained rise of inequality. The logic is as follows: the higher the capital/income ratio, the higher the earnings of capital relative to labor, at least at a constant rate of returns; then, if the rate of return on capital (r) is higher than the growth rate of the economy (g), the capital/income ratio will rise, eventually leading to a world where a class of owners would have perpetually increasing income from capital due to rising accumulated wealth. The author documents the strong rise of the capital/income ratio, especially in France, but also provides evidence of a similar trend in other countries and concludes to a worrisome accumulation of wealth and unbounded process of inequality increase.[1] The

[1] In more recent publications, the authors has been much less assertive in its conclusions and on the role of $r - g$ (Piketty (2015b), Piketty (2015a)). According to the author's ex post clarification, "$r - g$" is not the main tool to explain and forecast inequality and he seems to favor human capital instead.

conclusionshave been challenged in Krusel and Smith (2014), Rognlie (2014), Acemoglu and Robinson (2015), Stiglitz (2015a,b), Mankiw and Summers (2015), Weil (2015), Auerbach and Hassett (2015), Jones (2015), Kopczuk (2015). In short, there are mostly two lines of critics: critics related to the inability of the model to account for varying rates of return on capital especially in the limits of infinite accumulation; and critics on the data used to assess the theory.

Our paper and its early version (Bonnet et alii. 2014) belongs to the second line. It first questions the facts that capital is accumulating faster than income in the US, Canada and three European countries, France, Germany and the UK. First, capital is heterogeneous; second, capital (return value) should be distinguished from wealth (market value). What is measured by the author iswealth over income ; and it is based on only one of the components of capital, namely housing, or more precisely, land.

Indeed, housing is a very particular component of capital (see the literature review in Section 4). In particular, the market value of housing does not provide a good measure of actual direct returns on housing capital. Housing is both a consumption good, the price of which comes from rental or shelter costs, and an investment good, yielding an income corresponding to the rent. Only landlords (who represent a relatively small fraction of the population) effectively receive monetary income from their housing capital. This is not to say that owner-occupiers do not receive any income: they actually save on rent and therefore receive an implicit rent. It follows that returns on housing capital (the key ingredient in the "r" part of the "$r - g$" model) have to be measured by rent on housing, be it direct (actual rents) or only implicit (saved rents), in order to be possibly interpreted as a factor of rising capital accumulation.

The measurement issue is key here. Housing value measured on current prices and housing value based on rents are only equivalent in the absence of a divergence between sale prices and rental prices. And,precisely, this divergence was observed in several countries, especially France, since the late 1990's. We document the surge in housing sale prices over the recent decades has been much faster than income, rental prices and thus returns on housing capital in many countries. This may have arisen from a bubble in housing prices, but this is not necessarily the case, and the existence of a bubble is not necessary for our argument. When fundamentals (such as real interest rates or the specific utility derived from property) do change, this may also lead to higher prices relative to rent. We next show that if the value of housing capital was measured with rental prices, the capital/income ratio would actually remain fairly stable over the last four decades in four countries analyzed

(France, the US, the UK, and Canada) but not in Germany.

Note however, that our view is not that data in Piketty are biased for what they mean (they are a good measure of wealth) but they cannot be interpreted as evidence of explosive returns in capital. We do not deny however that the rise in housing price has had real consequences on access to housing and inequality.[2]

More specifically, we establish that the strong rise in housing prices in most countries is due to the rise in the value of land and explore the implications of this fact for optimal taxation. We follow the line of research arepresented by Judd (1985) and Chamley (1986)). Both of these seminal papers find that the optimal tax on capital should be equal to 0. However in these articles, capital is not heterogenous, yet taxing land could have a different impact than taxing productive capital.

Here, accounting for fixity of land and its consumption by households, our framework allows us to derive new results to the litterature. Since our primary goal is to study the effects of taxation on redistribution, we have extended the model of Judd (1985) to housing and land. Judd (1985) studies redistribution in a two-class economy with workers and capitalists producing one composite good . We extend his model to a situation where there are two kind of capitals, productive capital and land, and two types of consumptions, housing services and a composite good. The capitalists are the sole landlords of the economy whereas the workers are tenants. In the first best, we find a positive tax rate of land, set so as to reduce the inequalities of income, consumption and welfare between capitalists and workers-tenants. This tax will allow to compensate the wage-earners for the fact that they have no property right on capital and land. This contrasts with the fact that in Judd's model, according to assumptions, productive capital should not be taxed because taxation leads to an under-accumulation of capital and then to a decrease in the steady-state of output and then welfare. However, in our extended model, land should be taxed since land is fixed, property tax is lump-sum. In the second best, if property tax was not available, a tax on rent is possible; its optimal rate is given by a inverse elastictiy rule à la Ramsey. The optimal rate is increasing in the inverse of the supply elasticy of rental housing by landowners.

This extension of Judd's model shows that Piketty's proposition of a world tax on capital is misleading since it does not take into account for the heterogeneity of capital. If

[2] In particular, it has consequences on the wealth trajectories of individuals and dynasties: for instance, capital market imperfections make it is increasingly difficult for an individual without initial wealth to become a homeowner when housing prices increase. We come back on this important point in conclusion.

the objective is the reduction of wealth inequality, our conclusion is that the best instrument is a property tax worldwide, not a wealth tax.

The paper is organized as follows. Section 2 discusses the right measure of capital in particular housing capital. It provides the measure of capital consistent with a model of capital accumulation, and reaches clear conclusions regarding capital stability in France which also applies to the United Kingdom, the United States and Canada. The reversal does not hold in in Germany where housing prices actually dropped over the last decades. It then shows that the rise in the value of housing is primarily driven by a rise in land prices in France, the US, Canada, the UK and Germany. Section 3 introduces housing in an optimal taxation framework and discusses the issue of optimal taxation of various components of capital and their returns, both under the first best where all fiscal tools are available and notably the property tax, and the second best where only a tax on rents can be levied, but neither a property tax nor a tax on implicit rents served by landowners to themselves. We finally place our results into the different strands of literature. Section 4 offer concluding comments.

2 Comparison of housing capital and income dynamics

2.1 Definitions and discussion of concepts

Let us first define the empirical concepts used to measure wealth and capital. We will use the concept of wealth when the market value is used, or said otherwise, the liquidation value for its owner; we use the concept of capital when its value is based on its returns. Capital may be productive (fixed capital investments, residential investments) or unproductive (such as land), that is inherited from the past rather than from past efforts. In any event, separating away wealth from capital is essential and we therefore disagree with footnote 13 in Piketty and Zucman (2013) where both concepts are used interchangeably, following Giffen. The distinction is key when the two measurement methods do not coincide. We first point out these distinctions in K/Y series before proposing to correct these figures for the divergence between the market prices and the rental prices for housing.[3]

[3] An interesting question is whether the same exercice should be done for other assets. We leave this to future research, and focus here on housing, that is more specific. Everyone needs a shelter, so that one can always liquidate all other assets and consume this wealth, while, since all individuals prefer to live in a home rather than to be nomad, the consequence of which being that if you sell your home, you have to buy another one, which is not the case for other assets. We then argue that an homethetic price increases does not enrich the owner-occupiers which represents the vast majority of owners.

Another relevant discussion is whether housing is net wealth. This has always been subject to complex discussions. Buiter (2010), in a paper entitled *Housing wealth isn't wealth*, concludes that individual consumption is unchanged if housing prices increase and if the price increase follows the fundamentals (hence the exclusion of bubbles). If there is a housing bubble, there are real effects on consumption due to distortions. These effects are of a second order however and are very different from those described in Thomas Piketty's book (see below). Bajari, Benkard, and Krainer (2005) concluded that there is no first order impact on the price of existing housing stocks and the welfare of the economy: welfare gains due to the gains of sellers are compensated by welfare losses at the expenses of buyers. These transfers contribute to inequality but do not imply any systematic divergence of wealth since sales occur only once and even sellers need somewhere to live.

Regarding to measurement of housng, it is sometimes claimed (e.g. Haan and Diewert (2011)) that the market value of housing does not provide a good measure of actual direct returns on housing capital. The user cost (financial opportunity cost) might be used to valuate the the Owner Occupied Housing (OOH) Service in National account. However, national statistical agencies usually prefer the equivalent rental approach. Himmelberg, Mayer, and Sinai (2005) also use the user cost that they compare with actual rent to test for the presence of bubbles.[4]

2.2 Measurement issues of the original series K/Y

As a starting point, a decomposition of K/Y excluding housing capital shows that the capital income ratio has dropped due to the disappearance of agricultural rent in France (panel a in figure 1). In the US panel b), the capital income ratio net of housing capital was the same in 1770 as it was in 2010 and there is neither a long run trend nor a recent increase of this ratio. In Germany, the UK and Canada (panels c to e), excluding housing similarly leads to very small secular increases in K/Y or to stable evolutions. We do not claim that housing should be excluded from the analysis of capital income ratios: it should be included by all means. However, the particular way it is calculated must be in line with its interpretations and its normative implications. In the data in 1, the measurement of housing capital is the sum of two elements, dwellings and land with constructs. Appendices

[4] Note also that the measurement of the share of labor in national accounts is in itself a source of empirical debates due to necessary corrections for self-employment, the government sector and housing services. See Rupert (2012) and Gomme and Rupert (2004) for an example of a correction that changes the trends in labor shares."

A and B precisely describe the main method as well as a comparison with alternative methods used in national accounts.

We first illustrate why measurement matters in developing the French case first. This is the country where the rise in capital/income ratio is one of the most dramatic of all countries studied, before applying the same approach t other countries. In France, housing capital is estimated through two steps: a first step consists in estimating the total stock and value of housing in a reference year (1988) by INSEE, the French Institute of Statistics, using Housing Surveys and the last survey on land prices available. INSEE then follows over time the evolution of the number of buildings from aggregate housing investments, deflated by the housing construction index as well as the evolution of land with constructs using the evolution of the surface area covered by housing units and the development of the surface area covered by houses. The year-by-year value of housing capital stock is obtained in multiplying the above-described volume by the current price index of existing housing. Furthermore, new buildings are also evaluated at the price of existing housing units. For these two reasons, the estimation of housing capital follows closely the year-to-year evolutions of housing prices.

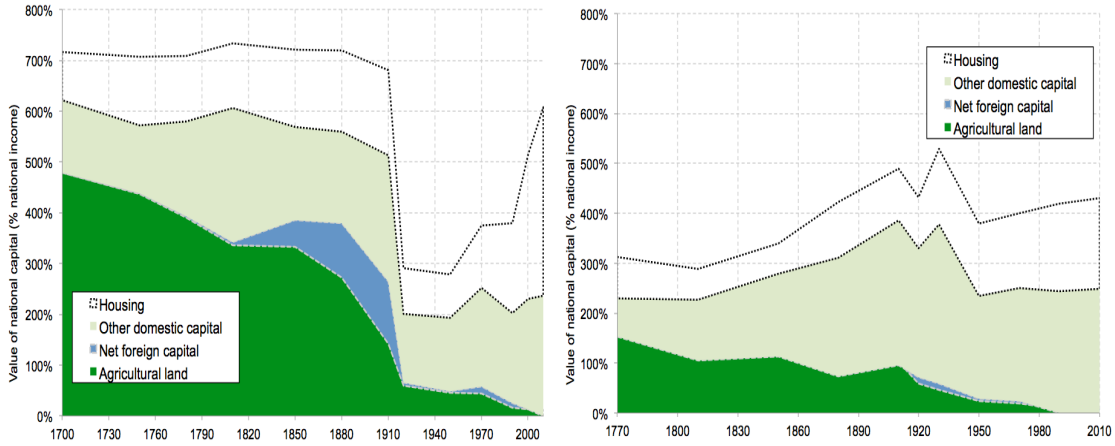
2.3 The steadiness of corrected housing capital in France

This method of calculation makes clear sense for national accounting, as a measure of national wealth. However, it cannot be used directly as an attempt to document dynamic accumulation of capital. This is a distinct issue since it involves returns on capital. The dynamics of housing capital accumulation must be measured using rents and their evolution, and not the evolution of housing prices. This does not matter that much if they follow the same pattern, as it should be the case in the long run.

Precisely, in France but also in other countries, there has been a rise over the last two decades in the ratio of housing prices relative to the disposable income of household. This rise has been approximately as high as 60% in France. Rents remained stable relative to income over the same period (see figure 2).[5]

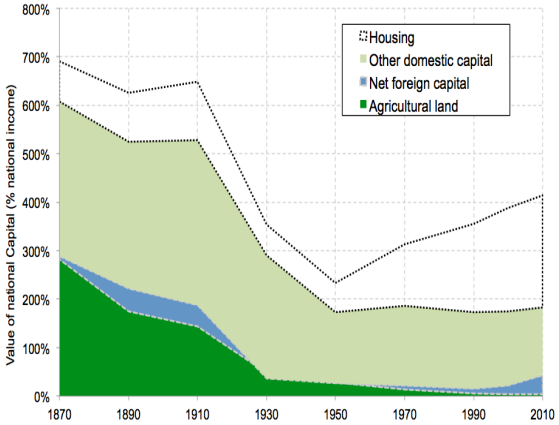
We therefore provide an alternative measure of housing capital accounting for this rent/price divergence. We do so in multiplying the initial housing capital by the rent-

[5] We do not try to explain why housing prices went up quickly here, but rather we will attempt to understand the consequences this has on income and savings for homeowners and households in the rental sector.

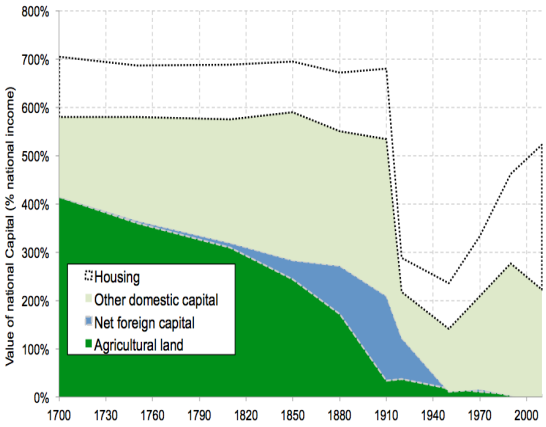


(a) France

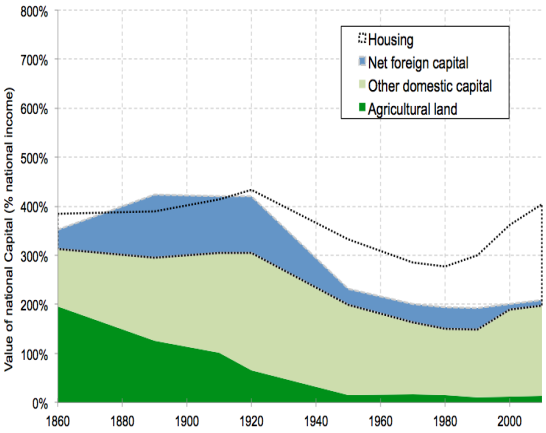
(b) US



(c) Germany



(d) UK



(e) Canada

Sources: Capital in the 21st Century, Figures 4.6 & Chart 3.2- see piketty.pse.ens.fr/capital21c.

Fig. 1: VERIFIER LA LARGEUR Secular decline in the capital/income ratio (excluding housing) in 5 OECD countries

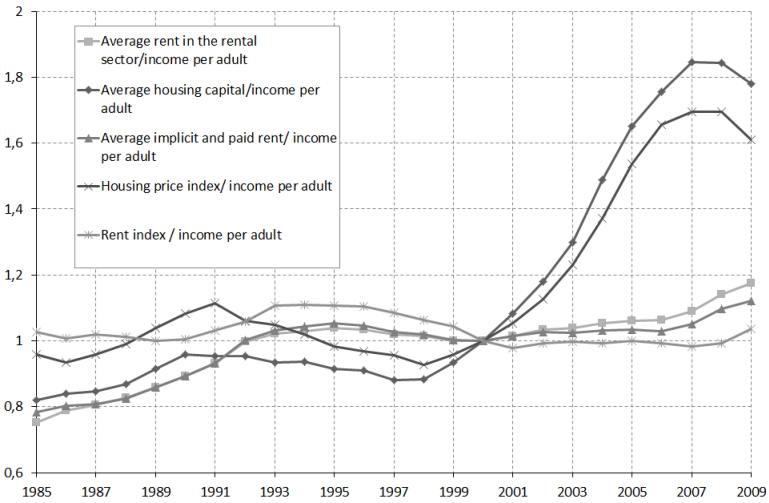
to-price ratio[6]. After correction, the recent trend in housing capital is replaced with a very modest evolution.[7] The new series K/Y in France is now steady after correction (series red and green), from 500% in 1950 to 500% in 2000 after a lower value (400%) in the 1980's. This is quite different from the original series in Piketty (represented by the dark line from 300% to 600%).

2.4 The stability of corrected housing capital in other countries

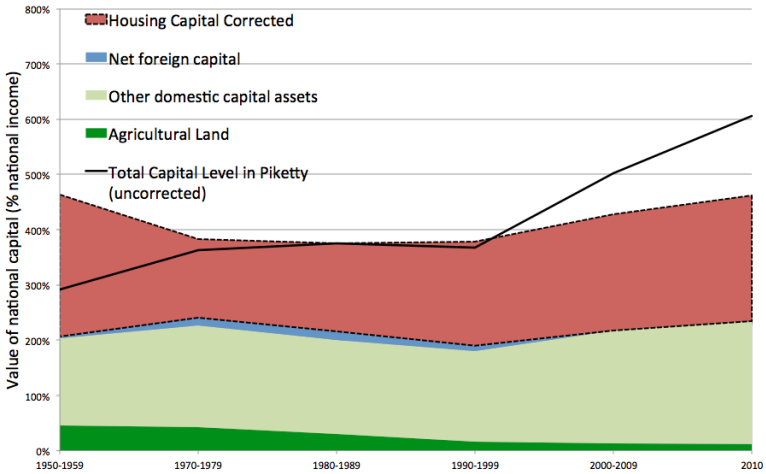
The correction method is applied to other countries where prices and rent data are available. For these countries, we used series of the price over rent ratio, available for the US, the UK, Canada and Germany (see rent to price ratios figure A.1 in Appendix).

[6] See Appendix B for a description of the method.

[7] Note also that if housing capital must be corrected to account for the housing price-rent divergence, such is not the case of GDP, the denominator of K/Y. Indeed, the value of GDP includes only the service of housing, and this value is calculated by national statistical agencies from imputed rents, themselves estimated from hedonic rent equations, therefore not on housing prices. Hereby, national statistical agency acknowledge that revenues from housing are aligned on rents (either actual or imputed rents) but not on fluctuating values of housing. Hence the value of GDP is immune to pur criticism that applies to the measurement of the stock of housing capital or more precisely, the interpretation one makes out of it.

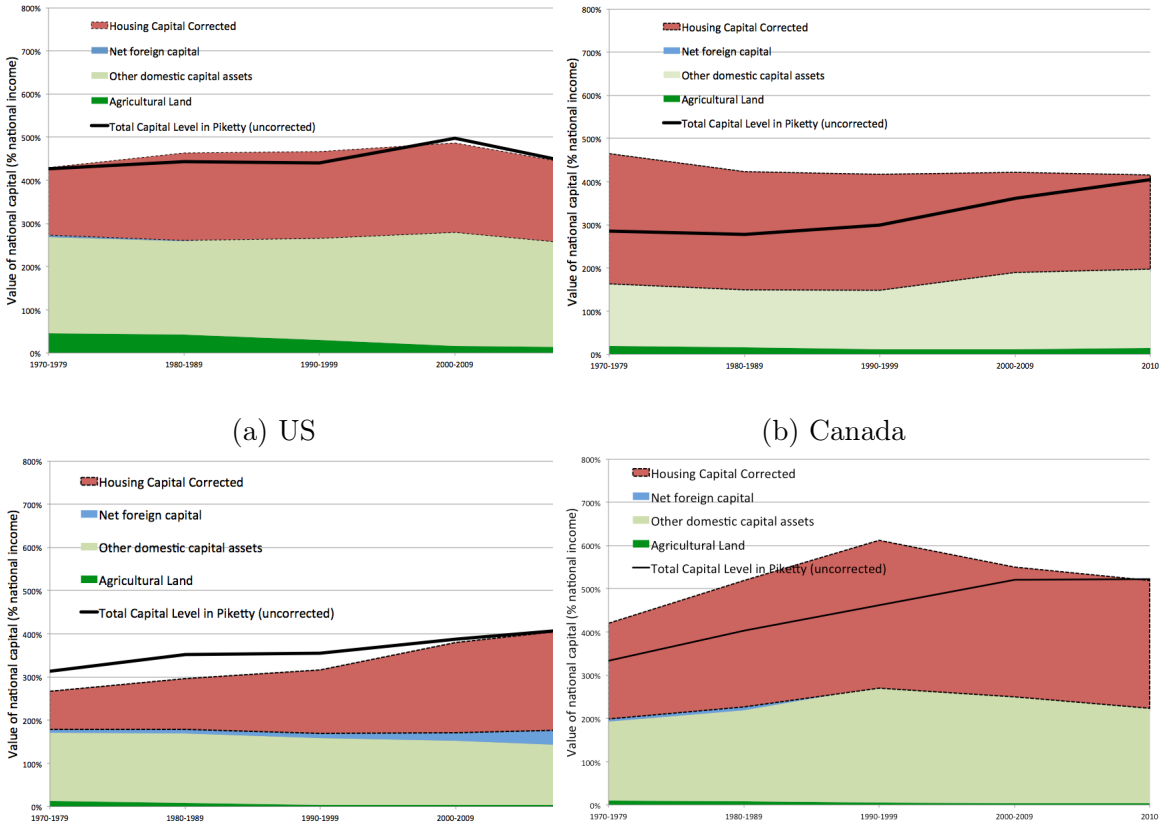


Sources: Friggit, http://www.cgedd.developpement-durable.gouv.fr/prix- immobilier-evolution-1200-a1048.html. "Compte du logement 2013". On the graph, we compare the changes of real estate prices, of average rents and indices. Average values take into account trends in the quality of housing, while indices do not correct for quality changes.



Sources: National accounts (decennial average), data from Thomas Piketty (figure 3.2 - see http://piketty.pse.ens.fr/en/capital21c) for Net Foreign Capital, Other Domestic Capital assets, Agricultural Land and Total Capital Level (uncorrected). For Housing Capital corrected, Housing capital is deflated from the housing price index and multiplied by the index of rental prices. Indices are chosen to represent 1 in 2000, which corresponds to one of the 'neutral years', where indexes of price and rents relative to income are close to their long term average or "regularity area" (Friggit, 2014). The choice of a different base year would vary the level of the corrected capital/labor ratio, but would not affect the relative evolutions after correction. The last year in this figure and next figures is 2010, but it should not be compared to other periods in this graph for the reasons given in footnote 5.

Fig. 2: Evolution of housing prices and rental prices in **France** since 1984 (up) and Capital in **France** with housing capital correctly measured, based on rent series (down)



(a) US (b) Canada

(c) Germany (d) UK

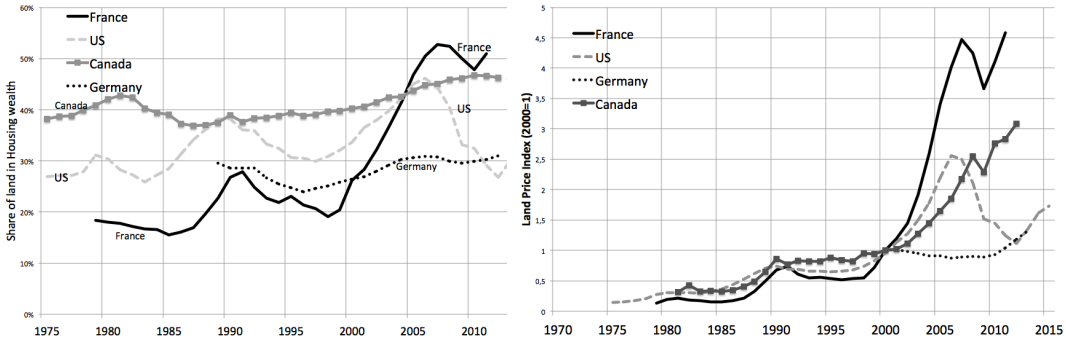
Sources: National accounts (decennial average), Thomas Piketty's database (figure 4.10 of the French edition of the book- See http://piketty.pse.ens.fr/en/capital21c) for Net Foreign Capital, other domestic assets, Agricultural Land and Total Capital Level (uncorrected) Housing capital corrected is calculated based on rents. The value of housing capital is deflated from a price index of real estate and then multiplied by an index of rents (index = 1 in 2010) from OECD data. The choice of a base year different from 2010 would not affect the relative evolutions after correction.

Fig. 3: Capital in four OECD countries with a corrected measure of housing capital based on rental price of housing

This is what we do next: we recalculate housing capital over the period 1970-2010, and obtain capital/income ratios which better measures the evolution of returns on capital. In the US, the corrected K/Y ratio is stable around 400%, while in the original series and over the same period it rose from 320% to 440%. In Canada, it even declined, while in the original series it went from 330% to 400%. In the UK, we still observe a slight increase in K/Y, whereast original series exhibited a strong rise, from 350% to 550%. The only exception to trend correction is Germany. In the original series, K/Y showed a light increase, but now it shows a significant increase, from 300% to 400%. Germany is an interesting case where real estate prices have declined relative to rents, over the period, which results in a raise of the corrected capital income ratio.

2.5 The respective value of land and of buildings

Another important question beyond the rise in price-to-rent ratios over the last decades is the role of land vs. construction costs, since the first component is "almost" a fixed factor (it is not easily reproductible) while the second is the outcome of residential investment. Therefore, before proceeding with a normative analysis, one must obtain insights on the role of each respective component. The perpetual inventory method used to build capital account can be adapted here. As in Davis and Heathcote (2007) (see Appendix A4 for details), we build and report a constant quality price index for France, Canada and Germany. Note that in panels (a) and (b), UK is not avaialble due to a lack of data, the ONS does not provide a decomposition between land and structure. However, series available for land price suggest similar patterns to France.



(a) Share of Land in Housing Capital

(b) Land Price Index

(c) House Price Index

(d) Buildings (Structure) Price Index

Sources:
(a) Piketty (http://piketty.pse.ens.fr/en/capital21c) for France, Canada and Germany and Davis and Heatthcote (2007) - https://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp - last consultation November 2015 for the US.
(b) Authors' computations described in the text for France, Germany and Canada*; Davis and Heatthcote (2007) for the US*
(c) Davis and Heatthcote (2007) for the US; CGEDD from INSEE Notaires index and repeated sales index of Jacques Friggit, downloadable on http://www.cgedd.developpement-durable.gouv.fr/rubrique.php3?id_ rubrique=138 for France; FED Dallas - Mack and Martinez-Garcia (2011) - http://www.dallasfed.org/institute/houseprice for the UK„ Germany and Canada.
(d) Davis and Heatthcote (2007) for the US; INSEE- ICC -http://www.insee.fr/fr/bases-de-donnees/bsweb/serie.asp?idbank=000008630 for France; *BIS - Output Price Index for New Construction (2010)- https://www.gov.uk/government/collections/price-and-cost-indices for the UK; Destatis- Baukostenindizes für Wohngebäude- -https://www.destatis.de/EN/FactsFigures/Indicators/ ShortTermIndicators/Prices/bpr110.html for Germany and Statcan - New House Price Index -Table 327-0046- House only for Canada.*

Fig. 4: Time evolution of the sub-components of housing capital

Figure 4b shows that the main share of the housing prices appreciation of the last decades is driven by land prices rather than dwellings. Similar conclusions on the importance of land price dynamics for a sample of 14 OECD countries were found in Knoll, Schularick, and Steger (2014). It thus appear that most of the appreciation in housing prices comes from an appreciation of land prices while construction costs only went through a moderate increase. The increasing importance of land in some countries appears to be of particular interest might be caused by the lack of innovation in the transportation sector

(Knoll, Schularick, and Steger (2014)) or a concentration of individuals within the national territory. The causes of these dynamics are left for further research.

3 A model of optimal taxation with housing and physical capital

We can summarize the empirical analysis as follows: i) capital is heterogeneous ; ii) the observed rise in the capital income ratio is due to a rise in housing prices (and not to an accumulation of the stock of capital); iii) and housing prices rose mostly due to the rise in prices of land, a quasi-fixed asset. This Section will study the implications for optimal taxation. To assess Piketty's view that a tax on capital (worldwide) is needed to cope with the increase in K/Y and its consequences on inequality, we develop a model where the heterogeneity in capital is central; Taxing productive capital and the housing land in the same way could be strongly misleading.

We extend Judd (1985) by adding housing and land to his framework. Our goal is to find optimal taxation policies for two types of capital: productive capital and housing. Judd's model is well adapted for discussing optimal taxation of capital in the perspective favored by Piketty of a dual world of capitalists and workers. The former are the only owners of capital and land for housing purpose. The workers do not save in the Judd model and then they can only rent.[8]

Note that if in the original framework, capital taxation was undesirable, in the presence of heterogeneous capital,it is possible and even worth to tax some of its components. This depends obviously of the social preference of the social decision maker for redistribution from capitalists to workers and whether we use first or second best instruments. We also briefly discuss the issue of taxation of consumption as well as other assumptions in lands and construct in the last part of this Section.

3.1 Extending Judd's model with housing

Judd (1985), there are two types of agents in mass 1 each, owners of capital (hereafter capitalists) and workers. Following the exposition and notations in Straub and Werning (2014a), capitalists consume an amount C_t and workers consume an amount c_t and subsequently, uppercase variables denote variables for capitalists, whereas lowercase variables denote variables for workers. Capitalists do not work and own physical capital K_t and pay

[8] We will assume that the conditions required for the conclusion of Judd's model, no taxation on capital, are valid. See for a thorough discussion on that point in Straub and Werning (2014a).

workers a market wage w_t. All workers are employed. The production function is denoted $F(K,1) = f(K)$. Capitalists inherit a stock of capital K_0, which depreciates at rate δ; they invest a quantity I_t so that

$$K_{t+1} = K_t(1-\delta) + I_t.$$

Capitalists and workers have a common discount factor denoted by β.

Our addition to the original model is first that capitalists and workers consume both housing and a composite good. Housing is only land (no construct) ; capitalists live in their own housing stock H and supply h in the rental market, with $h + H = \bar{H}$ which is fixed. Capitalists decide on how much housing they want for themselves H and rent the residuals h on the rental market to workers. The utility function of capitalists-landowners is

$$\sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \qquad \text{with} \qquad U(C,H) = \frac{[V(C,H)]^{1-\sigma}}{1-\sigma}$$

and the utility function of worker-tenant is

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \qquad \text{with} \qquad u(c,h) = \frac{[v(c,h)]^{1-\sigma}}{1-\sigma}$$

where $1/\sigma$ is the constant elasticity of subsitution and $V(C,H)$ and $v(c,h)$ are the utility combining a composite consumption good and the consumption of housing. The preferences are supposed to respect the Inada condition. The limit case of no housing studied in Straub and Werning (2014a) of Judd (1985) is when $v(c,h) = v(c)$, and $V(C,H) = V(C)$. We will consider the case where σ is lower than 1, (an intertemporal elasticity of substitution larger than 1) in order than Judd's result of asymptotic zero capital tax rate applies. See Straub and Werning (2014b) for developments on that point.

The resource constraint of the economy is:

$$c_t + C_t + G + K_{t+1} \leq f(K_t) + (1-\delta)K_t \tag{1}$$

where G is the constant government spending. Markets are perfectly competitive, and we define the wage w_t and the gross return on capital R_t^{Kgross}:

$$w_t = f(K_t) - f'(K_t)K_t \tag{2}$$

$$R_t^{Kgross} = f'(K_t) + 1 - \delta \tag{3}$$

The net, after-tax return on capital is

$$R_t^{Knet} = (1 - \tau_{Capital,t}) R_t^{Kgross} \tag{4}$$

where tax on capital are $\tau_{Capital,t}$. We also use the notation R_t^{Hgross} for the gross rent on land and $\tau_{Rent,t}$ for the tax rate of rents. To start in the simplest way, we do not introduce a property tax yet.

3.2 Decentralized economy

In the following, we solve the programs of the three representative agents: capitalist, worker and government.

3.2.1 Capitalist-landowner

The program of the capitalist is, in the absence of government bonds and thus other assets than housing and capital:

$$\underset{C_t, H_t, K_{t+1}}{Max} \sum_{t=0}^{\infty} \beta^t U(C_t, H_t)$$

$$s.t. \qquad C_t + K_{t+1} = R_t^{Kgross}(1 - \tau_{Capital,t}) K_t + R_t^{Hgross}(1 - \tau_{Rent,t})(\bar{H} - H_t)$$

In the general case, we obtain in the decentralized equilibrium the Euler equation:

$$U'_C(C_t, H_t) = \beta R_{t+1}^{Kgross}(1 - \tau_{Capital,t+1}) U'_C(C_{t+1}, H_{t+1}) \tag{5}$$

and intra-period space allocation:

$$U'_H(C_t, H_t) = R_t^{Hgross}(1 - \tau_{Rent,t}) U'_C(C_t, H_t) \tag{6}$$

The condition states that the opportunity cost of living in a better (or bigger) housing is the foregone utility of $R_t^{Hgross}(1 - \tau_{Rent,t})$ units of consumption.[9]

[9] One interesting way to rewrite the capitalist choice is to introduce wealth defined as $A_t = K_t + (H_t + h_t)$. In this case the resource constraint is such that: $C_t + A_{t+1} - \bar{H} = R_t^{Kgross}(1 - \tau_{Capital,t}) K_t + R_t^{Hgross}(1 - \tau_{Rent,t})(H_t + h_t) - H_t R_t^{Hgross}(1 - \tau_{Rent,t})$. If the net return on the two assets were identical, then the equation simplifies to $C_t + A_{t+1} - \bar{H} = R_t^{Knet} A_t - H_t R_t^{Knet}$. It has an interesting implication for our discussion: it shows why housing is a particular asset. Landowners have to sacrifice the foregone rent from living in their asset, the last term of the right hand side of equality, otherwise absent from the standard

3.2.2 Worker-tenant

The worker does not save. They live in h units of housing and consume from their income which is composed of their wage and of a government transfer T. Each worker's programm is:

$$\begin{aligned} &\underset{c_t,h_t}{Max} \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \\ s.t. \qquad & c_t + h_t R_t^{Hgross} = w_t + T_t \end{aligned}$$

The first order condition gives us the following intra-period allocation:

$$u'_h(c_t, h_t) = R_t^{Hgross} u'_c(c_t, h_t) \tag{7}$$

The condition states that the opportunity cost of one more unit of rental unit is the forgone utility of R_t^{Hgross} units of consumption.

3.2.3 Government

The government resource constraint equalizes public spending G and transfers to workers/tenants T_t to total revenues from taxation:

$$\tau_{Capital,t} R_t^{Kgross} K_t + \tau_{Rent,t} R_t^{Hgross} h_t = T_t + G \tag{8}$$

Hereafter in the analysis we will fix $G = 0$ with no implication.

3.3 First best allocation

The planner wants to maximize a weighted average of the utility of each agent, where the weight of capitalists/landowners is given by $\gamma \geq 0$. A weight of zero implies that the social planner only cares about those without property rights on land and capital (we will call this situation Rawlsian), a weight equal to 1 implies it cares equally about the two representative agents.

asset accumulation equation when housing is not modelled.

$$\underset{c_t,C_t,H_t,K_{t+1}}{Max} \sum_{t=0}^{\infty} \beta^t \left[u(c_t, \bar{H} - H_t) + \gamma U(C_t, H_t)\right]$$

$$s.t. \qquad c_t + C_t + G + K_{t+1} = f(K_t) + (1-\delta)K_t$$

$$\beta^t U'_C(C_t, H_t) K_{t+1} \to 0$$

The solution of this standard optimization problem is derived in Appendix D. Define the shadow value of numeraire λ_t as the marginal utility of consumption of the worker-tenant

$$\gamma U'_C(C_t, H_t) = u'_c(c_t, \bar{H} - H_t) = \lambda_t \tag{9}$$

$$u'_h(c_t, \bar{H} - H_t) = \gamma U'_H(C_t, H_t) \tag{10}$$

$$\lambda_t = \beta \lambda_{t+1} \left[f'(K_{t+1}) + (1-\delta)\right] \tag{11}$$

The first equation equalizes this shadow value of the marginal utility of consumption for the capitalist-landowner. It has a lower weight from the social planner for the utility of capitalists-landowners given the presence of γ. The second one equates the marginal utility of the consumption of housing between landowners and tenants, with again the weight from the social planner for that category. The last equation is obtained from the classical dynamic equation for the shadow value ; in the steady-state, it will deliver the steady-state value of K.

3.4 First best optimal taxation: Judd generalized (zero-tax on capital, zero-tax on rents, positive property tax)

In the following, we explain that the first-best (the solution of the planner's problem) can be attained through a positive tax on land ownership, but cannot be attained through taxation of physical capital nor through taxation of rents.

3.4.1 Zero tax on capital

At the optimal level of K, the optimality condition (11) in the steady-state and from the definition of R^{Kgross} that

$$R^{Kgross} = 1/\beta$$

From the first-order conditions on consumption of capitalist-landowners (equation 5), one then gets at steady state:

$$U'_C(C) = (1 - \tau_{Capital})U'_C(C)$$

so that tax on capital is zero or the marginal utility of consumption is zero. If we assume no-satiation, then $\tau_{Capital} = 0$. This is the classical zero taxation result of Judd.

3.4.2 Zero tax on rents

To show the zero tax on rents result, we eliminate R^{Hgross} from the two intra-period allocation conditions of the decentralized solution on housing of both types of agents, equations (6) and (7), to get:

$$\frac{U'_H}{u'_h} = (1 - \tau_{Rent})\frac{U'_C}{u'_c}$$

From the first best allocation defined in equations (9)-(11), one easily obtain: $\frac{U'_H}{u'_h} = \gamma^{-1} = \frac{U'_C}{u'_h}$. Therefore, optimality obviously requires a zero taxation on rents. A tax on rents would distort allocations: it would encourage landowners/capitalists to consume more housing services, and thus lead to sub-optimal consumption of housing services for workers/tenants.[10]

3.4.3 Optimal property tax

Now we extend the previous analysis to introduce a property tax $\tau_{property}$ affecting capitalist-landowners and redistributed to workers/tenants. Since the supply of land is inelastic, the constraint of the landowner/capitalist now includes the product of taxation denoted by $Q = \tau_{property}\bar{H}$. This is a constant term which barely modifies the decentralized solution and only changes the allocation of housing and goods consumption through transfers from landowners/capitalists to tenants-workers. The property tax on housing land is lump-sum and Q satisfies the government constraint resource:

$$G - T = \tau_{Capital}R^{Kgross}K + \tau_{Rent}R^{Hgross}h + Q$$

[10] Note in anticipation of a later discussion that this result actually comes from the fact that tax on rents applies only on the rents themselves, not the implicit rents that landowners/capitalits receive from leaving in housing stock $H = \bar{H} - h$. If one added a tax on implicit rents, the term $1 - \tau_{Rent}$ would show up on the first order condition of landowner/capitalists as well as of workers/tenants (thus on both h and H) and would therefore be equivalent to the property tax introduced later on.

One can also show that

$$Q = R^{Hgross}h + \frac{c - C - w - K + R^{Kgross}K}{2} \tag{12}$$

The correcting term is easier understood in the case where $\gamma = 1$: this implies that $c = C$ and $h = H$. In that case, the right hand side is half of the difference in gross income between the capitalist/landord and the worker/tenant, plus housing expenses. After a few easy steps using in particular equations (2) and (3), one has a formula for the optimal property tax which is an equal sharing rule of total resources of the economy net of depreciation:

$$Q = R^{Hgross}h + f'(K)K - \frac{1}{2}\left[f(K) + \delta K\right]$$

We do not calculate an explicit value of Q for other values of γ here but its value is calculated in the calibration exercise.

3.4.4 Taxing imputed rents as an alternative to a property tax

If instead We allow for the possibility to tax all housing rents, not only actual rents, but also imputed rents on owner-occupiers. This would completely remove the distortionary tax on housing rents in the allocation of housing, as in this case, the capitalist's constraints is

$$\begin{aligned} C_t + K_{t+1} &= R_t^{Kgross}(1 - \tau_{Capital,t}) + R_t^{Hgross}h_t - \tau_{Rent,t}R_t^{Hgross}(h + H) \\ &= R_t^{Kgross}(1 - \tau_{Capital,t}) + R_t^{Hgross}h_t - \tau_{Imp,t}R_t^{Hgross}\overline{H} \end{aligned}$$

To compute this tax, it is sufficient for the government to observe R_t^{Hgross} for owner-occupyers. Assuming perfect observability, both a tax property tax and a tax on rents including imputed rents are lump sum and then first-best instruments. The first order conditions will be the same as in the first best. When land is homogeneous, the tax on land is however easier to compute, and may be preferred to the tax on total (real and imputed) rents.

3.4.5 First best: a summary

A proposition makes stocks of findings:

Proposition: *in a Judd's world where capitalists/landowners own all capital of the economy, an optimal taxation implies :*

- *No taxation of effective rents in the absence of taxation of implicit rents, to avoid distortions on the decisions of supply housing in the rental market*
- *No taxation of physical capital so as to avoid underaccumulation of physical capital (in the case where capital and labor are imperfect substitutes, as emphasized in Straub and Werning 2015)*
- *A positive level of taxation of property. Its level compensates exactly for the absence of property rights on wealth (capital and housing) of tenants and workers and that shares the output and the costs induced by the depreciation of capital. In the case where we put the same weight on landowners and on workers, the level of taxation is such that the transfer ensures equal income between landowners and workers/tenants, as in equation (12).*

3.5 Second best and other possible extensions

All other taxes (tax on capital, tax on rents excluding imputed rents are second best) because they are distorsive. They may however be necessary in specific contexts. For example, in the absence of land registration or cadastre (that is, a systematic administrative record of land property by government authorities), land may be very difficult to tax. Van der Molen shows that 30 to 50 countries have or are about to have detailed land administration systems, while more than 140 countries don't have one. However, most developed countries belong to the group with cadastres. There might also be other types of constraints, such as political constraint as the cap on the property tax as in California proposition 13[11]. Here, we will abstract away from the exact nature of the constraints and explore the extreme alternative where $Q = 0$ but taxation of actual rents (and not imputed rents) is possible.

[11] On June 6, 1978, thirty-five years ago today, California voters passed Proposition 13, which cut property taxes down to 1 percent (for both homestead and commercial property) and limited the growth rate of future assessments to 2 percent.

We will start from the conjecture that capital taxation is not a second best taxation. At this stage, we can only prove it in the Rawlsian case $\gamma = 0$.

3.5.1 Second-best taxation of rents

The social planner can fix an ad valorem tax on rent $R^{gross} = R^{net}(1+\tau)$. Denote by

$$c^* = w^* + T - R^{net}(1+\tau)h$$

and

$$C^* = (R^K - 1)K + R^{net}\left(\overline{H} - H^*\right)$$

the levels of consumption of each representative agent at optimal factor prices. In the steady state, it maximizes the weighted sum of indirect utility function:

$$Max\Sigma(\tau)_{0<\tau<1} = u(c^*, h^*) + \gamma U(C^*, H^*)$$

Under the rationality constraints of workers/tenants and capitalists-owner, one obtains the following two first order conditions and the inelastic supply of total land respectively:

$$(4) R^{gross} = \frac{U'_h(c^*, h)}{U'_c(c^*, h)}$$
$$(5) R^{net} = \frac{U'_H(c^*, \overline{H} - h)}{U'_C(C^*, \overline{H} - h)}$$
$$(3) h^*(R^{net}K(1+\tau), y) = \overline{H} - H(R^{net}, Y)$$

where $T = \tau r^{net}(\overline{H} - H(r^{net}, y))$; $y = w^* + T - R^{net}(1+\tau)h$ and $Y = R^K K + R^{net}h = C + K$ are the total revenue of the two representative agents. We also add a constraint guaranteeing that $h^* > 0$:

$$(6)\, h^* > 0 \leftrightarrow R_N > \frac{U'_H(C_0, \overline{H})}{U'_C(C_0, \overline{H})}$$

where $C_0 = Y - K$ for $h = 0$. It is possible to show, in this context, that:

Proposition. *The optimal rent tax is given by*

$$\tau^*(\alpha) = \frac{1-\alpha}{\epsilon_s} \tag{13}$$

where $\alpha = \gamma U_C/u_c$ is the relative weight of the capitalist/landlord with respect to the worker/tenant in units of marginal utility of consumption and ϵ_s is the supply elasticity of rental housing with respect to the net rent.

The proof is in Appendix. The Proposition yields a simple formula to compute the optimal tax; the higher the social weight of tenants and the lower the elasticity of the supply of rental housing, a positive quantity, the higher the optimal second best tax on rents.

3.5.2 Constructs vs. land

The above discussion on the preference for taxing rents instead of capital dividends only holds however if housing is a fixed factor. If instead, housing is a combination of land and residential investment, then it follows that the absence of distortions on total housing, which is the comparative advantage of the tax on rents, tends to increase and may catch-up with the distortion on capital. However, since there will always be a fixed factor in housing (land), it is likely that the second-best result always holds in a world of equal returns on capital and residential investment.

3.5.3 Productive land

A last extension is when land can be divided into capitalist housing H, tenant housing h and "productive land" (e.g. office space) denoted by H_K, where the sum of H, h and H_K is equal to the fix quantity of land $\bar{H}$. Still in the case of the Rawlsian economy, it is possible to demonstrate that, starting from no taxation, an infinitely small tax on rents reduces less the welfare of tenants/workers than an infinitely small tax on capital, and an infinitely small tax on "productive land", all this at a constant fiscal revenue.

3.6 Simulation exercise

A simulation exercise is proposed to solve numerically the level of taxation a government should set to maximize overall welfare in a competitive economy at the steady state. Note that this exercise is not a calibration, rather a way to summarize the main lessons of previous Sub-sections and show the main comparative statics over γ, the share of capitalists/landlords in the social planner's program.

We make two series of simulations. In the first series, the government has one instruments: the taxation of property. In the second, it can not tax property. In each case, the

problem is solved for different values of γ (the weight of capitalists in the welfare function). With no loss of generality, we assume no government spendings.

The production function is a CES, normalizing the labor force to 1, as

$$Y = A\left(\alpha K^{\rho} + (1-\alpha)\right)^{1/\rho}$$

The coefficient $\rho = 0.5$ implies an elasticity of substitution capital-labor of 2. In order to compute first best and second best instruments depending on the weight γ, we have made a simulation exercise. We have chosen an identical log utility function for both capitalists and workers with a coefficient of land equal to 0.15 leading to a housing share in revenue of .13: housing spendings represent 20 to 25% of revenues for tenants or owner-occupyers (mortgage or implicit rents), and estimates of the share of land are between 40 and 50%. A value of 0.13 is thus the share of land. Alternatively, given that most of owner-occupyers have already repaid their mortgage, 13% is the GDP share of actual rents and mortgage. The exact value one should target is however open to discussion.

Each period, the agents maximize the following utility function:

$$u(c,h) = log(c) + 0.15 \times log(h)$$

3.7 Simulations in the first and second best

3.7.1 Property tax in the first best

As shown in Table 1, the optimal property tax varies with the weight γ. We report the tax rate as the revenue of the tax divided by the fundamental value of housing, that is the present discounted value of future rents. On doing so, we get a number for the tax rate that is above that in a world where housing prices would diverge from the fundamental value: the tax rate calculated on the market value would be lower, precisely by the correction factor of Section 2 (the index of rents to price ratio).

It amounts to 1.8% of the present-discounted value of rents (the fundamental component of housing prices) when the social planner cares equally about capitalists/landlords and workers/tenants. A lower value of the weight of capitalists, for instance $\gamma = 1/2$ implies instead a higher optimal tax rate at 8.2% of the present-discounted value of rents That value increases to more than 20% in the case of the smaller value of $\gamma = 0.025$.

Endogenous variables	$\gamma = 1$	$\gamma = 2/3$	$\gamma = 1/2$	$\gamma = 1/3$	$\gamma = 0.1$	$\gamma = 0.025$
Wage				1.63		
Wage + transfers to tenants/workers	1.81	2.17	2.41	2.71	3.28	3.52
Gross rents $h.R^{gross}$	0.24	0.28	0.31	0.35	0.43	0.46
Conso tenants/workers c	1.57	1.88	2.09	2.35	2.86	3.06
Conso capitalists/owners C	1.57	1.26	1.05	0.79	0.29	0.07
Housing tenants/workers h	0.50	0.60	0.67	0.75	0.91	0.98
Housing capitalists/owner H	0.50	0.40	0.33	0.25	0.09	0.02
Capital stock K				28.65		
K/F				4.77		
Optimal property tax rate as a fraction of the equilibrium value of land	0.018	0.057	0.082	0.11	0.175	0.2051

Tab. 1: Simulations of first best taxation of land. Simulation parameters (benchmark): discount factor $\beta = 0.95$; elasticity of substitution (K, L)=2; scale parameter production $A = 1$; share capital $\alpha_L = 1/3$; depreciation rate of capital $\delta = .1$; log utility of agents and capitalists; share of housing $a_H = 0.15$.

Also note that a tax on consumption can restore the first best as well: in the case $\gamma = 1$, in the absence of tax on property and capital, the first best levels of consumption is reached with a 11.4% tax on consumption. In this case, the government needs to transfer more to workers, e.g. wages+transfers are equal to 2.04 instead of 1.81, since later on the consumption is taxed through VAT. For this reason, the consumption tax rates goes up fast when γ is lower. With $\gamma = 1/2$, the consumption tax rate reaches 50%: the consumption tax base of the capitalists/owners goes down fast and the amount to be redistributed increases to almost 5.

3.7.2 Second best : taxing rents

Alternatively, we can calculate the optimal second best when the government can not tax property but can tax rents or productive capital. It is not possible to redistribute consumption across agents as efficiently as in the first best. For most weights γ, the tax on rents should be favored, since it does not decrease output and wages, but it does distort allocations.

We find that the aggregate quantities of the first best, K, w and K/Y can be achieved through the second best. However, it is not possible to redistribute consumption across agents as efficiently as in the first best.

Endogenous variables	$\gamma = 1$	$\gamma = 2/3$	$\gamma = 1/2$	$\gamma = 1/3$	$\gamma = 0.1$
Wage			1.63		
Wage + transfers to tenants/workers	1.70	1.78	1.81	1.84	1.88
Gross rents $h.R^{gross}$	0.217	0.220	0.222	0.223	0.224
Conso tenants/workers c	1.44	1.47	1.48	1.49	1.50
Conso capitalists/owners C	1.70	1.67	1.66	1.65	1.65
Housing tenants/workers h	0.43	0.40	0.38	0.37	0.36
Housing capitalists/owner H	0.57	0.60	0.62	0.63	0.64
Capital stock K			28.65		
K/F			4.77		
Second best tax on rents as a fraction of flow value of rents	0.12	0.26	0.30	0.34	0.39

Tab. 2: Simulations of second best taxation on rents. Simulation parameters (benchmark): discount factor $\beta = 0.95$; elasticity of substitution (K, L)=2; scale parameter production $A = 1$; share capital $\alpha_L = 1/3$; depreciation rate of capital $\delta = .1$; log utility of agents and capitalists; share of housing $a_H = 0.15$.

To conclude this simulation exercise, when $\gamma = 0.5$ (reflecting a moderate preference of the social planner for tenants-workers), one finds a 8.4% tax on the value of land, and alternatively, a 30% tax on gross rents. COMMENT

3.8 Synthesis

We followed here the line of research represented by Judd (1985) and Chamley (1986)). Both of these seminal papers find that the optimal tax on capital should be equal to 0. However in these articles, capital is not heterogenous, yet taxing land could have a different impact than taxing productive capital.

In another strand of literature, the urban literature, the classical Henry George Theorem states that, at an optimal city size, a land rent tax is the only tax needed; it is sufficient to finance a local public good. This was developed by Arnott and Stiglitz (1979) in this urban economics literature and subsequently reformulated in a dynamic and macroeconomics setting by Mattauch et al. (2013). The authors derive an optimal public investment formula in term of the land rent but where housing service does not enter the utility function.

Our approach is an attempt to account for the specificity of land or housing in such a framework. We found that productive capital, in the setup defined by Judd (1985), does

not need to be taxed, nor wealth as defined in Section 2. Instead, non-productive capital must.

This conclusions rejoins similar attempts that have been made. For example, Stiglitz (2015a) shows that taxing land would increase the consumption of workers, in a setting with only one consumption good and no housing services. Eerola and Määttänen (2013) shows in a model with a representative agent that derives utility from non-housing consumption, housing (composed by its structure and no Land), and leisure that "in the first-best, the tax treatment of business and housing capital should always be the same", and that in "the second-best, in contrast, the optimal tax treatment of housing capital depends on the elasticities of substitution between non housing consumption, housing, and leisure". Note however that none of the above papers take into account that Land is consumed by households through housing services. Some of the existing models include Land and but treat it as a Fixed factor entering the production function, but typically not household's consumption. Other papers include housing consumption but treat it as a structure, ignoring that Land is an important component of this capital . Our paper is the first one considering that land is fixed and enters separately in the utility function of households as housing service.

yet, other cases need to be developed, including the particular role of inheritance and bequests, along the line of Piketty and Saez (ECMA XXX). The very case of overlapping generation is indeed another important dimension. The presence of land and housing and their implication on optimal taxation also remains an open question in such a framework. While the optimal tax rate on capital income seems to be different from zero in OLG models with homogenous capital (Conesa, Kitao, and Krueger (2009)). The specificity of land and housing has also attracted some attention with the same shortcomings as before. For example, Chamley and Wright (1987) analyze the fiscal incidence of a Land tax where land does not enter the consumption function. In a similar setting, Kim and Lee (1997) suggest that a land tax is likely to create dynamic inefficiency distorting the no arbitrage condition. More recently, Nakajima (2010)developed an overlapping generation models taking into account the specific tax treatment of housing income and the difference across occupation status. In such a framework when imputed rents are not taxed, the author finds that Conesa et al's results do not hold and that "malleable" capital should almost not be taxed. The intuition is similar as Eerola and Määttänen (2013): housing and productive capital shouldn't be taxed differently *provided that land is not a component of housing*). In this framework housing is only composed by the structure and land is ignored, while we studied the polar case in the previous Section.

4 Conclusion

Résumé Housing as measured in national account could in part be an illusion if housing prices are away from the present discounted value of rents. The recent surge in the value of housing at least should lead to the question of which measure is the best. The data presented here show that for most countries, the positive trend in K/Y is replaced by a much flatter or even totally flat curve over the last decades, with the notable exception of Germany.

To better see this point, let us ask one question. What inequality would there be if each household owned one painting and kept it throughout its lifetime? The wealthiest households might own a pricey Manet or Kandinsky. The poorest might own a painting by a local artist. Now, if the price of art increased uniformly, would this contribute to an explosion of inequality in the sense of a divergent and exponential accumulation of capital? The answer is clearly it would not. Even if the paintings could be rented and generate some income for the owners, this revenue would need to increase proportionally to the selling price and faster than the labor earnings for capital to increase faster than total income.

The normative implications are subtle. If one is to interpret K/Y as an indicator of the capacity of an economy to provide returns to capitalists, our measure correcting for the rent/price divergence is better. If one is to interpret K/Y as an indicator of wealth relative to income, then the national accounting measure based on current price is a better measure.

Our analysis of the composition of the capital income ratio led us to highlight the heterogeneity of the types of capital. In terms of optimal taxation, it is crucial to distinguish between productive capital and housing (which is in part -and maybe mainly- made of land). Indeed, taxing land (or property) enables to make transfers from landowners to workers/tenants and to increase the income of the latter. However, taxing capital reduces total output and does not enable to increase the income of the workers.

In that case, our theory extending Judd's theory optimal capital taxation to housing implies that indeed, housing wealth must be taxed; we exhibit a formula for the optimal wealth tax that only depends on rents in the housing rental market. The analysis led us to conclude (more optimistically, or perhaps just less pessimistically) that long-run trends in wealth inequality are not dramatic.

Beyond, long term comparisons of housing capital do not necessarily make sense: currently, homeowners are the majority of households (56% in France, 70% in the UK). In

1950, this proportion was respectively 37% and 30%[12]. This is even further away from Karl Marx's description of 19th century England where, for 20 million inhabitants, he only counted 36,000 homeowners. Overall, the appropriate level of taxation of housing wealth, smoothly evolving as its current rental price, is advocated. Issues of inheritance and realized capital gain are however important. They are not addressed here because most of the value of housing capital (83% in France) belongs to owner-occupiers, who do not realize capital gain from a rise in sale prices on average. The debate on this issue requires a specific treatment and is left to future work.

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A The measurement of housing capital by statistical agencies : the example of INSEE

Piketty and Zucman's measurement of capital "*follows the most recent international guidelines as set forth in the 2008 System of National Accounts*" (Piketty and Zucman, 2013). They use series of statistical agencies to measure their capital/income ratio. The measurement of capital, and in particular housing capital, follows a particular methodology which is summarised and highlight to justify the adjustments proposed here.

According to OECD (2001), the framework used in several countries is the perpetual inventory methodology (PIM) briefly described by Piketty and Zucman: "*The goal of the perpetual inventory method (PIM) is to approximate the current market value of a number of capital assets when it cannot be directly observed. The general idea is that this value can be approximated by cumulating past investment flows and making suitable price adjustments.*" (Piketty and Zucman, 2013).

Our main interest will be in the "*suitable price adjustments*" for the housing market, namely the price index used to evaluate the value of the volume of capital. The perpetual inventory methodology used by the French institute of statistics (INSEE) to provide an estimate of the national housing capital stock is described as follows.

A.1 Measurement of the stock (volume) of housing capital in a reference year

"*In France, housing capital is estimated through a first step of estimating the total stock and value of housing in a reference year (1988). INSEE then follows over time the evolution of the number of buildings from aggregate housing investments, deflated the housing construction index; and the evolution of land with constructs using the evolution of the surface area covered by housing units and the development of the surface area covered by houses. To get the year- by-year value of housing capital stock, the above- described volume is multiplied by the price index of existing housing. Furthermore, new buildings were also evaluated at the price of existing housing units. Hence, housing capital follows year-to-year evolutions of housing prices, by contraction.*"

The assessment of the housing capital starts indeed from an initial survey-based assessment in the reference year. Two main surveys of 1988 (French Housing Survey and Survey on building land[13] were used to assess the housing capital stock divided between buildings and their underlying land.

From the surveys we calculate:

$$K_{1988}^{Housing} = K_{1988}^{Dwelling} + K_{1988}^{land}$$

where the housing capital which is broken down between dwellings and land.

A.2 The time evolution of the volume of housing capital

As we saw, housing capital is divided between land and dwellings, the evolutions of which are followed separately.

For dwellings, the stock in the following years is computed iteratively from the initial year[14] taking into account depreciation (δ)[15] and yearly capital increments (Gross Fixed Capital Formation (GFCF)) deflated by construction costs index (CCI)[16]:

[13] In 1988, two surveys are available: the Enquête Logement (EL) and information gathered by tax authority on land price (from the IMO file from the Direction Générale des Impôts which provides for the last time in 1988 an evaluation of the price of building land) (Baron, 2008).

[14] For each of these years after 1988, the value of the stock of housing will be calculated step by step.(Baron, 2008).

[15] From the CCF rate (that is, the depreciation rate) we compute net capital.(Baron, 2008)

[16] To evaluate the net capital of 1988 at 2000s prices, we deflate using the construction cost index as a price index for buildings (Baron, 2008).

$$Vol(K_{n+1}^{Dwelling}) = (1-\delta)Vol(K_n^{Dwelling}) + \frac{GFCF_{n+1}}{CCI_{n+1}}$$

For land, the statistical agency similarly follows the changes in developed land on the national territory with respect to the reference year using an index of the surface developed (S):

$$Vol(K_{n+1}^{land}) = S_{n+1} \times K_{1988}^{land}$$

Finally, the volume of housing capital is just the addition of both series for each year at the price of the reference year, here 1988[17]:

$$Vol(K_{n+1}^{Housing}) = Vol(K_{n+1}^{Dwelling}) + Vol(K_{n+1}^{land})$$

A.3 Pricing of the evolution of housing capital and decomposition

The volume of capital is then obtained multiplying its volume by the house price index[18] (HP):

$$K_{n+1}^{Housing} = HP_{n+1} \times Vol(K_{n+1}^{Housing})$$

This value (used in *Capital in the 21st Century*) is then broken down into land and dwellings. This step appears to be the most important to understand our reasoning since it shows that housing capital is evaluated at the market price of year n+1. The Value of the structure alone can be recovered by multiplying the Volume of dwelling with the Construction Price Index:

$$K_{n+1}^{Dwelling} = Vol(K_{n+1}^{Dwelling}) \times CCI_{n+1}$$

The developed Land Capital is the residual:

$$K_{n+1}^{land} = K_{n+1}^{Housing} - K_{n+1}^{Dwelling}$$

[17] E.g., the volume of housing capital at the end of 1989 (at 1988 prices) is found adding the net capital flow of end 1989 evaluated at 1988 prices (Baron, 2008).

[18] "The housing patrimony at the end of 1989 is obtained multiplying by the price index for the whole France " (Baron, 2008).

A.4 The Constant Quality Land Price index

As we can notice, the perpetual inventory method has an interesting property: it allows to decompose the housing stock between the value of the structure and Land. As a consequence, Davis and Heathcote (2007), are able to derive a constant quality land price index. We reproduce their methodology for France, Canada and Germany to derive a Constant Quality Land Price index. We start from the standard decomposition of the housing stock between Land and Structure.

$$HP_t \times Vol(K_t^{Housing}) = CCI_t \times K_t^{Dwelling} + p_t^l \times Vol(K_t^{land}) \tag{14}$$

where the price of a house $HP_t \times Vol(K_t^{Housing})$ is the sum of the structure $CCI_t \times K_t^{Dwelling}$ and the land $(p_t^l \times K_t^{land})$. From this equation the authors propose to construct a constant quality land price index as follows:

$$p_{t+1}^l = p_t^l(\frac{1}{w_t^l}(\frac{HP_{t+1}}{HP_t} - (1 - w_t^l)\frac{CCI_{t+1}}{CCI_t})) \tag{15}$$

where $w_t = \frac{K_t^{land}}{K_t^{Housing}}$ is the share of the land in the value of homes as plotted in Figure 4a.

B Reassessing the value of the housing capital: using a rent index vs. a housing index

B.1 Alternative measurement methods of housing capital

The methodology described above and used for France is however subject to debates. As it stands, it measures wealth to the extent that the current price of housing reflects the liquidation value of the stock of housing for households who would decide to sell and consume it. However, statisticians already pointed that several alternative approaches might be used to assess the value of the capital stock. In a recent report, the Canadian statistical agency pointed three different options[19]:

[19] http://www.statcan.gc.ca/pub/13-605-x/2013002/article/11782-eng.htm

1. *Cost approach: based on the idea that the value of an existing property is the value of the land plus the replacement cost of the structure, hence disconnected from the current housing price.*

2. *Sales comparison approach: it uses sales prices as evidence of the value of similar properties. The sales comparison approach is most suitable when there are sufficient sales of similar properties. This measure is closer to the one described in Appendix A, although not identical since the price index might mask divergences in the price of different categories of dwellings.*

3. *The income approach: used to estimate the rental income from a property and capitalise it into an estimate of present value. This is the one which precisely makes more sense in the logic of the study of Piketty, as it properly proxies the dynamics of divergence of wealth from the ownership of housing capital.*

In our paper, we focus on the difference between the sales comparison and the income approach which may have implication on the type of index that should be used to assess the market value of the capital stock.

B.2 Our choice of a correction method

B.2.1 Substituting the housing index by the rent index

As we already saw, the value of the housing capital in year n+1 is given by the following equation:

$$K_{n+1}^{Housing} = HP_{n+1} \times Vol(K_{n+1}^{Housing})$$

We divide by the national income of that year to get:

$$\frac{K_{n+1}^{Housing}}{Y_{n+1}} = \frac{HP_{n+1} \times Vol(K_{n+1}^{Housing})}{Y_{n+1}}$$

Our correction consists in multiplying by the rent/ prices ratio:

$$corrected(\frac{K_{n+1}^{Housing}}{Y_{n+1}}) = \frac{HP_{n+1} \times Vol(K_{n+1}^{Housing})}{Y_{n+1}} \times \frac{R_{n+1}}{HP_{n+1}}$$

$$corrected(\frac{K_{n+1}^{Housing}}{Y_{n+1}}) = \frac{R_{n+1} \times Vol(K_{n+1}^{Housing})}{Y_{n+1}}$$

B.2.2 An illustration with the French case

In this part, we propose to illustrate our method of correction presenting our computations for the french case. In table A.1, the modified series is obtained multiplying the original series by the rent/price index.

period	Agricultural Land	Other domestic capital assets	Net foreign assets	Original housing	rent price ratio	Corrected Housing
1950-1959	0.450	1.581	0.033	0.850	3.030	2.576
1970-1979	0.421	1.848	0.142	1.221	1.161	1.417
1980-1989	0.296	1.703	0.157	1.595	1.003	1.599
1990-1999	0.158	1.643	0.100	1.777	1.063	1.889
2000-2009	0.130	2.044	-0.002	2.853	0.740	2.112
2010	0.122	2.345	-0.127	3.715	0.615	2.284

Tab. A.1: Correction of the housing capital

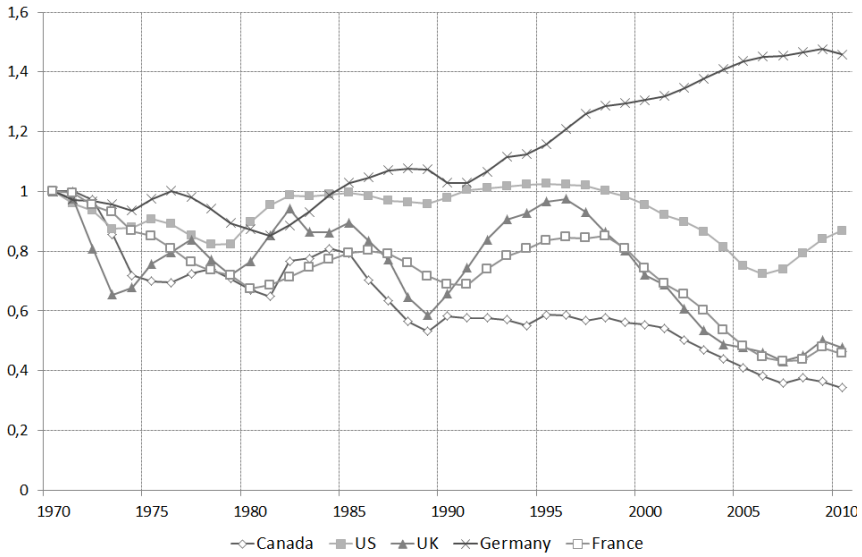
B.2.3 Correction in other countries and rent to price ratios in OECD countries

A similar methodology based on market prices is used in the United Kingdom (Barnard, 2012), in the United States (Cagetti et al., 2014) and in Canada[20]. Hence, we will apply the same correction method to these countries based on the rent-to-price ratios displayed in Figure A.1.

C The relation between the price of housing capital and the stability of its returns

Another way to rephrase the argument of Section 2 in theoretical terms is to make a distinction between the value of housing capital at the current housing price and its return

[20] http://www.statcan.gc.ca/pub/13-605-x/2013002/article/11782-eng.htm



Sources: OECD. Economic Outlook Database and Main Indicators database and www.econ.queensu.ca/files/other/House_Price_indices%20(OECD).xls

Fig. A.1: Rent-to-price ratios in various OECD countries

as a percentage of total national income. The latter is precisely the value of rents divided by the price. If the rise of the capital income ratio is only due to housing prices, then the share of capital income in total income should remain stable in the economy.

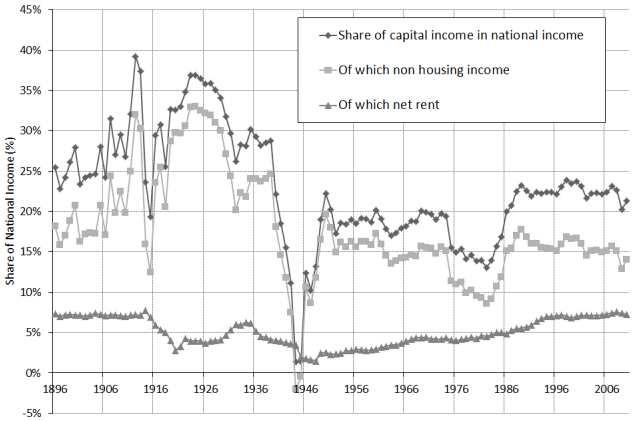
In *Capital in the 21st Century*, the author insists on the strong positive trend of the capital/income ratio over the last three decades in France as seen from Figure 1 where we also considered housing capital represented with the dashed line. The author cites the "first fundamental law of capitalism": if the return to capital is r, the share of income to capital in total income is α and the ratio K/Y is β, then:

$$\alpha = r \times \beta.$$

The secular rise in the value of β could lead us to think that the share of capital in total income must have increased as well. This is not the case however as can be seen in Figure A.2[21], *which shows the contrary, a secular decline in returns to capital in France (upper*

[21] Note that the implicit income of homeowners (56% of the population) is estimated based on rental and shelter costs of tenants in the private rental sector (20%) and does not take into account residents in the social housing sector (17%). Their rents are clearly lower than those in the private sector and the implicit income of homeowners is most likely already over evaluated.

curve), a secular decline in the returns to other capital and a moderate increase in net rents in national income since 1948, mostly a return to the values at the beginning of the 20th century which compensates for the decline of the returns to physical capital (not housing)[22]. If the rise in β is only due to housing prices, it is perfectly compensated by the decrease in returns $r_{\text{housing capital}} = \frac{\text{Value of rents}}{\text{Housing price}}$, and the share of capital income in total capital remains stable. In a purely accounting sense, a rise in housing prices has no direct impact on the share of capital income in total incomes. This share depends only on rent and on the quantity and quality of housing. The rise of housing prices increases wealth inequality, but has no impact on the sharing of total output between workers and owner of capital.



Sources: INSEE and Piketty (http://piketty.pse.ens.fr/en/capital21c).
Note: The upper series is the addition of implicit and monetary rents (lowest series) and non housing capital income (intermediate series). From the 1990s, we can see a decrease in capital income which is compensated by a rise in the amount of rents.

Fig. A.2: Decomposition of the share of capital income in French national income

Theory AppendixFirst best allocation

The solution to the maximisation problem defined in Section 4 is given, denoting by Σ the maximand (net of its multipliers):

[22] Also note that returns to capital series for France used in *Capital in the 21st Century* are based on rents estimated from the 2002 French Enquête Logement, which has been known to artificially overestimate the level of rent over the recent period. A correction in the 2006 Enquête Logement led to a downward reevaluation of this increase in the share of rents in national revenue over the last period. Figure A.2 uses the rent data corrected for this revision of national accounts. We thank Jacques Friggit for enlightening us on this point.

$$\begin{aligned}
\frac{\partial \Sigma}{\partial c_t} = 0 &\Leftrightarrow \lambda_t = u'_c(c_t, \bar{H} - H_t) \\
\frac{\partial \Sigma}{\partial C_t} = 0 &\Leftrightarrow \lambda_t = \gamma U'_C(C_t, H_t) \\
\frac{\partial \Sigma}{\partial H_t} = 0 &\Leftrightarrow u'_h(c_t, \bar{H} - H_t) = \gamma U'_H(C_t, H_t) \\
\frac{\partial \Sigma}{\partial K_{t+1}} = 0 &\Leftrightarrow \lambda_t = \beta \lambda_{t+1} \left(f'(K_{t+1}) + (1 - \delta) \right)
\end{aligned}$$

C.1 Optimal property tax

Substract the resource constraint for capitalists to the resource constraint for workers leads to:

$$\begin{aligned}
c + G - C - K + hR^{Hgross} + R^{Kgross}(1 - \tau_{Capital})K + R^{Hgross}(1 - \tau_{Rent})h \\
= w + \tau_{Capital} R^{Kgross} K + \tau_{Rent} h R^{Hgross} + 2Q \\
\Leftrightarrow 2Q = c - C - w - K + R^{Kgross}(1 - 2\tau_{Capital})K + 2R^{Hgross}(1 - \tau_{Rent})h \\
\Leftrightarrow Q = R^{Hgross}(1 - \tau_{Rent})h + \frac{c - C - w - K + R^{Kgross}(1 - 2\tau_{Capital})K}{2}
\end{aligned}$$

We also know that the optimal allocation of space implies no tax on rents and on capital. The equation therefore simplifies to:

$$Q = R^{Hgross} h + \frac{c - C - w - K + R^{Kgross} K}{2}$$

C.2 Second best allocation

Start from the general case. The solution to the problem defined in Section 3. 6 leads to the first order condition, using more compact notations $R'_N = \frac{\partial R^{net}}{\partial \tau}$ and $h'_{R_N} = \frac{\partial h}{\partial R^{net}}$:

$$u_c(R^{net} h + \tau(R'_N h + R^{net} h'_{R_N} R'_N)) - R'_N (1 + \tau) h - R^{net} h) + \gamma U_C.(R'_N \overline{H} - R'_N H^*) = 0$$

or

$$u_c(\tau(R^{net} h'_{R_N} R'_N) - R'_N h) + \gamma U_C(R'_N h)) = 0$$

Dividing by $u_c \neq 0$ and defining $\alpha < 1$ the social marginal welfare weight of the capitalist with respect to the worker, one has

$$\tau(R^{net} h'_{R_N} R'_N) - R'_N h + \alpha R'_N h = 0$$

and therefore

$$\tau^*(\alpha) = \frac{1-\alpha}{\epsilon_s} \tag{16}$$

where $\alpha = \gamma U_C / u_c$and $\epsilon_s = h'_{R_N} R^{net}/\tau$.

Proposition. *The optimal rent tax is given by formula (16)*

D Simulations : robustness check

In the next table, we have increased the weight of housing in the utility function : $a_H = 0.25$. We observe that the transfers to the tenants/workers are higher than in the initial case.

Endogenous variables	$\gamma = 1$	$\gamma = 2/3$	$\gamma = 1/2$	$\gamma = 1/3$	$\gamma = 0.1$	$\gamma = 0.025$
Wage				1.63		
Wage + transfers to tenants/workers	1.96	2.36	2.62	2.95	3.56	3.83
Gross rents $h.R^{gross}$	0.39	0.47	0.52	0.59	0.71	0.77
Conso tenants/workers c	1.57	1.88	2.09	2.35	2.86	3.06
Conso capitalists/owners C	1.57	1.26	1.05	0.79	0.29	0.07
Housing tenants/workers h	0.50	0.60	0.67	0.75	0.91	0.98
Housing capitalists/owner H	0.50	0.40	0.33	0.25	0.09	0.02
Capital stock K				28.65		
K/F				4.77		
Optimal property tax rate as a fraction of the equilibrium value of land	0.02	0.046	0.06	0.08	0.12	0.14

Tab. A.2: Simulations of first best taxation of land. Simulation parameters (benchmark): discount factor $\beta = 0.95$; elasticity of substitution (K, L)=2; scale parameter production $A = 1$; share capital $\alpha_L = 1/3$; depreciation rate of capital $\delta = .1$; log utility of agents and capitalists; share of housing $a_H = 0.25$.