# The Choice Channel of Financial Innovation*

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**Abstract**

Financial innovations in recent decades have vastly expanded investors' portfolio choice. We theoretically analyze the effect of gerater choice on investors' savings and asset returns in a model in which investors have possibly heterogeneous beliefs about asset payoffs. Under mild assumptions, we establish a *choice channel* by which greater portfolio choice increases investors' (perceived) return from saving, and induces them to save more. We then investigate the asset pricing implications of the choice channel, which depend on the type of financial innovation. Our main result shows that *portfolio customization*, which we capture with improved ability to trade risky assets other than the market portfolio, reduces the expected return on every asset. This result is consistent with the decline in the risk-free interest rate since the early 1980s, and is in contrast with the "precautionary savings" literature that would make the opposite prediction. In contrast, *market participation* (improved ability to trade the market portfolio) reduces the risk premia but typically increases the risk-free rate. We also analyze *securitization*, which we capture with a relaxation of constraints to issue risk-free debt in an environment in which some investors, which we label "emerging markets," have a high demand for safe assets. Securitization mitigates the decline in the interest rate driven by emerging markets' savings, and exacerbates the global savings imbalances.

**JEL Classification:** E21, E43, E44, G11, G12

**Keywords:** financial innovation, choice channel, customization, participation, securitization, risk premium, interest rate, secular stagnation, belief disagreements, speculation

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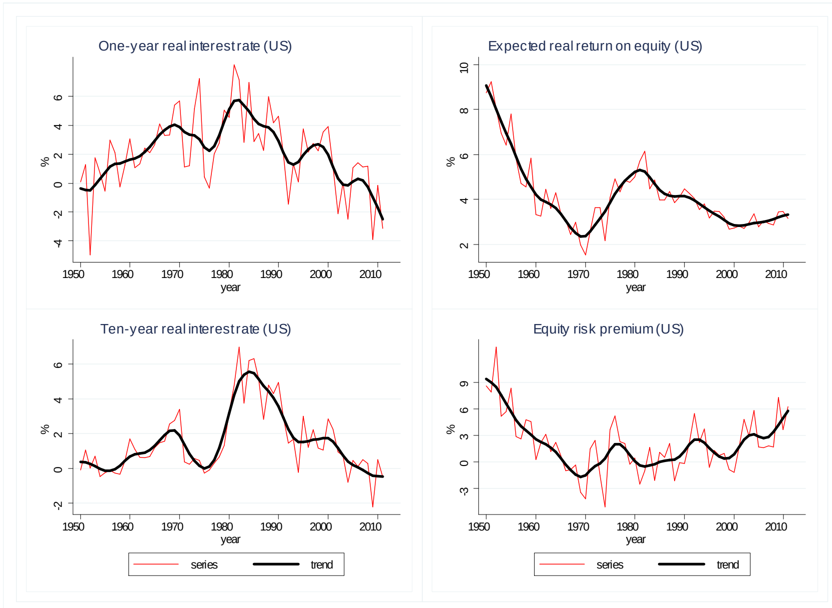


Figure 1: The plots are based on the authors' calculations using the methodology described in Blanchard (1993) and annual returns data from Robert Shiller (available at http://www.econ.yale.edu/ shiller/data.htm). The expected return on equity is calculated by using the dividend yield and the (model based) expected dividend growth.

# 1 Introduction

A key macroeconomic fact of recent decades is the decline in returns of various asset classes. The panels on the left side of Figure 1 illustrate that the short and long term risk-free real interest rates in the US have been on an increasing trend before the 1980s, but they have been declining since the early 1980s. The same trends also apply to the interest rates in most developed economies. The panels on the right side of Figure 1 illustrate the returns on risky assets display similar trends with some differences. The (model based) expected return on US stocks has been declining for most of the post-war period, except for an upwards swing in the 1970s. The equity premium—the difference between the expected return on equity and the risk-free rate—also declined in the earlier half of the period, but it appears to be sideways or slightly increasing in recent decades.[1]

Why are asset returns declining? This question is important for macroeconomic policy, because low interest rates can induce or exacerbate liquidity traps in which monetary policy is constrained by the zero lower bound (Krugman (1998), and Eggertsson and Woodford (2003)). Recent research has emphasized various factors that might have contributed to

[1] These trends in the expected risk premium have also been documented in Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2001), Pástor and Stambaugh (2001), and Fama and French (2002). King and Low (2014) also document the declining trend in real interest rates.

low returns. An aging population or rising income inequality in developed economies might have increased the demand for savings, thereby exerting downward pressure on returns (see, for instance, Summers (2014), Eggertsson and Mehrotra (2014)). High demand for assets—especially safe assets—by fast-growing emerging markets might have also contributed to this pattern (see, for instance, Bernanke (2005), Caballero (2006), and Caballero, Farhi, and Gourinchas (2008)).[2] In this paper, we supplement these explanations for high savings and low returns with a new rationale: financial innovation that expands investors' portfolio choice. Our analysis can help to explain, among other things, why the interest rates have been declining since the 1980s but not in earlier decades.

Our starting point is that financial innovation in the post-war years has vastly increased the trading opportunities in financial markets. The changes has been especially dramatic since the early 1980s. In the mid-1970s, the round trip cost of buying and selling a typical stock was about 5% of the stock price (Turley (2012)), whereas it declined to about few cents in recent years. New financial assets, such as futures, options, and other derivatives, enabled trades that were either impossible or too costly in previous years. While these changes were driven by multiple factors, the information technology revolution—which accelerated in the early 1980s—arguably played an important role.

These developments have in turn increased households' portfolio choice. In the 1950s, a typical household in the US arguably did not have much choice in constructing her savings portfolio. She could hold bank deposits, and perhaps buy a house (or durable assets), but she did not have access to many other financial assets. These days, a comparable household can also hold stocks and other risky assets. Figure 2 shows that the stock market participation has indeed increased from about 10% of households in the early 1950s to more than 50% by the end of the 1990s. More importantly, households in recent years can also hold highly customized portfolios. They can choose from a plethora of mutual funds, hedge funds, retirement funds, and ETFs. They can also construct their own portfolios by trading individual stocks, bonds, or more exotic derivatives. Figure 3 shows that mutual funds and exchange traded derivatives, both of which facilitate portfolio customization, has been growing rapidly since the early 1980s (until the recent financial crisis).

Motivated by these observations, we theoretically investigate how financial innovation that increases portfolio choice affects investors' savings and asset prices. In our model, investors with standard Epstein-Zin preferences hold assets to transfer wealth to a future period. Investors optimally choose savings portfolios that consist of the risk-free asset and various risky assets. Each investor has access to the risk-free asset, but investors have limited

[2] Other rationales include increased uncertainty (see Caballero and Farhi (2014)), a slowdown in productivity, or a reduction in the relative price of investment goods (see Summers (2014)).

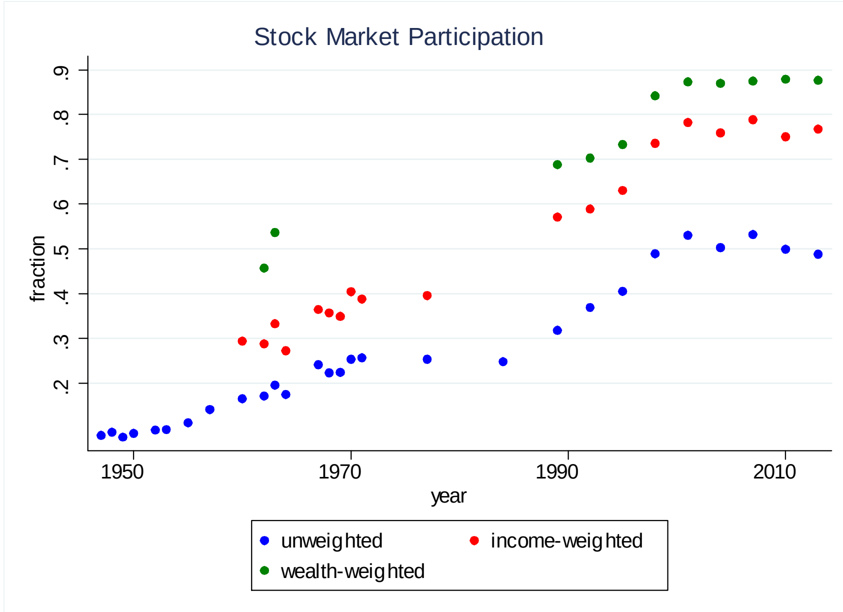


Figure 2: The figure shows the fraction of households in the US that invest in stocks over the period 1947-2013. The plots are based on the authors' calculations using data from the Michigan Survey of Consumer Finances (1947-1977), the PSID (1984), and the Survey of Consumer Finances (1989-2013).

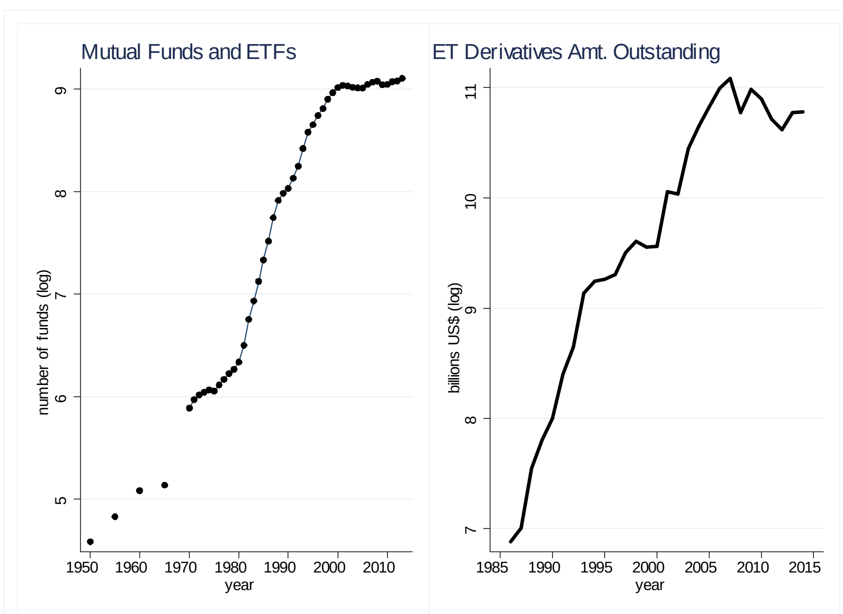


Figure 3: The left panel illustrates the changes in the number of mutual funds and the exchange traded funds in the US (source: Investment Company Institute). The right panel illustrates the changes in outstanding exchange traded derivatives (source: the Bank for International Settlements). Amounts are in constant year 2000 US dollars.

and (possibly) heterogeneous access to risky assets. We capture financial innovation as an improvement in investors' access to risky assets.

Our key assumption is that investors have heterogeneous beliefs about the payoffs of risky financial assets, which provides one rationale for portfolio customization. In fact, in our setup, as well as in many related models, investors with homogeneous beliefs (and without background risks) would only need one risky asset—namely, the market portfolio—to construct their optimal portfolios. Hence, the proliferation of investment funds with heterogeneous strategies is prima facie consistent with our heterogeneous beliefs assumption. The assumption can also be viewed as capturing some other sources of heterogeneity in investors' asset valuations, such as institutional restrictions to hold certain types of assets.

In this setup, we first characterize an investor's savings in partial equilibrium (that is, taking the asset prices as given). Under relatively weak assumptions, we establish a *choice channel* by which financial innovation induces the investor to save more. Intuitively, greater portfolio choice increases the investor's (perceived) certainty-equivalent return on her savings portfolio. This creates substitution and income effects that are similar to those created by an increase in the risk-free interest rate. When the substitution effect dominates, which we believe is the empirically relevant case for the majority of investors (see Section 4.2), then the investor increases her savings. With greater choice in financial markets, saving becomes more attractive, and investors do more of it.

Although the choice channel sounds intuitive, it counters a strand of the "precautionary savings" literature that makes the opposite prediction. This view posits that uninsured background risks induce agents to save for precautionary reasons. The implication is that financial innovation that improves the sharing of background risks should reduce savings, and increase the risk-free interest rate in equilibrium (see Section 1.1 for references and further discussion). We formally replicate this result in our setting and clarify the differences with our result. The choice channel dominates the precautionary savings channel as long as investors do not face significant background risks, or do not consider these risks when they make portfolio decisions. As we discuss further in Section 1.1, empirical studies often find that investors hold financial portfolios that are quite different from what would be required to hedge background risks. Motivated by these studies, as well as the declining trend in the risk-free interest rate, we shut down the precautionary channel in much of our analysis.

We also analyze the general equilibrium implications of the choice channel. To this end, we consider a canonical case of our model in which the available financial assets consist of a market portfolio of all cash flows, and several other risky assets in zero net supply. For analytical tractability, we assume a log-normal approximation for portfolio returns (as in Campbell and Viceira (2002)). The choice channel, which increases investors' savings in partial equilibrium, exerts an upwards pressure on asset prices in general equilibrium. However, financial innovation might also generate relative price effects that interfere with the choice channel. The net effect depends on the type of innovation, which we explore in empirically relevant settings.

Our main result concerns portfolio customization, which we capture with improved access to an arbitrary subset of the risky assets other than the market portfolio. Under mild symmetry assumptions on investors' beliefs, we show that greater customization reduces the risk-free rate while leaving the risk premia unchanged. In particular, portfolio customization reduces the expected return on each (risk-free or risky) asset. This result suggests that financial instruments that facilitate portfolio customization, which have become widespread starting in the early 1980s (see Figure 3), can be a contributing factor to the secular decline in the risk-free interest rate as well as the expected return on equity since the early 1980s (see Figure 1).

To understand the intuition, imagine financial assets as a forest that contains several trees. The trees could be a metaphor for individual stocks, industries, or mutual funds with different strategies or styles. Suppose each investor is optimistic for certain trees' yields relative to the average investor. Customization enables investors to trade individual trees as opposed to buying or selling claims on the forest. As a response, investors expand their investments in the trees they like the most, while reducing their positions in the trees they like less. Moreover, for every (relatively) optimistic investor that buys a particular tree, there are (relatively) pessimistic investors that sell that tree. Consequently, investors collectively like the forest more, in view of the choice channel, but the relative appeal of individual trees remain unchanged. We show that this logic is general, and implies that greater customization increases the valuation (and reduces the expected return) of each tree in tandem.

We also analyze the pricing implications of market participation, which provides a useful contrast with customization. We capture participation as an improvement in investors' access to the market portfolio. This tends to increase asset prices in view of the choice channel, but it also increases the demand for risky assets relative to the safe asset. We find that these relative demand effects are strong, whereas the choice channel is relatively weak in this context. In particular, for empirically relevant parameters, greater participation increases the risk-free rate—in contrast to customization. We also find that participation reduces the risk premium (and the expected return) on the market portfolio. These results suggest that improvements in market participation between the 1950s and the early 1980s (see Figure 2) can be a contributing factor to the trends in the risk-free rate and the equity premium over this period (see Figure 1).

We also analyze securitization, which is another important financial innovation in recent

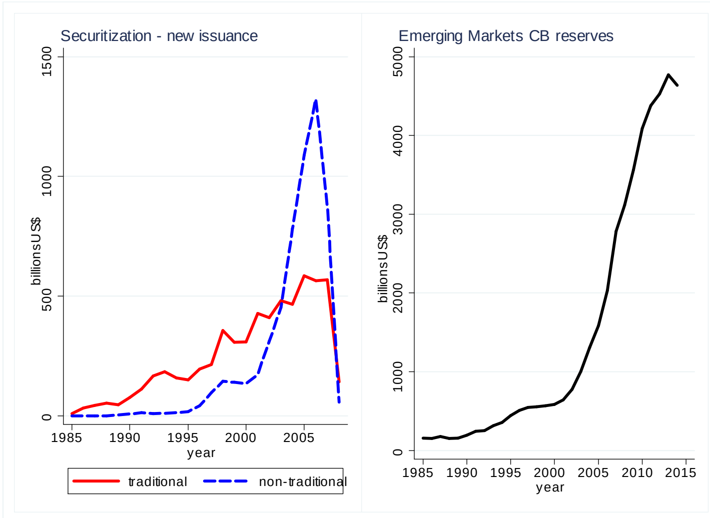


Figure 4: The left panel is from Chernenko, Hanson, and Sunderam (2014), based on data from the Securities Data Company. Traditional securitizations include commercial mortgage backed securities (MBS), prime residential MBS, and asset backed securities. Nontraditional securitizations include nonprime residential MBS and collateral debt obligations. The right panel illustrates the evolution of the reserve assets held by the central banks in emerging markets (source: the World Bank). Amounts are in constant year 2000 US dollars.

decades that converts traditionally illiquid assets (such as mortgages) into marketable securities. Nontraditional forms of securitization (such as CDOs) additionally split the securities into safer and riskier tranches. This type of securitization arguably facilitates portfolio choice by meeting investors' demand for safe assets. In fact, a recent literature has argued that these securities were introduced precisely to meet the growing demand for safety from emerging markets and other sources (see, for instance, Gennaioli, Shleifer, and Vishny (2012)). Figure 4 shows that the growth of nontraditional securitization in early 2000s has indeed coincided with a dramatic increase in safe asset holdings by emerging market central banks.

We capture the trends in Figure 4 by expanding our model in two dimensions. First, we introduce "emerging market" investors that have a preference for safe assets, in addition to having a relatively high demand for assets. An increase in the relative mass of these investors decreases the risk-free rate, consistent with the conventional wisdom, while also increasing the risk premium. Second, we also introduce securitization as a relaxation of (other) investors' constraints to issue safe debt so as to make leveraged investments in risky assets. We show that greater securitization mitigates the pricing effects of high savings by emerging markets. In particular, the interest rate declines less compared to the counterfactual without securitization. By keeping the interest rate relatively high, securitization also increases the emerging market investors' (already high) savings. Intuitively, if the US didn't provide additional safe assets, the interest rates would decline considerably, and the emerging market investors would be reluctant to save as much as they did. Hence, in this context, the choice channel manifests itself as an increase in savings by emerging markets (or more broadly, investors that demand safe assets). In fact, we find that securitization can reduce other investors' savings in equilibrium, and exacerbate their current account deficits. These results suggest that the collapse of securitization in the aftermath of the recent financial crisis (see Figure 4) can be a contributing factor to the sharper decline in the riskless rate in recent years (see Figure 1), as well as the recent decline in the US current account deficit.

Our analysis with emerging market investors and securitization also helps to shed light on an aspect of our analysis that might otherwise seem puzzling. In the US, a major concern in recent decades is the decline in aggregate savings. At face value, this pattern seems to contradict our result that greater choice leads to greater savings. However, as our analysis with securitization illustrates, savings need not increase for all countries and all investors. In general equilibrium, a high propensity to save by some investors is likely to reduce the savings by the remaining investors (via a reduction in asset returns), even if all investors have greater portfolio choice. In fact, in the run-up to the recent crisis, these savings imbalances seemed sufficiently strong to push a fraction of households (in the US and elsewhere) into debt. We view our result on savings as best applying to households or investors that have positive (and relatively sizeable) financial wealth. These households face a meaningful choice in constructing savings portfolios as in our model. They also happen to command much of the aggregate financial wealth in the economy (see, for instance, Saez and Zucman (2014)). Our result looks much less puzzling if one focuses on these households' savings behavior. Saez and Zucman (2014) show that, consistent with our analysis, the savings rate by the 1% richest households in the US has remained roughly stable since the 1970s, despite the fact that asset returns have been declining over this period (see Figure 1).

The rest of the paper is organized as follows. Section 1.1 discusses the related literature. In Section 2 we present an example that illustrates the choice channel and motivates the rest of our analysis. Section 3 introduces the basic environment. Section 4 characterizes the effect of financial innovation on an individual's savings in partial equilibrium, and formalizes the choice channel. Section 5 extends the basic framework into a general equilibrium model, and its subsections characterize the pricing implications of specific types of financial innovation. Section 5.1 analyzes increased participation. Section 5.2 presents our main result on increased customization. Section 5.3 focuses on increased securitization. We summarize our findings in a concluding section.

## 1.1 Related literature

Our paper spans various segments of the literature. We contribute to a sizeable literature that investigates financial innovation and security design.[3] We focus on the asset pricing implications of financial innovation in an environment with belief heterogeneity, similar to Fostel and Geanakoplos (2012) and Simsek (2013a) (see Detemple and Selden (1991) for an earlier example). These papers typically take the risk-free rate as given, and characterize how certain types of financial innovations affect the relative price of a single risky asset. In contrast, we focus on the risk-free rate while also characterizing the relative asset prices. We also analyze a broader set of financial innovations, with a focus on portfolio customization, whereas the recent papers in this literature consider securitization and credit derivatives.[4]

Our paper is also related to a large "precautionary savings" literature, which emphasizes that incomplete markets tend to increase agents' idiosyncratic consumption risks and reduce the risk-free interest rate. It is useful to divide this literature into two strands that differ in terms of the sources of risks as well as the implications for aggregate investment.

The first strand focuses on consumption risks driven by idiosyncratic income or background risks (see, for instance, Leland (1968), Dreze and Modigliani (1972), Bewley (1977),Skinner (1988), Kimball (1990), Weil (1992), Huggett (1993), Aiyagari (1994)). As we described earlier, this literature suggests that financial innovation that facilitates the sharing of background risks should reduce savings and increase the interest rate (see Elul (1997) for a formalization and critical evaluation, and Carvajal, Rostek, and Weretka (2012) for a recent application). While we think background risks are clearly important, especially for understanding the financial decisions of economic agents that are net borrowers, we question their empirical relevance for the types of financial innovations we analyze. At the macro level, the interest rate has been declining since the early 1980s in an environment with rapid financial innovation, which counters the precautionary savings view. At the micro level, most investors (that are net savers) do not seem to be concerned by background risks when constructing their savings portfolios. They tend to overinvest in domestic stocks (French and Poterba (1991)), as well as own company or professionally close stocks (e.g., Benartzi (2001), Poterba (2003), Døskeland and Hvide (2011)). They also seem to trade and adjust their portfolios much more frequently than what could be justified by hedging or liquidity

[3] A non-exclusive list of contributions includes Allen and Gale (1988, 1991, 1994a), Detemple and Selden (1991), Elul (1995, 1997), Pesendorfer (1995), Calvet, Gonzalez-Eiras, and Sodini (2004), and more recently, Carvajal, Rostek, and Weretka (2012), Fostel and Geanakoplos (2012), and Simsek (2013b). See Duffie and Rahi (1995) and Tufano (2003) for reviews of the older literature.

[4] More broadly, our paper is also related to a large literature on asset pricing with heterogeneous beliefs (e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), Geanakoplos (2003), Geanakoplos (2010), He and Xiong (2012), Hong and Sraer (2012)).

needs (see Hong and Stein (2007)).

A second strand of the precautionary savings literature examines the implications of idiosyncratic investment or rate-of-return risks (e.g. Sandmo (1970), Devereux and Smith (1994), Obstfeld (1994), Krebs (2003), Angeletos and Calvet (2006), Angeletos (2007)). These risks are conceptually different than background risks because they are endogenously taken by economic agents. Building upon this observation, this literature emphasizes that financial innovation that facilitates the sharing of rate-of-return risks can actually increase aggregate investment. The logic of this result is similar to our choice channel, and relies on a relatively large elasticity of substitution. That said, our paper is also different from this strand of the literature for two main reasons. First, we analyze the savings decisions of households in financial markets, whereas this literature focuses on firms' (or entrepreneurs') physical investment decisions. In fact, recent contributions in this literature emphasize that financial innovation can raise aggregate investment while still increasing the risk-free interest rate—as in the case with background risks (see Angeletos and Calvet (2006) and Angeletos (2007)). In contrast, our main result delineates conditions under which financial innovation reduces the risk-free interest rate. Second, while this literature argues that investment increases in response to better risk sharing, we emphasize that savings increases because households have greater choice—which they might or might not use for risk sharing. In our main result, households with heterogeneous beliefs increase their savings because they speculate that their customized portfolios will yield high (risk-adjusted) returns—a phenomenon that could be viewed as the opposite of risk sharing (see Simsek (2013b)).

Our paper is also related to the recent work by Guzman and Stiglitz (GS, 2015), who investigate investors' consumption and savings behavior in an environment with belief disagreements. They emphasize that belief disagreements increase investors' perceived wealth, which they call pseudo-wealth, and contribute to macroeconomic fluctuations. Our paper has several differences. First, we establish comparative statics with respect to financial innovation, as opposed to changes in belief disagreements (and we also focus on the level of asset prices as opposed to consumption volatility). Second, and more importantly, we illustrate that belief disagreements—when unleashed by financial innovation—generate not only a wealth effect as emphasized by GS, but also a substitution effect that tends to induce greater savings. We focus on the cases in which the substitution effect dominates, whereas GS focus on the wealth effect by restricting attention to a class of preferences (quadratic). While our emphasis and results are very different, our model reinforces the broader conceptual point in Guzman and Stiglitz (2015) that belief disagreements affect investors' consumption and savings, with implications for macroeconomic outcomes.[5]

[5] The idea that belief disagreements increase investors' perceived portfolio returns also appears in Simsek

The part of our paper on participation is related to a large literature that documents limited participation in equity markets and examines its implications for asset prices.[6] Our result that greater participation reduces the risk premium has been noted by this literature, which used it as a potential explanation for the historically high levels of the equity premium (see, for instance, Mankiw and Zeldes (1991), Heaton and Lucas (1999), Favilukis (2013)). Our result that greater participation increases the risk-free rate (due to a shift of relative demand towards risky assets) appears to be more novel. Basak and Cuoco (1998) demonstrate a version of this result in a dynamic environment in which participants' and nonparticipants' consumption shares evolve endogenously, and nonparticipants are restricted to have log utility. Relative to Basak and Cuoco (1998), we work with a two period model with exogenous wealth shares and Epstein-Zin preferences for both types of agents. We show that the result is qualitatively robust, and holds for a large range of the elasticity of intertemporal substitution parameter (e.g., away from log utility).

Finally, our analysis contributes to the recent macroeconomics literature on secular stagnation that investigates the sources of low interest rates (see Teulings and Baldwin (2014) for a summary of the recent literature). We identify financial innovation as a novel factor that can lower the risk-free rate. The part of our paper that analyzes the demand for safe assets from emerging markets, together with securitization, is related to a growing literature on the savings glut hypothesis and asset shortages.[7] We argue that securitization counters some of the relative price effects of the demand from emerging markets, while exacerbating their (already high) savings. This is consistent with Justiniano, Primiceri, and Tambalotti (2015), who argue that securitization might have relaxed what they refer to as "lending constraints," and analyze the implications of increased savings for mortgage debt and house prices in the US.

(2013b). The idea that this generates income and substitution effects, and affect investors' savings, appears in Brunnermeier, Simsek, and Xiong (2014). They formalize the idea in the context of a model by Sims (2009), and use it as an example of how speculation generates behavioral distortions that can be detected (as inefficient) by their welfare criterion. We focus on the case in which the substitution effect dominates, and analyze the implications for various types of financial innovations and asset returns.

[6] An incomplete list includes Allen and Gale (1994b), Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), Vissing-Jorgensen (2002), Vissing-Jørgensen and Attanasio (2003), Calvet, Gonzalez-Eiras, and Sodini (2004), Cao, Wang, and Zhang (2005), Gomes and Michaelides (2008), Guvenen (2009).

[7] In addition to the papers mentioned earlier, see Caballero (2006), Caballero and Krishnamurthy (2009), Bernanke, Bertaut, DeMarco, and Kamin (2011), Blanchard, Furceri, and Pescatori (2014), Justiniano, Primiceri, and Tambalotti (2014).

# 2 A Motivating Example

We first present a simple example that illustrates the choice channel, and provides the motivation for our more general model. Consider an economy with two dates, $t \in \{0, 1\}$, and a single consumption good. At date 1, the economy can be in one of two states, denoted by $z \in \{shine, rain\}$. There is a single fundamental asset in unit supply, which we refer to as the market portfolio and denote by subscript $m$. The asset yields payoff only at date 1, denoted by $\overline{\varphi}_m$, which doesn't depend on the state (for simplicity).

There are two types of investors which we refer to as "optimists" and "pessimists," with heterogeneous prior beliefs about the state $z$, denoted by $\pi^{opt}(z)$ and $\pi^{pes}(z)$. Optimists assign a higher probability to the shine state, $\pi^{opt}(shine) > \pi^{pes}(shine)$. Investors are risk-neutral and have a discount factor of one between dates 0 and 1. Thus, they trade financial assets at date 0 to maximize the sum of their expected payoffs at dates 0 and 1. They can take long or short positions in the available financial assets, but they are subject to having nonnegative consumption at each date and state. Investors have large endowments of the asset at date 0, as well as symmetric endowments of the market portfolio, and they have zero endowments of the consumption good at date 1.

First suppose the only available financial asset is the market portfolio. In this case, the equilibrium price of the market portfolio is the same as investors' common valuation,

$$P_m = \overline{\varphi}_m. \tag{1}$$

At this price, investors are indifferent between consuming and saving. The aggregate savings in equilibrium is given by $P_m$, which clears the asset market.

Next suppose that, thanks to financial innovation, there is a second financial asset in zero net supply that has a positive payoff only in the shine state. The asset is denoted by $s$ and has payoff $\varphi_s(shine) = 1$ and $\varphi_s(rain) = 0$. Together with the market portfolio, this asset completes the financial market and enables investors to take flexible positions on the payoffs in the two states. In equilibrium, optimists hold the payoff in the shine state, as they assign a relatively high probability to this state, $\pi^{opt}(shine) > \pi^{pes}(shine)$. Similarly, pessimists hold the payoff in the rain state, since $\pi^{pes}(rain) > \pi^{opt}(rain)$. The asset prices are then respectively given by,

$$P_s = \pi^{opt}(shine) \text{ and } P_m = \pi^{opt}(shine)\,\overline{\varphi}_m + \pi^{pes}(rain)\,\overline{\varphi}_m. \tag{2}$$

Optimists' and pessimists' equilibrium savings are given by respectively $\pi^{opt}(shine)\,\overline{\varphi}_m$ and $\pi^{pes}(rain)\,\overline{\varphi}_m$, and the aggregate savings is given by $P_m$.

Comparing Eqs. (1) and (2) shows that financial innovation increases aggregate savings, as well as the equilibrium price of the market portfolio. Intuitively, providing investors with greater portfolio choice makes saving more attractive, since investors self-select into holding assets or portfolios that they like relatively more. We refer to this effect as *the choice channel* of financial innovation. The example also illustrates that, in equilibrium, greater savings exerts an upwards pressure on asset prices.

While this example illustrates the choice channel, it also raises several questions. The example features linear utilities, which implies an infinite elasticity of substitution between date 0 and 1 consumption. One could wonder whether the results are robust to allowing for lower elasticities. The example also does not feature risk aversion or background risks. A natural question, in view of the precautionary savings literature, is whether the presence of background risks can overturn these results. Finally, the example features a single asset (before financial innovation) whose price increases in view of the choice channel. In a more realistic environment with multiple assets, one could wonder how the choice channel affects different asset prices, e.g., whether it increases the price of the risky assets as well as the safe assets. In the rest of the paper, we systematically analyze a more general model, which will enable us to address these questions and deliver additional insights.

# 3 Environment and Equilibrium

Consider an economy with two dates, denoted by $t \in \{0, 1\}$, and a single consumption good which will be referred to as a dollar. The economy has financial assets denoted by $j \in \{f\} \cup \mathbf{J}$. Each financial asset is a mapping $\varphi_j : Z \to \mathbb{R}_+$ where $\varphi_j(z)$ denotes the payoff in state $z$. Each asset is in fixed supply denoted by $\eta_j \geq 0$ and is traded in a competitive market at price $P_j \in \mathbb{R}_+$. The asset $f$ captures the risk-free asset that makes a constant payment in all states, $\varphi_f(z) = \overline{\varphi}_f > 0$ for each $z$. The set $\mathbf{J}$ captures risky assets. We assume (until Section 5) that the state space $Z$ is finite, and the vectors, $(\varphi_j(z))_{z \in Z}$ for $j \in \mathbf{J}$, are linearly independent so that each risky asset is nonredundant.

There are several types of investors denoted by $i \in \{1, .., |I|\}$, each of which has population mass $n^i \geq 0$. We normalize the total population mass to 1, so that, $\sum_i n^i = 1$. Each type $i$ investor starts with some endowment of the consumption good at date 0, denoted by $Y_0^i > 0$, as well as some positions on financial assets, $\{x_{-1,j}^i\}_j$. Thus, the investor's financial wealth at date 0 is given by $W_0^i = Y_0^i + \sum_j x_{-1,j}^i P_j$. The investor also receives some endowment of the consumption good in state $z$ of date 1, denoted by $L_1^i(z)$. We use these endowments to capture the investors' background risks, such as changes in her labor income.

Investors make consumption and savings decisions with incomplete financial markets.

Specifically, all investors have access to the risk-free asset (for simplicity). However, an investor with type $i$ has access to only a subset of the risky financial assets, denoted by $J^i \subset \mathbf{J}$. She chooses her consumption and total asset holdings at date 0, denoted by $C_0$ and $A_0$, as well as positions in financial assets, denoted by $\left\{x_j^i\right\}_{j\in\{f\}\cup J^i}$, to solve:

$$
\begin{aligned}
& \max_{C_0, A_0, \{x_j\}_{j\in\{f\}\cup J^i}} U_0^i\left(C_0, \left(W_1(z)\right)_Z\right) && (3)\\
\text{s.t.} \quad & C_0 + A_0 = W_0^i, \text{ where } W_0^i = Y_0^i + \sum_j x_{-1,j}^i P_j, \\
& \sum_{j\in\{f\}\cup J^i} P_j x_j = A_0 \\
\text{and} \quad & W_1(z) = L_1^i(z) + \sum_{j\in\{f\}\cup J^i} x_j \varphi_j(z) \text{ for each } z \in Z.
\end{aligned}
$$

Here, $W_1(z)$ denotes the investor's financial wealth in state $z$ of date 1. The second line captures her budget constraint at date 0 in terms of consumption and asset holdings. The investor allocates her savings portfolio among various assets in $J^i$, and receives returns from these assets in the next period.[8]

We assume the investor has recursive Epstein-Zin preferences. In particular, the investor's utility function can be written as,

$$
\begin{aligned}
U_0^i &= C_0^{1-1/\varepsilon^i} + \beta^i \left(V_1^i\right)^{1-1/\varepsilon^i} && (4)\\
\text{where } V_1^i &= \left(\sum_{z\in Z} \pi^i(z) W_1(z)^{1-\gamma^i}\right)^{1/\left(1-\gamma^i\right)}.
\end{aligned}
$$

Here, the parameter $\varepsilon^i$ captures the investor's elasticity of intertemporal substitution (EIS), which will play a central role for our analysis. The parameter $\gamma^i$ captures the investor's coefficient of relative risk aversion. The variable, $V_1^i$, captures the certainty equivalent of future consumption. The special case, $\varepsilon^i = 1/\gamma^i$, corresponds to time separable CRRA preferences. Note that investors can hold potentially heterogeneous beliefs denoted by, $\{\pi^i(z)\}_z$, and expectations denoted by $E^i[\cdot]$. We assume investors know each others' beliefs, that is, they agree to disagree. We will use belief disagreements of this type to capture investors' demand for customized assets.

We formally define and analyze the general equilibrium in this environment in Section 5. Our fist goal is to investigate how financial innovation affects an individual's saving in

[8] For simplicity, the investor can take unrestricted long or short positions. We investigate the implications of short-selling constraints in ongoing work.

partial equilibrium, which we turn to next.

# 4 Financial Innovation and Saving

The common denominator of much financial innovation in recent years is that it provides investors with greater choice of saving portfolios. To capture this, we model financial innovation as an expansion of the access set of an individual from some $J^{i,old}$ to a greater set $J^{i,new} \supset J^{i,old}$. In this section, suppose the individual in consideration is infinitesimal so that this change does not affect asset prices. To facilitate the analysis, we also assume there is a risk-neutral belief distribution, $\{\pi^n(z)\}_Z$, that prices all assets, that is,

$$P_j = P_f \sum_{z \in Z} \pi^n(z) \varphi_j(z) \text{ for each } j \in \mathbf{J}. \tag{5}$$

This assumption holds, for instance, if there is a positive mass of investors that can access all assets (which implies no arbitrage).

Under the assumptions we made, there is a unique solution to the investor's problem (3) for a given access set. We let $\left(C_0^{i,old}, A_0^{i,old}, \left\{x_j^{i,old}\right\}_{j \in \{f\} \cup J^i}\right)$ and $\left(C_0^{i,new}, A_0^{i,new}, \left\{x_j^{i,new}\right\}_{j \in \{f\} \cup J^i}\right)$ denote the solution corresponding to respectively the access sets $J^{i,old}$ and $J^{i,new}$. We characterize how financial innovation affects the investor's savings, defined as [cf. Eq. (3)],

$$S_0^i = Y_0^i - C_0^i = A_0^i - \sum_j x_{-1,j}^i P_j.$$

Note that savings are equal to the change in individuals' asset holdings within the period. Since asset prices are constant, it suffices to characterize the effect of financial innovation on asset holdings, $A_0^i$.

In this model, the investor values financial assets in part because these assets might help to hedge her background risks. In practice, investors value financial assets (and greater access to them) for many other reasons. We accommodate these reasons by allowing the investor's belief, $\{\pi^i(z)\}_Z$, to be different than the risk-neutral distribution, $\{\pi^n(z)\}_Z$. As we will see in subsequent sections, this difference will naturally emerge in a general equilibrium environment once we incorporate motives for trade such as sharing the aggregate risk or speculation. Our main result in this section applies as long as $\{\pi^i(z)\}_Z$ and $\{\pi^n(z)\}_Z$ are different, regardless of where these differences come from.

## 4.1 Precautionary channel

We start by formalizing the precautionary channel in our context, which applies under the following assumption.

**Assumption 1$^{\text{P}}$.** $\pi^i(z) = \pi^n(z)$ for each $z \in Z$, which also implies $E^i[\cdot] = E^n[\cdot]$.

This assumption is quite strong, as it implies that the investor's (perceived) expected return on every asset is the risk-free interest rate. Under this assumption, a risk-averse investor without background risks would not want to hold any risky financial assets. Hence, the assumption ensures that the investor demands financial assets only to hedge her background risks.

**Proposition 1** (Precautionary Channel)**.** *Suppose there is a risk-neutral distribution [cf. (5)], Assumption 1$^P$ holds, and the investor has CRRA preferences, $\gamma^i = 1/\varepsilon^i$. Suppose also that financial innovation completes the market in the sense that, for each $z \in Z$, there exists $j_z \in J^{i,new}$ such that $\varphi_{j_z}(z) > 0$ and $\varphi_{j_z}(\tilde{z}) = 0$ for each $z \neq \tilde{z}$. Then, financial innovation reduces the investor's asset holdings (and thus, savings), $A_0^{i,new} \leq A_0^{i,old}$, with strict inequality if $W_1^{i,old}(z_1) \neq W_1^{i,old}(z_2)$ for some $z_1, z_2 \in Z$.*

Hence, consistent with much of the precautionary savings literature (see Section 1.1), financial innovation induces the investor to save less, and strictly so when she faces some income risks before innovation. Intuitively, when the market is incomplete, the investor saves for precautionary reasons. This is because she faces some background risks, and the time-separable CRRA preferences satisfy the prudence condition. Financial innovation enables the investor to hedge her risks. By doing so, it alleviates the precautionary demand for saving, thereby reducing savings.

This intuition also illustrates the fragility of the result. The argument relies on the fact that the agents demand financial assets mainly to reduce their portfolio risks. If instead new financial assets increase the investor's portfolio risks, perhaps because they enable her to participate in sharing the aggregate risk, then the argument does not hold.

## 4.2 Choice Channel

We next establish the choice channel, which applies under the following assumption that shuts down the precautionary channel.

**Assumption 1$^{\text{C}}$.** There exists scalars, $\{l_j^i\}_{j \in \{f\} \cup J^{i,old}}$, such that $L_1^i(z) = \sum_{j \in \{f\} \cup J^{i,old}} l_j^i \varphi_j(z)$ for each $z \in Z$.

The assumption is satisfied when the investors' future endowment is constant. It is also satisfied if the investor's future endowment is perfectly correlated with a combination of the risky assets in her access set.[9] In either case, the investor effectively does not face any background risks, as she can hedge those risks fully using the set of available assets. Hence, under this assumption, the investor does not need the new risky assets to alleviate her precautionary concerns. However, she might still demand those assets to improve her portfolio return, since Assumption $1^{\mathrm{P}}$ does not necessarily hold.

**Proposition 2** (Choice Channel)**.** *Suppose there is a risk-neutral distribution [cf. (5)], Assumption $1^{C}$ holds, and $\varepsilon^i > 1$ (so that the investor's savings is increasing in the interest rate). Then, financial innovation increases the investor's asset holdings (and thus, savings), $A_0^{i,new} \geq A_0^{i,old}$, with strict inequality if $x_j^{i,new} \neq 0$ for some $j \in J^{i,new} \setminus J^{i,old}$.*

Unlike in Proposition 1, financial innovation that increases investors' portfolio choice also induces her to save more. Moreover, the inequality is strict as long as the investor takes a nonzero position on some new asset—so that the assets are not completely redundant from her perspective.

We provide a sketch-proof for this result, which is also useful to understand the intuition. In view of Assumption $1^{\mathrm{C}}$, the investor can be equivalently thought of as having zero future endowment, but starting with implicit initial financial positions in assets $\{f\} \cup J^{i,old}$. More specifically, we can define investors' effective financial wealth and the effective asset holdings as,

$$\tilde{W}_0^i = Y_0^i + \sum_{j \in \{f\} \cup \mathbf{J}} \tilde{x}_{-1,j}^i P_j \text{ and } \tilde{A}_0^i = \sum_{j \in \{f\} \cup J^i} P_j \tilde{x}_j^i.$$

Here, the variable $\tilde{x}_{-1,j}^i$ is defined as $x_{-1,j}^i + l_j^i$ for $j \in \{f\} \cup J^{i,old}$ and as $x_{-1,j}^i$ otherwise. It reflects the investors' effective initial position on asset $j$ after incorporating the positions that are implicit in her future income. The variable, $\tilde{x}_j^i$, is defined similarly and reflects the investors' effective chosen position.

With this new notation, the investor's problem can be split into two parts. Conditional on effective asset holdings, $\tilde{A}_0^i$, the investor maximizes her certainty-equivalent payoff at date 1: That is, she solves the portfolio problem,

$$V_1^i\left(\tilde{A}_0^i\right) = \max_{\{\tilde{x}_j\}_{\{f\} \cup J^i}} E\left[W_1(z)^{1-\gamma^i}\right]^{\frac{1}{1-\gamma^i}}, \tag{6}$$
$$\text{s.t. } \sum_{\{f\} \cup J^i} P_j \tilde{x}_j = \tilde{A}_0^i \text{ and } W_1(z) = \sum_{\{f\} \cup J^i} \tilde{x}_j \varphi_j(z).$$

[9] For instance, if an individual's income is perfectly correlated with a broad market index portfolio, and she has access to the index portfolio, then Assumption $1^{\mathrm{C}}$ holds.

In turn, given the value function, $V_1^i(\cdot)$, she chooses her effective asset holdings, $\tilde{A}_0^i$, by solving the intertemporal problem,

$$\max_{\tilde{A}_0} \left(\tilde{W}_0^i - \tilde{A}_0\right)^{1-1/\varepsilon^i} + \beta\left(V_1\left(\tilde{A}_0\right)\right)^{1-1/\varepsilon^i}. \tag{7}$$

The result then follows from three observations. First, the portfolio problem is linearly homogeneous, which implies that the value function is linear in effective (as well as actual) asset holdings,

$$V_1^i\left(\tilde{A}_0\right) = R_{ce}^i \tilde{A}_0. \tag{8}$$

We refer to $R_{ce}^i$ as the investor's certainty-equivalent marginal return. Second, and most importantly, financial innovation increases the certainty-equivalent return, $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, because it expands the choice set of feasible portfolios. Third, in the intertemporal problem, a greater risk-adjusted return implies an increase in asset holdings in view of the assumption $\varepsilon^i > 1$.

Intuitively, with greater portfolio choice, the investor's certainty-equivalent portfolio return can only increase. This creates substitution and income effects. On the one hand, the investor finds savings more attractive, which induces her to save more. On the other hand, the investor also feels richer, which induces her to consume more and save less. The substitution effect dominates, and financial innovation increases savings, whenever the EIS is relatively high.

As this intuition suggests, the result can further be generalized. The particular comparative statics we focus on, the expansion of the access set from $J^{i,old}$ to some $J^{i,new}$, does not play an important role beyond ensuring that the investor has greater choice. Any other financial innovation that expands the investor's choice would also induce the investor to save more.[10]

The result requires two key assumptions: the absence of background risks (Assumption $1^{C}$) and a relatively high elasticity of intertemporal substitution. As we discuss in Section 1.1, we believe the first assumption is reasonable in our context. Likewise, we also believe a relatively high EIS is appropriate for our context. Using different methodologies, empirical studies find a wide range of estimates for the EIS (see Hall (1988), Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), Vissing-Jorgensen (2002), Vissing-Jørgensen and Attanasio (2003), Gruber (2013)). Most of the studies assume that investors with separable or Epstein-Zin preferences fully observe the changes in the interest rate and make optimal decisions. Even though we also make the same assumptions, some of these features

[10] In ongoing work, we analyze financial innovations that relax short selling constraints, which increase the investor's savings even though they cannot be mapped into Proposition 2.

are not central for our analysis. What is important is that investors have an asset holding (or saving) function that is increasing in their perceived portfolio return. We believe this assumption is plausible, and it can also accommodate some behavioral biases such as limited attention.

To illustrate this, consider an alternative setting in which an investor with Epstein-Zin preferences with $\varepsilon^i > 1$ makes consumption and saving decisions over several periods. Suppose the investor has limited attention and observes the asset returns only with some probability. In any period, if she observes the asset returns, then she makes a fully optimal consumption plan as in our model. If she does not observe the returns, then she follows a default rule: say, she consumes and saves according to her earlier plan (many other default rules would also work). This investor's average asset holdings and savings would also increase in response to financial innovation, and our qualitative results would continue to apply in this setting. However, the investor's consumption growth would not increase much after a (surprise) increase in the interest rate. Thus, the empirical strategies that focus on consumption growth can easily (mis-)estimate that $\varepsilon^i < 1$.[11] The key point is that the investor's consumption level would not react much to the changes in the interest rate either. The income effects would still be weaker than the substitution effects, and the investor would actually choose a lower level of consumption and a higher level of savings—albeit not as much as in the full attention case. Moreover, over longer horizons—which is the range over which we apply the comparative statics of our model—the investor's attention would eventually catch up and her savings would arguably increase further.

In view of this discussion, we view the assumption, $\varepsilon^i > 1$, as a simple way of generating an increasing asset holding function in our setting. In the numerical simulations, we use $\varepsilon^i = 2$, which is the estimate provided by Gruber (2013) based on plausibly exogenous variations in the interest rate that come from tax changes. In the rest of the paper, we investigate the general equilibrium implications of Propositions 1 and 2 for asset prices.

# 5 Financial Innovation and Asset Returns

We next investigate how financial innovation affects asset returns in general equilibrium. For analytical tractability, we impose additional structure on the model. Since our focus is to understand the implications of the choice channel, we maintain a version of Assumption $1^{\mathrm{C}}$ as well as a relatively high EIS for each investor $i$.

**Assumption $1^{\mathrm{G}}$.** $L_1^i = 0$ and $\varepsilon^i > 1$ for each investor $i$.

[11] See Chetty (2012) for a formalization of this type of argument and an application to the estimation of labor supply elasticities.

We normalize the future endowment of each investor to zero to simplify the notation (see Section 4.2).

We also impose some structure on asset payoffs. The uncertainty in this economy is now described by a $K \times 1$ vector of continuous random variables, $\mathbf{z} = (z_1, ..., z_K)'$ (in particular, the state space is now given by, $\mathbf{Z} = \mathbb{R}^K$). The log payoff of a risky asset $j \in \mathbf{J}$ can be written as a linear combination of the underlying uncertainty,

$$\log \varphi_j(\mathbf{z}) = \mathbf{F}_j' \mathbf{z}, \tag{9}$$

where $\mathbf{F}_j$ is a $K \times 1$ vector. We assume investors' beliefs for $\mathbf{z}$ are normally distributed, and thus, their beliefs for asset payoffs are log-normally distributed.

**Assumption 2.** Investor $i$'s prior belief for $\mathbf{z}$ has a Normal distribution, $N(\boldsymbol{\mu}_{\mathbf{z}}^i, \Lambda_{\mathbf{z}})$, where $\boldsymbol{\mu}_{\mathbf{z}}^i \in \mathbb{R}^K$ is the mean vector and $\Lambda_{\mathbf{z}}$ is the $K \times K$ positive definite covariance matrix. In addition, the $K \times |\mathbf{J}|$ matrix of asset loadings, $\mathbf{F} = [\mathbf{F}_j]_{j \in \mathbf{J}}$, has full rank.

Note that investors can disagree on the mean of the asset payoffs but they agree on the variance of log payoffs (for simplicity). The full rank assumption ensures that risky assets are not redundant.

The log-normality of payoffs provides a tractable approximation to the investor's portfolio problem (6) over sufficiently short horizons. To see this, first consider the gross and log asset returns, respectively given by $R_j = \varphi_j / P_j$ and $r_j = \log R_j$, for each $j \in \{f\} \cup \mathbf{J}$. Investor $i$ believes that risky asset returns are jointly log-normally distributed, with the mean and the variance of log returns respectively given by,

$$\begin{aligned} E^i[r_j] &= \mu_j^i - \log P_j \text{ for } j \in \mathbf{J} \text{ and } var\left(\{r_j\}_{j \in \mathbf{J}}\right) = \Lambda, \\ &\text{where } \mu_j^i = (\mathbf{F}_j)' \boldsymbol{\mu}_{\mathbf{z}}^i \text{ and } \Lambda = \mathbf{F}' \Lambda_{\mathbf{z}} \mathbf{F}. \end{aligned} \tag{10}$$

As before, investors know (and agree on) the safe asset return, $r_f = \mu_f - \log P_f$ (where $\mu_f = \log \overline{\varphi}_f$).

Next consider the investor's portfolio return,

$$R_p^i(z) = W_1^i(z) / A_0^i = \sum_{j \in \{f\} \cup J^i} \omega_j^i R_j(z).$$

Here, $\omega_j^i \equiv x_j^i P_j / A_0^i$ denotes the investor's portfolio weight in asset $j$. The investor can be thought of as choosing the vector of weights on risky assets, $\boldsymbol{\omega}_{J^i} = (\omega_j)_{j \in J^i}$, with the residual weight invested in the safe asset. In discrete time, the portfolio return can have a complex distribution, even though the underlying asset returns have log-normal distributions. We proceed by approximating the investor's perceived portfolio distribution with its counterpart that would obtain in a continuous time problem. The approximation becomes increasingly accurate as the time horizon between dates 0 and 1 is shortened (see Campbell and Viceira (2002) for the details).

Specifically, we assume the investor solves the following analogue of the earlier portfolio problem [cf. (11)],

$$r_{ce}^i - r_f = \max_{\boldsymbol{\omega}_{J^i}, \omega_f} \zeta_p^i - \frac{\gamma^i \Lambda_p}{2}, \tag{11}$$

$$\text{s.t. } \zeta_p^i = \sum_{j \in J^i} \omega_j \zeta_j^i \text{ and } \zeta_j^i = E^i [r_j] + \frac{\Lambda_j}{2} - r_f \text{ for each } j, \tag{12}$$

$$\Lambda_p = \boldsymbol{\omega}'_{J^i} \Lambda_{J^i} \boldsymbol{\omega}_{J^i}, \text{ where } \boldsymbol{\omega}_{J^i} = (\omega_j)_{j \in J^i} \text{ and } \omega_f = 1 - \sum_{j \in J^i} \omega_j. \tag{13}$$

Here, the variable, $r_{ce}^i = \log R_{ce}^i$, denotes the log of investor's certainty equivalent return from asset holdings [cf. Eq. (8)]. The variables, $\zeta_p^i$ and $\zeta_j^i$, respectively denote the log of the mean portfolio and asset returns net of the risk-free rate. With a slight abuse of terminology, we refer to these variables as the risk premium on the portfolio and the asset.[12]

If the portfolio return were log-normal, then the objective function in (11) would be exact. Hence, the investor behaves as if the portfolio return is log-normal. She trades off the risk premium on the portfolio, $\zeta_p^i$, with the variance of the (log) portfolio return denoted by $\Lambda_p$. Eqs. (12) and (13) relate the portfolio premium and the portfolio variance to the premia and the variance of the underlying assets.[13]

Given the certainty equivalent return from the portfolio problem, the investor solves the same intertemporal problem (7) as before. The solution can be written as $A_0^i = a^i (r_{ce}^i) W_0^i$, where the function $a^i (r_{ce}^i)$ describes the investors' effective asset holding as a fraction of wealth. Importantly, $a^i (\cdot)$ is an increasing function for each $i$, that is, the investor's saving is increasing in her certainty-equivalent return, in view of the assumption $\varepsilon^i > 1$.

The asset market clearing conditions can then be written as,

$$\eta_j P_j = \sum_{\{i \quad | \quad j \in \{f\} \cup J^i\}} n^i \omega_j^i a^i \left( r_{ce}^i \right) W_0^i \text{ for each } j \in \{f\} \cup \mathbf{J}, \tag{14}$$

[12] The terminology becomes exact over a very short horizon since the log of the expected return (resp. log of the risk-free rate) can be replaced with the expected return (resp. the risk-free rate).

[13] To obtain these expressions, imagine that log asset returns followed a joint diffusion process over continuous time with instantenaous drifts $\{\mu_j^i\}_J$ and volatility $\Lambda$ (starting at $r_j = 0$ for each $j$). Then, the log portfolio return would also follow a diffusion process. Its mean and variance can be characterized using Ito's Lemma, which leads to the expressions in (12) and (13).

where $W_0^i = Y_0^i + \sum_j P_j x_{-1,j}^i$ for each $i$. The condition says that the supply of each asset $j$ equals its demand, which is determined by investors' savings as well as their asset allocations. To avoid trivial cases, we assume that each asset that is in positive supply, $\eta_j > 0$, lies in at least one investor's access set. We also assume the economy is closed, so that the assets are initially held by the investors in the economy, that is, $\sum_i n^i x_{-1,j}^i = \eta_j$ for each $j$.

**Definition 1** (Equilibrium). *Under Assumptions $1^G$ and 2, an (approximate) equilibrium, $\left\{\left(\omega_{J^i}^i, A_0^i\right)_i, P_j\right\}$, is a collection such that portfolio shares solve problem* (11)*, asset holdings satisfy $A_0^i = a^i\left(r_{ce}^i\right) W_0^i$, and markets clear [cf. Eq.* (14)*].*

To characterize the equilibrium, let $\boldsymbol{\zeta}_{J^i}^i = \left(\zeta_j^i\right)_{j \in J^i}$ denote the vector of risk premia for the assets in an investor's access set. Solving problem (11), the investor's portfolio shares and certainty-equivalent return are given by,

$$\boldsymbol{\omega}_{J^i}^i = \frac{1}{\gamma^i}\Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i}^i \text{ and } r_{ce}^i = r_f + \frac{1}{2\gamma^i}\left(\boldsymbol{\zeta}_{J^i}^i\right)' \Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i}^i. \tag{15}$$

Loosely speaking, increasing the risk premia of an asset, while keeping all else constant, tends to shift the investor's portfolio weight towards this asset. Greater risk premium also enables the investor to obtain a greater certainty-equivalent return, which increases her savings. Likewise, keeping risk premia constant, increasing the risk-free rate raises the certainty-equivalent return and savings. Note that the risk free rate as well as the risk premia are endogenous and depend inversely on asset prices according to Eq. (10) and the equation for $r_f$. The prices (and returns) adjust, so as to satisfy the market clearing conditions in (14).

Proposition 8 in the Appendix establishes the existence of an equilibrium. At this level of generality, we cannot characterize the equilibrium much further. In the rest of this section, we analyze a canonical case that can accommodate the key aspects of various recent financial innovations.

**Assumption 3.** There exist $K$ risky assets in total, $\mathbf{J} = \{m, 1, ..., K-1\}$. The asset $m$ is in positive supply, $\eta_m > 0$, while the remaining risky assets, as well as the risk-free asset are in zero net supply, $\eta_j = 0$ for $j \neq m$.

The first part ensures that the risky assets collectively complete the market, since the underlying uncertainty is $K$ dimensional (see Assumption 3). The second part says that there exists a single asset, denoted by $m$, that represents all of the cash flows in positive supply. This asset, which is typically referred to as *the market portfolio*, enables investors to obtain exposure to all assets in proportion to their market valuations. Its practical counterpart could be broad equity or bond indices that proxy for this type of exposure. The remaining risky assets, $j \in \{1, .., K-1\}$, enable investors to customize their exposures according to their specific preferences or beliefs. Their counterparts could be individual stocks, bonds, investment funds, or various types of derivatives. In the rest of the section, we analyze how innovations that improve access to the assets in $\mathbf{J}$ affect the equilibrium returns.

Throughout, we also maintain the following symmetry assumption.

**Assumption 4.** Investors start with the same endowment of the consumption good, $Y_0^i = Y_0 > 0$ for each $i$, as well as the risky assets, $x_{-1,j}^i = x_{-1,j}$ for each $j$ and $i$.

This assumption ensures that investors have the same wealth, $W_0^i = W_0$ for each $i$. Hence, it enables us to abstract away from the effects of financial innovation on wealth redistribution, which is not our focus. In view of this assumption, the wealth shares of different investor groups are captured by the exogenous relative mass parameters, $\{n^i\}_i$.

## 5.1 Market Participation

We start by investigating increased market participation, which we capture with improved access to asset $m$. In particular suppose there are two types of investors, $i \in \{1, 0\}$, with market access sets respectively given by $J^1 = \{m\}$ and $J^0 = \emptyset$. Type 1 investors have access to asset $m$, in addition to the risk-free asset. In contrast, type 0 investors have access to only the risk-free asset. To simplify the exposition, we also assume investors are identical in all other dimensions. In particular, they share the same beliefs, $\boldsymbol{\mu}_{\mathbf{z}}^i = \boldsymbol{\mu}_{\mathbf{z}}$, which implies $\mu_m^i = \mu_m$, for each $i$.[14] Investors also have the same preference parameters, $\beta^i = \beta, \varepsilon^i = \varepsilon$, and $\gamma^i = \gamma$, which implies, $a^i(\cdot) = a(\cdot)$, for each $i$.

In this setting, we capture financial innovation as an increase in the relative mass of type 1 investors, $n^1$ (while keeping the total mass unchanged, $n^1 + n^0 = 1$). In view of the symmetry assumptions, this is equivalent to expanding the access set of some of type 0 investors from $J^{old} = J^0$ to $J^{new} = J^1 \supset J^0$, similar to Section 4. The difference is that financial innovation applies to a positive mass of investors, with potential general equilibrium effects. The following lemma characterizes the equilibrium.

**Lemma 1.** *Consider the above setup with common beliefs and limited participation in the market portfolio. There exists a unique equilibrium in which $\zeta_m$ and $r_f$ jointly solve,*

$$\zeta_m = \gamma \Lambda_m \left(1 + \frac{1-n^1}{n^1} \frac{a(r_f)}{a(r_{ce}^1)}\right), \tag{16}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \left(1 - n^1\right) a(r_f) + n^1 a\left(r_{ce}^1\right), \tag{17}$$

[14] This assumption ensures that the market portfolio $m$ is sufficient to construct an efficient portfolio for each investor, providing a justification for the absence of assets $j \in \{1, .., K-1\}$ from investors' access sets. We analyze the introduction of these assets in the next subsection.

*where* $P_m = \exp\left(\mu_m + \frac{\Lambda_m}{2} - r_f - \zeta_m\right)$*, and*

$$r^1_{ce} = r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m}. \tag{18}$$

To understand Eq. (16), note that the participants' share of the market portfolio satisfies, $\omega^1_m = \frac{\zeta_m}{\gamma\Lambda_m}$ [cf. Eq. (15)]. With full participation, the equilibrium share of the market portfolio would satisfy, $\omega^1_m = 1$, which would lead to the risk premium, $\zeta_m = \gamma\Lambda_m$. With limited participation, the equilibrium share of each participant is greater, $\omega^1_m > 1$, since the aggregate risk is shared among fewer investors. This leads to a greater risk premium relative to the full participation benchmark, $\zeta_m > \gamma\Lambda_m$. Moreover, the degree by which the risk premium exceeds the benchmark depends on the (wealth-averaged) mass of participants, as captured by Eq. (16). Intuitively, the premium must be sufficiently large to compensate the participants for the additional risks they hold in equilibrium.

Eq. (17) is a market clearing condition for all assets. The left side captures the supply of all assets, which is decreasing in the expected return on the market portfolio, $r_f + \zeta_m$. The right side captures the average demand for assets, which is increasing in $r_f$ as well as $\zeta_m$. The equilibrium is found by jointly solving Eqs. (16) and (17). The following result describes the comparative statics with respect to financial innovation.

**Proposition 3** (Increased Participation)**.** *Consider the equilibrium characterized in Lemma 1. Financial innovation that increases the relative mass of participants,* $n_1$*, decreases the risk premium,* $\zeta_m$ *(increases* $P_m/P_f$*), and decreases the expected return on the market portfolio,* $r_f + \zeta_m$ *(increases* $P_m$*).*

The result says that greater participation increases the relative price of the market portfolio, $P_m/P_f$, as well as its absolute price, $P_m$. In our numerical simulations, we also find that it also typically decreases the price of the safe asset, $P_f$, and increases the risk-free rate, $r_f$, even though this effect is theoretically ambiguous.

The relative price effect follows from Eq. (16). With greater participation, the aggregate risk is shared among a greater set of investors. This reduces the premium for risky assets and raises their relative price. The absolute price effect follows from the choice channel. Investors with access to the market portfolio find saving more attractive, $r^1_{ce} > r_f$, since they can earn the risk premium. This increases the average demand for assets, as captured by Eq. (17), which in turn increases the asset valuations in equilibrium.

The effect on the price of the safe asset is ambiguous because the relative and the absolute price effects push in opposite directions. For empirically relevant parameters, we find that the relative price effect dominates and increased participation reduces $P_f$ and increases $r_f$.

Intuitively, increased participation creates a large reduction in the relative demand for the safe asset: Some investors who used to invest only in the safe asset move some of their wealth to the risky asset. Investors also increase their asset holdings, which counters the demand shift away from the safe asset to some extent, but cannot fully undo it.

#### 5.1.1 Numerical Illustration

We next illustrate these results using a numerical example. We use the preference parameters, $\gamma = 5$ and $\varepsilon = 2$, along with a yearly calibration. To mitigate the equity premium puzzle, we consider a relatively high level for the standard deviation of the market portfolio, $\sqrt{\Lambda_m} = 5\%$.[15] We also assume 1% growth rate for log output, and calibrate $\beta$ so that the risk-free rate is equal to its historical average, $r_f = 1\%$, for $n_1 = 0.5$ (conditional on all other parameters). Figure 2 in the introduction suggests that the wealth-weighted participation in the US increased from about 50% in 1960s to about 90% in the 1990s. We therefore investigate the effect of varying $n_1$ in our model over the range, $[0.5, 0.9]$.

The left panel of Figure 5 illustrates the results of this exercise. The solid lines show that increased participation reduces the risk premium and the expected return on the market portfolio, consistent with Proposition 3, while also increasing the risk-free rate. The dashed lines illustrate the alternative case with $\varepsilon = 1$, which provides a useful benchmark for our results. When $\varepsilon = 1$, the relative price effects are still active but the choice channel is not operational (because the substitution effects of increased choice are exactly countered by strong income effects). Comparing this benchmark with our calibration, $\varepsilon = 2$, shows that the choice channel from increased participation is quantitatively weak. It reduces $r_f$ by a small amount—less than 0.5 percentage points—which does not overturn the increase in $r_f$ due to the relative price effect.

The choice channel is quantitatively weak partly because of crowd-out effects that tend to lower the (marginal) benefit from participation in general equilibrium. Greater $n_1$ implies a smaller risk premium, and thus a smaller certainty-equivalent return for investors that already participate [cf. Eq. (18)]. These investors react by reducing their asset holdings, which mitigates the effect of increased choice on asset prices. The top right panel of Figure 5 illustrates this crowd-out effect by plotting the gains from participation, $r_{ce}^1 - r_{ce}^0$. The panel also plots the average certainty-equivalent return, $\overline{r}_{ce}$, defined as the solution to $a(\overline{r}_{ce}) = (1 - n^1)\, a(r_{ce}^0) + n^1 a(r_{ce}^1)$. Note that the average return, $\overline{r}_{ce}$, increases by a very small amount: This is because the gain from participation, $r_{ce}^1 - r_{ce}^0$, decreases in the level of

[15] In the data, the volatility of consumption growth is around 1%, which leads to the equity premium puzzle (with relatively standard parameters such as $\gamma = 5$). Our calibration with higher volatility can be thought of as capturing factors omitted from our model, e.g., long-run risk, that could help to explain the equity premium puzzle (Bansal and Yaron (2004)).

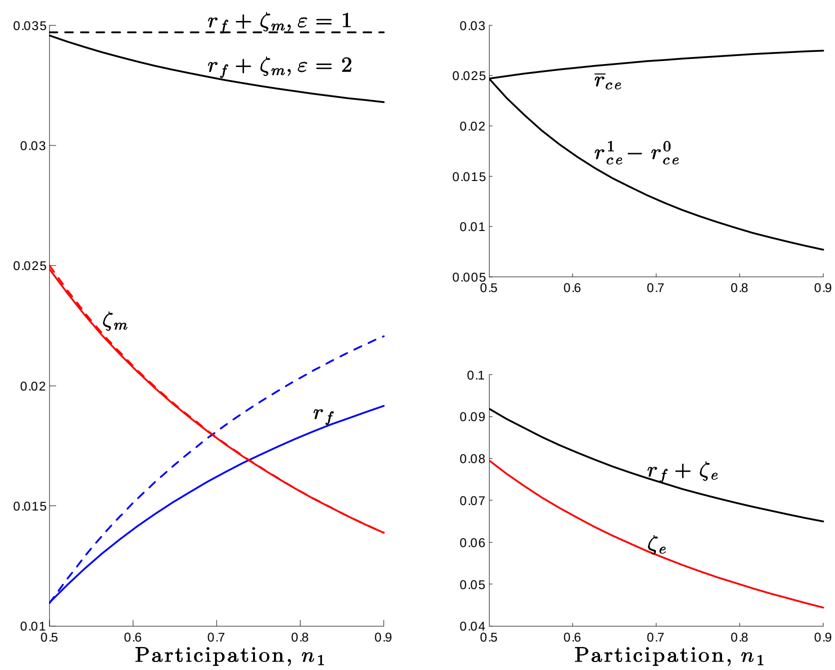


Figure 5: The left panel illustrates the effect of increased participation on asset returns for the main case, $\varepsilon = 2$ (solid lines) and the benchmark case, $\varepsilon = 1$ (dashed lines). The top right panel illustrates the return-gain from participation, as well as the avreage certainty-equivalent return (when $\varepsilon = 2$). The bottom right panel illustrates the effect of participation on the equity premium and the expected return on equity.

participation. For $n_1 = 0.5$, participants earn 2.5pp greater certainty-equivalent return than nonparticipants, but this difference falls to less than 1pp for greater levels of participation. In equilibrium, a relatively small increase in $\overline{r}_{ce}$ translates into a relatively small decrease in the expected return on the market portfolio.

We finally investigate the implications of our analysis for the pricing of equity, which we model as a proxy for the market portfolio. Specifically, suppose assets $e$ and $m$ are perfectly correlated (otherwise, the equity premium puzzle becomes even deeper), and calibrate the volatility of $e$ to match its historical average, $\sqrt{\Lambda_e} = 16\%$. By no arbitrage, the risk premium and the expected return on equity are respectively given by $\zeta_e = \zeta_m \frac{\sqrt{\Lambda_e}}{\sqrt{\Lambda_m}}$ and $r_f + \zeta_e$. The bottom right panel of Figure 5 shows that, as we increase participation, the equity premium declines from about 8% to 4%, and its expected return declines from about 9% to 6.5%.

## 5.2 Portfolio Customization

We next present our main result on customization. We capture increased portfolio customization with improved access to assets $j \in \{1, .., K-1\}$. The practical counterpart of these assets can be thought of as direct trading of individual stocks and bonds; investment funds that specialize in certain industries or styles; or derivatives such as futures, options, and ETFs. These financial instruments, like assets $j \in \{1, .., K-1\}$ in our model, enable investors to construct customized portfolios according to their needs or beliefs.

We model investors' demand for specific cash flows by allowing them to disagree about the underlying uncertainty, that is, $\mu^i_{\mathbf{z}}$ can be different for different $i$. Investors can also differ in their access to financial assets. Formally, investors' types have two dimensions, $\{i = (i_A, \mathbf{i}_B)\}_i$. The sub-type $i_A \in I_A$ captures the variation in investors' market access, while the sub-type $\mathbf{i}_B \in I_B$ (which itself is a vector) captures the variation in beliefs. Investors' beliefs are drawn independently of their market access. More specifically, the mass of type $i = (i_A, \mathbf{i}_B)$ investors is given by $n^i = n^{i_A} \times n^{\mathbf{i}_B}$, where $n^{i_A}$ denotes the mass of investors with market access type $i_A$, with $\sum_{i_A} n^{i_A} = 1$, and $n^{\mathbf{i}_B}$ denotes the mass of investors with belief type $\mathbf{i}_B$, with $\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} = 1$. Investors are identical in all dimensions other than possibly their market access and beliefs.

The access types are given by, $I_A = \{0, ..., K-1\}$, such that $J^{i_A} = \{m, 1, .., i_A\}$ for each $i_A \in I_A$. Hence, $i_A$ denotes the number of the nonmarket assets the investor has gained access to (in increasing order). Note that all investors have access to the market portfolio (for simplicity). We model increased customization as a shift of mass from a type with less access to one with more access, that is, $\tilde{n}^{i^1_A} = n^{i^1_A} + \Delta n$ and $\tilde{n}^{i^0_A} = n^{i^0_A} - \Delta n$ where $i^1_A > i^0_A$ and $\Delta n > 0$. This is equivalent to expanding the access set of a positive mass of investors to include the new financial assets $j \in \{i_A^0 + 1, .., i_A^1\}$.

The belief types are a collection of $K$-dimensional vectors, $I_B \subset \mathbb{R}^K$, such that type $\mathbf{i}_B$ investors have the belief, $\boldsymbol{\mu}_{\mathbf{z}}^{\mathbf{i}_B} = \boldsymbol{\mu}_{\mathbf{z}} + \mathbf{i}_B$, for the underlying uncertainty. Here, the type describes the deviation of the investor's belief from the average belief, $\boldsymbol{\mu}_{\mathbf{z}} \in \mathbb{R}^K$. We assume that for each type, $\mathbf{i}_B \in I_B$, the opposite type, $-\mathbf{i}_B \in I_B$, also exists and has equal mass,

$$n^{\mathbf{i}_B} = n^{-\mathbf{i}_B} \text{ for each } \mathbf{i}_B \in I_B. \tag{19}$$

This is a mild symmetry assumption that is satisfied for standard distributions. We also make the following assumption that shuts down belief disagreements on the market portfolio,

$$(\mathbf{F}_m)' \, \mathbf{i}_B = 0 \text{ for each } \mathbf{i}_B \in I_B, \text{ which also implies } \mu_m^{\mathbf{i}_B} = \mu_m. \tag{20}$$

This assumption provides analytical tractability but otherwise does not play an important role, as we illustrate below with a numerical example.

The upshot of these assumptions is a closed form characterization of equilibrium, which we present next. To state the result, we define the expected return on a risky asset according to the average belief as $E[r_j] = \mu_j - \log P_j + \frac{\Lambda_j}{2}$, where $\mu_j = \mathbf{F}_j' \boldsymbol{\mu}_z$, and the risk premium according to the average belief as $\zeta_j = E[r_j] + \frac{\Lambda_j}{2} - r_f$.

**Lemma 2.** *Consider the above setting with limited customization of portfolios, full participation in the market portfolio, and belief disagreements that satisfy* (19) *and* (20)*. There exists an equilibrium in which:*

*(i) The average risk premium on each risky asset satisfies,* $\zeta_j = \frac{\Lambda_{jm}}{\Lambda_m} \zeta_m$*, where* $\zeta_m = \gamma \Lambda_m$*.*

*(ii) The risk-free rate,* $r_f$*, is the unique solution to*

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \sum_{i \in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right), \tag{21}$$

*where the certainty equivalent return for an investor with type* $(i_A, \mathbf{i}_B)$ *is,*

$$r_{ce}^{(i_A, \mathbf{i}_B)} = r_f + \frac{1}{2\gamma} \frac{\zeta_m^2}{\Lambda_m} + \frac{1}{2\gamma} \left(\mathbf{F}'_{J^{i_A}} (\mathbf{i}_B)\right)' \Lambda_{J^{i_A}}^{-1} \left(\mathbf{F}'_{J^{i_A}} (\mathbf{i}_B)\right). \tag{22}$$

The first part says that the average risk premium on a risky asset is determined by its "beta" with the market portfolio. It also characterizes the risk-premium on the market portfolio. These are standard asset pricing conditions that would also obtain in a version of our model without any heterogeneity in beliefs or any customization (beyond access to asset $m$, which we assume). Hence, for the purposes of characterizing the risk premia, or relative asset prices, heterogeneity in beliefs or the degree of customization can be ignored. Loosely speaking, in view of the symmetry assumption (19), for every "optimist" whose portfolio shares deviate from the average portfolio share in a particular direction, there is a "pessimist" whose portfolio deviates in exactly the opposite direction. Since belief heterogeneity does not influence investors' portfolio shares on average, it also does not influence relative asset prices. Customization does not influence relative prices either because, absent belief heterogeneity, the market portfolio $m$ is sufficient to construct efficient portfolios in equilibrium.

The second part shows that, although heterogeneity or customization do not affect relative prices, they can influence absolute asset prices as well as the risk-free interest rate. The market clearing condition (22) characterizes the risk free rate in terms of investors' certainty-equivalent returns. Eq. (22) characterizes the certainty-equivalent returns in terms of investors' beliefs and access sets. Our main result, which we present next, describes the effects of greater customization.

**Proposition 4** (Customization)**.** *Consider the equilibrium characterized in Lemma 2. Consider financial innovation that increases the scope of customization for some market participants, $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ and $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ where $i_A^1 > i_A^0$ and $\Delta n > 0$. This change reduces the risk free rate $r_f$, leaves unchanged the average risk premia, $\left\{\zeta_j\right\}_{j \in \mathbf{J}}$ (and relative prices, $\{P_j/P_f\}$), and decreases the average expected return on risky assets, $\left\{r_f + \zeta_j\right\}_{j \in \mathbf{J}}$ (increases $\{P_j\}$).*

The intuition follows from Eq. (22), which implies the investor's certainty-equivalent return, $r_{ce}^{(i_A, \mathbf{i}_B)}$, is increasing in the number of available assets, $i_A$ (see the appendix for a proof, and Eq. (23) below for a special case). Hence, consistent with the choice channel [cf. Proposition 2], greater customization increases investors' savings, $a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right)$. This in turn increases asset prices, as in Section 5.1. The difference is that increased customization does not generate relative price effects. Thus, unlike increased participation, greater customization increases the absolute price of all assets, while leaving relative asset prices unchanged.

In this context, financial innovation increases investors' certainty equivalent returns by enabling them to construct customized portfolios that feature speculative positions. We illustrate this point further in a special case, which we also use to numerically illustrate the results.

### 5.2.1 Special Case and Numerical Illustration

Suppose that the market portfolio satisfies, $\log \varphi_m = z_K$, and the sources of risk, $\{z_1, .., z_K\}$, are linearly independent. Hence, the source, $z_K$, captures systematic risk factor, and the remaining sources, $\{z_1, .., z_{K-1}\}$, capture nonsystematic (e.g., idiosyncratic) risk factors that are orthogonal to the market portfolio as well as one another. For simplicity, suppose each investor is optimistic or pessimistic about one nonsystematic risk factor. Specifically, for each $k < K$, there are two belief types, $\mathbf{i}_{B,k}$ and $-\mathbf{i}_{B,k}$ that respectively think that the mean of $z_k$ is given by $\mu_k + \Delta_k$ and $\mu_k - \Delta_k$, whereas they agree on the objective mean of the remaining risk factors. Thus, type $\mathbf{i}_{B,k}$ investors are optimistic about $z_k$ (and only $z_k$), whereas type $-\mathbf{i}_{B,k}$ investors are pessimistic, and the degree of disagreement is captured by the parameter, $\Delta_k \geq 0$. All investors agree on the objective mean of the systematic risk factor, $z_K$. Note that investors' beliefs satisfy conditions (19) and (20).

Suppose also that there is one (nonmarket) asset per nonsystematic risk factor, that is, $\log \varphi_j = z_j$ for $j \in \{1, .., K-1\}$. These assets can be thought of as the nonsystematic component of stocks, bonds, or other similar financial assets.[16] We also assume investors have symmetric access to these assets, that is, $n^{\bar{i}_A} = 1$ for some $\bar{i}_A \leq K - 1$ (and $n^{i_A} = 0$ for $i_A \neq \bar{i}_A$). Hence, the parameter, $\bar{i}_A$, captures the total number of nonmarket assets available to any investor. Using Eq. (22), the investors' certainty-equivalent return can be written as,

$$r_{ce}^{(\bar{i}_A, \mathbf{i}_{B,k})} = r_{ce}^{(\bar{i}_A, -\mathbf{i}_{B,k})} = \begin{cases} r_{ce}^{nonspec} \equiv r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} & \text{if } \bar{i}_A < k, \\ r_{ce}^{spec} \equiv r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} + \frac{\Delta_k^2}{2\gamma\Lambda_k} & \text{if } \bar{i}_A \geq k. \end{cases} \tag{23}$$

Here, $r_{ce}^{nonspec}$ is the "nonspeculative" return the investor can obtain by only trading the market portfolio, whereas $r_{ce}^{spec} > r_{ce}^{nonspec}$ is the "speculative" return she can obtain by combining the market portfolio with a position in the risk factor, $z_k$. The investor is able to obtain the greater speculative return only if the asset $k$ is available for trade. Hence, greater customization (greater $\bar{i}_A$) increases investors' certainty equivalent returns by allowing more of them to construct customized portfolios with speculative positions.

Eq. (23) also implies that increased customization increases the asset holdings, $a\left(r_{ce}^{spec}\right) > a\left(r_{ce}^{nonspec}\right)$, by both the optimists and the pessimists about the risk factor, $z_k$. It might sound intuitive that optimists increase their savings and cut their consumption, as they need funds to take a long position in the asset. It is perhaps more surprising that the pessimists also increase their savings, since they could in principle consume the funds they generate from the short sale of the asset. As problem (11) illustrates, pessimists use the funds from the short sale to increase their holdings of the safe asset. These safe asset positions, combined with their short positions, is what enables them to obtain a high certainty-equivalent return on their portfolios. They cut consumption and increase asset holdings even further, because

[16] When an asset is introduced for trade, investors can combine it with (a proxy of) the market portfolio to make a pure trade on its nonsystematic component, which is similar to trading the assets $j \in \{1, .., K-1\}$ in our example.

they perceive a high return on their overall portfolios.[17]

We next numerically illustrate Proposition 4. Consider the same baseline parameters as in Section 5.1.1. To calibrate disagreements, let $\sqrt{\Lambda_k} = 50\%$ for each $k < K$, which roughly corresponds to the volatility of an average stock return. We assume $\Delta_k = 25\%$ for each $k < K$, so that optimists' perceived Sharpe ratios on asset $k$ is equal to the historical Sharpe ratio on the market portfolio, that is, $\frac{\Delta_k}{\sqrt{\Lambda_k}} = 0.5$. This calibration makes our analysis comparable to the previous subsection on participation, by ensuring the access to assets $k$ and $m$ increase investors' certainty equivalent return by the same amount [cf. Eq. (23)]. In practice, investors are likely to disagree on multiple sources of risks, and arguably to a greater degree, which would lead to even higher speculative Sharpe ratios.

We investigate the effects of varying the degree of customization, $\bar{i}_A/(K-1)$, over the range, $[0,1]$. Figure 6 illustrates the results of this exercise. The left panel shows that increasing customization reduces the risk-free rate and the expected return on the market portfolio, while leaving the risk premia constant, consistent with Proposition 4. The dashed lines illustrate the solution with $\varepsilon = 1$, which provides a useful benchmark by shutting down the choice channel. Note that, in contrast to Figure 6 on increased participation, the change in returns is entirely driven by the choice channel. Moreover, the choice channel is quantitatively stronger: It reduces the risk-free rate by about 1.5 percentage points, as opposed to less than 0.5pp for increased participation.

The choice channel is relatively strong in this case because, unlike increased participation, increased customization does not feature crowd-out effects. The top right panel of Figure 5 illustrates this by plotting the certainty-equivalent return from speculation for investors who disagree on the return of the newly introduced asset $\bar{i}_A$, which can be thought of as the marginal gain from customization. Note from Eq. (23) that the marginal gain is given by $r_{ce}^{spec} - r_{ce}^{nonspec}$, and does not depend on the level of customization. Consequently, greater customization induces a larger increase on the average certainty-equivalent return and asset holdings compared to greater participation [cf. Figures 6 and 5]. Intuitively, enabling speculation on an asset does not change relative asset prices. Thus, it does not preclude other investors from taking speculative positions on other (or even the same) assets. The absence of crowd-out effects induces a greater reduction in equilibrium asset returns.

We finally illustrate that condition (20) does not play an important role for our analysis beyond facilitating analytical tractability. To this end, suppose investors also disagree about the market portfolio. Specifically, an investor who is optimistic (resp. pessimistic) about a

[17] Short selling in practice also requires saving in relatively safe assets, because of the margin collateral lenders require for both short and long positions. Collateral in the margin account, just like the investors' safe asset holdings in the model, provides protection for potential losses from the short position.

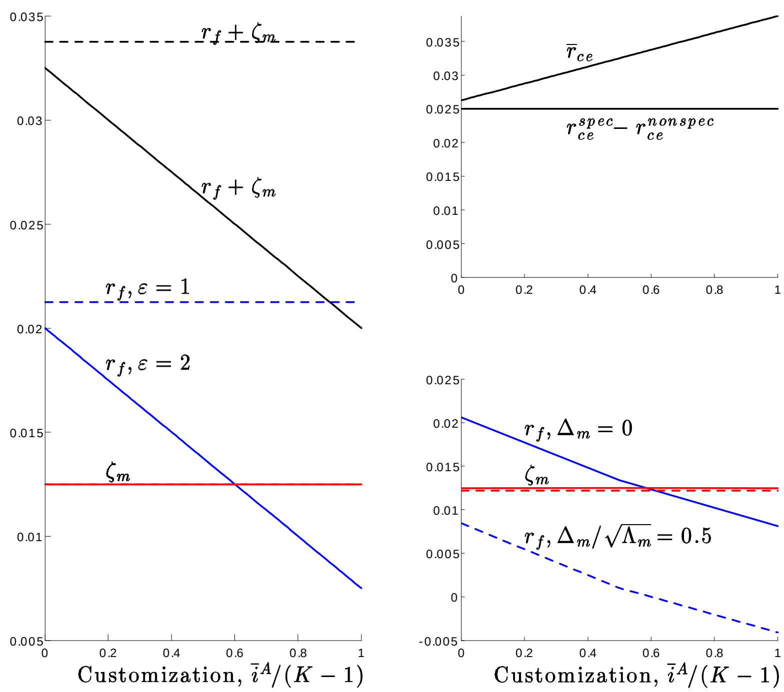


Figure 6: The left panel illustrates the effect of increased customization on asset returns for the main case, $\varepsilon = 2$ (solid lines) and the benchmark case, $\varepsilon = 1$ (dashed lines). The top right panel illustrates the marginal return-gain from customization, as well as the avreage certainty-equivalent return (when $\varepsilon = 2$). The bottom right panel illustrates the asset returns for the main case with no disagreement on the market portfolio (solid lines) and the alternative case with some disagreement on the market portfolio (dashed lines).

nonsystematic risk $k < K$ is also optimistic (resp. pessimistic) about the systematic risk $K$, and thus, the market portfolio, $\log \varphi_m = z_K$. We calibrate the level of disagreement on the market portfolio, $\Delta_m = \Delta_k \sqrt{\frac{\Lambda_m}{\Lambda_k}}$, so that investors are "equally" optimistic about systematic and nonsystematic risks (after normalizing by their relative volatilities). The bottom right panel of Figure 6 illustrates the results of increased customization in this case. Compared to the earlier case with $\Delta_m = 0$, the risk-free rate is uniformly lower. The risk premium is also slightly lower, but the difference is not discernible. More importantly, increased customization reduces the risk-free rate and does not have a discernible effect on the risk premium, as in Proposition 4, even though condition (20) is violated.

Absent condition (20), investors take speculative positions on the market portfolio as well as the nonsystematic risk factors. This generates an additional increase in their certainty-equivalent returns, and reduces the risk-free rate, as illustrated by Figure 6. The difference is that speculation on the market portfolio breaks the symmetry between optimists' and pessimists' returns in Eq. (23). Since the asset $m$ is in positive supply, all investors are its natural buyers. Even if optimists did not adjust their positions (relative to the average investor), their perceived return would be higher simply because they are already holding the market portfolio. Therefore, in equilibrium, optimists obtain a greater certainty-equivalent return—and hold more assets—relative to pessimists. This asymmetry implies that belief disagreements can potentially also affect relative asset prices and risk premia, which makes a theoretical characterization difficult. However, for empirically relevant parameters, these effects are very small, as illustrated by Figure 6, and the results remain qualitatively unchanged.

## 5.3 Emerging Market Savings and Securitization

Another important innovation in recent years is structured finance, or securitization, which helps to distribute risky cash flows according to investors' risk preferences or beliefs. The growth of securitization in recent years is often linked with another phenomenon: the growing demand for safe assets from fast-growing emerging market economies such as China as well as other sources (see Gennaioli, Shleifer, and Vishny (2012)). In this section, we use a variant of the setup in Section 5.1 to investigate the effect of securitization in an environment with high demand for safe assets.

Suppose there are three types of investors, denoted by $\{(1, +), (1, -), 2\}$, that have common beliefs (so it suffices to restrict attention to access to the market portfolio, $m$). Type 2 investors participate only in the safe asset, $J^2 = \emptyset$, similar to type 0 investors in Section 5.1. We now interpret these investors as corresponding to emerging market investors that have a preference for safe assets (due to unmodeled factors), in addition to having a high demand for assets. To capture the latter feature, we allow these investors to have a different asset holdings function, $a^2(\cdot)$, driven by different parameters, $\beta^2, \varepsilon^2$. We consider parameters that ensure that type 2 investors hold more assets than the remaining investors in equilibrium (see below for a precise statement).

To model securitization, we also depart from Section 5.1 by assuming that the remaining investors might face an additional constraint that prevents them from short selling the risk-free asset. Securitization relaxes this constraint, and enables a greater fraction of them to borrow and expand their investments in the market portfolio. We view this as capturing the essence of securitization in practice, which enables some investors (or banks) to make leveraged investments in risky assets by issuing relatively safe debt claims.

Specifically, type $(1,+)$ and $(1,-)$ investors are identical in all dimensions except for their ability to securitize. They have access to all (relevant) risky assets, $J^{(1,+)} = J^{(1,-)} = \{m\}$. However, type $(1,-)$ investors face an additional constraint, $\omega_f^{(1,-)} \geq 0$, that prevents them from short-selling the safe asset, whereas $(1,+)$ investors, which we refer to as "the securitizers," do not face this constraint.

Let $n^2$ and $n^1 = 1 - n^2$ denote the relative mass (or wealth share) of respectively type 2 and type 1 investors, and $n^+ \in [0,1]$ denote the fraction of securitizers within type 1 investors. In this context, our next result characterizes the equilibrium for fixed masses. We then describe the comparative statics of increasing the relative mass of emerging market investors, $n^2$, as well as the relative mass of securitizers, $n^+$. Our main analysis endogenizes the level of securitization, $n^+$, and characterizes how an increase in $n^+$ that is driven by an increase in $n^2$ affects the equilibrium. We then illustrate these results with a numerical example, which also shows that securitization has a different effect on type 1 and 2 investors' asset holdings, with implications for their net savings (or current accounts).

**Lemma 3.** *Consider the above setup with common beliefs, type 1 investors that differ in their ability to securitize, and type 2 investors that invest only in the safe asset. There exists an equilibrium in which $\zeta_m$ and $r_f$ jointly solve,*

$$\zeta_m = \gamma \Lambda_m \left(1 + \frac{n^2}{(1-n^2)\, n^+} \frac{a^2(r_f)}{a^1\left(r_{ce}^{(1,+)}\right)}\right), \tag{24}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = n^2 a^2(r_f) + \left(1 - n^2\right) \overline{a}^1, \text{ where } \overline{a}^1 = n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1 - n^+\right) a^1\left(r_{ce}^{(1,-)}\right). \tag{25}$$

*and the certainty-equivalent returns of type* 1 *investors satisfy* $r_{ce}^{(1,+)} \geq r_{ce}^{(1,-)}$, *where,*

$$r_{ce}^{(1,+)} = r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} \text{ and } r_{ce}^{(1,-)} = r_f + \zeta_m - \frac{\gamma}{2}\Lambda_m. \tag{26}$$

The result is similar to Lemma 1 with limited participation.[18] The main difference is that the aggregate risk that is not held by type 2 investors is now absorbed by a smaller fraction of investors that are able to securitize. Eq. (24) says that the risk premium is determined by the compensation required by the securitizers. Another difference is that type 2 and 1 investors have different asset holding functions, which is captured by the market clearing equation (25). The final difference is that securitizers obtain a greater certainty-equivalent return than non-securitizers. Both of these returns are characterized by Eq. (26). We next describe how an increase in the relative wealth share of type 2 investors affects the equilibrium.

**Proposition 5** (Emerging Market Savings)**.** *Consider an equilibrium characterized in Lemma 3 that features greater savings by type* 2 *investors,* $a^2(r_f) \geq \overline{a}^1$, *and satisfies the condition* $\left.\frac{da^2(r_{ce})/dr_{ce}}{a^2(r_{ce})}\right|_{r_{ce}=r_f} \geq \left.\frac{da^1(r_{ce})/dr_{ce}}{a^1(r_{ce})}\right|_{r_{ce}=r_{ce}^{(1,+)}}$. *Then an increase in the relative mass of type* 2 *investors,* $n^2$, *increases the risk premium,* $\zeta_m$, *and decreases the risk-free rate,* $r_f$.

The result applies for an equilibrium in which type 2 investors hold more assets despite their lower certainty-equivalent returns. This is the case as long as $\beta^2$ is sufficiently greater than $\beta^1$. The result also requires the technical condition that type 2 investors' asset holdings are more reactive to changes in return relative to the remaining investors. This condition is typically satisfied if $\varepsilon^2$ is sufficiently greater than $\varepsilon^1$, but it does not play an important role beyond facilitating analytical tractability.[19] Under these conditions, the proposition says that an increase in $n^2$ increases the price of the safe assets, $P_f$, while reducing the relative price of risky assets, $P_m/P_f$.

The intuition for the relative price effect is the same as in Proposition 3. The increase in $n^2$ is similar to a reversal of increased participation, with the implication that it reverses the decline of the risk premium. Unlike in Proposition 3, however, the increase in $n^2$ has an unambiguous effect on the risk-free rate. This is because type 2 investors not only prefer safe assets but they also demand more assets relative to the remaining investors. The combination of these features ensures that the increase in their relative wealth share decreases the risk-free

[18] Unlike Lemma 3, we are unable to establish the uniqueness of equilibrium, since $a^2(\cdot)$ and $a^1(\cdot)$ are potentially different, although the equilibrium is unique in all of our numerical simulations.

[19] In particular, the results in Proposition 5 continue to hold in our numerical simulations even if we assume $\varepsilon^2 = \varepsilon^1$.

rate, $r_f$, consistent with the savings glut hypothesis (see, for instance Bernanke (2005)). We next describe how financial innovation that expands securitization affects the equilibrium in this environment.

**Proposition 6** (Securitization)**.** *Consider an equilibrium that satisfies the conditions in Proposition 5. An increase in the relative mass of securitizers, $n^+$, decreases the risk premium, $\zeta_m$, and decreases the expected return on the market portfolio, $r_f + \zeta_m$.*

Comparing Propositions 3 and 6 illustrates that increased securitization has the same qualitative effects on equilibrium prices as increased participation. It increases the relative price of the risky assets, $P_m/P_f$, as well as the absolute price of all assets, $P_m$. In our numerical simulations, it also typically decreases the price of the safe asset, $P_f$. Intuitively, similar to participation, securitization increases the relative demand for the risky asset, which reduces the risk premium and tends to increase the risk-free rate.

### 5.3.1 Endogenous Securitization

Securitization in recent years has arguably increased in response to the growing asset demand from emerging markets. We next incorporate this feature into the model by endogenizing the level of securitization, $n^+$, via free entry. Specifically, suppose all type 1 investors are initially non-securitizers. However, each one of them can become a securitizer by paying a fixed cost $c > 0$ per unit of assets. The optimality condition to become a securitizer can then be written as,

$$r_{ce}^{(1,+)} - r_{ce}^{(1,-)} \geq c \text{ with strict inequality only if } n^+ = 1, \tag{27}$$

where $r_{ce}^{(1,+)}$ and $r_{ce}^{(1,-)}$ are given by Eq. (26). The condition says that the marginal benefit of securitization is equated to its marginal cost, except possibly for a corner solution in which all type 1 investors become securitizers. The equilibrium is defined as before, with the additional requirement that $n^+$ is endogenous and condition (27) holds. Our next result characterizes this equilibrium.

**Lemma 4.** *Consider the setup in Lemma 3 with endogenous entry by securitizers. There exists an equilibrium in which $\zeta_m, r_f, n^+$ jointly solve Eqs. (24) − (25) and $(1 - n^+)\left(\zeta_m - \overline{\zeta}_m\right) = 0$, along with $\zeta_m \geq \overline{\zeta}_m$, where $\overline{\zeta}_m$ is the unique positive solution to,*

$$\frac{\overline{\zeta}_m^2}{2\gamma\Lambda_m} - \overline{\zeta}_m + \frac{\gamma}{2}\Lambda_m = c. \tag{28}$$

Here, $\overline{\zeta}_m \geq \gamma \Lambda_m$ is the break-even level of the risk premium which ensures that condition (27) holds as an equality. The result says that, as long there is an interior level of securitization, the level of the risk premium is given by the break-even level. We next present the main result in this section, which describes the effect of increasing the relative wealth share of type 2 investors in an environment with endogenous securitization.

**Proposition 7** (Emerging Market Savings with Endogenous Securitization)**.** *Consider an equilibrium characterized in Lemma 4 which also satisfies* $n^+ < 1$, *greater savings by type* 2 *investors,* $a^2(r_f) \geq \overline{a}^1$, *and the condition* $\left.\frac{da^2(r_{ce})/dr_{ce}}{a^2(r_{ce})}\right|_{r_{ce}=r_f} \geq \left.\frac{da^1(r_{ce})/dr_{ce}}{a^1(r_{ce})}\right|_{r_{ce}=r_{ce}^{(1,+)}}$. *Then, an increase in the relative mass of type* 2 *investors,* $n^2$, *increases the level of securitization,* $n^+$. *In addition, it leaves unchanged the risk premium,* $\zeta_m = \overline{\zeta}_m$, *decreases the the risk-free rate,* $r_f$, *and decreases the expected return on all risky assets.*

The result captures the conventional wisdom that the growing demand for safe assets from emerging markets induces greater securitization. It also illustrates that the combined effect of greater asset demand and greater securitization is to reduce the risk-free rate while having a smaller impact (in fact, in our stylized model, no impact) on the risk premium. Put differently, endogenous securitization fully mitigates the impact of emerging market savings on the risk premium, while only partially mitigating its impact on the risk-free rate. Intuitively, for interior levels of securitization, the risk premium is determined by the marginal cost of securitization as illustrated by Eq. (28).[20] Consequently, the increased asset demand—driven by both greater $n_2$ and greater $n^+$—translates into a reduction in the risk-free rate.

### 5.3.2 Numerical Illustration

We next numerically illustrate these results, along with some other features of the equilibrium. We use the baseline parameters as in Section 5.1, e.g., $\gamma = 5$, $\sqrt{\Lambda_m} = 5\%$, and $\varepsilon^1, \beta^1$ are the same as before. We let $n^2$ vary over the range, $[1\%, 10\%]$, which captures the roughly 10-fold growth of the emerging market central bank reserves since late 1990s (cf. Figure 4). We assign a relatively high discount factor to type 2 investors, $\beta^2 = 1.2\beta^1$, along with the elasticity of substitution, $\varepsilon^2 = 1.2\varepsilon^1 = 2.4$. We calibrate the cost of securitization $c$ so that the implied equity premium (with $\sqrt{\Lambda_e} = 0.16$) is equal to 5%, which is close to its level in Section 5.1 with full participation (cf. Figure 5). We also let $n^+_{init}$ denote the endogenous

[20] Our setup with a fixed marginal cost of securitization is admittedly extreme. However, securitization would also greatly—if not fully—mitigate the impact on the risk premium in alternative specifications, as long as the marginal cost is not increasing too fast in the level of securitization.

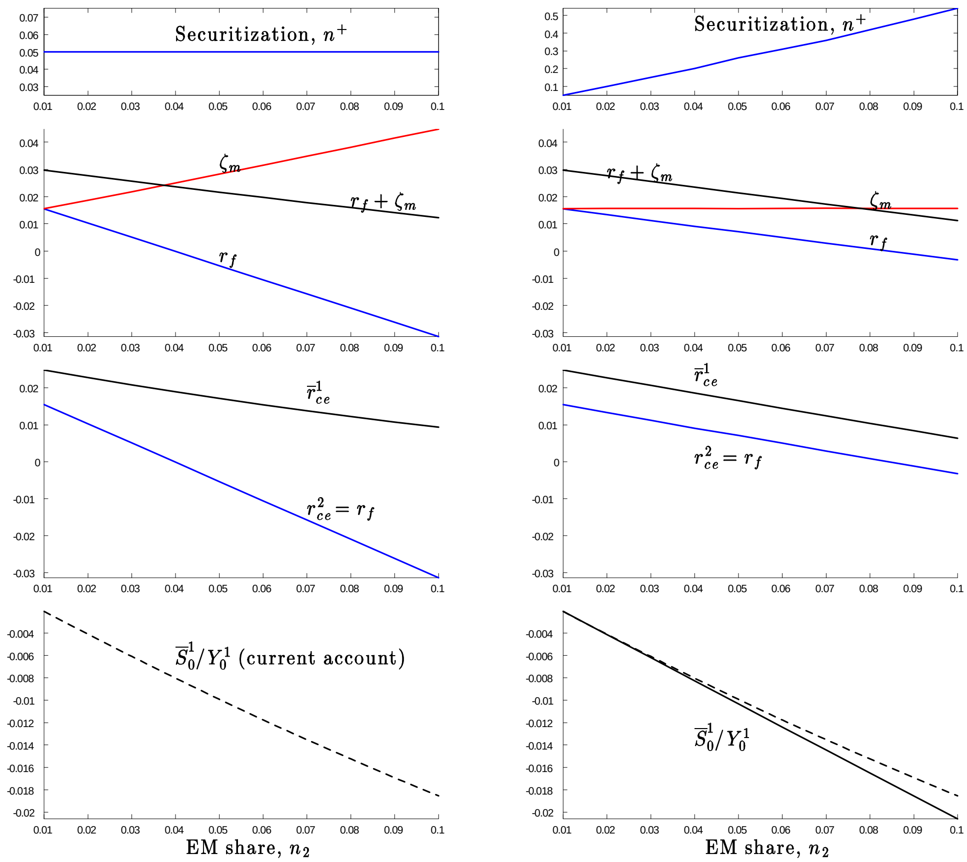


Figure 7: The left panels illustrate the effects of increasing the emerging market share, $n_2$, on equilibrium variables when the level of securitiation, $n^+$, remains constant. The right panels illustrate the corresponding effects when the level of securitization endogenously adjusts to the increase in the emerging market share.

level of securitization that obtains with this cost level and the initial level of type 2 investors, $n^2_{init} = 1\%$.

The panels on the left side of Figure 7 illustrate the effect of increasing $n_2$ while keeping the securitization constant at its initial level, $n^+_{init}$. The second panel shows that, consistent with Proposition 5, the risk premium increases and the risk-free rate declines. The third panel illustrates the certainty-equivalent return for type 2 investors as well as the average certainty-equivalent return for type 1 investors.[21] The general decline in asset returns reduces the certainty-equivalent return for both types. However, the decline is dampened for type 1 investors since they benefit from the rising risk premium. The bottom panel illustrates

[21] Here, $\bar{r}^1_{ce}$ is defined as the solution to $a\left(\bar{r}^1_{ce}\right) = \left(1 - n^+\right) a\left(r^{(1,-)}_{ce}\right) + n^+ a\left(r^{(1,+)}_{ce}\right)$ (see Section 5.1).

that the average (net) savings of type 1 investors as a fraction of their income, $\overline{S}_0^1/Y_0^1 = \left( \left(1-n^+\right) S_0^{(1,-)} + \left(1-n^+\right) S_0^{(1,+)} \right)/Y_0^1$, which corresponds to their current account in this model. Type 1 investors are running a current account deficit driven by the high asset demand by type 2 investors.

The panels on the right side of Figure 7 illustrate the effect of increasing $n_2$ in the model with endogenous securitization. Consistent with Proposition 7, greater emerging market share induces greater securitization. The second panel shows that the combined effect leaves the risk-premium unchanged while reducing the risk-free rate. Note, however, that the risk-free rate is greater than what it would be in the absence of the securitization response. The third panel illustrates that this also implies an increase in the certainty-equivalent return for type 2 investors, and therefore, their asset holdings. In contrast, the average certainty-equivalent return of type 1 investors is similar to the case without the securitization response. Intuitively, while greater securitization increases the certainty-equivalent return for type $(1,-)$ investors, it also decreases the certainty-equivalent return for type $(1,+)$ investors due to a crowd-out effect as in Section 5.1. The bottom panel illustrates further that greater securitization exacerbates the current account deficit of type 1 investors, because it increases type 2 investors' asset holdings without affecting much type 1 investors' average asset holdings.

The example also illustrates the additional conceptual point that financial innovation can increase the savings of some investors even when it does not directly expand their portfolio choice. Greater securitization increases the asset holdings of type 2 investors, despite the fact that the safe asset is available to them before and after financial innovation. Intuitively, type 1 and 2 investors would like to split the cash flows from risky assets according to their heterogeneous preferences. Securitization, which expands the portfolio choice of type 1 investors, facilitates the splitting of cash flows and raises the savings of type 2 investors in equilibrium. Hence, the choice channel, which we formalized for an investor in partial equilibrium (cf. Proposition 2), can also have spillover effects on other investors' savings.

# 6 Conclusion

Rapid financial innovation in recent years has vastly expanded the portfolio choice of investors. We theoretically investigate the implications of financial innovation for investors' savings and asset returns. We establish a choice channel by which, under mild assumptions, an investor that gains access to greater portfolio choice increases her savings.

In equilibrium, greater savings exerts a generally downward pressure on asset returns, but the precise effects also depend on the type of financial innovation. Our main result shows that, under mild assumptions, greater portfolio customization reduces the expected return on all assets, including the risk-free rate, without affecting the risk premia. In contrast, for empirically relevant parameters, greater participation increases the risk-free interest rate, and reduces the risk premium (as well as the expected return) on the market portfolio. Greater securitization has similar pricing implications as greater participation, which mitigate—but does not completely undo—the effects of increased savings from emerging markets. We also find that greater securitization facilitates the absorption of emerging market savings, and can exacerbate global current account imbalances.

Our results are broadly consistent with various trends in financial innovation and asset returns in the US over the last half century. Between 1950 and the early 1980s, market participation has considerably increased, which might have contributed to the decrease in the risk premium and the increase in the risk-free rate over this episode. Starting in the early 1980s, financial instruments that facilitate portfolio customization have become widespread, which might have contributed to the secular decline of the risk-free rate and other asset returns since the 1980s. Securitization started to accelerate in the early 2000s, arguably in response to the increasing demand for safe assets from emerging markets, but collapsed with the recent financial crisis. The collapse of securitization might have contributed to the sharp reduction in the risk-free interest rate, the sharp increase in the risk premium, as well as the reduction in the current account deficit of the US since the financial crisis.

Our main result also sheds some light on the low interest rates in recent years. These rates are worrisome from a macroeconomic policy point of view, as they increase the likelihood of liquidity trap episodes in which the monetary policy is constrained. Our main result suggests that financial innovation that facilitates portfolio customization might be a contributing factor to low interest rates. We also show that other types of financial innovation that facilitate participation or securitization might help to increase the interest rates.

Even though our analysis has been purely positive, our model also has policy implications. For instance, from the lens of our model, restricting portfolio customization or subsidizing securitization might be beneficial by reducing the incidence of liquidity traps. More generally, our analysis highlights that financial innovation affects investors' consumption and savings decisions, with implications for aggregate demand. Economic agents that introduce or adopt these financial innovations do not internalize their effects on aggregate demand, which might create inefficiencies (see Korinek and Simsek (2014)). We leave a more complete analysis of the interaction between financial innovation and aggregate demand externalities for future work.

# A Appendix A: Omitted Proofs

## A.1 Proofs for the partial equilibrium analysis in Section 4

**Proof of Proposition 1.** Let $u^i(c) = c^{1-\gamma^i}$ denote the investor's state utility function. First consider the investor's allocation before financial innovation. The optimality condition for the risk-free asset can then be written as,

$$u'\left(C_0^{i,old}\right) = \left(\beta^i/P_f\right) E\left[u'\left(W_1^{i,old}(z)\right)\right].$$

The key observation is that the CRRA preferences satisfy the prudence condition, $u'''(c) > 0$. In view of this observation (and Jensen's inequality), the optimality condition implies,

$$u'\left(C_0^{i,old}\right) \geq \left(\beta^i/P_f\right) u'\left(E\left[W_1^{i,old}(z)\right]\right). \quad \text{(A.1)}$$

This expression illustrates the precautionary savings motive. When $\beta^i/P_f = 1$, the investor would like to have greater average consumption in the future compared to the current period.

Next consider the investor's allocation after financial innovation. In view of Assumption $2^{\mathrm{P}}$, and the assumption that financial assets complete the market, the investor chooses the perfect risk sharing allocation, that is, $W_1^{i,new}(z) \equiv \overline{W}_1^{i,new}$ for each $z \in Z$. Consequently, the optimality condition for the safe asset implies,

$$u'\left(C_0^{i,new}\right) = \left(\beta^i/P_f\right) u'\left(\overline{W}_1^{i,new}\right). \quad \text{(A.2)}$$

Finally, in view of Assumption 1, the investor's consumption in either case satisfies her lifetime budget constraint,

$$C_0^i + P_f E^n\left[W_1^i(z)\right] = W_0^i + P_f E^i\left[L_1^i(z)\right].$$

Combining Eqs. $(A.1)$ and $(A.2)$ with the lifetime budget constraint implies that $C_0^{i,new} \geq C_0^{i,old}$, or equivalently, $A_0^{i,new} \leq A_0^{i,old}$. Moreover, the inequality is strict whenever the investor's consumption with old assets features less than perfect risk sharing, completing the proof. □

**Proof Proposition 2.** Included in the main text. □

## A.2 Proofs for the general equilibrium analysis in Section 5

### A.2.1 Proofs of results in section 5.1

We start with useful lemma about the behavior of the asset holding function.

**Lemma 5.** *Whenever $\epsilon > 1$, the semi-elasticity $\frac{a'(r_{ce})}{a(r_{ce})}$ is decreasing in $r_{ce}$.*

*Proof.* From the Euler Equation in logarithmic form

$$\log a\left(r_{ce}\right)-\log\left(1-a\left(r_{ce}\right)\right)=\epsilon\log\beta+\left(\epsilon-1\right)r_{ce}$$

thus differentiating with respect to $r_{ce}$ and simplifying

$$\frac{a^{'}\left(r_{ce}\right)}{a\left(r_{ce}\right)}=\left(\epsilon-1\right)\left(1-a\left(r_{ce}\right)\right) \tag{A.3}$$

so $\frac{a^{'}\left(r_{ce}\right)}{a\left(r_{ce}\right)}$ is decreasing in $s$ and therefore in $r_{ce}$, whenever $\epsilon>1$. □

**Proof of Lemma 1.** To simplify notation, we leave implicit the dependence of $\omega_1\left(\zeta_m\right)$, $r^1_{ce}\left(r_f,\zeta_m\right)$ and $P_m\left(r_f+\zeta_m\right)$ on $\left(r_f,\zeta_m\right)$. Using Eq. (15), type 1 investors' portfolio share and return are given by,

$$\omega^1_m=\frac{\zeta_m}{\gamma\Lambda_m}\text{ and }r^1_{ce}=r_f+\frac{1}{2\gamma\Lambda_m}\zeta^2_m,$$

establishing Eq. (18).

Notice that the market clearing condition for the safe asset can be written as,

$$0=n^1\left(1-\omega^1_m\right)a\left(r^1_{ce}\right)+n^0a\left(r_f\right).$$

Rearranging this expression implies Eq. (18). Finally, Eq. (25) follows by adding all of the market clearing conditions (14).

It remains to show that the system in $(24)-(25)$ has a unique solution. Towards that end let us first define the average level of savings out of wealth as $\overline{a}\left(r_f,\zeta_m,n^1\right)\equiv n^1a\left(r^1_{ce}\right)+\left(1-n^1\right)a\left(r_f\right)$, and the relative value of the asset endowment as $v\left(r_f+\zeta_m\right)\equiv\frac{\eta_mP_m}{e_0+\eta_mP_m}$. Combined they characterize

$$\varphi_1\left(r_f,\zeta_m,n_1\right)\equiv\overline{a}\left(r_f,\zeta_m,n^1\right)-v\left(r_f+\zeta_m\right).$$

Notice that $v^{'}\left(r_f+\zeta_m\right)=-e_0v\left(r_f+\zeta_m\right)<0$. As a consequence, $\frac{\partial\varphi_1\left(r_f,\zeta_m,n_1\right)}{\partial r_f}=\frac{\partial\overline{a}}{\partial r_f}-v^{'}>0$, and $\frac{\partial\varphi_1\left(r_f,\zeta_m,n_1\right)}{\partial\zeta_m}=\frac{\partial\overline{a}}{\partial\zeta_m}-v^{'}>0$.

Additionally, we define

$$\varphi_2\left(r_f,\zeta_m,n_1\right)\equiv n^1\left(1-\omega^1\right)a\left(r^1_{ce}\right)+\left(1-n^1\right)a\left(r_f\right).$$

An equilibrium then a solution to $\varphi_1\left(r_f,\zeta_m,n_1\right)=\varphi_2\left(r_f,\zeta_m,n_1\right)=0$.

Notice then that, $\frac{\partial\varphi_2}{\partial r_f}=n^1\left(1-\omega^1\right)a^{'}\left(r^1_{ce}\right)+\left(1-n^1\right)a^{'}\left(r_f\right)$. Additionally, $\varphi_2\left(r_f,\zeta_m,n_1\right)=0\implies\left(1-\omega^1\right)=-\frac{\left(1-n^1\right)}{n^1}\frac{a\left(r_f\right)}{a\left(r^1_{ce}\right)}$ and $\frac{\partial\varphi_2}{\partial r_f}=\frac{\left(1-n^1\right)}{a\left(r_f\right)}\left[\frac{a^{'}\left(r_f\right)}{a\left(r_f\right)}-\frac{a^{'}\left(r_{ce}\right)}{a\left(r_{ce}\right)}\right]$ which is positive whenever $\epsilon>1$, given Lemma 5. Last, $\frac{\partial\varphi_2}{\partial\zeta_m}=-\frac{\partial\omega^1}{\partial\zeta_m}n^1a\left(r^1_{ce}\right)+n^1\left(1-\omega^1\right)a^{'}\left(r^1_{ce}\right)\frac{\partial r_{ce}}{\partial\zeta_m}<0$ since $\left(1-\omega^1\right)<0$

whenever $\varphi_2 = 0$.

As a consequence, locus $\varphi_1(r_f, \zeta_m, n_1) = 0$ is downward slopping on $(r_f, \zeta_m)$-space while locus $\varphi_2(r_f, \zeta_m, n_1) = 0$ is upward slopping. Both loci are characterized by continuous functions. We can use $\varphi_1(r_f, \zeta_m, n_1) = 0$, with $\frac{\partial \varphi_1}{\partial \zeta_m} \neq 0$, and the Implicit Function Theorem to define a decreasing function $\zeta_m^{\varphi_1}(\cdot)$ of the interest rate $r_f$ over the first locus. We then look for a solution to $\varphi_2\left(r_f, \zeta_m^{\varphi_1}(r_f), n_1\right) = 0$, where the left-hand side strictly increasing function of $r_f$. The existence of a solution is guaranteed by Proposition **8** and uniqueness follows from strict monotonicity. □

**Proof of Lemma 3** Let $J \equiv \begin{bmatrix} \frac{\partial \varphi_1}{\partial r_f} & \frac{\partial \varphi_1}{\partial \zeta_m} \\ \frac{\partial \varphi_2}{\partial r_f} & \frac{\partial \varphi_2}{\partial \zeta_m} \end{bmatrix}$ and $\Delta_J < 0$ denote its determinant. Then,

$$\begin{bmatrix} \frac{dr_f}{dn^1} \\ \frac{d\zeta_m}{dn^1} \end{bmatrix} = -\frac{1}{\Delta_J} \begin{bmatrix} \frac{\partial \varphi_2}{\partial \zeta_m} & -\frac{\partial \varphi_1}{\partial \zeta_m} \\ -\frac{\partial \varphi_2}{\partial r_f} & \frac{\partial \varphi_1}{\partial r_f} \end{bmatrix} \begin{bmatrix} a\left(r_{ce}^1\right) - a\left(r_f\right) \\ -\frac{a\left(r_f\right)}{n_1} \end{bmatrix}.$$

Therefore, $\frac{d\zeta_m}{dn^1} < 0$. Also,

$$\begin{aligned} \frac{dE\left[r_f + \zeta_m\right]}{dn^1} &= \propto \left(a\left(r_{ce}^1\right) - a\left(r_f\right)\right)\left(\frac{\partial \varphi_2}{\partial \zeta_m} - \frac{\partial \varphi_2}{\partial r_f}\right) + \left(\frac{\partial \varphi_1}{\partial \zeta_m} - \frac{\partial \varphi_1}{\partial r_f}\right)\frac{a\left(r_f\right)}{n_1} \\ &= \left(a\left(r_{ce}^1\right) - a\left(r_f\right)\right)\left(\frac{\partial \varphi_2}{\partial \zeta_m} - \frac{\partial \varphi_2}{\partial r_f}\right) + \left(\frac{a'\left(r_{ce}^1\right)}{a\left(r_{ce}^1\right)} - \frac{a'\left(r_f\right)}{a\left(r_f\right)}\right)\frac{\left(1 - n^1\right)}{n_1}\left(a\left(r_f\right)\right)^2 < 0 \end{aligned}$$

again using Lemma 5. □

### A.2.2 Proofs of results in section 5.2

**Proof of Lemma 2.** We define the average portfolio share of an asset $j$ among all investors that have market access $i_A \in I_A$ as,

$$\omega_j^{i_A} = \frac{\sum_{I_B} n^{\mathbf{i}_B} \omega_j^{(i_A, \mathbf{i}_B)} a\left(r_{ce}^{(i_A, -\mathbf{i}_B)}\right)}{\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right)}. \tag{A.4}$$

We will establish the existence of an equilibrium in which prices are uniquely characterized by parts (i)-(ii), investors' certainty-equivalent returns are given by Eq. (22), and their average portfolio shares are given by,

$$\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\zeta}_{j^{i_A}} = \left[\omega_m, 0, ..0\right]' \text{ for each } i_A \text{, where } \omega_m = \frac{\zeta_m}{\gamma\Lambda_m}. \tag{A.5}$$

Here, $[\omega_m, 0, ..0]$ is a $\left|J^i\right|$-dimensional vector whose first entry is $\omega_m$ and the remaining entries are zero. Hence, in addition to the properties in the lemma, we claim that investors' average portfolio shares are independent of the heterogeneity in beliefs or market access.

We first establish Eq. $(A.5)$, given the prices characterized by parts (i)-(ii) and the certainty-equivalent returns in (22). To prove this, consider an investor's perceived risk premium for a risky asset $j$, which can be written as,

$$\zeta_j^{(i_A,\mathbf{i}_B)} = (\mathbf{F}_j)' \boldsymbol{\mu}_{\mathbf{z}}^i + \frac{\Lambda_j}{2} - \log P_j - r_f = \zeta_j + \mathbf{F}_j'\mathbf{i}_B. \tag{A.6}$$

Using Eq. (15), her demand for the risky assets $J^{i_A}$ (as a proportion of her wealth) is given by the vector,

$$\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\left(\boldsymbol{\zeta}_{j^{i_A}} + \mathbf{F}_j'\mathbf{i}_B\right) a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

In view of Eq. (22), investors of types $(i_A, \mathbf{i}_B)$ and $(i_A, -\mathbf{i}_B)$ obtain exactly the same certainty equivalent return. Combining these observations, the average demand across belief types $\mathbf{i}_B$ and $-\mathbf{i}_B$ is given by,

$$\frac{\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) + \boldsymbol{\omega}_{J^{i_A}}^{(i_A,-\mathbf{i}_B)} a\left(r_{ce}^{(i_A,-\mathbf{i}_B)}\right)}{2} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\zeta}_{j^{i_A}} \times a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

Averaging across all belief types, and using Eq. (19), we further obtain,

$$\sum_{\mathbf{i}_B} n^{\mathbf{i}_B}\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,-\mathbf{i}_B)}\right) = \left(\frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\zeta}_{j^{i_A}}\right)\left(\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)\right).$$

Using the definition of the average portfolio share in $(A.4)$, we obtain $\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\zeta}_{j^{i_A}}$. Next note that,

$$\left(\Lambda_{J^{i_A}}\left[\omega_m, 0, ..0\right]'\right)_j = \Lambda_{mj}\omega_m = \frac{1}{\gamma}\frac{\Lambda_{mj}\zeta_m}{\Lambda_m} = \frac{1}{\gamma}\zeta_j,$$

where the last equation uses part (i). Applying $\Lambda_{J^{i_A}}^{-1}$ to both sides of the expression implies, $\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\zeta}_{j^{i_A}} = [\omega_m, 0, ..0]'$, proving Eq. $(A.5)$.

We next check that the investors' certainty-equivalent returns are given by Eq. (22). Using Eqs. (15) and $(A.6)$, we have,

$$\begin{aligned}
r_{ce}^i &= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\zeta}_{J^i} + \mathbf{F}'_{J^i}\mathbf{i}_B\right)'\Lambda_{J^i}^{-1}\left(\boldsymbol{\zeta}_{J^i} + \mathbf{F}'_{J^i}\mathbf{i}_B\right) \\
&= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\zeta}'_{J^i}\Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i} + 2\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\left(\Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i}\right) + \left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)'\Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\right) \\
&= r_f + \frac{1}{2}\left(\boldsymbol{\zeta}'_{J^i}\left[\omega_m, 0, ..0\right]'\right) + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)'\Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right) + \left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\left[\omega_m, 0, ..0\right]' \\
&= r_f + \frac{1}{2\gamma}\frac{\zeta_m^2}{\Lambda_m} + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)'\Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right),
\end{aligned}$$

verifying Eq. (22). Here, the third line uses Eq. $(A.5)$, and the last line uses the assumption (20) that there is no disagreement on the market portfolio, so that $(\mathbf{F}_m)'\,\mathbf{i}_B = 0$.

Next note that parts (i)-(ii) uniquely characterize the equilibrium prices of all assets. We finally check that these prices satisfy the $|\mathbf{J}|+1$ market clearing conditions (14). The conditions for $j \neq m$ hold because $\omega_j^{i_A} = 0$ for each $i_A$ and $j \neq m$. To check the remaining conditions, substitute $\omega_m = 1$ in view of part. After this substitution, the market clearing condition for asset $f$ holds since each investor has a zero weight on the risk-free asset, $\omega_f = 1 - \omega_m = 0$. The market clearing condition for asset $m$ also holds, since the condition becomes identical to Eq. (21) in part (ii). This establishes the existence of an equilibrium that satisfies the conditions in the lemma along with Eq. $(A.5)$, completing the proof. □

### A.2.3 Proofs of results in section 5.3

**Proof of Lemma 3.** Type $1s$ investors' portfolio share and return are the same as before,

$$\omega_m^{(1,+)} = \frac{\zeta_m}{\gamma \Lambda_m} > 1 \text{ and } r_{ce}^{(1,+)} = r_f + \frac{1}{2\gamma\Lambda_m}\zeta_m^2.$$

In view of the short selling constraint for the safe asset, type $1n$ investors' portfolio share is given by $\omega_m^{1n} = \max\left(\frac{\zeta_m}{\gamma\Lambda_m}, 1\right)$. In the conjectured equilibrium (and in fact, in any equilibrium), we have $\zeta_m > \gamma\Lambda_m$. Thus, the constraint binds and we have,

$$\omega_m^{(1,-)} = 1 \text{ and } r_{ce}^{(1,-)} = r_f + r_f + \zeta_m - \frac{\gamma}{2}\Lambda_m.$$

Note that $\zeta_m > \gamma\Lambda_m$ also implies $r_{ce}^{1s} > r_{ce}^{1n}$, establishing Eq. (26).

Next note that the market clearing condition for the safe asset can be written as,

$$0 = n^{(1,+)}\left(1 - \omega_m^{(1,+)}\right) a\left(r_{ce}^{1s}\right) + n^2 a\left(r_f\right).$$

Rearranging this expression implies Eq. (24). Finally, Eq. (25) follows by adding all of the market clearing conditions (14). Existence is ensured by Proposition the 8 the proof of which is not altered by short-selling constraints that only apply to a subset of agents. □

We use the following definitions across the proof of both propositions that follow. First, we define the average levels of savings as $\overline{a}\left(r_f, \zeta_m, n\right) = n^2 a^2\left(r_f\right) + n^1\left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1 - n^+\right) a^1\left(r_{ce}^{(1,-)}\right)\right)$ and the relative value of asset endowments as $v\left(r_f + \zeta_m\right) \equiv \frac{\eta_m P_m}{e_0 + \eta_m P_m}$.Therefore, $v'\left(r_f + \zeta_m\right) = -e_0 v\left(r_f + \zeta_m\right)$.

The market clearing conditions can be rewritten as

$$\varphi_1\left(r_f, \zeta_m, n\right) \equiv \overline{a}\left(r_f, \zeta_m, n\right) - v\left(r_f + \zeta_m\right) = 0$$

and

$$\varphi_2\left(r_f, \zeta_m, n\right) \equiv n^2 a^2\left(r_f\right) + n^1 n^+\left(1 - \omega^{1,+}\right) a^1\left(r_{ce}^{(1,+)}\right) = 0.$$

We have $\frac{\partial \varphi_1}{\partial r_f} = \frac{\partial \overline{a}}{\partial r_f} - v' > 0$ and$\frac{\partial \varphi_1}{\partial \zeta_m} = \frac{\partial \overline{a}}{\partial \zeta_m} - v' > 0$. Notice that $\frac{\partial \varphi_1}{\partial r_f} - \frac{\partial \varphi_1}{\partial \zeta_m} =$

$n^2 a^2(r_f)\left(\frac{a^{0\prime}(r_f)}{a^2(r_f)} - \frac{a^{1\prime}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) > 0.$

Also, $\frac{\partial\varphi_2}{\partial r_f} = n^2 a^2(r_f)\left(\frac{a^{0\prime}(r_f)}{a^2(r_f)} - \frac{a^{1\prime}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) > 0$ and $\frac{\partial\varphi_2}{\partial\zeta_m} = -n^1\frac{a^1\left(r_{ce}^{(1,+)}\right)}{\gamma\Lambda_m} - n^2 a^2(r_f)\frac{a^{1\prime}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\omega^{1,+} < 0$. Let $J \equiv \begin{bmatrix} \frac{\partial\varphi_1}{\partial r_f} & \frac{\partial\varphi_1}{\partial\zeta_m} \\ \frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_2}{\partial\zeta_m} \end{bmatrix}$ and $\Delta_J$ denote its determinant.

**Proof of Proposition 5.** We have $\frac{\partial\varphi_1}{\partial n_0} = a^2(r_f) - \left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + (1-n^+)\, a^1\left(r_{ce}^{(1,-)}\right)\right) > 0$, and $\frac{\partial\varphi_2}{\partial n_0} = a^2(r_f) - n^+\left(1-\omega^{1,+}\right) a^1\left(r_{ce}^{(1,+)}\right) > 0$.Then,

$$\begin{bmatrix} \frac{dr_f}{dn_0} \\ \frac{d\zeta_m}{dn_0} \end{bmatrix} = -\frac{1}{\Delta_J}\begin{bmatrix} \frac{\partial\varphi_2}{\partial\zeta_m} & -\frac{\partial\varphi_1}{\partial\zeta_m} \\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f} \end{bmatrix}\begin{bmatrix} \frac{\partial\varphi_1}{\partial n_0} \\ \frac{\partial\varphi_2}{\partial n_0} \end{bmatrix}.$$

The condition on savings rates ensures that $\Delta_J < 0$. Then $\frac{dr_f}{dn_0} < 0$. Additionally,

$$\begin{aligned} \frac{d\zeta_m}{dn_0} &\propto -\frac{\partial\varphi_2}{\partial r_f}\frac{\partial\varphi_1}{\partial n_0} + \frac{\partial\varphi_1}{\partial r_f}\frac{\partial\varphi_2}{\partial n_0} \\ &= \frac{\partial\varphi_1}{\partial\zeta_m}\frac{\partial\varphi_1}{\partial n_0} + \frac{\partial\varphi_1}{\partial r_f}\left(\frac{\partial\varphi_2}{\partial n_0} - \frac{\partial\varphi_1}{\partial n_0}\right) > 0. \end{aligned}$$

□

**Proof of Proposition 6.** First notice that $\frac{\partial\varphi_1}{\partial n^+} = n^1\left(a^1\left(r_{ce}^{(1,+)}\right) - a^1\left(r_{ce}^{(1,-)}\right)\right) > 0$ and $\frac{\partial\varphi_2}{\partial n^+} = n^1\left(1-\omega^{1,+}\right) a^1\left(r_{ce}^{(1,+)}\right) = -n^2 n^+ a^2(r_f) < 0$. So,

$$\begin{bmatrix} \frac{dr_f}{dn^+} \\ \frac{d\zeta_m}{dn^+} \end{bmatrix} = -\frac{1}{\Delta_J}\begin{bmatrix} \frac{\partial\varphi_2}{\partial\zeta_m} & -\frac{\partial\varphi_1}{\partial\zeta_m} \\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f} \end{bmatrix}\begin{bmatrix} \frac{\partial\varphi_1}{\partial n^+} \\ \frac{\partial\varphi_2}{\partial n^+} \end{bmatrix}$$

where $\Delta_J$ is the determinant defined previously. Under the condition on savings rates, $\Delta_J < 0$ and $\frac{\partial\varphi_2}{\partial r_f} > 0$, so it follows that

$$\frac{d\zeta_m}{dn^+} \propto -\frac{\partial\varphi_2}{\partial r_f}\frac{\partial\varphi_1}{\partial n^+} + \frac{\partial\varphi_1}{\partial r_f}\frac{\partial\varphi_2}{\partial n^+} > 0$$

and

$$\begin{aligned} \frac{d(r_f+\zeta_m)}{dn^+} &\propto \left(\frac{\partial\varphi_2}{\partial\zeta_m} - \frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_1}{\partial n^+} + \left(-\frac{\partial\varphi_1}{\partial\zeta_m} + \frac{\partial\varphi_1}{\partial r_f}\right)\frac{\partial\varphi_2}{\partial n^+} \\ &= \left(\frac{\partial\varphi_2}{\partial\zeta_m} - \frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_1}{\partial n^+} + \left(\frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_2}{\partial n^+} < 0. \end{aligned}$$

□

**Proof of Lemma 4.** To show existence, first note that Lemma 3 and an application of the implicit function theorem imply that $\zeta_m$ is continuous in $n^+$ for $0 < n^+ \leq 1$. Next, optimality condition for securitization, (27), and the definition of $\overline{\zeta}_m$, (28), imply that $\zeta_m \geq \overline{\zeta}_m$, for any $0 < n^+ \leq 1$. These ensure that equation

$$\left(1-n^+\right)\left(\zeta_m - \overline{\zeta}_m\right) = 0$$

has at least one solution $n^+$ with $0 < n^+ \leq 1$. Finally, by Lemma 3 there exists an equilibrium for the economy with exogenous $n^+$ for the value(s) of $n^+$ that solve the above equation. This ensures existence of an equilibrium with an endogenous value of $n^+$.

Next, we define the average levels of savings as $\overline{a}\left(r_f, \zeta_m, n\right) = n^2 a^2\left(r_f\right) + \left(1-n^2\right)\left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1-n^+\right) a^1\left(r_{ce}^{(1,-)}\right)\right)$ and the relative value of asset endowments as $v\left(r_f+\zeta_m\right) \equiv \frac{\eta_m P_m}{e_0+\eta_m P_m}$.Therefore, $v'\left(r_f+\zeta_m\right) = -e_0 v\left(r_f+\zeta_m\right)$.

We have now a system characterized by

$$\varphi_1\left(r_f, \zeta_m, n\right) \equiv \overline{a}\left(r_f, \zeta_m, n\right) - v\left(r_f+\zeta_m\right) = 0,$$

$$\varphi_2\left(r_f, \zeta_m, n\right) \equiv n^2 a^2\left(r_f\right) + n^1 n^+\left(1-\omega^{1,+}\right) a^1\left(r_{ce}^{(1,+)}\right) = 0,$$

and

$$\varphi_3\left(\zeta_m, n^+\right) \equiv \left(1-n^+\right)\left(\zeta_m - \overline{\zeta}_m\right) = 0$$

where the first condition follows from market-clearing across both markets, the second is simply market clearing in the riskless asset market and the third reflects endogenous entry, where either $n^+ = 1$ and $\zeta_m \geq \overline{\zeta}_m$ or $n^+ < 1$ and $\zeta_m = \overline{\zeta}_m$. □

**Proof of Proposition 7.** If a solution features $n^+ < 1$, then by continuity, locally any solution to $\varphi_1\left(r_f, \zeta_m, n\right) = \varphi_2\left(r_f, \zeta_m, n\right) = \varphi_3\left(\zeta_m, n^+\right) = 0$ features $\zeta_m = \overline{\zeta}_m$. That leads to $\varphi_1\left(r_f, \overline{\zeta}_m, n\right) = \varphi_2\left(r_f, \overline{\zeta}_m, n\right) = 0$.

We define the matrix

$$\hat{J} \equiv \begin{bmatrix} \frac{\partial \varphi_1}{\partial r_f} > 0 & \frac{\partial \varphi_1}{\partial n^+} > 0 \\ \frac{\partial \varphi_2}{\partial r_f} > 0 & \frac{\partial \varphi_2}{\partial n^+} < 0 \end{bmatrix}.$$

The condition that $\left(\frac{a^{0\prime}\left(r_f\right)}{a^2\left(r_f\right)} - \frac{a^{1\prime}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) \geq 0$ is sufficient for $\Delta_{\hat{J}} < 0$, where $\Delta_{\hat{J}}$ denotes the determinant of $\hat{J}$ . Then,

$$\frac{\partial r_f}{\partial n^2} \propto \frac{\partial \varphi_2}{\partial n^+}\left(a^2 - \overline{a}_1\right) + \frac{\partial \varphi_1}{\partial n^+} \frac{n^+\left(1-\omega^{1,+}\left(\overline{\zeta}_m\right)\right) a^1\left(r_{ce}^{(1,+)}\right)}{n^2} < 0.$$

□

# B Appendix B: Omitted Results

**Proposition 8** (Existence)**.** *Under Assumptions $2^G$ and 3, there exists an approximate equilibrium with $P_j > 0$ for each $j \in \{f\} \cup \mathbf{J}$. The portfolio weights and the certainty equivalent returns are characterized by Eq.* (15)*, and the prices are characterized as the solution to the demand system* (14)*.*

**Proof.** Let $P = \{P_j\}_{j \in \{f\} \cup J}$ denote the asset price vector. We work with a truncated economy, where prices satisfy $P_j \leq \alpha$ for each asset $j \in \{f\} \cup J$. We are only interested in sufficiently large $\alpha$ so that the truncation becomes inconsequential. First, let extended portfolio weights be also defined over assets that agent $i$ cannot trade, so that

$$\hat{\omega}_j^i(P) \equiv \begin{cases} \omega_j^i(P), \text{ whenever } j \in \{f\} \cup J^i \\ 0, \text{ otherwise.} \end{cases}$$

For $P \gg 0$ we have individual excess demand for asset $j \in \{f\} \cup J$ defined as

$$z_j^i(P) \equiv \frac{\hat{\omega}_j^i(P)}{P_j} A_0^i(P) - x_{-1,j}^i \tag{B.1}$$

and we analogously define the excess demand for consumption at date $t = 0$ as $z_0^i(P) \equiv c_0^i(P) - e_0^i$. Aggregate excess demands are then simply defined as $z_j(P) \equiv \sum_i n^i z_j^i(P)$ and $z_0(P) \equiv \sum_i n^i z_0^i(P)$. Walras' Law, i.e., $z_0(P) + \sum_{j \in J} P_j z_j(P) = 0$ can be trivially verified from individual optimality.

First, we impose a lower bound on prices $\hat{\epsilon} > 0$, which we successively relax later. Define $S_{\hat{\epsilon}} \equiv \left\{P \in \mathbb{R}_{++}^{|J|} | P_j \geq \hat{\epsilon} \text{ and } P_j \leq \alpha, \forall j \in \{f\} \cup J\right\}$ which is compact and convex. We are only interested in $\alpha > \hat{\epsilon}$ as to ensure the non-emptiness of $S_{\hat{\epsilon}}$.

We next define a continuous price updating function. Let each entry, which describes the update to the price of asset $j \in J$, be defined by

$$P_j^{upd}(P, \hat{\epsilon}) \equiv \begin{cases} \hat{\epsilon},\, if\, z_j(P) < \hat{\epsilon} - P_j \\ P_j + z_j(P),\, if\, \hat{\epsilon} - P_j \leq z_j(P) \leq \alpha \\ \alpha,\, if\, z_j(P) > \alpha \end{cases}. \tag{B.2}$$

Then, let the function $P^{upd}(P, \hat{\epsilon}) : S_{\hat{\epsilon}} \to S_{\hat{\epsilon}}$ be defined as $P^{upd}(P, \hat{\epsilon}) = \left\{P_j^{upd}(P, \hat{\epsilon})\right\}_{j \in \{f\} \cup J}$. As excess demand functions are continuous, so is the function $P^{upd}(\cdot, \hat{\epsilon})$, which maps the non-empty, convex, and compact set $S_{\hat{\epsilon}}$ into itself. From Brouwer's Fixed Point Theorem, there exists $P^{\hat{\epsilon}} \in S_{\hat{\epsilon}}$ such that $P_j^{upd}\left(P^{\hat{\epsilon}}, \hat{\epsilon}\right) = P^{\hat{\epsilon}}$.

We now take a sequence $\{\hat{\epsilon}_k\}_{k \in \mathbb{N}}$ such that $\hat{\epsilon}_k \to 0$. Let $\left\{P^{\hat{\epsilon}_k}\right\}_{k \in \mathbb{N}}$ be the associated sequence of fixed points. As each price lies in $[0, \alpha]$ that sequence is bounded and admits a converging subsequence. To save on notation, assume we have selected such subsequence from the start.

Define its limit by $P^* = \left(P_1^*, P_2^*, ..., P_{|J|}^*\right)$. Naturally $P^* \in \overline{\cup_k S_{\hat{\epsilon}_k}} = \left\{P \in \mathbb{R}_+^{|J|} | P_j \leq \alpha, \forall \{f\} \cup J\right\}$. We now show that $P^* \in \mathbb{R}_{++}^{|J|}$.

Consider the case with $P_j^* = 0$ for risky assets, which w.l.o.g. we call assets $1, .., m$, while the riskless rate remains bounded away from zero. In this case, the risk premia for assets $1, .., m$ approach $+\infty$, and the risk premia for the remaining assets remain finite. Consider all investors that have access to at least one of the assets $1, .., m$ and call that set $I_{r\to\infty}$. It is easy to check that each of these investors have $r_{ce} \to \infty$, and thus, they save all their wealth.

Now consider the net demand for assets that comes from these investors only, $z_j^{I_{r=\infty}} \equiv \sum_{i \in I_{r\to\infty}} n^i z_j^i(P)$. We claim that regardless of how the prices for $1, .., m$ approach 0 (or conversely, regardless of the risk premia approach infinity), there exists at least one asset within $1, .., m$ such that the total demand from these investors for that asset becomes unboundedly positive. Since the demand from the other investors is finite, this will provide a contradiction.

Let us rewrite risk premia along the sequence. Take a given agent $i \in I_{r\to\infty}$, then the (individually perceived) risk-premium $\zeta_j^{i,k}\left(P^{\hat{\epsilon}_k}\right)$ on any asset $j \in J$ can be appropriately rewritten as $\zeta_j^{i,k} = \left\|\zeta^{i,k}\right\| \hat{\zeta}_j^{i,k}$ where $\left\|\zeta^{i,k}\right\| := \sum_j \left|\zeta_j^{i,k}\right|$ denotes a norm and

$$\hat{\zeta}_j^{i,k} \equiv \frac{\zeta_j^{i,k}}{\left\|\zeta^{i,k}\right\|}$$

denotes the $j-th$ entry of a normalized risk-premium vector.[22] The vector $\hat{\zeta}^{i,k} = \left\{\hat{\zeta}_j^{i,k}\right\}_{j \in J}$ belongs to the surface of the unit ball centered at zero.

As that surface is a compact set, $\left\{\hat{\zeta}^{i,k}\right\}_{k \in \mathbb{N}}$ admits a converging subsequence, which we can index by $k_i \in \mathbb{N}$. That forms another price sequence $\left\{P^{\hat{\epsilon}_{k_i}}\right\}_{k_i \in \mathbb{N}}$, from which we can extract a subsequence to ensure that the analogously defined vector $\hat{\zeta}^{i',k_i}$ converges for any second agent $i' \in I_{r\to\infty}$. Given that $I_{r\to\infty}$ is finite, this step can be iteratively repeated until a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, is extracted and ensures that each $\hat{\zeta}^{i,\tilde{k}}$ converges. Additionally, for each $i \in I_{r\to\infty}$, $\lim_{\tilde{k}\to\infty} \hat{\zeta}^{i,\tilde{k}} = \hat{\zeta}$, i.e., the limit of the normalized risk-premia are the same and independent of $i \in I_{r\to\infty}$, since disagreements are bounded, while at least one return goes to infinity.

Take a given agent $i \in I_{r\to\infty}$. Define $\hat{\zeta}_{J_i}^{i,\tilde{k}}$ and $\hat{\zeta}_{J_i}$ to be respectively the restriction of the normalized risk premia vectors $\hat{\zeta}^{i,\tilde{k}}$ and $\hat{\zeta}$ to the assets that agent $i$ can trade. Notice that along that subsequence portfolio weights of the form $\omega_{J^i}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \frac{1}{\gamma^i} \Lambda_{J^i}^{-1} \hat{\zeta}_{J_i}^{i,\tilde{k}} \left\|\zeta^{i,\tilde{k}}\right\|$ are optimal from equation 15. Therefore, we take the following limit of an inner product

$$\lim_{\tilde{k}\to\infty} \left\langle \hat{\zeta}_{J_i}^{i,\tilde{k}}, \frac{\omega_{J^i}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|} \right\rangle = \frac{1}{\gamma^i} \hat{\zeta}_{J_i}' \Lambda_{J^i}^{-1} \hat{\zeta}_{J_i} > 0$$

[22] As prices are converging to zero, there are finitely many elements with $\sum_j \zeta_j^{i,k} = 0$. We can move to a subsequence that disregards these.

from the positive-definiteness of $\Lambda_{J^i}^{-1}$ and the fact that $\hat{\zeta}_{J_i}$ is not null. It follows that it is possible to find $\delta > 0$ and a sufficiently high element $\overline{k}$ such that

$$\left\langle \hat{\zeta}, \frac{\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|} \right\rangle > \delta,$$

whenever $i \in I_{r\to\infty}$ and $\tilde{k} > \overline{k}$. Given that $A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ is bounded from below for sufficiently high $\tilde{k}$ for all $i \in I_{r\to\infty}$, there exists $\delta_1 > 0$

$$\left\langle \hat{\zeta}, \sum_{i\in I_{r\to\infty}} n^i A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \frac{\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|} \right\rangle > \delta_1, \tag{B.3}$$

for all $\tilde{k} > \overline{k}$. This directly implies that there exists one asset $j \in \{1,...,m\}$ such that $\sum_{i\in I_{r\to\infty}} n^i A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ grows without bounds. It follows that excess demand for that asset is unbounded along the subsequence that is indexed by $\tilde{k}$. From B.2 this means that $P_j^{upd}\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times as $k \to \infty$, reaching a contradiction with $P_j^* = 0$.

Suppose now, towards a different contradiction, that $r_f \to \infty$. Using arguments similar to the previous ones, it is possible to select a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, in which the risk premium, $\zeta_j^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$, perceived by each agent $i \in I$ for each asset $j \in J$ either converges to a finite constant, diverges to $+\infty$ or diverges to $-\infty$. Also, a premium can only diverge for all agents at the same time and in the same direction.

First, we deal with the case in which no premium diverges. In this situation, each asset price converges to zero. Adding equations B.1 over agents and assets, properly multiplied by prices and individual population shares, we get

$$\sum_{i,j} P_j^{\hat{\epsilon}_{\tilde{k}}} n^i z_j^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_{i,j} n^i\left[\hat{\omega}_j^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P_j^{\hat{\epsilon}_{\tilde{k}}} x_{-1,j}^i\right],$$

which after simplifications leads to

$$\sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_i n^i A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - \sum_{i,j} P_j^{\hat{\epsilon}_{\tilde{k}}} n^i x_{-1,j}^i.$$

As $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, the right hand side converges to $\sum_i n^i e_0^i > 0$. As a consequence, the excess demand for at least one asset $j$ needs to approach $+\infty$ along a subsequence. Along this subsequence then $P_j^{upd}\left(P, \hat{\epsilon}_{\tilde{k}}\right) = \alpha$ infinitely often, leading to a contradiction of the zero price limit.

For the case in which some premia diverge, we still obtain

$$\lim_{\tilde{k}\to\infty} A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P_j^{\hat{\epsilon}_{\tilde{k}}} n^i x_{-1,j}^i = e_0^i > 0$$

and

$$\sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to \sum n^i e_0^i > 0$$

If $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, we find the same contradiction as before. Therefore, for at least one asset $j \in J$, we need to have $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* \neq 0$ which implies that $\zeta_j^{i,\tilde{k}} \to -\infty$ for each $i \in I$. We can therefore follow all the previous steps leading to B.3, with the exception that $\hat{\zeta}$ can now have negative entries. This means that we can find a subsequence and an asset $j' \in J$, such that either $\zeta_{j'}^{i,\tilde{k}} \to -\infty$ and $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ or $\zeta_{j'}^{i,\tilde{k}} \to +\infty$ and $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$. For the latter case, we would reach the same contradiction as before since $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$ implies that $P_j^{upd}\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times which contradicts positive infinity limits for both the riskless rate and the risk premium on $j'$. Therefore, we need to rule out the former situation. Given that $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* > 0$, $\zeta_j^{i,\tilde{k}} \to -\infty$ and $\hat{\zeta}_{j'} \neq 0$ together imply that $P_{j'}^* > 0$. But from (B.2), $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ implies $P_j^{\hat{\epsilon}_{\tilde{k}}} = \hat{\epsilon}_{\tilde{k}}$ infinitely many times with $\hat{\epsilon}_{\tilde{k}} \to 0$, reaching a contradiction with $P_{j'}^* > 0$.

We have, therefore, ruled out any possibility that $P_j^* = 0$ for some asset $j \in J \cup \{f\}$. We still need to show that for sufficiently high $\alpha$, market clearing is ensured in all markets at prices $P^*$. Given that $P_j^* \gg 0$, it is possible to find a sufficiently high $\hat{k}$ and $\delta_2 > 0$, such that

$$P_j^{\hat{\epsilon}_k} > \delta_2 > \hat{\epsilon}_k,$$

for all $k > \hat{k}$. As a consequence, from (B.2), for $k > \hat{k}$, $P_j^{\hat{\epsilon}_k} \geq 0$ and $z_j\left(P_j^{\hat{\epsilon}_k}\right) \geq 0$.

Additionally, for each $i \in I$, $c_0^i\left(P_j^{\hat{\epsilon}_k}\right) \in \left[0, e_0^i + \alpha \sum x_{-1}^i\right]$ implying that

$$-\alpha \sum_i n^i x_{-1}^i \leq \sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \leq \sum_i n^i e_0^i.$$

For $\alpha^2 > \sum_i n^i e_0^i$, it follows that $z\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to z\left(P^*\right) = 0$ ensuring market-clearing in the limit and existence of a Walrasian Equilibrium.

□

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