Productivity Shocks, Dynamic Contracts and Income Uncertainty*

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Abstract

This paper examines how employer and worker specific productivity shocks transmit to wage and employment in an economy with search frictions and firm commitment. I develop an equilibrium search model with worker and firm shocks and characterize the optimal contract offered by competing firms to attract and retain workers. In equilibrium risk-neutral firms offer risk-averse workers contingent contracts where payments are back-loaded in good times and front-loaded in bad ones: the combination of search frictions, productivity shocks and private worker actions results in partial insurance against firm and worker shocks. I estimate the model on matched employer-employee data from Sweden, using information about co-workers to separately identify firm specific and worker specific earnings shocks. Preliminary estimates suggest that firm level shocks are responsible for about 20% of permanent income fluctuations, the remaining being accounted for by individual level shocks (30% to 40%) and by job mobility (40% to 50%). The wage contract attenuates 80% of individual productivity shocks but passes through 30% of firm productivity fluctuations.

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1 Introduction

What are the drivers behind the observed earnings and employment uncertainty faced by workers in the labor market? How is this uncertainty mitigated by contracts between workers and firms in equilibrium? How is this transmission mechanism affected by policies? To address these questions, I develop a framework where workers face uncertainty about both their productivity and ability to locate new job opportunities and where firms choose optimally how wages respond to shocks. Changes in aggregate, firm level and worker specific productivities affect the value of a worker to an employer. At the same time, employed workers cannot immediately switch firms when current productivity decreases and unemployed workers might require several periods to locate a job opportunity. I show theoretically that in equilibrium, firms offer contracts that smoothly track worker's productivity in his current match, while responding with different intensity to different sources of shocks. I estimate the model using match employer-employee data and find that firm shocks accounts for 20% of a worker's permanent income uncertainty and that only about a third of underlying productivity gets passed through into wages. Employment transitions to unemployment and other jobs (40%) and worker shocks (40%) are the main sources of uncertainties since those are not insured by the firm. This confirms that unemployment insurance plays an important role in providing insurance that cannot be insured by the wage contract since the firm is unable to insure the worker when the employment relationship ends. However if generous unemployment insurance reduces earning risk, it also affects the employment level and the total output of the economy.

Earnings and employment uncertainty have important implications for welfare. A large body of literature has studied both theoretically and empirically the nature of the income process and quantified how it translates into consumption and wealth inequalities[1]. However, the income process itself is the observed part of the complex employment agreement that links a worker to a job. The mechanism that defines this agreement in equilibrium, how the payments are delivered over time and how they respond to underlying productivity shocks has long been of high interest to the literature both theoretically and empirically. Knight (1921) first pointed out that one of the roles of the firm is to insure workers against productivity shocks. Baily (1974) and Azariadis (1975) formalized the idea and showed theoretically that when firms can sign long-term contracts, they fully insure their work force and offer fixed wage contracts even in the presence of demand shocks. Yet empirically income processes

[1](MaCurdy, 1982; Blundell, Pistaferri, and Preston, 2008; Attanasio and Pavoni, 2011; Low, Meghir, and Pistaferri, 2009)

feature growth and employment risks (Altonji, Smith, and Vidangos, 2009; Low, Meghir, and Pistaferri, 2009).

Empirical evidence for the transmission of firms' shocks to workers' wages is provided by Guiso, Pistaferri, and Schivardi (2005). Using employer-employee matched data from Italy, they estimate how permanent and transitory productivity shocks of firms enter the wage equation of continuing workers. They report that full insurance of firms permanent shocks is rejected by the data. Their paper, however, uses a sample of continuing workers and does not control directly for the selection of workers in and out of firms. If workers who suffer the most from a drop in firm performance are also the ones leaving the sample, the effect of firm shocks is underestimated. Roys (2011) uses French firm data to estimate a model with homogenous workers and firm adjustment costs. He finds that firm permanent shocks affect employment and transitory shocks affect wages. The result however might be driven by the assumption that wages are set according to Nash bargaining which means that they are continuously renegotiated.

Contract theory offers answers to the apparent failure of the first best allocation. Harris and Holmstrom (1982) show that in a competitive market without worker commitment firms continue to insure against downward risk but have to increase the wage whenever productivity increases in order to retain the worker. Thomas and Worrall (1988) introduce the idea of a shock to job productivity by developing a model where a match between a firm and worker enjoy rents that can vary over time. They derive the optimal contract in an environment where the outside option is exogenous and show that, in a way similar to Harris and Holmstrom (1982), the wage remains constant until either the firm's or the worker's participation constraint binds. Burdett and Coles (2003) and Shi (2009) characterize the optimal contract when outside offers come from competing firms and the worker's decision is private; firms offer wages that increase with tenure to retain workers even-though they are risk averse and would prefer flat wages. Menzio and Shi (2010) extends this equilibrium framework by reintroducing match shocks and aggregate fluctuations, but do not characterize the optimal contract. Schaal (2010) does characterize the contract in a similar model but with homogeneous risk neutral workers. Rudanko (2009) derives the optimal contract with two sided lack of commitment but without on-the-job search or any private action from the worker, wage changes when outside options bind. To my knowledge, the current paper is the first to characterize the long term optimal contract offered in equilibrium by firms in an economy with search frictions, on-the-job search, firm and worker shocks and risk averse workers.

This paper makes three contributions to the existing literature. First, I document new findings about the co-movement of wages among co-workers which suggests larger transmission of firm shocks to wages than previously reported. Second, I characterize the optimal contract offered by competing firms in a directed-search equilibrium. Third, I estimate and evaluate quantitatively the model using linked employer-employee data.

The model builds on the directed search equilibrium of Menzio and Shi (2009), which allows for stochastic heterogeneity of firms and workers as well as worker risk aversion. Workers can search for new positions when employed and when unemployed. When on the job, the search decision is not observed and outside offers are not contractible by the firm. Firms can commit to any history-contingent long-term contract but the worker can walk away at any time. This contract flexibility is crucial because picking a particular form of wage setting might impose a specific level of insurance between the firm and the worker, whereas here it is determined by profit maximization. Flows of workers into firms is modeled using directed search, when searching for a new job, workers observe all contract offers and choose one to apply to. Each contract has a queue associated with it and each worker chooses the queue that maximizes the product between the return of the contract and the probability of getting picked from that queue. Directed search is a very natural extension of the competitive labor market that directly generates all the endogenous movement of workers in, out and across firms[2].

I show that in equilibrium firms post contracts that can be represented by a target wage that corresponds to the certainty equivalent of the current match productivity. Wages below that target wage will increase and wages above will decrease. The optimal contract presented here shares features of both Burdett and Coles (2003) and Hopenhayn and Nicolini (1997): firms back-load wages to incentivize workers to search less when profits are positive and front-load wages when profits are negative to incentivize the worker to search for a better position. When the match experiences a negative shock, the firm does not want to layoff the worker right away and decides to insure her in way similar to an optimal unemployment insurance scheme.

The empirical strategy of this paper utilizes the property that the wage smoothly tracks the target wage, which is subject to both worker and firm productivity shocks. Assuming that shocks to the worker and shocks to the firm are independent and that co-workers share the same firm productivity, shocks to the firm should affect all workers, whereas idiosyncratic

[2] The pioneering work in directed search is due to (Montgomery, 1991; Peters, 1991; Moen, 1997; Shimer, 1996; Burdett, Shi, and Wright, 2001; Shimer, 2001).

shocks should affect them in an uncorrelated way. Using the auto-covariance and co-variance of co-workers' wages, I can identify how much of the wage movement is due to the firm relative to the worker and estimate the productivity process of both.

In Section 1, I present auxiliary models that will be used for estimation. This section also motivates the economic question with evidence of risk transmission at the firm level in the Swedish matched employer-employee data. In Section 2 I formally present the equilibrium search model and I characterize the optimal contract. In Section 3, I present the estimation strategy and I discuss the identification of the model. This section also reports the estimation results. In Section 4, I put the model to work to answer the question of how much of income uncertainty is due to worker shocks and how much is due to the firm. A summary of the notation and all proofs are in the Appendix.

2 Earnings dynamics and participation

2.1 Data

The employer-employee matched data from Sweden links three administrative data-sets: the employment data, the firm data and the benefits data that track workers who are currently unemployed. The sample runs from 1993 to 2007 and covers around 6 million individuals. The firm data covers around 100,000 firms in four industries. The sample only covers firms with more than 10 employees, which means that some workers covered in the data work in a firm for which we do not have an identifier. On the worker side, all self-employed are dropped from the original sample, as well as some specific industries such as fisheries and the financial sector. I first de-trend the data with time dummies to remove any non stationary effects. I select individuals under 50 years of age, and, for moments computed at the firm level, I limit the data to firms with at least 25 employees.

2.2 Wage growth for job-stayers

In order to give an intuitive interpretation to moments computed from the data I introduce the following statistical model for residual log earnings of continuing workers :

$$\begin{aligned} w_{ijt} &= \beta Z_t + \tilde{w}_{ijt} + v_{ijt} \\ \tilde{w}_{ijt} &= \tilde{w}_{ijt-1} + \delta_{jt} + \xi_{ijt}, \end{aligned}$$

Table 1. Residual income variance

	HS dropout	HS grad	Some college
residual wage variation σ_w^2	0.1274	0.1159	0.2033
	(0.000208)	(0.000132)	(0.000338)
worker transitory σ_v^2	0.0128	0.0123	0.014
	(0.000158)	(0.000117)	(0.000179)
worker permanent σ_ξ^2	0.0198	0.0173	0.0193
	(0.000238)	(0.000161)	(0.000242)
co-worker permanent σ_δ^2	0.0012	0.00146	0.00174
	(3.83e-05)	(3.11e-05)	(4.93e-05)
shared by co-workers	6.07%	8.41%	8.97%
	(0.19)	(0.173)	(0.309)
equivalent lottery	±3.47%	±3.82%	±4.17%
	(0.0555)	(0.0407)	(0.0591)

Standard errors are computed using clustered resampling. Wage differences are taken year on year. The equivalent lottery represents fair lottery over a permanent wage raise or cut in percent that would be equivalent to the share of variance common to co-workers.

where i is the individual, j is the firm and t is time. Z_t is a yearly dummy, $\tilde{w}_{ijt}$ is the permanent wage, ξ_{ijt} is an idiosyncratic permanent shock to the wage and δ_{jt} is a permanent shock shared by all the workers in firm j. Wage growth shared by co-worker should be thought of as a firm specific event. The model parameters can be estimated using simple moments from individual wage growth and average wage growth within a firm (See Appendix A.1). I report the estimates for each education group in Table 1.

The value of σ_δ is of interest as it represents the risk that co-workers share. To understand its monetary value, it is useful to think of the equivalent lottery that delivers a permanent percentage wage raise or cut. For instance for college graduates, every year, co-workers in a firm face the same lottery draw that delivers with 50 percent chance a wage raise of 4.17 percent and with 50 percent chance a 4.17 percent wage cut. This wage growth lottery is permanent and consequently 4.17 percent is economically significant. This provides evidence that part of the wage growth uncertainty is shared at the firm level.

2.3 Wage growth and value added

Quantitatively, the numbers presented in the previous section are larger than the one reported previously in the literature that focused on the link between value added and wages such as Guiso, Pistaferri, and Schivardi (2005) and Roys (2011). I replicate here a procedure similar to those papers to compare the Swedish economy to the French and Italian ones. I consider a

Table 2. Income variance and value added shocks

	HS dropout	HS grad	Col grad
τ	0.0287***	0.0217***	0.0181***
	(0.000955)	(0.000643)	(0.000679)
equivalent lottery	±0.537%***	±0.453%***	±0.399%***
	(0.0179)	(0.0134)	(0.015)

Standard errors are computed using clustered resampling. Wage differences are taken year on year. The equivalent lottery represents fair lottery over a permanent wage raise or cut in percent that would be equivalent to the share of variance common to co-workers.

simple unit-root model for the log value added per worker. The innovation shock μ_{jt} is then linked to the shock of permanent income among co-workers δ_{jt} from the previous section by the parameter τ:

$$\begin{aligned} y_{jt} &= \beta X_t + \tilde{y}_{jt} + u_{jt} \\ \tilde{y}_{jt} &= \tilde{y}_{jt-1} + \mu_{jt} \\ \delta_{jt} &= \tau \mu_{jt} + \nu_{jt} \end{aligned}$$

Table 2 contains the estimates for τ for each eduction group as well as the equivalent lottery implied by the amount of log wage growth uncertainty explained by shocks to value added. As in Guiso, Pistaferri, and Schivardi (2005) we see that the link between value added and wages is significantly different from zero. This provides evidence against the hypothesis of full insurance of firm shocks. The magnitude of the transmission of value added shocks to worker is economically small and similar to the values reported previously in the literature. Tables 1 and 2 suggest that shocks to value added can only explain a small part of the risk co-workers jointly share at the firm level. Consequently, I will focus the empirical analysis on wages within the firms rather than on value added.

2.4 Worker transitions

Finally it is also of interest to measure how changes at the firm level affect transitions of workers to unemployment and to other firms. When a firm receives a bad productivity shock, transitioning to another firm is a good way to insure income. This is precisely why studying the impact of search friction on the provision of insurance is important. Table 3 reports a linear probability models of worker transitions to unemployment and to other

Table 3. Transition probabilities to unemployment and other jobs

	to another firm			to unemployment		
	HS dropout	HS grad	Col grad	HS dropout	HS grad	Col grad
(Intercept)	0.0134***	0.0185***	0.0245***	0.0163***	0.0154***	0.0101***
	(0.000118)	(6.48e-05)	(0.000107)	(0.000124)	(5.74e-05)	(6.76e-05)
worker wage	0.0329***	0.0468***	0.028***	-0.0142***	-0.0033***	-0.00178***
	(0.000386)	(0.000219)	(0.00027)	(0.000408)	(0.000194)	(0.000171)
firm wage change	0.0162***	0.0165***	0.0035***	0.0186***	0.0208***	0.0202***
	(0.00122)	(0.000782)	(0.0011)	(0.00129)	(0.000693)	(0.000695)
firm wage	-0.032***	-0.0522***	-0.0281***	-0.0115***	-0.0222***	-0.0132***
	(0.000673)	(0.000422)	(0.000494)	(0.00071)	(0.000374)	(0.000312)
N	2,450,855	9,788,831	4,246,564	2,450,855	9,788,831	4,246,564

Linear probability model of the probability for currently employed workers. The dependent variable is an indicator for moving to another firm for the first three columns, and an indicator for transition to unemployment for the last three columns. Regressors include in order the change in worker's log wage, the cahnge in the mean log wage within current firm and the mean wage in the firm.

firms. Regressors include the mean wage of the firm and the mean wage change in the firm.

First we see that the mean wage in the firm affects negatively both transitions. This suggests that better firms pay higher wages and keep their workers longer. Interestingly however the worker's wage affects positively the probability to change firms. This can be explained by the fact that higher wages are more difficult to increase to prevent the worker form leaving, or it could be that higher earners move more frequently. We also note that an increase in firm average wage while keeping worker's wage constant affects positively the mobility of the worker. The results from Table 3 tell us that the risk of job loss is affected by changes in the firm. Not only do firms that pay higher wage seem to retain their workers longer, it also seems that a change in the firm's average wage does affect the rate at which workers loose their job. This source of risk associated with job loss can't be captured by the log wage models presented in the previous section that only looked at continuing workers. The model introduced in the rest of the paper will account for this additional employment risk.

3 An equilibrium search model of risk sharing and income dynamics

I present here an equilibrium model with search frictions and private worker actions. The key feature of the model is to embed the bilateral relationship between the firm and the worker, with productivity uncertainty, inside a competitive search equilibrium where firms compete to attract and retain workers.

In this model, ex-ante identical firms compete by posting long-term contracts to attract heterogeneous workers. Employed and unemployed workers observe the menu of contracts offered in equilibrium and decide which one to apply to. This process forms sub-markets of workers applying to particular contracts and firms offering them. Within each queue the matching between firms and workers is random. When choosing which sub-market to participate in, both firms and workers take into account the value of matching and the probability of matching. This probability is driven by how many firms and workers participate in a particular sub-market.

When matched, the contract specifies the wage after each possible history of shocks for the firm and workers. Given his wage profile, the worker chooses which sub-market to visit while employed and chooses effort, which directly affects the probability the current match remains intact. Both of these actions are private and so unobserved by the firm. Firms take this into account and post contracts that incentivize the worker's action in an optimal way. This will mean that in some cases the wage will adjust downward albeit in a smooth way. I now formally introduce the model.

3.1 Environment

Agents and preferences

Time is discrete, indexed by t and continues for ever. The economy is composed of a discrete uniform distribution of infinitely lived workers with ability indexed by $x \in \mathbb{X} = \{x_1, x_2 ... x_{n_x}\}$. Workers want to maximize expected lifetime utility, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(w_t) - c(e_t)\right)$ where utility of consumption $u : \mathbb{R} \to \mathbb{R}$ is increasing and concave and cost of effort $c : \mathbb{R} \to \mathbb{R}$ is increasing and convex with $c(0) = 0$. Worker's ability x changes over time according to Markov process $\Gamma_x(x_{t+1}|x_t)$. Unemployed workers receive flow value of unemployment $b(x)$. The other side of the market is composed of a uniform distribution of ex-ante identical firms with active jobs and vacancies. Vacancies live for one period and become active jobs if matched with a worker.

An active job is characterized by the current worker ability x and the current match quality z. The match quality z evolves with an innovation ι_t drawn at the firm level such that $z_{t+1} = g(z_t, \iota_t)$. ι_t is a firm level shock that affects all continuing workers' TFP. New hires all start with $z = 0$. The function $g(\cdot, \cdot)$ is assumed to generate a monotonic transition rule. Every period a match (x_t, z_t) has access to a technology that produces $f(x_t, z_t)$. Worker's effort e affects the probability that the technology continues to exists next period. This captures the idea that a negligent worker might loose a client or break the machine and cause the job to disappear. The firm cares about the total discounted expected profit of each created vacancy.

Firms here operate constant return to scale production functions and can be thought of as one worker per firm. However, empirically one cannot aggregate firms with the same output as the history of productivity shocks affects the distribution of workers. For instance whether or not a firm had a very bad shock in the last period will affect the current distribution of workers beyond the current productivity. To pin down the distribution of workers in a given firm one needs to know the entire history of shocks.

Search markets

The meeting process between workers and firms vacancies is constrained by search frictions. The labor market that matches workers to vacancies is organized in a set of queues indexed by $(x, v) \in \mathbb{X} \times \mathbb{V}$ where x is the type of the worker and v is the value promised to her in that given queue. Firms can choose in which (x, v) lines they want to open vacancies and workers can choose in which v line associated with their type x they want to queue[3]. Each visited sub-market is characterized by it's tightness represented by the function $\theta : \mathbb{X} \times \mathbb{V} \to \mathbb{R}_+$ which is the ratio of number of vacancies to workers. The tightness captures the fact that a high ratio of vacancies to workers will make it harder for firms to hire. In a directed search model like the one presented here, the tightness is queue specific which means that different worker types could be finding jobs at different rates. In queue (x, v) a worker of type x matches with probability $p(\theta(x, v))$ and receives utility v. Firms post vacancies at unit cost η and when posting in market (x, v) the vacancy is filled with probability $q(\theta(x, v))$. $\phi(x, v)$ will denote the mass of vacancies created in market (x, v).

States and actions

A worker is either employed or unemployed and enters each period with a given ability x. When unemployed she collects benefit $b(x)$ and can search every period. When searching she chooses which sub-market (x, v) to visit, in which case she gets matched with probability $p(\theta(x, v))$ and if matched joins a job and receives lifetime utility v.

[3]Menzio and Shi (2009) Theorem 3 tells us that workers will separate by type in equilibrium if markets are indexed by the value that each type x would get in a particular sub-market ($\mathbf{v} = (v(x_1), v(x_2)...v(x_{n_x})) \in \mathbb{R}^{n_x}$,) and workers can apply to any. At equilibrium only a given type x visits a particular market. This market can then be represented directly by (x, v) as done in the current paper.

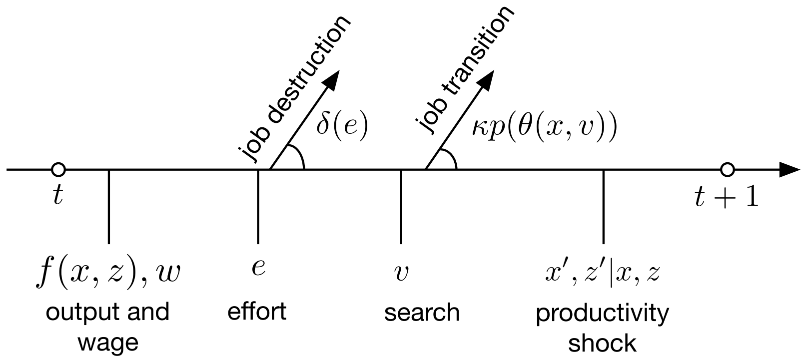


Figure 1: within period time line

An employed worker is part of a match and starts the period with a given ability level x and a current match quality z. The period is then divided in four stages as illustrated in Figure 1, first is production, the firm collects output $f(x,z)$ and pays the wage w to the worker. The worker cannot save, consumes all of w, chooses effort e and gets flow utility $u(w)-c(e)$. With probability $(1-\delta(e))$, where $\delta(e)$ is decreasing in e, the employment persists to the next period. With probability $\delta(e)$ the worker moves to unemployment. In the search stage, the worker is allowed to search with efficiency κ. When searching she chooses which sub-market (x,v) to visit and gets matched with probability $\kappa p(\theta(x,v))$. If matched she moves to a new match where she will enjoy v and the current job will be destroyed. If the worker is not matched to a new job, the current job persists, a new x' is drawn conditional on the old one, and a firm level shock ι is drawn to update z. In summary, in every period an active job chooses the wage w, and the worker chooses effort e and which sub-market (x,v) to search in. Because $c(0)=0$ the worker can quit in every period if the firm does not promise enough. By choosing v and e the worker controls his transition to other jobs and to unemployment.

Informational structure and contracts

A contract defines the transfer and actions for the worker and the firm within a match for all future histories. Call $s_\tau=(x_\tau,z_\tau)\in\mathbb{S}=\mathbb{X}\times\mathbb{R}$ the state of the match τ periods in the future and call $s^\tau=(s_1...s_\tau)\in\mathbb{S}^\tau$ a given history of realizations between s_1 the state today and s_τ, the state in τ periods.

The history of productivity is common knowledge to the worker and the firm and fully contractible. However the worker's actions are private information and transitions to other firms or to unemployment are assumed to be not contractible. This rules out side payments as well as countering outside offers[4]. Here, the contract offered by the firm to the worker is then represented by:

$$\mathcal{C} := (\mathbf{w}, \sigma); \quad \text{with } \mathbf{w} := \{w_\tau(s^\tau)\}_{\tau=0}^{\infty}, \text{ and } \sigma := \{v_\tau(s^\tau), e_\tau(s^\tau)\}_{\tau=0}^{\infty}, \tag{1}$$

I explicitly separate the firm's choice from the worker's response. The firm chooses the wage w_τ paid at every history and the worker responds by choosing (v_τ, e_τ) the search and effort decision[5]. σ can be thought as the action suggested by the contract and I will focus on contracts where the recommendation is incentive compatible. The contract space is completely flexible in the way it responds to tenure and any productivity history. In particular it leaves the firm free to chose how the wage should respond to productivity shock, which is the central question of this paper.

3.2 Worker choice

An unemployed worker of type x chooses optimally which sub-market (x, v_0) she applies to. The only value she cares about is the value she will get, specifically v_0 and the tightness of the market $\theta(x, v_0)$. Higher v_0 sub-markets deliver higher values but have longer average waiting times. I can write the value $\mathcal{U}(x)$ of being unemployed as follows:

$$\mathcal{U}(x) = \sup_{v_0 \in \mathbb{R}} b(x) + \beta p(\theta(x, v_0))v_0 + \beta\left(1 - p(\theta(x, v_0))\right)\mathbb{E}_{x'|x}\mathcal{U}(x'). \tag{W-BE}$$

We follow by writing the problem of the employed worker and the firm as a recursive contract. As presented in Spear and Srivastava (1987) the state space is augmented with V, the promised utility to the worker. The recursive contract is characterized at each (x, z, V) by $\{\pi_i, w_i, e_i, v_{1i}, W_{ix'z'}\}_{i=1,2}$ where $\pi_i : \mathbb{S} \times \mathbb{V} \to [0, 1]$ is a randomization, $w_i : \mathbb{S} \times \mathbb{V} \to \mathbb{R}_+$ is the wage, $e_i : \mathbb{S} \times \mathbb{V} \to [0, \bar{e}]$ is effort choice, $v_i : \mathbb{S} \times \mathbb{V} \to [0, \bar{v}]$ is the search choice and $W_{ix'z'} : \mathbb{S} \times (\mathbb{X} \times \mathbb{R}) \to \mathbb{V}$ is the utility promised for each realization next period.

The worker optimally chooses the action (v, e), when promised next period expected

[4] Lentz (2013) develops a model with optimal contracts and countering of outside offers, but without productivity shocks, and shows that firms continue to backload wages.

[5] Derivations will later require a randomization which means that the contract can specify simple probability over actions instead of actions themselves. This is left implicit at this point but will be clarified in the recursive formulation of the problem.

utility $W = \mathbb{E}_{x'z'} W_{x'z'}$, she solves the following problem:

$$\sup_{v,e} u(w) - c(e) + \delta(e)\beta \mathbb{E}_{x'|x} \mathcal{U}(x') + (1-\delta(e))\beta\kappa p(\theta(x,v))v + \beta(1-\delta(e))(1-\kappa p(\theta(x,v)))W,$$

for which we define the associated worker policies $v^* : \mathbb{X} \times \mathbb{V} \to [0, \bar{v}]$ and $e^* : \mathbb{X} \times \mathbb{V} \to [0, \bar{e}]$. Because of the properties of $p(\cdot)$, $\theta(\cdot,\cdot)$ and $c(\cdot)$, those functions are uniquely defined. Note that those policies only depend on the promised utility for next period and not on the current (z_t, V) as stated in the following definition.

Definition 1. We defined the composite transition probabilities $\tilde{p} : \mathbb{X} \times \mathbb{V} \to \mathbb{R}$ and the utility return to the worker $\tilde{r} : \mathbb{X} \times V \to \mathbb{R}$ as functions of the promised utility W (using short-hand $e^* = e^*(x, W)$ and $v^* = v^*(x, W)$):

$$\begin{aligned}
\tilde{p}(x,W) =& (1-\delta(e^*))\left(1-\kappa p(\theta(x,v_1^*))\right) \\
\tilde{r}(x,W) =& -c(e^*) + \beta\kappa\left(1-\delta(e^*)\right) p(\theta(x,v_1^*))\left(v_1^* - W\right) \\
& \qquad\qquad + \delta(e^*)\beta\mathbb{E}_{x'|x} U(x') + \beta\left(1-\delta(e^*)\right) W.
\end{aligned}$$

These functions capture everything the firm needs to know about the consequences of setting the wage dynamically. We now turn to the firm's problem.

3.3 Firm profit, optimal contracting problem

I can now describe the firm problem in terms of promised utilities. The firm chooses a lottery over promised values and wages which then determines the participation probabilities. The expected profit of a match to the firm can be expressed recursively as

$$\begin{aligned}
\mathcal{J}(x,z,V) = & \sup_{\pi_i, w_i, W_i, W_{ix'z'}} \sum_{i=1,2} \pi_i \Big(f(x,z) - w_i + \beta \tilde{p}(x, W_i) \mathbb{E}_{x'z'} \mathcal{J}(x', z', W_{ix'z'}) \Big) \\
s.t \qquad & V = \sum_i \pi_i \left(u(w_i) + \tilde{r}(x, W_i)\right), \qquad\qquad \text{(BE-F)} \\
& W_i = \mathbb{E} W_{ix'z'}, \quad \sum \pi_i = 1.
\end{aligned}$$

The firm chooses the current period wage w_i and the promised utilities $W_{ix'z'}$ for each lottery realization i and state of (x', z') tomorrow. These control variables must be chosen to maximize expected returns subject to the promise keeping constraint. This constraint makes sure that the choices of the firm honors the promise made in previous periods to deliver the value V to the worker. The right hand side of the constraint is the lifetime utility of the worker given the choices made by the firm. The lottery is present only to insure concavity of the function.

The incentive compatibility of the worker is embedded in the $\tilde{r}$ and $\tilde{p}$ functions that we defined previously. By increasing future promises the firm can increase the probability that the match continues. However at given V, larger promised utilities go together with lower current wage w. Since the utility function is concave, there will be a point where too low of a wage is just not efficient. This is the classic insurance incentive tradeoff.

Finally firms choose how many vacancies to open in each (x, v) market. Given vacancy creation cost η and the fact that the match quality z starts at 0, the return to opening a vacancy is given by:

$$\Pi_0(x, V) = q(\theta(x, V))\mathcal{J}(x, 0, V) - \eta,$$

and firms will open vacancies in a given market if and only if expected profit is positive. The vacancy creation cost is linear, which means that if Π_0 is positive the firm will create an infinity of vacancies, if it's negative it won't create any and if it's zero the firm is indifferent.

3.4 Equilibrium definition

Free entry condition

We now impose a free entry condition on the market. Firms will open vacancies in each markets until the the expected profit is zero or negative:

$$\forall (x, V) \in \mathbb{X} \times \mathbb{V} : \quad \Pi_0(x, V) \leq 0. \tag{EQ1}$$

This will pin down the tightness of each market. $\phi(x, v)$ will denote the total mass of vacancies posted in market $(x.v)$.

Market clearing

Markets for labor must clear, in the sense that the equilibrium distribution must be generated by the equilibrium decisions. Given an equilibrium stationary distribution $h(x, y, z, V)$ of workers assigned to matches with a given promised utility, given the mass $\phi(x, V)$ of vacancies, the following clearing condition must be satisfied:

$$\forall x, v \quad \phi(x,v) = \theta(x,v)\Big[u(x)\mathbf{1}[v_0^*(x)=v] \\ + \sum_{x\in\mathbb{X}} \int_z \sum_i \pi_i(x,z,v') \int_{V'} p(\theta(x,W_i))\mathbf{1}[v_i^*(x,W_i)=v] \, \mathrm{d}H(x,z,V')\Big]. \quad \text{(EQ2)}$$

There is one last market clearing equation for ϕ and it states that ϕ in the next period is consistent with itself, all the equilibrium decisions, and law motions such as the shocks on x, z and the endogenous separation. This is left for the appendix.

Definition 2. A **stationary competitive search equilibrium** is defined by a mass of vacancies $\phi(x,v)$ across sub-markets (x,v), a tightness $\theta(x,v) \in \mathbb{R}$, an active job distribution $h(x,z,V)$ and an optimal contract policy $\xi = \{\pi_i, w_i, e_i, v_j, W_{ix'z'}\}_{i=1,2}$ such that:

(a) ξ solves the firm optimal contract problem BE-F and so satisfies worker incentive compatibility.
(b) $\theta(x,v)$ and $\phi(x,v)$ satisfy the free entry condition EQ1 for all (x,v)
(c) $\theta(x,v)$, $\phi(x,v)$ and $h(x,z,V)$ solve the market clearing condition EQ2
(d) $h(x,z,V)$ is generated by $\phi(x,v)$ and ξ

The equilibrium assigns workers to firms with contracts in a way where neither workers or firms have an incentive to deviate. The distributions ϕ and h represent the equilibrium allocation.

3.5 Equilibrium and contract characterization

Lemma 1 (existence)**.** *A stationary competitive search equilibrium exists.*

Proof. See appendix A.2 □

Menzio and Shi (2010) gives us the important results that a block recursive equilibrium exists in the version of this model with aggregate shocks and no worker effort or heterogeneity, and Tsuyuhara (2013) proves the existence with effort but without shocks or firm heterogeneity. The existence continues to be true when the incentive problem and the shocks are combined. The equilibrium is also well defined when adding aggregate shocks.

Lemma 2. *The Pareto frontier $\mathcal{J}(x,z,V)$ is continuously differentiable, decreasing and concave with respect to V and increasing in z.*

Proof. See appendix A.4 □

Concavity is a direct implication of the use of the lottery. I then adapt the sufficient condition from Koeppl (2006) for differentiability in two-sided limited commitment models. From the free entry condition, the tightness function is a continuously differentiable and concave function of $\mathcal{J}(x,z,V)$, which implies that the composite search function $p(\theta(x,v))$ inherits those properties for all $x \in \mathbb{X}$.

I am interested in how firms decide to compensate workers over time given that they face the classic trade-off between insurance and incentives. The following proposition provides a clear prediction for how wages move dependent on the current state of the match:

Proposition 1 (optimal contract)**.** *For each viable match (x,z), independent of the lottery realization, the wage policy is characterized by a* ***target wage*** *$w^*(x,z)$, which is increasing in z such that:*

$$\begin{aligned} w_t \le w^*(x_t,z_t) \quad &\Rightarrow \quad w_t \le w_{t+1} \le w^*(x_t,z_t) \quad \textit{incentive to search less} \\ w_t \ge w^*(x_t,z_t) \quad &\Rightarrow \quad w^*(x_t,z_t) \le w_{t+1} \le w_t \quad \textit{incentive to search more} \end{aligned}$$

where the target wage is characterized by the zero expected profit condition for the firm:

$$\forall x,z \quad \mathbb{E}_{x'z'|xz}\mathcal{J}(x',z',W_{x'z'}) = 0$$

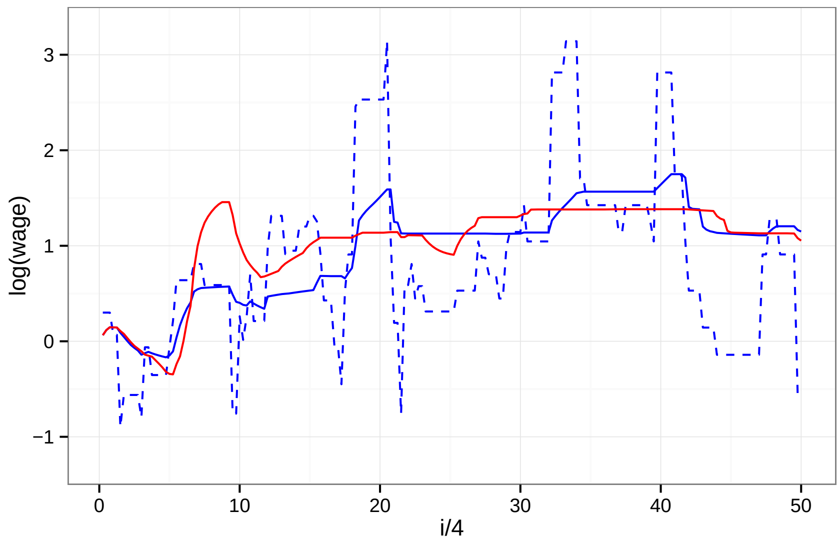
Proof. See Appendix A.5. □

The optimal contract links wages to productivity. For all histories of shocks, the change in wage growth will be in the direction of the target wage which is itself tied to the productivity of the match. This means that workers' wages will respond to any shock affecting the expected productivity (Figure 2 shows an example wage path). In particular it will respond to both worker specific and firm productivity shocks. The exact change in the wage is characterized by the first order conditions of the firm problem (BE-F) and reads:

$$\forall x,z,x',z' \quad \frac{\tilde{p}_v(x,W_i)}{\tilde{p}(x,W_i)} \cdot \mathbb{E}_{z''y''}\mathcal{J}(x'',z'',W_{ix''z''}) = \frac{1}{u'(w_{x'z'})} - \frac{1}{u'(w)}.$$

The right hand side represents the change in marginal utilities and tells us that risk aversion affects how rapidly wages adjust. On the left hand-side the first term represents the severity of the moral-hazard problem and the second term is the discounted expected profit of the firm. This expression resembles the main equation in Rogerson (1985) and captures the

Figure 2: Wage and target wage example



Notes: This figure represents the target wage (dotted blue) and the actual wage (plain blue) for a worker. The red line represents a second worker sharing same firm specific shocks, but a different worker specific productivity.

incentive problem the firm is facing when paying the worker. When in a match the worker and the firm are part of a locally monopolistic bilateral relationship as in the original paper. However, the incentive problem here is precisely on the availability of the outside option. In Rogerson (1985), workers effort affects the output of the match whereas here, the effort affects its duration and the availability of outside options.

The fact that wages adjust downward even though firms can commit is the consequence of the existence of rents and the presence of an incentive problem. In a competitive market without rents and with full commitment, even in the presence of productivity uncertainty, the firm will fully insure workers and the wage will be constant until the relationship is exogenously destroyed. The wage paid to the worker is the certainty equivalent of the present value of the firm output.

Harris and Holmstrom (1982) show that when allowing for only one-sided limited commitment, the wage will have to adjust when worker's productivity increases so as to retain her (Figure 3.a1). However negative shocks continue to be fully insured (Figure 3.a2). It is the lack of commitment that prevents the firm from offering the worker full insurance. Workers would want to commit ex-ante but can't and so the lack of commitment is a constraint, not a relaxation.

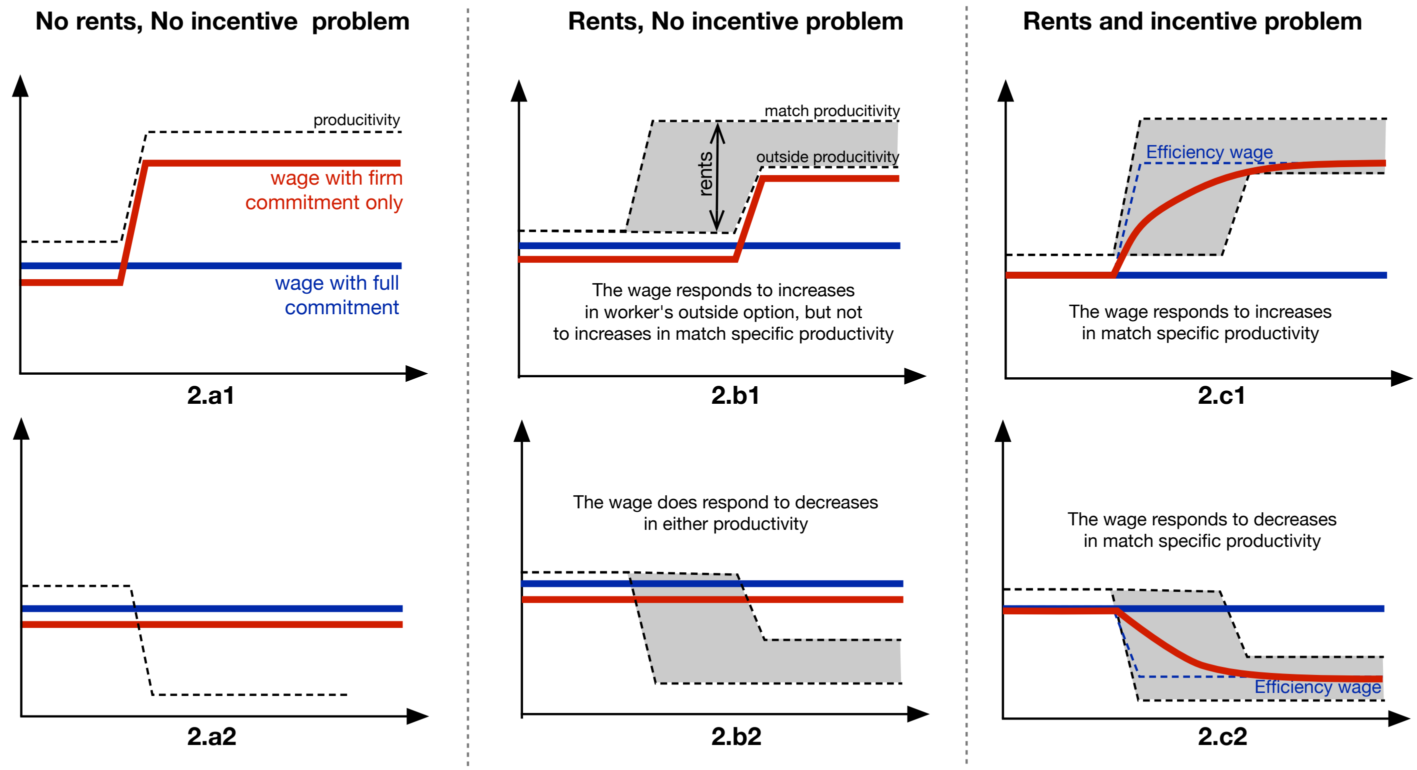


Figure 3: productivity and wages with rents, lack of commitment and incentive problems

In the presence of rents the outside option of the worker and the productivity in the current match might vary separately. The outside option is linked to characteristics the worker caries with her when she moves to another firm, while the match rent also depends on the firm specific characteristics. Retaining the worker only requires offering more than her outside offer and and so only depends on worker-specific characteristics. This means that with rents only, the worker's wage does not respond to firm specific productivity shocks (Figure 3.b). Thomas and Worrall (1988) take the outside offers and the match rents as exogenous and add firm-side lack of commitment and show that in that case downwards adjustment will happen when the firm participation constraint binds. However in the interior region of the surplus, the contract fully insures the worker and the wage is constant.

The final ingredient is the incentive problem (Figure 3.c) which implies an unique efficient transfer from the firm to the worker instead of a full set. The worker chooses where to search and applies to increasingly long queues when promised higher values. Whenever the worker is getting less than the total value of the match, she will tend to leave the current job with a higher than efficient probability (inversely when the workers gets more, she does not search enough). Ex-ante, it is more efficient to sign a contract that will give up some of the insurance to come closer to the efficient worker decision. This dynamic was fully described in the extreme case of bilateral monopoly of Rogerson (1985) and continues to apply here in an equilibrium with firm competition and rents.

The continuum of queues available to the worker in the directed search equilibrium can be thought of as a probabilistic version of the constraint faced by firms in Harris and Holmstrom (1982). In their competitive version workers can find their $\bar{v}$ with probability one, whereas with directed search they can access any $v \leq \bar{v}$ with decreasing probability $p(v)$. In the presence of search frictions the firm-worker relationship becomes a temporary bilateral monopoly with an incentive problem determined by the equilibrium. As the strength of the friction varies we get a continuum of contracts à la Rogerson (1985), with the property that as search frictions vanish, the contract becomes Harris and Holmstrom (1982) (See Figure 4).

Since rents and incentives are sufficient for the transfer of firm shocks to wages, search frictions are only one of several possible mechanisms. In the present model, there are two sources of rents, search frictions and match specific TFP, and two incentive problems, on the job search and effort choice e. This means that even when frictions are completely shut down, we would still see some firm level shocks in the earning dynamics[6]. Search frictions are an interesting feature not only because they allow us to consider employment risk but also because they generate both the rents and the incentive problem at the same time. It is also interesting to note that the shape of the meeting probability function creates some downward rigidities as in Harris and Holmstrom (1982).

Finally, firing never happens right away. First the firm decreases the wage of the worker over time because forcing her to search elsewhere is the most effective way for the firm to deliver ex-ante utility. The firm, when attracting the workers, can commit to paths where they keep the worker on payroll for a given amount of time even though it means negative expected profit.

[6] I am in the process of estimating a frictionless version of the model on a subset of the moments that should be included in future version.

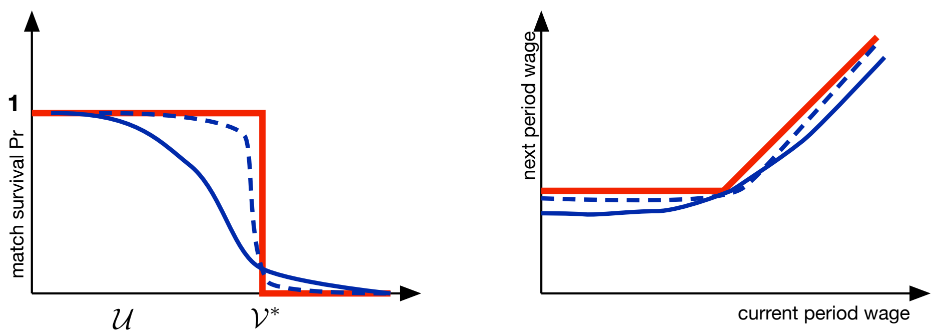


Figure 4: Meeting probability

4 Estimation

4.1 Model specification and identification

I estimate the model using indirect inference and a parametrized model. I present in Table 4 the specification I use in the next sections. I use the constant relative risk aversion utility function. The discount rate for the worker and the interest rate for the firm are set to an annual 5% and the model is solved quarterly. The production function is parametrized by γ_a a scale parameter, γ_z and γ_x that control the dispersions in ability and match productivity. The worker effort function is such that $c(0) = 0, c'(\cdot) > 0, c''(\cdot) > 0$ and $\lim_{e \to 1} c(e) = \infty$. For the time being I set the flow value of unemployment to 30 percent of the starting productivity and I fix $c_1 = 0.3$ and $\gamma_z = 1$. I normalize the mean wage in the economy which pins down the value of γ_a. I also set an absolute lower bound of $-f(\bar{x}, z_0)/(10 \cdot r)$ on the negative surplus that firms can commit to. This leaves 6 parameters to estimate as shown in Table 6.

The vacancy cost η affects the meeting rate through the free entry condition (EQ1) and κ affects the relative efficiency of on-the-job search. The probability of exiting unemployment and the probability of job-to-job transitions pin down η and κ.

The effort cost function $c(\cdot)$ affects both the average rate at which workers loose their jobs and how this rates is linked to their current wage. c_0 and c_1 can be measured by fitting the slope and intercept of a logistic regression on the probability of employment to unemployment (E2U) transition conditional on current wage.

The parameter γ_x of the production function affects the return to worker ability x. Normalizing x to be uniform on $[0, 1]$ (at discrete uniformly spaced support), the production

Table 4: functional form specifications

matching function	$p(\theta) = \theta(1-\theta^{\nu})^{-1/\nu}$
utility function	$u(w) = \frac{w^{1-\varsigma}}{1-\varsigma}$
production function	$f(x,z) = \gamma_a \cdot \exp\left(\gamma_z \Phi^{-1}(z) + \gamma_x \cdot \Phi^{-1}(x)\right)$
worker cost function	$c(e) = c_0\left((1-e)^{-c_1} - 1\right)$
	$\delta(e) = 1 - e$
unemployment benefits	$b(x) = f(x, Q_z(b))$
worker type	$\Gamma_x(x_{t+1}\|x_t)$ is a Gaussian copula with parameter ρ_x
match TFP	$\Gamma_z(z_{t+1}\|z_t)$ is a Gaussian copula with parameter ρ_z
	updates to z_t are computed via ι_t shared at firm level

function f can be interpreted as the quantile function of worker specific heterogeneity. Using the normal distribution Φ^{-1} gives the simple interpretation that workers' productivity is distributed as a log-normal distribution with log-variance γ_x. The mean of that distribution is defined by γ_a which, as mentioned before, is normalized to match the mean log-wage in the economy.

The parameter of risk aversion controls how quickly changes in productivity get transmitted into wage changes. Every else kept equal, matching the total value added growth variance and the total wage growth variance within the firm gives an indication of how risk averse workers are.

Finally let's consider the parameters of the worker and match productivity processes. The values of ρ_x and ρ_z are learned from the variance of wage growth and the auto-covariance of wage growth among co-workers. The statistical model presented in the first section of the paper illustrates how the growth variance of worker is composed of both the worker specific growth and the firm specific growth and that the auto-covariance between co-workers' wage growth is mostly due to the common firm specific innovation. Matching both workers' wage growth variance and co-variance between co-workers allows to pin down ρ_x and ρ_z.

4.2 Solving the model

The model is estimated by method of simulated moments. For each parameter value I solve for the equilibrium, which is then used to simulate a representative sample. I create the moments from the simulated data and compute the weighted distance between the simulated moments and the moments measured from the Swedish data.

This approach requires resolving the model for each parameter set. I use a nested fixed point method where I jointly solve for the worker's problem, the firm's problem and the equilibrium constraint. The main difficulty resides in solving the firm problem where tackling directly (BE-F) requires finding the promised utilities $W_{z'x'}$ in each state of the world for the next period. This becomes infeasible as soon as reasonable supports are considered for $\mathbb{X}$ and $\mathbb{Z}$. However, the first order condition with respect to W reveals that the utility promised in different states are linked to each other. Call $\lambda\beta p(x, W)$ the multiplier for the $W = \sum W_{z'x'}$ constraint, then the first order condition for $W_{x'z'}$ is

$$\frac{\partial \mathcal{J}}{\partial V}(x', z', W_{x',z'}) = \lambda,$$

where given λ, if $\mathcal{J}$ is strictly concave, then all the $W_{x'z'}$ are pinned down. This reduces the search to one dimension. The simplification comes from the fact that the firm always tries to insure the worker as much as possible across future states, and does this by keeping her marginal utility constant across realizations. Indeed, we know that the derivative of $\mathcal{J}$ is the inverse marginal utility. One difficulty however is that $\mathcal{J}$ might be weakly concave in some regions. In that case one needs to keep track of a set of possible feasible promised utilities $W_{x'z'}$. Given the concavity of $\mathcal{J}$ this set will be an interval fully captured by its two extremities. This means that at worst the number of the control variables is augmented by one.

Using the marginal utility in the state space is known as the recursive Lagrangian approach as developed by Kocherlakota (1996); Marcet and Marimon (2011); Messner, Pavoni, and Sleet (2011); Cole and Kubler (2012). The problem of non-strict concavity persists in this formulation but Cole and Kubler (2012) show how to overcome this difficulty by keeping track of the upper and lower bound of the set of solutions. Numerically I solve the firm problem using recursive Lagrangian and do not find any such flat region. The recursive Lagrangian for the firm problem is derived in Appendix A.7 and is given by:

$$\begin{aligned}\mathcal{P}(x, z, \rho) = \inf_{\gamma} \sup_{w,W} f(x, z) - w + \rho\left(u(w_i) + \tilde{r}(x, W)\right) \\ - \beta\gamma\tilde{p}(x, W) + \beta\tilde{p}(x, W)\mathbb{E}\mathcal{P}(x', z', \gamma),\end{aligned} \quad (2)$$

where

$$\mathcal{P}(x, z, \rho) := \sup_{v} \mathcal{J}(x, z, v) + \rho v.$$

4.3 Estimation and standard errors (Preliminary)

Estimation of the parameters is achieved using a minimum distance estimator based on a set of moments m_n. The method is close to simulated moments, however because of the moments are based on individual data and some are based on aggregation at the firm level, I present it as an indirect inference estimator.

Definition 3. Given a vector m_n of moments such that $\sqrt{n}\,(m_n - m(\theta_0)) \xrightarrow{d} \mathcal{N}(0,\Sigma)$ where θ_0 is the true parameter, and for a given weighting matrix $W_n = O(1)$, I define the following criterion:

$$L_n(\theta) = -\frac{n}{2}\left[m_n - m(\theta)\right]^T W_n \left[m_n - m(\theta)\right],$$

and the associated minimum distance estimate $\hat{\theta}_n = \inf_\theta L_n(\theta)$.

Because some of the moments are defined at the firm level, such as the correlation between co-worker wage growth, n refers to the number of firms. Point estimates are computed using a parallel version of differential evolution, see Das and Suganthan (2011) for a complete survey. In the first stage I use a weighting matrix constructed from the inverse diagonal of an estimate from the data of Σ which ignores the serial correlation and the fact that the same worker appears in several firms:

$$W_n = \left(\mathrm{diag}\left[\hat{\Sigma}\right]\right)^{-1}.$$

The computation of standard errors is based on the pseudo-likelihood estimator presented in Chernozhukov and Hong (2003). Using MCMC rejection sampling, I can perform the estimation in parallel, without having to compute derivatives and still obtain standard errors on the parameters. Given the criterion $L_n(\theta)$, with moments m_n, true parameter θ_0 and weighting matrix W_n, the asymptotic variance for the minimum distance estimator $\hat{\theta_n}$ is distributed according to

$$\sqrt{n}\left(\widehat{\theta}_n - \theta_0\right) \xrightarrow{d} \mathcal{N}(\theta_0, J^{-1}\Omega J^{-1})$$

where

$$\begin{aligned}
\Omega &= \lim_{n\to\infty} \quad \left[\frac{\partial m(\theta_0)}{\partial \theta^T}\right]^T W_n \Sigma W_n \frac{\partial m(\theta_0)}{\partial \theta^T} \\
J &= \lim_{n\to\infty} \quad \frac{1}{n}\frac{\partial^2 L_n(\theta)}{\partial \theta^T \partial \theta}
\end{aligned}$$

The full procedure requires two steps. In a first step I acquire a consistent estimate of

Table 5: Within sample model fit (Preliminary)

	HS dropout		HS grad		Some college	
	model	data	model	data	model	data
Pr_{U2E}	0.131	0.152 (2.69e-04)	0.214	0.184 (1.53e-04)	0.209	0.191 (3.36e-04)
Pr_{J2J}	0.0224	0.0223 (4.01e-05)	0.0284	0.0267 (2.27e-05)	0.0338	0.0331 (3.21e-05)
Pr_{E2U}	0.0202	0.0249 (6.25e-05)	0.0199	0.0223 (2.67e-05)	0.0164	0.0143 (3.05e-05)
$E(\Delta \log w_{it} \| EE)$	0.0145	0.0125 (1.73e-04)	0.03	0.0153 (8.69e-05)	0.0257	0.0335 (1.24e-04)
$E(\Delta \log w_{it} \| J2J)$	0.0329	0.0274 (8.36e-04)	0.0738	0.0306 (3.95e-04)	0.0875	0.0506 (5.60e-04)
$Var(\log w_{it})$	0.163	0.127 (2.09e-04)	0.141	0.116 (1.32e-04)	0.204	0.203 (3.38e-04)
$Var(\Delta \log w_{it} \| EE)$	0.0186	0.0198 (2.38e-05)	0.0171	0.0173 (1.61e-05)	0.0173	0.0193 (2.42e-05)
$Var(\Delta \log w_{it} \| J2J)$	0.0448	0.0206 (5.36e-04)	0.0353	0.018 (2.14e-04)	0.0466	0.0186 (2.49e-04)
$Var(\Delta \log y_{it})$	0.375	0.103 (1.24e-03)	0.158	0.119 (1.10e-03)	0.102	0.132 (1.65e-03)
$Cov(\Delta \log w_{it}, \Delta \log w_{jt} \| EE)$	0.00154	0.00126 (2.64e-06)	0.00169	0.00167 (1.80e-06)	0.0023	0.00235 (3.32e-06)

$\hat{\theta}_n$ using an approximate weighting matrix $\hat{\Sigma}_n$ using bootstrap. Given a good value of $\hat{\theta}_n$ I compute a Markov chain from the posterior of the pseudo likelihood of $L_n(\theta)$ as described Chernozhukov and Hong (2003) and extended to parallel chains as presented in Baragatti, Grimaud, and Pommeret (2011). The Markov chain allows construction of an estimate of Ω and J^{-1}. J^{-1} is obtained by taking the variance covariance matrix of the parameters generated by the chain. Ω can be computed by finite differences around the optimal value $\hat{\theta}_n$ by selecting draws from the chain that are close to it. A consistent estimate of Σ can then be constructed by simulating the model at $\hat{\theta}_n$ and computing the covariance matrix.

4.4 Moments and estimates

I present here the set of moments used for estimation on the different education groups. Table 5 reports the moments in the data with their measured standard deviation and the value of the moments in the model at the estimated parameter values. Table 6 presents the estimated parameters for each education group.

The model matches transition probabilities and variances quite precisely across education

Table 6: Parameter estimates (Preliminary)

		HS dropout	HS grad	Some college
scale (log wage)		0.127 (0.000209)	10.4 (0.0003)	10.4 (0.0003)
risk aversion	ς	1.12 (0.124)	1.62 (0.0408)	1.42 (0.0586)
vacancy cost	$\eta^{-}1$	1.34 (0.34)	0.646 (0.0753)	0.605 (0.0532)
OTJ efficiency	κ	0.586 (0.15)	0.617 (0.0238)	0.687 (0.0387)
effort cost	c_0	0.0779 (0.0244)	0.0498 (0.0202)	0.0418 (0.0229)
worker heterogeneity	γ_x	2.03 (0.303)	1.27 (0.123)	1.5 (0.0797)
worker type auto-cor	ρ_x	0.749 (0.0365)	0.802 (0.0206)	0.879 (0.0274)
match type auto-cor	ρ_z	0.765 (0.06)	0.962 (0.0502)	0.978 (0.0215)

groups. However at this time the model performs poorly on the average wage growth on the job and the mean wage gain on job-to-job transitions. Those moments are related to each other because the job-to-job transition rate, mean gain on moving and on the job mean wage growth are linked to each other because wages increase on-the-job to lower the worker search decision. This is a common limitation of search model which suggests that some human capital accumulation might be happening in the data. This is absent from the current model.

5 Empirical implications

5.1 Decomposition of permanent wage growth

I can now utilize the model to decompose observed variances into better defined welfare measures. Our concern is with the sources of uncertainty in the change of lifetime utility, however to get measures in monetary form, I define the wage growth variance of log permanent wage as:

$$\mathbb{E}_t \left(\bar{w}_{t+1} - \bar{w}_t\right)^2 \quad \text{where} \quad \bar{w}_t := \log \Big(u^{-1}\left(rW_t\right)\Big),$$

where $\bar{w}$ represents the annuity wage that delivers the current level of lifetime utility, the permanent wage equivalent to the expected lifetime utility. This is a meaningful measure since W_t includes all possible future risk of loosing the job or the opportunities to find new ones. Similarly we can measure equivalent permanent output that I will denote $\bar{y}$. Considering employed workers, five mutually exclusive events can happen to them over the course of a period: i) job loss, ii) job transition, iii) firm shock, iv) worker shock or v) none of the above. We can decompose the permanent earning growth variance into the contributions of those five events:

$$\mathbb{E}_t \left(\bar{w}_{t+1} - \bar{w}_t\right)^2 = \sum_{i=1}^{5} p(\mathrm{ev}_i) \cdot \mathbb{E}_t \left[\left(\bar{w}_{t+1} - \bar{w}_t\right)^2 | \mathrm{ev}_i \right] = \sum_{i=1}^{5} V_i.$$

To get the average risk in the population, I integrate the V_i over the stationary distribution. Table 7 reports this variance decomposition for the three education groups. Including $p(\mathrm{ev}_i)$ in the computation of V_i directly accounts for the likelihood of the event.

To get an idea of the overall underlying uncertainty I compute a pass through measure that links the growth variance in productivity to the growth variance in earnings:

$$\frac{Cov(\bar{w}_{t+1} - \bar{w}_t, \bar{y}_{t+1} - \bar{y}_t)}{Var(\bar{y}_{t+1} - \bar{y}_t)}$$

and report this value conditional on receiving a worker shock and firm shock and unconditional.

The results first tell us that the total uncertainty associated with mobility is of the same magnitude as the uncertainty associated with productivity shocks. For high school drop out mobility accounts for 50 percent of uncertainty and for 24 percent for college graduates. Within mobility, job loss takes a bigger share for high school drop outs than for college graduates. This seems intuitive given the $J2J$ and $E2U$ transition rates of the two groups. Among job stayers, firm productivity shocks represent the main source of uncertainty.

Finally the pass through measure indicates that even though different education group suffer differently from firm and worker shock in terms of total earning uncertainty, the way in which those uncertainty transmit seems to be the same. For both education groups, a 10 percent change productivity due to a firm shock generates a 3 percent drop in permanent earnings. Similarly a 10 percent drop in productivity due to a worker shocks translates into a 2 percent drop on average.

Table 7: Permanent wage growth variance decomposition

	HS dropout		HS grad		Some college	
	Growth variance shares					
firm shock	4.6e-04	19.6%	1.5e-04	17.8%	3.1e-04	19.2%
worker shock	1.3e-03	54.2%	2.9e-04	34.5%	7.1e-04	44.4%
job change	1.7e-04	7.13%	1.2e-04	13.8%	1.8e-04	11.3%
job loss	4.2e-04	18.1%	2.8e-04	32.4%	3.9e-04	24.3%
no shock	2.3e-05	0.968%	1.2e-05	1.36%	1.5e-05	0.933%
	Passthrough coefficents					
overall	0.369		0.243		0.282	
worker shock	0.388		0.179		0.215	
firm shock	0.348		0.271		0.328	

5.2 Optimal contract and first best allocation

The failure to implement full insurance against productivity shocks comes from the lack of commitment. To investigate how close the optimal contract comes to the first best solution I solve the firm's problem with observable worker actions. In this first best solution the firm dictates to the worker the search decision and the effort provision. In this case, the wage is flat and the firm insures against all possible future productivity uncertainty. There still is some mobility as it can be optimal for the worker to move if the match productivity is too low.

The transmission of risk in this model is caused by the presence of the incentive constraint of the worker. When the worker actions are not private, the firm can chose efficiently his decisions by punishing heavily any deviation. In that case we return to full insurance of workers against all productivity shocks. So capturing the monetary loss associated with the risk transmission can be thought of as the monetary loss from making the worker actions private. This loss can be computed by solving the optimal contract problem under full commitment but keeping the equilibrium meeting rates from the second best contract. Given the first best Pareto frontier I compute how much more profit the first best firms would make while promising the same value to the worker. I do this across the stationary distribution of the economy and compute the total insurance value loss:

$$\text{insurance loss} := \int \Big(\mathcal{J}^{FB}(x,z,V) - \mathcal{J}^{SB}(x,z,V)\Big)\, h(x,z,V)\ \mathrm{d}x\ \mathrm{d}z\ \mathrm{d}V.$$

Table 8 reports this value across educational group as a share of total output. Table 9 reports the change in the stationary allocation when all firms implement the first best contract. Surprisingly unemployment changes only by 0.2 points for the high education group. The total amount of wage paid decreases by 16 percent, but this should not be surprising since the contract is now fully insuring the worker, firms can pay a lower average wage. Indeed even though mean wage decreases, overall welfare increases by about 1 percent. Also welfare for the unemployed increases for every type and on average by 4.97 percent.

Table 8: Insurance loss due to moral-hazard, keeping equilibrium fixed

	HS dropout	Some college
Insurance loss	1.56%	1.91%

Table 9: Second and first best

	HS dropout	HS grad	Some college
unemployment	-	-	-0.21pt
output	-	-	+5.76%
wage bill	-	-	-16.10%
welfare	-	-	+1.03%
welfare at birth	-	-	+4.97%

5.3 Competitive equilibrium and the cost of search frictions

To analyze the cost associated with search frictions, I use the estimated parameters and solve for the equilibrium solution without search frictions. This competitive equilibrium is an extension of Harris and Holmstrom (1982) with match specific heterogeneity, moral-hazard and a job creation cost. The moral-hazard comes from the effort on the job to keep the match specific productivity.

As discussed in the previous section, this model will have downwards wage adjustments because of the incentive problem and the wage will also respond to firm productivity shocks, however it does not allow for unemployment risk unless a worker is more productive at home than in a firm.

Table 10 gives the change in several measures between the second best equilibrium and the frictionless equilibrium. The most striking difference is the large 40 percent welfare gain.

Table 10: Cost of search frictions

	Some college
gdp	12.4%
welfare	40.1%
prod	2.22%
wage	22.3%
mwage	13.3%

Part of this can be explained by the fact that wages go up by 22 percent, but a lot of it is also explained by the fact that lower ability workers are now able to work and collect wages which are higher than the flow monetary value of unemployment. Job loss takes up a bigger share for high school dropouts since the probability of the event itself is higher, yet that can't account for the 15point

5.4 Policy analysis

I analyze the effect of a revenue neutral government policy that redistributes from high wages to lower wages. I parametrize the policy as follows:

$$\tilde{w} = \lambda w^{\frac{1}{\tau}}.$$

I use the highest education group for the analysis, fix $\lambda = 1.2$ and solve for $\tau = 1.25$ to make the policy revenue neutral. To get a better understanding of the effect of the policy, I report four sets of numbers: i) the model solved at the estimated parameters, without any transfer, ii) use the same solution and apply transfers without adjusting decisions, iii) solve the model again with agents knowing about the transfers, and report pre-transfer moments and iv) post-tax moments. Figure 5 represents graphically the transfer and Table 11 reports the computed results.

The goal of the policy is to reduce both the uncertainty in earnings growth and the cross-sectional inequality. When applied directly on the equilibrium solution we see that total log wage variance is reduced by 36%, and the wage growth variance is reduced by 35%. However agents react to the introduction of the policy in a way that attenuates its direct effect. Re-solving the model including those transfers gives a reduction in log wage variance of only 10% and for wage growth of 30%.

The policy however also affects unemployment which goes from 4.96% to 4.56%. This

Table 11: Revenue neutral policy

Agents	do not expect transfers		expect transfers	
Transfers	before	after	before	after
Output	1		0.978	
Unemployment	4.96%		4.69%	
Total wage	1	0.992	0.962	0.961
Wage variance	0.205	0.131	0.238	0.152
Growth variance	0.0174	0.0111	0.021	0.0134

happens because the policy makes lower productivity jobs marginally more productive than without transfers, favoring workers coming out of unemployment who apply to lower paying, highly accessible jobs. On the other hand total output is reduced for a similar reason, worker reallocation is not as critical and in the economy with transfers, worker will reallocate less efficiently.

6 Conclusion

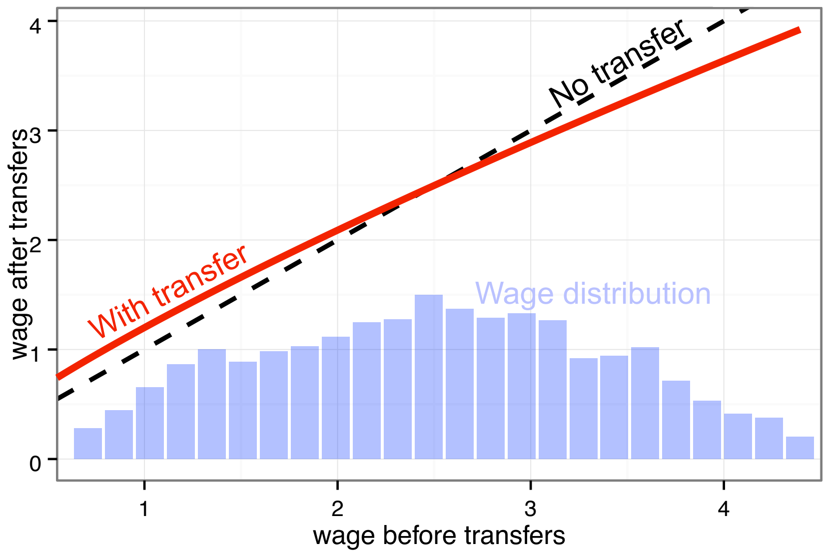
In this paper I study the different sources of uncertainty faced by workers in the labor market. Workers are subject to individual productivity shocks and their earnings may also be affected by the performance of their employer because of search frictions in the labor market. To understand the way shocks get transmitted and how this might affect welfare and labor market policy I develop an equilibrium model with search frictions, risk averse workers, firm and worker productivity shocks. In this model I show that the optimal contract pays a wage that smoothly tracks the joint match productivity. This implies that both worker and firm level shocks transmit to wages, albeit only partially. In contrast to the perfectly competitive model, on one hand firm may insure workers' productivity shocks but on the other hand they are able to transmit firm level shocks to wages.

I estimate the model on matched employer-employee data to estimate the relative importance of different sources of uncertainty. Firm productivity shocks can account for 20% for the overall permanent wage uncertainty, leaving mobility and worker shocks as the main sources of risk. Firms are unable to insure workers once the employment relationship ends making publicly provided unemployment benefits an important source of insurance. To quantify the underlying source of uncertainty I compute a pass-trough measure of productivity shocks to earning shocks and find that 20% of worker shocks and 30% of firm shocks get transmitted to wages. The implication of those findings is that policies should focus on transitions in and out of work. This is because when employed the firm will provide some source of insurance, but the firm can't continue to insure the worker when the relationship ends.

An important extension to this model is to allow individuals to hold assets, which would allow them to self insure. The inclusion of observable assets would depart only slightly from the current version of the model but a more realistic environment would allow workers to privately save. This creates many interesting economic questions such as how do firms recruit among workers with different asset holdings? In preliminary analysis of such an extension I find that firms try to hire workers with higher assets because they are easier to incentivize: firms can backload even more or get them to pay a bond, improving retention. Upfront payment by the worker to the firm is observed in high skill labor markets such as partnerships in law and consulting firms.

Another extension is to allow firms to counter outside offers. Inefficient poaching happens rarely in the estimated version of the model, but it would be more realistic to have a mechanism by which firms could optimally decide whether to counter outside offers. This type of negotiations happen in practice in high skills markets such as CEO and academics.

Figure 5: Policy



Notes: This figure represents the pre and post tranfer wages together with the distribution of wages for the polocy experiment considered here $\tilde{w} = \lambda w^{\frac{1}{\tau}}$.

A Appendix

A.1 Auxiliary model

Recall the auxiliary model described in the first section of the paper. Note that δ_{jt} appears alone in the very first model, but is then decomposed into two different components when value added is introduced.

$$
\begin{aligned}
w_{ijt} &= \beta Z_t + \tilde{w}_{ijt} + v_{ijt} \\
\tilde{w}_{ijt} &= \tilde{w}_{ijt-1} + \delta_{jt} + \xi_{ijt}, \\
y_{jt} &= \beta X_t + \tilde{y}_{jt} + u_{jt} \\
\tilde{y}_{jt} &= \tilde{y}_{jt-1} + \mu_{jt} \\
\delta_{jt} &= \tau \mu_{jt} + \nu_{jt}
\end{aligned}
$$

The auxiliary model presented can be recovered from the following moments:

$$\mathbb{E}_j\left[(\mathbb{E}_i\Delta w_{ijt})^2\right] = \sigma_\delta^2 = \sigma_\nu^2 + \tau^2\sigma_\mu^2 \tag{m1}$$

$$\mathbb{E}_{ij}\left[(\Delta w_{iit})^2\right] = \sigma_\xi^2 + \sigma_\delta^2 + 2\sigma_v^2 \tag{m2}$$

$$\mathbb{E}_{ij}\left[(\Delta y_{it})^2\right] = \sigma_\mu^2 + 2\sigma_u^2 \tag{mw1}$$

$$\mathbb{E}_j\left[\Delta y_{jt}\cdot\Delta y_{jt-1}\right] = -\sigma_v^2 \tag{mw2}$$

$$\mathbb{E}_j\left[\Delta y_{ijt}\cdot\Delta w_{ijt}\right] = \tau\sigma_\mu^2 \tag{mw2}$$

where $\mathbb{E}_i$ represents the expectation over co-workers within firm j.

A.2 Existence of the equilibrium

The model presented here is similar to the one presented in Menzio and Shi (2010). The differences are the composite functions $\tilde{r}$ and $\tilde{p}$ that now include the effort decision and the fact that workers are now heterogenous. This means that I can apply their proof here as long as I can derive the necessary properties on $(\tilde{p}, \tilde{r})$ and show that heterogeneity does not break any of the Lipschitz bounds.

Lemma 1 (existence)**.** *A stationary competitive search equilibrium exists.*

Definition. call $\mathbb{J}$ the set of functions $\mathcal{J} : \mathbb{X}\times\mathbb{Z}\times\mathbb{V}\to\mathbb{R}$ such that

(a) $\mathcal{J}$ is strictly decreasing in V,
(b) bi-Lipschitz continuous in V
(c) bounded
(d) concave

Lemma. *The operator T defined in (BE-F) is self-mapping on $\mathbb{J}$.*

Proof of Lemma 1 . Consider a function $\mathcal{J}\in\mathbb{J}$ and its image $\hat{\mathcal{J}} = T\mathcal{J}$. We start by noting that the lottery gives us that $\hat{\mathcal{J}}$ is concave which gives continuity and almost everywhere differentiability. Given that, we can apply the envelope theorem to find that the derivative of $\hat{\mathcal{J}}$ is almost everywhere $-1/u'(w^*(x,y,V))$. Given that we have established that the offered wage has to be bounded, it gives that the derivative of $\hat{\mathcal{J}}$ is also bounded in $[-1/\bar{u}', -1/\underline{u}']$. Given that $\mathbb{V}$ is itself bounded it gives us that $\hat{\mathcal{J}}$ is also bounded. The derivative is also strictly negative and so $\hat{\mathcal{J}}$ is a one to one mapping. $\hat{\mathcal{J}}$ is then also bi-Lipschitz. That concludes the fact that $\hat{\mathcal{J}}\in\mathbb{J}$. $\square$

Lemma. *Bounds on $\tilde{p}$,$\tilde{r}$ [incomplete]*

First I report a result from Menzio and Shi (2010) which applies directly here and states that given $\mathcal{J}_n, \mathcal{J}_r$ such that $||\mathcal{J}_n - \mathcal{J}_r|| < \rho$ we have that $\forall x, v$

$$\begin{aligned} \|p(\theta(x, v^*_{1n})) - p(\theta(x, v^*_{1r}))\| &< \alpha_P(\rho) = \max\{2\bar{B}_P + p'(0)\alpha_\theta \rho, 2\alpha_R \rho^{1/2}\} \\ \|p(\theta(x, v^*_{1n}))\,(v^*_{1n} - v) - p(\theta(x, v^*_{1r}))\,(v^*_{1r} - v)\| &< \alpha_R \rho \end{aligned}$$

that we need to use to show that it continues to apply when the effort choice of the worker is added. Given the policy for job search the effort choice is given by $\delta = e = c'^{-1}(p(\theta(x, v^*_{1n}))\,(v^*_{1n} - v) + v - \mathcal{U}(x))$ and so given that v itself is bounded we find new bounds on the $\tilde{p}$ and $\tilde{r}$ functions:

$$\begin{aligned} \|\tilde{r}_n - \tilde{r}_r\| &< \alpha_r \rho \\ \|\tilde{p}_n - \tilde{p}_r\| &< \alpha_P(\rho) \end{aligned}$$

Lemma. *The operator T is continuous on $\mathbb{J}$*

Proof. This boils down to showing that T is K-lipschitz. Let's take two functions $\mathcal{J}_1, \mathcal{J}_2 \in \mathbb{J}$ and their respective image $\hat{\mathcal{J}}_1, \hat{\mathcal{J}}_2$. We already know that they are part of $\mathbb{J}$. Then we need to find a constant K such that $||\hat{\mathcal{J}}_1 - \hat{\mathcal{J}}_2|| \leq K||\mathcal{J}_1 - \mathcal{J}_2||$. We substitute in the $\hat{\mathcal{J}}_1$ and $\hat{\mathcal{J}}_2$ by their definition. We then bound each element separately:

$$\begin{aligned} ||\hat{\mathcal{J}}_1(x, z, V) - \hat{\mathcal{J}}_2(x, z, V)|| \leq{}& ||u(w_1) - u(w_2)|| \\ &+ ||\tilde{p}_1(x, W_1)\mathbb{E}\mathcal{J}(x', z', W_{1x'z'}) - \tilde{p}_2(x, W_2)\mathbb{E}\mathcal{J}(x', z', W_{2x'z'})|| \end{aligned}$$

where we now want to bound each term.

[TBD] but extremely similar to Tsuyuhara (2013) and Menzio and Shi (2010). □

A.3 Properties or worker search functions

Lemma 3. *Given (x, W), $v^*(x, W)$ and $e^*(W)$ are uniquely determined, $\tilde{p}(x, W)$ is continuous and decreasing, $\tilde{r}(x, W)$ is increasing in W, continuously differentiable and $\frac{\partial \tilde{r}}{\partial W}(x, W) = \beta \tilde{p}(x, W)$.*

Proof. remember the definitions

$$\begin{aligned}
v^*(x, W) =& \arg\max_v p(\theta(x, v))(v - W) \\
e^*(x, W) =& \arg\max_e -c(e) + \delta(e)\beta\mathbb{E}W_0(x') \\
&+ \beta(1 - \delta(e))\left(p(\theta(x, v^*))v^* + \beta(1 - \delta(e))(1 - p(\theta(x, v^*)))W\right),
\end{aligned}$$

and the definition of the composite functions

$$\begin{aligned}
\tilde{p}(x, W) =& (1 - \delta(e^*(x, W)))\left(1 - p(\theta(x, v_1^*(x, W)))\right) \\
\tilde{r}(x, W) =& - c(e^*(x, W)) + \beta(1 - \delta(e^*(x, W)))p(\theta(x, v_1^*(x, W)))\left(v_1^*(x, W) - W\right) \\
&+ \delta(e^*(x, W))\beta\mathbb{E}_{x'|x}U(x') + \beta(1 - \delta(e^*(x, W)))(x, W)W
\end{aligned}$$

I first normalize $\delta(e) = 1 - e$ (or equivalently redefine c and e such that $c(e) = c(\delta^{-1}(e))$), where $c(e)$ is increasing and concave. The maximization problem for v gives the following first order condition

$$p'(\theta(x, v))(v - W) + p(\theta(x, v)) = 0$$

where given the property of p and q and the equilibrium definition of θ we have that the function $v \mapsto p(\theta(x, v))$ is decreasing and strictly concave. This gives that the maximum is unique and so $v^*(x, W)$ is uniquely defined. The first order condition for e is given by

$$c'(e) = \beta p(\theta(x, v_1^*(x, W)))\left(v_1^*(x, W) - W\right) + \beta W - \beta\mathbb{E}_{x'|x}U(x')$$

and given the assumption that c is strictly convex, we get that $e^*(x, W)$ is also uniquely defined.

Finally we can use the envelope condition to compute the derivative of $\tilde{r}$ with respect to W. By definition we have

$$\tilde{r}(x, W) = \sup_{v,e} u(w) - c(e) + (1 - e)\beta\mathbb{E}_{x'|x}W_0(x') + e\beta p(\theta(x, v))v + e\beta(1 - p(\theta(x, v)))W,$$

and so we get

$$\frac{\partial\tilde{r}}{\partial W}(x, W) = \beta e^*(x, W)(1 - p(\theta(x, v^*(x, W))) = \beta\tilde{p}(x, W)$$

which proves that $\tilde{r}$ is continuously differentiable as long as $\tilde{p}$ is continuous. □

A.4 Regularity properties for equilibrium functions

Lemma 2. *The Pareto frontier $\mathcal{J}(x,z,V)$ is continuously differentiable, decreasing and concave with respect to V and increasing in z.*

Proof of Lemma 2 . Consider the optimal contract equation:

$$\begin{aligned}
\mathcal{J}(x,z,V) = \sup_{\pi_i, W_i, W_{ix'y'}} & \sum \pi_i \Big(f(x,z) - w_i + \beta \tilde{p}(x,W_i) \mathbb{E}\mathcal{J}(x',z',W_{ix'y'}) \Big) \\
s.t \quad (\lambda) \quad & 0 = \sum_i \pi_i \left(u(w_i) + \tilde{r}(x,W_i) \right) - V, \\
& (\gamma_i) \quad 0 = W_i - \mathbb{E}W_{ix'y'}, \\
& \sum \pi_i = 1.
\end{aligned}$$

We already know that $\mathcal{J}$ is concave because of the two point lottery. That tells us that it is continuous and differentiable almost everywhere. Let's then show that it is differentiable everywhere. I follow the steps of the derivation presented in Koeppl (2006) where he shows that in the problem with two sided limited commitment it is sufficient to have one state realization where neither participation constraint binds to achieve differentiability of the Pareto frontier. Given that the current problem is one sided the result works almost right away, it just needs to be extended to include a search decision.

For a fixed $s = (x,z)$, let's consider a point $\tilde{V}$ where it's not differentiable and call $(\tilde{w}, \tilde{\pi}_1, \tilde{W}_{ix'z'}, \tilde{W}_i)$ the firm's action at that point. This action is by definition feasible and delivers $\tilde{v}$ to the worker. From that strategy I am going to construct a continuum that delivers any V around $\tilde{V}$. Keeping $(\tilde{\pi}_1, \tilde{W}_{ix'z'}, \tilde{W}_i)$ the same, I defined $w^*(V) = u^{-1}(V - \tilde{V})$.

I then define the function $\tilde{\mathcal{J}}(s,v)$ as the value that uses strategy $\Big(w^*(V) = u^{-1}(V - \tilde{V}), \tilde{\pi}_1, \tilde{W}_{ix'y'}, \tilde{W}_i\Big)$. It is the case that the strategy is feasible since all constraints remain satisfied. By definition of $\mathcal{J}$ we have that $\tilde{\mathcal{J}}(s,V) \leq \mathcal{J}(s,V)$ together with $\tilde{\mathcal{J}}(s,\tilde{V}) = J(s,\tilde{V})$. Finally because $u(\cdot)$ is concave, increasing and twice differentiable, $\tilde{\mathcal{J}}(s,\tilde{V})$ is also concave and twice differentiable.

We found a function concave, continuously differentiable, lower than $\mathcal{J}$ and equal to $\mathcal{J}$ at $\tilde{V}$ we can apply Lemma 1 from Benveniste and Scheinkman (1979) which gives us that $\mathcal{J}(s,v)$ is differentiable at $\tilde{v}$. We then conclude that $\mathcal{J}$ is differentiable everywhere. Finally let's show that $\mathcal{J}(x,z,v)$ is increasing in z.

Let's consider two different values $z_1 < z_2$. Call ξ_i the history contingent policy starting at (x,z_i,v). Policy ξ_1 will deliver identical utility to the worker in all histories independently of whether it started at z_1 or z_2. I then compare the value of using ξ_1 at (x, z_1, v) and (x, z_2, v). Given that the worker will be promised the same utility in both cases and given that the process on x and z are independent we can write the probability of each history h^t as the the product on the probability on the history on z and the probability on x

$$\mathcal{J}(x, z, v|\xi_1) = \sum_t \sum_{(x^t, z^t)} \beta^t \left(f(x_t, z_t) - w^t\right) \pi_{x,t}(x^t|x)\pi_{z,t}(z^t|z)\pi_{\delta,t}(\xi_1),$$

where $\pi_{x,t}$ is the productivity process on x generated by Γ_x , $\pi_{z,t}$ is the process on z generated by $g(z, \iota)$, and $\pi_{\delta,t}(\xi_1)$ is the composition of the leaving probabilities $\tilde{p}(x^t, W^t)$ prescribed by the policy ξ_1. We can then compare the following difference:

$$\mathcal{J}(x, z_2, v|\xi_1) - \mathcal{J}(x, z_1, v|\xi_1) = \\ \sum_t \sum_{(x^t, z^t)} \beta^t f(x^t, z^t) \left(\pi_{z,t}(z^t|z_2) - \pi_{z,t}(z^t|z_1)\right) \pi_{x,t}(x^t|x)\pi_{\delta t}(\xi_1),$$

where we finally use the fact that the transition matrix on z is assumed to be monotonic, in which case we get that all future distributions conditional on z_2 will stochastically dominate distributions conditional on z_1. Given the stochastic dominance of $\pi_{z,t}(z^t|z_2)$ over $\pi_{z,t}(z^t|z_2)$ and the monotonicity of $f(x, z)$ in z we get:

$$\forall t, x^t \quad \sum_{z^t} f(x^t, z^t) \left(\pi_{z,t}(z^t|z_2) - \pi_{z,t}(z^t|z_1)\right) \geq 0$$

which gives the result. See Dardanoni (1995) for more on properties of monotonic Markov chains. □

A.5 Characterization of the optimal contract

Lemma. *For a given (x, z), a higher wage always means higher lifetime utility.*

Proof. This is a direct implication of the concavity of $\mathcal{J}$ and the envelope condition:

$$\frac{\partial \mathcal{J}(x, z, v)}{\partial v} = \frac{1}{u'(w)},$$

and given also the concavity of $u(\cdot)$, we get that w and v_s are always moving in the same direction. □

Proposition 1 (optimal contract)**.** *For each viable match (x, z), independent of the lottery realization, the wage policy is characterized by a **target wage** $w^*(x, z)$, which is increasing in z such that:*

$$\begin{aligned} w_t \leq w^*(x_t, z_t) \quad &\Rightarrow \quad w_t \leq w_{t+1} \leq w^*(x_t, z_t) \quad \textit{incentive to search less} \\ w_t \geq w^*(x_t, z_t) \quad &\Rightarrow \quad w^*(x_t, z_t) \leq w_{t+1} \leq w_t \quad \textit{incentive to search more} \end{aligned}$$

where the target wage is characterized by the zero expected profit condition for the firm:

$$\forall x, z \quad \mathbb{E}_{x'z'|xz} \mathcal{J}(x', z', W_{x'z'}) = 0$$

Proof of Lemma 1 . We start again from the list of first order conditions and we want to find a relationship for wage change.

$$\begin{aligned} \mathcal{J}(x, z, V) = \sup_{\pi_i, W_i, W_{ix'z'}} & \sum \pi_i \Big(f(x, z) - w_i + \beta \tilde{p}(x, W_i) \mathbb{E} \mathcal{J}(x', z', W_{ix'z'}) \Big) \\ s.t \quad (\lambda) \quad & 0 = \sum_i \pi_i \left(u(w_i) + \tilde{r}(x, W_i) \right) - V, \\ (\gamma_i) \quad & 0 = W_i - \mathbb{E} W_{ix'z'}, \\ & \sum \pi_i = 1. \end{aligned}$$

From the envelope theorem and the f.o.c. for the wage, we get that the wage in the current period is given by

$$i = 1, 2 \quad u'(w_i) = \frac{1}{\lambda} = -\left(\frac{\partial \mathcal{J}}{\partial v}(x, z, v) \right)^{-1}.$$

Now that also means that the wage next period in state (x', z') will be given by

$$\frac{1}{u'(w_{ix'z'})} = -\frac{\partial \mathcal{J}}{\partial v}(x', z', W_{ix'z'}).$$

I then look at the first order condition with respect to W_i

$$\pi_i \beta \tilde{p}_v(x, W_i) \mathbb{E} \mathcal{J}(x', z', W_{ix'y'}) + \beta \lambda \pi_i r'(x, W_i) + \pi_i \gamma_i = 0,$$

where I substitute $r'(x, W) = \tilde{p}(x, W)$, derived in Lemma (A.3):

$$\pi_i \beta \tilde{p}_v(x, W_i) \mathbb{E} \mathcal{J}(x', z', W_{ix'y'}) + \beta \lambda \pi_i \tilde{p}(x, W_i) + \pi_i \gamma_i = 0.$$

Using the f.o.c. for $W_{ix'z'}$, which is

$$\beta\tilde{p}(x, W_i)\frac{\partial \mathcal{J}}{\partial v}(x', z', W_{ix'y'}) - \gamma_i = 0,$$

I get the following expression:

$$\pi_i\beta\tilde{p}_v(x, W_i)\mathbb{E}\mathcal{J}(x', z', W_{ix'z'}) + \beta\lambda\pi_i\tilde{p}(x, W_i) + \pi_i\beta\tilde{p}(x, W_i)\frac{\partial \mathcal{J}}{\partial v}(x', z', W_{ix'z'}) = 0.$$

Focusing on $p_1(x, W) > 0$ and $\pi_i > 0$ since otherwise, the worker is leaving the current firm and the next period wage is irrelevant, we first rewrite:

$$\frac{\tilde{p}_v(x, W_i)}{\tilde{p}(x, W_i)}\mathbb{E}\mathcal{J}(x', z', W_{ix'z'}) + \lambda + \frac{\partial J}{\partial v}(s', v_{s'}) = 0.$$

I finally use the envelope condition to extract the wage next period from the last term on the right

$$\frac{\tilde{p}_v(x, W_i)}{\tilde{p}(x, W_i)}\mathbb{E}\mathcal{J}(x', z', W_{ix'z'}) = \frac{1}{u'(w_{x'z'})} - \frac{1}{u'(w)},$$

where since $\tilde{p}_v(x, W_i) > 0$ the inverse marginal utility and consequently wages move according to the sign of expected surplus to the firm. This shows that within each realization of the lottery, the wage will move according to expected profit.

Randomizing over increase and decrease: the next step is to investigate if it is ever optimal for the firm to randomize over a wage increase and a wage decrease at the same time. If the lottery is degenerate then the result holds directly. We are left with non-degenerate lotteries. In that case the first order condition with respect to π must be equal to zero (otherwise we are at a corner solution, which is degenerate). Taking the first order condition with respect to π gives:

$$\beta\tilde{p}(x, W_1)\mathbb{E}\mathcal{J}(x', z', W_{1x'z'}) \ + \ \lambda\beta\tilde{r}(x, W_1) \ = \ \beta\tilde{p}(x, W_2)\mathbb{E}\mathcal{J}(x', z', W_{2x'z'}) \ + \ \lambda\beta\tilde{r}(x, W_2),$$

which we can reorder in

$$\beta\tilde{p}(x, W_1)\mathbb{E}\mathcal{J}(x', z', W_{1x'z'}) \ - \ \beta\tilde{p}(x, W_2)\mathbb{E}\mathcal{J}(x', z', W_{2x'yz'}) \ = \ \lambda\beta\left(\tilde{r}(x, W_2) - \tilde{r}(x, W_1)\right).$$

Now, suppose that the randomization is over two expected profits of opposite sign for the firm where 1 is positive and 2 is negative. The left hand side is then positive. But in that case we know that $W_2 < V < W_1$ because higher wages give higher utilities in all states of the world, and so they do so also in expectation. This gives us that $\tilde{r}(x, W_2) < \tilde{r}(x, W_1)$. Given that λ is equal to inverse marginal utility it is positive. But then the right hand side is negative, so we have a contradiction. So independent of the randomization, the wage will move according to the sign of the expected profit.

Monotonicity in z: the final step is to show that the efficiency wage is increasing in z. We already know that $\mathcal{J}(x, z, V)$ is increasing in z and decreasing and concave in V. Let's consider $z_1 < z_2$ and associated efficiency wage $w^*(x, z_1)$. We want to show that $w^*(x, z_1) < w^*(x, z_2)$. Call ξ_1 the optimal policy starting at state $\mathcal{J}(x, z_1, V_1)$ where V_1 delivers $w^*(x, z_1)$ and using ξ_1 at (x, z_2), the worker receives V_1 and is paid $w^*(x, z_1)$. The firm makes more profit than at z_1 since $f(x, z)$ is increasing in z and $\mathbb{E}\mathcal{J}$ is larger as well. The optimal policy at (x, z_2, V_1) will pay a higher wage than $w^*(x, z_1)$ to trade some output for a longer expected lifespan, but continue to choose positive $\mathbb{E}\mathcal{J}$. So we found a wage $w_3^* \geq w^*(x, z_1)$ such that $\mathbb{E}\mathcal{J}$ is still positive. This last point implies that $w_3^* \leq w^*(x, z_2)$ and concludes. $\square$

A.6 From matching function to tightness

I use the following matching function

$$
\begin{aligned}
p(\theta) &= \theta^{\nu} \\
q(\theta) &= p(\theta)/\theta = \theta^{\nu-1}
\end{aligned}
$$

this gives us that

$$
p = q^{\frac{\nu}{\nu-1}},
$$

and we have the following equilibrium equality for $q(\cdot)$ from the free entry condition: we end up with

$$
p(x, v) = \left(\frac{1}{k_e} J(x, y, z, v) \right)^{\frac{\nu}{1-\nu}}.
$$

Now since I am worried about keeping this function sufficiently concave to insure uniqueness of the worker search decision, I use $\nu < 1/2$.

A.7 Recursive Lagrangian formulation

Ignoring the lottery for now, we have the following recursive formulation for $\mathcal{J}$

$$
\begin{aligned}
\mathcal{J}(x,z,V) = \sup_{\pi_i, W_i, W_{ix'y'}} & f(x,z) - w_i + \beta \tilde{p}(x, W_i)\mathbb{E}\mathcal{J}(x', z', W_{ix'y'}) \\
s.t \quad (\lambda) \quad & 0 = u(w_i) + \tilde{r}(x, W_i) - V, \\
(\gamma_i) \quad & 0 = W_i - \mathbb{E}W_{ix'z'}.
\end{aligned}
$$

From which we can construct the Pareto problem

$$
\mathcal{P}(x,z,\rho) = \sup_v \mathcal{J}(x,z,v) + \rho v.
$$

Formally, $\mathcal{P}$ is also the Legendre–Fenchel transform of $\mathcal{P}$, see Villani (2003). We seek a recursive formulation. I first substitute the definition of $\mathcal{J}$ and the constraint on λ in $\mathcal{P}$ to get

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) = \sup_{V,w,W,W_{x'z'}} & f(x,z) - w + \beta \tilde{p}(x, W)\mathbb{E}\mathcal{J}(x', z', W_{x'z'}) + \rho V \\
s.t \quad (\lambda) \quad & 0 = u(w_i) + \tilde{r}(x, W) - V, \\
(\gamma) \quad & 0 = W - \mathbb{E}W_{x'z'}.
\end{aligned}
$$

at which point I can substitute in the V constraint:

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) = \sup_{V,w,W,W_{x'z'}} & f(x,z) - w + \beta \tilde{p}(x, W)\mathbb{E}\mathcal{J}(x', z', W_{x'z'}) + \rho\left(u(w_i) + \tilde{r}(x, W)\right) \\
s.t \quad (\gamma) \quad & 0 = W - \mathbb{E}W_{x'z'}.
\end{aligned}
$$

then I append the constraint $(\gamma)$with weight $\beta\gamma\tilde{p}(x,W)$

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) \quad = \quad & \inf_{\gamma} \sup_{V,w,W,W_{x'z'}} f(x,z) - w + \rho\left(u(w_i) + \tilde{r}(x, W)\right) \\
& -\gamma\beta\tilde{p}(x,W)(W - \mathbb{E}W_{x'z'}) \\
& +\beta\tilde{p}(x,W)\mathbb{E}\mathcal{J}(x', z', W_{x'z'})
\end{aligned}
$$

which finally we recombine as

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) &= \inf_{\gamma} \sup_{V,w,W,W_{x'z'}} f(x,z) - w + \rho\left(u(w_i) + \tilde{r}(x,W)\right) \\
& \quad -\beta\gamma\tilde{p}(x,W) \\
& \quad +\beta\tilde{p}(x,W)\mathbb{E}\mathcal{J}(x',z',W_{x'z'}) + \gamma\mathbb{E}W_{x'z'}
\end{aligned}
$$

the final step is to move the sup to the right hand side to get:

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) &= \inf_{\gamma} \sup_{w,W} f(x,z) - w + \rho\left(u(w_i) + \tilde{r}(x,W)\right) \\
& \quad -\beta\gamma\tilde{p}(x,W) \\
& \quad +\beta\tilde{p}(x,W)\mathbb{E}\left[\sup_{W_{x'z'}} \mathcal{J}(x',z',W_{x'z'}) + \gamma W_{x'z'}\right]
\end{aligned}
$$

where we recognize the expression for $\mathcal{P}$ and so we are left with solving the following saddle point functional equation (SPFE):

$$
\begin{aligned}
\mathcal{P}(x,z,\rho) = \inf_{\gamma} \sup_{w,W} f(x,z) - w + \rho\left(u(w_i) + \tilde{r}(x,W)\right) \\
- \beta\gamma\tilde{p}(x,W) + \beta\tilde{p}(x,W)\mathbb{E}\mathcal{P}(x',z',\gamma). \quad \text{(SPFE)}
\end{aligned}
$$

From the solution of this equation we can reconstruct the lifetime utility of the worker, and the profit function of the firm

$$
\begin{aligned}
\mathcal{V}(x,z,\rho) &= \frac{\partial \mathcal{P}}{\partial \rho}(c,z,\rho) \\
\mathcal{J}(x,z,v) &= \mathcal{P}(x,z,\rho^*(x,z,v)) - \rho^*(x,z,v)\cdot v.
\end{aligned}
$$

A.8 Notations

Here is a summary of the notations used in the paper:

β is discount factor

$u : \mathbb{R} \to \mathbb{R}$ is utility function

$c : \mathbb{R} \to \mathbb{R}$ is effort function

e is effort level of the worker

w is wage

x is worker productivity

Table 12: Uncertainty at firm level per industry and education group

	Construction etc.	Manufacturing	Retail trade	Services
educ1				
σ_f	0.00498	0.00365	0.00298	0.00471
	(0.000336)	(0.000151)	(0.000215)	(0.000376)
σ_w	0.0245	0.0212	0.0197	0.0279
	(0.00297)	(0.00107)	(0.00176)	(0.00228)
firm perc	16.9	14.7	13.2	14.5
educ2				
σ_f	0.0059	0.00321	0.00431	0.00481
	(0.000434)	(0.000164)	(0.000401)	(0.000509)
σ_w	0.024	0.0185	0.02	0.0254
	(0.00302)	(0.00125)	(0.00269)	(0.00334)
firm perc	19.8	14.8	17.7	15.9
educ3				
σ_f	0.00558	0.00225	0.00521	0.00757
	(0.000768)	(0.000122)	(0.000547)	(0.000299)
σ_w	0.0267	0.0187	0.0224	0.0231
	(0.00342)	(0.00124)	(0.00328)	(0.00173)
firm perc	17.3	10.7	18.9	24.6

z is match productivity

$f(x, z)$ is output of worker x in match z

κ is search efficiency on the job

η is vacancy cost

θ is market tightness for market (x, v)

v is value a worker will get in a given submarket

V is value promised to the worker when entering a period

W_i is expected value promised to the worker in realization i of the lottery

$W_{x'z'}$ is value promised to the worker in realization (x', z') of the shock

$v_1(x, z, v)$ is the search policy of the worker

$e(x, z, v)$ is the effort policy of the worker

B Additional Data information

C Model extensions

C.1 firm heterogeneity [TBC]

If worker shocks are small enough, firm permanent heterogeneity can be added to the model and the equilibrium continues to exists. I briefly show this here.

C.2 severance payments [TBC]

I present here an extended version of the model with side payments when the worker loses his job. The firm is allowed to choose a value g delivered to the worker when he moves to unemployment.

I start from the recursive form and

$$f(s) - w - g(1-q) + \beta p_1(e,g)\mathbb{E}J(s', v_{s'}) + \\ \rho\left(u(w) + r(e,g)\right) - \mu\beta p_1(e,g)(e - \mathbb{E}v_{s'})$$

where

$$r(e,g) = \sup_{v,q} -c(q) + (1-q)\beta\mathbb{E}U(x',g) + q\beta p(v)v + q(1-p(v))\beta e.$$

and so we get

$$\begin{aligned} r_e(e) &= -q^*\beta(1-p^*) = -\beta p_1(e) \\ r_g(e) &= (1-q^*)\beta\mathbb{E}U_g(x',g) \end{aligned}$$

which we can recombine in

$$f(s) - w + \rho(u(w) + r(e)) - \mu\beta p_1(e)e + \beta p_1(e)\mathbb{E}P(s',\mu)$$

and we get 3 FOC: w μ and e and g

$$e = \mathbb{E}P_\rho(s,\mu)$$

$$-\rho r_e(e) - \mu\beta p_e(e,g)e - \mu p_e(e,g) + \beta p_e(e,g)\mathbb{E}P(s',\mu) = 0$$

$$-(1-q)+\rho r_g-\mu\beta p_g e+\beta p_g\mathbb{E}P=0$$

I should combine the terms in $p'(e)$ to get $\mathbb{E}P-(\rho+\mu)e$

$$(\mu-\rho)p_1(e)=\beta p_1'(e)\mathbb{E}\Pi_1(s',\mu)$$

and we can recombine the equation in g to find the optimal severance package:

$$(1-q)(\beta\underbrace{\mathbb{E}U_g}_{<0}-1)=-\frac{\beta p_g}{1-q}\mathbb{E}\Pi_1$$

References

Altonji, J., A. Smith, and I. Vidangos (2009): "Modeling earnings dynamics," *NBER*.

Attanasio, O. P., and N. Pavoni (2011): "Risk sharing in private information models with asset accumulation: Explaining the excess smoothness of consumption," *Econometrica*, 79(4), 1027–1068.

Azariadis, C. (1975): "Implicit contracts and underemployment equilibria," *The Journal of Political Economy*, pp. 1183–1202.

Baily, M. N. (1974): "Wages and employment under uncertain demand," *The Review of Economic Studies*, 41(1), 37–50.

Baragatti, M., A. Grimaud, and D. Pommeret (2011): "Parallel Tempering with Equi-Energy Moves," *Arxiv preprint*.

Benveniste, L., and J. Scheinkman (1979): "On the differentiability of the value function in dynamic models of economics," *Econometrica: Journal of the Econometric Society*, pp. 727–732.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption inequality and partial insurance," *The American Economic Review*, pp. 1887–1921.

Burdett, K., and M. Coles (2003): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 71(5), 1377–1404.

Burdett, K., S. Shi, and R. Wright (2001): "Pricing and matching with frictions," *Journal of Political Economy*, 109(5), 1060–1085.

Chernozhukov, V., and H. Hong (2003): "An MCMC approach to classical estimation," *Journal of Econometrics*, 115(2), 293–346.

Cole, H., and F. Kubler (2012): "Recursive contracts, lotteries and weakly concave pareto sets," *Review of Economic Dynamics*.

Dardanoni, V. (1995): "Income distribution dynamics: monotone Markov chains make light work," *Social Choice and Welfare*, 12(2), 181–192.

Das, S., and P. N. Suganthan (2011): "Differential evolution: A survey of the state-of-the-art," *Evolutionary Computation, IEEE Transactions on*, 15(1), 4–31.

Guiso, L., L. Pistaferri, and F. Schivardi (2005): "Insurance within the Firm.," *Journal of Political Economy*, 113(5), 34.

Harris, M., and B. Holmstrom (1982): "A theory of wage dynamics," *The Review of Economic Studies*, 49(3), 315–333.

Hopenhayn, H., and J. Nicolini (1997): "Optimal unemployment insurance," *Journal of political economy*, 105(2), 412–438.

Knight, F. H. (1921): "Risk, uncertainty and profit," *New York: Hart, Schaffner and Marx*.

Kocherlakota, N. (1996): "Implications of efficient risk sharing without commitment," *The Review of Economic Studies*, 63(4), 595.

Koeppl, T. (2006): "Differentiability of the efficient frontier when commitment to risk sharing is limited," *Topics in Macroeconomics*, 6(1), 1–6.

Lentz, R. (2013): "Optimal Wage-Tenure Contracts without Search Intensity Commitment," *Working Paper*.

Low, H., C. Meghir, and L. Pistaferri (2009): "Wage risk and employment risk over the life cycle," *NBER*.

MaCurdy, T. E. (1982): "The use of time series processes to model the error structure of earnings in a longitudinal data analysis," *Journal of econometrics*, 18(1), 83–114.

Marcet, A., and R. Marimon (2011): "Recursive contracts," *Unpublished Manuscript*.

Menzio, G., and S. Shi (2009): "Efficient search on the job and the business cycle," *Unpublished Manuscript*.

——— (2010): "Block recursive equilibria for stochastic models of search on the job," *Journal of Economic Theory*, 145(4), 1453–1494.

Messner, M., N. Pavoni, and C. Sleet (2011): "Recursive methods for incentive problems," *Working Papers*.

Moen, E. (1997): "Competitive search equilibrium," *Journal of Political Economy*, pp. 385–411.

Montgomery, J. (1991): "Equilibrium wage dispersion and interindustry wage differentials," *The Quarterly Journal of Economics*, 106(1), 163–179.

Peters, M. (1991): "Ex ante price offers in matching games non-steady states," *Econometrica: Journal of the Econometric Society*, pp. 1425–1454.

Rogerson, W. P. (1985): "Repeated moral hazard," *Econometrica*, pp. 69–76.

Roys, N. (2011): "Persistence of Shocks, Employment and Wages," *Unpublished Manuscript*.

Rudanko, L. (2009): "Labor market dynamics under long-term wage contracting," *Journal of Monetary Economics*, 56(2), 170–183.

Schaal, E. (2010): "Uncertainty, Productivity and Unemployment during the Great Recession," *unpublished, December*.

Shi, S. (2009): "Directed Search for Equilibrium Wage–Tenure Contracts," *Econometrica*, 77(2), 561–584.

Shimer, R. (1996): "Contracts in a frictional labor market," Discussion paper, mimeo.

Shimer, R. (2001): "The assignment of workers to jobs in an economy with coordination frictions," *NBER*.

Spear, S. E., and S. Srivastava (1987): "On repeated moral hazard with discounting," *The Review of Economic Studies*, 54(4), 599–617.

Thomas, J., and T. Worrall (1988): "Self-enforcing wage contracts," *The Review of Economic Studies*, 55(4), 541–554.

Tsuyuhara, K. (2013): "Dynamic Contracts with Worker Mobility via Directed On-the-Job Search," *Working Paper*.

Villani, C. (2003): *Topics in optimal transportation*, vol. 58. AMS Bookstore.