

Elementary Forms of Economic Organization

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Abstract

This paper presents a model where societies whose population expands, experience advances in specialization at the cost of diluted monitoring, which limits enforcement and hinders trade possibilities. Money may replace monitoring. Advances in specialization may make it feasible to replace agriculture with manufacturing. The model generates three stable regimes with different combinations of productive and trade systems. In small societies, agriculture is accompanied by mutual sharing arrangements with gift exchange; in medium sized societies, agriculture is accompanied by monetary trade; in large societies, manufacturing replaces agriculture.

Keywords: specialization, anonymity, gift exchange, money, technological choice

JEL codes: E40, O1

1 Introduction

In his book *Institutions, Institutional Change and Economic Performance*, Douglass North, examining the historical records, has pointed out that the degree of specialization in production and the ability to enforce exchange agreements appear to be inversely related. In the traditional, village societies, with tightly knit, rural communities, specialization is rudimentary and exchange personalized; in the commercial societies with a still large agricultural sector and a nascent monetary economy, specialization is more developed and exchange agreements more difficult to enforce, due to the traders anonymity; in the industrial societies, with a large manufacturing and trading sector, specialization and the anonymity of exchange are at their highest.¹

¹Douglass North (1990), pages 118-122.

This paper presents a model in which several forms of economic organization, with different productive and exchange structures, may emerge and even coexist side by side, as a result of the interplay of specialization and anonymity. The model combines elements of the monetary framework by Nobuhiro Kiyotaki and Randall Wright (1993), where specialization and anonymity motivate the use of media of exchange to lubricate the process of trade, and the real one by Gary Hansen and Edward Prescott (2002), where a land intensive, Malthus technology is progressively replaced by a capital intensive, Solow technology as the economy expands. I build into the model the idea that larger societies tend to be *more* specialized, but also display *less* social control over individuals, relative to smaller ones. Small, tightly knit communities feature a low degree of specialization, typically accompanied by low productivity, but, thanks to a pervasive social control, are able to exert enough pressure on their members to enforce mutual sharing arrangements in which exchange is based on reciprocal gifts. Large, anonymous societies, instead, feature a high degree of specialization, leading to high productivity, but are unable to monitor their members. Once social control evaporates, the enforcement of mutual sharing agreements becomes problematic, and the incentives to exchange have to be induced differently. Individuals can resort, when needed, to fiat money. As regards production, either an agricultural, land-intensive technology or an industrial, capital-intensive technology can be adopted. As the economy expands, specialization-induced productivity gains have decreasing effects in agriculture, but constant in manufacturing, due to a factor of production in fixed supply, land, which is needed in the former but not in the latter. Finally, the population growth rate depends on income per capita, consistently with the so-called demographic transition, whereby population growth stops at high income per capita.

The interplay of specialization and anonymity, generates three steady state equilibria, or regimes, with different combinations of productive and trading forms. In small, tightly knit societies, agriculture is accompanied by gift exchange, at low productivity levels; in medium sized societies, agriculture, with some productivity improvements, is accompanied by monetary trade; in large societies, manufacturing, with notable productivity advances, replaces agriculture. These three regimes coexist in a region

of the parameters space. The first regime may represent the traditional societies observed by French anthropologist Marcel Mauss who describes them as "rural societies where exchanges and contracts take place in the form of presents". According to Mauss, "gift exchange was the primitive form of credit and buying and selling by cash as well as by lending arose through a system of presents given and reciprocated".² The other two regimes may represent a commercial and an industrial monetary economy, respectively. These latter two regimes are characterized by higher income per capita, but are also more unequal, than the first one. The main implication of the model is that a prevalent role of manufacturing relative to agriculture and monetary trade relative to mutual sharing should be observed at higher levels of income per capita. The basic framework is extended to capture private money systems and the advances in transportation and information technologies that have been observed to accompany the process of industrialization.

Relative to Hansen and Prescott (2002), where the declining value of land is identified as the crucial element in the early modern European transition toward an industrial society, the present model suggests that the concomitant transition toward a monetary system might have been an essential part of the same process. Imperfect monitoring has been identified, by the literature *à la* Kiyotaki and Wright, as key to create a beneficial role for money in helping anonymous agents transact. In the present paper, the extent of monitoring is not exogenously given, but determined in equilibrium. In a related paper, Townsend (1983) has compared three exchange arrangements, where productivity increases as society moves from less to more connected market structures. Here, instead, monitoring and trade become *more* problematic as the economy expands. As a result, productivity and the effectiveness of trading systems do not necessarily move together. Kiyotaki and John Moore (2005) have analyzed the process of financial deepening, also finding three regimes with different financial instruments, but absent any role for different productive activities.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 derives the equilibria. Section 4 presents three extensions. Section 5 concludes.

²Marcel Mauss (1954), *The Gift. The Form and Reason for Exchange in Archaic Societies*, p. 5.

2 The Model

Fundamentals Time, indexed by $t = 0, 1, \dots$, is discrete and continues for ever. There is a continuum of potential locations arranged on a circle. Initially a number N_0 of locations - equally spaced on the circle- are inhabited by a continuum of mass one of infinitely lived agents each, while the rest are empty. The population may grow over time. The number of (continua of mass one of infinitely lived) agents at time t is given by $N_t = n_{t-1}N_{t-1}$. The gross growth rate of population depends on income per capita and will be specified shortly. Each (continuum of mass one, infinitely lived) new-born agent is sent out to occupy new locations on the circle, as follows. First, all the locations exactly in between locations inhabited at time zero are occupied; when all the middle locations have been occupied, and, thus, the number of inhabited locations has doubled, then, all the locations exactly in between the locations inhabited at that time are occupied, and so on. The number of inhabited locations at time t is N_t , and, at each date t , inhabited locations are re-indexed consecutively by $j_t = 1, \dots, N_t$. Thus, N_t refers to both the population and the number of specialization types at time t . In each location one type of input is produced. Agents living in location j_t produce input $j_t - 1$ and need input j_t , modulo N_t . Define $d_{j_t} \equiv |(j_t + 1) - j_t| \pmod{N_t}$ as the distance between the specialization type of agent j_t and the specialization type of the agent who provides the necessary input, j_t . Notice that j_t depends on the calendar date: at any point in time, the best input available is used. Let $\delta_{j_t} \equiv d_{j_t}^{-1}$ be the inverse of the distance between specialization types, for every j_t . This constitutes the measure of specialization of the inputs. To summarize, as society becomes larger, new ideas are generated, leading to new, more specialized inputs and to a more effective use of old inputs: as population increases, new types are generated and spread around the circle of specialization types. The input j_t is denoted by k_{j_t} , capital, and enters in the production of the final good. The production cost of the inputs is measured in utils and is equal to $c(k_{j_t}) = (k_{j_t})^2$. Every agent is endowed at birth with one unit of land and two technologies of which only one can be operated. It is the choice of the agent which one to operate. The technology is chosen at birth before trading and it is prohibitively costly to replace, hence, once selected, the agents stick to it. Agents can

use only the plot of land that has been allotted to them at birth. The capital input k_{j_t} can be used by the agent j_t at time t - together with land l_{j_t} - in the technology

$$f(k_{j_t}, l_{j_t}) = (\delta_{j_t} k_{j_t})^\alpha (l_{j_t})^{1-\alpha}, \quad (1)$$

with $\alpha \in (0, 1)$ or it can be used in the technology

$$g(k_{j_t}) = \delta_{j_t} k_{j_t}. \quad (2)$$

Therefore, the agents have the initial choice between two constant returns to scale technologies, one that needs a factor of production in fixed supply, land, $l_{j_t} = 1$ for all j_t , and a land-free technology. The first represents agriculture, the second manufacturing. The productivity of the capital input changes with the specialization measure, δ_{j_t} . This captures the closeness in specialization types as an index of how tailored the inputs are for production: the more closely specialized, the more productive they will turn out to be. Final output of any technology can be consumed by the agent who produces it. Utility is linear in consumption. Agents discount future payoffs at a rate $\beta \in (0, 1)$. Inputs and final output decay fully at the end of each period. Land does not depreciate.

Exchange There is no centralized Walrasian market for the capital inputs. Instead, trade of inputs happens in two-person meetings. Every period agents are assigned to a trading partner according to a strongly anonymous matching process, whereby any two agents cannot be matched more than once in their lifetime and do not meet each other's trading partners. With constant probability $\sigma \in (0, 1)$, there is a single coincidence of wants, in the sense that one of the two trading partners - the buyer-wants to purchase the input the other - the seller- is offering. Agents bargain over the terms of trade. Buyers have all the bargaining power. The trades of the N_0 agents belonging to the initial generation can be monitored and recorded. Agents born at $t \geq 1$, if any, are anonymous and their trades are private knowledge. This captures the idea that, as a bigger population spreads out in space, social control tends to evaporate. A constant fraction $M \in (0, 1)$ of each specialization type is endowed at birth with one unit of an indivisible object called fiat money, which is worthless for

either consumption or production purposes, but potentially useful to lubricate trade. The model, in this respect, is similar to Alberto Trejos and Wright (1995). Agents can hold at most one unit of fiat money at any time. In a meeting, holdings of money and banknotes can be observed. Agents cannot issue IOUs backed by land, since holdings of land cannot be verified.

Growth Rate of Population The population gross growth rate, $\frac{N_{t+1}}{N_t} = n(y_t)$ at all t , depends on income per capita, y_t , defined over $[y_{\min}, \infty)$, where $y_{\min} \in \mathbb{R}_{++}$ is the subsistence level, as follows (with $\theta \in (1, \infty)$)

$$n(y_t) = \begin{cases} 1 & \text{if } y_t = y_{\min}, \\ 2 & \text{if } y_t \in (y_{\min}, \theta y_{\min}), \\ 1 & \text{if } y_t \in [\theta y_{\min}, \infty). \end{cases} \quad (3)$$

This functional form conveniently preserves the symmetry of the environment as the number of locations changes. A similar function, broadly consistent with the available evidence on the demographic transition,³ is used by Hansen and Prescott (2002).

3 Three Stable Regimes

The focus is on steady state symmetric (Nash) equilibria in pure strategies with a constant population - henceforth called *regimes*, ignoring no-trade equilibria, which always exist given the fiduciary nature of the equilibria. The analysis will proceed through a series of bootstrap arguments, whereby a combination of trading and productive systems will be guessed to form part of an equilibrium at a given population level, then, the guess will be confirmed and it will be shown that the population remains constant at such a level. Next, the steady state equilibria will be shown to be Pareto undominated by the feasible alternative combinations of trading and productive systems at the same population level. Finally, the existence of a region of the space of parameters where the three regimes coexist will be established.

³See Michael Kremer (1993).

The symmetry-preserving assumptions made above simplify considerably the analysis. Given (3), the number of occupied locations at any point in time is either the same or twice as before. The process through which new locations are occupied, guarantees that occupied locations are always equally spaced on the circle. Therefore, the distance between any two adjacent locations is given by the circumference $2\pi r$, where r is the radius of the circle of specialization types, divided by the number of occupied locations at any point in time. Hence, the measure of specialization is uniform across types and varies only if the number of types changes, i.e. $\delta_{j_t} = \delta_t = \frac{N_t}{2\pi r}$, for any j_t and $t \geq 0$. The figure below depicts graphically how specialization and monitoring change with the population. On the one hand, the specialization circle becomes more

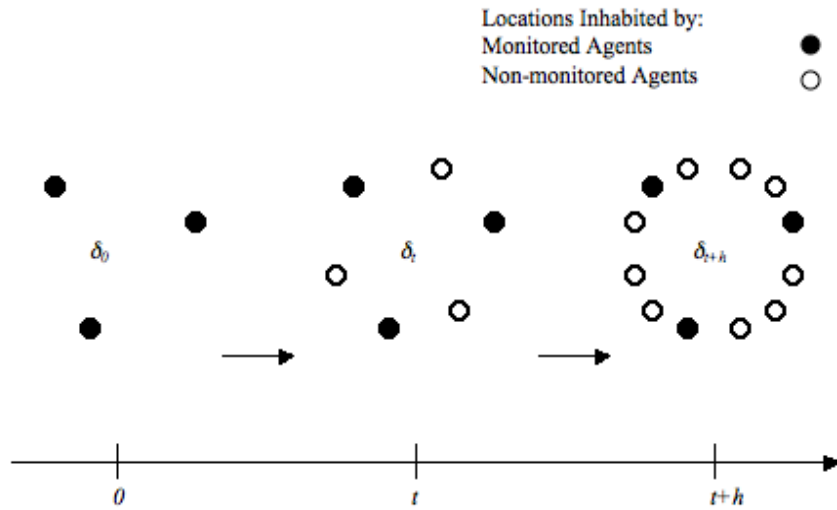


Figure 1: Specialization and Monitoring

densely populated, with new varieties of inputs becoming available. The agents use always the closest input available at any time. In turn, closer inputs are more productive, $\delta_{t+h} > \delta_t > \delta_0$. On the other hand, new generations of agents (represented by white circles) cannot be monitored, unlike the initial population (represented by black circles), and the proportion of monitored agents in the economy diminishes as the population expands. Notice that the picture is drawn assuming that the popula-

tion increases, which may not happen in equilibrium. There are equilibria (e.g. the first regime, below) in which the economy remains for ever in the situation depicted in the left-most part of the figure.

To simplify the notation, in what follows, let $\Lambda \equiv \frac{1-\beta}{\beta\sigma}$, $\Sigma \equiv (\Lambda + 1)^{-1}$, $\Psi \equiv \frac{\Lambda+1-M}{1-M}$ and $\Delta \equiv \Sigma\Psi (M - M^2)^{\frac{\alpha-2}{\alpha}}$. Henceforth, normalize the size of the specialization circle, $r = \frac{N_0}{2\pi} \Delta^{\frac{1}{2}}$. Normalize, also, the subsistence income in (3), $y_{\min} = \sigma (\Sigma\Delta^{-1})^{\frac{\alpha}{2-\alpha}}$.

3.1 Village Regime

Suppose the steady state population in this first regime is $N^{VR} = N_0$ and, thus, all the existing specialization types are monitored. In this case, trade can be based on gift giving. The process unfolds as follows. Inputs are exchanged without any immediate reward: they are handed out as gifts. A gift is given out by an agent in the expectation of receiving a gift in return by somebody else in the future. The agents are induced to participate in the gift exchange arrangement by the threat of being excluded from it. Should an agent refuse to give a gift at some point, he would be punished most severely by all other agents who would, thenceforth, refuse to trade with him. The monitoring of all agents makes the detection of a deviation and its punishment feasible. Should somebody defect from the gift exchange arrangement, the entire community would instantly know the identity of the defector and exclude him from any future dealings. Money is not accepted. As regards production, suppose all agents engage in agriculture. The measure of specialization is $\delta_0 \equiv \frac{N_0}{2\pi r} = \Delta^{-\frac{1}{2}}$. The value function for an agent is

$$V = \sigma (\delta_0 k)^\alpha - \sigma k^2 + \beta V, \quad (4)$$

since, with probability σ , the agent meets a trading partner either as a giver or a receiver of a gift, and an amount k is handed out. The amount of the input given out as a gift is determined by the following condition, which incorporates the assumption that producers obtain zero surplus,

$$-k^2 + \beta V = 0, \quad (5)$$

where the disutility of producing and the future benefit of remaining in the gift exchange arrangement is equated to the value of refusing to give the gift, which implies opting out of the system. The allocation and the value function should constitute a stationary equilibrium. Hence, agents should - at least weakly- prefer participation in the gift exchange system to inactivity, hence $V \geq 0$, and they should have no incentive to switch to manufacturing, i.e. the amount of final output obtained with agriculture should be at least as large as with manufacturing, given that k is determined by (5),

$$(\delta_0 k)^\alpha \geq \delta_0 k. \quad (6)$$

Finally, stationarity requires the population to remain constant at the original level, i.e. the gross growth rate (3) should be equal to 1,

$$n(y) = 1, \quad (7)$$

where $y = \sigma (\delta_0 k)^\alpha$. This is a *Village Regime*, *VR*.

Definition 1 *VR is a vector $(k, V, y) \in \mathbb{R}_+^3$ that satisfies (4) to (7).*

The following Proposition proves the existence of this equilibrium.

Proposition 1 *VR exists.*

Proof. (4)-(5) $\Leftrightarrow k = (\Sigma \Delta^{-\alpha})^{\frac{1}{2-\alpha}} > 0$, $V = \beta^{-1} \Sigma (\Sigma \Delta^{-1})^{\frac{\alpha}{2-\alpha}} > 0$. (6) $\Leftrightarrow \Sigma \Delta^{-1} \leq 1$, which holds, since $M \in (0, 1)$ and $\alpha < 1$. Since $y = y_{\min} \Rightarrow (7)$. ■

Since the population remains equal to N_0 , all agents can be monitored, no medium of exchange needs to be used to trade inputs and exchange can happen through reciprocal gifts. This regime emerges as an equilibrium since the initial degree of specialization is low relative to the potential expansion of specialization, implying a low productivity of the input, which, in turn, renders manufacturing less attractive than agriculture. Income per capita is rather meager, remaining at the subsistence level, consistently with the population being equal to N_0 . To summarize, this is a regime with a small population, a primitive degree of specialization and low productivity, a

productive sector based on agriculture and a mutual sharing system based on gift exchange. The productive structure is rudimentary at best, but the distribution system is quite effective.

Compare, now, the former steady state symmetric equilibrium with the feasible alternative regimes at the same population level, $N^{VR} = N_0$. The criterion adopted for the comparison is Pareto dominance. There are two possibilities: a system with the same technology of production, but a different trading system, or viceversa. In terms of exchange, the monetary system cannot dominate. The use of money implies that some trading opportunities - crucially, those in which the buyer has no money - would have to be missed, in order to preserve the incentive to acquire money. Hence, agents are treated unequally in a monetary equilibrium. The essential feature of gift exchange, instead, is that, whenever a trading opportunity arises, it is never missed and agents are treated equally. Some agents are bound to be worse off in a monetary equilibrium than in a gift exchange one. As regards production, a system with manufacturing cannot dominate, when the specialization is limited.

Proposition 2 *VR is undominated.*

Proof. In a monetary system, agents without money get $V_0 = 0 < V$. In a system with (2), the value $\beta^{-1}\Sigma(\Sigma\Delta^{-1}) \leq \beta^{-1}\Sigma(\Sigma\Delta^{-1})^{\frac{\alpha}{2-\alpha}} \Leftrightarrow \Sigma\Delta^{-1} \leq 1$. ■

3.2 Commercial Regime

Suppose that the population is $N^{CR} > N_0$. When the population is higher than N_0 , the trading partners of the N_0 monitored agents are always non-monitored agents. Gift exchange is not feasible anymore in these circumstances, since non-monitored agents cannot be punished for refusing to give gifts. Hence, the monitored agents would never see any of their gifts ever reciprocated and, knowing this, would never give any gift in the first place. Agents can still trade, though, thanks to money. At the same time, the productivity of the input is higher, since the process of specialization has improved as a consequence of the increase in the population. Suppose the agents use fiat money to trade inputs and adopt the agricultural technology to produce

output. Monetary trade works as follows. When an agent with money meets a suitable trading partner who does not hold money, offers a unit of money in exchange for an amount of the input. An agent without money may be induced to accept the offer in the expectation of being able to spend the money in the future with somebody else. A situation with both money and some gifts from the monitored to the non-monitored agents, although feasible in principle, is excluded by the symmetry of equilibrium. Next, I state formally the conditions corresponding to this system. The specialization index is $\underline{\delta} \equiv \frac{N^{CR}}{2\pi r} = \frac{N^{CR}}{N_0\sqrt{\Delta}}$. The value function for a money holder is

$$V_M = \sigma(1 - M)[(\underline{\delta}k_M)^\alpha + \beta V_P] + [1 - \sigma(1 - M)]\beta V_M, \quad (8)$$

since, with probability $\sigma(1 - M)$, the agent meets a producer of the input without money and makes an offer which, if accepted, entails the exchange of k_M units of the input for a unit of money, resulting in $(\underline{\delta}k_M)^\alpha$ units of output produced and consumed, while, with the complementary probability, the agent does not trade. The value function for a potential producer is

$$V_P = \sigma M(-k_M^2 + \beta V_M) + (1 - \sigma M)\beta V_P, \quad (9)$$

since, with probability σM , the agent meets a buyer with money, an offer is made, which, if accepted, leads to the exchange of k_M units of the input for a unit of money, resulting in a disutility k_M^2 , while, with the complementary probability, the agent does not trade. The terms of trade are determined by the buyer who makes a take-it-or-leave-it offer to the seller, who, consequently, obtains zero surplus,

$$-k_M^2 + \beta(V_M - V_P) = 0, \quad (10)$$

where the first term on the LHS reflects the disutility of producing the input and the second the future net benefit of becoming a money holder.

The allocation and the values should constitute a stationary equilibrium. For the agents to be willing to participate in the exchange process, they should obtain non-negative benefits, otherwise they would prefer to remain inactive, hence, $V_M, V_P \geq 0$. Agents should not be willing to substitute manufacturing to agriculture, i.e.

$$(\underline{\delta}k_M)^\alpha \geq \underline{\delta}k_M. \quad (11)$$

As a steady state requirement, population should remain constant, i.e.

$$n(y_M) = 1, \quad (12)$$

where $y_M = \sigma M (1 - M) (\underline{\delta} k_M)^\alpha$. This is a *Commercial Regime*, *CR*.

Definition 2 *CR is a vector $(k_M, V_M, V_P, y_M) \in \mathbb{R}_+^4$ that satisfies (8) to (12).*

Define $\underline{\mathbf{N}} \equiv N_0 \Delta^{\frac{1}{2}}$, $\overline{\mathbf{N}} \equiv N_0 (\Delta \Psi)^{\frac{1}{2}}$ and $\underline{\Theta} \equiv \frac{y_M}{y_{\min}}$. Notice that $\overline{\mathbf{N}} > \underline{\mathbf{N}}$, since $\Psi > 1$, and $\underline{\mathbf{N}} > N_0$, since $\Delta > 1$. The next Proposition shows the necessary and sufficient conditions for the existence of the Commercial Regime.

Proposition 3 *CR exists iff $N^{CR} \in (\underline{\mathbf{N}}, \overline{\mathbf{N}}]$ and $\theta \in (1, \underline{\Theta})$.*

Proof. (8)-(10) $\Leftrightarrow k_M = [\Psi^{-1} \underline{\delta}^\alpha]^{1/(2-\alpha)} > 0$, $V_M = \beta^{-1} [\Psi^{-1} \underline{\delta}^\alpha]^{2/(2-\alpha)} > 0$, $V_P = 0$.
(11) $\Leftrightarrow \underline{\delta} \leq \Psi^{\frac{1}{2}} \Leftrightarrow N^{CR} \leq \overline{\mathbf{N}}$. $\theta \in (1, \underline{\Theta}) \Leftrightarrow (12)$. $N^{CR} > \underline{\mathbf{N}} \Leftrightarrow y_M > y_{\min} \Leftrightarrow \underline{\Theta} > 1$.

■

An intermediate population level leads, at this stage, to a moderate degree of specialization which entails some improvements in the productivity of the input. On the other hand, gift exchange is not feasible, since the new generations of agents cannot be monitored, and the economy has to resort to fiat money. In a monetary economy, the transactions in which the buyer has no money, have to be foregone, to preserve the incentive to acquire it in the first place. The agents adopt the agricultural technology, as in the previous regime. The improvement in the productivity of capital is not large enough to lead to a switch to the industrial technology, but guarantees an intermediate level of income per capita, which is enough to sustain an intermediate population level. The amount of output produced in each transaction is higher than in the first regime, because of the specialization-induced productivity gains, but the number of transactions in the economy is lower, since the exchange system is less effective than in the first regime. If the specialization effect is sufficiently strong, which occurs when the population has increased enough, total output per person, i.e. income per capita, is higher than in the previous regime. For the population to remain constant at the equilibrium income per capita, the demographic transition should

not happen for too large values of income per capita. To summarize, the second regime is characterized by an intermediate population size, with an intermediate degree of specialization and productivity, with agriculture and a trading system based on money. The productive sector is more developed than in the previous regime, but the trading system is less effective.

Assume that Proposition 3 holds, and compare the commercial regime with the alternative one, for the same population, namely, monetary trade with manufacturing. This is clearly dominated, since the process of specialization has not advanced enough to make the adoption of manufacturing attractive.

Proposition 4 *CR is undominated.*

Proof. In the alternative regime: buyers get $\beta^{-1} [\Psi^{-1}\underline{\delta}]^2 \leq \beta^{-1} [\Psi^{-1}\underline{\delta}^\alpha]^{\frac{2}{2-\alpha}} \Leftrightarrow \underline{\delta} \leq \Psi^{\frac{1}{2}}$; sellers are indifferent. ■

3.3 Industrial Regime

Consider now a scenario where population is at an even higher level, say N^{IR} . The only difference with respect to the previous regime is that manufacturing is adopted instead of agriculture. The specialization index is $\bar{\delta} \equiv \frac{N^{IR}}{2\pi r} = \frac{N^{IR}}{N_0\sqrt{\Delta}}$. The value functions have the same interpretation as in the commercial regime. The value function for a money holder is

$$V_m = \sigma(1 - M)(\bar{\delta}k_m + \beta V_p) + [1 - \sigma(1 - M)]\beta V_m, \quad (13)$$

while the value function for an agent without money is

$$V_p = \sigma M(-k_m^2 + \beta V_m) + (1 - \sigma M)\beta V_p. \quad (14)$$

A seller obtains zero surplus,

$$-k_m^2 + \beta(V_m - V_p) = 0. \quad (15)$$

Equilibrium requires the value functions of the agents to be non-negative, to guarantee participation. Also, agents should not be willing to give up the manufacturing

technology for the alternative one,

$$\bar{\delta}k_m \geq (\bar{\delta}k_m)^\alpha. \quad (16)$$

Moreover, the population should remain constant, in a steady state,

$$n(y_m) = 1, \quad (17)$$

where $y_m = \sigma M(1 - M)\bar{\delta}k_m$. This equilibrium is an *Industrial Regime*, *IR*.

Definition 3 *IR is a vector $(k_m, V_m, V_p, y_m) \in \mathbb{R}_+^4$ that satisfies (13) to (17).*

Define $\bar{\Theta} \equiv \frac{y_m}{y_{\min}}$. The following Proposition shows the necessary and sufficient conditions for the existence of the Industrial Regime.

Proposition 5 *IR exists iff $N^{IR} \geq \bar{N}$ and $\theta \in (1, \bar{\Theta})$.*

Proof. (13)-(15) $\Leftrightarrow k_m = \Psi^{-1}\bar{\delta} > 0$, $V_m = \beta^{-1}(\Psi^{-1}\bar{\delta})^2 > 0$, $V_p = 0$. (16) $\Leftrightarrow \bar{\delta} \geq \Psi^{\frac{1}{2}} \Leftrightarrow N^{IR} \geq \bar{N}$. $\theta \in (1, \bar{\Theta}) \Leftrightarrow (17)$. $y_m > y_{\min} \Rightarrow \bar{\Theta} > 1$. ■

A large population fosters specialization and the productivity of the input, which, in turn, creates the incentive to adopt the manufacturing technology together with a trading sector based on money. The productive system has undergone dramatic advances but the trading system works still less smoothly than with gift exchange. Overall, the productivity improvement is large enough to give rise to a level of income per capita which is higher than in the other regimes.

Compare this regime, when it exists, with the alternative one, where the agricultural technology is adopted and trade is conducted with money, at the same population level. Manufacturing always dominates agriculture, since the process of specialization has developed sufficiently in this regime.

Proposition 6 *IR is undominated.*

Proof. In the alternative regime: buyers get $\beta^{-1}(\Psi^{-1}\bar{\delta}^\alpha)^{\frac{2}{2-\alpha}} \leq \beta^{-1}(\Psi^{-1}\bar{\delta})^2 \Leftrightarrow \bar{\delta} \geq \Psi^{\frac{1}{2}}$; sellers are indifferent. ■

3.4 Coexistence of the Three Regimes

The three regimes can be shown to coexist, under two assumptions. Assume that the demographic transition happens for values of income per capita which are not too high.

A1 $\theta \in (1, \underline{\Theta})$.

Assume, also, that the population is at an intermediate and high level in the commercial and industrial regime, respectively.

A2 $N^{IR} \geq \bar{N} \geq N^{CR} > \underline{N}$.

The next Proposition proves the coexistence, assuming A1 and A2.

Proposition 7 *VR, CR and IR exist and are undominated.*

Proof. A2 \Rightarrow $\underline{\Theta} < \bar{\Theta}$. All the requirements in Propositions 1-6 hold. ■

Hence, the three equilibria identified above could be observed simultaneously in different societies, rather than for the same society in different time periods. Three societies, at three different levels of income per capita, with three different population and specialization levels, coexist side by side. The first represents a tightly knit, rural village in which people share their produce communally; the second, a more anonymous commercial society still largely based on agriculture; the third one, an industrial society. It is interesting to notice that the village society is more egalitarian than the commercial and industrial one. Indeed, in the latter two, traders with and without money are treated differently in equilibrium, while in the former all traders are treated identically. A welfare comparison across regimes would be conceptually problematic since it would have to involve different population levels and, thus, a comparison of the welfare of agents who are not even all alive in all regimes. It is clear, however, that, although output is larger in the third regime, the full potential is not achieved even then, since the trading system is not entirely effective.

4 Extensions

In this section, I examine three extensions of the basic framework. In the previous part of the paper, I have considered only trading systems based on public money. Throughout history, systems based on private money have been frequently observed. The first extension features one such system, in which the agents are allowed to print circulating private instruments. Next, I suggest a way to capture the improvements in transportation technologies often observed in the course of industrialization. Finally, I consider an extension where information technologies, that become available at high income levels, make it feasible to develop new credit instruments.

4.1 Private Money

Suppose every agent can issue uniform, indivisible banknotes which differ from the banknotes issued by other agents, as in Ricardo Cavalcanti and Neil Wallace (1999). Consider a scenario where population is at a level N^{PM} , larger than N_0 . Suppose that the N_0 monitored agents issue banknotes every time they meet the suppliers of their input who are not already holding a banknote. The inputs are handed out in exchange for the banknotes which will be used in subsequent purchases. The banknotes - all treated symmetrically- circulate in the economy until, eventually, somebody will spend them with some other monitored agents, who will produce in exchange for the banknotes and withdraw them from circulation, thus placing a limit on the amount of banknotes in circulation at any given time. Monitored agents can be made to redeem banknotes, since their activities are monitored. Should they refuse to redeem banknotes, they would face punishment in the form of permanent exclusion from trade. There are B banknotes in circulation per specialization type. No other medium of exchange is accepted. In particular, non-monitored agents' banknotes are not accepted, since non-monitored agents have an incentive to issue a banknote whenever a trading opportunity arises. Should their banknotes be accepted, they would never have any incentive to produce and acquire one. Fiat money is not accepted either. In a symmetric equilibrium there will be no gift giving alongside

trade with banknotes, as before. All agents adopt manufacturing. The interpretation of the value functions is similar to the previous cases. The specialization measure is $\tilde{\delta} \equiv \frac{N^{PM}}{2\pi r} = \frac{N^{PM}}{N_0\sqrt{\Delta}}$. The value function when a non-monitored agent holds a banknote is

$$U_B = \sigma(1-B) \left(\tilde{\delta}k_B + \beta U_P \right) + [1 - \sigma(1-B)] \beta U_B. \quad (18)$$

The value function for a non-monitored agent without a banknote is

$$U_P = \sigma B \left(-k_B^2 + \beta U_B \right) + (1 - \sigma B) \beta U_P. \quad (19)$$

A seller obtains zero surplus,

$$-k_B^2 + \beta(U_B - U_P) = 0. \quad (20)$$

A monitored agent expects to obtain

$$U = \sigma(1-B) \tilde{\delta}k_B - \sigma B k_B^2 + \beta U, \quad (21)$$

since, with probability $\sigma(1-B)$, meets a seller of the input without a banknote, issues a banknote, buys the input, while, with probability σB , produces for a banknote and withdraws it from circulation. Notice that, although, in principle, the amounts produced in meetings involving non-monitored agents only or both non-monitored and monitored agents should be different, in a symmetric equilibrium, they have to be the same. Stationarity requires that, at each point in time, the amount of banknotes created equals the amount destroyed,

$$\sigma(1-B) = \sigma B, \quad (22)$$

since with probability $\sigma(1-B)$ monitored agents meet an agent without a banknote and willing to accept one and with probability σB they meet an agent with a banknote, redeem it and dispose of it. Equation (22) immediately implies $B = \frac{1}{2}$. Equilibrium requires the value functions of the agents to be non-negative, to guarantee participation. To make sure that monitored agents are willing to redeem their banknotes, the following incentive condition needs to hold,

$$-k_B^2 + \beta U \geq 0, \quad (23)$$

since redeeming a banknote requires a monitored agent to produce, which gives rise to a disutility, but allows him to remain in business, while the punishment for not redeeming banknotes consists in the permanent exclusion from trade. Moreover, the expected value of a monitored agent should be higher than the expected value for a non-monitored agent,

$$U \geq BU_B + (1 - B)U_P, \quad (24)$$

otherwise the agent would want to act as a user rather than an issuer of banknotes. Also, agents should not be willing to give up the manufacturing technology for the alternative one,

$$\tilde{\delta}k_B \geq \left(\tilde{\delta}k_B\right)^\alpha. \quad (25)$$

Moreover, the population should remain constant,

$$n(y_B) = 1, \quad (26)$$

where $y_B = \frac{\sigma}{4} \left(1 + \frac{N_0}{N^{PM}}\right) \tilde{\delta}k_B$. This equilibrium is a *Private Money Regime*, *PM*.

Definition 4 *PM is a vector $(k_B, U_B, U_P, U, y_B, B) \in \mathbb{R}_+^6$ that satisfies (18) to (26).*

Let $\Phi \equiv 2\lambda + 1$, $\tilde{\mathbf{N}} \equiv N_0 (\Delta\Phi)^{\frac{1}{2}}$ and $\tilde{\Theta} \equiv \frac{y_B}{y_{\min}}$. Notice that $\tilde{\mathbf{N}} > \underline{\mathbf{N}}$, since $\Phi > 1$. The following Proposition shows under what conditions this equilibrium exists.

Proposition 8 *PM exists iff $N^{PM} \geq \tilde{\mathbf{N}}$ and $\theta \in (1, \tilde{\Theta})$.*

Proof. (18)-(20) $\Leftrightarrow k_B = \Phi^{-1}\tilde{\delta} > 0$, $U_B = \beta^{-1} \left(\Phi^{-1}\tilde{\delta}\right)^2 = U > 0$, $U_P = 0$. Hence, (23) \Leftrightarrow (20), and, (24) $\Leftrightarrow U \geq 0$. (25) $\Leftrightarrow \tilde{\delta} \geq \Phi^{\frac{1}{2}} \Leftrightarrow N^{PM} \geq \tilde{\mathbf{N}}$. $\theta \in (1, \tilde{\Theta}) \Leftrightarrow$ (26). $y_B > y_{\min} \Rightarrow \tilde{\Theta} > 1$. ■

A large population fosters specialization and the productivity of the input, which, in turn, creates the incentive to adopt the manufacturing technology in this regime. Some agents - those who can be monitored- act as private intermediaries, issuing trade instruments that circulate in the economy, and agreeing to redeem each other's obligations. Monitoring, whose role reemerges here, makes it possible to sanction intermediaries who refuse to stick to the agreement. Non-monitored agents accept

banknotes as payment in the expectation that others will do the same in the future. Per capita income is higher than in the monetary regimes, since the productivity is higher, thanks to a more specialized economy, and the trading system works better. Indeed, unlike in the commercial regime with public money, the amount of banknotes is always the "right" one for the users, and the issuers trade more often than in the monetary case. To summarize, this regime features a large population, high degree of specialization and high productivity, with manufacturing and a banking sector that provides the economy with circulating instruments that facilitate transactions. The productive system has undergone dramatic advances and the trading system is more effective than with money, although it works still less smoothly than with gift exchange.

4.2 Transportation Technologies

So far, the probability with which the agents meet a trading partner has been assumed to be exogenous and constant. The model can easily be extended to make the probability endogenous and, specifically, depend on the distance between adjacent types, $\sigma(d_t)$. The effect of an increase in population and specialization on the chance of meeting a trading partner could go either way. The increase in the number of specialization types over time might make a single coincidence meeting less likely. On the other hand, the closer trading partners get to each other, as it happens in the model, the higher might become the probability of a single coincidence meeting. When the latter effect prevails, and, thus, $\sigma(d_t)$ is decreasing in d_t , the model generates the prediction that the agents are more likely to find suitable trading partners as income per capita increases, capturing the improvements in transportation technologies observed in economies with higher levels of income per capita. The previous results continue to hold, with minor changes.

4.3 Information Technologies

It has been noted above that there is room to improve upon the allocation reached in the industrial regime, since the trading system is not fully effective due to the limitations in monitoring. The model can be extended to consider an information technology that may allow the economy to reach a better monitoring of the otherwise non-monitored agents in a way similar to Narayana Kocherlakota and Wallace (1998). Suppose a meeting of agents can be monitored every period with probability φ_t , where φ_t is assumed to depend on income per capita, for instance, as follows

$$\varphi_t = \begin{cases} 0 & y_t \in [y_{\min}, \lambda y_{\min}), \\ \phi & y_t \in [\lambda y_{\min}, \infty), \end{cases} \quad (27)$$

with $\phi \in (0,1)$. This technology is available only at a sufficiently high level of income per capita since it is costly to develop: λy_{\min} in (27) is higher than the income per capita in the third regime, i.e. $\lambda \in \left(\frac{y_m}{y_{\min}}, \infty\right)$. In such a world, when income is sufficiently high, the trades of agents can be monitored, at least with some probability, and a form of credit, based on the monitoring of transactions, may be viable. Specifically, the monitoring technology works as follows. Every period, any meeting between two agents is either monitored or not. When the agents trade in a monitored meeting, their actions are recorded and revealed to everybody else in the economy, becoming common knowledge, while the actions in a non-monitored meeting remain private knowledge of the two agents in the meeting. Whether a meeting is monitored or not is revealed at the beginning of the period. Agents whose actions are going to be common knowledge in the current period, may be induced to produce without any immediate payment, i.e. on credit. An agent who expects to receive goods on credit in the future, may be willing to give goods on credit now. The threat of being excluded from the credit arrangement, provides the reason to oblige, assuming the identity of a defector remains common knowledge indefinitely. In turn, the threat is credible when the actions are monitored. When the actions are not monitored, the threat of exclusion is not credible, since nobody would know what the agent has done at that particular point in time, hence, punishment could not be

carried out. In those situations, money turns out to be useful. Let $\widehat{\delta} \equiv \frac{N^{IT}}{2\pi r} = \frac{N^{IT}}{N_0\sqrt{\Delta}}$. The value for an agent with money is

$$U_1 = \phi\sigma \left(\widehat{\delta}k_c - k_c^2 \right) + (1 - \phi)\sigma(1 - M) \left[\widehat{\delta}k_1 + \beta(U_0 - U_1) \right] + \beta U_1, \quad (28)$$

since, with probability ϕ , the agent is in a monitored meeting and trade happens on credit, while, with the complementary probability, the meeting is non-monitored and the agent can spend money for an input; the value for an agent who does not hold money is

$$U_0 = \phi\sigma \left(\widehat{\delta}k_c - k_c^2 \right) + (1 - \phi)\sigma M \left[-k_1^2 + \beta(U_1 - U_0) \right] + \beta U_0, \quad (29)$$

with a similar interpretation; the amount traded in non-monitored meetings are determined by the take-it-or-leave-it offers from the buyers,

$$-k_1^2 + \beta(U_1 - U_0) = 0. \quad (30)$$

As regards the amount traded in monitored meetings, even the efficient amount of trade could be achieved, i.e. the one that satisfies the equality of marginal productivity and marginal cost,

$$\widehat{\delta} = 2k_c. \quad (31)$$

The agents should prefer manufacturing to agriculture,

$$\widehat{\delta}k_i \geq \left(\widehat{\delta}k_i \right)^\alpha, \quad (32)$$

for $i = c, 1$. Finally, population should remain constant, in a stationary equilibrium,

$$n(\widehat{y}) = 1, \quad (33)$$

where $\widehat{y} = \sigma \left[\phi\widehat{\delta}k_c + (1 - \phi)M(1 - M)\widehat{\delta}k_1 \right]$. This equilibrium is an *Information Technology Regime, IT*.

Definition 5 *IT is a vector $(k_c, k_1, U_1, U_0, U, \widehat{y}) \in \mathbb{R}_+^6$ that satisfies (28) to (33).*

Let $\Omega \equiv \max \left\{ 2, \frac{\Lambda + (1 - \phi)(1 - M)}{(1 - \phi)(1 - M)} \right\}$, $\widehat{\mathbf{N}} \equiv N_0(\Delta\Omega)^{\frac{1}{2}}$ and $\widehat{\Theta} \equiv \frac{\widehat{y}}{y_{\min}}$. Notice that $\widehat{\mathbf{N}} > \overline{\mathbf{N}}$, since $\Omega > \Psi$. The next Proposition shows under what conditions this equilibrium exists.

Proposition 9 *IT exists iff $N^{IT} \geq \widehat{\mathbf{N}}$ and $\theta \in (1, \widehat{\Theta})$.*

Proof. (28)-(31) $\Leftrightarrow k_1 = \frac{(1-\phi)(1-M)\widehat{\delta}}{\Lambda+(1-\phi)(1-M)} > 0, k_c = \frac{\widehat{\delta}}{2} > 0, U_0 = \frac{\phi\sigma\widehat{\delta}^2}{(1-\beta)^4} > 0, U_1 = U_0 + \frac{(1-\phi)(1-M)\widehat{\delta}^2}{\beta[\Lambda+(1-\phi)(1-M)]}$. (32) $\forall i \Leftrightarrow \widehat{\delta} \geq \Omega^{\frac{1}{2}} \Leftrightarrow N^{IT} \geq \widehat{\mathbf{N}}$. $\theta \in (1, \widehat{\Theta}) \Leftrightarrow (33)$. $\widehat{y} > y_m > y_{\min} \Rightarrow \widehat{\Theta} > 1$. ■

This is an advanced regime where the development of information technologies, obtained at high income levels, makes it feasible to achieve very good allocations, in which the use of media of exchange plays an ancillary role to credit, while adopting the manufacturing technology. Should the monitoring technology become extremely sophisticated - analytically, when $\phi \rightarrow 1$, all trade would be based on credit and the allocation would be the efficient one.

5 Conclusion

In the model presented above, the population plays a dual role in fostering productivity and limiting monitoring. A larger economy has higher levels of specialization which, in turn, increases the productivity of the inputs, relative to a smaller economy. Since specialization exhibits decreasing returns in agriculture, but constant in manufacturing, any improvement in specialization has progressively smaller effects on agriculture but always the same on manufacturing. Hence, eventually, manufacturing ends up dominating, when the economy is sufficiently large and specialized. At the same time, though, a larger economy has a harder time monitoring its members, than a smaller one. Gift exchange is replaced by money in larger societies. Thus, the model predicts a prevalent role of manufacturing relative to agriculture and cash relative to mutual sharing, at higher levels of income per-capita. Notice, finally, that the result, obtained by Luis Araujo and Braz Camargo (2012) in a similar environment, that trade based on a social norm would outperform trade based on media of exchange, for a sufficiently large population, does not apply to the present environment, since the proportion of monitored to non-monitored agents *shrinks* as the population increases.

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