Ambiguous Policy Announcements

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Abstract

We study the effects of monetary policy announcements in a New Keynesian model, where ambiguity-averse households with heterogenous net financial wealth use a worst-case criterion to assess the credibility of announcements. The announcement of a future loosening of monetary policy leads to the rebalancing of financial asset positions, it can cause credit crunches, and it may prove to be contractionary in the interim before implementation. This is because the households with positive net financial wealth (creditors) are those that are most likely to believe the announcement, due to the potential loss of wealth from the prospective policy easing. And when creditors believe the announcement more than debtors, their expected wealth losses are larger than the wealth gains that debtors expect. So aggregate net wealth is perceived to fall, and the economy can contract owing to lack of aggregate demand, which is more likely when the inequality in wealth is more pronounced. We evaluate the importance of this mechanism, focusing on the start of the ECB's practice of offering forward guidance in July 2013. The inflation expectations of households have responded in accordance with the theory. After matching the entire distribution of European households' net financial wealth, we find that the ECB's announcement is contractionary in our model. In general, redistributing expected wealth may have perverse effects when agents are ambiguity-averse.

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1 Introduction

Policy makers often use announcements of future reform of economic institutions or changes in fiscal or monetary policy to stimulate the economy in the short run. These policies typically carry important redistributive implications. For example during the Great Recession, with nominal short-term interest rates at the zero lower bound, central banks have relied extensively on announcements of future monetary policy changes to raise current inflation and stimulate the economy, a practice generally known as *forward guidance*. And it is well known that inflation tends to redistribute wealth from creditors to debtors (Fisher 1933, Doepke and Schneider 2006, and Adam and Zhu 2015). In this paper we show that when agents are ambiguity-averse, policy announcements can have unintended effects in the period before the new policy is actually implemented. Generally the effect of the announcement depends on (i) the amount of redistribution that the policy change will induce, (ii) the concentration of future hypothetical wealth losses, and (iii) the (endogenous) correlation between agents' wealth and the change in their expectations with the announcement.

We consider the impact of monetary policy announcements in a New Keynesian model, where ambiguity-averse households holding different amounts of net financial wealth use a worst-case criterion in assessing the credibility of announcements, according to the Maximin preference specification proposed by Gilboa and Schmeidler (1989). Ambiguity aversion is a natural paradigm for characterizing the behavior of households who do not know the probability distribution of outcomes, which is likely when they have to deal with news about unfamiliar contingencies, such as announcements about future unconventional policies in an unusual economic environment. We analyze the effects of announcements of future changes in real interest rates, owing to changes in inflation or nominal rates, in an economy that is initially in a steady-state equilibrium. Creditors are more likely than debtors to believe the announcement of a future monetary easing, because their worst-case scenario—at least for those with sufficiently great financial wealth— is that real rates, hence financial wealth, will actually fall. And if creditors ascribe greater credibility to the announcement than debtors do, the wealth losses they expect to incur are larger than the gains that debtors expect, so expected aggregate net wealth falls. We call this the *forward misquidance* effect, which can be strong enough to dominate the conventional substitution effect and thereby to lead to a contraction in activity due to lack of aggregate demand. The decline in demand is more probable when wealth inequality is great so that the capital losses induced by future monetary policy are concentrated among a small group of wealthy households. Generally, when a policy easing is announced, the new real rate expected by creditors is lower than that expected by debtors. This produces a rebalancing in the financial asset positions and can even cause credit crunches, which happen because agents undo their positions in order

to be fully insured against future monetary policy changes.

In the case of an announcement of a future monetary policy tightening (a rise in future real rates), debtors are the most likely to take the announcement as credible and for them the increase in future rates reduces consumption through both substitution and income effects. So aggregate consumption and output unambiguously fall. When wealth inequality is sufficiently marked, the fall in output is sharper than it would be in a hypothetical equilibrium in which the announcement is fully believed by all agents.

We evaluate the importance of this mechanism by focusing on the start of forward guidance by the ECB on 4 July 2013.¹ After the announcement, long-term government bond yields and EONIA swap rates fell by 10-15 basis points at maturities between 2 and 4 years (see Coeuré (2013), ECB (2014), and Section 5, below). There is evidence that households' inflation expectations have responded according to the *misguidance* effect. Figure 1 shows the evolution of households' expected inflation in panel (a) and realized inflation in panel

Figure 1: Forward Guidance in the Euro 11: Expected and realized inflation



(a) Expected change in trend inflation

(b) Actual Inflation

Notes: Core Inflation is yearly log differences in consumer prices excluding energy and seasonal food multiplied by 100. Expectations are calculated in terms of balances—differences between respondents giving positive and negative answers. Price expectations come from the following question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall". Probabilities are calculated in terms of balances: differences between those saying that the answer is very likely versus those saying the answer is unlikely. Price expectations are calculated as equal to $(f_i + 1/2f_{ii} - 1/2f_{iv} - f_v) \times 10$, where f_j is the fraction of individuals who opted for option j = i, ii, iii, iv, v in the survey. Creditors are Austria, Finland, Germany, Luxembourg, and Netherlands. Debtors include all the remaining countries in the group of Euro 11 countries. Source of data: ECB, Joint Harmonized Programme of Business and Consumer Surveys by European Commission and External Wealth of Nation Mark II (EWN).

¹On that date the ECB Governing Council announced that "it expected the key ECB interest rates to remain at present or lower levels for an extended period of time."

(b) in the Euro 11 countries, separately for creditor countries (dashed blue line) and debtor countries (dotted green line). We define creditor countries as those with a positive net foreign asset position at the time of the ECB announcement—Austria, Finland, Germany, Luxembourg, and the Netherlands. The debtor countries are all the others. The difference between the two lines corresponds to the solid red line in the figure. The area in grey identifies the quarter in which the announcement was made. Inflation kept falling after the announcement, which is consistent with the relatively muted effects of forward guidance estimated for the US (Del Negro, Giannoni, and Patterson 2015). Interestingly, after the announcement, inflation kept falling faster in the creditor than in the debtor countries (panel (b)). Still, relative to trend, inflation expectations have been revised upwards only in creditor countries, remaining stable in debtor countries (panel (a)). As a result the difference in inflation expectations between creditor and debtor countries has increased, and together with the foregoing evidence on actual inflation this implies that the wedge between expected and realized future inflation has widened in creditor countries relative to debtor countries.

We provide further evidence of the misguidance effect by using a Difference-in-Differences identification strategy based on data for Italian provinces, which differ substantially in their net financial wealth and are subject to the same country-specific shocks. We exploit a unique quarterly dataset on both expected and realized inflation at a highly disaggregated level. For each province we construct a measure of the inflation expectation bias of households by calculating the difference between their current expectations of future inflation and realized future inflation. We classify provinces according to the fraction of households with positive net financial wealth (creditor households). In response to the ECB announcement, the provinces with more creditor households experience a relative increase in their inflation expectation bias: roughly, a one-standard deviation increase in the share of creditor households is associated with an after-the-announcement relative increase of 9 basis points in the expectation bias.

To study the quantitative importance of this effect, we calibrate our heterogenous-agents new Keynesian model to match the data from the Household Finance and Consumption Survey (HFCS) on the entire distribution of European households' net financial wealth. We calibrate the ECB announcement to match the response on 4 July 2013 of the yield curve of interest rates at all maturities from 2 and 10 years. We find that in our model the effect of the ECB announcement on output is considerably muted by comparison with the standard benchmark situation in which all households accord full credit to the announcement. Under our preferred parametrization, indeed, the announcement is contractionary, with a cumulative output loss of 1.6% in the fifteen months before implementation, against a gain of almost 6% under the standard benchmark. The literature. Forward guidance has become a central tool of monetary policy as a result of the Great Recession, because conventional monetary expansion was no longer available, with short-term rates at the zero lower bound. There is a growing literature on optimal monetary policy in a liquidity trap (Eggertsson and Woodford 2003) as well as on the effects of forward guidance (Del Negro, Giannoni, and Patterson 2015, Swanson 2016). For conventional new Keynesian sticky-price models it is a puzzle why forward guidance has been so ineffective in stimulating the economy and getting it out of the liquidity trap. Some papers have proposed explanations for this puzzle, see Andrade, Gaballo, Mengus, and Mojon (2015), Caballero and Farhi (2014), Kaplan, Moll, and Violante (2016b), McKay, Nakamura, and Steinsson (2015) and Wiederholt (2014). In this paper we abstract from the reason why the monetary authority has come to rely on announcements. We observe that ambiguity aversion can contribute to solving the forward guidance puzzle since announcements of a future monetary easing are more likely to affect the expectations of agents with high net financial wealth than those of agents with negative wealth, and because of this the economy may even contract in the presence of high financial imbalances.

At least since Fisher (1933) it has been known that expansionary monetary policy redistributes wealth from creditors to debtors. It has also been observed that such redistribution could expand aggregate demand because agents may differ in marginal propensity to consume out of wealth (as first posited by Tobin (1982)), or in portfolio liquidity or term structure, as in Kaplan, Moll, and Violante (2016a) and Auclert (2015) respectively. However, Doepke, Schneider, and Selezneva (2015) postulate an overlapping-generations model in which this redistribution decreases aggregate consumption. Here we focus on the redistribution of expected wealth induced by news about future policies, which, under ambiguity aversion, is a negative-sum game because the net losers tend to believe the news more strongly than the net winners.

Other papers have shown the relevance of ambiguity aversion to business cycle analysis. Ilut and Schneider (2014) show that shocks to the degree of ambiguity can be an important source of cyclical volatility. Ilut, Valchev, and Vincent (2016) study the implications of ambiguity aversion for sticky prices. Backus, Ferriere, and Zin (2015) examine asset pricing, and Ilut, Krivenko, and Schneider (2016) devise methods suitable for dynamic economies where ambiguity-averse agents differ in their perception of exogenous shocks and study the implications for precautionary savings, asset premiums and insurance gains. Here instead we focus on the effects of announcements, and more generally news, about the future and how they interact with wealth inequality and redistribution.

Recent research has emphasized that the inflation expectations of agents are quite heterogeneous, see for example Coibion, Gorodnichenko, and Kumar (2015). Here we show that changes in agents' inflation expectations are related to their wealth position, which arises as a natural consequence of aversion to ambiguity.

Section 2 sets out some evidence. Section 3 characterizes the economy. Section 4 studies the effect of monetary policy announcements in a simple case. Section 5 quantifies the effect of the ECB announcement on July 2103, Section 6 studies robustness, and Section 7 concludes. The Appendix contains details on data and model computation.

2 Additional evidence of the misguidance effect

Figure 1 indicates that in response to the ECB's forward guidance announcement households' inflation expectations responded more sharply in creditor than in debtor countries, which is evidence consistent with a *misquidance* effect. However, aggregate national data provide no information on the distribution of net financial assets within countries. Moreover, asymmetric country-specific shocks have played a crucial role in the crisis. To address these concerns, we rely on a Difference-in-Differences identification strategy, exploiting quarterly data on realized and expected inflation in Italian provinces. Italian provinces vary substantially in the net financial wealth of households and by construction are subject to the same country-specific shocks. We construct a measure of the average inflation expectation bias of households within a province by calculating the difference between agents' expectations for inflation and subsequent realized inflation in the province. We test whether, in response to the ECB announcement, the expectation bias of households increased more in provinces with a larger fraction of creditor households. This, as we will see, is a major implication of the model described in Section 3 if one allows the fraction of creditor households to vary. We next discuss briefly the construction of the dataset; for full details, see the Appendix.

2.1 Data

Our province-level data come from three different sources: realized inflation is from the official Italian statistical institute (ISTAT); expected inflation is from confidential data collected in the Survey on Inflation and Growth Expectations conducted by the Bank-of-Italy and *il Sole 24 Ore*; Net Financial Assets (NFA) of households are calculated using the Bank of Italy's Survey of Household Income of Wealth (SHIW). The data are quarterly and our regressions cover the sample period 2012:I-2014:II. The end of the sample is dictated by the start of the ECB's Quantitative Easing program in 2015:I. Realized inflation in each province is the yearly log-difference of the provincial general price index, which is consistent with the ECB practice of monitoring price stability annually. Expected inflation is measured two quarters ahead, averaging the estimates of all survey respondents in the province. The NFA of a household is calculated as the difference between total holdings of

money, postal deposits, saving certificates, pension funds, government securities, and other securities less all financial liabilities to banks or other financial institutions. A creditor household is one with positive NFA. For each province we calculate the pre-announcement fraction of creditor households, increasing sample size by using the last two waves of SHIW before the ECB announcement, which refer to 2010 and 2012. Table 1 describes our sample. The average NFA of Italian households is 14,589 Euros at constant 2010 prices. The fraction

 Table 1: Descriptive statistics

	(1)	(2)	(3)	(4)	(5)
VARIABLES	mean	sd	Ν	\min	max
Pre-announcement fraction of creditor households	0.67	0.13	1078	0.26	0.97
Pre-announcement fraction of creditor households divided by SD, F_i	5.47	1	1078	2.56	7.75
Inflation rate in province π_{it}	1.77	1.24	1078	-0.47	4.76
Two quarters ahead expected inflation, $E_{it}[\pi_{it+2}]$	2.02	1.23	1078	-10	8.72
Two quarters ahead realized inflation, π_{it+2}	1.15	1.16	1078	-9.62	4.53
Inflation expectation bias, $\hat{\pi}_{it}$	0.86	0.74	1078	-3.61	6.79
Year	2012.80	0.75	1078	2012	2014

Notes: Quarterly data over the sample period 2012:I-2014:II. Realized inflation comes from ISTAT. Data on expected inflation are based on confidential data from the Bank of Italy-Sole 24Ore survey on expectations. The Net Financial Asset position of households is calculated using the 2010 and 2012 waves of the Survey of Household Income of Wealth (SHIW).

of all Italian households with positive NFA is 67%, but it varies considerably by province, from 26% to 97%. In each province *i* and quarter *t*, we calculate the difference between expected inflation, $E_{it}[\pi_{it+2}]$, and realized future inflation, π_{it+2} ,

$$\hat{\pi}_{it} \equiv E_{it}[\pi_{it+2}] - \pi_{it+2},$$
(1)

which is a measure of the (average) inflation expectation bias of agents in province i at time t; $\hat{\pi}_{it}$ is positive on average in our sample.

2.2 Evidence

To evaluate whether, in response to the ECB announcement, the inflation expectation bias changed more in provinces with a higher proportion of creditor households, we run the following regression:

$$\hat{\pi}_{it} = \sum_{n=-6}^{5} \phi_n \left(F_i \times \mathbb{I}_{n=t-t_0} \right) + \beta X_{it} + \varepsilon_{it}, \qquad (2)$$

where F_i is the pre-announcement fraction of creditor households in the province, divided by its standard deviation (0.13). $\mathbb{I}_{n=t-t_0}$ is a dummy equal to one just at quarter n since the announcement which was made at time $t_0=2013$:III. The set of controls X_{it} include a full set of time dummies. The coefficient ϕ_n , whose profile is reported in Figure 2, measures the difference in the inflation expectation bias of two provinces that differ by a one-standard-deviation amount of creditor households in quarter n post-announcement. The point estimates of ϕ_n for n < 0 correspond to the blue bullet point in Figure 2, those for $n \ge 0$ to the red bullet point. Vertical bars denote the 90% confidence interval using Robust Standard Errors. The vertical dotted black line indicates the quarter of the

Figure 2: Expected inflation bias of creditor households before and after FG



Notes: Profile of the coefficient ϕ_n in regression (2). The dependent variable is the inflation expectations bias $\hat{\pi}_{it}$ in (3). The independent variable is the pre-announcement proportion of creditor households in the province divided by its standard deviation, F_i . The vertical dotted black line indicates the quarter of the announcement, n = 0. Vertical bars denote the 90% confidence interval using Robust Standard Errors.

announcement, n = 0. There is clear indication that coefficient ϕ_n increased at the time of the announcement and in the subsequent quarters; that is, the inflation expectation bias increased more in the provinces with a larger share of creditor households.

To evaluate the effect of the ECB announcement on the expectation bias $\hat{\pi}_{it}$, we can calculate the difference between the average value of the coefficient ϕ_n in the quarters after and before the shock. We follow the common practice in the Dif-in-Dif literature, measuring this difference by the following regression:

$$\hat{\pi}_{it} = \overline{\phi} F_i + \phi F_i \times \mathbb{I}_{t \ge t_0} + \beta X_{it} + \varepsilon_{it} \,. \tag{3}$$

Again F_i is equal to the (standardized) proportion of creditor households in the province. The set of controls X_{it} includes a full set of time and province dummies. $\mathbb{I}_{t\geq t_0}$ is a dummy equal to one in the quarter of the announcement and in all subsequent quarters, zero in previous quarters. The coefficient ϕ measures the average effect of F_i on the expectation bias in the province. The Difference-in-Differences coefficient ϕ measures the increase in the effect of F_i on the inflation expectation bias in the quarters after the announcement. Roughly, ϕ measures the difference between the average value of the red and the blue bullet points in Figure 2. The results from estimating (3) are reported in Table 2. Column 1 refers to the specification with no province fixed effects, column 2 after controlling for the full set of province dummies. According to these estimates, after the ECB announcement, provinces with a share of creditor households one-standard-deviation higher experienced a relative increase of around 9 basis points in their inflation expectation bias.

VARIABLES	(1)	(2)
Fraction of creditors, F_i (coefficient $\overline{\phi}$)	-0.02	-0.23
	(0.03)	(0.15)
Announcement-dummy $\times F_i$ (coefficient ϕ)	0.09^{***}	0.09^{***}
	(0.03)	(0.03)
R^2	0.35	0.49
No. of observations	1078	1078
No. of provinces	108	108
Year FE	Υ	Υ
Province FE	Ν	Υ

Table 2: Effects of FG on the inflation expectation bias of creditor households

Notes: Results from regression (3). The dependent variable is $\hat{\pi}_{it}$ in (3). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA in the province. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

3 The model

First we consider a simple economy in discrete time (extended in Section 5, for a quantitative analysis). The economy is populated by a unit mass of households, indexed by $x \in [0, 1]$, who are ambiguity-averse and differ only in net financial wealth, $a_{xt} \in [\underline{a}_t, \overline{a}_t]$, which is invested in one-period bonds. There is a unit mass of firms that demand labor to produce intermediate goods sold under monopolistic competition; prices are sticky. The nominal interest rate is adjusted continuously to achieve the inflation target set by a monetary policy authority which has an unambiguous mandate to maintain price stability. The monetary authority has always complied with this mandate, fully stabilizing prices over the years. We focus on the short run response of the economy, when the monetary authority suddenly and unexpectedly announces a future change in the inflation target, which makes households doubt whether the authority will actually deviate from its historical mandate, as announced. Hereafter the convention is that, unless otherwise specified, variables are real—measured in units of the final consumption good.

Households Household $x \in [0, 1]$ is infinitely-lived, with a subjective discount factor $\beta < 1$ and per period preferences over consumption c_{xt} and labor l_{xt} given by

$$U(c_{xt}, l_{xt}) = \frac{\left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma},$$
(4)

with $\psi_0, \psi > 0$ and $\sigma > 1$. When all households share the same beliefs, these preferences (Greenwood, Hercowitz, and Huffman 1988) guarantee that the economy is characterized by a representative household, which is a canonical benchmark in the New Keynesian literature. Financial markets are incomplete, in that households can only invest in a oneperiod bond, which, at time t, pays (gross) return r_t per unit invested. Households can borrow freely by going short on the asset. The labor market is perfectly competitive, so households take the wage w_t as given. At each point in time t, household x chooses the triple $\{c_{xt}, l_{xt}, a_{xt+1}\}$ subject to the budget constraint

$$c_{xt} + a_{xt+1} \le w_t \, l_{xt} + r_t \, a_{xt} + \lambda_t,\tag{5}$$

where a_{xt+1} measures the units invested in bonds at time t that will yield return r_{t+1} at time t + 1, while λ_t denotes (lump sum) government transfers (specified below).

Monetary policy rule The monetary authority sets the (gross) nominal interest rate R_{t+1} between time t and t+1 (paid at t+1) according to

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{\Pi_t}{\Pi_t^*}\right)^{(1-\rho_r)\phi} \left(\frac{R_t}{\bar{R}}\right)^{\rho_r},\tag{6}$$

where $\phi > 1$ and $\rho_r \in [0, 1)$, \bar{R} is the steady state interest rate, $\Pi_t \equiv p_t/p_{t-1}$ is gross inflation, and Π_t^* is the time-*t* inflation target, which is equal to one in steady state, $\bar{\Pi}^* = 1$. **Firms** The final consumption good is produced by a (representative) competitive firm, which uses a continuum of varieties $i \in [0, 1]$ as inputs according to

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}},\tag{7}$$

where y_{it} is the amount of variety *i* used in production. The variety *i* is produced only by a firm *i*, which uses a linear-in-labor production function, so that $y_{it} = \ell_{it}$, where ℓ_{it} denotes firm *i*'s demand for labor whose unit cost is w_t . Firm $i \in [0, 1]$ sets the nominal price for its variety p_{it} to maximize expected profits at the beginning of the period, $d_{it} \equiv y_{it} (p_{it}/p_t - w_t)$, taking as given the demand schedule by the competitive firm, the aggregate nominal price, p_t , and the wage rate, w_t . We assume firm *i* chooses its nominal price at time *t*, p_{it} , after the monetary authority has set the inflation target Π_t^* , but before any time-*t* policy announcement. Finally, we posit initially that the government owns all the firms in the economy and rebates profits back to households in lump-sum fashion, so that $\lambda_t = \int_0^1 d_{it} di$. We relax this assumption in Section 5.

Market clearing In equilibrium, output Y_t is equal to aggregate consumption $C_t \equiv \int_0^1 c_{xt} dx$, so that $Y_t = C_t$, and labor demand is equal to labor supply, $\int_0^1 \ell_{it} di = \int_0^1 l_{xt} dx$. Since bonds are in zero net supply, clearing the financial market requires that $\int_0^1 a_{x,t} = 0$ at the return $r_t = R_t/\Pi_t$, where the nominal interest rate R_t satisfies (6).

Steady state At t = 0, the economy is initially in a steady state, where a monetary authority with an unambiguous mandate for price stability has always set $\Pi_t^* = 1$, and households expect Π_t^* to remain equal to one also in any future t, implying $\bar{r} = \bar{R} = 1/\beta$ and $\bar{\Pi} = 1$, where the upper bar denotes the steady state value of the corresponding quantity.

Policy announcement At t = 0 (after firms have set their nominal price), the monetary authority announces that in period T > 0, and only at T, the inflation target will deviate from full price stability, implying that $\Pi_T^* = \varepsilon$, and $\Pi_t^* = 1$ for all $t \neq T$. If $\varepsilon > 1$, the announcement is *inflationary*; if $\varepsilon < 1$, it is *deflationary*. On the basis of the announcement, household $x \in [0, 1]$ makes her decisions on consumption, labor supply and saving, while firm $i \in [0, 1]$ supplies any amount demanded at its set price.

Ambiguity aversion There is ambiguity about whether the monetary authority will actually deviate from its mandate of price stability and households are ambiguity-averse. Households doubt that the monetary authority can commit to the announced inflation target at time T, Π_T^* , and do not know how to calculate the probability distribution of

 $\Pi_T^{*,2}$ Thus, $\forall t < T$, household x perceives Π_T^* as a random variable with a probability distribution about which the household must form her own subjective beliefs. To model ambiguity aversion, we rely on the multiple priors utility model of Gilboa and Schmeidler (1989), whose axiomatic foundations are provided by Epstein and Schneider (2003). The utility of household x is then given by the sum of the felicity from time-t consumption and labor plus the expected continuation utility, which is evaluated for the household's worst-case scenario on the realizations of the inflation target. Formally, we assume that preferences at time t order future streams of consumption, $\mathbf{C_t} = \{c_s(h^s)\}_{s=t}^{\infty}$, and labor supply, $\mathbf{L_t} = \{l_s(h^s)\}_{s=t}^{\infty}$, so that utility is defined recursively as

$$V_t(\mathbf{C_t}, \mathbf{L_t}) = U(c_t, l_t) + \beta \min_{\Omega \subseteq \mathcal{S}_t, \ G \in \mathcal{P}(\Omega)} \int_{\Omega} V_{t+1}(\mathbf{C_{t+1}}, \mathbf{L_{t+1}}) G(d\Pi_{t+1}^*),$$
(8)

where $h^t = \{\Pi_{-\infty}^*, ..., \Pi_{t-1}^*, \Pi_t^*\}$ denotes history up to time t, and Ω is the support of the probability distribution G that household x ascribes to the realizations of the inflation target one period ahead, Π_{t+1}^* . Expected utility arises when the household is forced to take Ω and the associated probability distribution G as given. Under ambiguity aversion, to rank the utility from future streams of consumption and labor, the household chooses a support Ω and an associated probability distribution G so as to minimize the continuation utility V_{t+1} (worst case criterion). The support Ω is chosen among the possible realizations of the inflation target at t+1, denoted by \mathcal{S}_t . A non-degenerate set of beliefs captures the household's lack of confidence in probability assessments, with a larger set implying greater ambiguity. The probability distribution G is chosen from the set of all probability distributions $\mathcal{P}(\Omega)$ that assign positive probability to all values in the support Ω . We assume that, $\forall t$, household $x \in [0, 1]$ can condition her choices to the entire history up to time t, h^t , which is fully characterized by the observed realizations of Π_t^* up to t. Household x chooses consumption plans, $c_t(h^t)$, labor supply $l_t(h^t)$ and savings $a_{t+1}(h^t)$ to maximize (8). Notice that if the realizations of the inflation target affect the consumption and labor streams of different households differently, these preferences will give rise to actions that are taken under heterogeneous beliefs.

In our experiment, the household faces ambiguity only about the realization of Π_T^* , so the set S_t is non-degenerate just at t = T - 1. In particular, we assume that $S_{T-1} = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}]$: when the announcement is inflationary, $\varepsilon > 1$, we have $S_{T-1} = [1, \varepsilon]$; when it is deflationary, $\varepsilon < 1$, we have $S_{T-1} = [\varepsilon, 1]$. The reason for this specification is that a monetary authority with a price stability mandate that has been successfully pursued over a long past history makes household x doubt whether the authority will actually deviate

²For simplicity, we assume that households face ambiguity only about the intensity of the policy implementation, ε , but not about its time horizon, T.

from that mandate, as announced, and, if so, by how much.³ There is no ambiguity about the inflation target at t < T - 1 or at $t \ge T$. So we have $S_t = 1 \quad \forall t \ne T - 1$. Finally, notice that since the set S_t is common to all households $x \in [0, 1]$, they all face the same ambiguity.⁴ We can now define an equilibrium as follows:

Equilibrium An equilibrium is a set of beliefs, quantities, and prices such that, $\forall t$,

- 1. Each household $x \in [0, 1]$ chooses c_{xt} , l_{xt} , and a_{xt+1} to maximize the utility in (8), which also determines her beliefs about the support for the next-period realizations of the inflation target, $\Omega_x \subseteq S_t$, and the associated probability distribution $G_x \in \mathcal{P}(S_t)$;
- 2. The monetary authority sets the nominal interest rate R_t as in (6);
- 3. Each firm $i \in [0, 1]$ sets the price $p_{it} = p_t$ optimally, after the inflation target for the period has been determined (but before any policy announcement);
- 4. The labor market, the goods market, and the financial market all clear at wage w_t , inflation Π_t , and return r_t .

4 Solution of the model

We start by assuming that the policy announcement at t = 0 is about the next-period inflation target Π_1^* , so that T = 1. We further assume that there are only two types of households differing only in initial financial wealth.⁵ A fraction (half) of households are *creditors*, j = c, with wealth equal to $a_{x0} = a_{c0} = B > 0 \ \forall x \in [0, 1/2]$, and the remaining fraction are *debtors*, j = d, with financial wealth $a_{x0} = a_{d0} = -B < 0, \ \forall x \in [1/2, 1]$. Here B denotes the amount of initial *financial imbalances* in the economy. First we prove two simple preliminary results that clarify the functioning of the model. Then we solve for the equilibrium in three steps: determining the allocation of the economy for given households' degenerate beliefs about Π_1^* , endogenizing beliefs by using (8), and fully characterizing the equilibrium.

 $^{^{3}}$ As will become clear below, this is a conservative assumption because any larger support implies more heterogeneity in equilibrium households' beliefs, which would generally strengthen our results.

⁴There is empirical evidence suggesting that more educated individuals and those with greater financial literacy are characterized by smaller ambiguity when investing in financial markets and dealing with financial institutions, see Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016). Here we do not allow for exogenous differences in ambiguity to better isolate the effects of wealth inequality on the formation of households' expectations, which endogenously generate heterogeneity in beliefs.

⁵Both assumptions are relaxed in the quantitative model of Section 5. To keep the notation consistent throughout the paper, we have described the economy for general T and for an arbitrary distribution of households' assets a_{xt} . In this simple model the assumption T = 1 entails only a minor loss of generality, because firms adjust prices in every period so output can respond just at t = 0. The time horizon of the announcement will matter in the quantitative model because in that case prices are adjusted slowly.

4.1 Two preliminary results

Figure 3 shows the time line of the experiment. At the announcement, t = 0, prices are predetermined at a value normalized to one, $p_0 = 1$. The analysis focuses on characterizing output at time zero, Y_0 which is determined, given sticky prices, by the saving decisions of creditors, a_{c1} , and debtors, a_{d1} . Clearing the financial market implies that $a_{c1} = -a_{d1} = B'$, where B' denotes the amount of financial imbalances at the end of period zero. In the

Figure 3: Timing

$\Pi_0^* = 1$ $p_0 = 1$ $R_0 = \bar{R}$	Announcement HHs form beliefs Y_0 and B'	$\begin{array}{c} Y_{1} = \bar{Y} \\ R_{1} = \bar{R} \\ \Pi_{1} = \Pi_{1}^{*} \\ r_{1} = \bar{R} / \Pi_{1}^{*} \end{array}$	$\begin{array}{l} Y_t = \bar{Y} \\ R_t = \bar{R} \\ \Pi_t = 1 \\ r_t = \bar{R} \end{array}$
$\mathbf{t} = 0^-$	$\mathbf{t}=0$	t=1	$ extbf{t} \geq 2$

following periods, $t \ge 1$, firm $i \in [0, 1]$ sets its price p_{it} to maximize expected profits at the beginning of period, $d_{it} \equiv y_{it} (p_{it}/p_t - w_t)$, taking as given the demand for the variety of the competitive firm, which has the conventional form:

$$y_{it} = Y_t \left(\frac{p_{it}}{p_t}\right)^{-\theta}$$

The resulting optimal nominal price is a markup over firm i's expected nominal wage:

$$p_{it} = \frac{\theta}{\theta - 1} E_{it}[w_t p_t] \quad \forall i \in [0, 1],$$
(9)

which immediately implies $p_{it} = p_t \ \forall i$. Also, since firms set their price after observing Π_t^* , pricing decisions are taken under perfect information $\forall t \geq 1$, allowing us to conclude that

$$w_t = \frac{\theta - 1}{\theta}, \ \forall t \ge 1.$$
 (10)

The utility in (4), together with the preferences in (8), further implies that the labor supply of a household of type j = c, d solves a simple static maximization problem, yielding the familiar condition

$$\psi_0 l_{jt}^{\psi} = w_t. \tag{11}$$

This implies that all households (independently of wealth and beliefs) supply the same labor, which given that aggregate labor supply equals output yields $l_{jt} = Y_t$, $\forall j$. This together with (10) and (11), immediately implies that: **Lemma 1** Output Y_t converges back to steady state at t = 1, so that $Y_t = \overline{Y} \ \forall t \ge 1$.

In the Appendix we use Lemma 1 together with the interest rate rule in (6) to prove that

Lemma 2 At any point in time $t \ge 0$, inflation is equal to the inflation target, $\Pi_t = \Pi_t^*$, and the nominal interest remains unchanged at its steady state value, $R_t = \bar{R}$.

In Section 5 we extend the model to allow for more general dynamics in nominal interest rates.⁶

4.2 Output and the financial market for given beliefs

We now solve for time-zero output Y_0 , when a household j = c, d has degenerate beliefs about the realization of Π_1^* represented by a point $\varepsilon^{\tau_j} \in S_0$ with $\tau_j \in [0, 1]$. It is useful to define $\bar{\tau} \equiv (\tau_c + \tau_d)/2$ and $\rho \equiv (\tau_c - \tau_d)/(2\bar{\tau}) \in [-1, 1]$, which are related to τ_c and τ_d as follows: $\tau_c \equiv \bar{\tau} (1 + \rho)$ and $\tau_d \equiv \bar{\tau} (1 - \rho)$. $\bar{\tau}$ measures the *average credibility* of the announcement; while ρ measures the *correlation* between households' wealth and their perception of the announcement's credibility. When $\rho > 0$, creditors believe the announcement more than debtors; and conversely when $\rho < 0$; $\rho = 0$ means that all households share the same beliefs. The problem of a household of type j = c, d at t = 0 is then given by

$$\max_{\{c_{js}, l_{js}, a_{js+1}\}_{s \ge 0}} E_{j0} \left[\sum_{s=0}^{\infty} \beta^s U(c_{js}, l_{js}) \right],$$

subject to the budget constraint in (5). The expectation operator is indexed by j, since households of different types may hold different beliefs. The first order condition for the consumption choices of household j yields the Euler condition

$$\left(c_{jt} - \psi_0 \frac{l_{jt}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta E_{jt} \left[r_{t+1} \left(c_{jt+1} - \psi_0 \frac{l_{jt+1}^{1+\psi}}{1+\psi} \right)^{-\sigma} \right],$$
(12)

which, together with the budget constraint in (5) and the labor supply condition (11), fully determines household j's consumption, c_{jt} , savings, a_{jt+1} , and labor supply, l_{jt} , $\forall t$.

Output Y_0 can be obtained using the market clearing condition for final consumption

$$Y_0 = \frac{c_{c0} + c_{d0}}{2},$$

⁶In practice, the unchanged nominal interest rates mimic a situation where nominal rates cannot move (say because they have hit the zero lower bound) and the monetary authority tries to stimulate the economy today by promising higher inflation in the future, as implied by the announcement that nominal rates will remain low for an "extended period of time". The unchanged nominal interest rates also imply that neglecting long-term bonds (with predefined nominal rates) does not entail a loss of generality.

where c_{j0} and c_{j1} satisfy the equilibrium budget constraint of the type-*j* household at t = 0and t = 1, which, using Lemma 1, can be shown to be equal to

$$c_{j0} = Y_0 + \bar{R} a_{j0} - a_{j1}$$
 and $c_{j1} = \bar{Y} + (\bar{R} - 1)\varepsilon^{-\tau_j} a_{j1} \quad \forall j = c, d.$ (13)

Notice that under degenerate beliefs and given Lemma 2, $\varepsilon^{-\tau_j} a_{j1}$ can be interpreted as the expected real wealth of household j after the inflation target in period one has been realized, which will yield real return \bar{R} in all remaining periods. We now use equation (13) to substitute for c_{j0} and c_{j1} in the corresponding Euler equation (12) at t = 0. In the resulting expression, we use the conditions for financial market clearing at t = -1, $a_{c0} = -a_{d0} = B$, and at t = 0, $a_{c1} = -a_{d1} = B'$. Since $l_{jt} = Y_t$ and, under degenerate beliefs, $E_{j0} [r_1] = \bar{R} \varepsilon^{-\tau_j}$, we can finally obtain the following two conditions:

$$\frac{\bar{N} + (\bar{R} - 1)\varepsilon^{-\bar{\tau}\,(1+\rho)}\,B'}{N_0 + \bar{R}B - B'} = \varepsilon^{-\bar{\tau}\,\frac{1+\rho}{\sigma}}\,,\tag{DA}$$

$$\frac{\bar{N} - (\bar{R} - 1)\varepsilon^{-\bar{\tau}(1-\rho)} B'}{N_0 - \bar{R}B + B'} = \varepsilon^{-\bar{\tau} \frac{1-\rho}{\sigma}} , \qquad (SA)$$

where $\bar{N} \equiv N(\bar{Y})$ and $N_0 \equiv N(Y_0)$, with

$$N(Y) \equiv Y - \psi_0 \frac{Y^{1+\psi}}{1+\psi}$$

representing output net of the effort cost of working, which in equilibrium is just a monotonically increasing transformation of output Y.⁷ Equation (DA), which corresponds to the positively sloped straight blue line in Figure 4, can be interpreted as creditors' demand for assets: the demand for assets B' is increasing in time-zero net output N_0 , because creditors want to save more when output increases temporarily, to smooth consumption. By the same logic, equation (SA), which corresponds to the negatively sloped straight red line in Figure 4, characterizes the supply of assets by debtors: the supply of assets B' is decreasing in N_0 , as debtors want to borrow less (save more) when time-zero output is higher. The financial market clears at the point where the two schedules cross, which is unique and corresponds to point A in the figure. The associated value of time-zero net output N_0 , is obtained by combining (DA) with (SA) which yields

$$N_0 = N_0(\varepsilon, \bar{\tau}, \rho) \equiv \bar{N} \left[\omega \,\tilde{\varepsilon}^{\frac{1+\rho}{\sigma}} + (1-\omega) \,\tilde{\varepsilon}^{\frac{1-\rho}{\sigma}} \right] + B \,\kappa \left[\tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma}-1\right)} - \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma}-1\right)} \right], \quad (14)$$

⁷Notice that N'(Y) > 0 when w < 1, which is implied by (10).

where $\tilde{\varepsilon} \equiv \varepsilon^{\bar{\tau}}$ measures the announcement rescaled by its average credibility while

$$\omega \equiv \frac{1 + (\bar{R} - 1) \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma} - 1\right)}}{2 + (\bar{R} - 1) \left[\tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma} - 1\right)} + \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma} - 1\right)}\right]} \in [0, 1],$$

$$\kappa \equiv \frac{\bar{R}(\bar{R} - 1)}{2 + (\bar{R} - 1) \left[\tilde{\varepsilon}^{(1+\rho)\left(\frac{1}{\sigma} - 1\right)} + \tilde{\varepsilon}^{(1-\rho)\left(\frac{1}{\sigma} - 1\right)}\right]} > 0.$$

This discussion can be summarized in the following Lemma:

Lemma 3 When the beliefs of households are given and degenerate, as characterized by $\bar{\tau} \in [0,1]$ and $\rho \in [-1,1]$, net output N_0 and financial imbalances B' are determined by the point where the demand for assets (DA) and the supply of assets (SA) intersect. The intersection is unique and the resulting N_0 is given by the function $N_0(\varepsilon, \bar{\tau}, \rho)$ in (14).

The first term on the right-hand side of (14) is always positive and characterizes the intertemporal substitution effect on consumption. The second term characterizes the effects on consumption of redistributing expected future wealth from one household type to the other. This second term is zero when B = 0, because no wealth is redistributed. It is also zero when $\rho = 0$, because in this case the wealth losses expected by the household type that losses from the redistribution (creditors when $\varepsilon > 1$, debtors when $\varepsilon < 1$) are exactly equal to the gains expected by the other type. And zero-sum transfers of wealth between household types have no effect on aggregate consumption, because all households have the same marginal propensity to consume—due to the utility function in (4) and the absence of financial constraints. So when B = 0 or $\rho = 0$, net output is $N_0 = \tilde{\varepsilon}^{\frac{1}{\sigma}} \bar{N}$, as in a standard representative-household New Keynesian model in response to an announcement $\tilde{\varepsilon} = \varepsilon^{\bar{\tau}}$. The canonical New Keynesian model in which all households fully believe the announcement corresponds to $\bar{\tau} = 1$ and $\rho = 0$. Substituting these values into (14), we immediately obtain $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$, which can be substituted back into (DA) to get

$$B' = \frac{\bar{R}B}{(\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}-1}+1} \; .$$

This implies that the new steady state imbalances at $t \ge 1$ when the announcement is implemented, B'/ε , are such that $B'/\varepsilon - B < 0$ if $\varepsilon > 1$, and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$, which leads to the following Proposition: **Proposition 1 (The full credibility benchmark)** If all households fully believe the announcement, $\bar{\tau} = 1$ and $\rho = 0$, then $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$. Thus output Y_0 is a strictly increasing function of ε and is independent of initial imbalances B. The new steady-state financial imbalances after implementation, B'/ε , decrease if the announcement is inflationary, $\varepsilon > 1$, and increase if it is deflationary, $\varepsilon < 1$.

We now use the diagram representation in Figure (4) to clarify how the initial financial imbalances, B, and the correlation between a household's wealth and its beliefs, ρ , affect output at time zero. First, while the income and substitution effects on consumption both shift the supply of assets by debtors in the same direction, they have opposite effects on creditors' demand for assets. To see this, notice that the substitution effect is represented by the term on the right-hand side of equation (DA), for households of type j = c, and of equation (SA), for households of type j = d respectively. In response to an inflationary announcement, $\varepsilon > 1$, these terms stimulate output by shifting both the (DA) and the (SA) schedule upward. But $\varepsilon > 1$ also redistributes expected future wealth from creditors to debtors, which explains why ε has a negative effect in the numerator of the left-hand side of (DA), while it has a positive effect in that of the left-hand side of (SA). Larger B implies larger expected wealth losses for creditors and gains for debtors, but under heterogeneous beliefs $\rho \neq 0$, expected losses are different from expected gains: the redistribution of expected wealth is no longer a zero-sum game.

To illustrate the implications of these observations, let us posit an inflationary announcement ($\varepsilon > 1$) that is believed only by creditors ($\rho = 1$). In this case, the supply of assets (SA) remains unchanged, while the demand (DA) can shift up or down: if *B* is small, the substitution effect prevails and (DA) shifts up (at least locally), as in panel (a) of Figure 4; if *B* is large, the income effect prevails and (DA) shifts down (locally), as in panel (b). Case (b) arises because creditors, expecting a lower return on assets, feel poorer and consume less, leading to a contraction in current aggregate net income. This, in turn, induces debtors to borrow more to smooth consumption, which increases their supply of assets and allows the financial market to clear, even if debtors' expected cost of debt service does not change. This corresponds to point *A'* in panel (b). Formally, we see that an inflationary announcement $\varepsilon > 1$ and $\rho > 0$, the right-hand side of (14) is a linear function in *B*, with an intercept higher than \overline{N} and a negative slope.

For debtors, the income and the substitution effects both work in the same direction, so the shift in asset supply (SA) is unambiguously signed. Panels (c) and (d) of Figure 4 illustrate this point for a deflationary announcement ($\varepsilon < 1$) that is believed only by debtors ($\rho = -1$). In this case (DA) remains unchanged, while the supply of assets (SA) always moves down, the more so the larger is *B*. As debtors expect the cost of debt service





to increase, they borrow less, so their consumption and hence aggregate output fall. The fall in income at t = 0 reduces the creditors' demand for assets even if their expected return is unchanged, allowing the financial market to clear. This corresponds to point A' in panels (c) and (d). Generally we can see that, when $\rho < 0$, a deflationary announcement $\varepsilon < 1$ is always contractionary in output: under $\varepsilon < 1$ and $\rho < 0$, the right-hand side of (14) is linear in B, with an intercept lower than \bar{N} and a negative slope. The next proposition summarizes this discussion:

Proposition 2 (Output with heterogeneous beliefs) For given beliefs, $\bar{\tau} \in [0, 1]$ and $\rho \in [-1, 1]$, net output N_0 is given by the function $N_0(\varepsilon, \bar{\tau}, \rho)$ in (14), which implies that

- 1. If $\rho > 0$, greater initial imbalances, B, reduce the response of time-zero output Y_0 to an inflationary announcement, $\varepsilon > 1$; and if they are large enough, Y_0 contracts;
- 2. When $\rho < 0$, Y_0 always falls in response to a deflationary announcement, $\varepsilon < 1$; and it falls more when the initial imbalances, B, are greater.

Numerical illustration In Figure 5 we plot the response of output to a 1-percentagepoint inflationary announcement for various parameter values. The solid blue line takes the values of Table 3 when $\rho = 1$ and $\bar{\tau} = 1/2$ —which will turn out to be the equilibrium beliefs. *B* is set equal to the standard deviation of the ratio of wealth to yearly labor income in the euro area at the time of the ECB's forward guidance announcement in July 2013. Panel (a) characterizes the effects of changing ρ ; panel (b) of changing *B*; panel (c) of changing σ . The lower the correlation between households' wealth and their perception of the credibility of the announcement ρ , the greater the output response: output falls by 0.3% under $\rho = 1$, and increases by 1.1% when $\rho = -1$. Panel (b) shows that smaller initial imbalances *B* lead to larger increases in output: when B = 0, output increases by 0.4%, whereas in the baseline specification it falls by 0.3%. Panel (c) shows that greater elasticity of intertemporal substitution (smaller σ) leads to a larger increase in output.

Figure 5: Output response to a 1% inflationary announcement



Notes: Output response to an announcement $\varepsilon = 1.01$. The blue solid line corresponds to $\rho = 1$, $\overline{\tau} = 1/2$, and $B/(4 \times w\ell) = 15$, while $\overline{R} = 1.015$, $\sigma = 2$, $\theta = 3$, $\psi = 1/2$, and $\psi_0 = 2/3$, as in Table 3. Dashed and dotted lines are the analogous impulse response after changing one parameter at a time.

4.3 Endogenous beliefs

We now characterize the way in which household j forms her beliefs about Π_1^* . Her problem is given by

$$V(a_{j0}) = \max_{c,l,a'} \left\{ U(c,l) + \beta \min_{\Omega \subseteq \mathcal{S}_0, \ G \in \mathcal{P}(\mathcal{S}_0)} \left[\int_{\Omega} \bar{V}\left(\frac{a'}{\Pi_1^*}\right) \ G(d\Pi_1^*) \right] \right\}$$
(15)

s.t.
$$c + a' \le w_0 l + \bar{R} a_{j0} + \lambda_0,$$
 (16)

where $S_0 = [\min(1, \varepsilon); \max(1, \varepsilon)]$, while, using Lemma 1, the continuation utility is

$$\bar{V}(s) = \frac{\left[\bar{N} + (\bar{R} - 1)s\right]^{1-\sigma}}{(1-\sigma)(1-\beta)},\tag{17}$$

which is an increasing function of the household's wealth at the beginning of period one. Generally, higher $\Pi_1^* \in \mathcal{S}_0$ lowers continuation utility when a' > 0, and increases it when a' < 0. If a' = 0, households' utility is unaffected by $\Pi_1^* \in \mathcal{S}_0$. We conclude that:

Proposition 3 (Individual beliefs) A household-j's beliefs depend on the announcement, ε , and her end-of period savings, a'. When a' = 0, beliefs are indeterminate. If $a' \neq 0$, they are degenerate and equal to $\varepsilon^{\tau(a',\varepsilon)}$ where

$$\tau(a',\varepsilon) = \mathbb{I}(\varepsilon > 1) \times \mathbb{I}(a' > 0) + \mathbb{I}(\varepsilon < 1) \times \mathbb{I}(a' < 0), \tag{18}$$

in which \mathbb{I} denotes the indicator function.

Figure 6 characterizes the function $\tau(a', \varepsilon)$, which measures (in percentage) how much of the announcement ε the household expects to be implemented in period one. If the announcement is inflationary, $\varepsilon > 1$, $\tau(a', \varepsilon) = 1$ if a' > 0, and zero otherwise, which corresponds to panel (a) of Figure 6. If the announcement is deflationary, $\varepsilon < 1$, $\tau(a', \varepsilon) = 1$ if a' < 0, and zero otherwise, which corresponds to panel (b).

4.4 Equilibrium

We now solve for the aggregate beliefs $\bar{\tau}$ and ρ , and for time-zero output Y_0 . First we replace c and l in (15), using (16) and (11), and then use the labour market clearing condition to write $l_0 = Y_0$. Given (18), and the definition of λ_0 , the problem of household j in (15) can be written:

$$V(a_{j0}) \equiv \max_{a'} F(a_{j0}, a').$$
 (19)





Here $F(a_{j0}, a')$ denotes household j's value for given a', which is equal to

$$F(a_{j0}, a') = \frac{\left(N_0 + \bar{R}a_{j0} - a'\right)^{1-\sigma}}{1-\sigma} + \beta \bar{V}\left(\frac{a'}{\varepsilon^{\tau(a',\varepsilon)}}\right),\tag{20}$$

where the functions \overline{V} and τ are defined as in (17) and (18), respectively. $F(a_{j0}, a')$ is continuous in a' and its derivative, $F_2(a_{j0}, a')$, is globally strictly decreasing in a' with a discontinuity point at a' = 0. So $F(a_{j0}, a')$ is concave in a', which guarantees a unique solution to the problem in (19). Moreover we have that the marginal value of a household's savings, a', is strictly increasing in her beginning-of-period wealth a_{j0} :

$$F_{12}(a_{j0}, a') = \sigma \bar{R} \left(N_0 + \bar{R} a_{j0} - a' \right)^{-\sigma - 1} > 0,$$
(21)

which immediately proves:

Proposition 4 (No reversal in households' net financial assets) In equilibrium, creditors and debtors never switch their net financial asset position: if B > 0, then $B' \ge 0$.

Combining Propositions 3 and 4, we can now fully characterize the aggregate equilibrium beliefs:

Proposition 5 (Aggregate beliefs) In a credit crunch equilibrium, B' = 0, households' beliefs are indeterminate. In any other equilibrium, B' > 0, only one type of household believes the announcement, $\bar{\tau} = 1/2$: if the announcement is inflationary, creditors believe it, $\rho = 1$; if it is deflationary, debtors believe it, $\rho = -1$. So in general we have

$$\rho = \rho(\varepsilon) \equiv 1 - 2\mathbb{I}(\varepsilon < 1). \tag{22}$$

To characterize equilibrium output, we calculate the intercept on the y-axis of debtors' asset supply and creditors' asset demand, both evaluated at the equilibrium beliefs of Proposition 5. The intercept of (SA) is given by

$$N_0^A = \min\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} + \bar{R}B,\tag{23}$$

that of (DA) by

$$N_0^B = \max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}B.$$
(24)

Clearly $N_0^B < N_0^A$ is equivalent to

$$B > \frac{|\varepsilon^{\frac{1}{\sigma}} - 1|\bar{N}|}{2\bar{R}},\tag{25}$$

which is a necessary and sufficient condition to guarantee that (SA) and (DA) intersect at B' > 0. If this happens, we have an equilibrium where financial markets remain active after the policy announcement. This is the case in panels (a) and (b) of Figure 4, for example, which deal with an inflationary announcement $\varepsilon > 1$, and in panels (c) and (d), which deal with a deflationary announcement $\varepsilon < 1$. But when (25) fails, we have $N_0^A < N_0^B$, which implies that (SA) and (DA) would intersect at a point where B' < 0, as in panel (a) of Figure 7. In this case, given Proposition 4, we have a credit crunch equilibrium, B' = 0, where both households types j = c, d completely undo their financial positions. This equilibrium arises because of the endogenous beliefs of household j (see Proposition 3), which cause a discontinuous fall in the expected return on assets when the household j's savings switch from negative to positive. So at a' = 0 the value of household j's savings $F(a_{i0}, a')$ has a kink and the marginal value of savings, $F_2(a_{i0}, a')$, falls discontinuously. In a credit crunch equilibrium, $F_2(a_{i0}, a')$, changes sign at a' = 0, as in panel (b) of Figure 7, which guarantees that household j will find it optimal neither to borrow—which means higher consumption today in exchange for lower consumption tomorrow— nor to lend lower consumption today and higher tomorrow. On these premises, in the Appendix we prove that:

Figure 7: Credit crunch equilibrium



Lemma 4 If (25) fails, then $N_0^A < N_0^B$, and the equilibrium features a credit crunch B' = 0.

Intuitively, credit crunches arise because of a *zen effect*, due entirely to the endogenous formation of households' beliefs under ambiguity aversion: due to the kink in the value of their savings $F(a_{j0}, a')$, households naturally tend to choose a financial position that assures them "complete peace of mind" about future monetary policy choices, which in this simple model is attained when a' = 0.

Lemma 4 together with the foregoing considerations immediately implies:

Proposition 6 (Equilibrium output) An equilibrium always exists. If (25) holds, then the financial market is active, B' > 0, and net output, N_0 , is given by (14) evaluated at the equilibrium beliefs of Proposition 5, so that $N_0 = N_0(\varepsilon, 1/2, \rho(\varepsilon))$. If (25) fails, households' beliefs are indeterminate and the equilibrium features a credit crunch, B' = 0, where net output, N_0 , can be any value in the range $[N_0^A, N_0^B]$, N_0^A and N_0^B being given by (23) and (24), respectively.

In general a credit crunch equilibrium requires that output at t = 0 be neither too low (so that debtors do not find it optimal to borrow) nor too high (so that creditors do not find optimal to lend). For example, in a credit crunch equilibrium caused by an inflationary

announcement $\varepsilon > 1$, output never falls, $N_0^A > \bar{N}$, and it is always lower than the output level of the benchmark New Keynesian model characterized in Proposition 1, as $N_0^B < \varepsilon^{\frac{1}{\sigma}} \bar{N}$.

Combining Proposition 6 with Propositions 1 and 2, we immediately obtain the following corollary:

Corollary 1 After an inflationary announcement $\varepsilon > 1$, output Y_0 increases less than in the full credibility benchmark. This difference is increasing in B, and Y_0 can even decrease if B is large enough. In response to a deflationary announcement, $\varepsilon < 1$, Y_0 always decreases. The decrease is larger the larger is B; and if B is large enough, Y_0 decreases more than in the full credibility benchmark.

Finally, in the Appendix, we compare the steady state imbalances that result when the announcement is implemented, B'/ε , with the corresponding imbalances in the canonical New Keynesian model, where the announcement is fully credited by all households:

Proposition 7 (Steady state imbalances) After an inflationary announcement $\varepsilon > 1$, the new steady state financial imbalances after implementation, B'/ε , always decrease $(B'/\varepsilon < B)$, and they decrease more than in the full-credibility benchmark. After a deflationary announcement $\varepsilon < 1$, there are two (strictly positive) thresholds \tilde{B}_1 and \tilde{B}_2 , with $\tilde{B}_1 < \tilde{B}_2$, such that for $B < \tilde{B}_1$, B'/ε falls; for $B \in [\tilde{B}_1, \tilde{B}_2]$, B'/ε increases, but less than in the full-credibility benchmark; and for $B > \tilde{B}_2$, B'/ε increases, and more than in the benchmark.

The effects on the end-of-period imbalances B' depend on who stands to gain from the redistribution of expected wealth. To see this, observe that (13) implies that B' is a function of the relative consumption of debtors j = d and creditors j = c:

$$B' = \frac{c_{d0} - c_{c0}}{2} + \bar{R}B$$

When $\varepsilon > 1$, debtors do not believe the announcement (see Propositions 5 and 6) so their relative consumption c_{d0} increases less then under full credibility, which generally makes B' smaller than under full credibility, independently of B. When $\varepsilon < 1$ and B is small, the zen effect (induced by the kink in the value of households' savings) leads to credit crunches (see Proposition 6); and it makes the end-of-period imbalances B' smaller than in the full credibility benchmark. If $\varepsilon < 1$ and B is large, creditors do not believe the announcement and neglect the large wealth gain associated to $\varepsilon < 1$ (again following from Propositions 5 and 6), so c_{c0} is (relatively) smaller, hence B' larger, than under full credibility. This leads to the unintuitive result that when B is large enough and $\varepsilon < 1$, ambiguity aversion induces larger imbalances.

5 Quantitative analysis

We now analyze the quantitative response of euro area output to the ECB's announcement in July 2013. We extend the model to allow for (i) a non-zero net supply of financial assets, (ii) convex price adjustment costs à la Rotemberg (1982), and (iii) a more general monetary announcement. Extension (i) is needed to match the observed distribution of European households' assets; (ii) is needed to generate persistent effects of monetary policy; and (iii) is needed to match the observed response of interest rates to the announcement. First we characterize the extended model and its equilibrium properties, before calibrating and reaching our quantitative results.

5.1 Extended model

Financial markets are characterized as a competitive mutual fund that collects all interest payments by borrowers and all firm profits (so $\lambda_t = 0 \forall t$) and pays interest on the financial assets owned by households. The fund also pays an amount Υ to external agents, who represent foreign holders of euro-area assets. The interest rate at time $t r_t$ is equal for lenders and borrowers, equalized across assets, and is set so that, at every point in time, the financial flows out of and into the fund are exactly equal. The flow budget constraint thus equates aggregate net interest payments, $(r_t-1) \int_0^1 a_{xt} dx + \Upsilon$, to the sum of dividends, D_t , and the net new supply of assets, $\int_0^1 (a_{xt+1} - a_{xt}) dx$, which yields the condition

$$r_t \int_0^1 a_{xt} \, dx + \Upsilon = D_t + \int_0^1 a_{xt+1} \, dx.$$
 (26)

Each firm $i \in [0, 1]$ can adjust nominal prices subject to convex adjustment costs, as in Rotemberg (1982). Adjustment costs are quadratic in the rate of price change and are scaled by aggregate output, Y_t :

$$\Theta_t(\pi_{it}) = \frac{\kappa}{2} (\pi_{it})^2 Y_t, \qquad (27)$$

where $\pi_{it} = (p_{it} - p_{it-1})/p_{it-1}$ denotes the inflation rate for firm *i* and $\kappa > 0$.

The Taylor rule is still given by (6) where the (logged) inflation target Π_t^* follows an AR(1) process: $\ln(\Pi_t^*) = \rho^* \ln(\Pi_{t-1}^*)$. For simplicity we assume that households face no ambiguity about ρ^* , T and all the values of Π_t^* up to T - 1. The only ambiguity for households before T concerns Π_t^* at t = T, which can be any value in the set $S_{T-1} = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}], \varepsilon$ again being the monetary announcement. Moreover, households at time $t \geq T$ face no ambiguity about the current or future values of Π_t^* .

5.2 Equilibrium conditions

In each period t, after Π_t^* has been realized, firm i sets its nominal price p_{it} so as to maximize

$$\max_{\{p_{it}\}} E_{ft} \left\{ \sum_{s=0}^{\infty} q_{t,t+s} \left[\left(\frac{p_{it+s}}{p_{t+s}} - w_{t+s} \right) Y_{t+s} \left(\frac{p_{it+s}}{p_{t+s}} \right)^{-\theta} - \Theta_{t+s} \left(\pi_{it+s} \right) \right] \right\},$$
(28)

where $q_{t,t+s} \equiv (\prod_{n=1}^{s} r_n)^{-1}$ is the discount factor between period t and period t+s. The expectation operator is indexed by f to denote firms' beliefs about future output, inflation and interest rate, which we assume are common to all firms $i \in [0, 1]$. The solution to the firm's problem in (28) implies symmetric pricing, $p_{it} = p_t$ and $\pi_{it} = \pi_t \forall i$, which can be used to derive the following New-Keynesian Phillips curve:

$$1 - \kappa (\Pi_t - 1) \Pi_t + \kappa E_{ft} \left[q_{t,t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = \theta (1 - w_t).$$
(29)

The aggregate dividend payments, which are rebated back to the mutual fund, are obtained by aggregating all firms' profits, which yields

$$D_{t} = \left[1 - w_{t} - \frac{\kappa}{2} \left(\Pi_{t} - 1\right)^{2}\right] Y_{t}.$$
(30)

At every point in time, aggregate output clears the goods market, so that

$$Y_t = \frac{C_t + \Upsilon}{1 - \frac{\kappa}{2} \left(\Pi_t - 1\right)^2},\tag{31}$$

where $C_t = \int_0^1 c_{xt} dx$ denotes aggregate consumption obtained by summing, $\forall x \in [0, 1]$, the optimal consumption of all households x, c_{xt} , as determined below. Given the definition of the interest rate, $r_t = R_t/\Pi_t$, and the output path Y_t that is implied by (31), the nominal interest rate R_t , the wage, w_t , and inflation, Π_t , are jointly (implicitly) determined by the Taylor rule in (6), the labour supply condition in (11) and the Phillips curve in (29). We now turn to the characterization of household x's consumption c_{xt} .

The economy after T At $t \ge T$, household x maximizes the utility $\sum_{s=0}^{\infty} \beta^s U(c_{xt+s}, l_{xt+s})$ under perfect foresight. She chooses consumption c_{xs} and labor supply l_{xs} subject to the budget constraints in (5) and the path of aggregate prices w_s , π_s and r_s , which yields the first order conditions (11)-(12). Clearly (11) together with labour market clearing implies that

$$\psi_0 Y_t^\psi = w_t. \tag{32}$$

We then use (12) and (32) together with the intertemporal budget constraint of household x to solve for c_{xt} . Aggregating the consumption choices of all households $x \in [0, 1]$, we obtain that, $\forall t \geq T$, aggregate consumption is equal to

$$C_{t} = \frac{\psi_{0}}{1+\psi}Y_{t}^{1+\psi} + \left(\sum_{s=0}^{\infty}\beta^{\frac{s}{\sigma}}q_{t,t+s}^{1-\frac{1}{\sigma}}\right)^{-1}\sum_{s=0}^{\infty}q_{t,t+s}\left(\frac{\psi_{0}\psi}{1+\psi}Y_{t+s}^{1+\psi} + D_{t+s} - \Upsilon\right),\tag{33}$$

the consumption level that would be chosen by a representative household of the aggregate economy. In general we have proved that

Lemma 5 The aggregate dynamic of the economy $\forall t \geq T$ is fully characterized by the tuple $[D_t, Y_t, w_t, C_t, \Pi_t, R_t, r_t]$, where (i) D_t and Y_t are given by (30) and (31); (ii) aggregate labour supply and consumption solve a representative household problem that yields (32) and (33); (iii) inflation Π_t satisfies the Phillips curve in (29) under perfect foresight; (iv) the nominal interest rate R_t follows the Taylor rule in (6); and (v) the interest rate satisfies the identity $r_t = R_t/\Pi_t$.

After using Lemma 5, the Taylor rule in (6) and the definition of a steady state, we can conclude:

Proposition 8 (Equilibrium after T) The equilibrium dynamic before T affects aggregate dynamics at $t \ge T$, only through the nominal interest rate that prevails between T - 1and T, R_T . In particular, if R_T and Π_T^* are both at their steady state value, $R_T = \overline{R}$ and $\Pi_T^* = 1$, then the economy is back to steady state at t = T.

The economy before T At t < T, household x solves the following problem:

$$V(a_{x0}) = \max_{\{c_{xt}, l_{xt}\}_{t=0}^{T-1}} \left\{ \sum_{t=0}^{T-1} \beta^t U(c_{xt}, l_{xt}) + \min_{\Omega \subseteq \mathcal{S}_{T-1}, \ G \in \mathcal{P}(\Omega)} \beta^{T-1} \int_{\Omega} \bar{V}(a_{xT}, \Pi_T^*) G(d\Pi_T^*) \right\},$$
(34)

where a_{x0} is household x's initial wealth, and the function $\bar{V}(a_{xT}, \Pi_T^*)$ is the household's continuation utility at T, when ambiguity is fully resolved. Continuation utility is a function of the household's wealth at time T, a_{xT} , and the inflation target realized at time T, Π_T^* . This function determines the formation of household x's beliefs and can be easily constructed using Lemma 5 to solve the equilibrium of the economy at $t \ge T$ for different realizations of Π_T^* . Figure (8) characterizes how the household forms her beliefs in the case of an inflationary announcement $\varepsilon > 1$. It uses the parameter values of Table 3 to plot (solid blue line) the difference between $\bar{V}(a_{xT}, \varepsilon)$ and $\bar{V}(a_{xT}, 1)$ as a function of a_{xT} scaled by steady state yearly labor income. There is a strictly positive threshold value of wealth a^* , approximately five times steady state yearly labor income, at which the difference function crosses the zero line. In contrast with the simple model of Section 3, here a^* is strictly positive, because expansionary monetary policy increases labour income, which is beneficial to the household. So, given the worst case criterion, only sufficiently wealthy households believe that the monetary authority will implement the announcement. The threshold a^* changes with the announcement ε . The dashed red line in Figure (8) corresponds to the difference function $\bar{V}(a_T, \varepsilon) - \bar{V}(a_T, 1)$ for a higher ε . If implemented, a higher ε implies a stronger monetary shock, which causes both a sharper rise in labour income and a sharper fall in the interest rate. In our calibration, the first effect dominates and a^* moves to the right.⁸



Figure 8: Determination of beliefs, $\bar{V}(a_T, \varepsilon) - \bar{V}(a_T, 1)$

Notes: The solid blue line plots the difference in continuation values at t = T, $\bar{V}(a_T, \varepsilon) - \bar{V}(a_T, 1)$, as a function of a_T scaled by steady state yearly labor income, using the parameter values in Table 3. The dashed red line plots $\bar{V}(a_T, \varepsilon) - \bar{V}(a_T, 1)$ for a larger announcement, $\varepsilon = 1.03$. Vertical lines identify the threshold a^* at which $\bar{V}(a_T, \varepsilon) - \bar{V}(a_T, 1)$ crosses the zero line.

The threshold a^* allows us to partition households into three groups depending on their wealth a_{xt} at time t = T: i) households with $a_{xT} > a^*$ will believe the announcement for sure

$$\frac{da^*}{d\varepsilon} \simeq -\varphi_{a^*} + (1 - \varphi_{a^*})\eta_{wr}$$

⁸One can show that, up to a first order approximation,

where " \simeq " means "same sign as", $\varphi_{a^*} \equiv \frac{ra^*}{ra^*+wl}$ is the steady state ratio of capital to total income of a household with wealth a^* and $\eta_{wr} \equiv |d \ln w/d \ln r|$ is the elasticity of wages to changes in interest rates in the representative household model, which characterizes the economy after T (Lemma 5). In our baseline calibration, φ_{a^*} is about 0.3 and η_{wr} is about 5.

(trusting households); (ii) households with $a_{xT} < a^*$ will not believe it at all (untrusting households); (iii) households with $a_{xT} = a^*$ will be indifferent about any future choices by the monetary authority and have indeterminate beliefs (zen households). Notice that $\forall t < T$ there is perfect foresight about the realization of interest rates r_t , output Y_t and wages w_t before T. However, at t < T, trusting and untrusting households have different beliefs about the realization of r_s , Y_s and w_s at $s \ge T$. In particular, for all t < T, trusting households believe that r_s , Y_s and $w_s \forall s \ge T$ will be the determined under the equilibrium of the model conditional on $\Pi_T^* = \varepsilon$, while untrusting households believe that they will solve the equilibrium of the model under $\Pi_T^* = 1$.

5.3 Calibration

The model is calibrated at quarterly frequency. Table 3 reports the parameter values used in our baseline calibration.

Nominal interest rates and firms' beliefs We start assuming that in the interim before T, nominal interest rates stay at their steady state value, $R_t = \overline{R} \forall t < T$, whereas at time T and thereafter they are determined by the Taylor rule in (6). This profile of nominal interest rates reflects the fact that short-term rates could not fall at the time of the ECB announcement because they were practically at the zero lower bound. We also assume that firms fully believe the policy announcement. This is a reasonable benchmark, because firms are typically owned by wealthy households, who tend to trust the monetary announcement. In Section 6, we relax both assumptions and show that they are conservative, in that they tend to increase the expansionary effects of the monetary policy announcement.

Preferences and technologies We set β to match a steady state return on savings of 6%, which is the approximate real return from investing in the stock market in the euro area. We target an elasticity of intertemporal substitution (EIS) of 0.5 and a Frisch elasticity of labor supply of 2, which are in the range of values commonly used in the literature; see Guvenen (2006) and Keane and Rogerson (2012) for a review of the empirical estimates of EIS and Frisch elasticity, respectively. The parameter governing the elasticity of substitution across varieties θ is set to target a steady state labor share of 2/3. The resulting value, $\theta = 3$, is in the range of values typically used in macro models.⁹ The parameter governing the cost of price adjustment κ is used to match the elasticity of inflation to current marginal cost in the Phillips curve θ/κ , which we set at a value of 0.1, quite closely in line with the literature (see Schorfheide 2008). We normalize steady-state

⁹For instance, Midrigan (2011) assumes $\theta = 3$, as we do. Instead Golosov and Lucas (2007) work with $\theta = 7$, which in our model would yield a labour share greater than 2/3, while in reality the labor share has fallen below 2/3 over the last decade; see Karabarbounis and Neiman (2014).

Model		Data	
Parameter	Value	Moment	Value
β	0.985	Yearly real stock market return	0.06
σ	2	Elasticity of intertemporal substitution	0.5
ψ	0.5	Frisch elasticity of labor supply	2
ψ_0	0.66	Labor supply normalization in steady-state	1
θ	3	Labor share	0.66
κ	30	Slope of the Phillips curve	0.1
Υ	0.25	Mean net financial assets to yearly average labor income	2
ϕ	1.5	Taylor rule response to inflation	1.5
$ ho_r$	0.8	Taylor rule inertia of nominal interest rate	0.8
T	6	Max fit to forward rate response at 2-10 years maturity	-
$ ho^*$	0.9	Max fit to forward rate response at 2-10 years maturity	-
ε	1.01	Max fit to forward rate response at 2-10 years maturity	-

Table 3: Baseline Calibration

labour supply to one, which determines the scaling factor of the utility function ψ_0 .

The initial distribution of assets Let G(a) denote the initial distribution function (CDF) of households' assets a_{x0} in the model. The distribution has support over 1,000 discrete points, $a_1, a_2, \ldots a_{1000}$, which are set to match the mean and the 999 permilles of the distribution of euro-area net financial assets (NFA), scaled by the average yearly household labor income. NFA data are from the euro-area Household Finance Consumption Survey (HFCS). NFA is the difference between total financial assets (deposits, bonds, mutual funds, voluntary pension funds) and total financial liabilities (mortgages plus non-mortgage debt); see the Appendix for further details. We set Υ to match a ratio of 2 between aggregate households' initial assets and yearly labor income, which corresponds to the value found in the HFCS. Panel (a) of Figure 9 plots the CDF of the resulting initial assets distribution, scaled by average yearly labor income. The standard deviation is high, equal to 23, and

skewness is substantial, at 116. The vertical red bar indicates the set of initial assets $[\underline{a}_0^*, \overline{a}_0^*]$ identifying zen households. So (i) households with $a_{x0} > \overline{a}_0^*$ believe the inflationary announcement; (ii) those with $a_{x0} < \underline{a}_0^*$ do not; (iii) and those with $a_{x0} \in [\underline{a}_0^*, \overline{a}_0^*]$ have undetermined beliefs. Around 10% of households believe the announcement, whereas the number of zen households is tiny. Panel (b) of Figure 9 plots, as a solid blue line, the fraction of total consumption accounted for by the wealthiest households of the (calibrated) economy indicated on the x-axis, which corresponds to the Lorenz curve of consumption. The top wealthiest 10% of households, who believe the announcement, account for 20% of aggregate consumption, which reflects the high concentration of wealth among European households.¹⁰



Figure 9: The distribution of net financial assets from HFCS

Notes: Panel (a) plots the CDF of euro-area households' net financial assets scaled by average annual labor income; panel (b) plots the fraction of aggregate consumption that is accounted for by the wealthiest households of the calibrated economy described in Table 3. The vertical red bar identifies zen households.

Monetary policy The parameters of the Taylor rule in (6), ϕ and ρ_r , are standard. We choose the horizon of the announcement T, its intensity ε , and the persistence of the

¹⁰In HFCS, the wealthiest 10 percent of European households account for around 15 percent of aggregate food consumption expenditure, which is the only measure of consumption available in this survey. In practice food consumption is a small fraction of these households' consumer spending, so we believe our calibration underestimates the contribution of the top decile of wealth to aggregate consumption expenditures. For example using data from the Consumer Expenditure Survey for 2010 (also available at http://www.bls.gov/cex/2010/Standard/higherincome.pdf), we calculate that households in the top decile of the income distribution account for about 23 percent of aggregate US consumption expenditure.

inflation target ρ^* , to match the response of the nominal interest rate curve at the time of the ECB announcement on 4 July 2013. We focus on the responses of the instantaneous forward rates of euro-area AAA government bonds between 4 July and 2 July. The instantaneous forward rate is the data equivalent of the expected nominal short-term interest rate in our model.¹¹ Since the empirical evidence suggests that only financially sophisticated investors, with low ambiguity aversion and substantial wealth operate in this market (Dimmock, Kouwenberg, Mitchell, and Peijnenburg 2016), we make the simplifying assumption that the instantaneous response of forward rates measures the forward guidance announcement. Panel (a) of Figure 10 plots the profile of instantaneous forward rates on 2 July, before the announcement. The solid blue line in panel (b) shows the changes in forward rates immediately after the announcement: they are negative and very persistent, with a spike of around 10 basis points at the three-year maturity. We search for the model profile of nominal short-term rates in deviation from steady state, $R_t - \bar{R}$, when the announcement is implemented, that minimizes the square distance from the actual response of forward rates at all maturities of at least two years. The dashed red line in panel (b) shows the resulting profile. The best fit is with the horizon of the announcement at 6 quarters, T = 6, intensity at $\varepsilon = 1.01$, and persistence at $\rho^* = 0.9$, which implies a half-life of the monetary expansion of about six quarters.

5.4 Impulse responses

We obtain an exact solution of the model by global non-linear methods (see the Appendix for details). Figure 11 plots the impulse responses of selected economic variables. The solid blue lines correspond to our baseline calibration, the dashed black lines to the responses that would arise in a hypothetical benchmark model where all households accord full credibility to the announcement. The vertical red line pinpoints the time of the announcement T. For simplicity we assume that the announcement is actually implemented, but this is without loss of generality since the focus is on the response of the economy in the interim before T. At $t \ge T$, nominal and real interest rates fall owing to the monetary expansion (see panels (a) and (e)), which leads to an expansion in output (panel (b)) and a rise in inflation (panel (f)). The effects in the model and in the alternative full-credibility benchmark are identical, which follows from Proposition 8. At t < T, the two economies behave very differently: in

¹¹The data are obtained from the datawarehouse of the ECB. The time-t instantaneous forward rate f(t, u) at maturity $u \ge t$ is such that the price at t of a zero coupon bond maturing at T is equal to $\exp(-\int_t^T f(t, u) du)$. The ECB calculates the instantaneous forward rate using the methodology of Svensson (1995). Using the Eonia Swap rate curve, the effects are similar. We focus on government bonds rather than Eonia swap rates because the yield curve of government bonds is also available at maturities greater than two years, while publicly available data for Eonia swap rates are available only for maturities up to two years.



Figure 10: The effects of forward guidance on the yield curve





Notes: All values are in basis points. The left panel plots the instantaneous forward rates extrapolated from the yield curve of AAA Government bonds on 2 July 2013. The solid blue line in the right panel plots the difference between the instantaneous forward rates on 4 July and those on 2 July 2013. The dashed red line is the change in the path of short-term interest rates implied by (6), which arises in the model if the announcement is implemented, and that minimizes the square distance from the actual response of forward rates at all maturities of at least two years.

the model, output falls on impact at the time of the announcement and continues to fall further; in the full-credibility benchmark, output increases on impact and remains above its steady state level throughout the interim period up to T. Quantitatively, in the five quarters before implementation the model predicts a cumulative output loss relative to steady state of 1.5%, while the full-credibility benchmark predicts a cumulative gains of 5.4%. In the model, the fall in output is driven by the fall in the consumption of wealthy households (those with $a_{xT} > a^*$) who believe the announcement, see panel (c). However, panel (d) shows that the main difference between model and benchmark is the lack of increase in the consumption of relatively poorer households (those with $a_{xT} \leq a^*$, who in the model do not believe the announcement). In the model their consumption increases only slightly at the time of the announcement and then falls throughout the interim period, while in the full credibility benchmark it increases by 2.5% on average throughout the interim period. Finally, notice that in the interim period real rates decrease (see panel (e)), as nominal rates are fixed and firms are posited as fully believing the announcement, so they start increasing prices before T in order to smooth adjustment costs. This fall in real rates stimulates the economy, attenuating the decline in output (see also below). Since real rates keep falling throughout the interim period, households substitute consumption intertemporally, which



Figure 11: Response to forward guidance announcement

explains why output falls less at the time of the announcement than in the quarters just before implementation T (see panel (b)).

6 Robustness and extensions

We now discuss some robustness exercises.

6.1 Government debt into households liabilities

At least since Ricardo (1888) and Barro (1974), there is genuine doubt whether households perceive government bonds as net wealth. Accordingly, we have recalibrated the model on the assumption that European households feel liable for their country's public debt. We now calculate a household's net financial wealth as the difference between the corresponding Net Financial Assets (NFA) in HFCS and the country's government debt per household. Per household government debt, in the data, is the country's net government debt per capita, as reported in Table 1 of Adam and Zhu (2015), multiplied by the number of people in the household older than 16 years of age according to HFCS. The resulting distribution of NFA has similar standard deviation and skewness to those of the baseline calibration, but the mean is now lower, at 0.35 times the average euro-area yearly labor income. The distribution of initial wealth in the model G is calibrated as before, to match the mean and the 999 permilles of the new NFA distribution, and we set $\Upsilon = 0.32$ to match the ratio of mean NFA to average yearly labor income, which is now equal to 0.35 rather than 2 as in the baseline. Due to the lower NFA, the fraction of trusting households falls from 10% to 8%, and they now account for 18% of aggregate steady state consumption expenditure rather than 20% as in the baseline. We recalibrate the parameters governing the monetary policy announcement, ε , T, and ρ^* to match the observed changes in forward rates in panel (b) of Figure 10. Panel (a) of Figure 12 shows the response of output in the recalibrated economy, and panel (b) shows the new profile of the inflation target, both as dashed black lines. The solid blue line in the two panels corresponds to the baseline specification of Figure 11. The estimated path of the inflation target now falls more at the time of implementation, T = 6, which explains the stronger response of output at T = 6 (see panel (a)). In the interim period, t < T, output now falls less than in the baseline specification, and it even increases at the time of the announcement, although it is below the steady-state level in the quarter before T. The cumulative output gain in the five quarters following the announcement is now 0.6%, about nine times less than under the full-credibility benchmark (not displayed). Overall we can conclude, that when households realize that government bonds do not really constitute wealth, monetary policy announcements are more expansionary. This implies that getting households to realize the true cost of servicing the public debt is important for monetary policymaking.



Figure 12: Output response to Forward Guidance: robustness and extensions

(c) Y: firms don't believe annoucement



6.2 Firms do not believe the announcement

We now assume that firms do not trust the monetary authority and assign zero probability to the implementation of the policy announcement at T. Again we recalibrate the announcement parameters, ε , T, and ρ^* to match the observed changes in forward rates. Panel (c) shows the responses of output and panel (d) plots the recalibrated profile of the inflation target, both as dashed black lines. The solid blue lines correspond to the baseline specification. At $t \ge T$, the output response is unchanged relative to baseline. In the interim up to T, output drops more than in the baseline specification at the time of the announcement and then remains at this lower level throughout the entire interim period. Since in the interim firms do not believe the announcement, they no longer raise prices to smooth adjustment costs, so the interest rate no longer falls. Thus output falls more on impact and remains at this lower level until T.

6.3 Other robustness exercises

In the Appendix we study the effects of changing the elasticity of intertemporal substitution (parameter σ) and the time horizon of the announcement T, as well as of allowing the Taylor rule to be effective also before T. A higher σ reduces the substitution effect of reducing

interest rates. So output falls more before T and increases less thereafter. With a higher T, households have a stronger incentive to substitute consumption intertemporally to take advantage of the decreasing profile of interest rates throughout the interim period. This produces a smaller decline in output on impact and a larger one just before T. When the Taylor rule in (6) is always effective, the nominal interest rate increases in the interim period, whereas in the baseline it holds constant. So output falls more at the time of the announcement, and then stabilizes at this lower level throughout the interim period.

6.4 Deflationary announcement

We now consider the effects of a deflationary announcement in our baseline economy. All parameters are as in Table 3, but we now set $\varepsilon = 0.99$. Panel (a) in Figure 13 shows the responses of output and panel (b) reports the profile of the inflation target, both as solid blue lines. The dashed black lines in the two panels correspond to the response in the full-credibility benchmark, where all households believe the announcement. In the interim up to T, output falls and falls more sharply than in the full-credibility benchmark: the cumulative output loss in the first five quarters is 46% greater in the model than in the benchmark. This is because wealthy households, who will benefit from the future monetary tightening, do not believe the announcement and so consume less than under full credibility.



Figure 13: Response to a deflationary announcement

7 Conclusions

We have characterized the equilibrium of a New Keynesian model in which ambiguity-averse households with heterogenous net financial wealth use a worst-case criterion to judge the credibility of monetary policy announcements. An announcement of a future tightening is always contractionary. An announcement of loosening is less expansionary in our framework than under full credibility, and it can even be contractionary when the inequality in wealth is sufficiently pronounced. This is because wealthy creditor households are more prone to believe the announcement of loosening than poor, indebted households. Hence there is a fall in perceived aggregate wealth, which if large enough causes a contraction in aggregate demand. To gauge the importance of this mechanism, we have considered the start of forward guidance by the ECB in July 2013. Calibrating the model to match the entire distribution of European households' net financial wealth, we find that forward guidance is contractionary, and particularly so when households do not feel liable for the public debt. We have analysed the effects of monetary policy announcements, but the same logic would apply to announcements about any future policy that, if implemented, would generate winners and losers, such as pension reform, or revisions to competition, innovation or fiscal policy, or changes to labor market institutions like unemployment insurance and job protection. Generally, the announcements of future reforms that will redistribute wealth if implemented, tend to have unintended perverse effects when agents are ambiguity-averse, because the net losers tend to give more credit to announcements than the net winners.

Throughout the analysis, we have maintained some simplifying assumptions that it would be interesting to relax in future research. For example, we allowed households to trade just a one-period bond with a predefined nominal interest rate. In practice financial markets allow households to buy a variety of assets, including long-term nominal bonds and stocks, and to save in real assets, such as human capital and real estate, which might also require some borrowing. All this is relevant because the effects of monetary policy on the real return on investment could differ across assets, which would imply that monetary policy losers and winners are not perfectly identified by the sign of their net financial asset position. Allowing households to face a more complex portfolio problem might generate further insights into the interaction between redistribution and ambiguity aversion. Moreover, in our model, households cannot trade *real* interest rate swaps, which would insure them against future changes in monetary policy. This assumption is realistic, because the market for real interest rate swaps is tiny and only a very few financially sophisticated households hold swaps (Lusardi and Mitchell 2014). Yet real interest rate swaps would allow households to exploit trade opportunities induced by differences in their beliefs and would generally increase the effectiveness of monetary policy announcements.

In our model we have also abstracted from the role of fiscal policy in the transmission of monetary policy and from heterogeneity in households' marginal propensity to consume, ambiguity, and income. Both these issues are important and would interact with our mechanism. For example Kaplan, Moll, and Violante (2016b) emphasize that monetary policy has an impact on fiscal transfers, which in turn affect households' disposable income and hence consumption and aggregate demand. But in our model, fiscal transfers, and in particular their timing, would also affect the formation of households' beliefs, so governments could use them strategically to enhance the credibility of monetary policy. Finally, in our model households differ only in initial financial wealth, but in reality households also differ in marginal propensity to consume (Werning 2015), degree of ambiguity aversion (Dimmock, Kouwenberg, Mitchell, and Peijnenburg 2016), labour income, and human capital. Under ambiguity aversion, this heterogeneity has a first-order effect on the formation of households' beliefs and thereby on the effect of policy announcements.

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APPENDIX

Section A describes the data, Section B presents some proofs, Section C discusses computational details, and Section D provides further details on the robustness exercises in Section 6.

A Data appendix

We describe the sources of our data for realized and expected inflation in the euro area, and in the Italian provinces, as well as the net financial assets of European households.

A.1 Euro Area Data: realized and expected inflation

The data are for the Euro 11 countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

Core Inflation is the yearly log differences in the Harmonised Index of Consumer Prices (HICP), net of energy and unprocessed food, multiplied by 100, taken from the Eurostat data warehouse available at "http://ec.europa.eu/eurostat/".

Price expectations come from the European Commission's Business and Consumer Surveys. The key advantage of the Consumer Survey is that it directly asks households for their expectations about future inflation, which distinguishes it sharply from the commonly used Survey of Professional Forecasters. Sample size varies with country. Price expectations are derived from the question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall. Probabilities are calculated in terms of net balance (the difference between responses of "very likely" and "unlikely") so that price expectations are calculated as $(f_i + 1/2f_{ii} - 1/2f_{iv} - f_v) \times 10$, where $f_j = i$, ii, iii, iv, v is the fraction of individuals selecting option j. The series are seasonally adjusted by the Commission.

Creditor vs debtor countries The creditor countries are Austria, Finland, Germany, Luxembourg, and the Netherlands. The others Euro 11 countries are debtors. Countries are classified by net foreign asset position, measured by the External Wealth of Nation Mark II (EWN) as in Lane and Milesi-Ferretti (2007). The only exception is Austria, which by the latest available EWN observation for 2011, had a net foreign asset position of -4.5% of GDP. But since Austria has run large current account surpluses averaging 2 percent of GDP in all the years since 2011, we count as a creditor country. In any case, the results are robust to Austria's classification.

A.2 Italian data

Our Italian data come from ISTAT's Survey of Inflation Expectations conducted by the Bank of Italy and Sole24Ore (Italy's main daily business paper), and from the Bank of Italy's Survey of Household Income and Wealth.

Realized inflation at province level is taken directly from ISTAT's "I.Stat" online archive. We use the general price index, pgen in the ISTAT database. Realized inflation in the province corresponds to the yearly log-difference of pgen in the province. We take yearly log-differences because the ECB monitors price stability on the basis of the annual rate of change in HICP and because of the working of the inflation expectations question (see below).

Expected inflation measures 2 quarters ahead expected inflation, averaging the reported estimates of all observations in the province in the Survey of Inflation Expectations. The disaggregated province level data are confidential data kindly made available to us by the Bank of the Italy. The Survey has been conducted quarterly since 1999, in March, June, September and December. The sample comprises about 800 companies, operating in all industries including construction. Individuals are asked to predict the price inflation 6 months ahead, answering the following question: "[If the survey is conducted in June 2013] What do you think consumer price inflation in Italy, measured by the 12-month change in the Harmonized Index of Consumer Prices (HICP), will be in December 2013?". Note that the individuals in the survey are all asked to predict the evolution of the same index (HICP at the national level). In practice, therefore we are assuming that the replies of respondents in the survey in that province reflect the average beliefs of agents in the province.

Net Financial Assets (NFA) Our data on the Italian households' NFA come from the Survey of Household Income and Wealth (SHIW), administered by the Bank of Italy on a representative sample of Italian households. The survey, which is biannual, collects detailed data on households' finances. Each wave surveys about 8,000 households, which, applying weights provided by SHIW (mnemonic Pesofit in SHIW), are fully representative of the Italian resident population. To increase sample size, we use both the 2010 and the 2012 waves. NFA is calculated as the difference between the sum of households' holdings of postal deposits, saving certificates and CDs (mnemonic shiwaf1 in SHIW), government securities (mnemonic shiwaf2) and other securities (mnemonic shiwaf3) minus the sum of their financial liabilities to banks and other financial companies (mnemonic shiwpf1), trade debt (mnemonic shiwpf2) and liabilities to other households (mnemonic shiwpf3).

Creditor households are those with positive NFA (see the construction of the variable NFA for details).

Fraction of creditor households For each province we calculate the pre-announcement fraction of creditor households, based on the 2010 and 2012 waves of SHIW, weighting each household according to the weights provided by SHIW (mnemonic Pesofit).

Inflation expectation bias In each province i and quarter t, we calculate the difference between expected inflation and future realized inflation, which corresponds to equation (1) in the main text.

A.3 European households' Net Financial Assets in HFCS

The Eurosystem Household Finance and Consumption Survey (HFCS) collects fully harmonized data on households' portfolio asset allocation of households and consumption expenditures in the Euro-11 countries (except Ireland). Wealthy individuals are over-sampled for better characterization of the right tail of the income and wealth distribution. Within each country, the sum of the estimation weights equals the total number of households, so that the sum of weights in the entire dataset equals the total number of households in the ten countries of Euro 11 we consider. The structure of the HFCS resembles that of the US Survey of Consumer Finances. To account for measurement error and missing observations, HFCS reports five separate imputation replicates (implicates) for each record. All statistics are calculated by the procedure recommended by HFCS: for each implicate we calculate the desired statistic using HFCS weights (mnemonic hw0010) and then average across the five implicates (mnemonic im0100). The survey was carried out in 2010 except in Finland and the Netherlands, where it was done in 2009, and in Spain (2008). All statistics are at constant 2010 prices.

Net Financial Assets (NFA) is calculated as the the difference between total financial assets and total financial liabilities. Financial assets are (i) deposits (mnemonic da2101); (ii) mutual funds (mnemonic da2102); (iii) bonds (mnemonic da2103); (iv) non self-employment private business (mnemonic da2104); (v) value of self-employment business (mnemonic da1140); (vi) shares of publicly traded companies (mnemonic ds2105); (vii) managed accounts (mnemonic da2106); (viii) money owed to households (mnemonic da2107); (ix) other assets (mnemonic da2108); and (x) voluntary pensions plus whole life insurance (mnemonic da2109). Financial liabilities are the sum of (i) outstanding balance of mortgages on household's main residence (mnemonic dl1110); (ii) outstanding balance of mortgages on other properties (mnemonic dl1120); and (iii) outstanding balance of other non mortgage debt (mnemonic dl1200).

Net Financial Assets net of public debt is obtained by subtracting the country's per household government debt from the household's NFA. The household's country of residence is obtained from mnemonic sa0100. Per household government debt is the country-specific level of net government debt per capita as reported in Table 1 of Adam and Zhu (2015) multiplied by the average number of household members older than 16 as obtained by HFCS (mnemonic dh0006).

Consumption expenditures is the sum of the expenditures during the last 12 months on food and beverages at home (mnemonic hi0100) and on food and beverages outside the home (mnemonic hi0200).

Average labor income in the euro area is the average of the employee income of all household members (mnemonic di1100) for all households whose head is aged 20-65 (mnemonic ra0300). The resulting average labor income is EUR 21,631.

B Proofs

Proof of Lemma 2. $R_0 = \bar{R}$ because the economy is initially in a steady state. Given the timing of the monetary announcement, prices do not respond at t = 0 so $\Pi_0 = \Pi_0^* = 1$, which given (6) yields $R_1 = \bar{R}$. Lemma 1 implies that the economy is back to steady state starting from t = 1 so it must be that $r_t = \bar{R} \ \forall t \ge 2$, which obviously follows from the Euler equation of consumption in (12). By assumption we also have $\Pi_t = \Pi_t^* = 1, \ \forall t \ge 2$ so we have $R_t = r_t = \bar{R} \ \forall t \geq 2$, which immediately gives $R_t = \bar{R} \ \forall t$. And this together with (6) also implies that $\Pi_t = \Pi_t^* \ \forall t \geq 0$.

Proof of Lemma 4. Panel (b) of Figure 7 implies that a credit crunch requires that $\forall j = c, d$, the following conditions should hold:

$$F_2^-(a_{j0},0) = -\left(N_0 + \bar{R}a_{j0}\right)^{-\sigma} + \frac{\beta \bar{V}'(0)}{\min(1,\varepsilon)} > 0$$
(35)

$$F_2^+(a_{j0},0) = -\left(N_0 + \bar{R}a_{j0}\right)^{-\sigma} + \frac{\beta V'(0)}{\max(1,\varepsilon)} < 0$$
(36)

where we used the expression for individual beliefs in (18). Given (17), the condition $a_{c0} = -a_{d0} = B$ and doing some simple algebra it is confirmed that inequalities (35) and (36), evaluated both at j = c and at j = d, are equivalent to the condition $N_0^A < N_0^B$, or alternatively that the inequality in (25) fails.

Proof of Proposition 7. The proof proceeds in three steps. We characterize (i) the full-credibility (FC) benchmark ($\bar{\tau} = 1$ and $\rho = 0$), (ii) an inflationary announcement $\varepsilon > 1$, and (iii) a deflationary announcement $\varepsilon < 1$.

FC benchmark The properties of the FC benchmark are given in Proposition 1, which implies that $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$; B' > 0; $B'/\varepsilon - B < 0$ if $\varepsilon > 1$; and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$. When $\bar{\tau} = 1$ and $\rho = 0$, (SA) also implies that

$$B'/\varepsilon - B = \frac{\varepsilon^{-\frac{1}{\sigma}} N_0 - \bar{N} + B \left[\varepsilon^{-\frac{1}{\sigma}} \left(\bar{R} - \varepsilon\right) - (\bar{R} - 1)\right]}{\bar{R} - 1 + \varepsilon^{1 - \frac{1}{\sigma}}}$$
(37)

with $N_0 = \varepsilon^{\frac{1}{\sigma}} \overline{N}$.

Case $\varepsilon > 1$ If (25) fails we have a credit crunch equilibrium, $B'/\varepsilon = 0$, which immediately implies a larger fall in B'/ε than in the FC benchmark. If (25) holds, then B' > 0 and, from Proposition 5, we have $\bar{\tau} = 1/2$ and $\rho = 1$, which can be substituted into (SA) to show that B'/ε still satisfies (37). After substituting $\bar{\tau} = 1/2$ and $\rho = 1$ into (14) we obtain

$$N_0 = \bar{N} \left(\omega \, \varepsilon^{\frac{1}{\sigma}} + 1 - \omega \right) - B \, \kappa \left(1 - \varepsilon^{\frac{1}{\sigma} - 1} \right) < \bar{N} \, \varepsilon^{\frac{1}{\sigma}},$$

which, together with (37), proves in general that, $\forall B, B'/\varepsilon$ falls more than in the FC benchmark.

Case $\varepsilon < 1$ Proposition 6 implies that if (25) fails, we have a credit crunch equilibrium, $B'/\varepsilon = 0$. If (25) holds, $B'/\varepsilon > 0$, and, from Proposition 5, we have $\bar{\tau} = 1/2$ and $\rho = -1$, which can be substituted into (SA) and (14) to obtain

$$B'/\varepsilon - B = \varepsilon^{-1} \frac{N_0 - \bar{N}}{\bar{R}} + (\varepsilon^{-1} - 1) B,$$

and

$$N_0 = \bar{N} \left(\omega + (1 - \omega) \varepsilon^{\frac{1}{\sigma}} \right) + B \kappa \left(1 - \varepsilon^{\frac{1}{\sigma} - 1} \right).$$

By combining the two expressions, we conclude that $B'/\varepsilon - B < 0$ if

$$B < \tilde{B}_1 \equiv \frac{\bar{N}\left(1 - \varepsilon^{\frac{1}{\sigma}}\right)}{2\,\bar{R} - \left(\bar{R} - 1\right)\varepsilon^{\frac{1}{\sigma}} - \varepsilon\left(1 + \bar{R}\right)},\tag{38}$$

where $B = \tilde{B}_1$ satisfies (25), which generally implies that, $\forall B < \tilde{B}_1, B'/\varepsilon$ falls. Moreover, (DA) evaluated at $\bar{\tau} = 1/2$ and $\rho = -1$ yields

$$B'/\varepsilon - B = \frac{\bar{N} - \varepsilon^{-\frac{1}{\sigma}} N_0 + B \left[\varepsilon^{-\frac{1}{\sigma}} \left(\bar{R} - \varepsilon\right) - (\bar{R} - 1)\right]}{\bar{R} - 1 + \varepsilon^{1 - \frac{1}{\sigma}}}.$$
(39)

Comparing (37) with (39), we immediately conclude that B'/ε increases less than in the FC benchmark if and only if $N_0 > \bar{N} \varepsilon^{\frac{1}{\sigma}}$. Proposition 2 implies that (under $\varepsilon < 1$ and $\rho < 0$, which is implied by Proposition 5) N_0 is strictly decreasing in B, whereas N_0 in the FC benchmark ($\rho = 0$) is invariant to B. So we conclude that B'/ε increases less (more) than in the FC benchmark if and only if $B < \tilde{B}_2$ ($B > \tilde{B}_2$) where

$$\tilde{B}_2 \equiv \bar{N} \frac{\left[1 + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma} - 1}\right] (1 - \varepsilon^{\frac{1}{\sigma}})}{\bar{R} (\bar{R} - 1) (1 - \varepsilon^{\frac{1}{\sigma} - 1})}$$

is the value of B at which $N_0 = \bar{N} \varepsilon^{\frac{1}{\sigma}}$. Remember that at $B = \tilde{B}_1$ we have $B'/\varepsilon - B = 0$, so from (39) we conclude that $N_0 > \bar{N} \varepsilon^{\frac{1}{\sigma}}$. Thus the definition of \tilde{B}_2 together with the fact that N_0 is strictly decreasing in B immediately implies that $\tilde{B}_2 > \tilde{B}_1$.

C Computational details

We solve the extended model of Section 5 in four steps. In the first step we guess the nominal interest rate at t = T, $R_T = \hat{R}_T$. In the second step, we take the guess for R_T as given and construct the function $\bar{V}(a_{xT}, \Pi_T^*)$ in (34) by solving for the equilibrium of the economy at $t \geq T$, for different values of Π_T^* . In the third step we take the function $\bar{V}(a_{xT}, \Pi_T^*)$ as given from step 2 and solve for the equilibrium of the economy at t < T. This yields a new value for R_T equal to \hat{R}'_T . In the fourth step we check convergence. We describe the four steps below.

Step 1 We guess a value for R_T , say \hat{R} .

Step 2 Given $R_T = \hat{R}$, we solve the equilibrium of the model at $t \ge T$ under two scenarios: $\Pi_T^* = \varepsilon$ and $\Pi_T^* = 1$. Notice that Π_T^* fully determines $\Pi_t^* \forall t \ge T$. We index equilibrium quantities by the superscript 1 if $\Pi_T^* = \varepsilon$; by the superscript 0 if $\Pi_T^* = 1$. Then, $\forall m = 0, 1$, we guess a path of output, $\{\hat{Y}_t^m\}_{t\ge T}$. Notice that when $R_T = \bar{R}, Y_t^0 = \bar{Y} \forall t \ge T$ (see Proposition 8). Given an output path, (11) together with the labor market clearing condition yields a path for wages, $\{\hat{w}_t^m\}_{t\ge T}$. Given output and wages, (6) and (29) jointly determine the path of inflation $\{\hat{\Pi}_t^m\}_{t\ge T}$ and nominal interest rates $\{\hat{R}_t^m\}_{t\ge T}$, where interest rates satisfy $\hat{r}_t^m = \hat{R}_t^m/\hat{\Pi}_t^m$. The path of dividends $\{\hat{D}_t^m\}_{t\ge T}$ is obtained using (30). Then we obtain aggregate consumption $\{\hat{C}_t^m\}_{t\geq T}$ from (33). Given the path of inflation $\{\hat{\Pi}_t^m\}_{t\geq T}$ and aggregate consumption $\{\hat{C}_t^m\}_{t\geq T}$, we apply (31) to obtain a new sequence of output, denoted by $\{Y_t^m\}_{t\geq T}$. If $\max_{t\geq T} |Y_t^m - \hat{Y}_t^m| < |\varepsilon - 1| \times 10^{-5}$ we stop, and the initial guess for the output sequence $\{\hat{Y}_t^m\}_{t\geq T}$ is verified; otherwise we use $\{Y_t^m\}_{t\geq T}$ to update the guess for $\{\hat{Y}_t^m\}_{t\geq T}$ and reiterate until convergence. After achieving convergence for m = 0, 1, we construct the function $\bar{V}(a_{xT}, \Pi_T^*)$ in (34), find a^* such that $\bar{V}(a^*, \varepsilon) = \bar{V}(a^*, 1)$, and then go to step 3.

Step 3 For all t < T, we conjecture a path of output, $\{\hat{Y}_t\}_{t < T}$. Given $\{\hat{Y}_t\}_{t < T}$, (11) together with the labor market clearing condition yields a path of wages, $\{\hat{w}_t\}_{t < T}$. Let $\{\tilde{w}_{xt}\}_{t \geq 0}$ and $\{\tilde{Y}_{xt}\}_{t \geq 0}$ denote household x's beliefs about the path of wages and output, respectively. Since households share the same beliefs about all variables $\forall t < T$, we have that $\{\tilde{w}_{xt}\}_{t \geq 0} = \{\hat{w}_0, \hat{w}_1, ... \hat{w}_{T-1}, \hat{w}_T^1, \hat{w}_{T+1}^1, ...\}$ if $a_{xT} > a^*$ and $\{\tilde{w}_{xt}\}_{t \geq 0} = \{\hat{w}_0, \hat{w}_1, ... \hat{w}_{T-1}, \hat{w}_T^0, \hat{w}_{T+1}^0, ...\}$ if $a_{xT} < a^*$. Given $\{\tilde{w}_{xt}\}_{t \geq 0}$, (6) and (29) determine household x's beliefs about the path of the interest rate $\{\tilde{r}_{xt}\}_{t \geq 0}$ and inflation $\{\tilde{\pi}_{xt}\}_{t \geq 0}$. There are then three cases to consider: (i) household x believes the announcement, which requires $a_{xT} < a^*$; (ii) she does not believe the announcement, which requires $a_{xT} < a^*$; (ii) she has degenerate beliefs, which requires $a_{xT} = a^*$. In cases (i) and (ii), household x's consumption can be obtained by integrating forward the Euler condition in (12) to yield

$$c_{xt} = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \left(\sum_{s=0}^{\infty} \beta^{\frac{s}{\sigma}} \tilde{q}_{xt,t+s}^{1-\frac{1}{\sigma}}\right)^{-1} \left(a_{xt} r_t + \frac{\psi_0 \psi}{1+\psi} \sum_{s=0}^{\infty} \tilde{q}_{xt,t+s} \tilde{Y}_{xt+s}^{1+\psi}\right), \quad (40)$$

where $\tilde{q}_{xt,t+s} = (\prod_{n=1}^{s} \tilde{r}_{xt+n})^{-1}$. We use (40) to calculate c_{xt} both under the assumption that household x believes the announcement, m = 1, and under the assumption that it does not, m = 0. Applying the household's budget constraint, we obtain an associated value of a_{xT} denoted by a_{xT}^m , $\forall m = 0, 1$. If $a_{xT}^1 > a^*$, then household x believes the announcement (case (i)); if $a_{xT}^0 < a^*$, it does not believe the announcement (case (ii)). Notice that $a_{xT}^1 > a^*$ and $a_{xT}^0 < a^*$ cannot both hold. If neither $a_{xT}^1 > a^*$ nor $a_{xT}^0 < a^*$ is verified, we have case (iii), so $a_{xT} = a^*$, which can be used together with (12) to determine the path of consumption of household $x \forall t < T$. Once we have $\{c_{xt}\}_{t<T} \forall x \in [0, 1]$, we calculate $C_t = \int_0^1 c_{xt} dx$ and use (31) together with (29) to obtain a new sequence of output $\forall t < T - 1$, which is denoted by $\{\hat{Y}'_t\}_{t<T}$. If $\max_{t<T} |\hat{Y}'_t - \hat{Y}_t| < |\varepsilon - 1| \times 10^{-5}$ we stop, use (6) to calculate R_T , set $\hat{R}'_T = R_T$, and go to step 4; otherwise we use $\{\hat{Y}'_t\}_{t<T}$ to update the guess for $\{\hat{Y}_t\}_{t<T}$ and iterate until convergence.

Step 4 If $|\hat{R}'_T - \hat{R}_T| > 10^{-4} |\varepsilon - 1|$ we go back to step 1 and use \hat{R}'_T to update our guess for R_T to a new \hat{R}_T ; otherwise the algorithm finally converges and we stop.

D Further robustness checks

D.1 Intertemporal elasticity of substitution

In this Section we increase σ from 2 to 2.5, corresponding to a drop in the elasticity of intertemporal substitution from 0.5 to 0.4. Figure A1 shows that a lower EIS makes forward guidance less expansionary: relative to the baseline specification, output falls more $\forall t < T$, and increases less $\forall t \geq T$.





D.2 Horizon of the announcement

In this section we consider an announcement at a time horizon of 2 years, T = 8. Figure A2 shows that output falls less on impact than in the baseline specification (T = 6). This is because the expected cumulative decline in the interest rate before T is mechanically larger when T is farther ahead. Thus, the substitution effect on consumption is relatively stronger, which makes output increase more (or decrease less) on impact. As we approach T, untrusting households expect the real rate to go back to steady state sooner and reduce their consumption more, decreasing output relative to the baseline specification.





D.3 Taylor rule at t < T

In this section we assume that the nominal interest rate follows the Taylor rule in (6) also in the interim period period up to T. We reestimate the path of the inflation target to match the observed response of the yield curve. The new profile of the inflation target is shown in panel (b) of Figure A3, the response of output in panel (a). Output now falls immediately at the time of the announcement and remains at this lower level in all periods until T. This is due to the Taylor rule in (6) and the increased inflation in the interim period before T, which leads to an increase in nominal interest rates above their steady state level in all periods before T.



