

Uncertainty Shocks in a Model of Effective Demand*

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Abstract

Can increased uncertainty about the future cause a contraction in output and its components? This paper examines uncertainty shocks in a one-sector, representative-agent general-equilibrium model. When prices are flexible, uncertainty shocks cannot produce business-cycle comovements among key macroeconomic variables. However, uncertainty shocks can generate business-cycle fluctuations with countercyclical markups through sticky prices. If the central bank is constrained by the zero lower bound, higher uncertainty has even more negative effects on the economy. We calibrate our model using fluctuations in the VIX and find that increased uncertainty about the future may have played a role in worsening the Great Recession.

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1 Introduction

Economists and the financial press often discuss uncertainty about the future as an important driver of economic fluctuations, and a contributor in the Great Recession and subsequent slow recovery. For example, Diamond (2010) says, “What’s critical right now is not the functioning of the labor market, but the limits on the demand for labor coming from the great caution on the side of both consumers and firms because of the great uncertainty of what’s going to happen next.” Recent research by Bloom (2009), Bloom, Foetotto, Jaimovich, Saporta-Eksten, and Terry (2011), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2011), Born and Pfeifer (2011), and Gilchrist, Sim, and Zakrajšek (2010) also suggests that uncertainty shocks can cause fluctuations in macroeconomic aggregates. However, most of these papers experience difficulty in generating business-cycle comovements among output, consumption, investment, and hours worked from changes in uncertainty. If uncertainty is a contributing factor in the Great Recession and persistently slow recovery, then increased uncertainty should reduce output and its components.

In this paper, we show why competitive, one-sector, closed-economy models generally cannot generate business-cycle comovements in response to changes in uncertainty. Under reasonable assumptions, an increase in uncertainty about the future induces precautionary saving and lower consumption. If households supply labor inelastically, then total output remains constant since the level of technology and capital stock remain unchanged in response to the uncertainty shock. Unchanged total output and reduced consumption together imply that investment must rise. If households can adjust their labor supply and consumption and leisure are both normal goods, an increase in uncertainty also induces “precautionary labor supply,” or a desire for the household to supply more labor for an given level of the real wage. As current technology and the capital stock remain unchanged, the competitive demand for labor remains unchanged as well. Thus, higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple neoclassical models subject to uncertainty fluctuations.

We also show that non-competitive, one-sector models with countercyclical markups through sticky prices can easily overcome the comovement problem and generate simultaneous drops in output, consumption, investment, and hours worked in response

to an uncertainty shock. An increase in uncertainty induces precautionary labor supply by the representative household, which reduces firm marginal costs of production. Falling marginal costs with slowly-adjusting prices imply an increase in firm markups over marginal cost. A higher markup reduces the demand for consumption, and especially, investment goods. Since output is demand-determined in these models, output and employment must fall when consumption and investment both decline. Thus, comovement is restored, and uncertainty shocks cause fluctuations that look qualitatively like a business cycle. Returning to Diamond’s (2010) intuition, simple competitive business-cycle models do not exhibit movements in “the demand for labor” as a result of an uncertainty shock. However, uncertainty shocks easily cause fluctuations in the demand for labor in non-competitive, sticky-price models with endogenously-varying markups. Thus, the non-competitive model captures the intuition articulated by Diamond. Understanding the dynamics of the demand for labor explains why the two models behave so differently in response to a change in uncertainty. Importantly, the non-competitive model is able to match the estimated effects of uncertainty shocks in the data by Bloom (2009) and Alexopoulos and Cohen (2009), while the competitive model cannot.

To analyze the quantitative impact of uncertainty shocks under flexible and sticky prices, we calibrate and solve a representative-agent, dynamic stochastic general equilibrium model with nominal price rigidity. We examine uncertainty shocks to both technology and household discount factors, which we interpret as cost and demand uncertainty. We calibrate our uncertainty shock processes using the Chicago Board Options Exchange Volatility Index (VIX), which measures the expected volatility of the Standard and Poor’s 500 stock index over the next thirty days. Using a third-order approximation to the policy functions of our calibrated model, we show that uncertainty shocks can produce contractions in output and all its components when prices adjust slowly. In particular, we find that increased uncertainty associated with future demand can produce significant declines in output, hours, consumption, and investment. Our model predicts that a one standard deviation increase in the uncertainty about future demand produces a peak decline in output of about 0.2 percent.

Finally, we examine the role of monetary policy in determining the equilibrium effects of uncertainty shocks. Standard monetary policy rules imply that the central bank usually offsets increases in uncertainty by lowering its nominal policy rate. We show that increases in uncertainty have larger negative impacts on the economy if the monetary authority is

constrained by the zero lower bound on nominal interest rates. In these circumstances, our model predicts that an increase in uncertainty causes a much larger decline in output and its components. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that greater uncertainty may have plausibly contributed significantly to the large and persistent output decline starting at that time. Our results suggest that about one-fourth of the drop in output that occurred in late 2008 can plausibly be ascribed to increased uncertainty about the future.

Our emphasis on the effects of uncertainty in a one-sector model does not mean that we deprecate alternative modeling strategies. For example, Bloom, Foetotto, Jaimovich, Saporta-Eksten, and Terry (2011) examine changes in uncertainty in a heterogeneous-firm model with convex and non-convex adjustment costs. However, this complex model is unable to generate positive comovement of the four key macro aggregates following an uncertainty shock. Furthermore, heterogeneous-agent models are challenging technically to extend along other dimensions. For example, adding nominal price rigidity for each firm and a zero lower bound constraint on nominal interest rates would be difficult in the model of Bloom, Foetotto, Jaimovich, Saporta-Eksten, and Terry (2011). We view our work as a complementary approach to modeling the business-cycle effects of uncertainty. The simplicity of our underlying framework allows us to tackle additional issues that we think are important for understanding the Great Recession.

2 Intuition

This section formalizes the intuition from the introduction using a few key equations that characterize a large class of one-sector business cycle models. We show that the causal ordering of these equations plays an important role in understanding the impact of uncertainty shocks. These equations link total output Y_t , household consumption C_t , investment I_t , hours worked N_t , and the real wage W_t/P_t . The following key equations consist of a “demand” equation, an aggregate production function, and a static first-order condition for a representative consumer to maximize utility:

$$Y_t = C_t + I_t, \tag{1}$$

$$Y_t = F(K_t, Z_t N_t), \quad (2)$$

$$\frac{W_t}{P_t} U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \quad (3)$$

Typical partial-equilibrium results suggest that an increase in uncertainty about the future decreases both consumption and investment. When consumers face a stochastic income stream, higher uncertainty about the future induces precautionary saving by risk-averse households. Recent work by Bloom (2009) argues that an increase in uncertainty also depresses investment, particularly in the presence of non-convex costs of adjustment. If an increase in uncertainty lowers consumption and investment in partial equilibrium, Equation (1) suggests that it should lower total output in a general-equilibrium model. In a setting where output is demand-determined, economic intuition suggests that higher uncertainty should depress total output and its components.

However, the previous intuition is incorrect in a general-equilibrium neoclassical model with a representative firm and a consumer with additively time-separable preferences. In this neoclassical setting, labor demand (the partial derivative of Equation (2) with respect to N_t) is determined by the current level of capital and technology, neither of which changes when uncertainty increases. The first-order conditions for firm labor demand derived from Equation (2) and the labor supply condition in Equation (3) can be combined to yield:

$$Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \quad (4)$$

Equation (4) defines a positively-sloped “income expansion path” for consumption and leisure for given levels of capital and technology. If higher uncertainty reduces consumption, then Equation (4) shows that increased uncertainty must increase labor supply. However, Equation (2) implies that total output must rise. A reduction in consumption and an increase in total output in Equation (1) means that investment and consumption must move in opposite directions.¹

In a non-neoclassical setting, especially one with a time-varying markup of price over marginal cost, Equations (1) and (3) continue to apply, but Equation (4) must be modified, and becomes:

$$\frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t) \quad (5)$$

¹This argument follows Barro and King (1984). Jaimovich (2008) shows that this prediction may not hold for certain classes of preferences that are not additively time-separable.

where μ_t is the markup of price over marginal cost.

In such a setting, Equation (1) is causally prior to Equations (2) and (3). From Equation (1), output is determined by aggregate demand. Equation (2) then determines the necessary quantity of labor input for given values of K_t and Z_t . Finally, given C_t (determined by demand and other factors), the necessary supply of labor is made consistent with consumer optimization by having the markup taking on its required value. Alternatively, the wage moves to the level necessary for firms to hire the required quantity of labor, and the variable markup ensures that the wage can move independently of the marginal product of labor.

The previous intuition can also be represented graphically using simplified labor supply and labor demand curves in real wage and hours worked space. Figures 1 and 2 show the impact of an increase in uncertainty under both flexible prices with constant markups and sticky prices with endogenously-varying markups. An increase in uncertainty induces wealth effects on the representative household through the forward-looking marginal utility of wealth denoted by λ_t . An increase in the marginal utility of wealth shifts the household labor supply curve outward. With flexible prices and constant markups, the labor demand curve remains fixed for a given level of the real wage. In the flexible-price equilibrium, the desire of households to supply more labor translates into higher equilibrium hours worked and a lower real wage. When prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases their labor supply. For a given level of the real wage, an increase in markups decreases the demand for labor from firms. Figure 2 shows that equilibrium hours worked may fall as a result of the outward shift in the labor supply curve and the inward shift of the labor demand curve. The relative magnitudes of the changes in labor supply and labor demand depend on the specifics of the macroeconomic model and its parameter values. The following section shows that in a reasonably calibrated New-Keynesian sticky price model, firm markups increase enough to produce a decrease in equilibrium hours worked in response to an increase in uncertainty.

3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that we use in our analysis of uncertainty shocks. Our model provides a specific quantitative

example of the intuition of the previous section. The baseline model shares many features with the models of Ireland (2003), Ireland (2011), and Jermann (1998). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers both technology shocks and household discount rate shocks. Both shocks have time-varying second moments, which have the interpretation of cost uncertainty and demand uncertainty.

3.1 Households

In our model, the representative household maximizes lifetime utility given Epstein-Zin preferences over streams of consumption, C_t , and leisure, $1 - N_t$. The household solves its optimization problem subject to its risk aversion over the consumption-leisure basket σ and its intertemporal elasticity of substitution ψ . The parameter $\theta_V \triangleq (1 - \sigma)(1 - 1/\psi)^{-1}$ controls the household's preference for the resolution of uncertainty.² The household receives labor income W_t for each unit of labor N_t supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares S_t and one-period riskless bonds B_t issued by representative intermediate goods firm. Equity shares pay dividends D_t^E for each share S_t owned, and the riskless bonds return the gross one-period risk-free interest rate R_t^R . The household divides its income from labor and its financial assets between consumption C_t and the amount of financial assets S_{t+1} and B_{t+1} to carry into next period. The discount rate of the household β is subject to shocks via the stochastic process a_t . Since our model is a standard dynamic general-equilibrium model without government, any non-technological source of shocks must come from changes in preferences. Therefore, we interpret changes in the household discount factor as demand shocks hitting the economy.

The representative household maximizes lifetime utility by choosing C_{t+s} , N_{t+s} , B_{t+s+1} , and S_{t+s+1} for all $s = 0, 1, 2, \dots$ by solving the following problem:

$$V_t = \max \left[a_t (C_t^\eta (1 - N_t)^{1-\eta})^{\frac{1-\sigma}{\theta_V}} + \beta (\mathbb{E}_t V_{t+1}^{1-\sigma})^{\frac{1}{\theta_V}} \right]^{\frac{\theta_V}{1-\sigma}}$$

²Our main results are robust to using expected utility preferences over consumption and leisure. The use of Epstein-Zin preferences allows us to calibrate our model using stock market data. Section 6.1 explains the details of our calibration method.

subject to its intertemporal household budget constraint each period,

$$C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left(\frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\frac{\partial V_t}{\partial C_t} = \lambda_t \quad (6)$$

$$\frac{\partial V_t}{\partial N_t} = \lambda_t \frac{W_t}{P_t} \quad (7)$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\} \quad (8)$$

$$1 = R_t^R \mathbb{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) \right\} \quad (9)$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. The utility function specification implies the following stochastic discount factor M_{t+1} :

$$M_{t+1} \triangleq \left(\frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \right) = \left(\beta \frac{a_{t+1}}{a_t} \right) \left(\frac{C_{t+1}^\eta (1 - N_{t+1})^{1-\eta}}{C_t^\eta (1 - N_t)^{1-\eta}} \right)^{\frac{1-\sigma}{\theta_V}} \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{V_{t+1}}{\mathbb{E}_t [V_{t+1}^{1-\sigma}]} \right)^{1-\frac{1}{\theta_V}}$$

Using the stochastic discount factor, we can eliminate λ and simplify Equations (7) - (9) as follows:

$$\frac{1-\eta}{\eta} \frac{C_t}{1-N_t} = \frac{W_t}{P_t} \quad (10)$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ M_{t+1} \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\} \quad (11)$$

$$1 = R_t^R \mathbb{E}_t \{ M_{t+1} \} \quad (12)$$

Equation (10) represents the household intratemporal optimality condition with respect to consumption and leisure, and Equations (11) and (12) represent the Euler equations for equity shares and one-period riskless firm bonds.

3.2 Intermediate Goods Producers

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The intermediate-goods firms own the capital stock $K_t(i)$ for the economy and face adjustment costs for adjusting its rate of investment. Each firm issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm i chooses $N_t(i)$, $I_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production Φ .

Each intermediate goods-producing firm maximizes discounted cash flows using the household stochastic discount factor:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s} \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t \leq K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} - \Phi,$$

and subject to the capital accumulation equation:

$$K_{t+1}(i) = \left(1 - \delta - \frac{\phi_K}{2} \left(\frac{I_t(i)}{K_t(i)} - \delta \right)^2 \right) K_t(i) + I_t(i)$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta_\mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

The behavior of each firm i satisfies the following first-order conditions:

$$\frac{W_t}{P_t} N_t(i) = (1 - \alpha) \Xi_t K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} \tag{13}$$

$$\frac{R_t^K}{P_t} K_t(i) = \alpha \Xi_t K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} \quad (14)$$

$$\begin{aligned} \phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[\frac{P_t}{\Pi P_{t-1}(i)} \right] &= (1 - \theta_\mu) \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} + \theta_\mu \Xi_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu - 1} \\ &+ \phi_P \mathbb{E}_t \left\{ M_{t+1} \frac{Y_{t+1}}{Y_t} \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\} \end{aligned} \quad (15)$$

$$q_t = \mathbb{E}_t \left\{ M_{t+1} \left(R_{t+1}^K + q_{t+1} \left(1 - \delta - \frac{\phi_K}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi_K \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \right) \right\} \quad (16)$$

$$\frac{1}{q_t} = 1 - \phi_K \left(\frac{I_t}{K_t} - \delta \right) \quad (17)$$

where Ξ_t is the marginal cost of producing one additional unit of intermediate good i , and q_t is the price of a marginal unit of installed capital. R_t^K/P_t is the marginal revenue product of capital, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by Jermann (1998) and Ireland (2003), and allows Tobin's q to vary over time.

Each intermediate goods firm finances a percentage ν of its capital stock each period with one-period riskless bonds. The bonds pay the one-period real risk-free interest rate. Thus, the quantity of bonds $B_t(i) = \nu K_t(i)$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left(K_t(i) - \frac{1}{R_t^R} K_{t+1}(i) \right). \quad (18)$$

Since the Modigliani and Miller (1958) theorem holds in our model, leverage does not affect firm value or optimal firm decisions. Leverage makes the payouts and price of equity more volatile and allows us to define a concept of equity returns in the model. We use the volatility of equity returns implied by the model to calibrate our uncertainty shock processes in Section 6.

3.3 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is

transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_\mu - 1}{\theta_\mu}} di \right]^{\frac{\theta_\mu}{\theta_\mu - 1}} \geq Y_t$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t . The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_\mu} di \right]^{\frac{1}{1-\theta_\mu}}$$

3.4 Monetary Policy

We assume a cashless economy where the monetary authority sets the net nominal interest rate r_t to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (r + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t), \quad (19)$$

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta y_t = \ln(Y_t/Y_{t-1})$. Changes in the nominal interest rate affect expected inflation and the real interest through the Fisher relation $\ln(R_t) = \ln(\mathbb{E}_t \Pi_{t+1}) + \ln(R_t^R)$. Thus, we include the following Euler equation for a zero net supply

nominal bond in our equilibrium conditions:

$$1 = R_t \mathbb{E}_t \left\{ \left(\frac{M_{t+1}}{\Pi_{t+1}} \right) \right\} \quad (20)$$

3.5 Equilibrium

The assumption of Rotemberg (1982) (as opposed to Calvo (1983)) pricing implies that we can model our production sector as a single representative intermediate goods-producing firm. In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose to hold the same amount of capital $K_t(i) = K_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. With a representative firm, we can define the unique markup of price over marginal cost as $\mu_t = 1/\Xi_t$, and gross inflation as $\Pi_t = P_t/P_{t-1}$.

3.6 Shock Processes

In our baseline model, we are interested in capturing the effects of independent changes in the level and volatility of both the technology process and the preference shock process. The technology and preference shock processes are parameterized as follows:

$$\begin{aligned} Z_t &= (1 - \rho_z) Z + \rho_z (Z_{t-1}) + \sigma_t^z \varepsilon_t^z \\ \sigma_t^z &= (1 - \rho_{\sigma^z}) \sigma^z + \rho_{\sigma^z} \sigma_{t-1}^z + \sigma^{\sigma^z} \varepsilon_t^{\sigma^z} \\ a_t &= (1 - \rho_a) a + \rho_a a_{t-1} + \sigma_t^a \varepsilon_t^a \\ \sigma_t^a &= (1 - \rho_{\sigma^a}) \sigma^a + \rho_{\sigma^a} \sigma_{t-1}^a + \sigma^{\sigma^a} \varepsilon_t^{\sigma^a} \end{aligned}$$

ε_t^z and ε_t^a are first moment shocks that capture innovations to the level of the stochastic processes for technology and household discount factors. We refer to $\varepsilon_t^{\sigma^z}$ and $\varepsilon_t^{\sigma^a}$ as second moment or ‘‘uncertainty’’ shocks since they capture innovations to the volatility of the exogenous processes of the model. An increase in the volatility of the shock process increases the uncertainty about the future time path of the stochastic process. All four stochastic shocks are independent, standard normal random variables.

3.7 Solution Method

Our primary focus of this paper is to examine the effects of increases in the second moments of the shock processes. Using a standard first-order or log-linear approximation to the equilibrium conditions of our model would not allow us to examine second moment shocks, since the approximated policy functions are invariant to the volatility of the shock processes. Similarly, second moment shocks would only enter as cross-products with the other state variables in a second-order approximation to the policy functions, and thus we could not study the effects of shocks to the second moments alone. In a third-order approximation, however, second moment shocks enter independently in the approximated policy functions. Thus, a third-order approximation allows us to compute an impulse response to an increase in the volatility of technology or discount rate shocks, while holding constant the levels of those variables.

To solve the baseline model, we use the Perturbation AIM algorithm and software developed by Swanson, Anderson, and Levin (2006). Perturbation AIM uses Mathematica to compute the rational expectations solution to the model using n th-order Taylor series approximation around the nonstochastic steady state of the model. We find that a third-order approximation to the policy functions is sufficient to capture the dynamics of the baseline model. As discussed in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011), approximations higher than first-order move the ergodic distributions of the endogenous variables of the model away from their deterministic steady-state values. In the following analysis, we compute the impulse responses in percent deviation from the ergodic mean of each model variable.

4 Calibration and Baseline Results

4.1 Calibration

Table 1 lists the calibrated parameters of the model. We calibrate the model at a quarterly frequency, using standard parameters for one-sector models of fluctuations. Since our model shares many features with the estimated models of Ireland (2003) and Ireland (2011), we calibrate our model to match the estimated parameters reported in those papers. We use the estimates in these papers to calibrate the steady-state volatilities for the technology and preference shocks, σ^z and σ^a . We calibrate the steady-state level

of the discount factor and technology processes a and Z to both equal one. To assist in numerically calibrating and solving the model, we introduce constants into the period utility function and the production function to normalize the value function V and output Y to both equal one at the deterministic steady state. We choose steady-state hours worked N and the model-implied value for η such that our model has a Frisch labor supply elasticity of 1. Our calibration of ϕ_K implies an elasticity of the investment-capital ratio with respect to marginal q of 2.0. The household IES is calibrated to 0.50, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm Φ is calibrated to eliminate pure profits in the deterministic steady state of the model. Risk aversion over the consumption and leisure basket σ is set to 60, which is inline with the estimated values of van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2010) and Swanson and Rudebusch (2012). We discuss our calibration of the uncertainty shock stochastic processes in depth in Section 6. In the following analysis, we compare the results from our baseline sticky-price calibration ($\phi_P = 160$) with a flexible-price calibration ($\phi_P = 0$).

4.2 Uncertainty Shocks & Business Cycle Comovements

Holding the calibrated parameters fixed, we analyze the effects of an exogenous increase in uncertainty associated with technology or household demand. Figure 3 plots the impulse responses of the model to a technology uncertainty shock and Figure 4 plots the responses to a demand uncertainty shock. The results are consistent with the intuition of Section 2 and the labor market diagrams in Figures 1 and 2. Uncertainty from either technology or household demand both enter Equation (4) or Equation (5) through the forward-looking marginal utility of wealth. An uncertainty shock associated with either stochastic process induces wealth effects on the household which triggers precautionary labor supply. Thus, the responses and time paths for the endogenous variables look qualitatively similar for both types of uncertainty shocks.

Households want to consume less and save more when uncertainty increases in the economy. In order to save more, households optimally wish to both reduce consumption and increase hours worked. Under flexible prices and constant markups, equilibrium labor supply and consumption follow the path that households desire when they face higher uncertainty. On impact of the uncertainty shock, the level of capital is predetermined,

the level of the shock process is held constant, and thus labor demand is unchanged for a given real wage. Under flexible prices, the outward shift in labor supply combined with unchanged labor demand increases hours worked and output. After the impact period, households continue to save, consume less, and work more hours. Since firms own the capital stock, higher household saving translates into higher capital accumulation for firms. Throughout the life of the uncertainty shock, consumption and investment move in opposite directions, which is inconsistent with basic business-cycle comovements.

Under sticky prices, households also want to consume less and save more when the economy is hit by an uncertainty shock associated with technology or household demand. On impact, households increase their labor supply and reduce consumption to accumulate more assets. With sticky prices, however, increased labor supply decreases the marginal costs of production of the intermediate goods firms. A reduction in marginal cost with slowly-adjusting prices increases firm markups. An increase in markups lowers the demand for household labor and lowers the real wage earned by the representative household. The decrease in labor demand also lowers investment in the capital stock by firms. In equilibrium, these effects combine to produce significant falls in output, consumption, investment, hours worked, and the real wage, which are consistent with business-cycle facts. Thus, the desire by households to work more can actually lead to lower labor input and output in equilibrium.

5 Discussion and Connections

5.1 Specific Example of General Principle

The differential response of our economy under flexible and sticky prices to uncertainty fluctuations is a specific instance of the general proposition established by [Basu and Kimball \(2005\)](#). They show that “good” shocks that cause output to rise in a flexible-price model generally tend to have contractionary effects in a model with nominal price rigidity. [Basu and Kimball \(2005\)](#) also show that the response of monetary policy is critical for determining the equilibrium response of output and other variables. If monetary policy follows a sensible rule, for example the celebrated [Taylor \(1993\)](#) rule, then the monetary authority typically lowers the nominal interest rate to offset the negative short-run effects of the shock. Our results show, however, this effect is not strong enough for standard

parameter values. Even though the monetary authority in our model lowers interest rates when uncertainty rises, it does not succeed in offsetting the contractionary effects of uncertainty with nominal rigidities. In keeping with the bulk of the literature, we do not model why the monetary policy rule does not react more aggressively to uncertainty in normal times. However, we do investigate in depth one particular barrier to expansionary monetary policy that is critical for understanding the Great Recession: the zero lower bound constraint on nominal interest rates. If uncertainty increases when the monetary authority is unable to lower the nominal interest rate further because the policy rate is essentially zero, as was the case in late 2008 and early 2009, then the short-run contractionary effect of the “good” shock dominates, and the equilibrium response of output becomes robustly negative. We explore this issue in Section 7.

5.2 Extension to Sticky Nominal Wages

Our exposition so far suggests that the mechanism we have identified works only in the special case where nominal prices are sticky but wages are flexible. Indeed, our intuition for the channel through which an increase in uncertainty raises the markup has emphasized these two elements. We argued that higher uncertainty induces households to work at lower wages, the reduction in the wage reduces firms marginal costs, but since their output prices are fixed, lower marginal costs translate to higher markups, which are contractionary. However, various types of evidence suggests that nominal wages are sticky, not flexible, especially at high frequencies. At the macro level, [Christiano, Eichenbaum, and Evans \(2005\)](#) find that nominal wage stickiness is actually more important than nominal price stickiness for explaining the observed impact of monetary policy shocks. At the micro level, [Barattieri, Basu, and Gottschalk \(2010\)](#) find that the wages of individual workers are often unchanged for long periods of time (with wages changed, on average, less than once a year).

In this subsection, we show that our results extend readily to the case where either or both nominal prices and wages are sticky. Rather than writing down an extended model with two nominal frictions, we make our point heuristically, using the graphical labor supply-labor demand apparatus of Section 2. As we argued above, if households act competitively in the labor market:

$$U_2(C_t, 1 - N_t) = \lambda_t W_t, \tag{21}$$

where W is the nominal wage and λ is the shadow value of nominal wealth (the utility value of the marginal dollar). Assuming firms have market power, cost-minimization implies that

$$W_t = \frac{P_t}{\mu_t^P} Z_t F_2(K_t, Z_t N_t). \quad (22)$$

Thus,

$$\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t), \quad (23)$$

where μ_t^P is the price-markup over marginal cost.

Now assume a new model, where households also have market power, and set wages with a markup over their marginal disutility of work:

$$W_t = \mu_t^W \frac{U_2(C_t, 1 - N_t)}{\lambda_t} \quad (24)$$

Then,

$$\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^W} \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t) \quad (25)$$

In our labor market diagrams, suppose we replace the labor supply curve with $U_2(C_t, 1 - N_t)/\lambda_t P_t$. This quantity has the interpretation of being the disutility faced by the household of supplying one more unit of labor, expressed in units of real goods (the real marginal cost of supplying labor). On the vertical axis, put the equilibrium level of the real marginal disutility of work. Note that this ‘supply curve’ is shifted in exactly the same way by uncertainty as the standard labor supply curve of Figures 1 and 2 – higher uncertainty raises λ , which shifts the supply curve out. But now the ‘demand curve’ (the right-hand side of (25)) is shifted by both price and wage markups – only the product of the two matters. Take the polar opposite of the case we have analyzed so far: Assume perfect competition in product markets, but Rotemberg wage setting by monopolistically competitive households in the labor market. Then the price markup is always fixed at 1, but the wage markup would jump up in response to an increase in uncertainty (since the marginal cost of supplying labor falls but the wage is sticky), making the qualitative outcome exactly the same as in our current case with only sticky prices and flexible wages. Thus, while introducing nominal wage stickiness would certainly affect quantitative magnitudes, it would not change our qualitative results.

5.3 Connections with Existing Literature

Our framework can be used to understand the economic mechanisms at work in some recent papers in the literature. Recent work by Bloom, Foetotto, Jaimovich, Saporta-Eksten, and Terry (2011), Chugh (2010), and Gilchrist, Sim, and Zakrajšek (2010) uses flexible-price models to show that shocks to uncertainty can lead to fluctuations that resemble business cycles. Their modeling approach is to drop Equation (2) and use multi-sector models of production. Following the insight of Bloom (2009), the normal industry equilibrium in these models features resource reallocation from low- to high-productivity firms. Higher uncertainty impedes the reallocation process by reducing the necessary investment or disinvestment needed to move capital and labor to higher-productivity uses. These models use multi-sector production and costly factor adjustment to transform a change in the expected future dispersion of total factor productivity (TFP) into a change in the current mean of the TFP distribution.³ This approach may allow equilibrium real wages, consumption and labor supply to move in the same direction. However, all three papers experience difficulties in getting the desired comovements, at least for calibrations that are consistent with steady-state growth. We view these approaches as complementary to ours since both mechanisms (cyclical markups and cyclical reallocation) could be at work simultaneously. However, we view our approach as a realistic and tractable alternative, since non-linear heterogeneous-agent models are computationally difficult to analyze. Our model of time-varying markups allows us to analyze uncertainty in the same representative-agent DSGE framework used to study other real and monetary shocks.

A recent paper by Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) studies the effects of uncertainty in a small open economy setting, where they directly shock the exogenous process for the real interest rate. Since a small open economy analysis is effectively done in a partial-equilibrium framework, they experience no difficulties in getting business-cycle comovements from an uncertainty shock. As we show,

³This intuition also helps understand the recent work of Bidder and Smith (2012), which embeds stochastic volatility and preferences for robustness in a business-cycle model. In their setting, an increase in volatility of technology shocks affects the expected mean of the technology distribution by changing the conditional worst case distribution of the robustness-seeking agent. In a related paper, Ilut and Schneider (2011) embed ambiguity-averse agents in the model of Smets and Wouters (2007). They show that exogenous changes in the agents' beliefs about the worst-case scenario can produce business-cycle comovements.

the difficulties come when the real interest rate is endogenous in a general equilibrium framework. In this setting, our mechanism changes the qualitative predictions of baseline DSGE models, and makes the model predictions consistent with the empirical evidence.

Another recent paper by Gourio (2010) follows Rietz (1988) and Barro (2006) and introduces a time-varying “disaster risk” into an otherwise-standard real business cycle. This shock can be viewed as bad news about the future first moment of technology combined with an increase in the future dispersion of technology. Thus, a higher risk of disaster is a combination of a negative news shock and a shock that increases uncertainty about the future. However, a key difference between Gourio (2010) and our work is that a realized disaster affects the level of both technology and the capital stock. In our model, a realized innovation does not affect the level of capital at the impact of the shock. The additional assumption in Gourio (2010) implies that an increase in the probability of disaster directly lowers the risk-adjusted rate of return on capital. In order for investment to fall when the probability of disaster increases, Gourio must assume an intertemporal elasticity of substitution (IES) greater than one. With an IES greater than one, the substitution effect dominates the wealth effect when the probability of disaster increases. The lower risk-adjusted rate of return on investment induces the household to decrease investment. Since the return on investment is low, households supply less labor which lowers total output. Since leisure and consumption are normal goods, an increase in risk results in lower equilibrium output, investment, and hours, but higher equilibrium consumption. For the reasons we discuss in Section 2, his competitive one-sector model is unable to match basic business-cycle comovements. A key difference is that our mechanism is able to generate business-cycle comovement with any calibrated value for the IES.

In independent and simultaneous work, papers by Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2011) and Born and Pfeifer (2011) examine the role of fiscal uncertainty shocks in a model with nominal wage and price rigidities. Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2011) shows that uncertainty regarding future fiscal policy is transmitted to the macroeconomy primarily through uncertainty about future taxes on income from capital. As we discuss in the introduction, an increase in uncertainty with nominal rigidities changes markups and creates macroeconomic comovement. We view this work as highly complementary to our paper. Our work emphasizes the basic mechanism in a stripped-down model and shows why fluc-

tuations in uncertainty can create business cycle comovement. These two papers show that the mechanism we identify can have important economic effects in the benchmark medium-scale model of Smets and Wouters (2007). Other than sharing a mechanism for generating comovement, these two papers differ greatly from our work. We focus on technology and demand uncertainty, rather than policy uncertainty. In addition, we follow a very different calibration strategy, which we discuss in the next section. The object of our paper is to understand the role of increased uncertainty in generating the Great Recession and the subsequent slow recovery. We also analyze the interaction between the zero lower bound on nominal interest rates and uncertainty shocks, which we view as important for understanding the economics of this period.

6 Quantitative Results and the Great Recession

6.1 Uncertainty Shock Calibration

The intuition laid out in Sections 1 and 2, and the previous qualitative results suggest that uncertainty shocks can produce declines in output and its components when prices adjust slowly. This section uses the previous sticky-price model to determine if uncertainty shocks are quantitatively important for business cycle fluctuations. A related issue is determining the proper calibration of our shock processes for the uncertainty shocks associated with technology and household demand. The transmission of uncertainty to the macroeconomy in our model crucially depends on the calibration of the size and persistence of the uncertainty shock processes. However, aggregate uncertainty shocks are an *ex ante* concept, which may be difficult to measure using *ex post* economic data. To ensure our calibration of an unobservable process is reasonable, we want our model and uncertainty shock processes to be consistent with a well-known and observable measure of aggregate uncertainty.

We choose the Chicago Board Options Exchange Volatility Index (VIX) as our observable measure of aggregate uncertainty due to its prevalence in financial markets, ease of observability, and the ability to generate a model counterpart. The VIX is a forward-looking indicator of the expected volatility of the Standard and Poor's 500 stock index. To match the frequency of our model, we aggregate an end-of-month VIX series to quarterly frequency by averaging over the three months in each quarter. The top panel of Figure 5

plots our quarterly VIX series. Using our VIX data series, denoted V_t^D , we estimate the following simple reduced-form autoregressive time series model:

$$\ln(V_t^D) = (1 - \rho_V)\ln(V^D) + \rho_V\ln(V_{t-1}^D) + \sigma^{V^D}\varepsilon_t^{V^D}, \quad \varepsilon_t^{V^D} \sim N(0, 1). \quad (26)$$

The ordinary least squares regression results are $V^D = 20.4\%$, $\rho_V = 0.83$, and $\sigma^{V^D} = 0.19$ with an $R^2 = 0.68$. Using the estimated parameters, we can also compute a series of VIX-implied uncertainty shocks as the regression residuals divided by the sample standard deviation. Compared to its sample average of 20.4%, a one standard deviation VIX-implied uncertainty shock raises the level of the VIX to 24.27%. The bottom plot of Figure 5 shows the time series of the VIX-implied uncertainty shocks. We use this reduced-form time-series model to ensure a reasonable calibration for our technology and demand uncertainty shocks processes.

We want to create a model concept that is the counterpart to our observable measure of aggregate uncertainty. Therefore, we compute a model-implied VIX index as the expected conditional volatility of the return on the equity of the representative intermediate-goods producing firm. Using the third-order approximation to the policy functions of the model, we define our model-implied VIX V_t^M as follows:

$$V_t^M = 100 * \sqrt{4 * \text{VAR}_t(R_{t+1}^E)}, \quad (27)$$

where $\text{VAR}_t(R_{t+1}^E)$ is the quarterly conditional variance of the equity return.⁴ We annualize the quarterly conditional variance, and then transform the annual volatility units into percentage points.

Using our model-implied VIX, we calibrate leverage and the uncertainty shock parameters using a two-step process. Given the other parameters for the model and the unconditional shock variances σ^a and σ^z , we first choose the level of firm leverage such that the unconditional level of the model-implied VIX at the ergodic mean matches the average level of the VIX in the data, 20.4 percent.⁵ After matching the unconditional level of the model-implied VIX, we then choose our uncertainty shock parameters such

⁴Technically, the VIX is the expected volatility of equity returns under the risk-neutral measure. In the model, the results are quantitatively unchanged if we compute the model-implied VIX using the risk-neutral expectation.

⁵Since the Modigliani & Miller (1963) theorem holds in our model, the amount of leverage does not affect firm decisions or firm value.

that a one standard deviation uncertainty shock in our model, to either technology or household demand, generates an impulse response that closely matches our reduced-form estimate for the actual VIX in the data. For example, in our calibrated model a one standard deviation uncertainty shock to technology or household demand produces a 19 percent increase in the model-implied VIX and has a first-order autoregressive term of 0.83. Conditional on the values of the endogenous state variables, our model-implied VIX has an AR(1) representation in each of the two types of uncertainty shocks. Therefore, we are able to closely match the impulse response of the simple reduced-form model.

6.2 Quantitative Impact of Uncertainty Shocks

Figure 6 shows the impact of our calibrated uncertainty shock processes on the endogenous variables of the sticky-price model. Section 4.2 shows that the responses are qualitatively similar for both technology and household demand uncertainty shocks. In this section, we analyze the quantitative differences between technology and household demand uncertainty shocks. The bottom right plot of Figure 6 shows that both uncertainty shocks under sticky prices produce a similar law of motion in the model-implied VIX, which approximately matches the reduced-form VIX model. The bottom middle plot of each figure shows that the percentage increase in the volatility of the exogenous shocks to generate the same movement in the model-implied VIX differs between technology and household demand shocks. Household preference shocks require a 96 percent increase in volatility to produce the same movement in the model-implied VIX as a 37 percent increase in the volatility of technology.

In addition, the quantitative transmission of uncertainty to the macroeconomy differs greatly between the technology and household demand shocks. A one standard deviation technology uncertainty shock generates a peak drop in output of less than 0.05 percent. However, a one standard deviation household demand uncertainty shock produces a peak drop in output of about 0.17 percent. Much of the quantitative difference in the output fluctuations originates from the behavior of investment. When the uncertainty about future technology increases, higher capital provides a hedge against possible negative shocks to future marginal costs. This additional substitution effect, which is not present under a demand uncertainty shock, provides an incentive for the firm to not disinvest in the capital stock when uncertainty about future technology increases. Accordingly,

investment falls by only a few basis points after a technology uncertainty shock but falls by over 20 basis points after a demand uncertainty shock. Since capital and labor are complements in production, the time path of investment implies that equilibrium hours worked also falls by less after a technology uncertainty shock. Overall, our results suggest that household demand uncertainty shocks can cause quantitatively significant fluctuations in output and its components.

Our calibration strategy produces general-equilibrium results which are consistent with the empirical literature on the macroeconomic effects of stock market volatility. Alexopoulos and Cohen (2009) analyze the effects of stock market volatility on industrial production using a vector autoregression with a recursive identification scheme. They show that a one standard deviation increase in the VIX produces a statistically significant decline of output with a peak decline of approximately 0.25 percent. Our calibrated impulse responses of demand uncertainty shocks are close to this point estimate and well within its confidence interval, which provides additional evidence that our calibration strategy is reasonable.

6.3 The Role of Uncertainty Shocks in the Great Recession

The previous section shows that uncertainty shocks associated with household demand have quantitatively significant effects on output and its components. Many economists and the financial press believe the large increase in uncertainty in the fall of 2008 may have played a role in the Great Recession and subsequent slow recovery.⁶ The plot of the VIX in Figure 5 shows a large increase in expected stock market volatility around the collapse of Lehman Brothers in September of 2008. In particular, the bottom plot shows a three and a half standard deviation VIX-implied uncertainty shock during the end of 2008. In calibrating our model, one standard deviation uncertainty shocks to either household demand or technology generate one standard deviation movements in the model-implied VIX. Thus, we cannot easily identify or partition the contribution of demand or technology uncertainty shocks in our model in generating the large change in the VIX in the fall of 2008. However, the utilization-adjusted total factor productivity series of Fernald (2011) shows very little evidence of stochastic volatility, either during the Great Recession or over the entire postwar period. Thus, if we assume demand uncertainty shocks explain

⁶For example, Kocherlakota (2010) states, “I’ve been emphasizing uncertainties in the labor market. More generally, I believe that overall uncertainty is a large drag on the economic recovery.”

the bulk of the movement in the VIX during the fall of 2008, our baseline model predicts that the increase in uncertainty in the Fall of 2008 should have lowered output by about 0.6 percent.⁷

This decline in output may seem a small number relative to the size of the output drop in 2008-2009.⁸ However, as we show in the next section, the assumptions regarding monetary policy are crucial in determining the effects of changes in uncertainty on the macroeconomy. The Fed Funds target rate hit the zero lower bound on December 16, 2008. From then on, the Fed could no longer offset the contractionary effects of higher uncertainty on the economy. Under these circumstances, the predicted macroeconomic effects of uncertainty are substantially larger.

One potential criticism of using our model to determine the role of uncertainty shocks in the Great Recession is that our model lacks a realistic financial sector and abstracts from financial frictions. Thus, one might argue that what we term an exogenous uncertainty shock is actually due to a financial crisis. We are quite sympathetic to the idea that a financial crisis can raise uncertainty, but we believe that it is important to investigate the full set of channels through which financial market disruptions can affect the macroeconomy. A financial market disruption, such as the failure of Lehman Brothers in the Fall of 2008, is a single event which can have multiple effects, just as a war might increase government expenditure, raise distortionary taxes, and lead to rationing, each of which has different macroeconomic effects. Recent work by [Gertler and Karadi \(2011\)](#) and many others focuses on the first-moment effects of the financial market disruption, such as a higher cost of capital and tighter borrowing constraints for households and firms. In this paper, we analyze the likely effects of the concurrent rise in uncertainty and its effect on the economy during the Great Recession, which are second-moment effects. To analyze this independent mechanism and the effects of the increase in uncertainty, we choose to model uncertainty in a simple but reasonable macroeconomic model that abstracts from financial frictions. Our paper complements other work on the Great Recession, since one could easily combine the first-moment and second-moment analyses to obtain a complete picture of the effects of the financial crisis. Adding a detailed financial sector to our model would obscure the transmission mechanism of uncertainty to the macroeconomy, and we

⁷Given the AR(1) law of motion for volatility shocks in our third-order approximation to the policy functions, the impulse responses for the model scale approximately linearly in the size of the uncertainty shock.

⁸The CBO estimates that the output gap was -4.6 percent in 2008Q4.

eschew this course of action for the sake of clarity.

7 Uncertainty Shocks and the Zero Lower Bound

Finally, we examine the role of monetary policy in determining the general-equilibrium effects of uncertainty shocks. In our model, the monetary authority follows a standard interest-rate rule that responds to inflation and output growth. The impulse responses in Figure 4 show that the monetary authority aggressively lowers the nominal interest rate in response to a demand uncertainty shock. However, the calibrated interest rate rule does not decrease the policy rate enough to offset the negative impact on output and the other model variables. If the interest rate rule allowed the monetary authority to conduct policy optimally and replicate the flexible-price equilibrium allocations, then monetary policy could undo the negative effects of the uncertainty shock. However, if the monetary authority is constrained by the zero lower bound on nominal interest rates, then monetary policy cannot replicate the flexible-price outcome. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that the zero lower bound may have plausibly contributed significantly to the large and persistent output decline starting at that time. We show in this section that increases in uncertainty have much larger effects on output when monetary policy is constrained by the zero lower bound. Our results suggest that the second-moment effects of the financial crisis may be important for understanding the large declines in output and employment in late 2008 and 2009.

7.1 Solution Method and Calibration

To analyze the impact of the zero lower bound, we solve a modified version of our baseline model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This method discretizes the state variables and solves for the policy functions which satisfy all the equilibrium conditions of the model. Appendix A.1 contains the details of the policy function iteration algorithm. To make the model computationally feasible using policy function iteration, we simplify our baseline model by reducing the number of state variables and Euler equations. We remove technology shocks and examine only the impact

of shocks associated with household demand. Also, we eliminate two Euler equations by removing leverage and assuming that households receive firm dividends as a lump-sum payment.

7.2 Interactions of Uncertainty and the Zero Lower Bound

In addition to the difficulty of modeling changes in uncertainty at the zero lower bound, increases in uncertainty can produce an additional source of fluctuations beyond the precautionary working and saving channel. This additional amplification mechanism, which we refer to as the contractionary bias, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges from the interaction of uncertainty and the zero lower bound when monetary policy follows a standard Taylor (1993)-type policy rule. In this situation, an increase in uncertainty causes an increase in the average nominal interest rate since the distribution of the nominal interest rate is left-truncated by the zero lower bound. For any given level of inflation, a higher nominal interest rate raises the real interest rate, which discourages consumption and investment and depress output in economy. In Appendix B, we discuss this issue in detail and show this contractionary bias in the average nominal interest rate can dramatically affect the economy when uncertainty increases at the zero lower bound. In the main text, however, we choose to eliminate the contractionary bias mechanism from our results. We view the contractionary bias channel as a technical consequence of examining changes in uncertainty at the zero lower bound under a particular simple monetary policy rule, which probably does not represent the actual conduct of Fed policy at the zero lower bound.⁹ Note, however, that since we are removing an amplification mechanism, our results represent a lower bound on the effects of changes in uncertainty at the zero lower bound. Indeed, if we assumed that Fed policy follows the same simple Taylor rule at the zero lower bound that it does during normal times, then we could explain the entire output drop in the Great Recession as being due to increased uncertainty!

To remove the contractionary bias, we follow the conjecture of Mendes (2011) and assume that the monetary authority implements policy using the following history-dependent

⁹Our specific model is along the lines of the Fed announcing a loose path of future policy even after the economy emerges from the zero lower bound, which is something that it has arguably done. We assume that the expected future path of policy offsets the higher-than-desired nominal interest rates caused by the zero lower bound. Thus, the average expected nominal interest rate remains unchanged when uncertainty increases at the zero lower bound.

monetary policy rule:

$$r_t^d = r + \rho_\pi (\pi_t - \pi) + (r_{t-1}^d - r_{t-1}) \quad (28)$$

$$r_t = \max(0, r_t^d), \quad (29)$$

where r_t^d is the desired policy rate of the monetary authority, and r_t is the actual policy rate subject to the zero lower bound. When the monetary authority is unconstrained by the zero lower bound, the policy rule in Equation (28) responds exactly as a simple Taylor (1993)-type policy rule. However, when the monetary authority encounters the zero lower bound, the history-dependent monetary policy rule lowers future desired policy rates to offset the previous higher-than-desired nominal rates that obtained due to the zero lower bound. Since deviations from the desired path of the policy rate are offset exactly one-for-one, the average nominal policy rate remains unchanged when volatility increases. Thus, the history-dependent monetary policy rule removes the contractionary bias and allow us to isolate the effects of precautionary saving and working due to uncertainty at the zero lower bound.

7.3 Impulse Response Analysis

Figure 7 plots the impulse responses of a one standard deviation uncertainty shock for our simplified model at the ergodic mean of the model variables. These impulse responses replicate our previous experiments using this alternative model and calibration. Holding the level of the discount factor shock constant, an increase in uncertainty about the future decreases output by 0.16 percent. In our following analysis of the zero lower bound, we focus on the relative amount that the zero lower bound amplifies the effects of an uncertainty shock compared to this impulse response at the ergodic mean.

To compute the impulse response of an uncertainty shock at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate a large negative first moment demand shock, which causes the zero lower bound to bind for about two years. In the second time path, we simulate the same large negative first moment demand shock, but also simulate a one-standard-deviation uncertainty shock. We compute the percent difference between the time paths of variables in the two simulations as the impulse response to the uncertainty shock at the zero lower bound.

Figure 7 also shows the impulse response to a one-standard-deviation uncertainty

shock when the economy hits the zero lower bound constraint for two years. At the zero lower bound, a one standard deviation uncertainty shock produces a 0.35 percent drop in output on impact, and causes a much larger declines in consumption, investment, and hours worked. When compared with the impulse response at the ergodic mean, these results suggest that the zero lower bound more than doubles the decline in output and its components. The desire by households to work and save more translates into a larger drop in equilibrium hours worked and investment when the monetary authority cannot adjust its nominal interest rate. In addition to removing the contractionary bias, simple history-dependent rules like Equation (28) act as a form of commitment by the monetary authority to keep interest rates lower after encountering the zero lower bound. This promise of future lower nominal rates stimulates the economy throughout the zero lower bound episode, but the effect is not strong enough to prevent significant contractions in output and its components. As the monetary authority maintains zero policy rates during the beginning of the recovery, output and its components rise above the ergodic mean impulse responses. As the first moment demand shock subsides and the economy exits the zero lower bound, the time-paths for output and its components rebound sharply and closely follow the impulse response at the ergodic mean.

7.4 Revisiting Uncertainty Shocks in the Great Recession

The impulse responses suggest that adverse effects of uncertainty shocks are amplified at the zero lower bound. The peak drop in output in response to the uncertainty shock is about two times larger when the monetary authority is constrained. As we discuss in Section 6.3, the bottom plot of Figure 5 shows a three and a half standard deviation VIX-implied uncertainty shock during the end of 2008. Our larger baseline model, without accounting for the zero lower bound, suggests that this large uncertainty shock may explain up to a 0.6 percent drop in output during that period. The results of our zero lower bound experiments, however, suggest that the zero lower bound amplifies uncertainty shocks by at least a factor of two. Thus, our results suggest that the increase in uncertainty when the zero lower bound constraint was binding may have accounted for about a 1.3 percent drop in output during the Great Recession. The Congressional Budget Office estimates that the gap between actual and potential output for the fourth quarter of 2008 is negative 4.6 percent. Our results suggest that a non-trivial fraction of the decline

in output during the Great Recession can be explained by increased uncertainty about the future. Note again that due to our assumption that monetary policy succeeds in fully offsetting the contractionary bias, our results are a lower bound on the effects of uncertainty during the recent crisis. We view our findings as highly complementary to other work on the financial crisis, since our results can be combined with investigations of other channels through which financial crises affect the macroeconomy to obtain a complete picture of the Great Recession.

7.5 Computational Complexity and the Zero Lower Bound

Even after our simplifying assumptions, the problem of modeling uncertainty shocks at the zero lower bound remains computationally intensive in our model. Our alternative model of this section retains the Epstein-Zin preferences, endogenous capital accumulation, and stochastic volatility in the discount factor process from our baseline model of Section 3. Many other papers in the zero lower bound literature commonly make one of two simplifying assumptions to reduce the computational burden of the zero lower bound. Some papers, such as Nakov (2008) and Nakata (2011), examine the zero lower bound in a dynamic and stochastic environment using a simple New-Keynesian model without capital. This simple model often features only one exogenous state variable and no endogenous state variables. Other works, such as Erceg and Linde (2010), use a richer business-cycle model, but rely on a solution technique that imposes perfect foresight. Our paper shows that the transmission of uncertainty to the macroeconomy through precautionary saving and working requires capital accumulation, in a dynamic and stochastic setting where we cannot impose perfect foresight. Therefore, these two simplifications are inappropriate in our framework and we are required to solve a computationally more difficult problem.

8 Conclusion

This paper examines the transmission mechanism of uncertainty to the macroeconomy in a standard representative-agent general equilibrium model. Under reasonable assumptions, fluctuations in uncertainty can generate business cycle-like comovements in output, consumption, investment, and hours worked if nominal prices are sticky (or, more generally, if markups are countercyclical). We calibrate our model to be consistent with a

well-known and observable index of *ex ante* stock market volatility. We find that the dramatic increase in uncertainty during the fall of 2008, combined with the zero lower bound on nominal interest rates, may be an important factor in explaining the large and persistent decline in output starting at that time.

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Technical Appendix

A Model Solution with Zero Lower Bound Constraint

A.1 Numerical Solution Method

To analyze the impact of uncertainty shocks at the zero lower bound, we solve our model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a simple Taylor (1993)-type interest-rate rule. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: $\{K_t \times a_t \times \sigma_t^a\}$
2. Conjecture initial guesses for the policy functions of the model $N_t = N(K_t, a_t, \sigma_t^a)$, $I_t = I(K_t, a_t, \sigma_t^a)$, $\Pi_t = \Pi(K_t, a_t, \sigma_t^a)$, and $\mathbb{E}_t V_{t+1}^{1-\sigma} = EV(K_t, a_t, \sigma_t^a)$.
3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables a_t and σ_t^a to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space.
4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

We implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows the history-dependent policy rule in Equation (28), we include the lagged difference between the actual and desired policy rates ($r_{t-1} - r_{t-1}^d$) in the discretized state space.

B Contractionary Bias at the Zero Lower Bound

As we discuss in the main text, the interaction between uncertainty and the zero lower bound can produce an additional source of fluctuations beyond the precautionary working and saving channel. We refer to this additional amplification mechanism as the contractionary bias in the nominal interest rate distribution. In this Appendix, we show that the contractionary bias can dramatically affect the economy when uncertainty increases at the zero lower bound. In addition, we show that the assumptions regarding this new mechanism are crucial in assessing the general-equilibrium effects of changes in uncertainty at the zero lower bound. For Sections B.1-B.3 only, we reduce the unconditional volatility of demand shocks σ^a to 0.5 percent from our baseline calibration of 2.0 percent and decrease the standard deviation of uncertainty shocks, σ^{σ^a} , to 0.50. In Section B.4, we explain the rationale for temporarily reducing the volatility of the exogenous shocks hitting the economy.

B.1 Impulse Responses Under Simple Taylor (1993) Rule

We begin our analysis by assuming the monetary authority sets the nominal interest rate according to the following simple rule:

$$r_t^d = r + \rho_\pi(\pi_t - \pi) \quad (30)$$

$$r_t = \max(0, r_t^d), \quad (31)$$

where r_t^d is the desired policy rate of the monetary authority, and r_t is the actual policy rate subject to the zero lower bound. Figure B.1 plots the impulse responses of a one standard deviation uncertainty shock at the ergodic mean of the model variables. These impulse responses replicate our previous experiments using this simplified model and alternative calibration. Due to the considerably smaller calibration of the exogenous shocks, this alternative calibration produces an extremely small drop in output: Holding the level of the discount factor shock constant, a 50 percent increase in the volatility of the shock process decreases output by less than one basis point. Figure B.1 also plots the impulse responses of a one standard deviation uncertainty shock under a zero lower bound scenario similar to the simulation in Section 7.3. At the zero lower bound, a one standard deviation uncertainty shock produces a 0.35 percent drop in output. Compared

to the impulse responses at the ergodic mean, the decline in output due is magnified by over an order of magnitude when the monetary authority is unable to change its nominal policy rate. This result explains our claim in the text that we could explain all of the output drop in the Great Recession as being due to uncertainty alone if we did not remove the contractionary bias.

B.2 Contractionary Bias in the Average Nominal Interest Rate

The previous results suggest that the zero lower bound massively amplifies uncertainty shocks. However, our assumed monetary policy rule may be overstating the effects of the zero lower bound. In the model, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (30), the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates the *actual* policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure B.2 illustrates this effect by plotting the distribution of the nominal interest rate under both low and high levels of exogenous shock volatility. Figure B.2 shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule.¹⁰ We refer to this link between the volatility of the exogenous shocks and the level of the nominal interest rate as the contractionary bias in the actual policy rate distribution.¹¹

We argue that accounting for the contractionary bias is crucial in assessing the general-equilibrium effects of changes in uncertainty at the zero lower bound. Figure B.3 plots the average Fisher relation $\ln(R) = \ln(\Pi) + \ln(R^R)$ and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility

¹⁰Using a simple New-Keynesian model without capital, Mendes (2011) analytically proves that the average nominal interest rate is increasing in the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule.

¹¹Nakata (2011) and Nakov (2008) also use a New-Keynesian model to examine the zero lower bound in a dynamic and stochastic setting. Both papers also discuss this link between the volatility of the exogenous shocks and the average level of the nominal interest rate under a simple policy rule or optimal monetary policy under discretion.

thus raises average real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourage consumption and investment and depress output in the economy.

Using this intuition regarding the contractionary bias, we can identify two distinct sources of fluctuations in the impulse responses in Figure B.1. An increase in uncertainty induces precautionary saving and working, which we discuss in detail in the main text of the paper. In addition, the uncertainty shock temporarily increases the contractionary bias in the expected average nominal interest rate. The transitory increase in the contractionary bias implies higher expected nominal interest rates for any given level of inflation. Even though current nominal rates remain at the zero lower bound, an increase in expected nominal rates after the zero lower bound episode raises expected real interest rates. Higher future real interest rates reduce expected future output and inflation, which lowers current output and inflation through forward-looking consumption and investment decisions. Like the precautionary saving and working channel, the transitory increase in the contractionary bias produces declines in output and its components. Our previous impulse responses in Figure B.1 show the effects of both mechanisms. However, the previous results obscure the relative contribution of each mechanism in explaining the amplification of the uncertainty shock.

B.3 Impulse Responses Under History-Dependent Policy Rule

To quantify the contribution of each mechanism, we also examine the impact of an uncertainty shock at the zero lower bound under the history-dependent policy rule in Equation (28). As we discuss in the main text, this alternative specification for monetary policy removes the contractionary bias by promising to offset deviations from the desired policy rule caused by the zero lower bound. Figure B.1 also plots the impulse responses to an uncertainty shock for the history-dependent policy rule under the alternative shock calibration. A demand uncertainty shock at the zero lower bound produces a two basis point drop in output when the monetary authority follows the history-dependent policy rule. The differences in the impulse responses under each monetary policy rule allows us to quantify the relative contributions of the contractionary bias and the precautionary saving and working channels. Under the simple Taylor (1993) rule, Figure B.1 shows that the increase in the contractionary bias and the precautionary behavior channel combine

to produce a decline in output of 35 basis points. This decline is much larger than the 2 basis point decline under the history-dependent policy rule, which only features the precautionary saving and working channel. These results suggest that the increase in the contractionary bias explains much of the decline in output after an uncertainty shock when monetary follows a simple interest-rate rule.

B.4 Uncertainty, Contractionary Bias, & Equilibrium Existence

In addition to greatly amplifying fluctuations due to changes in uncertainty, this section provides evidence that the contractionary bias can even interfere with equilibrium existence under some calibrations. When the monetary authority follows the simple policy rule in Equation (30), Figure B.3 shows an increase in volatility shifts the policy rule to the left and increases the average nominal interest. For high levels of volatility, however, the policy rule shifts far enough to the left such that the policy rule no longer intersects the Fisher relation. In this situation, Mendes (2011) shows that a rational expectations equilibrium fails to exist because the contractionary bias is too large. Mendes (2011) also conjectures that a simple history-dependent rule like Equation (28) should remove the contractionary bias since the average nominal interest rate is no longer increasing in the volatility of the exogenous shocks.

Our computational experiments provide numerical support to the analytical results and conjectures of Mendes (2011). When monetary policy follows the simple Taylor (1993) rule in Equation (30), we are unable to solve our model numerically for our baseline calibration of $\sigma^a = 0.02$ and $\sigma^{\sigma^a} = 0.019$. This numerical failure suggests that the contractionary bias is large enough that a rational expectations equilibrium fails to exist for this calibration.¹² However, we are able to solve our model when we decrease the size of the exogenous shocks to $\sigma^a = 0.005$ and $\sigma^{\sigma^a} = 0.0025$. This result suggests that the smaller exogenous shock volatility decreases the size of the contractionary bias to a level consistent with a rational expectations equilibrium. However, when monetary policy follows the history-dependent rule in Equation (30), we are able to solve our model using our baseline calibration of $\sigma^a = 0.02$ and $\sigma^{\sigma^a} = 0.019$. This numerical result suggests that

¹²The contractionary bias only affects equilibrium existence when the monetary authority follows a simple Taylor (1993)-type rule subject to the zero lower bound. Without the zero lower bound, an increase in volatility increases the volatility of the nominal interest rate, but leaves the average level of the nominal interest rate unchanged.

the conjecture by Mendes (2011) is correct and the history-dependent rule removes the contractionary bias in the decision rules. Maintaining the considerably lower volatility calibration of $\sigma^a = 0.005$ and $\sigma^{\sigma^a} = 0.0025$ in Sections B.1-B.3 allows us to solve the model under both monetary policy specifications and decompose the relative contributions of the precautionary working and contractionary bias channels.

Even for small increases in uncertainty, the temporary increase in the contractionary bias produces large declines in output and its components. However, we choose to eliminate the contractionary bias channel and assume that the monetary authority follows the history-dependent rule in the main text of the paper. Mechanically, the history-dependent rule allows us to solve our model using our baseline volatility of Table 1. In addition, we believe the increase in the contractionary bias at the zero lower bound produces implausibly large declines in output and its components. We view the contractionary bias channel as a technical consequence of examining changes in uncertainty at the zero lower bound under a particular simple monetary policy rule. Therefore, we focus our main analysis of uncertainty at the zero lower bound on the more economically interesting precautionary working and savings channel.

Table 1: Baseline Calibration

Parameter	Description	Calibrated Value
α	Capital's Share in Production	0.333
β	Household Discount Factor	0.9987
δ	Depreciation Rate	0.025
ϕ_K	Adjustment Cost to Changing Investment	20.0
ϕ_P	Adjustment Cost to Changing Prices	160.0
Π	Steady State Inflation Rate	1.0062
ρ_r	Central Bank Interest Rate Smoothing Coefficient	0.50
ρ_π	Central Bank Reaction Coefficient on Inflation	1.50
ρ_y	Central Bank Reaction Coefficient on Output Growth	0.50
σ	Parameter Affecting Household Risk Aversion	60.0
ψ	Intertemporal Elasticity of Substitution	0.50
θ_μ	Elasticity of Substitution Intermediate Goods	6.0
ρ_a	First Moment Preference Shock Persistence	0.90
ρ_{σ^a}	Second Moment Preference Shock Persistence	0.83
σ^a	Steady-State Volatility of Preference Shock	0.02
σ^{σ^a}	Volatility of Second Moment Preference Shocks	0.019
ρ_z	First Moment Technology Shock Persistence	0.99
ρ_{σ^z}	Second Moment Technology Shock Persistence	0.83
σ^z	Steady-State Volatility of Technology	0.01
σ^{σ^z}	Volatility of Second Moment Technology Shocks	0.0037

Figure 1: Flexible Price Model Intuition

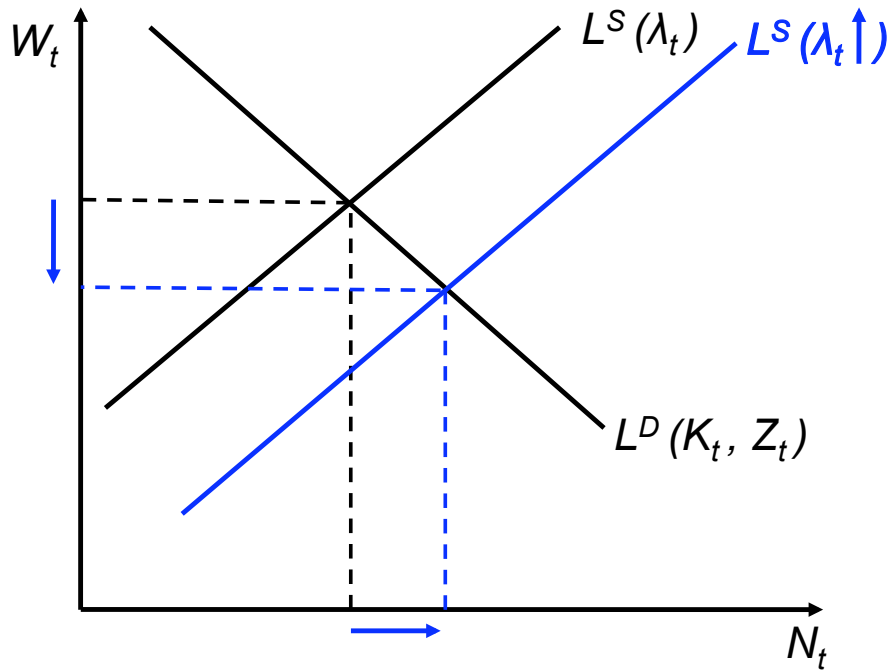


Figure 2: Sticky Price Model Intuition

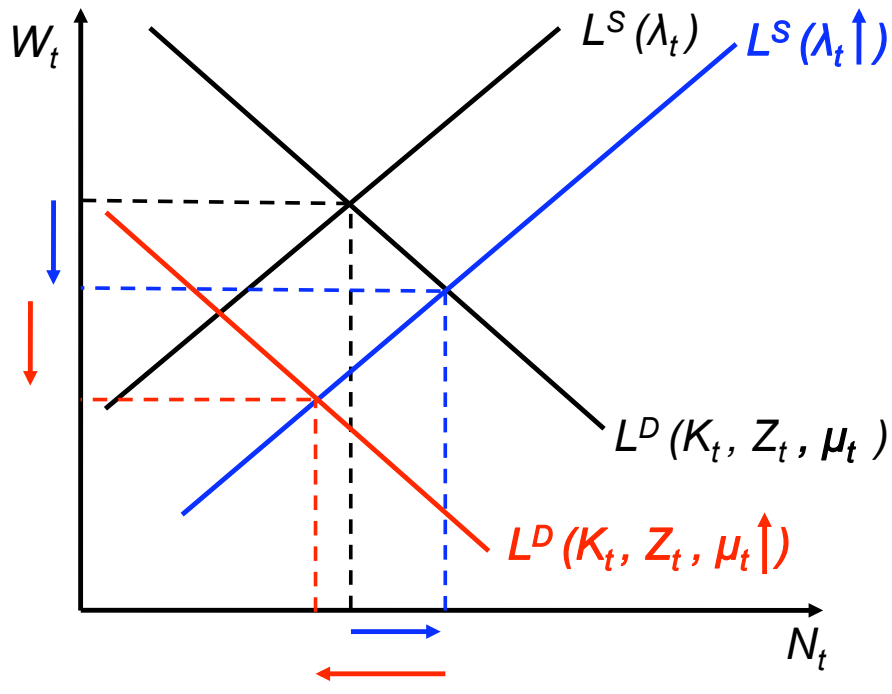
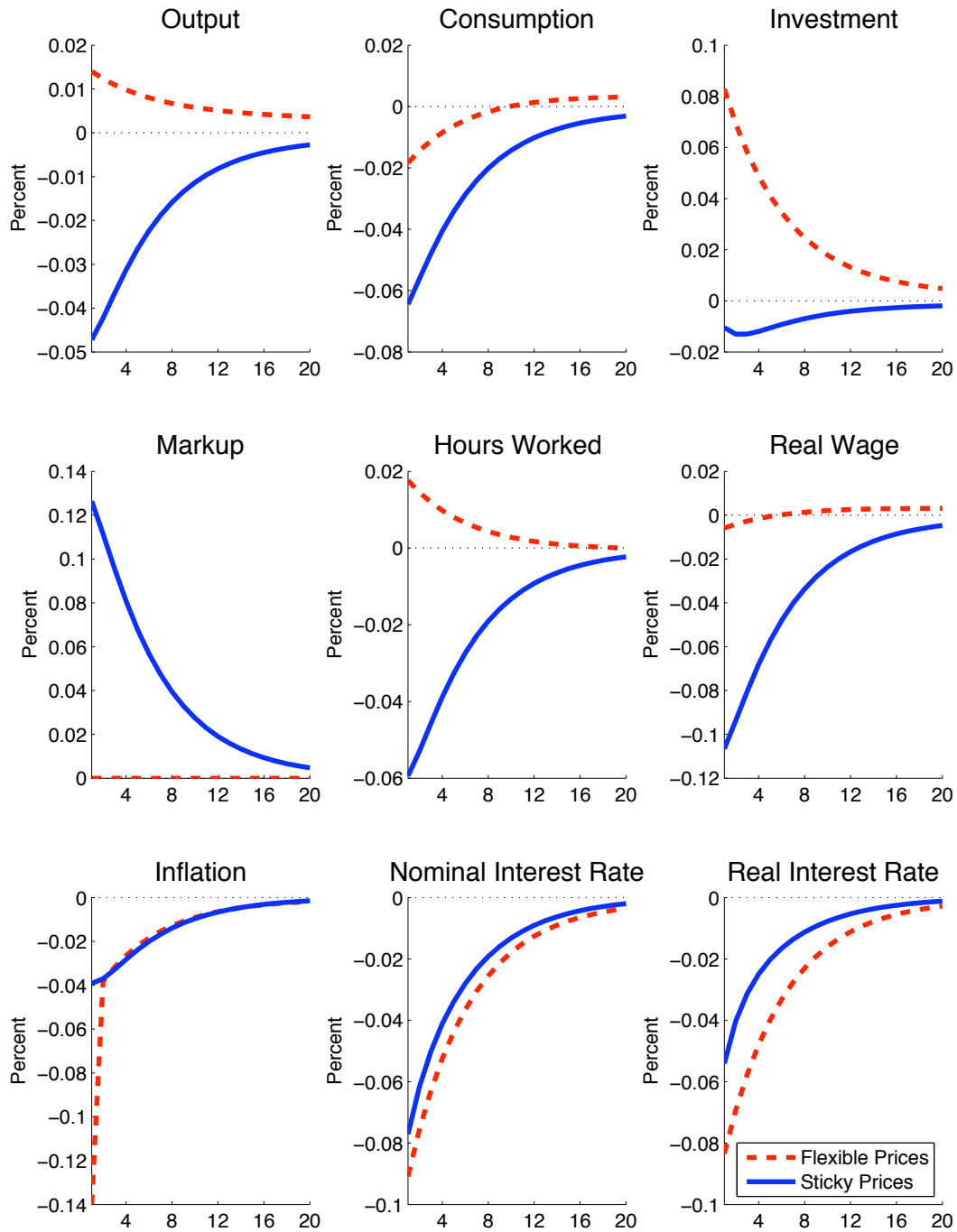
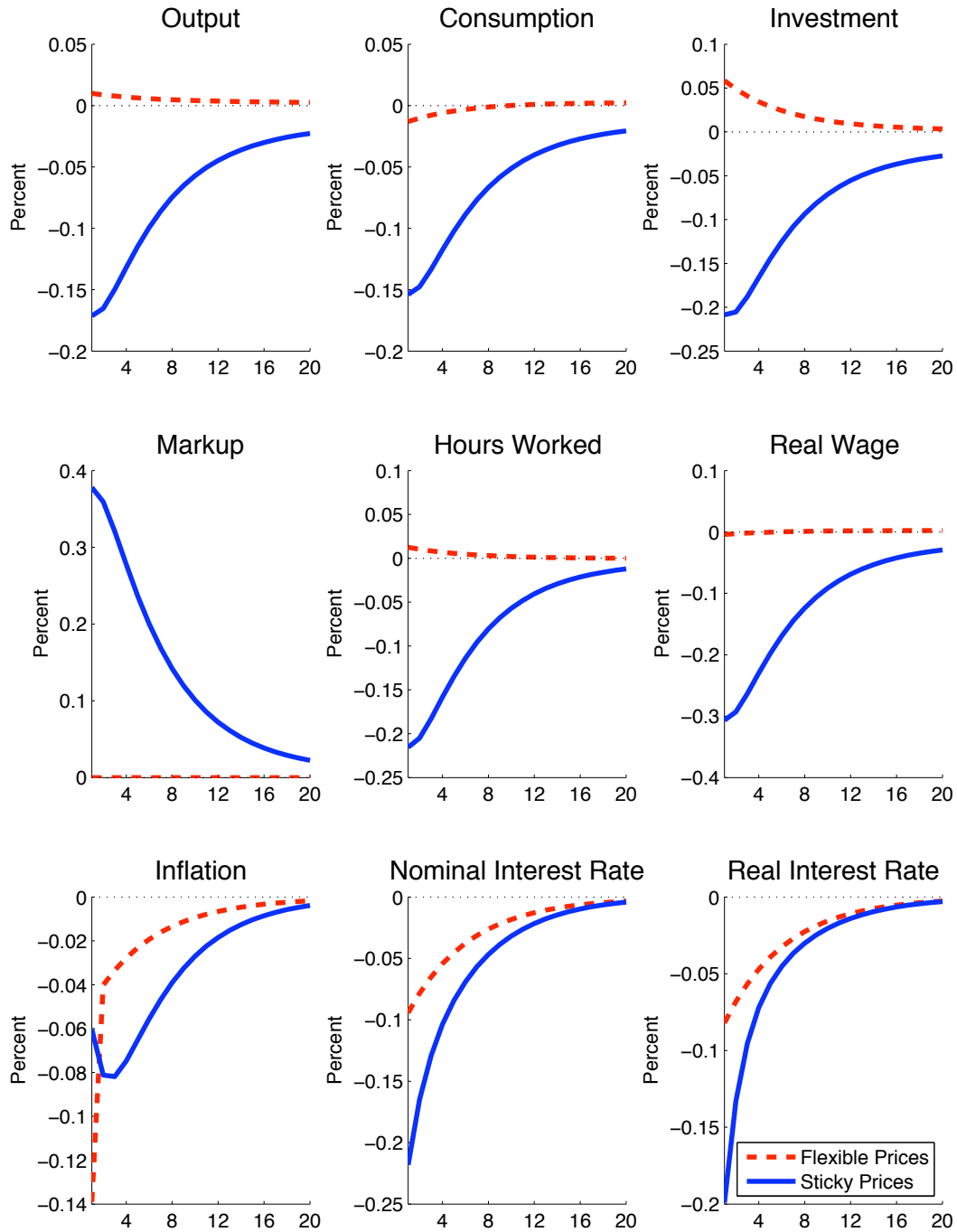


Figure 3: Impulse Responses to Second Moment Technology Shock



Note: The impulse responses for inflation and interest rates are plotted in annualized percent deviations from their ergodic mean. All other impulse responses are plotted as percent deviations from their ergodic mean.

Figure 4: Impulse Responses to Second Moment Preference Shock



Note: The impulse responses for inflation and interest rates are plotted in annualized percent deviations from their ergodic mean. All other impulse responses are plotted as percent deviations from their ergodic mean.

Figure 5: VIX and VIX-Implied Uncertainty Shocks

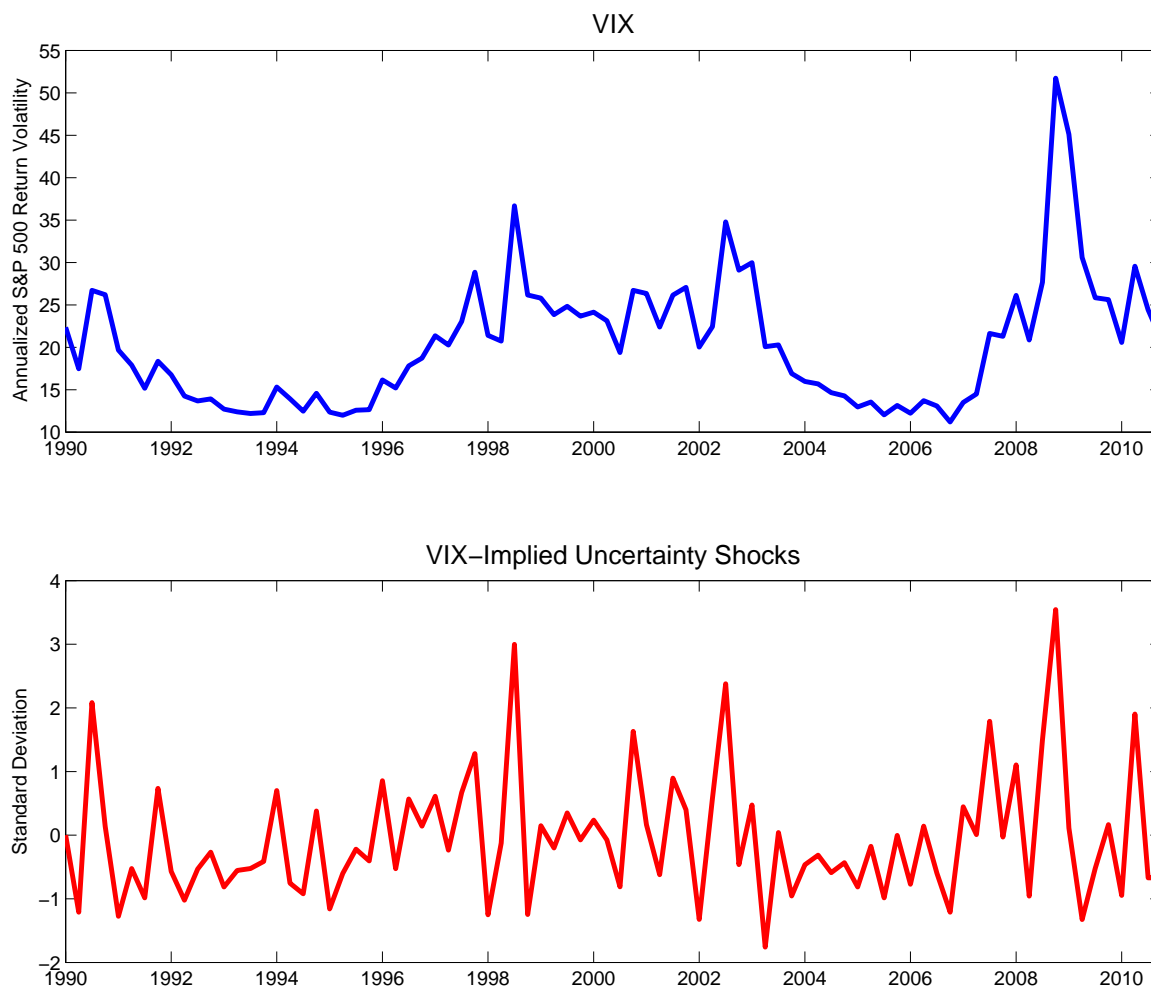
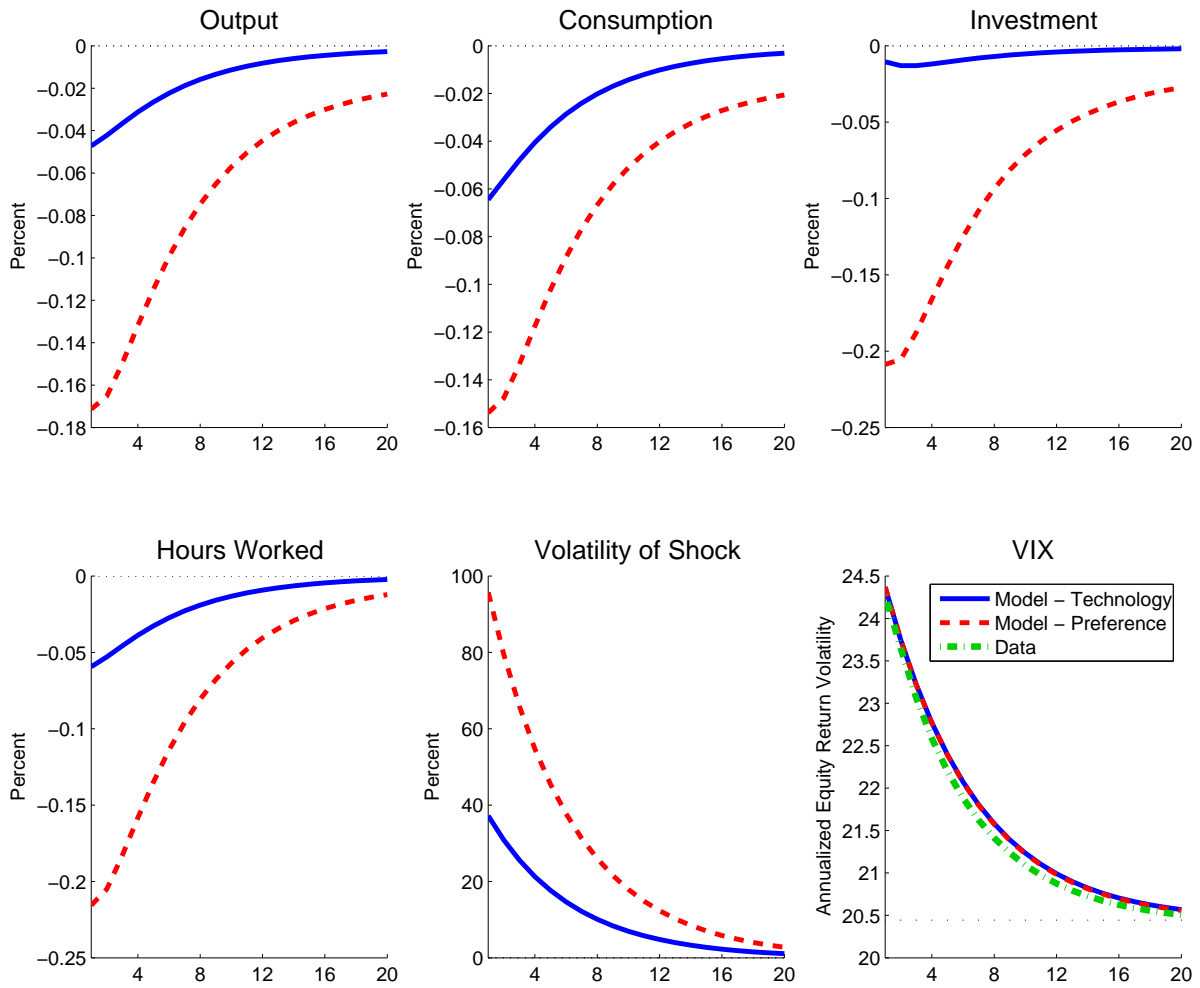
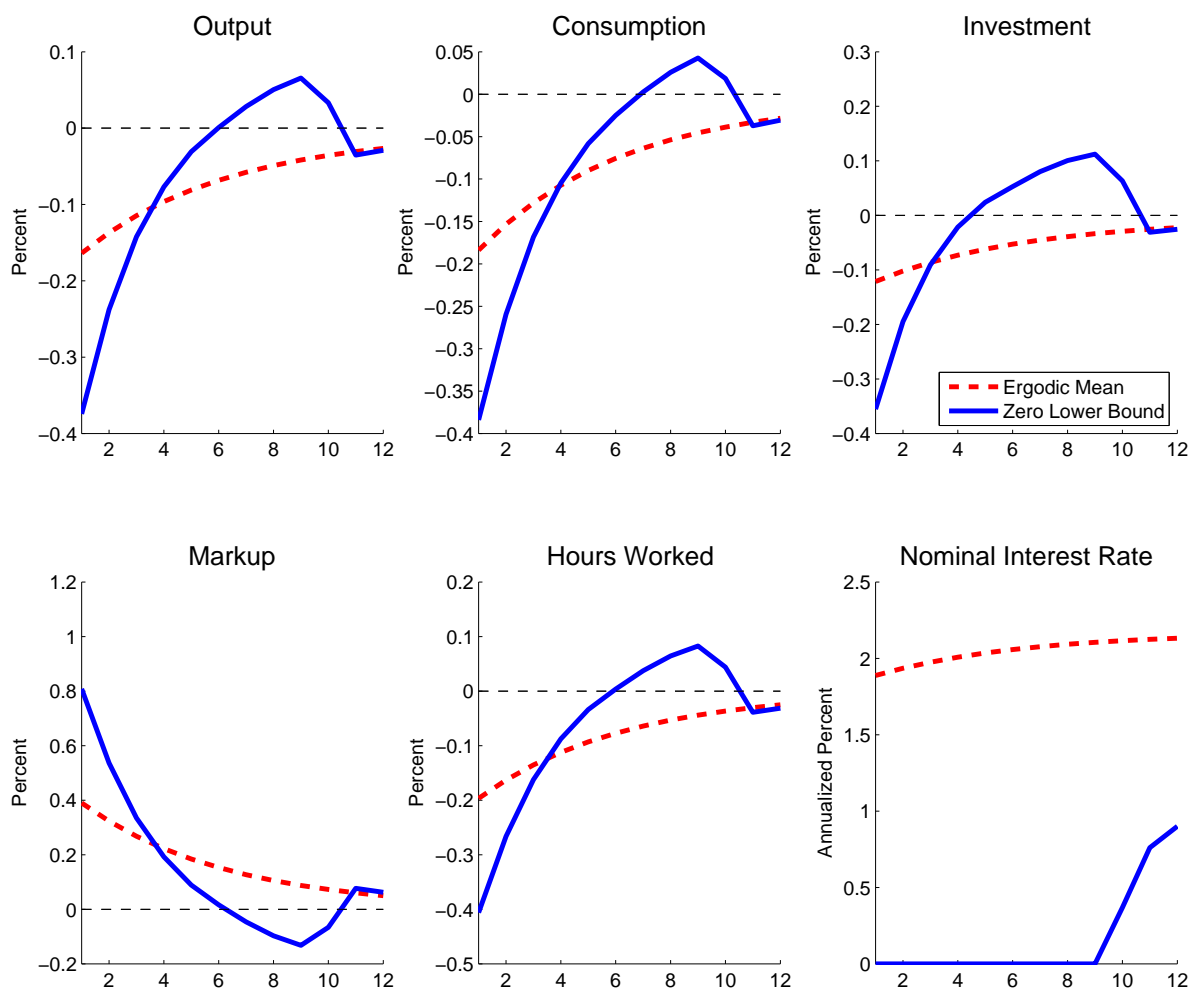


Figure 6: Model-Implied VIX and Uncertainty Shock Calibration



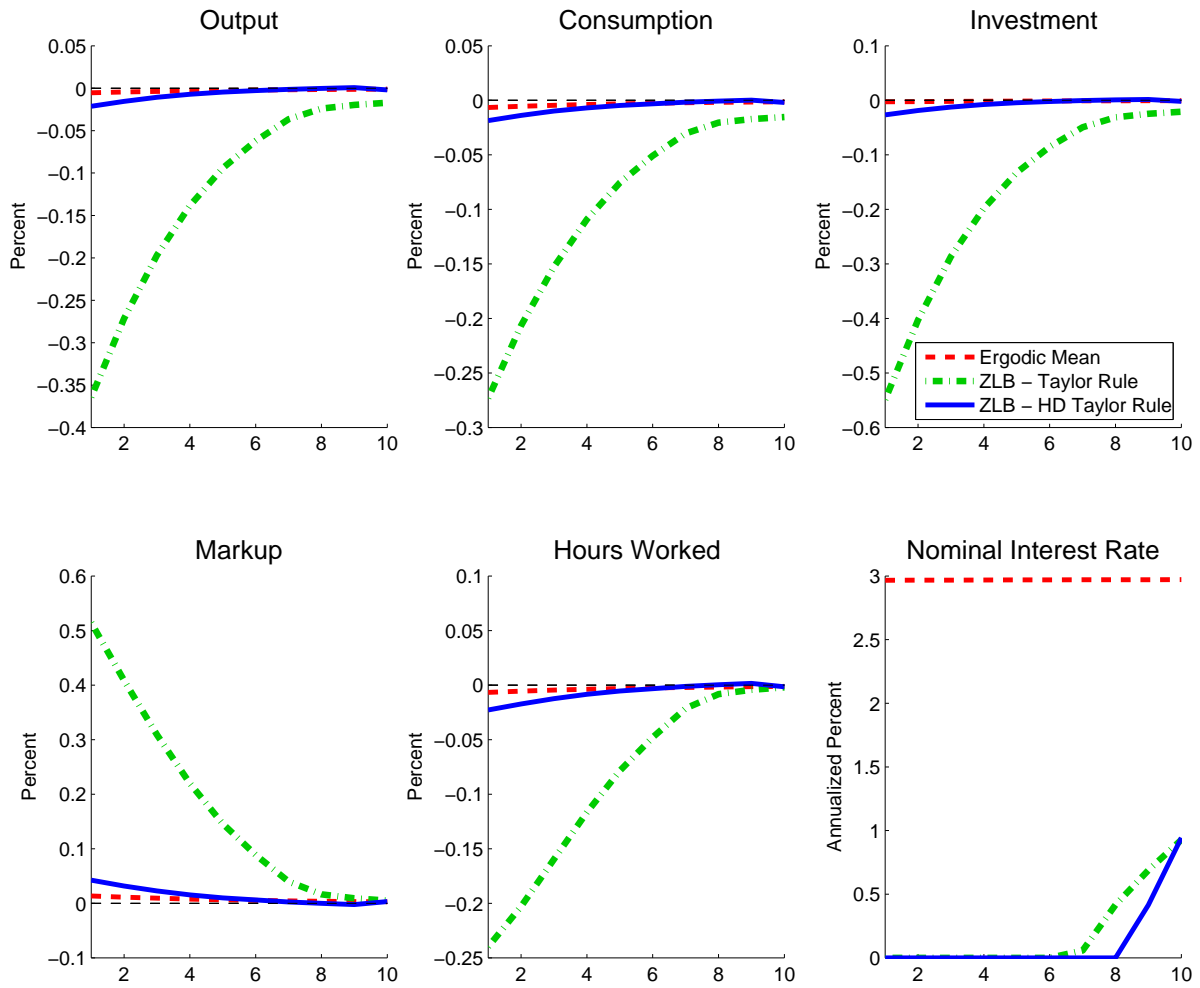
Note: The impulse response for the VIX is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.

Figure 7: Demand Uncertainty Shock at Zero Lower Bound



Note: The impulse response for the nominal interest rate is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.

Figure B.1: Uncertainty Shock at Zero Lower Bound Under Alternative Policy Rules



Note: The impulse response for the nominal interest rate is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.

Figure B.2: Nominal Interest Rate Distribution with Zero Lower Bound Constraint

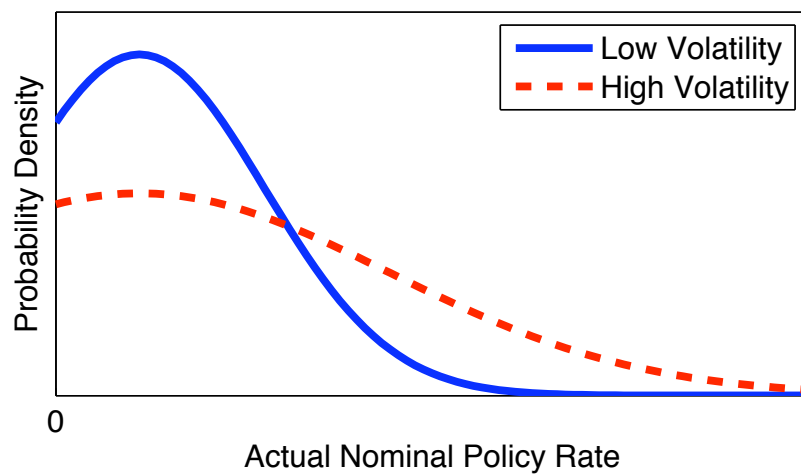


Figure B.3: Simple Policy Rules & Fisher Relation with Zero Lower Bound Constraint

