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Forward Guidance and Heterogeneous Beliefs

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Abstract

Nous analysons les effets des politiques de "forward guidance" lorsque l'engagement sur les actions futures qu'elles impliquent sont perçues de manière hétérogène par les agents privés. En utilisant des données d'enquête sur les anticipations, nous montrons que la "forward guidance" a diminué le désaccord sur le taux d'intérêt nominal à court terme à des niveaux historiquement bas, sans affecter, toutefois, outre mesure le désaccord sur l'inflation et la consommation futures. Nous introduisons des croyances hétérogènes sur les politiques et les fondamentaux dans un modèle néo-keynésien standard et nous montrons que, parce que la capacité d'engagement de la banque centrale n'est pas observable, un accord sur la trajectoire de taux d'intérêt peut coexister avec un désaccord sur la longueur de la trappe de liquidité. Une telle hétérogénéité de croyance peut nettement diminuer l'efficacité de la "forward guidance" et peut également modifier la politique optimale à la "zero lower bound" par rapport à une situation où les croyances sont homogènes.

Keywords: politique monétaire, forward guidance, communication, croyances hétérogènes, désaccord.

JEL Code: E31, E52, E65.

Abstract

We analyze the effects of forward guidance when agents have heterogeneous interpretations of whether forward guidance contains a commitment on future policy actions. Using survey expectations, we document that forward guidance lowered disagreement about future short-term interest rates to historically low levels while it did not affect much disagreement about future inflation and future consumption. We introduce heterogeneous beliefs on future policy and fundamentals in an otherwise standard New-Keynesian model. We show that, because the commitment type of the central bank is unobserved, agreement on the future path of interest rates can coexist with disagreement on the length of the trap. Such heterogeneity of beliefs can strongly mitigate the effectiveness of forward guidance. It also alters the optimal policy at the zero lower bound compared to a situation where beliefs are homogenous.

Keywords: Monetary policy, forward guidance, communication, heterogeneous beliefs, disagreement.

JEL Classification: E31, E52, E65.

Non-technical summary

When the current nominal policy interest rate cannot be lowered further because of the effective lower bound, a central bank can still stimulate aggregate demand by committing to future accommodative monetary stance when the economy will exit the liquidity trap. Since the unfolding of the Great Recession, many central banks implemented such Forward Guidance (FG) policies. They typically provided information on the period during which they expect the policy rate will be kept at the zero. We emphasize that these policies have been effective to coordinate expectations about future policy rates. However, as we also underline, that agents coordinated their expectations on future policy rates tells little about the efficiency of FG policies in the sense of Krugman (1998), Woodford and Eggertsson (2003) and Werning (2013).

We first show that FG policy in the US was not uniformly interpreted by the private sector. Disagreement among Professional Forecasters about 1 or 2 years ahead short-term interest rates started to drop to historically low levels right when the Fed strengthened its forward guidance with fix date commitments in August 2011. By contrast, disagreement about future consumption growth and future inflation rates stayed within their historical range. As a consequence, the period of FG corresponds to a dramatic break-up in the relationship between disagreements about future consumption and inflation relative to the disagreement about future short-term interest rates. This evidence is puzzling for standard macroeconomic analysis. Indeed, future inflation and demand determine future interest rates through the policy reaction function of the central bank, e.g. a Taylor rule. How can it be that, exactly at the time of explicit FG, agents started to disconnect the two, so that they agreed on future short-term interest rates but disagreed on future macroeconomic outcomes?

We show that this can occur when agents have different views on the policy stance implied by the expected path for future short-term interest rates. More precisely, we include heterogeneous beliefs on the policy stance in an otherwise standard New-Keynesian model of the ZLB à la Eggertsson and Woodford (2003). Households face a common discount factor shock pushing the economy towards the ZLB. Private agents observe the current discount rate shock and the resulting current allocation, but they do not know the number of periods this shock will last. Moreover, they cannot observe the commitment ability of the central bank. This information is not available until the economy reaches the actual end of the trap.

In this setup, at the equilibrium, agents can agree on the path of nominal interest rates, without agreeing on the length of the trap. Indeed, there are two macroeconomic scenarios that are consistent with a policy promising to keep interest rate at zero for an extended

period of time. Either agents believe the central bank is able to commit and will conduct an accommodative stance of monetary policy after the trap ended. They view the interest rate path as consistent with a more accommodative monetary policy stance and therefore have relatively optimistic forecasts of future macroeconomic conditions. Or agents believe the central bank cannot commit so that it cannot promise to keep interest rate at zero for a number of extra periods after the end of trap. They view the interest rate path as consistent with a monetary policy constrained to be at the ZLB for a long period of time and therefore have bleak forecasts of future macroeconomic conditions. Because, by definition, future fundamentals are not observed, private agents have no clear way to discriminate today between these two macroeconomic scenarios. Neither can they infer the length of the trap from the policy of the central bank. Indeed, the Zero Lower Bound (ZLB) prevents the central bank to signal its commitment as keeping interest rates at the ZLB is consistent with either a commitment to future deviations or an orthodox inflation-targeting policy. In sum, agents can agree on the path of future short-term interest rates for different reasons; and such difference shows up in their disagreement about medium-term expectations on consumption and inflation. Two implications follows from the potential ambiguity of the stance attached to the same path of future interest rate. First, it provides an explanation for why forward guidance was not as effective as standard models with homogeneous understanding of the policy would predict, as e.g. Del Negro et al. (2012) emphasize. Indeed, the different interpretations of such path imply different consumption choices that are offsetting. Optimistic agents consume more in anticipation of future higher inflation and lower real interest rates. Pessimistic agents consume less in anticipation of future lower inflation and higher real interest rates. As a consequence, aggregate consumption and thus aggregate inflation react less compared to a situation where every agent interprets the policy as a commitment to more accommodation. Second, the possibility that private agents interpret FG differently has important consequences for the efficiency of such a policy. Indeed, when the fraction of pessimists is small enough, we show that the optimal policy requires aiming at an accommodative stance by keeping rates at the ZLB for an extended period. However, when pessimists represent an important share of the population, FG can be detrimental with respect to status quo, so that the optimal policy has to follow a standard inflation-targeting rule. In the end, our analysis illustrates both the robustness and the frailty of FG: by strengthening the policy, one can undo imperfections in the interpretation of the announcements but only to the extent that these imperfections remain small.

More generally, our analysis provides insights on the effectiveness of FG policies that also broadly apply to any announcement about future policies. For example, the announcement

of the exact timing of Quantitative Easing policies is subject to the same kind of ambiguity. Is the policy in place because the authority wants to give an extra accommodative stimulus or because the situation has further deteriorated? Whether one or the other interpretation will prevail is going to determine the impact of the policy. We hope that this paper provides a useful framework to evaluate such a delicate aspect of institutional communication, which, we believe, lies at the heart of policy action.

1 Introduction

The FOMC has not been clear about the purpose of its forward guidance. Is it purely a transparency device, or is it a way to commit to a more accommodating future policy stance to add more accommodation today?

Charles I. Plosser, March 6, 2014.

When facing the Zero Lower Bound (ZLB) on its nominal policy rate, a central bank can still affect current allocations by making statements about future policy rates, indicating that they will remain low for a significant length of time.¹ In the aftermath of the Great Recession, several central banks implemented such forward guidance policies with somewhat mixed success: they succeeded in lowering expected future interest rates² but their resulting impact on the macroeconomy seemed limited.³ As Charles Plosser's quote above suggests, one possible reason is that an announcement that interest rates will remain at zero is ambiguous: it is consistent with anticipation of bad economic fundamentals – forward guidance is then said to be *Delphic* – or with anticipations of an expansionary monetary policy – forward guidance is then said to be *Odyssean*.⁴ In this paper, we investigate how heterogeneous interpretations of the same policy can change the effectiveness and the design of forward guidance policies.

More precisely, we make three main contributions. First, using survey data, we document that US forward guidance has been interpreted differently by private agents. Second, we introduce a New-Keynesian model with heterogeneous beliefs about the commitment type of the central bank and fundamentals. We show that, in line with the facts, an Odyssean and a Delphic interpretation of the same path of the policy rate can coexist at the equilibrium. We also underline that this can explain why forward guidance did not have a large impact on the average of macroeconomic expectations other than the interest rates. Third, we use the model to analyze the optimal forward guidance policy at the ZLB, i.e. the number of periods for which the central bank commits to keep its interest rate at zero. We show that with too few Odyssean believers, Odyssean forward guidance policy can be detrimental, so that it is optimal not to implement it.

We start by documenting new facts from the US Survey of Professional Forecasters. We observe that disagreement about future short-term interest rates dropped to historically low

¹See Krugman (1998), Eggertsson and Woodford (2003) or, more recently, Werning (2012).

²See e.g. Swansson and Williams (2014).

³See e.g. Del Negro et al. (2013).

⁴This terminology has been introduced by Campbell et al. (2012). See also Ellingsen and Söderström (2001) for a seminal contribution where monetary policy decisions are either related to new information about the economy or to changes in the preferences of the central banker.

levels after the Fed strengthened its forward guidance with fixed date commitments in August 2011. This evidence is consistent with forward guidance coordinating private agents' opinion about future monetary policy. However, disagreement about future consumption growth and inflation rates decreased by an order of magnitude less. The fall of 2011 marks a striking break in the correlation between disagreement on future interest rates and disagreement about other future macro outcomes (consumption, output and inflation). In sum, fixed date forward guidance was associated with disagreement on medium term values of fundamentals that was not related to disagreement on future policy rates. These facts would be hard to obtain in any model in which a monetary policy rule (either the normal times one or systematic deviations from such normal times rule) relates interest rates to inflation and activity. How can agents agree on future interest rates while they do not agree on future inflation and activity?

Our second contribution is to build an otherwise standard New-Keynesian economy where agents may (agree to) disagree both on the nature of forward guidance policy and on future fundamentals.⁵ In this setup, households face a common discount factor shock pushing the economy towards the ZLB. Private agents observe the current discount rate shock and the resulting current allocation, but they cannot observe the commitment ability of the central bank and they do not know the number of periods this shock will last. This information is not available until the economy reaches the actual end of the trap. We first show that, in this framework, agents can agree on the path of nominal interest rates, without agreeing on the length of the trap, provided they disagree on the commitment ability of the authority. Some agents anticipate a shorter liquidity trap associated with an accommodative stance of monetary policy after the trap (Odyssean forward guidance) while others only expect the trap to be longer (Delphic forward guidance). This disagreement about the possibility of future accommodation leads agents to have different views about the medium-run effects of monetary policy on aggregate demand and inflation. So, the model generates outcomes that are similar to the stylised facts above mentioned where we observe both disagreement about future consumption and inflation and consensus about future interest rates. Moreover, the actions induced by different beliefs can be offsetting, which can explain why the average expectation of future consumption and inflation did not react when forward guidance started to be implemented.

Finally, we use the model to investigate how heterogeneity of beliefs affects the efficiency of forward guidance. Agents who interpret the policy as Odyssean consume more in anticipation of future higher inflation and consumption, while agents who interpret it as Delphic consume

⁵See [Wiederholt \(2014\)](#) for an analysis of dispersed information about current fundamentals at the ZLB.

less anticipating less inflation.⁶ It follows that when a high enough proportion of agents take forward guidance as *Delphic*, the implementation of an Odyssean forward guidance may be inefficient, and even detrimental compared with the no forward guidance *status quo*. This is because these latter agents drag current aggregate consumption down. In contrast, Odyssean forward guidance can stimulate consumption and raise inflation expectations if a high enough proportion of agents believe that the central bank can commit to a time inconsistent policy *Odyssean*.⁷

Related literature [Gürkaynak et al. \(2005\)](#) show that FOMC announcements have strong effects on asset prices and in particular expected future policy rates while [Romer and Romer \(2000\)](#) provide evidence FOMC decisions convey Fed-specific information about the macroeconomic outlook so that private agents update their forecasts accordingly. [Campbell et al. \(2012\)](#) confirm such results in a sample that includes the Great Recession. Their results are consistent with market participants interpreting FOMC’s announcements as being *Delphic* rather than *Odyssean*. Our analysis complements these empirical exercises by analyzing the dispersion, rather than the average, of individuals’ macroeconomic forecasts. We show that the two interpretations of forward guidance announcements coexisted, we provide an explanation why this is so, and we analyse the consequences of this heterogeneity.

Our paper is also related to the literature on the effectiveness of forward guidance. [Carlstrom et al. \(2012\)](#) and [Del Negro et al. \(2013\)](#) underline that standard DSGE models predict incredibly high positive impacts of forward guidance policies on future inflation and activity. According to these models, announcements such as those made by the Fed should have led to a boom in demand much greater than what has been observed, a result [Del Negro et al. \(2013\)](#) dubbed the “forward guidance puzzle”. These papers consider that announcements were unambiguously perceived as sequences of deviations from the normal times reaction function of the central bank. We show that agents had different forecasts of such deviations and we analyze how this heterogeneity of interpretation reduces the aggregate impact of such policies. An alternative explanation for the “forward guidance puzzle” is provided by [McKay et al. \(2015\)](#) who rely on heterogeneous agents and borrowing constraints. In the same spirit, we show that disagreement about future monetary policy translates into some form of “discounting” in the aggregate Euler equation, thus explaining why the effect of future monetary shocks is

⁶We introduce intra-household transfers which induce wealth-sharing at the end of the trap. This permits to study the consequences of the mere heterogeneity of beliefs in the NK setup.

⁷Note that forward guidance may have additional positive effects compared with the framework of this paper, e.g. by reducing the uncertainty associated with future economic outcomes.

attenuated.

In addition, and in contrast to frequent policy discussions (e.g. [Filardo and Hofmann, 2014](#)), our results underline that gauging the efficiency of forward guidance announcements by merely looking at the reaction of expected future policy rates can be misleading as agents may disagree on the meaning of such a low future interest rate path. As [Woodford \(2012\)](#) emphasizes, for forward guidance to be effective, private agents should not only believe that interest will remain low in the future but they also should understand that the reason *why* they will is that the central bank will temporarily allow for more inflation than in normal times. [Engen et al. \(2014\)](#) provide survey evidence that it took time to convince private agents that the unconventional policies of the Fed implied a policy stance that would be more accommodative than it had been in the past.

Our paper is linked to the literature studying how the ZLB affects optimal monetary policy. [Krugman \(1998\)](#), [Eggertsson and Woodford \(2003\)](#) and [Werning \(2012\)](#) study the optimal policy at the ZLB in an infinite horizon model, emphasizing the associated commitment problem. [Coibion et al. \(2012\)](#) determines the optimal inflation target in the presence of occasional liquidity traps. [Bassetto \(2015\)](#) studies the optimal communication problem of central banks' forward guidance policies. We extend their analyses to a setup where beliefs about the commitment type of the central bank need not be identical across agents. [Bodenstein et al. \(2012\)](#) investigate quantitatively how imperfect credibility of future policy rate announcements in Sweden and in the US lowered the impact of forward guidance policies. They conclude from their analysis that the period of low interest rate should be extended – and future monetary stimulus increased – for forward guidance to remain fully efficient. In contrast, we show that the impact of forward guidance may not only be muted compared to the full commitment case, but that it can even be detrimental when the credibility of commitment is not shared broadly enough among agents.

We investigate disagreement in the case of liquidity traps provoked by exogenous natural rate of interest shocks as in [Eggertsson and Woodford \(2003\)](#) or [Werning \(2012\)](#). Similar insights can be obtained in the context of endogenously low natural rate of interest as in [Eggertsson and Krugman \(2012\)](#) or [Eggertsson and Mehrotra \(2014\)](#) as soon as agents can disagree on future monetary policies.

Several papers study monetary policy under limited information. [Melosi \(2010\)](#) considers a dispersed information setup where monetary policy decisions signals central bank's information to the private sector. [Paciello and Wiederholt \(2014\)](#) analyse optimal monetary policy when firms choose the attention they pay to aggregate conditions. Several papers investigate the role of imperfect information at the ZLB. [Wiederholt \(2014\)](#) develops a model where

agents have dispersed beliefs about the current aggregate shock generating the liquidity trap. He then explains why inflation expectations remained well anchored, even in the presence of deflationary risks. He also highlights the potentially detrimental effect of communication policies that may lead agents to revise downward these expectations. [Bianchi and Melosi \(2015a\)](#) emphasize that transparency on future deviations from active stabilization is welfare improving as it anchors medium to long-run macroeconomic expectations. [Bianchi and Melosi \(2015b\)](#) study the consequences of the private sector's uncertainty on whether a stabilizing monetary/fiscal policy mix regime will be abandoned when the economy reach the the ZLB. [Gaballo \(2014\)](#) and [Kiley \(2014\)](#) illustrate how imperfect information can reduce the efficiency of forward guidance in New-Keynesian models. We derive the optimal monetary policy at the ZLB when agents perfectly observe market variables but they disagree about the unobserved length of the trap. In contrast to our setup, such alternative modeling frameworks cannot replicate situations where agents agree on future interest rates but disagree about future activity and inflation.

Finally, we rely on survey expectations to infer how private agents form their expectations. We work with surveys of professional forecasters which [Carroll \(2003\)](#) shows influences the forecasts of households. [Coibion and Gorodnichenko \(2015\)](#) show that households forecasts behave on average similarly to the ones of professionals. [Andrade et al. \(2013\)](#) underline that, in normal times, professionals understand monetary policy in the sense that they rely on a Taylor rule to draw their interest rate forecasts. [Carvalho and Nechio \(2014\)](#) provide similar evidence from qualitative households' forecasts. Whether households adjust their consumption in reaction to changes in their inflation expectations and hence to the monetary policy stance they forecast is open to debate (see [Bachmann et al. \(2015\)](#) and [D'Acunto et al. \(2015\)](#)). In contrast to these recent contributions, we focus on how private agents understand future monetary policy when the economy is at the ZLB and the central bank is bound to deviate from its normal times rule.

The paper is organized as follows. We present the stylized facts specific to forward guidance in Section 2. We introduce a New-Keynesian model with heterogenous beliefs and characterize the optimal monetary policy in Section 3. Finally, in Section 4, we discuss two further implications of the model: why it can rationalize the forward guidance puzzle and why a central bank cannot signal its type at the ZLB.

2 Stylized facts

In this section, we present new facts on the cross-sectional dispersion of forecasts, or “disagreement”, observed in the US survey of professional forecasters precisely when the Fed started to conduct a date-based forward guidance policy in August 2011.⁸ More precisely, we look at disagreement on short-term interest rates and on macroeconomic determinants of future monetary policy such as inflation and consumption growth.

Consistent with the evidence in [Andrade et al. \(2013\)](#), we postulate that, in normal times, agents view the short-term interest rate to be predominantly determined by the reaction function of the central bank. Under this assumption, the cross-sectional dispersion of short-term interest rate forecasts should be linked to the cross-sectional dispersion of variables such as inflation or activity which are typical inputs of usual monetary policy rules.

We then show that, when the Fed started to conduct a date-based forward guidance, agents began to agree on future short-term interest rates but kept disagreeing on their fundamental determinants. The correlation between disagreements on various macroeconomic variables strikingly dropped precisely at that time. Furthermore, we provide evidence that the break that happened at the time of forward guidance was associated with an increase in disagreement about medium-term forecasts of such macroeconomic fundamentals.

2.1 Empirical framework

We assume that private agents consider that the monetary policy of the central bank can be described by the following rule:

$$r = f \cdot \Omega + \epsilon,$$

where r is the short-term nominal interest rate, Ω is the set of variables that are relevant to the central bank decisions, f is the (linear) reaction function of the central bank and ϵ is a non-systematic deviation from such a reaction function.

Let $E_{i,t}(\cdot)$ denote the expectation of an individual i conditional on its information set available at date t . At date t , individual i 's forecast of future nominal interest rate h periods ahead is given by $E_{i,t}(r_{t+h}) = E_{i,t}(f \cdot \Omega_{t+h}) + E_{i,t}(\epsilon_{t+h})$. We assume that, in normal times, every agent i agrees on the reaction function f and that future deviations from the rule are

⁸On August 8, 2011, the FOMC reinforced its forward guidance policy by stating that “The Committee currently anticipates that economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013.” Previous statements were looser, mentioning a period of zero interest rates that will last for “some time” or for a “long period of time”.

non-predictable, so that $E_{i,t}(\epsilon_{t+h}) = 0$. We then get that:

$$E_{i,t}(r_{t+h}) = f \cdot [E_{i,t}(\Omega_{t+h})]. \quad (1)$$

This expression makes clear that, in normal times, the disparity of opinions about future nominal interest rates is driven by the disagreement about future fundamentals Ω_{t+h} . This implication is in line with the evidence provided in [Andrade et al. \(2013\)](#). They show that, over the past 30 years, and for a large range of forecasting horizons, disagreement about future short-term interest rate can be explained by forecasters agreeing on the Fed's reaction function but disagreeing about fundamentals, namely future short-term interest rates, future inflation and future growth rates.

By contrast, in crisis time, other sources of disagreement may arise. First, people may start to believe that the central bank will deviate from its normal time rule in the future so that $E_{i,t}(\epsilon_{t+h}) \neq 0$. This can happen either because it chooses to do so (through unconventional policies) or because it is constrained to do so (because of the zero lower bound on nominal interest rates). In addition, agents may expect that the central bank will change its reaction function compared to normal times and have different views on what the new rule will be, $E_{i,t}(f \cdot \Omega_{t+h}) = f_i \cdot [E_{i,t}(\Omega_{t+h})]$. Note that considering that agents have their own view on the reaction function f_i , is equivalent to considering agents disagreeing about future deviations η from an average rule f . In that case, individuals' forecasts of future short-term nominal interest rates follow:

$$E_{i,t}(r_{t+h}) = f \cdot [E_{i,t}(\Omega_{t+h})] + [E_{i,t}(\eta_{t+h})] \quad (2)$$

with $E_{i,t}(\eta_{t+h}) = (f_i - f) \cdot [E_{i,t}(\Omega_{t+h})] + E_{i,t}(\epsilon_{t+h})$. This expression makes clear that, in non-conventional times, the disparity of opinions about future nominal interest rates is driven by two sources: disagreement about future fundamentals Ω_{t+h} , as in normal times, and, in addition, disagreement about future deviations from the usual reaction function η_{t+h} .

Equation (2) has important implications for the joint evolution of disagreement about future interest rates and fundamentals. For a given disagreement about future deviations from the policy reaction function ϵ_{t+h} , changes in the disagreement about future policy, r_{t+h} , should coincide with changes in future fundamentals, Ω_{t+h} . Alternatively, observing a change in the disagreement about future values of r_{t+h} that differs from the disagreement implied by future fundamentals Ω_{t+h} means that the disagreement about future deviations from the rule η_{t+h} has changed.

2.2 Forward guidance and disagreement on the macroeconomy

In this section, we document how disagreement about three macroeconomic variables, namely the short-term nominal interest rate, the inflation rate and the consumption growth rate, has evolved over recent years.⁹ We measure disagreement about several macroeconomic variables by relying on the individual forecasts provided in the US quarterly Survey of Professional Forecasters (SPF). More precisely, the measure of disagreement is the interquartile range in the distribution of individual forecasts, i.e. the difference between the 75th percentile and the 25th percentile in the cross-sectional distribution of individual forecasts for a given quarter. We work with a 1982Q2-2014Q4 sample. We focus on three forecast horizons – 1 quarter, 1 year and 2 years.¹⁰

2.2.1 Short-term interest rates

Figure 1 reports the evolution of disagreement about 1Q, 1Y, and 2Y ahead forecast of US short-term nominal interest rates. Campbell et al. (2012) and Swansson and Williams (2014) illustrate that US forward guidance announcements lowered expected future interest rates. Looking at Figure 1 reveals that, in addition, forward guidance was associated with a sharp drop in the dispersion of forecasts about future US short-term nominal interest rates. In particular “date-based” statements (which started in August 2011) and “state-contingent” statements (which started in December 2012) were associated with a strong coordination of opinions of short (1Q) but also medium term (1Y/2Y) nominal interest rate forecasts.¹¹

Importantly, Figure 1 also illustrates that disagreement on future nominal interest rates 1 year and 2 years ahead did not decline when the economy reached the ZLB, that is in 2008Q4. They declined markedly only when the Fed implemented fixed date forward guidance in August 2011. This more explicit commitment on future short-term interest rates was much more efficient in coordinating opinions about future interest rate for longer horizons than the

⁹Similar results hold when one considers disagreement on real GDP growth instead of consumption. We preferred to focus on consumption which is more consistent with models advocating forward guidance which usually rely arguments based on the Euler equation and intertemporal substitution of consumption. Looking at disagreement on future consumption growth is also consistent with the NK model we develop in the next section.

¹⁰More specifically, the survey collects each quarter individual forecasts for the 3-Month T-Bill rate, the headline CPI rate, and the private consumption growth rate. 1Q and 1Y ahead forecasts are quarterly averages. 2 year ahead forecasts are annual averages.

¹¹Similar signs of an increase in the agreement on nominal interest can also be found in the Michigan survey, where the fraction of surveyed households expecting interest rates to rise significantly decreased around date-based announcements.

“open-date” announcements used from 2008Q4 to 2011Q2.¹² In contrast, the ZLB combined with the so-called “open-date” forward guidance (which started in January 2009) only led to a coordination of opinions on nominal interest rates 1 quarter ahead.

2.2.2 Inflation and demand

Figure 2 displays the evolution of disagreement on 1Q, 1Y and 2Y ahead forecast of US consumption growth and inflation rates. It shows that forecasters’ disagreement about these two variables also decreased substantially starting 2009, in particular for short horizon forecasts. However, for both variables, this decline in disagreement was more an adjustment after the unusually high levels they reached at the peak of the Great Recession crisis. In any case, they did not substantially fall below historical standards. This stands in sharp contrast with what is observed for disagreement about future nominal interest rates.

Figure 3 digs a bit deeper into the characteristics of disagreement about future fundamentals observed in Summer 2011. More precisely, it plots the distribution of individual forecasters’ revisions of 2-year ahead inflation forecasts observed in 2011Q3, that is at the date when the Fed moved to the explicit date-based forward guidance. This distribution is compared with the empirical distribution of individuals’ inflation revisions observed for each quarter of the 2010Q1-2014Q4 period. As the chart illustrates, on average, individual revisions of future inflation are centered around zero over that period. By contrast, after the announcement of August 2011, two groups of forecasters clearly emerged. One of the groups had views on future inflation that was more optimistic than the other. Forward guidance was therefore coincident with two different views about future inflation. Note also that at that in 2011Q3, the probability of an extremely optimistic or pessimistic revisions were much more important than the average probability observed over the whole 2010Q1-2014Q4 sample. As we further illustrate, forward guidance coincided with an increase in disparity of opinions about medium-term inflation forecasts.

2.2.3 A structural break in August 2011

According to (1), in normal times, disagreement on fundamentals Ω_{t+h} – which determine the policy rate through the reaction function f – should map into disagreement about future nominal interest rates r_{t+h} . However, the previous results show that, for medium-term horizons, disagreement about Ω_{t+h} started to be disconnected from disagreement about r_{t+h}

¹²Forward guidance statements were also combined with quantitative easing operations which may have had an impact on future interest rate forecasts through a signaling channel. In this paper, we do not make the distinction between this signaling effects of QE and forward guidance policies.

precisely at the time when the Fed started its date-based forward guidance policy.¹³ Figure 4 illustrates this disconnection further. It presents the evolution of the disagreements of 1-year and 2-year ahead expected consumption and inflation scaled by the disagreement about the 1-year and 2-year ahead short-term interest rate forecasts. These relative disagreements strikingly spiked up to unprecedented values concomitantly with the reinforcement of forward guidance policy in Summer 2011. While disagreement about future short-term interest rates and future consumption growth and inflation rates usually fluctuate around a constant ratio, that normal-times relationship has been broken when the Fed strengthened its forward guidance.¹⁴

One may wonder whether, in line with our assumption of equation (1) disagreement on future nominal interest rates is related to disagreement on future fundamentals in normal times. If not, then disagreement about future nominal interest rates always evolves independently from such fundamentals and what we observe in Figure 4 might just be a pure random event. We therefore test whether the date-based forward guidance is associated with a significant shift in a relationship between disagreement that applies in normal times.

Table 1 displays the regression results of the (log) disagreement about 1-year ahead nominal interest rates on the (log) disagreement on a set of such potential fundamentals Ω_{t+h} , namely 1-year ahead inflation, consumption growth, and 1-quarter ahead interest rates (which captures policy smoothing in the reaction function). We contrast a pre date-based forward guidance sample (1982Q1-2011Q2) with a post date-based forward guidance sample (2011Q3-2014Q4). Two conclusions can be drawn. First, consistent with our assumption that agents rely on a Taylor-rule type of policy reaction function to forecast short-term interest rates, disagreement on future interest rate is significantly correlated with disagreement on future fundamentals in regular times.¹⁵ Second, that normal times relationship completely disappeared when forward guidance started to be implemented. Whatever the specification, the tests clearly reject the

¹³Crump et al. (2013) exploit the timing of the Blue Chip survey to document that there has been a sharp drop in the dispersion of individual short-term interest rate forecasts in between July and September 2011 while, at the same time, the dispersion of individual inflation or GDP growth forecasts remained broadly stable. So they conclude that the coordination of opinions about future short-term interest rates was mainly a result from the change change in monetary policy of August 2011.

¹⁴Looking at the evolution of disagreement about US consumption and inflation 1 year ahead relative to disagreement about future interest rates 1 year ahead for the whole 1982-2014 of data available makes clear that the forward guidance episode stands out as a clear outlier within more than 30 years of US economic history. So what was observed when the Fed implemented forward guidance has never been observed over the last three decades.

¹⁵See Andrade et al. (2013) for more evidence on a larger set of forecast horizons based on the Blue Chip survey data.

null of no change in the relationship between the two periods.

2.2.4 An increase in disagreement on medium-term fundamentals

We now show that the correlation between disagreement time series collapses when the Fed announces date-based forward guidance. Specifically, Figure 5 presents the residuals of a regression of disagreement about, respectively, the 1-year and 2-year ahead forecasts of inflation on (i) the disagreement about, respectively, the 1-year and 2-year ahead forecasts of short-term nominal interest rates and (ii) the disagreement about 1-quarter ahead inflation forecast. The regression is estimated on a pre-crisis sample (1982Q2-2008Q4).

The adoption of date-based forward guidance is a striking break. Before August 2011 the residuals are not different from zero. Controlling for disagreement on 1 quarter inflation forecasts, disagreement about future inflation stays in the range of what its correlation with disagreement on interest rate would predict. By contrast, disagreement on future inflation becomes much higher than its predicted value after August 2011. ¹⁶

A potential explanation of why there is a disconnect between disagreement on future short-term nominal interest rates and disagreement on future fundamentals is that people started to be more uncertain about fundamentals as the economy reached the ZLB. As Bianchi and Melosi (2015b) emphasize, everything else being constant, macroeconomic uncertainty should increase at the ZLB because one stabilization instrument – conventional monetary policy – cannot be used in such circumstances. However the evidence reported in Figure 4 weakens this explanation. This break does not occur when the economy reached the ZLB but when the Fed moved to the date-based forward guidance.

2.3 Summary and implications

We draw the following four main implications from our empirical analysis.

1. Before the Great Recession, agents forecast future nominal interest rates according to a rule akin to equation (1): $E_{i,t}(r_{t+h}) = f \cdot [E_{i,t}(\Omega_{t+h})]$. Disagreement on future nominal interest rates r_{t+h} is linked to disagreement about their fundamental determinants Ω_{t+h} .
2. When date-based forward guidance started to be implemented, agents agreed on future nominal interest rates r_{t+h} but disagreed on future fundamentals, e.g. future inflation and consumption, Ω_{t+h} . This requires that agents disagreed about the sequence of future

¹⁶We obtain similar evidence for disagreement on medium-term real consumption growth.

deviations from the normal times rule η_{t+h} in (2) $E_{i,t}(r_{t+h}) = f \cdot [E_{i,t}(\Omega_{t+h})] + [E_{i,t}(\eta_{t+h})]$.

$$\forall i, E_{i,t}(r_{t+h}) = 0 \text{ and } f \cdot E_{i,t}[(\Omega_{t+h})] = -E_{i,t}(\eta_{t+h}).$$

3. Controlling for short-term disagreement and disagreement on future interest rates, date-based forward guidance increased disagreement on fundamental determinants Ω of future interest rates for 1-year and 2-year ahead forecasts. This started to happen exactly when date-based forward guidance was implemented.
4. In addition, at the same time, forecasters could broadly be classified into two groups with views on medium-term fundamentals Ω_{t+h} that were relatively optimistic or pessimistic.

In the next section, we present a model that is consistent with these implications. In particular, agents share the same view about future policy rates but rationally disagree about the type of forward guidance that is implemented. More precisely, agents agree on the path of future nominal interest rates r_{t+h} but disagree on the deviations from the normal time policy reaction function η_{t+h} that this path conveys. To put it differently, agents disagree on the stance attached to such path.

3 Theory: Does agreeing on the interest rate path imply agreeing on policy?

The aim of this section is to clarify the implications of agents' agreement on the interest rate path in the context of a standard New Keynesian model of an economy at the zero lower bound, as in [Eggertsson and Woodford \(2003\)](#) or [Werning \(2012\)](#). We take as given agents' agreement on nominal interest rates and, in addition to the rest of the literature, we assume that the commitment-type of the central bank is not observable, which in general allows for disagreement on future monetary policy. We first present a benchmark analysis before we then extend it to the case of heterogeneous beliefs on such a commitment ability. In this latter situation, we show that agreement on nominal interest rates is still an equilibrium outcome, when agents have heterogeneous beliefs also on the length of the liquidity trap. We finally derive the optimal policy in this context.

3.1 NK-economy with heterogeneous beliefs

Our model relies on Lucas type families, with both risk sharing and different beliefs between members, in an otherwise standard New-Keynesian economy. The household family

endogenously¹⁷ produce heterogeneous consumption-saving paths as long as agents maintain different views about the conduct of the monetary policy. In particular, agents can anticipate that in the long run they will equalize their wealth, which is essential to the existence of a unique steady state equilibrium for each individual.¹⁸

Household. The household family is constituted by a continuum of agents of mass one indexed by $i \in [0, 1]$. Each agent decides how much to work, consume and save in order to maximally contribute to the household welfare

$$U_i = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \quad (3)$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t . The parameter $\beta \in (0, 1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock discussed below.

Each agent manages a portfolio representing a fraction of the household wealth. Between periods t and $t + 1$, agent i deals with the following flow budget constraint:

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_tL_{i,t} + D_t - P_tC_{i,t} + Z_{i,t}, \quad (4)$$

where $B_{i,t}$ are bond holdings of the agent between periods $t-1$ and t , R_{t-1} is the gross nominal interest rate on bond holdings between periods $t-1$ and t , W_t is the nominal wage rate in period t , D_t is the difference between nominal profits received and nominal lump-sum taxes paid, by each agent in period t (we assume here diffuse ownership), and P_t is the price of the final good in period t . The agent can borrow (formally, bond holdings can be negative), but the household is not allowed to run a Ponzi scheme. Finally, the term $Z_{i,t}$ denotes a nominal intra-household transfer voluntarily received or carried out by agent i .

Intra-Household risk sharing. Each period is divided into three stages. In the first stage, current shocks hit and agents observe them. At this stage agents form their beliefs on the state of the world. In the second stage of each period, agents can implement a feasible transfer plan $\{Z_{i,t}\}_{i=0}^1$ such that

$$\int_0^1 Z_{i,t} di = 0. \quad (5)$$

¹⁷See [Curdia and Woodford \(2010\)](#), among others, for alternative assumptions for dealing with heterogeneous agents in the New-Keynesian model.

¹⁸In [Appendix A.2](#), we show that the household family model is first-order equivalent to an economy with a representative agent uncertain about the type of monetary policy announced.

only if every agent agrees on it. Without loss of generality¹⁹, we assume that when no unanimity is reached, then no transfers are made; in such a case each agent owns the wealth resulting from her own portfolio management. Let us therefore introduce the following formal definition.

Definition 1. *An implementable transfer plan at time t is a feasible transfer plan $\{\hat{Z}_{i,t}\}_{i=0}^1$ such that*

$$E_{t,i}[U_t|\{\hat{Z}_{i,t}\}_{i=0}^1] \geq E_{t,i}[U_t|\{Z_{i,t}\}_{i=0}^1],$$

for each $i \in [0, 1]$ and each feasible transfer plan $\{Z_{i,t}\}_{i=0}^1$.

In the last stage, once intra-household wealth transfers are carried out, each agent decides on her own labor supply and consumption, based on their own individual beliefs and taking other agents' decisions as given. The crucial assumption we are making here is that agents cannot commit on future transfers: each period they decide under discretion. We also assume that the whole mechanism is common knowledge.

Firms. Production is implemented in the context of a standard monopolistic competition environment. The final good is produced by competitive firms using the technology: $Y_t = (\int Y_{j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}$. Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j . The parameter θ is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate good j : $Y_{j,t} = P_{j,t}^\theta P_t^{-\theta} Y_t$, where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies $P_t = (\int P_{j,t}^{1-\theta} dj)^{1/(1-\theta)}$. Each intermediate good j is produced by a monopolist using the technology $Y_{j,t} = L_{j,t}$ where $Y_{j,t}$ is output and $L_{j,t}$ is labor input of this monopolist. Monopolists producing intermediate goods are subject to a price-setting friction as in Calvo (1983). Each monopolist can optimize its price with probability $1 - \chi$ in any given period. With probability χ the monopolist producing good j sets the price $P_{j,t} = P_{j,t-1}$. As we said, each agent owns an equal share of each firm and so the firms choose the price $P_{j,t}$ so as to maximize

$$E_t \sum_{k=0}^{\infty} \chi^k Q_{t,t+k} (P_{j,t} Y_{j,t+k} - W_{t+k} L_{j,t+k})$$

where $Q_{t,t+k}$ is firms' discount factor from time t to time $t + k$. Finally, the price level satisfies: $P_t = [(1 - \chi)P_{t,*}^{1-\theta} + \chi P_{t-1}^{1-\theta}]^{1/(1-\theta)}$ i.e. they cannot adjust instantaneously to the

¹⁹To explain why is without loss of generality, we need to introduce a bit more structure. See footnote 24 below.

optimal reset price $P_{t,*}$ as long as there are nominal rigidities, i.e. $\chi \neq 0$. To simplify our exposition we will assume that the information sets of the producers is isomorphic to the one of the household members. Finally, we assume that firms' stocks are held by households in equal shares.

Shock and Information. As is standard in the literature of optimal policy at the ZLB, we will focus on liquidity traps triggered by the natural rate of interest being below steady state for a number of periods. Therefore, to induce a trap of *length* $T \in \mathbb{N}$ starting at time $t = 0$ we will assume a series of shocks $\{\xi_\tau\}_{\tau=0}^\infty$ to the households' discount factor such that $\xi_\tau - \xi_{\tau+1}$ takes value $-\xi$ with $\tau = 0, \dots, T-1$ and zero afterwards, so that $t = T$ is the first period out of the trap.²⁰

The beliefs of agent i about the number of crisis periods T is formed at time $t = 0$ and denoted by $E_{i,0}[T]$ where $E_{i,0}[\cdot]$ represents the expectation of agent i conditional on the information set and priors of agent i at time t . In practice, $E_{i,0}[T]$ implies $E_{i,0}[\xi_\tau - \xi_{\tau+1}] = -\xi$ for $\tau = t, \dots, t-1+T$ and zero afterwards, starting from period $t+T$. It is important to remark that the end of the trap has the feature of a “news” on a future shock. As such it can be only assessed ex-post, once its realization occurs. In particular, $t = T$ is the first date when agents can assess the end of the trap. We do not restrict agents' information on current aggregate variables, including realizations of current shocks.

Monetary policy. The central bank's monetary policy is to set a path of gross nominal interest rate $\{R_t\}_{t \geq 0}$ in order to maximize the household's utility (3). Yet, this policy faces a zero lower bound (ZLB), that is $R_t \geq 1$, which constraints the policy action. Without loss of generality, we restrict our attention to the following representation of the policy action:

$$R_t = \max\{R e^{\epsilon_t} \Pi_t^\phi (Y_t/Y_t^n)^{\phi_y}, 1\}. \quad (6)$$

where $R = (1/\beta)$ is the nominal interest rate in the non-stochastic steady state with zero inflation, $\Pi_t = (P_t/P_{t-1})$ denotes the inflation rate, Y_t^n is the natural level of output (i.e. the counter-factual flexible price outcome) and $\phi > 1$ and $\phi_y \geq 0$ the monetary authority's systematic response to inflation and output gap respectively.

ϵ_t denotes a deviation from a strict application of a Taylor rule, which instead holds setting $\epsilon_t = 0$ at all times. Any policy whereby the interest rate would differ from a strict application of a Taylor rule can be mapped into the representation above through a sequence of ϵ_t different

²⁰There is no loss of generality in considering deterministic traps. In a more general framework, the shock can follow a Markov process as in Eggertsson and Woodford (2003).

from 0. The policy choice of the authority can be summarized by a path $\{\epsilon_\tau\}_{\tau=0}^\infty$. In this respect, we consider two alternative types of central banker: the type \mathcal{C} can *commit* to set ϵ_t in advance, whereas the type $-\mathcal{C}$ cannot. In the latter case, the central bank will *ex post* re-optimize its policy each time. Importantly, the type $\varrho \in \{\mathcal{C}, -\mathcal{C}\}$ of the monetary authority is not observed by agents.

Finally, we assume that the central bank does not have more information on the length of the trap compared with the private sector. Conversely, it can observe the current allocation and agents' decisions. Note that we rule out the possibility for the central bank to make announcements.²¹

Fiscal policy. Regarding fiscal policy, we assume that the government implements a constant proportional tax on sales proceeds as in [Woodford \(2003\)](#), whose revenues are transferred lump sum to households. This ensures that the monetary authority has no inflation bias.²²

Equilibrium. We are now ready to define an (perfect bayesian) equilibrium:

Definition 2. *For a given sequence of shocks $\{\xi_\tau\}_{\tau=0}^\infty$, an equilibrium at time $t = 0$ is defined by the set of following conditions:*

i) given a sequence of policy deviations $\{\epsilon_\tau\}_{\tau=0}^\infty$ and a set of beliefs about the end of the trap and the type of the authority $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i \in [0,1]}$,

$$\{C_{i,t}, L_{i,t}, B_{i,t}, D_t, R_t, W_t, Z_{i,t}, P_t\}_{i \in [0,1], t \geq 0}$$

solves household's and firms' problems, satisfies the monetary policy rule (6) and so that markets clear;

ii) given a type of the central bank and given agents' optimal reaction, $\{\epsilon_0, \dots\}$ solves the central bank's problem;

iii) agents beliefs $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i \in [0,1]}$ are updated following Bayes' law.

²¹See [Bassetto \(2015\)](#) for an analysis of the resulting communication problem. See also Section 4.2 for further discussion.

²²In the benchmark model, we only consider shocks to the natural rate of interest in the presence of a zero lower bound associated with the time-inconsistency problem of forward guidance policy. Similar insights can be derived for cost-push shocks. Indeed, these shocks also require a time-inconsistent monetary policy response (known as the *stabilization bias*).

In this definition, we require that agents' actions maximize expected utility conditional on agents' beliefs about the length of the trap which have to be consistent with the observed current allocation, the size of the discount shock (not the length) and the optimal monetary policy given the disagreement in the economy. The equilibrium is defined in terms of time 0 posteriors of agents' beliefs and the actual allocation. Agents of each type anticipate, consistently with their own beliefs, that the other type will update once the truth unfolds.

Condition (iii) establishes that agents' beliefs must be rational expectations in the sense that any available observable produced in equilibrium will be used by agents to restrict their beliefs about the length of the trap. In this respect, *we do not assume any informational friction or ad-hoc asymmetry*. As said, the only two elements that are not directly observable to agents are the length of the trap and the commitment-type of the authority.

The set of equilibria satisfying this definition is large and so, in the following, we make some further assumptions on agents' prior beliefs about the type of the monetary authority to select equilibria in which agents agree on the number of periods with the interest rate at the zero lower bound.

Remark. Given that the type of the authority is not observable, a central banker of the commitment type would be willing to signal its type by current actions, which are observable. Such signaling is not implementable as we show in Section 4.2. The reason is that the non-commitment type could strategically mimic the commitment type, preserving the advantage of not paying the cost of an ex-post inefficient boom. Therefore, it is important to remark that, the presence of heterogeneous beliefs is possible because the equilibrium is pooling, by result and not by assumption.

3.2 Common knowledge on the commitment ability of the central bank

This subsection presents the analysis with homogeneous beliefs on the commitment-type of the central bank. In this case, which is equivalent to the situation in which the commitment-type is observable, if agents have the same expectations about the policy path, they also share the same views about the length of the trap.

Inflation targeting in normal times. Given individual beliefs and wealth distribution, agents' first order conditions yield the consumption Euler equation in any period $t \geq 0$:

$$c_{i,t} = -\gamma^{-1}(E_{i,t}[\xi_{t+1}] - \xi_t + r_t - E_{i,t}[\pi_{t+1}]) + E_{i,t}[c_{i,t+1}], \quad (7)$$

that we express here in lower case denoting log-linear deviations from steady state. Current consumption increases as the current interest rate decreases or future inflation or consumption increase. The labor-decision equation instead entails a static relation,

$$\gamma c_{i,t} = w_t - p_t - \psi l_{i,t}, \quad (8)$$

meaning that, for the same real wage, lower consumption increases labor supply. This determines a unique equilibrium as stated below. Manipulating the equilibrium relations as shown in appendix A.1, we can recover the well-known New-Keynesian Phillips curve

$$\pi_t = \kappa c_t + \beta \int_0^1 E_{i,t}[\pi_{t+1}] di, \quad (9)$$

linking current inflation to current aggregate consumption and the average expectation of future inflation.

In the absence of the ZLB constraint, the central bank would be able to perfectly smooth discount rate shocks. The first best allocation can be implemented both by the commitment- and the no-commitment type central banks by setting the interest rate to a level given by the rule (6) with $\epsilon_t = 0$ and $\phi_y = 0$. The result $\phi_y = 0$ follows directly from the observation that in presence of shocks to the discount factor only, there is no trade-off between inflation and the output gap so that, a strong response to inflation (i.e. a ϕ chosen sufficiently high with $\phi_y = 0$) is sufficient to fully stabilize the economy. The resulting allocation is the steady state $\{c_{i,t}, \pi_t, r_t\} = \{0, 0, 0\}$ at any t for each i .

Inflation targeting at the ZLB. Yet, when the discount rate shock is too large, the downward limit on interest rates, $R_t \geq 1$, may prevent such stabilization. Suppose agents have homogeneous beliefs that the trap will last $E_{i,0}[T] = E_0[T]$ periods and that the authority will take interest rates at zero from $t = 0$ until $E_{i,0}[T] - 1 = E_0[T] - 1$ included, then the resulting expected and current consumption is given for each i by,

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [0, E_{i,0}[T] - 1], \quad (10a)$$

$$E_{i,0}[c_{i,t}] = 0 \text{ for } t \geq E_{i,0}[T], \quad (10b)$$

where the inflation path expected is determined in accordance with the Phillips curve (9). The ZLB on the nominal interest rate imposes $\log R$ as an upper bound to the stimulative impulse that monetary policy can give under the restriction $\epsilon_t = 0$.

Note that when the authority follows an inflation targeting rule, then the number of periods at which the interest will stay at zero is *equal to* the length of the trap. We then define the following.

Definition 3. *When the authority cannot commit, i.e. its type is $-\mathcal{C}$, it sets $\epsilon_t = 0$ at any t , then, the policy has a Delphic nature, i.e. for a given T it will be $R_t = 1$ for $t \in \{0, \dots, T\}$ and $R_t = R$ for $t > T$.*

We use the term Delphic consistently with [Campbell et al. \(2012\)](#), with the meaning that, beliefs about the number of periods when the interest is at the ZLB correspond exactly to beliefs about the length of the trap.

The Odyssean FG-benchmark. As shown by [Krugman \(1998\)](#), [Eggertsson and Woodford \(2003\)](#) and [Werning \(2012\)](#), when the ZLB binds, the second-best policy prescribes policy rates at zero for longer than a Delphic central bank would. In fact, an Odyssean policy amounts to stimulate current consumption promising lower short-term rates once the trap ends. This case can be characterized by a sequence of ϵ_t sufficiently low such that, irrespective of the course of inflation, the policy rate satisfies $r_t = -\log R$, for an optimal number of periods $T_{cb} \geq T$.

As before, suppose agents have homogeneous beliefs that the trap will last $E_{i,0}[T]$ and that the authority will keep interest rates at zero for a number of periods $E_{i,0}[T_{cb}] \geq E_{i,0}[T]$, then the resulting expected and current consumption is given for each i by,

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [0, E_{i,t}[T] - 1], \quad (11a)$$

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [E_{i,0}[T], E_{i,0}[T_{cb}] - 1], \quad (11b)$$

$$E_{i,0}[c_{i,t}] = 0 \text{ for } t \geq E_{i,0}[T_{cb}], \quad (11c)$$

where the inflation path expected is determined in accordance with the Phillips curve (9). This policy generates an expansionary stimulus after the end of the trap, that boosts inflation and consumption in the future. The expectation of a future boom is partly reflected in current consumption. Therefore, this policy can, through an optimal choice of T_{cb} , deliver higher welfare than Delphic forward guidance which sticks to a time consistent inflation targeting.

When the authority implements this second-best policy, then the number of periods at which the interest is expected to stay at zero is *longer than* the length of the trap. We then define the following.

Definition 4. *When the authority is of the commitment type \mathcal{C} , i.e. it sets ϵ_t sufficiently low for $T_{cb} > T$ periods and $\epsilon_t = 0$ after, then, the policy has an Odyssean nature, i.e. for a given T it will be $R_t = 1$ for $t \in \{0, \dots, T, \dots, T_{cb} - 1\}$ and $R_t = R$ for $t \geq T_{cb}$.*

Again, we borrow the term Odyssean from [Campbell et al. \(2012\)](#), with the meaning that, beliefs about the number of periods at the ZLB do not directly match the length of the trap,

but rather they are induced by the optimal policy choice T_{cb} . It is important to note that the authority *must* be able to commit to this Odyssean policy - it ties its hands as Odysseus before the sirens. Let us clarify this point in the following.

Time-inconsistency of FG. Once the recovery occurs, inflation is no longer socially desirable, and the authority would be tempted to renege on her promise and set $\epsilon_t = 1$ from T onward, which corresponds to the time-consistent solution with perfect stabilization at steady state, after the end of the trap.

Therefore, the second-best policy at the ZLB implies a time-inconsistency problem so that the possibility to implement it relies on the central bank being of the commitment type ($\varrho = \mathcal{C}$).²³ When instead the authority cannot commit ($\varrho = \neg\mathcal{C}$), then the optimal (second best) policy corresponds to the one implemented in normal times as soon as the economy exits from the trap. This is common knowledge among agents and pins down their beliefs.

Proposition 1. *Suppose agents have homogeneous beliefs about both the number of periods with interest rates at the ZLB, T_{zlb} , and the type of authority, ϱ , then they have the same expectation on the length of the trap, in particular:*

- if $E_{i,0}[\varrho] = \neg\mathcal{C}$ for each i , then the policy is Delphic so that

$$T_{zlb} = E_{i,0}[T] \text{ for each } i,$$

and $c_0 \equiv \underline{c}$;

- if $E_{i,0}[\varrho] = \mathcal{C}$ for each i , then there exists an optimal Odyssean policy $T_{cb}(T)$ such that

$$T_{zlb} = E_{i,0}[T_{cb}(T)] > E_{i,0}[T] \text{ for each } i,$$

and $c_0 \equiv \bar{c} > \underline{c}$.

This proposition is the consequence of the results established by [Eggertsson and Woodford \(2003\)](#) and [Werning \(2012\)](#). It stresses that if agents agree on a path of interest rates and on the commitment ability of the central bank, then they should also agree on the length of the trap and (obviously) the current equilibrium allocation. This scenario represents the benchmark against which we will contrast the effects of heterogeneous beliefs.

Figure 6 plots the reaction of consumption and inflation to the sequence of shocks with homogeneous beliefs in the two scenarios with and without beliefs that the central bank can

²³One reason why agents believe that the central bank would stick to a time inconsistent policy is that they face a reputation cost if they renege on their commitment

commit. Odyssean policy (in blue) requires keeping interest rates at the ZLB for 4 quarters after the end of the trap. A Delphic policy would in contrast keep interest rate at zero only for the length of the trap. Odyssean policy avoids a major recession, whereas consumption and inflation drop if monetary policy cannot provide additional support and follows a standard inflation targeting rule.

Unfortunately, we neither observe beliefs about the type of the monetary authority nor on the length of the trap so we cannot conclude that agreement on the interest rate path implies agreement on these two dimensions. As we will see, although agents have homogeneous beliefs about the interest rate path and observe the current equilibrium allocation, they can still disagree on the length of the trap and the commitment ability. Let us discuss this scenario in the next subsection.

3.3 Agreement on interest rates with disagreement on policy

In this subsection, we maintain homogeneous beliefs on the path of interest rates and on the (observable) current allocation and show that this is compatible in equilibrium with heterogeneous beliefs jointly on both the commitment ability of the authority and the length of the trap.

Optimists and Pessimists. Let us consider the case in which agents agree that the interest rate will be at the ZLB for T_{zlb} periods. Agents can be of two types depending on their priors on the commitment ability of the authority: some agents think that the central bank can commit ($\varrho = \mathcal{C}$), while others do not think so ($\varrho = \neg\mathcal{C}$). The former hold a belief $E_{o,0}[T_{cb}] = T_{zlb}$, i.e. they expect the trap to be shorter than the period at the ZLB ($E_{o,0}[T] < T_{zlb}$) - we label them *optimists*, as they expect a short liquidity trap associated with expansionary monetary policy. Conversely, the latter, who do not believe in the commitment, will hold a belief $E_{p,0}[T] = T_{zlb}$, i.e. they expect the trap to be as long as the period at the ZLB - we label them *pessimists*, as they expect a longer liquidity trap without additional monetary policy accommodation. Finally, we denote by $\alpha \in [0, 1]$ the fraction of agents that are pessimists whereas the rest are optimists.

Both optimists and pessimists agree on the same path of nominal interest rates until date $T_{zlb} - 1$, but they expect different evolutions of the economy thereafter, in particular a different path of the real interest rate. Disagreement prevails at the equilibrium because the information available at time 0 cannot reveal the type of the monetary authority nor the length of the trap. The (observable) current allocation itself does not aggregate any “truth”.

In this sense, agents observe the distribution of beliefs but they have no reason to update their own opinion and, thus, they may agree to disagree.

However, it is common knowledge that at time $E_{o,t}[T]$, which is , at the optimistic date for the end of the trap, only one of the two types will be right. Optimists (pessimists) can be either right (wrong) or wrong (right) in case the trap is over (not over). At that point, heterogeneity of beliefs ends.

Remark. We can extend our analysis to the more general environment where the shock follows a Markov process. In this case, optimists' subjective beliefs on the probability of exiting the trap is higher than the ones of pessimists. This leads to a limited disagreement (rather than strict agreement as in the deterministic case) between optimists and pessimists on the whole path of nominal interest rates.

Risk-sharing with disagreement.

Disagreement has major consequences for the dynamics of intra-family transfers. At the second stage of each period, agents need to decide on the wealth transfers. In the absence of disagreement, this would optimally result in an even distribution of wealth.

Yet, the type of the central bank will be revealed only once date $E_{o,t}[T]$ is reached, as explained in the previous paragraph. Before that date, agents have different opinions on which transfer plan maximizes family welfare, even though they anticipate that they will share wealth in the future. This implies that no transfer plans can be implemented before date $E_{o,t}[T]$.

The following proposition states this formally.

Proposition 2. *Consider the case of heterogeneous beliefs about the end of the trap, namely $E_{o,0}[T] < E_{p,0}[T]$, then the only equilibrium sequence of implementable plans of transfers $\{\{Z_{i,t}^*\}_0^1\}_{t=0}^\infty$ is the one providing for $\{Z_{i,t}^*\}_0^1 = 0$ at each $t \neq E_{o,0}[T]$ and $\{Z_{i,E_{o,0}[T]}^*\}_0^1$ such that*

$$U_c(C_{i,T}) = U_c(C_{j,T}) \text{ for } t = E_{o,0}[T], \tag{12}$$

namely, the marginal utility of consumption is equal between types at the time where the truth unfolds, which implies $B_{i,t} = B_{j,t}, \forall (i, j)$ for $t \geq E_{o,0}[T]$.

Proof. See [Appendix](#). □

As no transfers are made during the period of the trap, the two types of agents then consume according to their beliefs, managing the share of wealth that they hold at the beginning of the trap.

It is worth to remark that proposition 2 relies on the assumption that households cannot commit to future transfers. As a consequence, agents of each type anticipate that, whatever their financial position, intra-household wealth will be equalized at a future date, when the truth will eventually unfold. Before that date, intra-family transfers, even if they were implemented,²⁴ cannot change agents' perceptions of their permanent income, and so cannot affect current consumption-saving choices. In other words, as they expect wealth to be equalized in the future – even though not at the same level – but anticipate different paths of real interest rates, pessimists and optimists select different paths of consumption. If different transfers are implemented, pessimists and optimists both modify their portfolio choices, keeping consumption paths unmodified and anticipating future transfers.

Allocation with heterogeneous beliefs. The effect of disagreement on the current and expected allocation is established by the following proposition.

Proposition 3 (Equilibrium beliefs). *Suppose agents have homogeneous beliefs about the number of periods with interest rates at the ZLB, T_{zlb} , and there exists a fraction $1 - \alpha$ of optimists anticipating a trap of length $E_{o,0}[T] \leq T_{zlb}$, then the unique path for each individual's expected consumption is given respectively by (11) with $i = o$ for the optimists, and (10) with $i = p$ for the pessimists, where the inflation path is expected by each type in accordance with the Phillips curve:*

$$E_{i,0}[\pi_t] = \kappa E_{i,0}[\alpha c_{p,t} + (1 - \alpha)c_{o,t}] + \beta E_{i,0}[\alpha E_{p,t}[\pi_{t+1}] + (1 - \alpha)E_{o,t}[\pi_{t+1}]], \quad (13)$$

where

$$E_{i,0}[c_{j,t}] = E_{j,0}[c_{j,t}] \text{ and } E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{j,0}[\pi_{t+1}] \text{ for } t \in [0, E_{o,0}[T] - 1], \quad (14a)$$

$$E_{i,0}[c_{j,t}] = E_{i,0}[c_{i,t}] \text{ and } E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{i,0}[\pi_{t+1}] \text{ for } t \geq E_{o,0}[T], \quad (14b)$$

with $(i, j) \in \{o, p\}^2$.

The interpretation of equilibrium beliefs is intuitive. Each type understands that until date $E_{o,0}[T]$ no information can lead the other type to change her beliefs. At that date, each type

²⁴ With different opinions about which plan achieves the first best, agents cannot implement any transfer plan. However, this rule has the mere role of selecting a unique feasible plan when agents disagree. That is, another backup rule would not change the results. For example, we could have equally assumed that, when agents disagree, a king decides on their transfers. Given that the king cannot enforce future transfers (no commitment), agents commonly know that, from some future date onward, they will agree again, and so, their wealth will be equalized. In this case, the king's transfers cannot affect the perceived permanent income of an agent, and so cannot change agents consumption-saving plans.

expects that the other will conform to her own expectations. In the short run, agents agree on the path of both inflation and consumption and they only disagree for periods after the date ($E_{o,0}[T]$), when optimistic expect the end of the trap. After that date, optimists believe that monetary policy will engineer a boom resulting in higher inflation and higher consumption, and that pessimists will finally share their views. Conversely, pessimists expect that the economy will still be experiencing the negative shock and that optimists will finally share their views as well. In sum, disagreement on the commitment ability of the authority tends to yield disagreement over medium-term inflation and consumption expectations, whereas it will have no impact on short-term expectations.

Figure 7 plots the reaction of consumption and inflation when beliefs about the nature of policy are heterogeneous. In this example, 75 per cent of households are convinced that the monetary policy will be Odyssean and the other 25 per cent believes that the policy is Delphic ($\alpha = .25$).

With this fraction of optimists and pessimists, the central bank is still better off implementing Odyssean forward-guidance. Yet, to compensate the presence of pessimists, it has to keep interest rates low for more periods (6 instead of 5). This results in a larger and longer boom in the end of the trap.

Figure 7 also illustrates how heterogeneity matters. Although agents agree on current and short-term future allocations, their heterogeneous beliefs about future effects of policy contemporaneously result in heterogeneous actions: optimists consume more in the short run as they expect higher inflation in the medium run than pessimists.

In addition, optimists expect pessimists to consume less than them in the short run as they know that pessimists do not share their beliefs, but they expect pessimists to revise their beliefs at date T and, then, to consume more in the future, catching up with optimists. This expected learning by pessimists will contribute to the optimists' anticipation of a boom.²⁵

In the end, heterogeneity in actions and in beliefs are asynchronous. Agents have different consumption paths in the short run – optimists consume more than pessimists – but they agree on both short-run aggregate consumption and inflation. In the medium-term, the opposite occurs: agents expect that they will consume the same, but they have different views about the state of the economy. Finally, agents agree on the long run because they know that the economy will go back to its steady state.

²⁵Symmetrically, pessimists expect optimists to consume more than them in the short run, but they also expect them to revise their expectations downward at date T , forcing the economy to a new recession.

Forward guidance puzzle. Figure 7 also makes clear that, when beliefs are heterogeneous, forward guidance is not as effective as in the Odyssean case with homogeneous beliefs. Pessimists attenuates the current impact of future monetary accommodation. [Del Negro et al. \(2013\)](#) show the effects US forward guidance policy on the economy has been much weaker than predicted by state of the art New Keynesian models (see Section 4.1 for further analysis).

Individual consumption and financial positions. As optimists consume more than pessimists, they also save less than pessimists. As a consequence, pessimists accumulate positive net positions against optimists. The difference between the budget constraints of the two types is:

$$B_{o,t} - B_{p,t} = R_t(B_{o,t-1} - B_{p,t-1}) + W_t(L_{o,t} - L_{p,t}) - P_t(C_{o,t} - C_{p,t}) - Z_{o,t} + Z_{p,t},$$

A higher consumption level by optimists imply they have a lower marginal utility of consumption and, as a consequence, they work fewer hours. As a consequence, optimists have a lower stock of bonds than pessimists all along the trap ($B_{o,t} \leq B_{p,t}$ for $t < E_{o,0}[T]$). Such a difference in financial wealth has also a tendency to grow at the prevailing interest rate, as long as agents do not agree to share resources again.

3.4 Optimal policy response with heterogeneous beliefs

In this subsection, we determine how the presence of optimists and pessimists affects the design of an optimal Odyssean forward guidance. The central bank's problem is to maximize utility (3) over the trap, fixing a path of interest rates such that monetary policy is described by (6) and given agents' optimal consumption, pricing decisions and beliefs. To ensure uniqueness of the equilibrium, we shall assume that the central banker knows α and does not need to infer it.²⁶

In Appendix B.2, we show that proceeding similarly to [Gali \(2008\)](#), we can approximate the utility of each agent as:²⁷

$$\mathbb{W}_i \equiv U_{i,t} - U \simeq U_c C \left(c_{i,t} + \frac{1-\gamma}{2} c_{i,t}^2 \right) + U_l L \left(l_{i,t} + \frac{1+\psi}{2} l_{i,t}^2 \right), \quad (15)$$

²⁶Given that the central bank observes α , it can commit to implement an Odyssean forward guidance whenever it is optimal. This way, we prevent the emergence of a coordination failure between the central bank and the optimists. Indeed, if optimists behaved as if they were pessimists, then it would be optimal for the central bank of the commitment type not to implement its commitment policy. In turn, this makes self-fulfilling optimists behaving like pessimists. Note that this kind of equilibrium is not a consequence of heterogeneity and can be also emerge in the classical framework of [Eggertsson and Woodford \(2003\)](#).

²⁷See also [Bilbie \(2008\)](#) for the computation of welfare functions in the case of heterogeneous agents.

where, labor is determined at the equilibrium by (8). Thus, social welfare can be approximated by $\mathbb{W} = \int_0^1 \mathbb{W}_i di$. In particular, given that there is a unique labor market, general equilibrium externalities make welfare depend on the dispersion of type specific levels of labor and consumption. This feature complicates the derivation of the optimal policy rule, which is carried out numerically in the following simulations. Nevertheless, we can show that, in the special case $\gamma = \psi$, (23) becomes identical to the textbook welfare approximation typical of New-Keynesian models with homogeneous beliefs. This is,

$$\tilde{\mathbb{W}}_i \equiv -\varpi\theta^{-1}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t(\lambda c_{i,t}^2 + \pi_t^2)\right] \quad (16)$$

where $\lambda = 2\gamma/\theta$ and ϖ is a positive constant. For the specification above we can derive general features of the optimal policy for given length of the trap T and the existence of a fraction α of pessimists. Our main result, as described by the following proposition, is a non-monotonic response of the central bank with respect to the share of pessimists. We will show later numerical simulations suggesting that these properties hold for the more general specification (15) in which $\gamma \neq \psi$.

Proposition 4. *In the special case $\gamma = \psi$, there exists two values $\underline{\alpha} >$ and $\bar{\alpha}$ where $\underline{\alpha} > \bar{\alpha}$ such that, for a given T , the policy map $T_{cb}(\alpha, T)$ is:*

- increasing in α , i.e. $T_{cb}(0, T) < T_{cb}(\alpha, T)$, for $\alpha < \underline{\alpha}$;
- decreasing in α , i.e. $T_{cb}(\alpha', T) > T_{cb}(\alpha'', T)$, for $\underline{\alpha} < \alpha' < \alpha'' < \bar{\alpha}$;
- equal to T , i.e. $T_{cb}(0, T) > T_{cb}(\alpha, T) = T$, for $\alpha > \bar{\alpha}$.

Proof. See [Appendix](#). □

The intuition is simple. When only a small share of agents misunderstand the Odyssean forward guidance, the central bank is better reinforcing this policy: a stronger expected monetary stimulus leads to a further drop in pessimists' consumption - as they wrongly interpret the additional periods of low interest rate as the sign of a longer trap - but, as optimists consume sufficiently more, aggregate consumption also increases. In the end, the central bank is marginally better off to increase the period of low interest rate T_{zlb} .

Yet, Proposition 4 suggests that a coordination on an extended period of low interest rates can also be detrimental when misunderstood. Indeed, misinterpretation can then have more dramatic effects than just mitigating the effect of Odyssean forward guidance. It can exacerbate the consequences of the ZLB. As a result the central bank can be better off not implementing an Odyssean forward guidance policy, no matter whether it is willing and able to commit to it.

Numerical illustration. Let us now show numerical simulations to discuss to what extent the features in the special case $\psi = \gamma$ extend to the general case $\psi \neq \gamma$.

Figure 8 plots the number of periods of extra accommodation as a function of the fraction of pessimists. In the upper panel we contrast policies reacting to a large shock ($\xi = -0.01$), with policies react to a smaller shock ($\xi = -0.007$) in the lower panel. In both cases we consider a shock lasting for 20 periods. In each panel there are three types of curves: solid, dashed and dotted.

The solid line reproduces the optimal policy in case $\lambda = 0$. This is a limit case when the authority only cares about inflation, so that ϕ and γ are extremely low. In this case, the relation is hump-shaped as described in Proposition 4: the presence of pessimists forces the central bank to extend its monetary stimulus, until the contractionary effects that are growing with the share of pessimist outweigh the benefits of additional stimulus. Then, the central bank starts to reduce the length of its stimulus and, ultimately, prefers not to implement Odyssean forward guidance.

The dotted and the dashed lines represents the optimal policy with $\lambda = 50$ for $\psi = \gamma$ and $4\psi = \gamma$, respectively. $\lambda = 50$ entails a case where the policy maker's loss function hinges mainly on the variance of output gap (this case approximates well higher values of λ). The two curves illustrate that the optimal length of extra accommodative periods becomes a monotonically decreasing function of α for a sufficiently high ratio γ/ψ . In particular, the higher the ratio γ/ψ , the higher the welfare loss due to dispersion of actions, and so the lower the incentive to generate disagreement by further reinforcing Odyssean forward guidance. In other words, intra-temporal distortions implied by heterogeneous beliefs provide an additional incentive against Odyssean forward guidance.

Finally, let us comment on how policy reactions change depending on the size of the shock. *Ceteris paribus*, with larger shocks, the contractionary effect of pessimists increases. First, the threshold value of pessimists after which the central bank will not use its commitment ability decreases with the size of the shock. Second, a longer period of accommodation is generally optimal with a larger shock, at least for sufficiently low values of α .

4 Further implications

In this section, we investigate two additional implications of our model: the mild effect of forward guidance on macroeconomic outcomes other than interest rates (labelled "forward guidance puzzle"), and whether monetary policy can affect the agents' beliefs, and so the share of pessimists, by revealing its type.

4.1 Forward Guidance Puzzle

In our model, disagreement on aggregate consumption and inflation arises if and only if $\alpha \in (0, \bar{\alpha})$, in which case the central bank implements a Odyssean forward guidance. Such a presence of heterogeneity can help to explain why the empirical effect of Odyssean forward guidance is usually found to be much lower than what the theory under homogeneous beliefs predicts (see Carlstrom et al., 2012; Del Negro et al., 2013, among others).

Similarly to McKay et al. (2015), our mechanism produces some form of discounting that limits the effects of future shocks on current aggregate consumption via the Euler equation. To highlight the mechanism, suppose the simplest case of a trap of one period and with two periods of Odyssean forward guidance, i.e. $T^{cb} = 2$, before monetary policy fully stabilizes inflation and output starting in period $t = 2$: $c_{o,2} = c_{p,2} = \pi_2 = 0$.

In period $t = 1$, individual consumption and inflation equal:

$$\begin{aligned} c_{o,1} &= \gamma^{-1}(\log R) > 0 \text{ and } c_{p,1} = \gamma^{-1}(\log R - r) \\ \pi_{o,1} &= \kappa\gamma^{-1}(\log R) > 0 \text{ and } \pi_{p,1} = \kappa\gamma^{-1}(\log R - r) \end{aligned}$$

Using the Euler equation for each household, we obtain date-0 consumption levels:

$$\begin{aligned} c_{o,0} &= c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1}) \\ c_{p,0} &= c_{p,1} - \gamma^{-1}(\log R - r - \pi_{p,1}) \end{aligned}$$

This allows us to write the aggregate Euler equation.

$$c_0 = (1 - \alpha)c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1}) + \alpha(c_{p,1} + \gamma^{-1}(\pi_{p,1} - \pi_{o,1})).$$

Finally, using the fact that $c_{p,1}$ and $\pi_{p,1}$ are expected to be negative by pessimists, aggregate consumption c_0 satisfies:

$$c_0 \leq (1 - \alpha)c_{o,1} - \gamma^{-1}(\log R - r - (1 - \alpha)\pi_{o,1}).$$

$(1 - \alpha)$ appears in the aggregate Euler equation as a discounting the future effects of forward guidance, as measured by optimists' beliefs. Therefore, the presence of pessimists affects the effectiveness of Odyssean forward guidance reducing the aggregate impact of the future boom on current aggregate consumption.

4.2 Can a Central Bank signal its own type by current actions?

So far, we restricted policies to be constrained by the ZLB during the trap and, in particular, before $E_{0,o}(T)$. Therefore, we prevented central bankers to signal their own type using current actions (i.e. interest rates).²⁸ In this subsection, we show that this *pooling* of central

²⁸Bear in mind that agents only observe current actions by the policy maker.

bankers is, in fact, an equilibrium outcome: the commitment-type central banker cannot signal itself using current actions and is forced to be pooled with the discretionary-type central banker.

Proposition 5. *The commitment central banker is always better off not to signal itself before $t \leq E_{o,0}(T)$, so that the equilibrium is pooling for any period $t \leq E_{o,0}(T)$. As a result, every share of pessimists $\alpha \in [0, 1]$ is an equilibrium outcome.*

Proof. See [Appendix](#). □

Signaling is not feasible by the commitment central banker's actions, as the other type can easily replicate the same signal, without bearing the costs of forward guidance. Hence, the central banker of the commitment type has no other options but to be pooled with the discretionary type, at least until $E_{o,0}(T)$ after which agents will learn the central banker's type and then will agree on the state of the economy.

As a result, the equilibrium is always pooling, at least until agents observe the length of the trap and update their beliefs, as in the benchmark model. The commitment-type central banker's policy has then to take into account the fraction of pessimists, as it cannot act in such a way that all agents become optimists.

Other tools for such a signaling may be available like: communication, transparency on central banks' beliefs (e.g. by releasing forecasts) or unconventional monetary policy instruments (as quantitative or loan policies, such as TLTROs by the ECB).

Communication. The first tool to modify the fraction of pessimists is for the central banker to communicate on his commitment. As argued by [Woodford \(2012\)](#), the announcement of a clear commitment by the central banker can be a way to make costly *ex post* deviations from this commitment ("to cause embarrassment" to borrow Woodford's words) and, so, to convince pessimists to change their views on the central banker's type. Yet, such announcements can be also made by no-commitment central bankers. They will not bear the cost of renegeing it but enjoy the ex ante gains related to the presence of more optimists.

In the end, communication on commitment is plagued by cheap talk problems: as it costs nothing more to the no-commitment central banker, such communication provides no information on central bankers' types. Such cheap talk view of forward guidance announcements can be found in [Bassetto \(2015\)](#).

Transparency. The second tool that may modify α is to communicate on fundamentals and to try to coordinate agents on shorter liquidity traps than what pessimists are expecting. This can be achieved by releasing forecasts of macro-economic variables, as frequently done by central banks. Yet, this can also be mimicked by no-commitment central bankers as well,

as it does not involve costs when deviating from the commitment, but only gains before the end of the trap.

Quantitative policies. Another tool that may allow for signaling on types is quantitative policies. Indeed, such policies can imply a cost to the central bank in case it deviates from its commitment: a rise in interest rate may lead to a depreciation of purchased assets and so to capital losses to the central bank (see [Bhattarai et al., 2014](#), for an investigation of this mechanism). Yet, such an explanation hinges on the central bank's aversion for capital losses and the extent to which it cannot be rescued by the fiscal authority in case of negative equity.

To conclude, the insights from Proposition 5 applies no matter what the signaling device. The reason is that the no-commitment central banker can strategically mimic the commitment-type central banker without bearing the cost of the commitment in the future. This is what makes the separating equilibrium unfeasible.

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Dependent variable: DIS(r)						
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-0.134**	-1.462***	-0.123**	-1.910***	0.232***	-0.122
	0.054	0.345	.0496	0.520	0.054	0.893
DIS(π)	0.706***	0.836	0.458***	1.261	0.317***	0.183
	0.130	0.751	0.121	0.793	0.094	0.949
DIS(c)			0.463***	-1.335	0.172	-1.136
			0.129	1.081	0.107	0.883
DIS(r_{-1})					0.365***	0.692*
					0.054	0.317
R2	0.287	0.073	0.382	0.195	0.581	0.364
Test of struct. break	26.79***		23.62***		6.19***	
Start	1982Q1	2011Q3	1982Q1	2011Q3	1982Q1	2011Q3
End	2011Q2	2014Q4	2011Q2	2014Q4	2011Q2	2014Q4

Table 1: The link between disagreement on future interest rates and on future fundamentals. Disagreement is measured by the 75/25 inter-quantile range in the distribution of 1-year ahead individual mean point forecasts for the 3-month T-Bill interest rate (DIS(r)), CPI inflation (DIS(π)), real consumption growth (DIS(c)) and 1-quarter ahead individual mean point forecasts for the 3-month T-Bill interest rate (DIS(r_{-1})). Regressions are in logs. Standard errors are obtained via a HAC Newey-West procedure. The sample covers 1982Q1-2014Q4 and is split into two regimes: before and after the date based forward guidance (2011Q3). The test of structural break reports Wald tests for the assumption that the coefficients are equal over the two periods. ***, **, * indicate significance at resp. the 1%, 5% and 10% level.

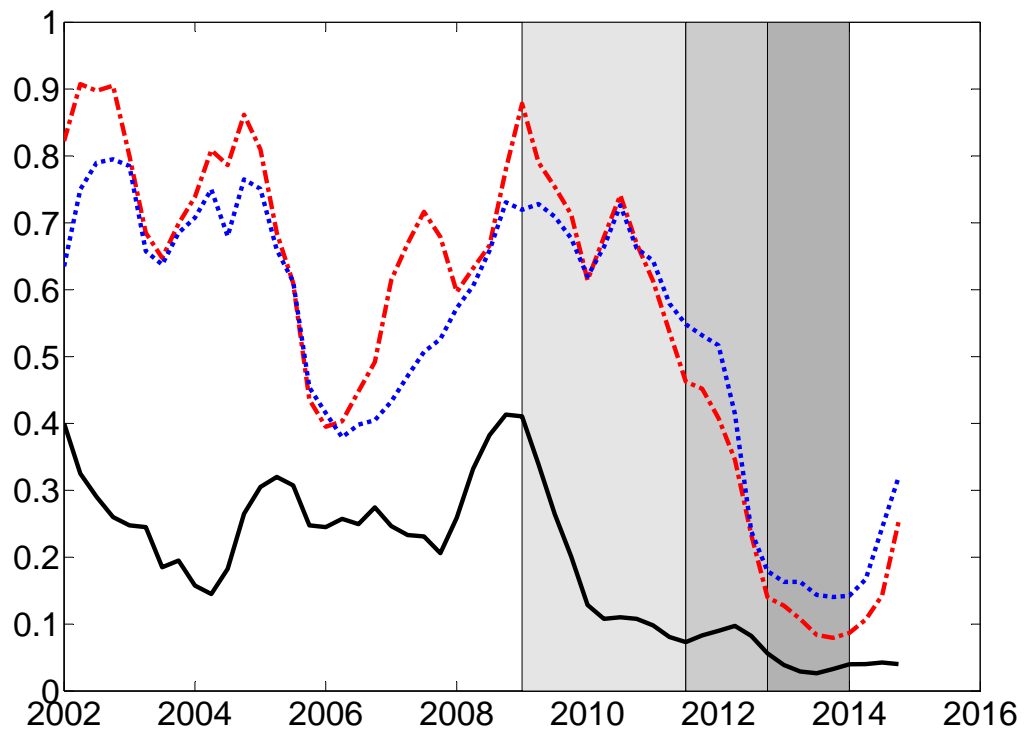
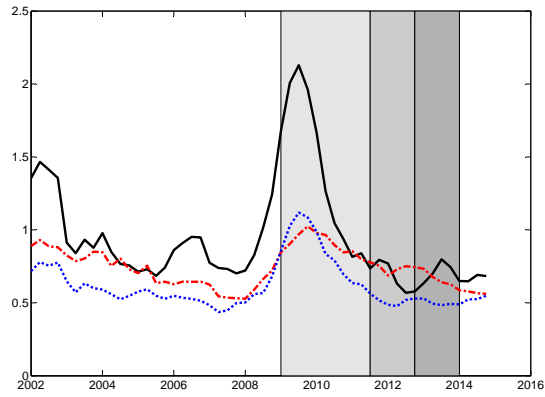


Figure 1: Disagreement about future short-term interest rates.

The chart displays the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

(a) Consumption



(b) Inflation

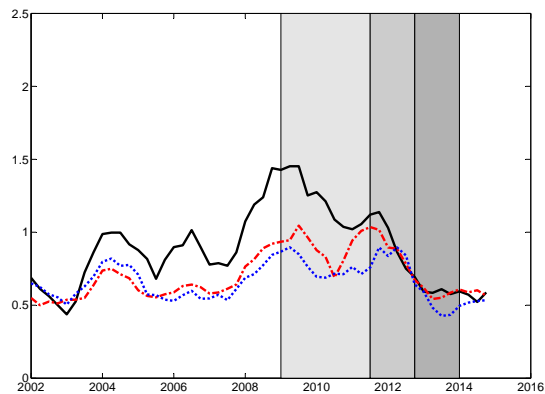


Figure 2: Disagreement about future consumption growth and inflation.

The figure shows the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for real consumption growth and CPI inflation. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

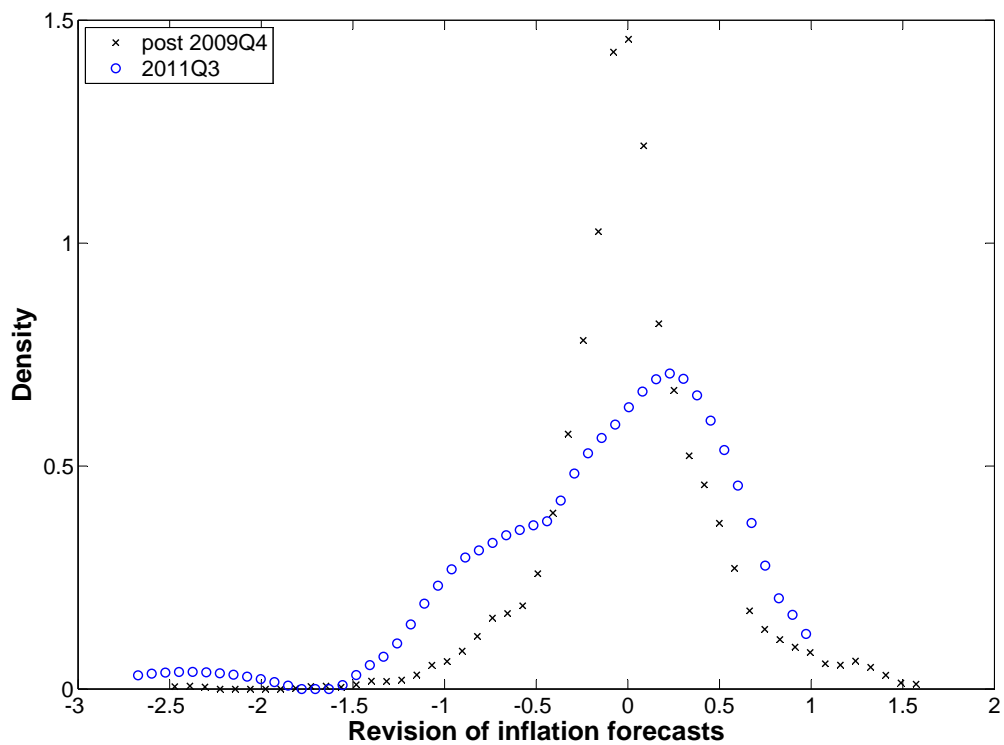
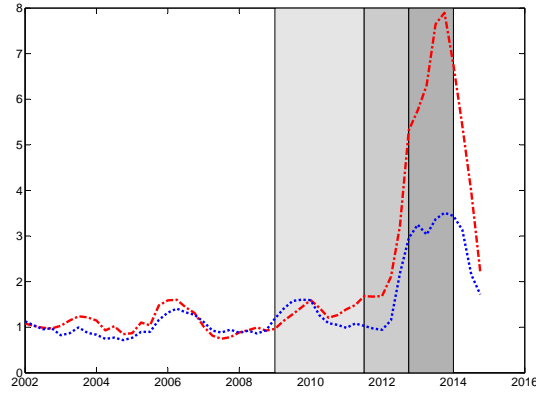


Figure 3: Cross-section distribution of revisions in inflation forecasts

The Figure plots the density of individual revisions of 2-year ahead inflation forecasts observed in the US SPF for two periods: (i) in the 2010Q1 to 2014Q4 surveys and (ii) in the 2011Q3 survey (i.e. the one collected just after the Fed moved to an explicit date based forward guidance policy). Densities are obtained through a non-parametric kernel estimation method with an Epanechnikov kernel.

(a) Consumption



(b) Inflation

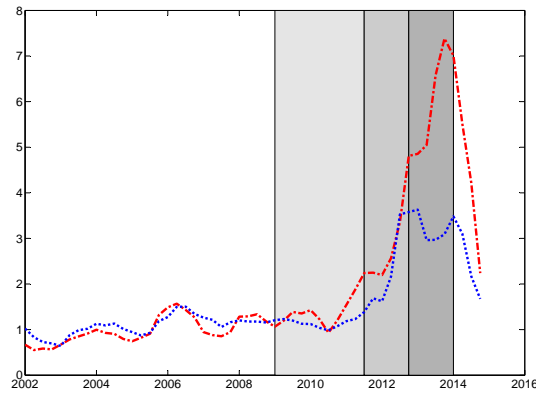
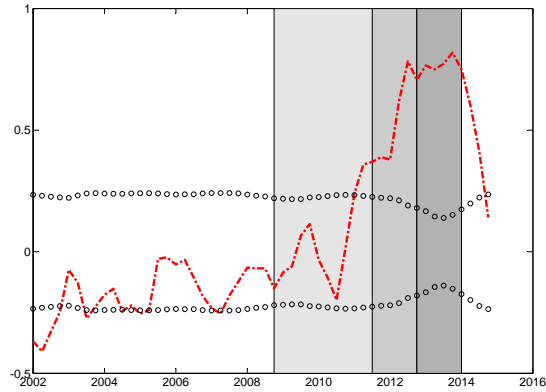


Figure 4: Disagreement about future consumption growth and inflation relative to disagreement about future short-term interest rates.

The figure provides the ratio of disagreement on 1-year (red line) and 2-year (blue line) ahead consumption growth and inflation to disagreement on 1-year and 2-year the short-term interest rates. Disagreements are measured as a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of corresponding individual mean point forecasts. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

(a) Inflation - 1 year ahead



(b) Inflation - 2 years ahead

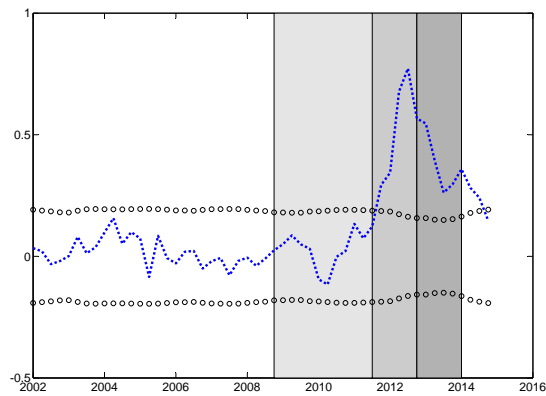


Figure 5: Excess disagreement about future inflation.

The Figure plots the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast. The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

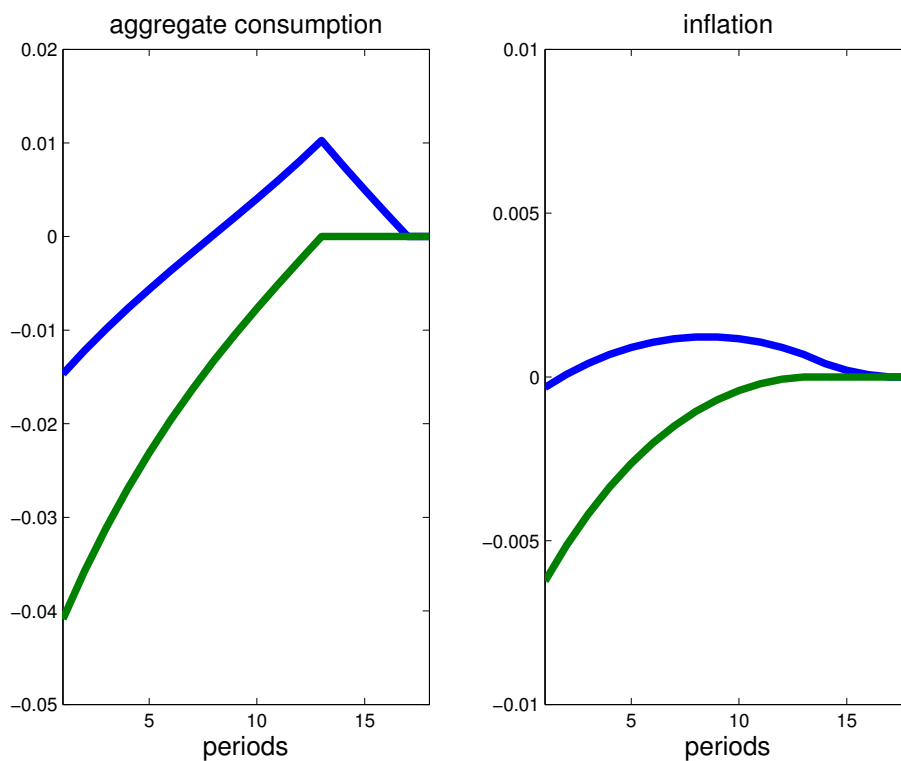


Figure 6: The effect of Delphic (green) and Odyssean (blue) policies.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

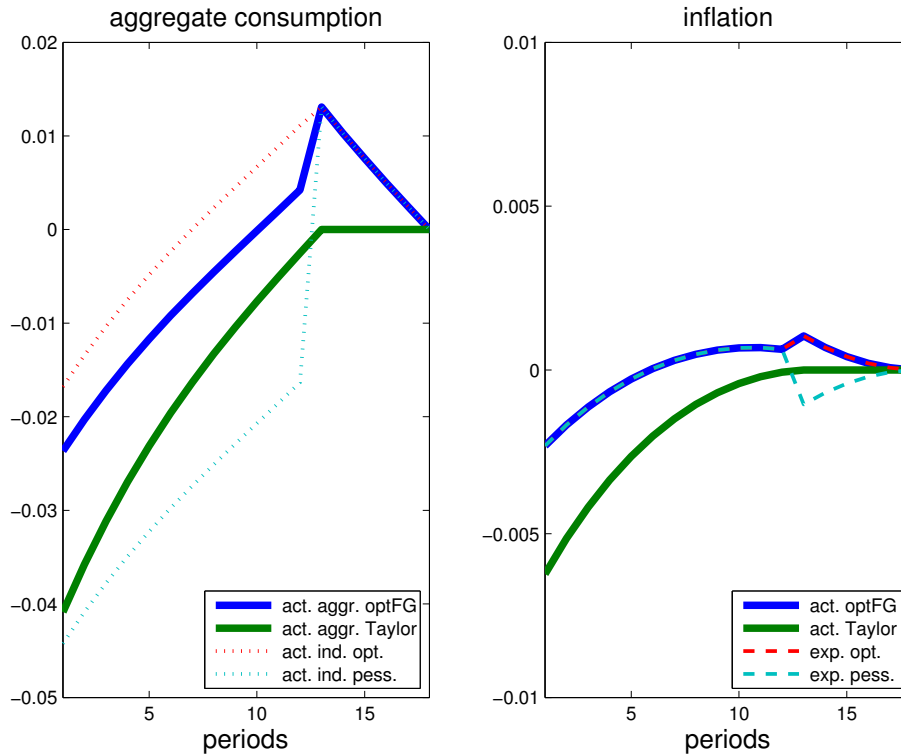


Figure 7: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

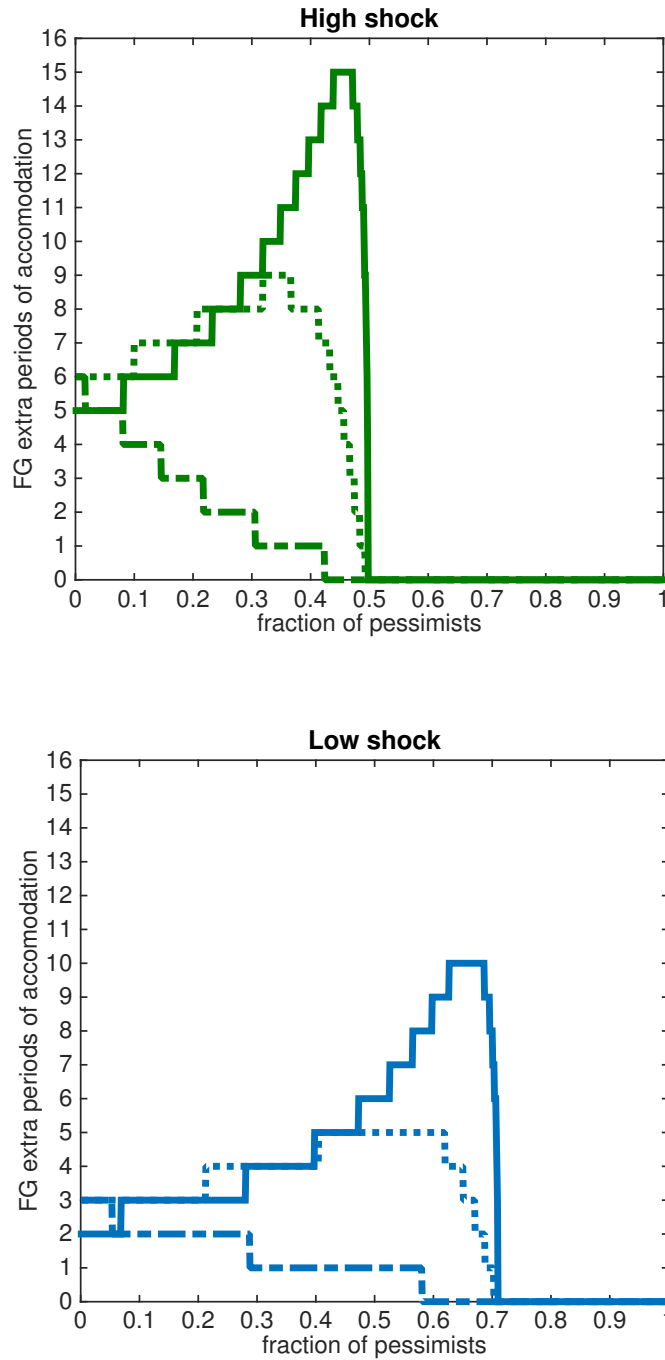


Figure 8: We plot the optimal $T_{cb}(\alpha, 20) - 20$ for $\xi = -.007$ (lower panel) and $\xi = -.01$ (upper panel) for: $\lambda = 0$ with a solid line; $\lambda = 50$ and $\gamma/\psi = 1$ with a dotted line; $\lambda = 50$ and $\gamma/\psi = 4$ with a dashed line. λ is the weight on the average volatility of working hours in the loss function of the central bank. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2%, The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

A Model derivation and equivalence with the representative agent model

A.1 Model derivation.

Following standard steps, we can write down the log-linearized versions of optimality conditions as:

$$c_{i,t} = -\frac{1}{\gamma} (E_{i,t}\xi_{t+1} - \xi_t + r_t - E_{i,t}\pi_{t+1}) + E_{i,t}c_{i,t+1}, \quad (17)$$

$$\gamma c_{i,t} + \psi l_{i,t} = w_t - p_t \quad (18)$$

Notice that that $\xi_t < 0$ in the trap and $\xi_t = 0$ out of the trap. This means that an exit from the trap, say at time $t+1$, implies $\xi = E_{i,t}\xi_{t+1} - \xi_t > 0$. So, the term $\xi = E_{i,t}\xi_{t+1} - \xi_t$ is positive at the time of reverting to normal times and equals 0 otherwise. As a result, the Euler equation (17) implies that consumption decreases at the beginning of the liquidity trap before it gradually increases during the trap.

The optimal price setting for producer j is given by:

$$x_{j,t} = (1 - \chi\beta) E_t^j \left[\sum_{\tau=t}^{\infty} (\chi\beta)^{\tau-t} w_{\tau} \right]$$

as standard in the sticky price literature.

Aggregate behavior. Assuming that ξ can be anticipated a period in advance and by solving forward, we obtain that individual consumption equals:

$$c_{i,t} = -\frac{1}{\gamma} E_{i,t} \left[\sum_{\tau=t}^{\infty} (r_{\tau} - \pi_{\tau+1} + \xi_{\tau+1} - \xi_{\tau}) \right]$$

and aggregate consumption equals:

$$c_t = -\frac{1}{\gamma} E_t \left[\sum_{\tau=t}^{\infty} (r_{\tau} - \pi_{\tau+1})_{\tau+1} + \xi_{\tau+1} - \xi_{\tau} \right]$$

Notice that as long as agents do not disagree on the size of the shock (this is the case as they observe it), but only on the future date on which it will unfold, it enters as a fix wedge in the IS curve. This wedge will disappear only at the optimistic date when agents will discover the truth.

Aggregating over producers yields:

$$x_t = (1 - \chi\beta) w_t + \chi\beta \int E_{i,t} x_{i,t+1} di.$$

By noticing that $\pi_t = p_t - p_{t-1}$, we obtain the following new-keynesian Phillips curve:

$$\pi_t = \frac{(1 - \chi)(1 - \chi\beta)}{\chi} (w_t - p_t) + \beta E_t \pi_{t+1}, \quad (19)$$

which is identical to the one under homogeneous beliefs. This result relies on the assumption that producers observe all current variables and only disagree about the future.

We obtain the New-keynesian Phillips Curve in the presence of heterogeneous beliefs as follows. By defining $\Delta_t \equiv \int E_{i,t} x_{i,t+1} di - E_t x_{t+1}$, we can write x_t recursively as:

$$x_t = (1 - \chi\beta) w_t + \chi\beta E_t x_{t+1} + \chi\beta \Delta_t$$

At the same time, $x_t = \frac{p_t - \chi p_{t-1}}{1 - \chi}$ and so, we can write

$$p_t - \chi p_{t-1} = (1 - \chi)(1 - \chi\beta)w_t + \chi\beta E_t(p_{t+1} - \chi p_t) + (1 - \chi)\chi\beta\Delta_t$$

Thus, we obtain:

$$\pi_t = \frac{(1 - \chi)(1 - \chi\beta)}{\alpha}(w_t - p_t) + \beta E_t\pi_{t+1} + (1 - \chi)\beta\Delta_t$$

By definition, $\Delta_t \equiv \int E_{i,t}x_{i,t+1}di - E_t x_{t+1}$ and $x_{i,t}$ is a function of current and future wages (w_{τ} s). As a result, we can rewrite Δ_t as follows:

$$\Delta_t = (1 - \chi\beta) \sum_{\tau=0}^{\infty} (\chi\beta)^\tau \int E_{i,t} \left(w_{t+\tau+1} - \int E_{t+1}^i [w_{t+\tau+1}] di \right) di$$

which equals 0 in this case, yielding (19).

A.2 Equivalence with the Representative Agent

So far, we consider both uncertain and heterogeneous beliefs about forward guidance. We now investigate how uncertainty differs from heterogeneity. To this purpose, let us consider the following two distributions:

- (i) $\lambda_i = 1$ for $i \leq \alpha$ and $\lambda_i = 0$ otherwise. Agents disagree but are certain about their beliefs
- (ii) $\lambda_i = \alpha$ for all i . Agents do not disagree but are uncertain, i.e. a representative agent fiction holds.

Given sequence of shocks $\{\xi_0, \xi_1, \dots\}$ and a sequence of policy deviations $\{\Delta_0, \dots\}$ and a policy announcement, let us consider the equilibrium path associated with the distribution of beliefs (i):

$$\{C_{i,t}^h, L_{i,t}^h, B_{i,t}^h, D_t^h, R_t, W_t^h, Z_{i,t}^h, P_t^h\}_{i \in [0,1], t \geq 0}$$

and the resulting aggregate variables:

$$C_t^h = \int_0^1 C_{i,t}^h di, L_t^h = \int_0^1 L_{i,t}^h di \text{ and } B_t^h = \int_0^1 B_{i,t}^h di.$$

Let us also consider the equilibrium path associated with the distribution of beliefs (ii):

$$\{C_{i,t}^u, L_{i,t}^u, B_{i,t}^u, D_t^u, R_t, W_t^u, Z_{i,t}^u, P_t^u\}_{i \in [0,1], t \geq 0}$$

and the resulting aggregate variables:

$$C_t^u = \int_0^1 C_{i,t}^u di, L_t^u = \int_0^1 L_{i,t}^u di \text{ and } B_t^u = \int_0^1 B_{i,t}^u di.$$

Proposition 6. *The aggregate outcome of the two equilibrium paths coincide at the first order, i.e.:*

$$c_t^u = c_t^h + o(\|\xi\|) \text{ and } l_t^u = l_t^h + o(\|\xi\|)di.$$

but not to higher-order approximations. More generally, $C_t^u \leq C_t^h$ for all $t \geq 0$.

B Proofs

B.1 Proof of Proposition 2

The proof is organized in five steps. First step. Consider an economy with homogeneous agents at the date $T_{cb} + 1$ just after the end of the zero-rate period, so that the steady state can be restored. Because of Ricardian equivalence holds, the present value of their life-utility is the same irrespective of the stock of bonds they hold at that time, which is a legacy of the realized states of the world. Therefore, because of the permanent income hypothesis, the level of homogenous individual consumption $C_{T_{cb}+1} = \bar{C}$ is pin down only by the forward evolution of the economy that will remain at steady state. Second step. At time T , as soon as agents become homogeneous, they would agree on a plan of transfers $\{Z_{i,t}^*\}_0^1$ such that $B_{o,t} = B_{p,t}$, that is, their stock bonds is equalized. In fact, as a consequence, consumption is equalized and so $U_{C_{o,T}} = U_{C_{p,T}}$, that is, social welfare is maximized. After that period, irrespective of whether or not the economy is already at steady state (preference shock does not hit), individual consumption will converge to $C_{T_{cb}+1} = \bar{C}$ because of what argued in the first step. Third Step. Consider now the sequence of transfers $\{\{Z_{i,t}^*\}_0^1\}_{t=0}^\infty$, then since step two and three are common knowledge, there is only one equilibrium consumption path associated to each state of the world as described in the proposition. Fourth step. Different transfers plans, which modify the path of consumption of the two types, imply, because of the permanent income hypothesis, different level of consumption at steady state. Given that agents anticipate step 2, no plan of this kind can be implemented. In other words, agents anticipate that at time t they will agree to equalize their wealth so that \bar{C} will be their steady state consumption that in turn determines the unique consumption path described at step three. Fifth step. Among all the transfer plans that can engineer an equalization in the stock of bonds at time T onwards, $\{Z_{i,t}^*\}_0^1$ is the only one that is implementable because before time T agents disagree on the actual transfer that will equalize bonds holding at time T as they expect different real interest rates paths, after time T they agree on no transfers.

B.2 Proof of Proposition 4.

The case $\gamma = \psi$ is a special case in which we can derive the welfare function quite easily. The welfare function is the expected representative utility function

$$U_\tau = \int_0^1 U_{i,\tau} di, \quad (20)$$

where

$$U_{i,\tau} = \sum_{t=\tau}^{\infty} \beta^t e^{\xi t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) \quad (21)$$

and $i \in \{o, p\}$. Following standard steps (see [Gali \(2008\)](#) pag. 87) we can write

$$U_{i,t} - U \simeq U_c C \left(c_{i,t} + \frac{1-\gamma}{2} c_{i,t}^2 \right) + U_L L \left(l_{i,t} + \frac{1+\psi}{2} l_{i,t}^2 \right). \quad (22)$$

The next step is to use the fact $L_t = Y_t \int (P_{j,t}/P_t)^{-\theta} di$ to derive

$$(1-\alpha)l_{o,t} + \alpha l_{p,t} = (1-\alpha)c_{o,t} + \alpha c_{p,t} + d_t$$

where the last price dispersion term is derived from as a direct implication of the Calvo assumption (for a proof see [Woodford \(2003\)](#) pag.399) as being proportional to the square of inflation π_t^2 . Given the foc

on labor supply, and in particular because of the assumption of homogeneous labor market, we have that $\gamma c_{o,t} + \psi l_{o,t} = \gamma c_{p,t} + \psi l_{p,t}$ that is

$$l_{p,t} - l_{o,t} = -\frac{\gamma}{\psi} (c_{p,t} - c_{o,t}).$$

Therefore we can rewrite

$$\begin{aligned} l_{o,t} + \alpha(l_{p,t} - l_{o,t}) &= c_{o,t} + \alpha(c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} + (1 - \alpha)(l_{o,t} - l_{p,t}) &= c_{p,t} + (1 - \alpha)(c_{o,t} - c_{p,t}) + d_t, \end{aligned}$$

or

$$\begin{aligned} l_{o,t} &= c_{o,t} + \alpha \left(1 + \frac{\gamma}{\psi}\right) (c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} &= c_{p,t} + (1 - \alpha) \left(1 + \frac{\gamma}{\psi}\right) (c_{o,t} - c_{p,t}) + d_t. \end{aligned}$$

In the special case $\gamma = \psi$ we can show that

$$\alpha l_{p,t}^2 + (1 - \alpha) l_{o,t}^2 = \alpha c_{p,t}^2 + (1 - \alpha) c_{o,t}^2$$

since

$$\alpha (c_{p,t} + 2(1 - \alpha)(c_{o,t} - c_{p,t}))^2 + (1 - \alpha)(c_{o,t} + 2\alpha(c_{p,t} - c_{o,t}))^2 = (1 - \alpha)c_{o,t}^2 + \alpha c_{p,t}^2.$$

We can proceed as in [Gali \(2008\)](#) and getting

$$\mathbb{W}_i = -\varpi \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t ((1 + \psi)l_{i,t}^2 - (1 - \gamma)c_{i,t}^2 + \theta\pi_t^2) \right] \quad (23)$$

where ϖ is a positive constant, so that finally social welfare can be approximated by $\mathbb{W} = \int_0^1 \mathbb{W}_i di$. In the special case, $\gamma = \psi$, (23) becomes

$$\tilde{\mathbb{W}}_i = -\varpi \theta^{-1} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t (\lambda c_{i,t}^2 + \pi_t^2) \right] \quad (24)$$

with $\lambda = 2\gamma/\theta$.

To enlighten the main intuition behind the proof, we firstly only consider a one-period trap that hits at time 0, in the case $\lambda = 0$. Let us then denote by $FG(k) = \sum_{t>0} \beta^t \pi_t^2$ when there is k of periods of Odyssean forward guidance. $FG(k)$ is increasing in k and does not depend on α . The last two properties are general all the periods after the end of the trap, irrespective of its length and the value λ . The reason is that for $t > T$ agents will not disagree and anticipate that at time 0.

Inflation and consumption at time 0 are given by

$$\begin{aligned} \pi_0 &= (\alpha(\beta\pi_{p,1} + \kappa c_{p,0}) + (1 - \alpha)(\beta\pi_{o,1} + \kappa c_{o,0})), \\ c_0 &= \alpha c_{p,0} + (1 - \alpha)c_{o,0} \\ c_{o,0} &= c_{o,1} - \gamma^{-1}(\rho_l - \pi_{o,1}) \\ c_{p,0} &= c_{p,1} - \gamma^{-1}(\rho_l - \pi_{p,1}) \end{aligned}$$

where $\pi_{i,1}$ is a short notation for the expectation of agent i about inflation at time 1 and $\rho_l = -\log R - \xi < 0$. Bear in mind that $c_{i,1}$ and $\pi_{i,1}$ do not depend on α as agents consistently expect homogeneous beliefs are restored after that date.

Let us investigate the conditions for which for $k > k'$ we can have $\pi_0^2(k) + FG(k) \leq \pi_0^2(k') + FG(k')$, i.e. forward guidance for k period is not less efficient of a forward guidance for k' periods. First note that

$$\frac{\partial c_0(k)}{\partial \alpha} = c_{p,0}(k) - c_{o,0}(k) = \gamma^{-1}(\pi_{p,1}(k') - \pi_{o,1}(k')) + c_{p,1}(k') - c_{o,1}(k')$$

as $\partial c_{p,0}/\partial \alpha = \partial c_{o,0}/\partial \alpha = 0$, and so

$$\begin{aligned} \frac{\partial c_0(k)}{\partial \alpha} &< \frac{\partial c_0(k')}{\partial \alpha} \\ \frac{\partial \pi_0(k)}{\partial \alpha} &= \beta(\pi_{p,1}(k) - \pi_{o,1}(k)) + \kappa(c_{p,0}(k) - c_{o,0}(k)) < \frac{\partial \pi_0(k')}{\partial \alpha} < 0. \end{aligned}$$

given the facts:

- i) $\pi_{p,1}(k) < \pi_{p,1}(k') < \pi_{p,1}(0) = 0$, $\pi_{o,1}(k) > \pi_{o,1}(k') > \pi_{o,1}(0)$,
- ii) $c_{p,1}(k) < c_{p,1}(k') < c_{p,1}(0) = 0$, $c_{o,1}(k) > c_{o,1}(k') > c_{o,1}(0)$.

The derivative of $\Pi(k, k', \alpha) = \pi_0^2(k) - \pi_0^2(k')$ with respect to α is:

$$\frac{\partial \Pi(k, k', \alpha)}{\partial \alpha} = 2 \left(\pi_0(k) \frac{\partial \pi_0(k)}{\partial \alpha} - \pi_0(k') \frac{\partial \pi_0(k')}{\partial \alpha} \right) \quad (25)$$

whereas, $\Phi(k, k') = FG(k') - FG(k) < 0$. By substitution we get:

$$\begin{aligned} \pi_0 \frac{\partial \pi_0}{\partial \alpha} &= \alpha \left((\beta + \kappa \gamma^{-1}) (\pi_{p,1} - \pi_{o,1}) + \kappa (c_{p,1} - c_{o,1}) \right)^2 + \\ &+ \left((\beta + \kappa \gamma^{-1}) \pi_{o,1} + \kappa c_{o,1} - \gamma^{-1} \rho_l \right) \left((\beta + \kappa \gamma^{-1}) (\pi_{p,1} - \pi_{o,1}) + \kappa (c_{p,1} - c_{o,1}) \right) \end{aligned}$$

where the term

$$\left((\beta + \kappa \gamma^{-1}) (\pi_{p,1}(k) - \pi_{o,1}(k)) + \kappa (c_{p,1}(k) - c_{o,1}(k)) \right),$$

is smaller than

$$\left((\beta + \kappa \gamma^{-1}) (\pi_{p,1}(k') - \pi_{o,1}(k')) + \kappa (c_{p,1}(k') - c_{o,1}(k')) \right),$$

for the facts i) and ii) above. As a result, when $\alpha = 0$, the derivative $\partial \Pi(k, k') / \partial \alpha$ is negative. In addition, $\partial \Pi(k, k') / \partial \alpha$ is a linear and increasing function of α .

Therefore, let us consider a situation in which $\Pi(k, k', \alpha) > \Phi(k, k')$ - i.e. forward guidance for k' is preferred to k , with $k > k'$, in the absence of pessimists. As α increases in the range $(0, 1)$, the inequality can switch sign either never or twice, given that by construction $\Pi(k, k', 1) > 0$ (all agents are Delphic). In particular, the upper threshold $\bar{\alpha}$ is such that $\Pi(1, 0, \bar{\alpha}) = \Phi(1, 0)$.

Let us go back now to the case $\lambda > 0$. In this case the relevant inequality becomes

$$(1 - \alpha) (c_{o,0}^2(k) - c_{o,0}^2(k')) + \alpha (c_{p,0}^2(k) - c_{p,0}^2(k')) + \Pi(k, k', \alpha) \leq \hat{\Phi}(k, k')$$

where $\hat{\Phi}(k, k')$, which preserves the properties of $\Phi(k, k')$, has been extended accordingly. As before with $k > k'$, we have facts i) and ii). To show that the additional term

$$c_{o,0}^2(k) - c_{o,0}^2(k') + \alpha (c_{p,0}^2(k) - c_{p,0}^2(k')) - \alpha (c_{p,0}^2(k') - c_{o,0}^2(k'))$$

is also increasing in α , notice that $0 > c_{p,0}(k') > c_{p,0}(k)$ and $c_{o,0}(k) > c_{o,0}(k') > 0$ implies

$$c_{o,0}(k) > c_{p,0}(k') + c_{o,0}(k')$$

so that

$$(c_{p,0}(k) + c_{o,0}(k))(c_{p,0}(k) - c_{o,0}(k)) > (c_{p,0}(k') + c_{o,0}(k'))(c_{p,0}(k') - c_{o,0}(k'))$$

can be easily shown given that $c_{p,0}(k) - c_{o,0}(k) > c_{p,0}(k') - c_{o,0}(k')$ from facts ii). Nevertheless, the additional term is positive at $\alpha = 0$. This implies that whereas all the qualitative feature of our analysis equally hold considering $\lambda > 0$, a longer forward guidance are ceteris paribus more efficient at low α .

Let us look at how the reasoning can be extended to multiple periods in the liquidity trap. Without loss of generality, let us go back to the simple case $\lambda = 0$. We add a period $t = -1$ that takes place just before period 0, then the reasoning can be extended recursively. We have then to compare:

$$1/\beta\pi_{-1}^2(k) - 1/\beta\pi_{-1}^2(k') + \Pi(k, k', \alpha) \leq \Phi(k, k').$$

Notice that the additional term is typically positive, so ceteris paribus, with a longer trap a longer forward is needed for low α . The derivative with respect to α of the additional terms $\Pi_{-1}(k, k', \alpha)$ is:

$$\frac{\partial \Pi_{-1}(k, k', \alpha)}{\partial \alpha} = \beta^{-1} \left(2\pi_{-1}(k) \frac{\partial \pi_{-1}(k)}{\partial \alpha} - 2\pi_{-1}(k') \frac{\partial \pi_{-1}(k')}{\partial \alpha} \right),$$

which has the same structure than (25) and can be expressed similarly as a linear combination of future actual aggregate consumption and inflation. In particular, we can show

$$\begin{aligned} \frac{\partial c_{-1}(k)}{\partial \alpha} &= \frac{\partial c_0(k)}{\partial \alpha} + \gamma^{-1} \frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial c_{-1}(k')}{\partial \alpha} < 0 \\ \frac{\partial \pi_{-1}(k)}{\partial \alpha} &= \kappa \frac{\partial c_{-1}(k)}{\partial \alpha} + \beta \frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial \pi_{-1}(k')}{\partial \alpha} < 0 \end{aligned}$$

using previous relations. Therefore, $\partial \pi_{-1}^2(k)/\partial \alpha$ is a linear downward sloping function of α . Given this result, we can then extend recursively the analysis to an arbitrarily number of periods.

B.3 Proof of Proposition 5.

Without loss of generality, we show this point for a one period liquidity trap and we denote by $Loss(\alpha)$ the discounted future loss of the commitment-type central banker when facing a share of pessimists α , and $Loss_D(\alpha)$ the one associated to the discretionary type. The incentive constraints can be summarized as follows:

$$\begin{aligned} (\pi(0)^2 + \lambda c(0)^2) + Loss(0) &\leq (\pi(\alpha)^2 + \lambda((1-\alpha)c_o(\alpha)^2 + \alpha c_p(\alpha)^2)) + Loss(\alpha) \\ (\pi(0)^2 + \lambda c_o(0)^2) + Loss_D(0) &\geq (\pi(1)^2 + \lambda c_p(1)^2) + Loss_D(1) \end{aligned}$$

The first one capture the incentive for the commitment central banker to signal itself and to gain from being in a separating equilibrium. The second constraint captures the need that the discretionary central banker has no incentive to mimic the commitment central banker.

Note that $Loss_D(0) = Loss_D(1)$, as the model is forward looking and the discretionary central banker does not care about past outcomes. In addition, the second IC constraint holds with equality, so as to reduce the cost of signaling beard by the commitment-type central banker. Finally, we obtain:

$$(\pi(1)^2 + \lambda c(1)^2) + Loss(0) \leq (\pi(\alpha)^2 + \lambda((1-\alpha)c_o(\alpha)^2 + \alpha c_p(\alpha)^2)) + Loss(\alpha)$$

This is not satisfied for $\alpha = 0$ and for $\alpha = 1$. Using the envelope condition with respect to α on the right hand term, we obtain that this term is decreasing in α and that the inequality never holds. In the end, the equilibrium is pooling.

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