# Durables and Size-Dependence in the Marginal Propensity to Spend<sup>\*</sup>

Martin Beraja MIT and NBER

Nathan Zorzi Dartmouth

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#### Abstract

Stimulus checks have become an increasingly important policy tool in recent U.S. recessions. How does the households' marginal propensity to spend (MPX) vary as checks become larger? To answer this question, we develop a quantitative model of durable and non-durable spending that accounts for a rich set of micro facts. We find that the MPX declines with the size of checks, albeit relatively slowly. In general equilibrium, a large check of \$2,000 increases output by 27 cents per dollar over one quarter during a typical recession, compared to 41 cents for a \$300 check.

<sup>\*</sup>Martin Beraja: maberaja@mit.edu. Nathan Zorzi: Nathan.G.Zorzi@dartmouth.edu. We thank Isaac Baley, Andres Blanco, David Berger, Ricardo Caballero, Greg Kaplan, Peter Ganong, Andrea Lanteri, Peter Maxted, Alisdair McKay, Benjamin Moll, Laura Pilossoph, Ludwig Straub, Gianluca Violante, Johannes Wieland, Tom Winberry and Christian Wolf for helpful discussions.

# 1 Introduction

Stimulus checks have become an increasingly important policy tool in recent U.S. recessions. Eligible individuals received a tax rebate of up to \$300 in 2001 and \$600 in 2008, and a payment of \$1,200 early in 2020 plus roughly \$2,000 in subsequent rounds. The government relied on these stimulus checks to boost spending and close the output gap during these episodes. Despite the importance of stimulus checks, we know surprisingly little about their effectiveness as they become larger. A large check of \$2,000 could be barely more effective than a smaller check of \$300 if households spend less and less of each additional dollar they receive.

How does the households' marginal propensity to spend (MPX) vary as stimulus checks become larger?<sup>1</sup> Measuring the *size-dependence* in the MPX is challenging.<sup>2</sup> Empirical studies obtain a wide range of estimates: the marginal propensity to spend can be decreasing, essentially flat, or even increasing (Souleles, 1999; Kueng, 2018; Fuster et al., 2021; Fagereng et al., 2021; Ganong et al., 2022). State-of-the-art models of the MPX focus on non-durables and predict that the marginal propensity to spend falls rapidly with the size of stimulus checks (Kaplan and Violante, 2014). The relevant quantity for policy, however, is total household spending including durables. Indeed, durable spending accounts for a large share of the MPX out of stimulus checks (Souleles, 1999; Parker et al., 2013; Orchard et al., 2022).<sup>3</sup> The empirical literature has conjectured that large checks could skew households' spending towards durables (Parker et al., 2013; Fuster et al., 2021), dampening or even reversing the decline in the MPX predicted by non-durable models.

In this paper, we develop a quantitative model of durable and non-durable spending that accounts for a rich set of micro facts which existing models of household spending cannot replicate jointly and which are key for disciplining the response of durables to checks. We use the model to quantify the response to stimulus checks of various sizes, and evaluate their effect in general equilibrium.

Our spending model augments a canonical incomplete markets model of lumpy dur-

<sup>&</sup>lt;sup>1</sup> Following the literature, we use the term "marginal propensity to spend" (MPX) to refer to the average spending response across individuals divided by the size of the income change (e.g., the check). The MPX includes spending on durables and non-durables (Laibson et al., 2022).

<sup>&</sup>lt;sup>2</sup> The MPX is notoriously difficult to estimate even in *levels*. Part of the reason is that the MPX varies with the state of the business cycle (Gross et al., 2020), the depth of the recession, etc. Estimating the size-dependence in the MPX is even more challenging, as we do not directly observe multiple checks for the *same* household at the *same* point in the business cycle. Lottery gains are typically much larger than stimulus checks (Fagereng et al., 2021; Golosov et al., 2021).

<sup>&</sup>lt;sup>3</sup> More generally, an extensive literature documents that durable spending responds strongly to income changes (Wilcox, 1989; Aaronson et al., 2012) and wealth shocks (Mian et al., 2013).

ables (Berger and Vavra, 2015) by allowing for time-dependent adjustments in a flexible way. Households are subject to linearly additive taste shocks for adjustment (McFadden, 1973). This formulation delivers a smoother adjustment hazard than the typical (s, S) bands produced by the canonical model where adjustments are purely state-dependent.<sup>4</sup> Smooth adjustment hazards are often used when studying firms' investment and price setting decisions (Caballero and Engel, 1993, 1999; Alvarez et al., 2023). We find that such a hazard is key for household spending and its response to stimulus checks.

Allowing for a smooth adjustment hazard is important for two reasons. First, it enables our model to match several pieces of evidence on household spending that a purely state-dependent model, a purely time-dependent model, or even a Calvo-Plus model (Nakamura and Steinsson, 2010; McKay and Wieland, 2021) cannot replicate jointly. Second, the shape of the adjustment hazard plays a crucial role for the size-dependence in the MPX since it controls how many households purchase durables as checks become larger. In particular, we show that the MPX can be decreasing, flat, or even increasing depending on whether the hazard is relatively flat (i.e., adjustments are mostly time-dependent) or sharply increasing (i.e., adjustments are mostly state-dependent).

To discipline the shape of the adjustment hazard, we match several pieces of evidence on household spending. In particular, our model (i) matches the evidence on the quarterly MPX on durables and non-durables out of small checks; (ii) generates a realistic short-run price elasticity of durable purchases; and (iii) fits the empirical distribution of durable adjustment sizes. We show that matching these moments is important for the response of durables to checks and how it varies as these checks become larger. Our calibrated model also matches several untargeted moments well; for example, the empirical probability of adjustment as a function of the time elapsed since the last adjustment, the annual MPX out of small and large lottery gains as well as the timing of the spending response in Fagereng et al. (2021), the fraction of hand-to-mouth agents in Kaplan and Violante (2022), and the skewed distribution of marginal propensities to spend (with many above 1) in Lewis et al. (2022). Overall, our model provides a realistic description of households' spending and matches the empirical responses to various shocks — both small and large.

Using our model, we find that the quarterly MPX is around 45% out of a \$100 check, 39% out of a \$1,000 check, and 35% out of a \$2,000 check. In contrast, a canonical twoasset model of non-durables (Kaplan and Violante, 2022) produces a smaller MPX which declines much more rapidly, whereas a version of our model with only state-dependent

<sup>&</sup>lt;sup>4</sup> The rest of the model includes non-durable spending, uninsured idiosyncratic risk, borrowing constraints, and durables depreciation and maintenance. We assume that households make a down payment in cash to purchase a durable, and borrow the rest with credit.

adjustments of durables (as in Berger and Vavra, 2015, for example) produces a much larger MPX which increases sharply initially. The MPX in our model neither surges as sometimes conjectured in the literature (Parker et al., 2013), nor does it fall sharply as in the canonical two-asset model of non-durables. Instead, the MPX declines relatively slowly with the size of checks (Fagereng et al., 2021; Ganong et al., 2022).

The extensive margin of durable adjustment plays an important role in this result. As stimulus checks become larger, more and more households adjust their stock of durables. This effect offsets the usual precautionary savings motive at the intensive margin which contributes to a rapidly decreasing MPX in non-durables models. The strength of the extensive margin depends on the shape of the adjustment hazard, i.e., how steep it is for most households. We discipline our model by matching various moments that are directly informative about this extensive margin and the response to small and large shocks. This results in much fewer adjustments in response to checks compared to a purely state-dependent model and does not surge as checks become larger. Households do tilt their spending towards durables when receiving larger checks (Fuster et al., 2021), which can contribute to a hump shape in the MPX on durables at first. However, this effect is not sufficiently strong to dominate, and the overall MPX declines slowly with the size of checks.

We conclude the paper with an application. We embed our spending model into an open-economy heterogeneous-agent New-Keynesian model. This allows us to account for endogeneous income changes in general equilibrium in response to checks, as well as forces that can dampen the effect of these checks such as inflation and relative price movements, the response of monetary policy, or international leakages through imports. We use this model to evaluate the effect of checks on output and inflation in various recessions driven by a mix of demand and supply shocks.

We first consider a purely demand-driven recession where output falls by 4% (or \$670 per capita) over three quarters and later recovers over two years. Starting from this recession, the government sends a stimulus check in the first quarter to elegible households. A large check of \$2,000 increases output by 27 cents per dollar in the quarter when it is sent, compared to 41 cents for a small check of \$300. Large checks thus remain effective, but extrapolating from the response out of small checks overestimates how much stimulus larger checks provide. A larger check of \$2,500 (or even \$3,000 depending on the specification) is required to fully close the output gap. For comparison, we then consider a recession that is coupled with an adverse supply shock and a non-linear Phillips curve. The effect of larger checks wears off more rapidly in this case. A government that

misdiagnoses the recession as being entirely demand-driven and attempts to close the perceived output gap by sending a \$3,000 stimulus check would overheat the economy and raise inflation meaningfully.

Our paper contributes to the literature on durable spending in incomplete markets economies. Berger and Vavra (2015) developed the canonical model that spearheaded this literature. Most notably, McKay and Wieland (2021, 2022) extend this canonical model to study monetary policy. They introduce several features to dampen the interest rate elasticity of durable purchases, including operating costs, exogenous adjustment shocks, and limited attention. Their model, however, produces an MPX that is substantially below typical empirical estimates. Gavazza and Lanteri (2021) build on the canonical model to study the effect of credit shocks, and Berger et al. (2023) analyze policies that subsidize durable purchases. We study different questions compared to this literature: the size-dependence in the MPX and the effect of stimulus checks in general equilibrium. We introduce one parsimonuous deviation from the canonical model, namely a smooth adjustment hazard, and show that the shape of this hazard is key for the size-dependence in the MPX.<sup>5</sup> We discipline this hazard empirically, and find that this single deviation from the canonical model can explain a rich set of micro facts on household spending.

We generate a smooth adjustment hazard by introducing a discrete choice problem with additive taste shocks à la McFadden (1973). This specification allows for purely time-dependent adjustment (constant hazard), purely state-dependent adjustment (binary hazard), and everything in between.<sup>6</sup> An important body of work in industrial organization uses this form of discrete choice to estimate the demand for durables both in static settings (Berry et al., 1995) and dynamic ones (Chen et al., 2013; Gowrisankaran and Rysman, 2012). We incorporate these preference shifters into an incomplete markets model with lumpy durables to match the evidence on household spending. We discipline both the mean and the variance of these shocks using micro evidence, and show that these moments are key for the shape of the adjustment hazard and hence the size-dependence in the MPX.<sup>7</sup> Our taste shocks are distributed according to a logistic distribution (McFad-

<sup>&</sup>lt;sup>5</sup> The present paper supersedes Zorzi (2020) who used an off-the-shelf (s, S) model to study the amplification of large sectoral shocks. As we show, such a purely state-dependent model cannot match the micro evidence, which is crucial to quantify the size-dependence in the MPX and evaluate the effects of stimulus checks.

<sup>&</sup>lt;sup>6</sup> This specification is rooted in the psychology literature and has axiomatic foundations (McFadden, 2001). It is used extensively in the context of consumption choices (Nevo, 2001), school choices (Agarwal and Somaini, 2020) and occupational choices (Artuç et al., 2010). Random monetary fixed costs of adjustment, which are sometimes used in the firm investment and price setting literatures, do not have a clear empirical counterpart for consumer durables.

<sup>&</sup>lt;sup>7</sup> Some papers in the heterogeneous-agent literature adopt taste shocks when studying discrete choices, e.g., Iskhakov et al. (2017) in the context of labor supply. They do so for numerical reasons only; the

den, 1973). We find that this distribution closely fits the evidence on durable spending. An alternative, coarser distribution that underlies Calvo-Plus models (Nakamura and Steinsson, 2010; McKay and Wieland, 2021) misses important moments in the data, resulting in a different response of durables to stimulus checks.

Our paper also adds to a literature that studies the effect of stimulus checks in general equilibrium. The existing work on tax rebates (e.g., Wolf, 2021; Wolf and McKay, 2022) or transfers in fiscal unions (e.g., Farhi and Werning, 2017; Beraja, 2023) abstracts from durables altogether and uses first order approximations in the aggregates. In contrast, durable spending is central to our analysis, and we show that it generates substantial non-linearities in the aggregate. Our general equilibrium application is also related to Orchard et al. (2022), who use a linearized two-agent model to show that changes in the relative price of durables can dampen the response to stimulus checks in general equilibrium. We focus on the non-linearities generated by our heterogeneous-agent model with lumpy durables. We confirm that movements in the relative price of durables attenuate general equilibrium multipliers, consistently with Orchard et al. (2022). When it comes to size-dependence, we find that the effect of checks wears off more rapidly as they become larger when we allow for relative price changes.

Finally, our analysis is related to a literature that explores how behavioral frictions affect the MPX. Laibson et al. (2021) find that MPXs can remain elevated for large shocks when households are present-biased. In an extension that builds on Laibson et al. (2022), they allow for a durable good whose adjustment is frictionless. In contrast, non-convex adjustment costs are key to our mechanism. Fuster et al. (2021) find that non-convex costs of attention or re-optimization can generate an MPX that increases with income changes. Their model allows for a single non-durable good, whereas durables are central to our analysis. We obtain a logit adjustment hazard in our model by introducing random taste shocks. Matějka and McKay (2015) provide a behavioral foundation for such hazard based on agents making mistakes due to costly information processing.

# 2 A Model With A Smooth Adjustment Hazard

We now introduce our model of household spending. Households consume non-durables and invest in durables, and they face uninsured earnings risk. Time is discrete, and there is no aggregate uncertainty. Periods are indexed by  $t \ge 0$ .

shocks have an arbitrary small variance and a zero mean. In contrast, we discipline both moments empirically, and show that they are key to match the micro evidence and for the size-dependence in the MPX.

#### 2.1 Goods and Preferences

Households consume  $c_t \ge 0$ , and invest in durables  $d_t \ge 0$ . Their utility is

$$\mathcal{U}_t \equiv u\left(c_t, d_t\right) + \beta \mathbb{E}_t\left[\mathcal{U}_{t+1}\right],$$

for some discount factor  $\beta \in (0, 1)$ . Inter- and intra-temporal preferences are

$$u(c,d) = \frac{1}{1-\sigma} U(c,d)^{1-\sigma} \quad \text{and} \quad U(c,d) = \left[\vartheta_{c}^{\frac{1}{\nu}} c^{\frac{\nu-1}{\nu}} + \vartheta_{d}^{\frac{1}{\nu}} d^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

where  $\sigma$  is the inverse elasticity of inter-temporal substitution,  $\nu$  is the elasticity of intratemporal substitution, and consumption weights satisfy  $\vartheta_c + \vartheta_d = 1$ .

#### 2.2 Durable Adjustment Hazard

We specify a flexible adjustment hazard that captures the time- and state-dependence in durable adjustment. Households are subject to linearly additive taste shocks for adjustment. These taste shocks  $\epsilon$  are independent over time and distributed according to a logistic distribution  $\mathcal{E}$ .<sup>8</sup> The mean and variance of this distribution are controlled by  $\kappa > 0$  and  $\eta^2 > 0$ , respectively.<sup>9</sup> The resulting durable adjustment hazard is

$$S(\mathbf{x}) = \frac{\exp\left(\frac{V^{\text{adjust}}(\mathbf{x}) - \kappa}{\eta}\right)}{\exp\left(\frac{V^{\text{adjust}}(\mathbf{x}) - \kappa}{\eta}\right) + \exp\left(\frac{V^{\text{not}}(\mathbf{x})}{\eta}\right)},$$
(2.1)

where  $V^{\text{adjust}}$  and  $V^{\text{not}}$  denote the continuation values when adjusting and not adjusting, respectively, and **x** denotes the household's idiosyncratic state which we define formally later in this section.

The scale parameter  $\eta$  controls the *shape* of the adjustment hazard while the location parameter  $\kappa$  controls its *position*. The model reduces to a fully state-dependent model when  $\eta \rightarrow 0$ , i.e., adjustment is deterministic conditional on **x**. The parameter  $\kappa$  controls the position of the (*s*, *S*) bands in this case. At the other extreme, the model boils down to a fully time-dependent model when  $\eta \rightarrow +\infty$ , i.e., adjustment is random and indepen-

<sup>&</sup>lt;sup>8</sup> This specification is common in the literature, as discussed in our introduction. Additional references in the context of automobile demand include Rust (1985), Copeland (2014) and Gillingham et al. (2022).

<sup>&</sup>lt;sup>9</sup> The literature typically normalizes the mean of these shocks to zero (Artuç et al., 2010). By letting the mean and variance be unrestricted, we introduce one extra degree of freedom which allows us to match the micro-level evidence (Section 3). Random monetary fixed costs of adjustment also produce a smooth hazard, although they do not have a clear economic interpretation in the context of consumer durables.

**Figure 2.1:** Adjustment hazard (fixing *d*)



dent of **x**. The parameter  $\kappa$  controls the probability of adjustment in this case.<sup>10</sup> Figure 2.1 illustrates two such hazards. The first (solid curve) is a very steep hazard. It resembles the discontinous adjustment hazard associated with (*s*, *S*) bands in the canonical model of lumpy durable spending, which is purely state-dependent. The second (dashed curve) is a much flatter hazard, which results from allowing for time-dependent adjustments. As we discuss after presenting the rest of the model, the shape of this adjustment hazard plays a key role in the size-dependence in the MPX (Section 2.6).

#### 2.3 Investment, Saving, and Down Payment

Households invest in durables. Their stock depreciates at rate  $\delta$  and requires a mandatory maintenance rate  $\iota$  between adjustments so  $d_t = (1 - (1 - \iota) \delta) d_{t-1}$  when the household does not adjust (Berger and Vavra, 2015). Households also save in a liquid asset  $m \ge 0$  (i.e., cash, deposits) with return  $r^m$ . They use this liquid asset to make a down payment when they purchase a durable and borrow the rest through credit at interest rate  $r^b \ge r^m$ . This credit equals a share  $1 - \theta$  of the value of the durable next period (before depreciation).<sup>11</sup> Households repay their outstanding credit at the same rate at which the value of their durable depreciates, so that credit effectively tracks the stock of durables d. This assumption allows us not to introduce credit as an additional state variable, which would

<sup>&</sup>lt;sup>10</sup> In the limit  $\eta \to +\infty$ , the parameter  $\kappa = \log (1/\phi - 1) \eta$  induces a constant hazard  $\phi \in (0, 1)$ .

<sup>&</sup>lt;sup>11</sup> Down payments are an important feature of durable goods purchases (Argyle et al., 2020), and are key to understand the response of durables to shocks (Luengo-Prado, 2006). In practice, the vast majority of down payments on cars — the largest component of consumer durables — does not exceed the minimum level required (Green et al., 2020). Refinancing and prepayment are relatively rare too for auto loans.

make the problem numerically intractable.<sup>12</sup> It is also fairly realistic. Households make pre-determined credit repayments in our model while they hold their stock of durables (as in Laibson et al., 2021), which mimicks the rule of thumb they appear to follow in practice (Argyle et al., 2020). Moreover, when it comes to cars — the largest component of consumer durables — most loans are repaid within 5–6 years and cars depreciate at a rate of roughly 20% per year so that outstanding credit effectively tracks durables. Finally, households repay any outstanding credit in full when purchasing a new durable.

Our formulation differs from existing models of durables, which assume a loan-tovalue constraint and do not make a distinction between cash and credit (Luengo-Prado, 2006; Berger and Vavra, 2015; McKay and Wieland, 2021). This presumes that households can refinance *continuously* and extract equity from their durables. As a result, the effective supply of liquidity in the economy (i.e., the average distance to the borrowing constraint) is much larger than in the data and the households' MPX is implausibly small (McKay and Wieland, 2021) particularly for non-durables.<sup>13</sup> Moreover, while refinancing is common for housing, it is virtually nonexistent for consumer durables which we focus on; auto loan prepayments are relatively rare too (Heitfield and Sabarwal, 2004).

#### 2.4 Earnings and Income

Households' earnings  $y_t Y_t^{\text{inc}}$  are the product of idiosyncratic productivity  $y_t$  and aggregate income  $Y_t^{\text{inc}}$ . The log-productivity log  $(y_t)$  follows an AR(1) process as in Berger and Vavra (2015) and McKay and Wieland (2021). We denote the associated transition kernel by  $\Gamma(dy';y)$ . Households' net income before interest rate payments is  $\psi_0 (y_t Y_t^{\text{inc}})^{1-\psi_1}$ , where  $\psi_0$  and  $\psi_1$  parametrize progressive taxation (Heathcote et al., 2017). Total income after interest rate payments is

$$\mathcal{Y}_{t}(\mathbf{x};T_{t}) \equiv \psi_{0,t}\left(yY_{t}^{\text{inc}}\right)^{1-\psi_{1}} + \left(1+r_{t-1}^{m}\right)m - r_{t-1}^{b}\left(1-\theta\right)d + T_{t},$$

where  $\mathbf{x} \equiv (d, m, y)$  is the household's idiosyncratic state (i.e., its stock of durables, holdings of liquid assets, and income shock), and  $T_t$  are stimulus checks.<sup>14</sup>

<sup>&</sup>lt;sup>12</sup> An *even* richer model could allow for refinancing (Berger et al., 2021; Laibson et al., 2021) or prepayments. Adding these features *in addition to* lumpy durables would be intractable.

<sup>&</sup>lt;sup>13</sup> For instance, Kaplan et al. (2018) report that the average stock of *net* durables equals 22% of annual GDP. Assuming that  $\theta = 20\%$  as in our calibration (Section 3), the conventional formulation would imply that the average household can draw liquidity at any point to  $22\%/\theta \times (1 - \theta) = 88\%$  of average annual income. This figure is much larger than usual values (e.g., Kaplan et al., 2018).

<sup>&</sup>lt;sup>14</sup> We assume for now that the stimulus check in the first period  $T_0 \ge 0$  is the same for all households. It acts as a one-time, unanticipated income shock. The spending response that we measure off of that is thus an average marginal propensity to spend, as in the literature (footnote 1). We allow for an asymmetric

#### 2.5 **Recursive Formulation**

We now state the household's problem recursively. The household first chooses whether to adjust its stock of durables or not. The value associated to the discrete choice problem is

$$\mathcal{V}_{t}(\mathbf{x};\epsilon) = \max\left\{V_{t}^{\mathrm{adjust}}(\mathbf{x}) - \epsilon, V_{t}^{\mathrm{not}}(\mathbf{x})\right\}$$

This discrete choice problem yields the adjustment hazard (2.1). When the household adjusts its stock of durables, it solves

$$V_{t}^{\text{adjust}}\left(\mathbf{x}\right) = \max_{c,d',m'} u\left(c,d'\right) + \beta \int \mathcal{V}_{t+1}\left(d',m',y';\epsilon'\right) d\mathcal{E}\left(\epsilon'\right) \Gamma\left(dy';y\right)$$
  
s.t.  $\theta d' + m' + c \leq \mathcal{Y}_{t}\left(\mathbf{x};T_{t}\right) + \left\{\left(1-\delta\right)-\left(1-\theta\right)\right\} d$   
 $m' \geq 0.$ 

The households' cash-on-hand consists of its total income  $\mathcal{Y}_t(\mathbf{x}; T_t)$  plus the value of the durable it sells  $(1 - \delta) d$  net of the outstanding credit it repays  $(1 - \theta) d$ . The household chooses its new stock of durables d' and makes a down payment  $\theta d'$ , and it decides how much to spend on non-durables c.<sup>15</sup> When holding on to its existing stock of durables, the household solves

$$V_{t}^{\text{not}}(\mathbf{x}) = \max_{c,m'} u(c,d') + \beta \int \mathcal{V}_{t+1}(d',m',y';\epsilon') dG(\epsilon') \Gamma(dy';y)$$
  
s.t.  $m' + c \leq \mathcal{Y}_{t}(\mathbf{x};T_{t}) - \iota \delta d - (1-\theta)(d-d')$   
 $m' \geq 0,$ 

where  $d' = (1 - (1 - \iota) \delta) d$  is the depreciated stock after maintenance. The household pays  $\iota \delta d$  for maintenance and repays  $(1 - \theta) (d - d')$  off of its outstanding credit. We explain in Appendix A how we solve this recursive problem numerically.

#### 2.6 Adjustment Hazard and Size-Dependence in the MPX

Having presented the model, we are now ready to discuss the role that the adjustment hazard plays in the size-dependence in the MPX. Following the literature, we will compute the MPX as the average spending response across individuals divided by the size of

incidence of checks in our general equilibrium model (Section 5).

<sup>&</sup>lt;sup>15</sup> The price of durables is fixed in the stationary equilibrium and normalized to 1. We allow for relative price movements in general equilibrium (Section 5). See Appendix A.1 for the version of the households' problem with relative price changes.



**Figure 2.2:** Hazard and intensive margin (fixing *d* and *y*)

the check. We focus momentarily on the marginal propensity to spend on durables since our adjustment hazard is particularly important for durables. Let *T* be a one-time unanticipated transfer and MPX<sup>*d*</sup>(*T*) be the associated average marginal propensity to spend on durables

$$MPX^{d}(T) \equiv \frac{1}{T} \int \int \underbrace{\mathcal{S}(m,d)}_{\text{extensive}} \underbrace{x(m+d)}_{\text{intensive}} \left\{ d\mu \left(m-T,d\right) - d\mu \left(m,d\right) \right\}, \quad (2.2)$$

where S(m, d) is the adjustment hazard, x(m + d) is spending conditional on adjustment for a household with cash-on-hand m and durable stock d, and  $\mu$  is the associated distribution. The expression above abstracts from households' idiosyncratic productivity y to save on notation. Stimulus checks shift the distribution of cash-on-hand in the economy (the last term in the expression). Households spend more on durables as a result. They adjust their stock of durables both at the extensive margin (as captured by the hazard S) and the intensive margin (as captured by spending conditional on adjustment x).

Figure 2.2 illustrates these two objects as a function of cash-on-hand *m*, fixing the other states *d* and *y*. The figure shows the same two hazards (in red) as in Figure 2.1, with the steeper hazard associated with more state-dependent adjustments. Finally, the spending conditional on adjustment (in blue) is concave due to a standard precautionary savings motive. We also plot the distribution of cash-on-hand (in black). A stimulus check T > 0 shifts this distribution to the right (dotted black curve). Households are more likely to adjust their stock of durables (they move along the hazard) and they spend more conditional on adjustment.

The shape of the adjustment hazard is key for the size-dependence in the MPX on

durables. To see this, suppose first that the model is purely state-dependent, i.e., S is discontinuous around some threshold  $m^*(d)$ . In this case, the extensive margin of adjustment is particularly strong and it dominates the intensive margin (McKay and Wieland, 2022; and Section 4.3). The marginal propensity to spend on durables becomes

$$\mathrm{MPX}^{d}\left(T\right) \propto \int \int_{m^{\star}\left(d\right)}^{+\infty} \frac{d\mu\left(m-T,d\right) - d\mu\left(m,d\right)}{T}$$

when the intensive margin is roughly constant. Thus, the marginal propensity to spend on durables *increases* with the size of stimulus checks T when the distribution of cashon-hand decreases with m (as in the data). The reason is that *proportionately* more and more households are pushed over their threshold and adjust at the extensive margin as the check T becomes larger. Next, consider the opposite polar case where the model is purely time-dependent, i.e., S is constant. In this case, there is no response at the extensive margin and the intensive margin dominates. After a simple change of variable, the marginal propensity to spend on durables becomes

$$MPX^{d}(T) \propto \int \int \left\{ x\left(m+d+T\right) - x\left(m+d\right) \right\} d\mu\left(m,d\right),$$

and households move along a concave spending function. In this case, the marginal propensity to spend on durables *decreases* with the size of stimulus checks.

Between these purely state- and time-dependent benchmarks, the adjustment hazard increases smoothly with cash-on-hand *m* and the scale parameter  $0 < \eta < +\infty$  governs its shape. This shape controls how many households purchase durables as checks become larger. As such, it determines the size-dependence in the MPX on durables and the total MPX more generally (which includes spending on non-durables too). In the following section, we will discipline the shape of this hazard by matching a rich set of micro facts, including moments that are directly informative about the strength of the extensive margin and the response to small and large shocks.

# **3** Bringing the Model to the Data

We interpret durables as consumer durables (cars, appliances, furniture). We assume that our single, composite durable good behaves as cars (in terms of frequency of adjustment, down payment, etc.) since they make up most of the spending on consumer durables. We abstract from housing purchases since these are unlikely to be affected by a stimulus check of a realistic size. Each period in the model is a quarter. We start by calibrating some parameters externally (Section 3.1), before disciplining the most important ones internally (Section 3.2). Tables 3.1 and 3.2 summarize the parametrization. We discuss alternative parametrizations in Section 4.1. Appendix A explains how to solve the model.

## 3.1 External Calibration

External parameters are set to standard values in the literature. The inverse elasticity of intertemporal substitution is  $\sigma = 2$ , which is usual in the literature on durables (Berger and Vavra, 2015; Guerrieri and Lorenzoni, 2017). We choose an elasticity of substitution between durables and non-durables of  $\nu \rightarrow 1$  to obtain a unitary long-run price elasticity for cars (Berry et al., 2004; Orchard et al., 2022). The quarterly depreciation rate is  $\delta = 5\%$ . We set  $\theta$  so the down payment share is 20%, which lies between the estimates of Adams et al. (2009) and Attanasio et al. (2008). The real return on the liquid asset is  $r^m = 1\%$  per year and the borrowing spread is  $r^b - r^m = 3.5\%$  for auto loans. We assume that idiosyncratic log-productivity follows an AR(1) process as in Berger and Vavra (2015) and McKay and Wieland (2021). We set the persistence of this process so as to obtain an annual persistence of 0.91 (Floden and Lindé, 2001). We set the standard deviation of the innovations to match a standard deviation of 0.92 in log-earnings (Auclert et al., 2018). We normalize earnings so that aggregate income is 1 in the stationary equilibrium. The elasticity of the tax schedule is  $\psi_1 = 0.181$  as in Heathcote et al. (2017), and we choose the intercept  $\psi_0 = 0.782$  so the marginal tax rate is 30% in the stationary equilibrium.

# 3.2 Matching the Evidence

We calibrate five parameters internally: (i) the discount factor  $\beta$ ; (ii) the relative weight on non-durables  $\vartheta_c$ ; (iii) the maintenance rate  $\iota$ ; (iv) the location parameter for preference shocks  $\kappa$ ; and (v) the scale parameter for preference shocks  $\eta$ . We choose the discount factor to match an average stock of liquid asset holdings m of 26% of average annual income (Kaplan et al., 2018). We calibrate the relative weight on non-durables to target a ratio of durables to non-durable expenditures x/c = 0.26 based on CEX data.<sup>16</sup> We set the maintenance rate to obtain a ratio of maintenance spending to gross investment of 32.6% as in the CEX for cars, which is similar to the value used in McKay and Wieland (2021, 2022). The rest of this subsection describes how we discipline the location parameter  $\kappa$  and the

<sup>&</sup>lt;sup>16</sup> As discussed, we exclude housing from both durables and non-durables. Durable spending in the CEX consists of: household furnishings and equipment; vehicle purchases; maintenance and repairs on vehicles; audio and visual equipment and services; and other entertainment supplies, equipment and services. Non-durable spending consists of total spending minus the categories above and housing.

Parameter	Description	Value	Source / Target
Preferences			
$\sigma$	Inverse EIS	2	Berger and Vavra (2015)
ν	CES parameter	1	Long-run price elasticity
Durables $\delta$	Doprociation rate	5%	ΝΠΡΑ
0	Depreciation rate	5 /0	NIFA
Earnings			
ρ	Persistence	0.977	Floden and Lindé (2001)
$\gamma$	Volatility	0.198	Auclert et al. (2018)
$\psi_0$	Tax intercept	0.782	Average marginal tax rate of 30%
$\psi_1$	Tax progressivity	0.181	Heathcote et al. (2017)
Financial asset			
$\theta$	Down payment	20%	Adams et al. (2009); Attanasio et al. (2008)
r <sup>m</sup>	Return on cash	1%	Real annual Fed funds rate
$r^b - r^m$	Borrowing spread	3.5%	Fed board (G.19 Consumer Credit)

### Table 3.1: External calibration

#### Table 3.2: Internal Calibration

Param.	Description	State-dep.	Our model	Target	Value
β	Discount factor	0.946	0.944	Liquid. / A inc.	26%
θ	Non-dur. pref.	0.711	0.687	<i>d</i> / <i>c</i> spending	26%
ι	Maintenance	0.255	0.257	Mainten. ratio	32.6%
κ	Location param.	0.239	0.803	Adjust. frequ.	23.8%
η	Scale param.	0	0.20	See Section 3.2	

*Notes*: The purely state-dependent model is a version of our model with  $\eta \rightarrow 0$ . We calibrate  $\eta = 0.2$  in our model as we discuss in Section 3.2. All other targets are matched exactly. The sources are described in Section 3.2.

scale parameter  $\eta$  which govern the adjustment hazard (2.1).

*Disciplining the hazard*. The location parameter  $\kappa$ , controls the *position* of the hazard and hence governs the frequency of adjustment. We pick  $\kappa$  to match an annual frequency of adjustment of 23.8% for vehicles in the PSID, which is in line with conventional estimates (McKay and Wieland, 2021).<sup>17</sup>

In turn, the scale parameter  $\eta$  controls the *shape* of the hazard, i.e., whether it is flat as in purely time-dependent models or sharply increasing as in purely state-dependent models. To understand the role that  $\eta$  plays in the response to stimulus checks and which moments we should target to discipline  $\eta$ , it is useful to decompose the MPX on durables (2.2) as follows

$$MPX^{d}(\mathbf{x}) = \frac{1}{T} \underbrace{\int \int [S(m+T,d) - S(m,d)] d\mu(m,d) \times \bar{x}}_{\text{Average slope of the hazard}} + \frac{1}{T} \underbrace{\int \int S(m,d) (x(m+d) - \bar{x}) d\mu(m-T,d)}_{\text{Convolution between spending and the distribution}} - \frac{1}{T} \times [1 - \underbrace{\int \int S(m,d) d\mu(m,d)}_{\text{Average frequency of adjustment}}] \times \bar{x}$$
(3.1)

where  $\bar{x} \equiv \int \int S(m, d) x(m+d) d\mu(m, d)$  is average durable spending in the stationary equilibrium.<sup>18</sup> Our calibration of the relative weight on durables  $1 - \vartheta_c$  already ensures that we match  $\bar{x}$ . Similarly, our choice of the location parameter  $\kappa$  ensures that we match the average frequency of adjustment, i.e., the last term in (3.1). Conditional on those, the scale parameter  $\eta$  controls the first two terms in (3.1), namely the average slope of the hazard and the convolution between spending and the distribution.

We discipline the scale parameter  $\eta$  using three joint moments: (i) the relative MPX on durables compared to non-durables out of small stimulus checks; (ii) the short run price elasticity of durables; and (iii) the distribution of adjustment sizes in the stationary equilibrium. As we show below, these moments respond strongly in our model to changes in  $\eta$ . Their empirical counterparts are thus very informative about this parameter. The first two moments provide us with an upper bound and a lower bound on  $\eta$ , respectively. We will pick a middle value between these bounds to match the distribution of adjustment

<sup>&</sup>lt;sup>17</sup> In Section 3.3, we describe how we estimate the empirical distribution  $\pi_k$  of the duration k between vehicle purchases. The frequency of adjustment is the inverse of the average duration  $1/\sum_{k\geq 0} k\pi_k$ .

<sup>&</sup>lt;sup>18</sup> We provide the derivation of (3.1) in Appendix **B**.

sizes closely. We confirm in Section 3.3 that our model and calibration perform well along several other dimensions, including the timing of the spending response, the response to small and large lottery gains, and the distribution of MPXs.

*Relative MPX on durables.* Matching the relative MPX on durables ensures that the model creates a realistically large role for durables, which is precisely the component of spending that can behave differently from non-durables as checks get larger.

The left panel of Figure 3.1 shows the marginal propensity to spend on durables and non-durables out of a \$500 check for different values of  $\eta$ . All other parameters are recalibrated as we change  $\eta$  to match the other targeted moments. The lower  $\eta$ , the more state-dependent the model; eventually it converges to the canonical model with (s, S) bands as  $\eta \rightarrow 0$ . The MPX on durables declines monotonically as  $\eta$  increases and the model becomes more time-dependent. The literature offers a wide range of estimates of the MPX on durables and non-durables; but it is generally agreed that the MPX on durables is larger (Havranek and Sokolova, 2020; Orchard et al., 2022).<sup>19</sup> For this reason, 0.45 is a plausible upper bound for the scale parameter  $\eta$ . That is, the model cannot be *too* time-dependent to match the evidence on the marginal propensity to spend on durables relative to the one on non-durables.

Short-run price elasticity. We next inspect the *short-run* price elasticity of durable purchases. We measure these durable purchases as the number of households who adjust their stock of durables in response to a transitory change in the price of durables. By definition, this price elasticity only captures the extensive margin of adjustment, i.e., the first term in (3.1). It is well-known that conventional (s, S) models with discontinuous hazards produce an excessively high elasticity of durable demand to changes in the user cost as too many households adjust at the extensive margin (House, 2014; McKay and Wieland, 2021, 2022). In contrast, purely time-dependent models with a flat hazard do not produce any response at the extensive margin so the price elasticity is zero in this case. The short-run price elasticity is thus informative about the average slope of the hazard in (3.1). Matching this moment also makes sense in order to quantify the response to checks in general equilibrium (Section 5): the MPX on durables (discussed above) controls the spending response to checks in partial equilibrium, whereas the price or user cost elasticity deter-

<sup>&</sup>lt;sup>19</sup> In their meta analysis, Havranek and Sokolova (2020) compile hundreds of micro estimates of the MPX from the literature. Using their data, the average total MPX is 58% across 100 micro estimates at the quarterly frequency from studies that report total spending. In turn, the average MPX on non-durables is 22% across 285 micro estimates at the quarterly frequency from studies that report it. This suggests that the MPX on durables is roughly one and a half times as large as the MPX on non-durables.





*Notes*: The left panel plots the marginal propensities to spend (quarterly) out of a \$500 check on durables and non-durables for various values of the scale parameter  $\eta$  in (2.1). These are computed as the response of average spending divided by the size of the check. The right panel plots the short-run price elasticity of durable purchases after a one-quarter increase in the price of durables by 1%. The dashed vertical line is our preferred estimate ( $\eta = 0.2$ ).

mines how price and interest rate changes dampen this response in general equilibrium.

The right panel shows the short-run elasticity of durable purchases after a one-quarter transitory increase in the price of durables by 1%. As expected, the fully state-dependent model with (s, S) adjustments bands  $(\eta \rightarrow 0)$  predicts an implausibly high elasticity of -113. Introducing a smooth adjustment hazard is a parsimonious way to dampen this elasticity.<sup>20</sup> There is much uncertainty in the empirical literature about the exact elasticity. Bachmann et al. (2021) find an elasticity of durable purchases that ranges from -10 to -15 following a short-run decrease in the VAT in Germany.<sup>21</sup> Gowrisankaran and Rysman (2012) estimate a short-run elasticity of -2.55 for camcoders. For this reason, 0.1 is a plausible lower bound for the scale parameter  $\eta$ . The model must be somewhat state-dependent (otherwise the elasticity would be zero).<sup>22</sup> But it cannot be *too* state-dependent

<sup>&</sup>lt;sup>20</sup> McKay and Wieland (2022) dampen the interest rate elasticity by introducing a combination of low elasticity of intertemporal substitution, low elasticity of substitution between durables and non-durables, various operating costs, exogenous mandatory adjustments, and limited attention.

<sup>&</sup>lt;sup>21</sup> Bachmann et al. (2021) find an elasticity of about -10 using their ex ante survey (a 10 percentage point change relative to a baseline of 71 percentage point, for an average perceived change of roughly 1.5%), and -10 to -15 using their ex post surveys (Table A.4, columns 9 and 10 in their paper). The ex post estimates should be thought of as upper bounds since they might capture some intensive margin as well. The literature evaluating the impact of the 2009 Cash for Clunkers program obtains relatively high elasticities too (Mian and Sufi, 2012; Green et al., 2020).

<sup>&</sup>lt;sup>22</sup> This rules out purely time-dependent models, including Calvo models where households have heterogeneous adjustment probabilities.

#### Figure 3.2: Distribution of adjustments and probability of adjustment

Conditional adjustment probability



*Notes*: The left panel plots the distribution of net investment rates (standardized) across two consecutive PSID waves between which households adjusted their stock of durables. The black curve is the data, while the red and blue bars are our calibrated model ( $\eta = 0.2$ ) and a version with purely state-dependent adjustments ( $\eta \rightarrow 0$ ), respectively. The right panel plots the adjustment probability conditional on a household not having adjusted so far. The black, red and blue curves are the same models as on the left panel. The dashed black curve is a version of our model with purely time-dependent adjustments ( $\eta \rightarrow +\infty$ ). The confidence intervals are bootstrapped (10%).

to match the evidence on the elasticity of durable purchases.

Distribution of net investment rates

*Distribution of adjustment sizes*. Households are constantly subject to idiosyncratic income shocks and adjust their stock of durables as a result. Some of these shocks are much larger than the \$500 checks that we discussed so far — some even amount to several thousand dollars.<sup>23</sup> These idiosyncratic income shocks perpetually move households along the distribution  $\mu$  (m, d). The distribution of adjustments reveals whether households who are pushed into regions with a higher probability of adjustment S tend to purchase small or large durables compared to the average x (·)  $- \bar{x}$ , i.e., the second term in the decomposition (3.1). As explained in Section 2.6, this convolution between the distribution of states  $\mu$  and spending  $S \times x$  is at the heart of the size-dependence of the MPX on durables. Getting the shape of this distribution right is thus important for our question.

The left panel of Figure 3.2 plots the empirical distribution of net investment rates in vehicles by households who adjust their stock across two consecutive PSID waves w

<sup>&</sup>lt;sup>23</sup> At the quarterly frequency, the standard deviation of income shocks is almost 20% of households' incomes (Section 3.1). We assume throughout that mean annual income is \$67,000 at the steady state, as in Kaplan and Violante (2022).

(in black). To measure net investment, we restrict our sample to household heads (or reference persons) who are male, aged 21 or above, and own at least one vehicle in at least three PSID waves between 1999 and 2019. An adjustment (Adj<sub>w</sub> = 1) occurs in two cases. Either the number of vehicles owned by the household changes. Or the household reports that the vehicle that was last purchased (vehicle "#1") was acquired more recently than the one reported in the previous wave Purchase<sup>#1</sup><sub>w</sub> > Purchase<sup>#1</sup><sub>w-1</sub>, and at most two years before the interview date Purchase<sup>#1</sup><sub>w</sub>  $\geq t_w - 2$  (since the PSID waves are bi-annual).<sup>24</sup> We denote the year of the most recent purchase by Year<sub>w</sub>. We measure the net investment rate upon a purchase as  $\log(d_w^{net}) - \log(d_{w-1}^{net})$  when  $Adj_w = 1$ , where  $d_w^{net}$  is the value of the stock of vehicles *net* of liabilities reported by the household.<sup>25</sup> Lastly, we standardize the distribution of net investment rates by de-meaning it and normalizing it by its standard deviation (Alvarez et al., 2016b). We trim the top and bottom 1% of the distribution when standardizing.

The left panel of Figure 3.2 also plots the distribution of net investment rates in our model with a smooth adjustment hazard ( $\eta = 0.2$ , in red) and in a version of our model with only state-dependent adjustments ( $\eta \rightarrow 0$ , in blue).<sup>26</sup> To ensure that the data and models are comparable, we discretize our model-simulated series into PSID waves and treat those identically to the actual data. We divide time into years, as our model is set up quarterly. For each individual and wave, we compute Year<sub>w</sub> as the year of the most recent purchase. The value of the stock of durables in the simulated PSID *w* is  $d_{T(w)}$ , where T(w) is the last quarter in that wave. The value *net* of credit is  $d_w^{net} \equiv \theta d_{T(w)}$  in the model.

The purely state-dependent model ( $\eta \rightarrow 0$ ) fails to reproduce the empirical distribution. As expected from a (*s*, *S*) model, the distribution has two clear modes: households who are subject to positive income shocks tend to adjust upward, while those who are subject to sufficiently negative shocks adjust downward. The right mode is much larger since durables depreciate, i.e., there is a drift. The purely state-dependent model pro-

<sup>&</sup>lt;sup>24</sup> Some households sometimes skip a PSID wave. In this case, we say that  $Adj_w = 1$  if one of the two changes explained above occurs since the previous wave where the household is observed (w - 1). We also experimented with a version where we restricted the sample to households who never skip a wave while they appear in the PSID. This reduces the number of households in the sample by half and mechanically shortens the period over which they are observed. We obtained very similar results, but the moments are less precisely estimated due to fewer observations.

<sup>&</sup>lt;sup>25</sup> We do not attempt to back out the gross value of the stock by using imperfect information on liabilities, which would add another layer of measurement error. Instead, we directly compute the changes in the net stock, and we treat the model-generated data identically. Note that  $\log(d_w^{net}) - \log(d_{w-1}^{net})$  is exactly equal to the net investment rate in the stationary equilibrium of our model since the price of durables is constant.

<sup>&</sup>lt;sup>26</sup> As the model becomes even more time-dependent (larger  $\eta$ , not shown to keep the plot readable), it generates a larger share of small adjustments, which shifts the mode of the distribution to the left of its empirical counterpart. Section 3.5 elaborates on these small adjustments in time-dependent models.

duces too few negative adjustments and most adjustments are concentrated around the same value.<sup>27</sup> In contrast, introducing a smooth adjustment hazard allows the model to produce a bell-shaped distribution that fits its empirical counterpart closely. In particular, the model matches well the tails of the distribution — an important moment in models with lumpy adjustment (Alvarez et al., 2016b). In Section 3.5, we also consider a Calvo-Plus variant of our model and find that it misses the distribution of adjustments. As a result, that model produces a different response of durables to stimulus checks.

*Choosing a value for*  $\eta$ . Overall, our preferred value for the scale parameter is  $\eta = 0.2$ . It lies between the lower and upper bounds identified above, and minimizes the  $\mathcal{L}^2$  distance between the distribution of adjustments in the model and in the data. Our model with  $\eta = 0.2$  delivers a total MPX of 42% out of a \$500 windfall. This figure lies between the estimate of 34% in Orchard et al. (2022) and the mean estimate of 58% across micro studies compiled in the meta analysis of Havranek and Sokolova (2020) (footnote 19). The marginal propensity to spend on durables is 25% in our model.<sup>28</sup> This is comparable to the preferred estimate of 30% in Orchard et al. (2022), and it is one and a half times as large as the MPX on non-durables consistently with the meta analysis. We obtain a short-run price elasticity of durables of -7.34 in our calibration, which lies between the existing estimates. This calibration fits the empirical distribution of adjustment sizes well. Moreover, the next subsection shows that the model with  $\eta = 0.2$  matches well other important moments.

#### 3.3 Inspecting Other Moments

Our calibrated model performs well along several other dimensions. We start by inspecting the conditional probability of adjustment since the last purchase, which again highlights the importance of allowing for a smooth adjustment hazard. We also examine the timing of the spending response, the response to small and large lottery gains, and the distribution of MPXs.

<sup>&</sup>lt;sup>27</sup> The empirical distribution might contain some measurement error, i.e., households over- or underestimating the value of their cars for instance. To account for this possibility, we conducted an experiment where we introduced a measurement error of 10% in the model-generated investment sizes. The overall shapes of the resulting distributions are essentially unchanged compared to Figure 3.3.

<sup>&</sup>lt;sup>28</sup> Less than 1% of households adjust at the extensive margin but account for a disproportionate share of the response, which is consistent with the existing evidence on the response to stimulus checks (Parker et al., 2013; Misra and Surico, 2014). The small number of adjustments in the data typically results in noisy empirical estimates, which can be sensitive to measurement error. The extensive margin is much stronger for changes in the user cost (McKay and Wieland, 2022), e.g., a price change as in Figure 3.1, so this type of elasticities makes it easier to discriminate across models and compare them to the data.

*Probability of adjustment*. The right panel of Figure 3.2 plots (in black) the empirical probability that a household adjusts its stock of vehicles after a certain number of years *conditional* on not having adjusted so far, which is also known as the Kaplan-Meier hazard. This moment is important for the response to shocks in fixed cost models (Alvarez et al., 2021). A purely state-dependent model will tend to produce a steep hazard, whereas a purely time-dependent model produces a constant hazard.<sup>29</sup>

We construct this conditional probability using the purchase dates Year<sub>w</sub> as follows. The duration between two consecutive purchases is given by  $\text{Duration}_w = \text{Year}_w - \text{Year}_{w-2}$  whenever an adjustment occurs ( $\text{Adj}_w = 1$ ). We restrict attention to the first purchase by a given household.<sup>30</sup> This yields an empirical probability distribution  $\pi_k$  over durations  $k = 1, 2, \ldots$  expressed in years. Following Alvarez et al. (2021), we compute the conditional probability of adjustment as  $\text{Prob}_k = \pi_k / (1 - \sum_{j < k} \pi_j)$ .

The right panel of Figure 3.2 compares the empirical probability (in black) to the one implied by our model ( $\eta = 0.2$ , in red) and two alternative calibrations with, respectively, purely time-dependent adjustments ( $\eta \rightarrow +\infty$ , dashed) and state-dependent adjustments ( $\eta \rightarrow 0$ , in blue). The conditional probability is flat in the purely time-dependent model. On the contrary, the data suggests that vehicle adjustments are fairly state-dependent. This is intuitive: the longer a household owns a car and the more it depreciates, the more likely it is that the household will adjust next period. The model with  $\eta = 0.2$  matches the empirical profile quite well.<sup>31</sup> The overall pattern is roughly similar in the purely state-dependent model ( $\eta \rightarrow 0$ ), although the fit becomes somewhat poorer as the horizon increases and the conditional probability can be non-monotonic in this case.<sup>32</sup> Overall, this confirms that our calibrated model retains a substantial degree of state-dependence. This also means that the conditional probability of adjustment is only a *partially* informa-

<sup>&</sup>lt;sup>29</sup> The conditional probability could even be *decreasing* in a purely time-dependent model if households faced heterogeneous adjustment probabilities (Alvarez et al., 2021). The right panel of Figure 3.2 rules out such a model. The short-run price elasticity would be zero in that model, which would contradict the evidence as well (Section 3.2).

<sup>&</sup>lt;sup>30</sup> The reason is that subsequent purchases, if observed in the PSID's relatively short time dimension, are more likely to be of shorter duration which would bias our estimates. Focusing on the first adjustment allows us to circumvent this issue.

<sup>&</sup>lt;sup>31</sup> Note that all models match the *average* probability, by construction. The reason is that we target the empirical frequency of adjustment in our calibration, which is computed using the empirical probability of adjustment. The success of our model lies in the fact that it matches the *profile* well.

 $<sup>^{32}</sup>$  In numerical experiments, we have found that this monotonicity occurs for different parametrizations and larger values of  $\eta$  (but is still smaller than in our model). The non-monotonicity is not the result of a "numerical imprecision" since we average over thousands of simulations. Instead, it stems from a compositional change over time. The "surviving" households who still have not adjusted after 6 years tend to be richer and wealthier and buy larger durables that they adjust less frequently.

tive moment. It allows us to rule out very large values of  $\eta$  (a strong time-dependence), as did the evidence on the relative marginal propensity to spend on durables (left panel of Figure 3.1). But it does not allow us to discriminate between lower values of  $\eta$ . Very low values of  $\eta$  are instead ruled out by the evidence on the price elasticity (right panel of Figure 3.1) as well as the evidence on the distribution of net investment rates (left panel of Figure 3.2).

Timing of the spending response and large shocks. We have focused so far on the quarterly response to stimulus checks. Turning our attention to longer horizons, we find that households spend on average 65% out of a \$500 stimulus checks over 6 months, 75% over 9 months, and 92% over 12 months (Figure D.1 in Appendix D.1).<sup>33</sup> For comparison, Hausman (2016) estimates an MPX of 70% over 6 months, Agarwal and Qian (2014) obtain an MPX of 80% over 10 months, and Fagereng et al. (2021) find that winners of lottery prizes of less than \$2,000 tend to spend all these gains (or even more) in the year when they receive them. While most lottery gains are much larger than typical stimulus checks, we find it useful to compute the MPX out of these large gains and compare them to available estimates. We obtain an annual MPX of 67% out of the mean lottery gain (\$9,240) in the sample of Fagereng et al. (2021).<sup>34</sup> This value lies between their truncated estimate of 51% and their untruncated estimate of 72%.<sup>35</sup> The latter is more comparable to our value since we do not trim the distribution of MPXs in the model. The time profile of the spending response out of a check of the same size lines up closely with the estimates Fagereng et al. (2021). In particular, the MPX in year 1 is 260% as large as in year 2 in our model, compared to roughly 290% in their study.

*Share of hand-to-mouth*. We find that 42% of households are hand-to-mouth, i.e., their holdings of liquid assets are less than half of their monthly (gross) income (Kaplan et al., 2014). While untargeted, this figure turns out to be almost exactly identical to the estimates of Kaplan and Violante (2022) and Aguiar et al. (2023).

Secondary market. Our model makes no distinction between old and new durables.<sup>36</sup>

<sup>&</sup>lt;sup>33</sup> Households spend 52% of the check on durables over one year, and 40% on non-durables.

<sup>&</sup>lt;sup>34</sup> Fagereng et al. (2021) observe total spending, but are not able to directly break it down between durables and non-durables. However, they conjecture that durable spending could be important to explain the rate at which the MPX declines with the size of lottery prizes.

<sup>&</sup>lt;sup>35</sup> For comparison, Golosov et al. (2021) find an annual MPX of roughly 60% in their sample of US lottery winnings of at least \$30,000.

<sup>&</sup>lt;sup>36</sup> In particular, they have the same depreciation rate and are valued equally. Gavazza and Lanteri (2021) model the secondary market explicitly by allowing older cars to have a lower perceived quality.

However, suppose that households who adjust their stock of durables (upward or downward) first *sell* their existing stock. Part of households' gross purchases is then fulfilled by old durables on the secondary market. Under this assumption, used durables account for 53% of gross purchases in our model.<sup>37</sup> For comparison, used cars represent roughly 55% of total spending on cars in the US (Department of Transportation, 2023).<sup>38</sup>

*Distribution of MPX*. Figure D.2 in Appendix D plots the distribution of total MPXs produced by our model. We also compare this distribution to the ones produced by a purely state-dependent version of our model and a two-asset model of non-durable spending similar to one in Kaplan and Violante (2022).<sup>39</sup> The distribution of MPXs is skewed in our model and has a relatively long right tail. The overall shape of the distribution is consistent with the evidence in Fuster et al. (2021) and Lewis et al. (2022). In particular, a non-negligible share of households displays an MPX close to (or above) 1.<sup>40</sup> Lumpy adjustment and households' ability to pay only a fraction of the price as a down payment make such high MPXs possible. Turning to the purely state-dependent version of our model, the distribution of MPXs is bi-modal (with a second mode around 0.5), which is expected in a model with (*s*, *S*) adjustment bands. Finally, the two-asset model of non-durables struggles to generate MPXs larger than 1 as observed in the data.

#### 3.4 State- vs. Time-Dependent Adjustments

Our model with a smooth hazard has both state- and time-dependent features. Having calibrated the model, we can now quantify the degree of state-dependence more formally.

In the purely state-dependent model, durable adjustment is deterministic conditional on the household's idiosyncratic state  $\mathbf{x}$ , and it results exclusively from movements in  $\mathbf{x}$  along the state space. In the purely time-dependent model, durable adjustment is purely random and unrelated to  $\mathbf{x}$ . In our model, adjustment occurs  $\mathcal{A}(\mathbf{x}; \psi) = 1$  if  $\psi \leq \mathcal{S}(\mathbf{x})$ , where  $\mathcal{S}(\mathbf{x})$  is the adjustment hazard (2.1) and  $\psi$  is distributed uniformly on the line [0, 1]. No adjustment occurs  $\mathcal{A}(\mathbf{x}; \psi) = 0$  otherwise.

<sup>&</sup>lt;sup>37</sup> After an aggregate shock, any increase in durable purchases must be met by more production since the secondary market already clears in steady state. Our general equilibrium analysis allows for relative price movements between durables and non-durables, but not between new and old durables since we do not make a distinction between the two. Modelling the secondary market explicitly by allowing for a quality ladder is beyond the scope of this paper.

<sup>&</sup>lt;sup>38</sup> About 70% of car sales in the US involve a used car. However, used cars are cheaper than new ones in the data and hence account for a smaller share of total spending on cars.

<sup>&</sup>lt;sup>39</sup> We describe the two-asset model of non-durables in Appendix E.1.

<sup>&</sup>lt;sup>40</sup> Figure D.3 in Appendix D breaks down the MPX by quartile of the distribution of liquid assets.

Accordingly, we introduce the following measure of state-dependence

State-dependence (SD) 
$$\equiv \frac{\text{share with } \mathcal{A}(\mathbf{x}'; \psi) = 1 \text{ and } \mathcal{A}(\mathbf{x}; \psi) = 0}{\text{share with } \mathcal{A}(\mathbf{x}'; \psi') = 1 \text{ and } \mathcal{A}(\mathbf{x}; \psi) = 0}$$
 (3.2)

where households are tracked over consecutive periods as they move along the state space from **x** to **x**' and switch from a draw  $\psi$  to  $\psi'$ . Households decide to adjust for two reasons: either because they moved to **x**' or because they got a particular draw  $\psi'$ . Our measure of state-dependence captures the share of adjustments that occur *exclusively* for the first reason. By definition, SD = 1 in the purely state-dependent model, and SD = 0 in the purely time-dependent model.

We plot our measure of state-dependence in Figure D.4 in Appendix D, as a function of the scale parameter  $\eta$ . All other parameters are re-calibrated as we change  $\eta$ . We repeat this experiment at the quarterly and annual frequencies. As anticipated in Section 2.2, the model becomes less state-dependent as  $\eta$  increases. In our preferred calibration with  $\eta = 0.2$ , roughly 23% (50%) of all adjustments during a quarter (year) occur due to changes in households' idiosyncratic state **x**.

#### 3.5 Comparing to a Calvo-Plus Variant

We have assumed so far that taste shocks are distributed according a logistic distribution, which yields the adjustment hazard (2.1).<sup>41</sup> The literature on price setting sometimes uses an alternative specification. Agents face a deterministic fixed cost  $\kappa$ , but randomly get the chance to adjust freely. The distribution of shocks is much coarser in this case; it is degenerate over two points {0,  $\kappa$ }. This type of Calvo-Plus models can generate small price adjustments that a model with a fixed cost alone cannot account for (Nakamura and Steinsson, 2010).

While a distribution with two mass points might be less realistic, the choice of the distribution should be guided ultimately by its ability to match the share of small adjustments. We therefore calibrate a Calvo-Plus variant of our model to inspect its ability to fit the evidence. Our approach follows McKay and Wieland (2021) who introduce exogeneous adjustments to dampen the elasticity of durable spending to changes in interest rates, i.e., the user cost.<sup>42</sup> Accordingly, we choose the probability of free adjustments to

 $<sup>\</sup>overline{}^{41}$  This distribution is commonly used in the literature (see the introduction and footnote 8).

<sup>&</sup>lt;sup>42</sup> McKay and Wieland (2021) assume that households are *forced* to adjust, as opposed to being *allowed* to adjust for free. Effectively, households face a constant adjustment cost  $\kappa$  and occasionally experience an infinite disutility of *not* adjusting. This formulation is equivalent to the Calvo-Plus one: households who are given the choice to adjust freely do so with probability one.

dampen the user cost elasticity and match the same short-run price elasticity of durable purchases as in our model (Figure 3.1). All other parameters are re-calibrated to match the targets discussed in Section 3.2.

Unsurprisingly, the Calvo-Plus model misses the distribution of adjustment sizes. It generates a distribution that has many small changes due to the free adjustments (the left panel of Figure E.1 in Appendix E.2).<sup>43</sup> The reason is that the Calvo-Plus model requires a large share of exogeneous adjustments to dampen the (otherwise) large price elasticity induced by the deterministic fixed cost (Section 3.2). The resulting model behaves essentially as a purely time-dependent model with a flat adjustment hazard  $S(\mathbf{x})$ . Indeed, the state-dependence index is SD = 12% at the annual level, which is much lower than in our model (50%). The empirical distribution of adjustment sizes is not consistent with such a flat hazard; it calls for more state-dependence to generate medium-size changes and explain the hump shape in the distribution.

As a result of this strong time-dependence, the Calvo-Plus model generates a lower quarterly MPX on durables out of a \$500 check (18%) compared to our model (25%) and the preferred estimate of Orchard et al. (2022) (30%).<sup>44</sup> In particular, the MPX on durables is roughly equal to the one on non-durables (16%) in the Calvo-Plus model, whereas it is about one and a half times as large in our model and in the data. The difference is starker at the annual level: the MPX on durables is much lower (31%) compared to non-durables (43%) — the proportions are inversed compared to our model (52% and 40%, respectively).<sup>45</sup> Finally, the conditional probability of adjustment since the last purchase is overall flatter compared to our model and the data (the right panel of Figure E.1).

Summing up, preference shifters distributed according to a logistic distribution provide a parsimonuous deviation from the canonical model that allows us to match a rich set of micro moments on household spending. In contrast, a Calvo-Plus variant misses several of these moments, resulting in a different response of durables to checks. This explains why we adopted the hazard (2.1) in the first place.

<sup>&</sup>lt;sup>43</sup> This distribution is standardized, as usual (Alvarez et al., 2016b). This explains why the mode (corresponding to very small adjustments) is not centered.

<sup>&</sup>lt;sup>44</sup> The Calvo-Plus model requires a large share of exogeneous adjustments to match the same price elasticity as in our model. However, the extensive margin of adjustment is stronger for shocks that affect the user cost that encourage intertemporal substitution (McKay and Wieland, 2021, 2022), e.g., a price change, compared to an income change (as we confirm in Section 4.3). This explains why the response to stimulus checks is weaker in the Calvo-Plus model.

<sup>&</sup>lt;sup>45</sup> We obtained very similar results when calibrating the Calvo-Plus model to match the same relative MPX between durables and non-durables as in our model, instead of the same short run price elasticity. In particular, the distribution of adjustment sizes has too many small adjustments and the annual MPX on durables is very low in this variant of the model as well.

# **4** Size-Dependence in the MPX

In the previous section, we brought the model to the data and disciplined the hazard to match a rich set of micro moments. We are now ready to use our model to quantify how the MPX varies as stimulus checks become larger (Section 4.1). We then compare this size-dependence across models, and highlight the role of our smooth adjustment hazard (Section 4.2). Finally, we discuss the role of the extensive margin of adjustment (Section 4.3), and how aggregate conditions affect the MPX (Section 4.4).

#### 4.1 Durables, Non-Durables, and the Total MPX

The left panel of Figure 4.1 plots the marginal propensities to spend on durables and non-durables at the quarterly frequency following stimulus checks of varying sizes (red curves).<sup>46</sup> We also plot the MPX in a purely state-dependent version of our model (in blue) and in a canonical two-asset model of non-durables similar to Kaplan and Violante (2022) (in grey).<sup>47</sup>

Starting with durables, we find that the MPX is essentially flat in our model over the range \$100 to \$600, and then declines slowly. This contrasts with a purely state-dependent version of our model where the MPX starts much higher (as in Figure 3.1) and does increase substantially at first — the MPX and its surge are so large in this case that this requires a separate axis on the top panel. We elaborate on the difference between these models in Section 4.3 when discussing the role of the extensive margin of adjustment.

Turning to non-durables, the MPX is lower than the MPX on durables in our model and it declines more rapidly. Households therefore tilt their spending towards durables as checks get larger. For instance, the MPX on non-durables out of \$2,000 is about 1/3 lower than the one out of \$100, whereas the MPX on durables is only 15% lower. In contrast, the canonical two-asset model of non-durables produces a MPX on non-durables that declines at a much higher rate relative to our model: the response is essentially halved when comparing a \$100 and \$2,000 check.<sup>48</sup> This partly reflects the complementarity between durables and non-durables which is absent from the two-asset model of non-durables. As checks become larger, households spend more on durables in our model,

<sup>&</sup>lt;sup>46</sup> Figure D.1 in Appendix D plots the dynamic responses, i.e., beyond the first quarter. We discuss the annual responses later in this section. All responses are computed starting from the stationary equilibrium. We explore the role of aggregate conditions in Section 4.4.

<sup>&</sup>lt;sup>47</sup> Again, we describe the two-asset model of non-durables in Appendix E.1.

<sup>&</sup>lt;sup>48</sup> As we explain below, the rate at which the MPX declines (not the absolute change) is the relevant metric to compare the size-dependence across models. A standard, one-asset incomplete market model of nondurables also predicts that the MPX declines fast.



#### Figure 4.1: Size-dependence in the MPX

*Notes*: The left panel plots the MPX on durables and non-durables at the quarterly frequency as a function of the size of stimulus checks. The red curves are our model. The blue curves are a purely state-dependent version of our model. The grey curve is a canonical two-asset model of non-durables. The right panel plots the MPX as a function of the size of the checks in our model and in three alternative calibrations with lower liquidity (13% of annual income instead of 26%), higher frequency of adjustment (30% instead of 24%), and more down payment ( $\theta = 30\%$  instead of  $\theta = 20\%$ ).

and this raises the marginal utility of consuming non-durables and the associated MPX. Note that in the purely state-dependent model, however, the initial surge in the MPX on durables is so strong that the MPX on non-durables actually declines faster initially relative to our model.

The right panel plots the total MPX (the sum of the MPXs on durables and nondurables) in our model as a function of the size of stimulus checks (in red). We find that the MPX declines with the size of stimulus checks, albeit relatively slowly. In particular, it remains elevated even for large checks (Fagereng et al., 2021). The total MPX starts between the ones in the purely state-dependent version of our model and the two-asset model of non-durables.

How does the size-dependence in the MPX compare across these models? The relevant metric to assess how quickly the effect of stimulus checks wears off as they become larger turns out to be the *rate* at which the MPX declines (not the absolute change). To understand why, consider two scenarios. In the first one, the MPX declines from 42% to 35% as the check increases from \$500 to \$2,000 (as in our model); in the second one, the MPX declines from 9% to 2% over the same interval. The absolute change is the same, but the MPX falls at a higher rate in the second scenario. In the first case, spending increases as the check gets larger (by \$490), while in the second scenario spending actually *decreases* 

(by \$5).<sup>49</sup> This stylized example illustrates that the rate at which the MPX declines, or the *elasticity* of the MPX with respect to *T*, is the correct way to compare the size-dependence across models.

The total MPX decreases by a fourth when comparing \$100 and \$2,000 checks in our model. In contrast, it is almost constant in the purely state-dependent model when comparing the same amounts, as the changes in the MPX on durables and on non-durables offset each other. And the total MPX is almost halved in the canonical model of non-durables over the same interval.

We report the annual responses in our model in Figure D.5 in Appendix D. The MPX on durables is slightly hump-shaped at first. The initial increase is relatively modest, however, and there is no surge as sometimes conjectured in the literature (Parker et al., 2013). The MPX on non-durables declines slowly. Overall, the total annual MPX is mostly constant when comparing \$100 and \$2,000 checks. A Calvo-Plus model produces an annual MPX on durables that is much lower than in our model, that is lower relative to the MPX on non-durables (Section 3.5), and that does not have a meaningful hump (Figure E.2 in Appendix E.2).

Sensitivity. Finally, we perturbate various parameters to explore how they affect our results. The right panel of Figure 4.1 plots the total quarterly MPX as a function of the size of stimulus checks for three alternative calibrations with less liquidity (13% of annual income instead of 26%), a higher frequency of adjustment (30% annually instead of 24%) and a higher down payment (30% instead of 20%), respectively.<sup>50</sup> To make sure that the models are comparable, we calibrate the scale parameter  $\eta$  to match the same short-run price elasticity (Figure 3.1). All other parameters are re-calibrated to match the targets discussed in Section 3.2. The first two alternative calibrations raise the *level* of the MPX, but the overall profile is mostly unchanged and the MPX also falls by a fourth when comparing \$100 and \$2,000 checks. In the third calibration, the MPX declines more slowly, as larger checks provide households with the down payment to purchase durables. Figure **D.6** in Appendix **D** breaks down these responses between durables and non-durables.

#### 4.2 Concavity in the Spending Response

The left panel of Figure 4.2 plots the response of aggregate spending as a function of the size of stimulus checks in the three models that we have discussed so far. The concavity

<sup>&</sup>lt;sup>49</sup> We have used the fact that  $\Delta$ Spending  $\equiv$  MPX  $\times$  *T* (Section 2.6).

<sup>&</sup>lt;sup>50</sup> The lower level of liquidity lies between the mean and median holdings of liquid assets measured by Kaplan and Violante (2022).

#### Figure 4.2: Spending response



*Notes*: The left panel plots the response of aggregate spending as a function of the size of stimulus checks. The red curve is our model. The blue curve is a purely state-dependent version of our model. The grey curve is a canonical two-asset model of non-durables. The right panel reports the elasticity  $\gamma$  (see text) as a function of the scale parameter  $\eta$ . All other parameters are re-calibrated to match the targets discussed in Section 3.2. The vertical dashed line is our preferred calibration with  $\eta = 0.2$ .

in this function reflects the size-dependence in the MPX. As explained in the previous section, the elasticity of the spending response with respect to the size of checks provides a parsimonuous way to compare the size-dependence across models. Specifically, we fit a parametric function  $\Delta$ Spending =  $\alpha T^{\gamma}$  over the range \$100 to \$3,000, where  $\gamma$  is the elasticity of interest. The lower  $\gamma$ , the more concave the spending response, and the faster the MPX declines with the size of checks  $(1 - \gamma)$  is the rate at which it declines).<sup>51</sup>

The left panel of Figure 4.2 indicates the elasticity  $\gamma$  for each model. The elasticity is  $\gamma = 0.87$  in our preferred parameterization with  $\eta = 0.2$  (in red): the spending response is somewhat concave but it still remains relatively robust as stimulus checks become larger. In contrast, the two-asset model of non-durables (in grey) predicts that the effect of checks wears off more rapidly as they become larger. The spending response is very concave in the size of checks, with an elasticity  $\gamma = 0.73$ . That is, the MPX declines at a rate  $1 - \gamma$  that is twice as fast in the non-durables model relative to ours. Finally, a purely state-dependent model of durables (in blue) predicts a much stronger and more linear response. The elasticity  $\gamma = 0.94$  is much closer to unity in this model with  $\eta \to 0.52$  That

<sup>&</sup>lt;sup>51</sup> By definition, MPX  $\equiv \Delta$ Spending/*T* so  $\gamma - 1$  is elasticity of the MPX with respect to *T* and  $1 - \gamma$  is the rate at which the MPX declines.

<sup>&</sup>lt;sup>52</sup> Berger et al. (2023) build a purely state-dependent model of durable purchases. To compute total consumption, they add non-durable consumption and the imputed service flow from durables. While not the focus of their paper, they find in a numerical experiment that the marginal propensity to consume

is, the MPX declines at half the rate compared to our model with  $\eta = 0.2$ .

What role does the smooth adjustment hazard play in the size-dependence in the MPX in our model? The right panel of Figure 4.2 reports the elasticity  $\gamma$  as we vary the scale parameter  $\eta$ . The spending response becomes more concave as the scale parameter  $\eta$  (and hence time-dependence) increases. However, the elasticity  $\gamma$  is relatively flat around our preferred calibration  $\eta = 0.2$ . In other words, the size-dependence in our model is robust to changes in  $\eta$  between the lower bound ( $\eta = 0.1$ ) and the upper bound ( $\eta = 0.45$ ) that we discussed in Section 3.2.<sup>53</sup>

#### 4.3 Extensive and Intensive Margins

The smooth adjustment hazard in our model dampens the extensive margin of adjustment. To understand how it affects the size-dependence in the MPX, we decompose the marginal propensity to spend on durables into its extensive and intensive margins. The extensive margin captures changes in the adjustment hazard S, holding fixed the policy function conditional on adjustment. The intensive margin captures changes in this policy function, holding the hazard fixed. We define these components formally in Appendix B.2.

The left panel of Figure D.7 in Appendix D plots the extensive and intensive margins as a function of the size of stimulus checks in our model. The extensive margin explains a higher share of the MPX on durables than the intensive margin, but the two margins are roughly similar. This contrasts with a purely state-dependent model where the extensive margin is several times larger (right panel of Figure D.7). While the extensive and intensive margins are comparable in our model for *income* shocks, the extensive margin accounts for a larger share of the response to shocks that encourage intertemporal substitution (McKay and Wieland, 2022), e.g., a transitory change in the price of durables or a credit shock.<sup>54</sup>

Turning to the size-dependence, the intensive margin declines in our model as the stimulus checks become larger due a standard precautionary savings motive. Perhaps surprisingly, the extensive margin declines as well. Figure D.8 in Appendix D shows that more and more households are pushed into adjustment as stimulus checks become larger, initially at a constant rate and eventually at a lower pace. Overall we find that increasing

<sup>(</sup>MPC) declines more slowly compared to a canonical model of non-durables.

<sup>&</sup>lt;sup>53</sup> Changes in the scale parameter within these bounds still affect the level of the MPX and the other moments discussed in Section 3. We prefer  $\eta = 0.2$  for the reasons discussed in that section.

<sup>&</sup>lt;sup>54</sup> This is intuitive: a transitory price change can only be taken advantage of today, while a stimulus check can be used at any subsequent point in time. Consistently, we find that the extensive margin accounts for about two thirds of the response to a transitory price change.

the size of stimulus checks from \$100 to \$3,000 increases the mass of adjusters by roughly 2.5%, which is broadly consistent with the evidence of Fuster et al. (2021). The decline in the pace at which households adjusts accounts for most, but not all the decrease in the extensive margin. There is a selection effect too: households who do not adjust unless they receive a large check value durables less and thus buy smaller ones.<sup>55,56</sup>

## 4.4 Aggregate Conditions

Finally, we explore how aggregate conditions affect the MPX in our model. The left panel of Figure D.9 in Appendix D plots the MPX out of \$500 at various points of the business cycle. The stimulus checks are received unexpectedly after three quarters of constant expansion or contraction, followed by a linear mean-reversion over eight quarters. The MPX is somewhat countercyclical: it is larger in recessions, and even more so in deeper ones. This prediction is in line with the evidence of Gross et al. (2020). In contrast, a purely state-dependent model predicts a sharp decline in the MPX in deeper recessions (right panel of Figure D.9) through the mechanism proposed by Berger and Vavra (2015).

# 5 Stimulus Checks in General Equilibrium

In the rest of this paper, we evaluate the effect of stimulus checks in general equilibrium. We start by embedding our model of households' spending into an open-economy heterogeneous-agent New-Keynesian model (Section 5.1). Our model accounts for endogeneous changes in incomes in response to the stimulus checks, as well as various forces that can dampen the effect of these checks. We describe the parameterization in Section 5.2. We quantify the general equilibrium response to stimulus checks in Section 5.3. We allow for supply shocks and richer inflation dynamics in Section 5.4. We provide more details in Appendix C.

# 5.1 Environment

The economy has two sectors. The first produces a non-durable good and the second an investment good. The non-durable good can be used for consumption or as an intermediate for producing the investment good. The investment good can be used to build up

<sup>&</sup>lt;sup>55</sup> This selection effect is well known in the price setting literature (Golosov and Lucas, 2007).

<sup>&</sup>lt;sup>56</sup> The non-linear residual captures the interaction between the extensive and intensive margins. It is smaller than the other components. However, its contribution rises with the size of stimulus checks as house-holds who only adjust if they receive a large check are poorer on average and have higher marginal propensitites to spend.

the stock of durables or capital. The non-durable good is produced with labor. The investment good is produced with non-durables (as in McKay and Wieland, 2021), or with capital.

*Households*. The household block of the economy is identical to the one introduced in Section 2. The only difference is that we allow for inflation and relative price movements between durables and non-durables over time, as well as imports and exports of goods.

Households import part of their non-durable and investment goods. Accounting for imports is important to quantify the effect of stimulus checks accurately, as part of the extra spending from checks leaks abroad. This is especially important for durables for which the import share in US expenditures is roughly one fourth. Non-durable consumption  $c_t$  and investment  $x_t$  are given by

$$c_{t} = \left[\sum_{j \in \{H,F\}} \left(\alpha_{j}^{c}\right)^{\frac{1}{\rho}} \left(c_{t}^{j}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad x_{t} = \left[\sum_{j \in \{H,F\}} \left(\alpha_{j}^{d}\right)^{\frac{1}{\rho}} \left(x_{t}^{j}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \quad (5.1)$$

where  $c_t^H$  and  $c_t^F$  are the consumptions of the home and foreign non-durable goods, respectively, and the weights  $\alpha_H^c + \alpha_F^c = 1$  capture the corresponding spending shares.<sup>57</sup> The terms  $x_t^j$  and  $\alpha_j^d$  are defined similarly for investment in durables. In the following, we let  $P_t^c$  and  $P_t^d$  denote the price of the consumption and investment baskets (5.1) expressed in terms of the home non-durable good. All other prices and real quantities are also expressed in terms of that good (Appendix C.1).

The demands from the rest of the world are similar to (5.1). *Total* consumption of nondurables  $c_t^*$  and investment in durables  $x_t^*$  in the rest of the world are constant and equal to the steady state levels at home, so there are no net imports in steady state. However, countries can run a current account surplus or deficit after an aggregate shock. Domestic and foreign prices are normalized to 1 initially. Domestic prices respond to an aggregate shock, while foreign nominal prices are fixed throughout. Finally, the nominal exchange rate is pinned down by purchasing power parity in the long-run, and uncovered interest rate parity during the transition (Appendix C.1).

Non-durable goods. A firm produces non-durables using labor. Inflation in the price of the

 $\overline{}^{57}$  We will contrast the responses in closed ( $\alpha_F^c = \alpha_F^d = 0$ ) and open ( $\alpha_F^c, \alpha_F^d > 0$ ) economies.

non-durable good  $(\pi_t)$  is given by a Phillips curve

$$\pi_t = \kappa \log\left(\frac{Y_t^{\text{dom}}}{Y_t^{\text{potent}}}\right) + \beta \pi_{t+1},\tag{5.2}$$

where  $Y_t^{\text{dom}}$  is the aggregate demand for the non-durable good,  $Y_t^{\text{potent}}$  is potential output in that sector, and  $\kappa > 0$  is the slope of the Phillips curve.<sup>58</sup>

*Investment goods*. A firm can produce  $F(M) = A_0 M^{\frac{\zeta}{1+\zeta}}$  units of the investment good using M units of non-durables as in McKay and Wieland (2021, 2022), where  $1/\zeta > 0$  governs the decreasing returns in production (and hence the inverse supply elasticity) and  $A_0 > 0$  is productivity.

We assume that the firm can also produce the investment good using capital. This allows us to introduce firm investment shocks, which are the main driver of US business cycle fluctuations (Justiniano and Primiceri, 2008). These shocks will act as aggregate demand shifters in our economy.<sup>59</sup> Specifically, we assume that the firm can use  $K_{t-1}$  units of capital to produce  $A_1K_{t-1}$  units of the investment good, where  $A_1 > 0$  is productivity. New capital is produced with investment goods too. The stock of capital evolves as

$$K_{t} = \left\{ 1 - \delta^{K} + \Phi \left( I_{t} / K_{t-1} \right) + z_{t} \right\} K_{t-1},$$
(5.3)

with initial condition  $K_{-1} = K$  at the steady state, where  $I_t$  is investment,  $\delta^K$  is the depreciation rate of capital, and  $\Phi(x)$  is the investment technology which is increasing and concave.<sup>60</sup> As in Brunnermeier and Sannikov (2014), the shocks  $\{z_t\}$  are a source of aggregate fluctuations in our economy.

The firm maximizes its value and smoothes the dividends  $\text{Div}_t$  that it disburses to households (Leary and Michaely, 2011), which is important to account for the response to shocks in practice (Holm et al., 2021).<sup>61</sup> Profit maximization implies that, in equilibrium,

<sup>&</sup>lt;sup>58</sup> See McKay and Wieland (2021) for a microfoundation. This Phillips curve can result from sticky prices or wages. Distinguishing between the two would require taking a stance on workers' labor supply, which is not the focus of this paper. We consider an alternative, non-linear Phillips curve in Section 5.4.

<sup>&</sup>lt;sup>59</sup> An alternative would be to introduce discount rate shocks. However, this type of shock implies that the MPX is *lower* during a recession, which contradicts the evidence (Gross et al., 2020; Sokolova, 2023). Discount rate shocks also play a relatively minor role in explaining output fluctuations in the US (Justiniano and Primiceri, 2008; Auclert et al., 2020). We prefer investment shocks for these reasons.

<sup>&</sup>lt;sup>60</sup> This specification with a linear production function and a concave investment technology is common in the asset pricing literature (Jermann, 1998; Brunnermeier and Sannikov, 2014).

<sup>&</sup>lt;sup>61</sup> Absent dividend smoothing, dividends would be *countercyclical* during an investment boom which is counterfactual (Covas and Haan, 2011). Dividend smoothing ensures that investment booms raise house-holds' incomes in (5.8) and hence act as aggregate demand shifters. We describe the dividend smoothing

the relative price of the investment good is

$$p_t^d \equiv \left(\frac{X_t^{\text{dom}}}{X_t^{\text{potent}}}\right)^{1/\zeta},\tag{5.4}$$

where  $X_t^{\text{dom}}$  is the aggregate demand for durables produced domestically and  $X_t^{\text{potent}}$  is potential output in that sector. The potential outputs  $Y_t^{\text{potent}}$  and  $X_t^{\text{potent}}$  in (5.2) and (5.4) capture sectoral productivities (Appendix C.2). In Section 5.4, we allow for shocks to these potential outputs which are inflationary. In the following, we let  $\hat{y}_t \equiv \log(Y_t^{\text{dom}}/Y_t^{\text{potent}})$ and  $\hat{x}_t \equiv \log(X_t^{\text{dom}}/X_t^{\text{potent}})$  denote the sectoral output gaps.

*Policy*. The government sends nominal stimulus checks to eligible households in the first period. We assume that households who earned less than \$75,000 in the previous year are eligible to receive a check. The size of the check decreases linearly with income after that and reaches zero at \$80,000.<sup>62</sup> The government's flow budget constraint is

$$B_t^g + P_t^c G_t + \frac{T_t}{1 + \pi_t} = \frac{1 + r_t}{1 + \pi_t} B_{t-1}^g + \mathcal{T}_t + \Sigma_t,$$
(5.5)

where  $B_t^g$  are real asset holdings,  $G_t$  is government spending on non-durables,  $T_t$  are nominal stimulus checks,  $\mathcal{T}_t \equiv \int (yY_t^{\text{inc}} - \psi_{0,t}(yY_t^{\text{inc}})^{1-\psi_1}) d\mu_{t-1}$  are the revenues from progressive income taxation (Heathcote et al., 2017) with  $Y_t^{\text{inc}}$  denoting households' aggregate real income, and  $\Sigma_t$  are credit payments from households.<sup>63</sup> As in our baseline calibration, the government maintains a constant ratio of debt to output in the stationary equilibrium. Its real spending G > 0 on domestic goods balances the budget (5.5) in steady state. In period t = 0, the government sends a one-time nominal stimulus check to eligible households. These checks are deficit-financed. In later periods t > 0, the government maintains a constant spending  $G_t = G > 0$  and repays its new debt over time by raising the tax intercept  $\psi_{0,t}$  as we explain in Section 5.2.

Monetary policy follows a standard rule  $r_t^m = \max\{r^m + \phi_{\Pi}\pi_t, \underline{r}\}$  where  $r^m$  is the steady state interest rate on the liquid asset,  $\phi_{\Pi}$  is the coefficient on inflation, and  $\underline{r}$  is the effective lower bound.<sup>64</sup>

in more detail when we discuss the parametrization (Section 5.2).

<sup>&</sup>lt;sup>62</sup> This distribution of checks mimicks the one observed in 2020–2021. Again, we assume that mean annual income is \$67,000 at the steady state as in Kaplan and Violante (2022).

<sup>&</sup>lt;sup>63</sup> Instead of assuming that the government claims  $\Sigma_t$ , we could have introduced a separate financial sector. We assume that the government spends  $G_t$  on domestic and foreign varieties using the same aggregator (5.1) as the households.

<sup>&</sup>lt;sup>64</sup> We assume that the Taylor coefficient on the output gap is zero, as in Auclert et al. (2021). We also

*Outputs and incomes.* Market clearing requires that the amounts spent on the non-durable and durable goods equal the value of the production in these sectors

$$P_t^c \left( C_t + G_t \right) + F^{-1} \left( X_t^{\text{dom}} \right) + \mathsf{N} X_t^{c, \text{real}} = Y_t^{\text{dom}}, \tag{5.6}$$

and

$$P_t^d X_t + p_t^d I_t + N X_t^{d, real} = p_t^d \left( X_t^{dom} + A_1 K_{t-1} \right),$$
(5.7)

where  $C_t$  and  $X_t$  are the households' aggregate demands for the non-durable and investment good, respectively,  $F^{-1}(X_t^{\text{dom}})$  is the demand for intermediates used to produce  $X_t^{\text{dom}}$  units of the investment good,  $NX_t^{c,\text{real}}$  and  $NX_t^{d,\text{real}}$  are real net exports (Appendix C.1), and  $Y_t^{\text{dom}}$  and  $X_t^{\text{dom}} + A_1K_{t-1}$  are the sectoral outputs.<sup>65</sup>

Households' aggregate income before interest and tax payments is

$$Y_t^{\text{inc}} = Y_t^{\text{dom}} + \text{Div}_t, \tag{5.8}$$

where  $Y_t^{\text{dom}}$  are the payments from the firm producing the non-durable good and  $\text{Div}_t$  are the dividends disbursed by the firm producing the investment good.<sup>66</sup> Households' real net income before interest payments is

$$E_t^{\text{net}}\left(\mathbf{x}\right) = \psi_{0,t} \left(yY_t^{\text{inc}}\right)^{1-\psi_1},\tag{5.9}$$

where *y* are idiosyncratic income shocks, and  $\psi_{0,t}$  and  $\psi_1$  parametrize the tax schedule.

Finally, we will compute aggregate output as a quantity index

$$Y_t^{\text{GDP}} \equiv C_t + X_t + G_t + I_t + \text{TB}_t \tag{5.10}$$

using steady state prices ("chained dollars"), where  $TB_t$  is the quantity index for the trade balance (Appendix C.1).

assume that monetary policy responds to inflation in non-durables  $\pi_t$ , as in McKay and Wieland (2021), since this is the only good produced with labor. We have experimented with a version where the Taylor rule depends on CPI (or PPI inflation) instead of non-durable inflation  $\pi_t$ , and obtained similar results.

<sup>&</sup>lt;sup>65</sup> Households' consumption  $C_t$  and investment in durables  $X_t$  and government spending G all use both the local and foreign goods. On the contrary, the firm's investment  $I_t$  uses local goods only. Hence the different price indices in (5.7).

<sup>&</sup>lt;sup>66</sup> Households claim the revenue of the firm producing non-durables. We do not make a distinction between the wage bill and profits of that firm for the reasons discussed in footnote 58.

#### 5.2 Parametrization

As in our baseline calibration (Section 3), the real interest rate is  $r^m = 1\%$  in the stationary equilibrium, aggregate income is  $Y_t^{inc} \equiv 1$ , the government maintains a constant ratio of debt to annual aggregate income of 26%, and the tax intercept  $\psi_0$  ensures that the marginal tax rate is 30% in the long-run. Households import 23% of their durable spending at the steady state, and 9% of their non-durable spending (Hale et al., 2019). We set the elasticity of substitution between home and foreign varieties to  $\rho \rightarrow 1$ . This value lies between the short-run and long-run estimates of Boehm et al. (2023). We normalize the productivity  $A_0$  in the sector producing the investment good so the relative price of durables is  $p^d \equiv 1$  at the initial steady state. The investment technology is  $\Phi(x) = 1/\phi(\sqrt{1+2\phi x}-1)$  with  $\phi \equiv 2$  as in Brunnermeier and Sannikov (2014). The productivity of the investment firm  $A_1$  is chosen so there is no long-run growth.<sup>67</sup> The slope of the Phillips curve is  $\kappa = 0.0031$  based on the evidence of Hazell et al. (2022).<sup>68</sup> For now, we focus on the case  $\zeta \to +\infty$  where the relative price of durables is acyclical. We allow for relative price movements and a non-linear Phillips curve in Sections 5.3–5.4.69 The Taylor coefficient on inflation is  $\phi_{\Pi} = 1.5$ . The effective lower bound on the interest rate is 3 percentage points lower than the steady state (real) interest rate  $r^m$ , assuming a 3% nominal return on the liquid asset. Therefore, we set the effective lower bound to <u>r = -2%</u>. The government finances the stimulus checks by issuing debt, and repays it slowly by raising the tax intercept  $\psi_{0,t}$  uniformly over 15 years and letting it decay to its long-run value  $\psi_0$  over the next 5 years. Similarly, the firm producing the investment good disburses dividends Div<sub>t</sub> uniformly over 15 years, and then lets them decay back to their long-run level over the next 5 years.

#### 5.3 The Response to Stimulus Checks in General Equilibrium

We are now ready to quantify the effect of stimulus checks in general equilibrium. The economy experiences a demand-driven recession due to investment shocks  $\{z_t\}$ . We engineer these shocks so that aggregate output falls by 4% over three quarters, and then recovers linearly over the next two years (Appendix C.3). Starting from this recession, the government sends a nominal stimulus check in the first quarter to eligible house-

<sup>&</sup>lt;sup>67</sup> This is standard in models with AK technology. We normalize the level of capital in steady state to  $K \equiv 1$ . <sup>68</sup> Hazell et al. (2022) estimate that this slope is -0.0062 in terms of unemployment since 1980. The semi-

elasticity of unemployment with respect to output is roughly -0.5 over that period.

<sup>&</sup>lt;sup>69</sup> Empirically, the relative price of new consumer durables is relatively acyclical, even when using transaction prices (Gavazza and Lanteri, 2021) as in CPI data (McKay and Wieland, 2021; Cantelmo and Melina, 2018).




*Notes*: The left panel plots aggregate output in the first quarter in deviations from steady state as a function of the size of stimulus checks. The solid curve is our benchmark model (Sections 5.1–5.2). The dashed black curve is a version with relative price movements between durables and non-durables ( $\zeta \equiv 1/0.049$ ). The purple line extrapolates the response out of a \$300 check. We also indicate the output increase (in cents per dollar sent) after a \$2,000 stimulus check. The right panel reports the dynamic response of aggregate output in our benchmark model for stimulus checks of various sizes.

holds. We repeat this experiment for checks of various sizes.

*Aggregate output*. The left panel of Figure 5.1 plots aggregate output in the first quarter in deviations from steady state for various sizes of stimulus checks. We first focus on the benchmark model that we presented in Sections 5.1–5.2 (solid black curve). Output is 4% below potential absent stimulus checks, which amounts to a \$670 decrease in average quarterly income. A large check of \$2,000 increases output by 30 cents per dollar, whereas a smaller check of \$300 increases output by 41 cents per dollar. Overall, large checks remain effective but extrapolating from the response out of small checks overestimates their impact substantially (dashed purple curve).<sup>70</sup> A larger check of \$2,300 is required to fully close the output gap. We contrast the response of aggregate output to stimulus checks in closed and open economies in Figure D.10 in Appendix D. Accounting for imports dampens the response by roughly a fourth in an open economy since part of the extra spending leaks abroad, but the overall size-dependence is mostly unchanged.

<sup>&</sup>lt;sup>70</sup> The response out of the \$300 check is larger than the static Keynesian multiplier associated with an MPX of 44% out of small checks (Figure 4.1). Indeed: (i) only a fraction of households are eligible to the checks (66%); (ii) part of the extra spending leaks abroad through imports (17%); and (iii) labor incomes are taxed (at a marginal rate 30%). Accouting for (i)–(iii) yields a static Keynesian multiplier of 32 cents per dollar. This response is amplified by future incomes changes in general equilibrium, despite the response of monetary policy and higher future taxes.



#### Figure 5.2: Durables and non-durables

*Notes*: The left panel plots general equilibrium transfer multipliers as a function of the size of the average stimulus check sent. The red curves correspond to durables and the blue ones to non-durables. The solid curves correspond to our benchmark calibration and the dashed ones to our version with relative price movements. The right panel plots the sectoral output gaps over time for various stimulus checks.

Next, we extend our benchmark model to allow for relative price movements between durables and non-durables by lowering the supply elasticity of durables ( $\zeta < \infty$ ). Indeed, relative price movements can dampen the response to aggregate shocks (McKay and Wieland, 2022; Orchard et al., 2022). We set the supply elasticity to  $\zeta \equiv 1/0.049$  as in McKay and Wieland (2021, 2022). In this case, the effect of checks wears off more rapidly as they become larger (dashed black curve). A check of \$2,000 only increases output by 27 cents per dollar. Thus, extrapolating from the response out of small checks overestimates the impact of larger ones even more (about 50% for a \$2,000 check). A check of almost \$3,000 is needed to close the output gap.

The right panel of Figure 5.1 plots aggregate output over time in our benchmark model for two check sizes. A \$2,000 check closes most of the output gap in the first period, and about half of the cumulative output gap. A check of roughly \$4,000 closes the full cumulative output gap but stimulates output above potential in the short run (not shown).

*Durables and non-durables.* We now turn our attention separately to durables and non-durables. The left panel of Figure 5.2 plots general equilibrium transfer multipliers for durables and non-durables, which we measure as the spending responses divided by the total value of checks sent.<sup>71</sup> These multipliers are larger than the MPXs of Section 4.1 as

 $<sup>\</sup>overline{}^{71}$  These multipliers account for the fact that only 66% of households are eligible for checks.

they are sent to poorer households on average and also account for subsequent changes in incomes in equilibrium. Relative price movements attenuate the response of durables, leading to some substitution towards non-durables. Figure D.11 in Appendix D decomposes these responses along quartiles of the distribution of labor income. Lower-income households account for most of the aggregate spending responses, especially for durables, both because they have higher MPXs and because they are more likely to be eligible for checks. The right panel of Figure 5.2 plots the response of the sectoral output gaps for various stimulus checks.<sup>72</sup> The sector producing the investment good contracts proportionately more in the recession, both because households' durable spending is more cyclical and because of the demand shock that lowers the firm's investment. The two sectors recover roughly simultaneously.

## 5.4 Supply Shocks and Inflation

We conclude the paper with an exercise that creates a larger role for supply side effects. The goal is to quantify the extent to which these forces could further dampen the output response to large stimulus checks and create inflationary pressures.

We add two features to our model. First, we allow for contractions in potential outputs in the two sectors ( $Y_t^{\text{potent}}$  and  $X_t^{\text{potent}}$ ). Second, we introduce a non-linear Phillips curve

$$\pi_t = \kappa \hat{y}_t + \kappa^* \max{\{\hat{y}_t, 0\}}^2 + \beta \pi_{t+1},$$
(5.11)

when output is above potential.<sup>73</sup> With this specification, the Phillips curve remains relatively flat while the economy is below potential. It steepens endogenously when sectoral outputs exceed potential.<sup>74</sup> These features capture a rather extreme scenario: a "perfect storm" with both demand and supply shocks, and a strong inflationary response. While not representative of the typical recession, this scenario bears some resemblance to the 2020 recession and its recovery.<sup>75</sup>

We assume that potential outputs  $Y_t^{\text{potent}}$  and  $X_t^{\text{potent}}$  decrease for three quarters and

<sup>&</sup>lt;sup>72</sup> Note that these output gaps do not exactly average out to the aggregate output gap reported in Figure 5.1 since intermediaries  $F^{-1}(X_t^{\text{dom}})$  are counted in sectoral output (5.6) but not in aggregate output (5.10).

<sup>&</sup>lt;sup>73</sup> Microfounding this non-linear Phillips curve is beyond the scope of this paper. The literature has proposed various microfoundations based for instance on labor market slack or capacity constraints.

<sup>&</sup>lt;sup>74</sup> Higgins (2021) argues that the Phillips curve was flat early on in 2020 around the time when stimulus checks checks were sent. Cerrato and Gitti (2022) reach the same conclusion, and find that the Phillips curve steepened subsequently during the 2021-2022 recovery as output exceeded potential.

<sup>&</sup>lt;sup>75</sup> For example, US inflation was low during the 2001 recession and the Great recession whereas it rose in 2021. Our specification allows both for a *steepening* of the Phillips curve and an outward *shift* as the potential outputs contract (Hobijn et al., 2023; Ari et al., 2023).



## Figure 5.3: Aggregate output and inflation

*Notes*: The left panel plots aggregate output in the first quarter in deviations from steady state as a function of the size of stimulus checks. The black dashed curve is our model with relative price movements (as in Figure 5.1). The orange dotted curve is the version of our model where we add a supply shock and a non-linear Phillips curve. The purple dashed line is the same as in Figure 5.1. The right panel plots annualized CPI inflation against aggregate output in the first period for the same checks as the left panel as well as two larger ones (\$5,000 and \$6,000).

then mean revert linearly over the next 2 years (as aggregate output itself). We choose the initial drops so that potential output in each sector falls by half the contraction in actual output in that sector. Turning to the non-linear Phillips curve, there is much uncertainty in the literature about the appropriate value for  $\kappa^*$ . We purposefully choose a high value to allow inflation to play an important role. We set  $\kappa^*$  so the average slope of the Phillips curve is 0.1 as the output gap  $\hat{y}_t$  rises from zero to 2%. This output gap is similar to what we observed in the US in 2023 when inflation peaked (CBO). This slope lies at the upper end of conventional estimates in the literature (Mavroeidis et al., 2014) and is consistent with the findings of Cerrato and Gitti (2022) for the 2021-2022 recovery.<sup>76</sup>

The left panel of Figure 5.3 plots aggregate output in the first quarter in deviations from steady state as a function of the size of checks. In turn, the right panel plots annualized CPI inflation (right panel) against aggregate output.<sup>77</sup> The CPI price index averages the price indices for durables and non-durables using the households' steady state spending shares (Appendix C.1). In our model with demand shocks only (dashed back curve, as

<sup>&</sup>lt;sup>76</sup> Cerrato and Gitti (2022) estimate that the slope of annualized inflation with respect to the unemployment rate was -0.85 during 2021-2022 recovery. Expressing this estimate in terms of quarterly inflation and output gap leads to a slope of roughly 0.1, assuming an unemployment elasticity of -0.5 (footnote 68).

<sup>&</sup>lt;sup>77</sup> Figure D.12 in Appendix D plots the sectoral output gaps with and without supply shock. The response of inflation to stimulus checks is maximized in the first quarter since there is no built-in lags in inflation in our model (not shown).

in Figure 5.1) the economy starts in a deflation as output is below potential; the response of inflation to checks is relatively modest. Introducing the non-linear Phillips curve and the supply shock (dashed orange curve) raises the level of inflation and makes it more responsive to checks as output exceeds its now lower potential level (-2%) and approaches its steady state level. As a result, the effect of checks on output wears off more rapidly as they get larger. A \$2,000 check only increases output by 24 cents per dollar (instead of 27) in this case, as monetary policy responds to the rise in inflation. The difference is even more pronounced for larger checks. A government that misdiagnoses the recession as being entirely demand-driven could send a check as large as \$3,000 to close the perceived output gap, i.e., the full 4% decline in output from steady state, when the true gap is smaller due to the supply shock. This would raise inflation meaningfully.

# 6 Conclusions

We study how the households' marginal propensity to spend (MPX) varies as stimulus checks become larger. To do so, we augment a canonical incomplete markets model of durable spending by introducing a smooth adjustment hazard. This smooth hazard is essential to match a rich set of micro facts on household spending, and the shape of this hazard is key for the size-dependence in the MPX.

We find that the MPX declines with the size of checks, albeit more slowly than in a canonical two-asset model of non-durables. Households tilt their spending towards durables when receiving larger checks in our model, so the MPX on durables is relatively constant at first. This partly offsets the decrease in the MPX on non-durables, thereby slowing down the decline in the total MPX. We then use our model to evaluate the effect of checks on output during recessions. Large checks remain effective at stimulating output, but extrapolating from the response out of small checks overestimates substantially how much stimulus larger checks provide.

Our analysis provides a useful, though incomplete answer when deciding how large stimulus checks should be in recessions. In particular, the optimal size of checks depends on how the government trades off the benefits of stimulating output with the costs of higher inflation. Checks are also used to insure households when some of them are asymmetrically exposed to recessions. Therefore, the optimal size of stimulus checks depends on the government's tolerance for inflation and its preference for insurance. Future work can build on the model that we have developed in this paper to quantify the optimal size of stimulus checks.

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# Online Appendix for: Durables and Size-Dependence in the Marginal Propensity to Spend

This online appendix contains a detailed description of the quantitative model and its numerical solution, as well as additional results for the article "Durables and Size-Dependence in the Marginal Propensity to Spend."

Any references to equations, figures, or sections that are not preceded "A.," "B.," "C.," "D.", or "E." refer to the main article.

# A Consumption and Investment Problem

In this appendix, we discuss how to solve the households' consumption and investment problem. Section A.1 states the problem recursively. Section A.2 discusses the numerical implementation. Finally, Section A.3 provides details about our numerical implementation.

## A.1 Households' Problem

We now state the household's problem recursively. Relative to Section 2.5, the formulation below allows for movements in the price of durables  $(P^d)$  and non-durables  $(P^c)$  as in our general equilibrium analysis (Section 5). We also formulate the problem in a way that lends itself better to numerical implementation (Appendix A.2). All prices and real quantities are expressed relative to the domestic non-durable good (Appendix C.1). Households are still indexed by three idiosyncratic states: their stock of durables (d); their holdings of liquid asset (m); and their idiosyncratic income (y). We let  $\mathbf{x} \equiv (d, m, y)$  to save on notation.

*Continuation values*. The continuation values  $\{V_t(\cdot)\}$  can be characterized recursively.<sup>78</sup>

1. **Discrete choice.** The household chooses whether to adjust its stock of durables. The value associated to the discrete choice problem is

$$V_{t}(\mathbf{x}) \equiv \max\left\{V_{t}^{\text{adjust}}(\mathbf{x}) - \epsilon, V_{t}^{\text{not}}(\mathbf{x})\right\},$$
(A.1)

where  $V_t^{\text{adjust}}(\mathbf{x})$  is the value of adjusting the stock of durables,  $V_t^{\text{not}}(\mathbf{x})$  is the value of not adjusting, and  $\epsilon$  is a taste shifter that follows a logistic distribution whose mean and variance are controlled by  $\kappa > 0$  and  $\eta^2 > 0$ , respectively.<sup>79</sup> Therefore,

$$V_t(\mathbf{x}) = \eta \log \left( \exp \left( \frac{V_t^{\text{adjust}}(\mathbf{x}) - \kappa}{\eta} \right) + \exp \left( \frac{V_t^{\text{not}}(\mathbf{x})}{\eta} \right) \right)$$
(A.2)

<sup>&</sup>lt;sup>78</sup> The terminal condition for  $V_{t+1}(\cdot)$  is either an initial guess when solving for the stationary equilibrium, or the stationary value function without stimulus checks when solving for transitions.

<sup>&</sup>lt;sup>79</sup> An equivalent formulation consists of introducing two additive taste shifters  $e^{adjust}$  and  $e^{not}$  (one for each option) which are distributed according to a generalized extreme value distribution of type-I. See Artuç et al. (2010) for the derivation of (A.2) and (A.3) in this case.

The adjustment hazard associated to this discrete choice problem is

$$S_{t}(\mathbf{x}) = \frac{\exp\left(\frac{V_{t}^{\mathrm{adjust}}(\mathbf{x}) - \kappa}{\eta}\right)}{\exp\left(\frac{V_{t}^{\mathrm{adjust}}(\mathbf{x}) - \kappa}{\eta}\right) + \exp\left(\frac{V_{t}^{\mathrm{not}}(\mathbf{x})}{\eta}\right)}$$
(A.3)

The continuation values are given by

$$V_t^{\text{adjust}}\left(\mathbf{x}\right) \equiv W_t^D\left(\mathcal{Y}_t\left(\mathbf{x}; T_t\right) + \Delta_t^D d, y\right)$$
(A.4)

$$V_t^{\text{not}}(\mathbf{x}) \equiv W_t^C\left(\left(1 - (1 - \iota)\,\delta\right)d, \mathcal{Y}_t\left(\mathbf{x}; T_t\right) - \Delta_t^C d - \iota\delta P_t^d d, y\right)$$
(A.5)

The households gets to choose a new stock of durables if it decides to adjust, and maintains its stock otherwise by offsetting a share *i* of the depreciation (Berger and Vavra, 2015). These continuation values depend on the household's initial cash-on-hand after interest payment and stimulus check

$$\mathcal{Y}_{t}(\mathbf{x}; T_{t}) \equiv \psi_{0,t} \left( y Y_{t}^{\text{inc}} \right)^{1-\psi_{1}} + \frac{1+r_{t-1}^{m}}{1+\pi_{t}}m - r_{t-1}^{b} \left( 1-\theta \right) \hat{P}_{t}^{d}d + \frac{T_{t}}{1+\pi_{t}},$$
(A.6)

where  $Y_t^{\text{inc}}$  is real aggregate income and  $T_t$  are nominal stimulus checks. The interest rate on credit  $r_{t-1}^b$  is equal to the return on the liquid asset  $r_{t-1}^m$  plus a spread of 3.5% (Section 3.1). The inflation rate  $\pi_t$  accounts for the fact that the budget constraints are expressed in real terms, i.e., all prices are expressed relative to the one of the non-durable domestic good. The credit that a household contracts in period t - 1 depends on the expected nominal price of its durables next period (as in Gavazza and Lanteri, 2021). The real price  $\hat{P}_t^d$  is thus equal to the real price of durables  $P_t^d$  for all periods  $t \ge 1$  in our perfect foresight economy. However, these two prices need not be equal in the very first period t = 0 since the price of durables and inflation can jump after an aggregate shock. Therefore,  $\hat{P}_t^d \equiv \mathbb{E}_{t-1} [P_t^d] \times$  $(1 + \mathbb{E}_{t-1} [\pi_t]) / (1 + \pi_t)$  in t = 0. In turn, the remaining terms in (A.4)–(A.5) are

$$\Delta_t^D \equiv (1-\delta) P_t^d - (1-\theta) \hat{P}_t^d \tag{A.7}$$

$$\Delta_t^C \equiv (1 - \theta) \times \left\{ \hat{P}_t^d - P_{t+1}^d \left( 1 + \pi_{t+1} \right) \left( 1 - (1 - \iota) \, \delta \right) \right\}$$
(A.8)

which capture, respectively, the net profit that the household makes when selling its old durable (after repaying its outstanding credit) for  $\Delta_t^D$ , and the credit repayment

on the principal for  $\Delta_t^{C.80}$  In the fully state-dependent limit  $\eta \to 0$ , the value (A.2) and the hazard (A.3) become

$$V_{t}(\mathbf{x}) = \max\left\{V_{t}^{\text{adjust}}(\mathbf{x}) - \kappa, V_{t}^{\text{not}}(\mathbf{x})\right\}$$
(A.9)

and

$$S_{t}(\mathbf{x}) = \begin{cases} 1 & \text{if } V_{t}^{\text{adjust}}(\mathbf{x}) - \kappa > V_{t}^{\text{not}}(\mathbf{x}) \\ 0 & \text{otherwise} \end{cases}$$
(A.10)

2. **Durable adjustment.** If the household decides to adjust its stock of durables, it chooses how much durables to purchase

$$W_{t}^{D}(m,y) \equiv \max_{d',m'} W_{t}^{C}(d',m',y)$$
s.t.  $\left[P_{t}^{d} - (1-\theta) P_{t+1}^{d}(1+\pi_{t+1})\right] d' + m' \leq m,$ 
(A.11)

where *m* is real cash-on-hand before the household purchases its new stock of durables. As explained above, households' credit depends on the expected price of durables next period. The price index for durables  $P_t^d$  is expressed relative to the price of the domestic non-durable, which grows at rate  $\pi_{t+1}$  over time. The continuation value  $W_t^C$  reflects the subsequent optimal consumption-saving choice that occurs in the same period.

3. **Consumption-saving.** Finally, the household chooses how much to consume and save in liquid asset

$$W_t^C(d, m, y) \equiv \max_{c, m'} u(c, d) + \beta \int V_{t+1}(d, m', y') \Gamma(dy'; y)$$
(A.12)  
s.t.  $P_t^c c + m' \le m$  and  $m' \ge 0$ ,

where m is the household's real cash-on-hand when it chooses non-durable consumption c, and m' is the real holdings of liquid asset for next period.

<sup>&</sup>lt;sup>80</sup> While holding their stock of durables, households repay their outstanding credit at the same rate at which the value of their durables depreciates (Section 2.3).

## A.2 Numerical Implementation

We now describe how we solve numerically for the value functions defined above, and how we iterate on the associated policy functions to obtain aggregate quantities.

*Value functions*. We proceed as follows:

- 1. **Guess.** Fix  $\mathcal{V}_{T+1}(\mathbf{x}) \equiv \int V_{T+1}(d, m, y') \Gamma(dy'; y)$  for the terminal period.
- 2. **Consumption-saving.** Fix the continuation states (d, y). If the household's borrowing constraint  $m' \ge 0$  is not binding, a necessary condition for an optimum to (A.12) is

$$u_{c}(c,d) = \beta P_{t}^{c} \partial_{m} \mathcal{V}_{t+1}(d,m',y), \qquad (A.13)$$

together with the budget constraint  $c = (m - m') / P_t^c$ . This condition is not sufficient, however, since the problem is typically non-convex.<sup>81,82</sup> To recover policy functions, i.e., maps  $m \mapsto (c, m')$ , we proceed as follows. We first obtain a map  $m' \mapsto m$  using the endogenous grid method (EGM) of Carroll (2006). The (generalized) inverse of this map (as a function of m) might contain several points for m' since the problem is non-convex. These points define a set of candidates, together with the borrowing constraint m' = 0. The optimum is found by comparing the values of the objective in (A.12) associated to each candidate. More specifically, we recover the policy functions  $m \mapsto (c, m')$  using an approach similar to Druedahl and Jørgensen (2017). We split the map  $m' \mapsto m$  into monotonic segments, i.e., either increasing or decreasing. Fixing some m on the grid of interest, we interpolate linearly the value of m' at m using each segment. We add max  $\{m', 0\}$  to the set of candidates. The borrowing constraint m' = 0 and the upper bound of the grid for m also belong to this set of candidates for m'.<sup>83</sup> The policy function  $m \mapsto m'$  is the one that

<sup>&</sup>lt;sup>81</sup> The reason is that the continuation value involves the upper envelope (A.1). Random taste shocks for adjustment, i.e., the smooth hazard (A.3), can make continuation value *smooth* (i.e. no kinks) but not necessarily *concave*.

<sup>&</sup>lt;sup>82</sup> Condition (A.13) is necessary for an optimum (even when  $\eta = 0$ ). The argument is similar to the one in Clausen and Strub (2012). Consider a simplified version of the problem of interest:  $\max_c f(c) + G(-c)$  with  $f(\cdot)$  and  $G(\cdot)$  smooth except for a convex kink in  $G(\cdot)$  at  $\bar{c} \in \mathbb{R}$ . Suppose by contradiction that the optimizer is  $\bar{c}$ . Then,  $f'(\bar{c}) \ge G'_+(-\bar{c})$  and  $f'(\bar{c}) \le G'_-(-\bar{c})$ . However,  $G'_+(-\bar{c}) > G'_-(-\bar{c})$  since  $G(\cdot)$  admits a convex kink at  $\bar{c}$ . This leads to the desired contradiction. Therefore, the optimizer cannot be the point where the kink occurs, and condition (A.13) is necessary. The argument generalizes to multiple kinks and multiple assets.

<sup>&</sup>lt;sup>83</sup> An alternative approach is to focus only on the couple  $(m'_0, m'_1)$  such that *m* is bracketed by the couple

provides the highest value, and  $m \mapsto c$  is recovered using the budget constraint  $c = (m - m') / P_t^c$ . Using the resulting policy function  $m'_t(\cdot)$ , we compute the value  $W_t^C(\mathbf{x})$  using (A.12), and the marginal values

$$\partial_{d}W_{t}^{C}(\mathbf{x}) = u_{d}\left(\left(m - m_{t}'(\cdot)\right) / P_{t}^{c}, d\right) + \beta \partial_{d}\mathcal{V}_{t+1}\left(d, m_{t}'(\cdot), y\right)$$
(A.14)

$$\partial_m W_t^C(\mathbf{x}) = 1/P_t^c u_c\left(\left(m - m_t'(\cdot)\right)/P_t^c, d\right)$$
(A.15)

for the durable and the liquid asset.

3. Durable adjustment. A necessary condition for an optimum to (A.11) is

$$\partial_{d}W_{t}^{C}(d',m',y) - \left[P_{t}^{d} - (1-\theta)P_{t+1}^{d}(1+\pi_{t+1})\right]\partial_{m}W_{t}^{C}(d',m',y) = 0$$
(A.16)

where

$$m' = m - \left[ P_t^d - (1 - \theta) P_{t+1}^d \left( 1 + \pi_{t+1} \right) \right] d'$$
(A.17)

Again, (A.16) is typically not sufficient for an optimum.<sup>84</sup> We thus define a set of candidates d' that satisfy either (A.16) or  $d' = \bar{d}$  where  $\bar{d}$  is the upper bound of our numerical grid for durables. We compute the value (A.11) associated to these candidates. The policy function for d' is the one that provides the highest value. We compute the value  $W^D(\mathbf{x})$  using (A.11), and the marginal value

$$\partial_m W_t^D(m, y) = \partial_m W_t^C(d'(\cdot), m'(\cdot), y), \qquad (A.18)$$

and proceed to Step 4.

4. Continuation values. Compute the values (A.4)–(A.5) and the marginal values

$$\partial_d V_t^{\text{adjust}} \left( \mathbf{x} \right) = \left\{ -r_{t-1}^b \left( 1 - \theta \right) \hat{P}_t^d + \Delta_t^D \right\} \partial_m W_t^D \left( \cdot \right)$$
(A.19)

$$\partial_d V_t^{\text{not}} \left( \mathbf{x} \right) = \left( 1 - \left( 1 - \iota \right) \delta \right) \partial_d W_t^C \left( \cdot \right) + \tag{A.20}$$

$$\left\{-r_{t-1}^{b}\left(1-\theta\right)\hat{P}_{t}^{d}-\Delta_{t}^{C}-\iota\delta P_{t}^{d}\right\}\partial_{m}W_{t}^{C}\left(\cdot\right)$$

 $<sup>(</sup>m_0, m_1)$  that was recovered by the EGM for  $(m'_0, m'_1)$ . In principle, this approach might miss the correct value of m' if the grid for liquid asset is too coarse. Our approach does not suffer from this problem, but it requires considering a few more candidates for m'.

<sup>&</sup>lt;sup>84</sup> The solution is necessarily interior, however, since d' = 0 cannot be optimal.

for the durable stock, with  $\Delta_t^D$  and  $\Delta_t^C$  defined by (A.7)–(A.8), and

$$\partial_m V_t^{\text{adjust}}\left(\mathbf{x}\right) = \frac{1 + r_{t-1}^m}{1 + \pi_t} \partial_m W_t^D\left(\cdot\right) \text{ and } \partial_m V_t^{\text{not}}\left(\mathbf{x}\right) = \frac{1 + r_{t-1}^m}{1 + \pi_t} \partial_m W_t^C\left(\cdot\right) \quad (A.21)$$

for the liquid asset.

5. Discrete choice. Compute the value (A.2) and the marginal values

$$\partial_{z} V_{t} \left( \mathbf{x} \right) = \mathcal{S}_{t} \left( \mathbf{x} \right) \partial_{z} V_{t}^{\text{adjust}} \left( \mathbf{x} \right) + \left\{ 1 - \mathcal{S}_{t} \left( \mathbf{x} \right) \right\} \partial_{z} V_{t}^{\text{not}} \left( \mathbf{x} \right)$$
(A.22)

for the durable stock and the liquid asset  $z \in \{d, m\}$ , where  $S_t(\mathbf{x})$  is the adjustment hazard (A.3).

6. Update. Compute the expected utility  $\mathcal{V}_t(\mathbf{x}) \equiv \int V_t(d, m, y') \Gamma(dy'; y)$ . Similarly, compute the marginal utilities  $\partial_z \mathcal{V}_t(\mathbf{x}) \equiv \int \partial_z V_t(d, m, y') \Gamma(dy'; y)$  for the durable stock and the liquid asset  $z \in \{d, m\}$ . Finally, iterate on Step 2 until convergence when solving for the stationary equilibrium, or until t = 0 when solving for transitions.

### A.3 Computational Details

*Numerical parameters.* We use 175-point grids for the stock of durables *d* and the liquid asset *m*. We discretize the income process on a 7-point grid using the method of Rouwenhorst (1995). We use a stochastic simulation given the non-convexities inherent to our model.<sup>85</sup> To iterate on the distribution, we use the policy functions computed above, together with the income process  $\Gamma$  and we randomly assign households between adjustment and no adjustment according the adjustment hazard (A.3). The hazard and the policy functions are interpolated linearly between grid points. When computing our sta-

<sup>&</sup>lt;sup>85</sup> A non-stochastic simulation (e.g., Young, 2010) typically produces a different stationary distribution in presence of non-convexities. To understand why, consider a simplified example. Suppose that the stock of durables *d* is the only state and that there is no depreciation  $\delta = 0$ . There are three evenly-spaced points  $\underline{d} < \hat{d} < \overline{d}$ . Let us assume that the hazard satisfies  $S(\underline{d}) = 1$  and  $S(\hat{d}) = S(\overline{d}) = 0$ , and it is linear between these points. Conditional on adjustment, the policy function satifies  $d'(\underline{d}) = \overline{d}$  and  $d'(\hat{d}) = d'(\overline{d}) = \underline{d}$ , and it is linear between these points. Suppose that the household starts with a stock  $d = 1/3\underline{d} + 2/3\hat{d}$ . In this case, the stationary distribution is a mass point at *d*. Now, suppose that the functions *S* and *d'* are discretized on the three points  $\underline{d} < \hat{d} < \overline{d}$ . Starting at *d*, the probability of adjustment is 1/3, both for the stochastic simulation (when interpolating the hazard linearly between grid points) and for the non-stochastic simulation (where households are allocated to neighboring grid points based on their proximity to those). By construction, the stochastic simulation induces the correct stationary distribution with a mass point at *d*. On the contrary, the non-stochastic simulation induces a stationary distribution with two mass points at  $\hat{d}$  and  $\bar{d}$  which are both larger than *d*.

tionary moments (Section 3), we simulate 15,000 households over 3,000 quarters with a burn of 400 quarters. In general equilibrium, we sample 200,000 households from this stationary distribution, and simulate them over 125 quarters after a burn of 400 quarters.

Smoothing the responses. In Sections 3–4, we compare the properties of our model with a purely state-dependent version ( $\eta \rightarrow 0$ ). To obtain slightly smoother responses, we introduce a very small variance  $\eta = 0.0025$  in this case. In contrast to this state-dependent limit, we do not assume in our model that this variance is arbitrarily small. We discipline it to match a rich set of micro moments.

# **B** Omitted Expressions and Derivations

## **B.1** Decomposing the MPX on Durables

In this appendix, we show how to decompose the MPX on durables (2.2) into the components in expression (3.1). It is convenient to write the MPX on durables (2.2) as follows<sup>86</sup>

$$MPX^{d}(T) \equiv \frac{1}{T} \int \int \{ \mathcal{S}(m+T,d) x (m+d+T) - \mathcal{S}(m,d) x (m+d) \} d\mu(m,d) \quad (B.1)$$

Equivalently,

$$MPX^{d}(T) = \frac{1}{T} \int \int [S(m+T,d) - S(m,d)] d\mu(m,d) \times \bar{x} + \frac{1}{T} \int \int S(m+T,d) [x(m+d+T) - \bar{x}] d\mu(m,d) - \frac{1}{T} \int \int S(m,d) [x(m+d) - \bar{x}] d\mu(m,d)$$
(B.2)

Furthermore,

$$\int \int \mathcal{S}(m+T,d) \left[ x \left( m+d+T \right) - \bar{x} \right] d\mu(m,d)$$
$$= \int \int \mathcal{S}(m,d) \left[ x \left( m+d \right) - \bar{x} \right] d\mu(m-T,d), \qquad (B.3)$$

and

$$\int \int S(m,d) [x(m+d) - \bar{x}] d\mu(m,d) = \bar{x} \left[ 1 - \int \int S(m,d) d\mu(m,d) \right]$$
(B.4)

by definition of  $\bar{x}$ . Combining the expressions above yields (3.1) in the text.

 $<sup>\</sup>overline{}^{86}$  As in Section 2.6, we omit the third state variable (income *y*) to save on notation.

## **B.2** Extensive and Intensive Margins

This appendix defines formally the extensive and intensive margins of adjustment that we discuss in Section 4.3. The MPX on durables (2.2) can be expressed as

$$MPX^{d}(T) = \underbrace{\int \overline{\{S_{0}(m+T,d,y) - S_{0}(m,d,y)\}}_{\text{Extensive margin}} \times \underbrace{\frac{Selection}{x(m,d,y) \times d\mu(\mathbf{x})}_{\text{Extensive margin}}}_{T} + \underbrace{\int S_{0}(m,d,y) \times \{x(m+T,d,y) - x(m,d,y)\} \times d\mu(\mathbf{x})}_{\text{Intensive margin}} + \text{res} \qquad (B.5)$$

The extensive margin captures changes in the adjustment hazard S, holding fixed the policy function conditional on adjustment. The intensive margin captures changes in this policy function, holding the hazard fixed. Finally, the residual "res" captures the non-linear interaction between these two margins.

# C General Equilibrium

In this appendix, we describe the general equilibrium setup in more details. Section C.1 describes the price indices and the open economy features of our model (net exports, the real exchange rate, etc.). Section C.2 states and characterizes the firm's investment problem. Section C.3 explains how we construct efficiently the sequence of investment shocks that generates any particular recession of interest. Finally, Section C.4 discusses fiscal policy.

## C.1 Price Indices, Trade Balance and Exchange Rate

This appendix provides the expressions for the price indices, the trade balance, and the equilibrium exchange rate.

*Price indices*. We express the domestic prices and price indices, the exchange rate, and the trade balance relative to the price of the domestic non-durable good.<sup>87</sup> The real exchange rate is the cost of acquiring a non-durable good from the foreign country. The price indices

 $<sup>\</sup>overline{}^{87}$  Similarly, we express the foreign price indices (C.2) relative to the foreign non-durable good.

at home for the non-durable and investment goods baskets are

$$P_{t}^{c} \equiv \left[\alpha_{c} + (1 - \alpha_{c}) (e_{t})^{1 - \rho}\right]^{\frac{1}{1 - \rho}} \quad \text{and} \quad P_{t}^{d} \equiv \left[\alpha_{d} \left(p_{t}^{d}\right)^{1 - \rho} + (1 - \alpha_{d}) (e_{t})^{1 - \rho}\right]^{\frac{1}{1 - \rho}},$$
(C.1)

where  $e_t$  is the real exchange rate and  $p_t^d$  is the relative price of the domestic investment good, using the fact that the nominal prices of foreign goods are normalized to 1 (Section 5.1). Similarly, the price indices abroad are

$$P_{t}^{c,\star} \equiv \left[\alpha_{c} + (1 - \alpha_{c}) (1/e_{t})^{1-\rho}\right]^{\frac{1}{1-\rho}} \quad \text{and} \quad P_{t}^{d,\star} \equiv \left[\alpha_{d} + (1 - \alpha_{d}) \left(p_{t}^{d}/e_{t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$
(C.2)

The *level* of the price of the domestic non-durable good is

$$P_t^{\text{dom}} = \prod_{s=0}^t (1 + \pi_s), \qquad (C.3)$$

where the inflation rate  $\pi_t$  is given by the Phillips curve (5.2). The CPI price index is

$$CPI_{t} \equiv \left\{ \omega^{c,CPI} P_{t}^{c} + \left( 1 - \omega^{c,CPI} \right) P_{t}^{d} \right\} P_{t}^{dom}, \qquad (C.4)$$

where  $\omega^{c,CPI} \equiv 1/(1 + X/C)$  is the spending share of domestic households on the nondurable good in the stationary equilibrium.

Net exports and trade balance. Let

$$\mathrm{IM}_{t}^{c} \equiv (1 - \alpha_{c}) \left(\frac{e_{t}}{P_{t}^{c}}\right)^{-\rho} (C_{t} + G_{t}) \quad \text{and} \quad \mathrm{IM}_{t}^{d} \equiv (1 - \alpha_{d}) \left(\frac{e_{t}}{P_{t}^{d}}\right)^{-\rho} X_{t} \tag{C.5}$$

denote the quantities imported of the non-durable and investment good, respectively, using the fact that the nominal prices of foreign goods are normalized to 1. Similarly, let

$$\mathrm{EX}_{t}^{c} \equiv (1 - \alpha_{c}) \left(\frac{1/e_{t}}{P_{t}^{c,\star}}\right)^{-\rho} (C^{\star} + G^{\star}) \quad \text{and} \quad \mathrm{EX}_{t}^{d} \equiv (1 - \alpha_{d}) \left(\frac{p^{d}/e_{t}}{P_{t}^{d,\star}}\right)^{-\rho} X^{\star} \quad (C.6)$$

denote the quantities exported, where consumption  $C^*$ , government spending  $G^*$  and investment  $X^*$  in the rest of the world are constant and equal to the steady state levels at home, i.e.,  $C^* = C$ ,  $G^* = G$  and  $X^* = X$ , so there are no net imports initially. The quantity indices for net exports are  $NX_t^z \equiv EX_t^z - IM_t^z$  for the non-durable and investment goods

 $z \in \{c, d\}$ . The quantity index for the trade balance is  $TB_t \equiv NX_t^c + NX_t^d$ . Net exports in real terms are  $NX_t^{z,real} \equiv p_t^z EX_t^z - e_t IM_t^z$  for the non-durable and investment goods  $z \in \{c, d\}$ . Finally, the trade balance in real terms is  $TB_t^{real} \equiv NX_t^{c,real} + NX_t^{d,real}$ .

*Exchange rate*. The nominal exchange rate satisfies uncovered interest parity. Therefore, the real exchange rate follows

$$e_t = (1 + \pi_{t+1}) \frac{1 + r^*}{1 + r^m_t} e_{t+1}$$
(C.7)

where  $r^*$  is the foreign interest rate, which is constant and equal to the steady state level at home  $r^m = 1\%$ . The terminal condition is  $\lim_{t\to+\infty} e_t = 1$  by purchasing power parity and using the fact that the foreign nominal price is normalized to 1.<sup>88</sup>

#### C.2 Firm's Problem

The firm producing the investment good chooses how much to produce with intermediate (non-durable) goods, and how much to invest in capital to produce in the following period. These two problems are separable, so we characterize them sequentially.

Intermediates. The firm solves

$$\max_{X_t^{\text{dom}}} p_t^d X_t^{\text{dom}} - \left(\frac{X_t^{\text{dom}}}{A_0}\right)^{\frac{1+\zeta}{\zeta}}$$
(C.8)

since the production function is  $X_t^{\text{dom}} = A_0 M_t^{\frac{\zeta}{1+\zeta}}$  where  $M_t$  are intermediates. Therefore,

$$p_t^d = \left(\frac{X_t^{\text{dom}}}{X^{\text{potent}}}\right)^{1/\zeta},\tag{C.9}$$

which is expression (5.4) in the text, where  $X^{\text{potent}} \equiv \left(\frac{\zeta}{1+\zeta}\right)^{\zeta} A_0^{1+\zeta}$  is potential output in the sector producing the investment good.

<sup>&</sup>lt;sup>88</sup> We work with a finite horizon in our simulation and assume that  $e_t = 1$  after 20 years.

Investment. The firm's investment problem is

$$\max_{\{I_t, K_t\}} \sum_t Q_t p_t^d \{A_1 K_{t-1} - I_t\}$$
s.t.  $K_t \leq \{1 - \delta^K + \Phi(I_t / K_{t-1}) + z_t\} K_{t-1} \text{ and } K_t \geq 0$ 
(C.10)

with initial condition  $K_{-1} \equiv K$  where *K* is steady state capital. The price of the investment good  $p_t^d$  is expressed relative to the price of the non-durable good (Section C.1). The firm's stochastic discount factor  $Q_t$  is expressed in real terms and satisfies  $Q_{t+1}/Q_t \equiv (1 + \pi_{t+1}) / (1 + r_t)$  and  $Q_0 \equiv 1$ . At optimum,

$$\frac{1}{\Phi'(x_t)} \frac{1+r_t}{1+\pi_{t+1}} \frac{p_t^d}{p_{t+1}^d} = A_1 + \frac{1}{\Phi'(x_{t+1})} \left\{ 1 - \delta^K + \Phi(x_{t+1}) - x_{t+1} \Phi'(x_{t+1}) + z_{t+1} \right\}$$
(C.11)

with terminal condition  $\lim_{T\to+\infty} x_T = \Phi^{-1}(\delta^K)$ , where  $x_t \equiv I_t/K_{t-1}$  and where we have used the definition of the firm's stochastic discount factor. The initial value problem (i.e., finding  $x_0$ ) associated with this difference equation can be solved using a standard shooting algorithm.<sup>89</sup> The sequence of capital can then be constructed recursively using the law of motion of capital

$$K_{t} = \left\{ 1 - \delta^{K} + \Phi(x_{t}) + z_{t} \right\} K_{t-1},$$
 (C.12)

with initial condition  $K_{-1} \equiv K$ .

*Dividends*. The firm's dividends are  $\text{Div}_t = \text{Div} + \Psi_t \widehat{\text{Div}}$ , where Div is the steady state dividend, and  $\{\Psi_t\}$  takes the value 1 over 15 years and then decreases linearly to 0 over the next 5 years (Section 5.2). The change in dividends  $\widehat{\text{Div}}$  over that period ensures that  $\sum_t Q_t \text{Div}_t = \sum_t Q_t \Pi_t$  where  $\Pi_t$  are real profits. Therefore,

$$\operatorname{Div}_{t} = \operatorname{Div} + \Psi_{t} \frac{\sum_{s} Q_{s} \{\Pi_{s} - \operatorname{Div}\}}{\sum_{s} Q_{s} \Psi_{s}}$$
(C.13)

<sup>&</sup>lt;sup>89</sup> Expression (C.11) defines a unique map  $x_t \mapsto x_{t+1}$  since the right-hand side is increasing in  $x \ge 0$  as  $\Phi(x) - x\Phi'(x)$  is also increasing given our choice  $\Phi(x) = 1/\kappa (\sqrt{1+2\kappa x}-1)$  with  $\kappa \equiv 2$  and  $1 - \delta^K + z_{t+1} > 0$  when  $z_{t+1}$  is positive (during a recession) or sufficiently small. This is the case in our numerical simulations (Appendix C.3).

Finally, real profits are

$$\Pi_t \equiv p_t^d X_t^{\text{dom}} - \left(\frac{X_t^{\text{dom}}}{A_0}\right)^{\frac{1+\zeta}{\zeta}} + p_t^d \left(A_1 K_{t-1} - I_t\right), \qquad (C.14)$$

using the fact that  $p_t^c \equiv 1$  in the non-durables sector (Appendix C.1).

## C.3 Investment Shocks

We are interested in constructing a sequence of investment shocks  $\{z_t\}$  that produces a particular recession, i.e., a path for aggregate output

$$Y_t^{\text{GDP}} \equiv C_t + X_t + I_t + G + \text{TB}_t \tag{C.15}$$

as defined in Section 5.1. We show below that this sequence of shocks can be constructed in a straightforward way despite the non-linearities inherent to the demand side of our economy. In the following, we let  $C_t(\cdot)$ ,  $X_t(\cdot)$  and  $\text{TB}_t(\cdot)$  denote total demands and the quantity index for the trade balance as a function of households' aggregate income before interest and tax payments { $Y_t^{\text{inc}}$ }.

**Lemma 1.** Consider a sequence of aggregate output  $\{Y_t^{GDP}\}$  that converges to its steady state level  $Y_t^{GDP} \rightarrow Y^{GDP}$  as  $t \rightarrow +\infty$ . There exists a (unique) sequence of investment shocks  $\{z_t\}$  that induces this output in equilibrium. It can be contructed in four steps.

<u>Step 1</u> (Net investments). Fix an initial guess for incomes  $\{Y_t^{inc}\}$ , e.g.,  $Y_t^{inc} = Y^{inc}$  for each period  $t \ge 0$ . Back out the sequence of investments  $\{I_t\}$  residually from the resource constraint

$$I_t \equiv Y_t^{GDP} - C_t \left( \left\{ Y_t^{inc} \right\} \right) - X_t \left( \left\{ Y_t^{inc} \right\} \right) - G - TB_t \left( \left\{ Y_t^{inc} \right\} \right)$$

<u>Step 2</u> (Investment shocks). Fix an initial guess for capital  $\{K_t\}$ , e.g.,  $K_{t-1} = K$  for each period  $t \ge 0$ . Compute the investment rates  $x_t \equiv I_t/K_{t-1}$  using the sequence of investment from the previous step. Back out the sequence of investment shocks  $\{z_t\}$  from the firm's Euler equation

$$z_{t+1} = \frac{\Phi'(x_{t+1})}{\Phi'(x_t)} \frac{1+r_t}{1+\pi_{t+1}} \frac{p_t^d}{p_{t+1}^d} - (A_1 - x_{t+1}) \Phi'(x_{t+1}) - \left\{1 - \delta^K + \Phi(x_{t+1})\right\}$$

with the normalization  $z_0 \equiv 0.90$  Given this sequence of investment rates and investment shocks,

<sup>&</sup>lt;sup>90</sup> We normalize  $z_0 \equiv 0$  because the purpose of these investment shocks is to act as aggregate demand shifters (investment depends on the expected value of  $z_{t+1}$ ) not the initial stock of capital.

compute a new sequence of capital  $\{K'_t\}$  using the law of motion

$$K'_{t} = \left\{1 - \delta^{K} + \Phi\left(x_{t}\right) + z_{t}\right\}K'_{t-1},$$

for each  $t \ge 0$ , with initial condition  $K'_{-1} = K$ . Update the initial guess for capital  $\{K_t\}$  using  $\{K'_t\}$  and repeat Step 2 until convergence. This yiels a sequence of investment shocks  $\{z_t\}$  such that the firm chooses investments  $\{I_t\}$  given equilibrium prices.

<u>Step 3</u> (Incomes and prices). Update incomes, prices, taxes, and the interest rate: households' aggregate income  $Y_t^{inc}$  is given by (5.8) where  $Div_t$  is given by (C.13); prices are computed using equations (5.2), (5.4) and (C.1); taxes are given by the constraints (C.18)–(C.19); and the interest rate satisfies the Taylor rule. Repeat the previous steps until convergence. The resulting sequence of investment shocks  $\{z_t\}$  is the one that implements the sequence of aggregate output  $\{Y_t^{GDP}\}$  in equilibrium.

*Proof.* The sequence of shocks  $\{z_t\}$  induces aggregate outputs  $\{Y_t^{\text{GDP}}\}$  in equilibrium if and only if the following conditions are satisfied: (i) the firm's Euler equation (C.11); (ii) the law of motion of capital (C.12); (iii) incomes, prices and taxes are given by the expressions described in Step 2 above; and (iv) aggregate output satisfies (C.15) with consumption, investment and the trade balance given by  $C_t(\cdot)$ ,  $X_t(\cdot)$  and  $\text{TB}_t(\cdot)$ .<sup>91</sup> The result simply combines these equilibrium conditions.

### C.4 Fiscal Policy

Budget constraint. The government's budget constraint is

$$B_t^g + P_t^c G + \frac{T_t}{1 + \pi_t} = \frac{1 + r_{t-1}}{1 + \pi_t} B_{t-1}^g + \int \left( y Y_t^{\text{inc}} - \psi_{0,t} \left( y Y_t^{\text{inc}} \right)^{1 - \psi_1} \right) d\mu_{t-1} + \Sigma_t \quad (C.16)$$

Instead of introducing passive financial intermediaries, we suppose that the government claims the net payments on credit from households

$$\Sigma_t \equiv (1 - \theta) \times \left\{ \left( 1 + r_{t-1}^b \right) \hat{P}_t^d D_{t-1} - P_{t+1}^d \left( 1 + \pi_{t+1} \right) D_t \right\}$$
(C.17)

The pre-determined stock of durables is  $D_{t-1} \equiv \int d \times \mu_{t-1} (d\mathbf{x})$ . The price  $\hat{P}_t^d$  was defined in Appendix A.1.

 $<sup>\</sup>overline{^{91}}$  Necessity uses the fact that the firm's problem (C.10) is convex.

*Taxes.* The tax intercept is  $\psi_{0,t} = \psi_0 + \Psi_t \hat{\psi}_0$ , where  $\psi_0$  is the intercept at steady state and  $\{\Psi_t\}$  was defined in Appendix C.1. The change  $\hat{\psi}_0$  ensures that the government's tax revenues are equal to its spending in present discounted value. Therefore, the tax intercept is

$$\psi_{0,t} = \psi_0 + \Psi_t \frac{\sum_t Q_t \Omega_t + \frac{1+r_{-1}}{1+\pi_0} B_{-1}^g}{\sum_t Q_t \Psi_t \int (yE_t)^{1-\psi_1} d\mu_{t-1}}$$
(C.18)

where

$$\Omega_t \equiv \int y E_t d\mu_{t-1} - \psi_0 \int (y E_t)^{1-\psi_1} d\mu_{t-1} + \Sigma_t - \frac{T_t}{1+\pi_t} - P_t^c G_t$$
(C.19)

# **D** Additional Quantitative Results

#### Figure D.1: Dynamic responses in our model

Quarterly MPX

Annual MPX



*Notes*: The left panel plots the total MPX over time to a check received in the first quarter. We repeat this experiment for checks of \$500 and \$9,240 (the average lottery gain in Fagereng et al., 2021). The right panel reports the associated annual MPXs.





*Notes*: This figure plots the distribution of MPXs out of \$500 in our model (in red), in the fully statedependent model (in blue), and in the two-asset model of non-durables (in grey).



**Figure D.3:** MPX and liquid assets

*Notes*: The left panel plots the total MPX in the first quarter in our model for each quartile of the distribution of liquid assets. We repeat this experiment for checks of \$500 and \$9,240 (the average lottery gain in Fagereng et al., 2021). The right panel reports the same at the annual frequency.

Figure D.4: State- vs. time-dependent adjustments



*Notes*: The figure plots our state-dependence index SD in (3.2) as a function of the scale parameter  $\eta$ . All other parameters are re-calibrated to match the targets discussed in Section 3.2. The vertical dashed line is our preferred calibration with  $\eta = 0.2$ .

#### Figure D.5: Size-dependence in the MPX in our model (annual)



*Notes*: This figure plots the MPX on durables and non-durables at the annual frequency in our model as a function of the size of stimulus checks.

#### Figure D.6: Sensitivity analysis



*Notes*: This figure plots the MPX on durables (left) and non-durables (right) at the quarterly frequency in our model (red) and in three alternative calibrations with lower liquidity (13% of average annual income instead of 26%), a higher frequency of adjustment (30% instead of 24%), and more down payment ( $\theta = 30\%$  instead of  $\theta = 20\%$ ).



Figure D.7: Extensive and intensive margins

*Notes*: The left panel decomposes the MPX on durables in our model. The solid and dashed curves are the extensive and intensive margins. The dotted curve is the non-linear residual that captures the interaction between these two margins. The right panel is the same for the purely state-dependent model.



Figure D.8: Decomposing the extensive margin in our model

*Notes*: This figure decomposes the extensive margin into the two components in the first term of expression (B.5). The solid curve is the extensive margin. The dashed curve captures the rate at which households adjust  $\{S(d, m + T, y) - S(d, m, y)\} X/T$  where X ensures that the two curves coincide for a check of \$100. By construction, the difference between these two curves captures the selection effect.



Figure D.9: Aggregate conditions (MPX out of \$500)

*Notes*: The left panel plots the MPX out of \$500 in our model at various points of the business cycle. The stimulus checks are received unexpectedly after three quarters of constant expansion (or contraction), following by a linear mean-reversion over eight quarters. The right panel plots the same for the purely state-dependent model.



**Figure D.10:** Aggregate output (t = 0) in closed and open economies

*Notes*: This figure plots aggregate output in the first quarter in deviations from steady state as a function of the size of stimulus checks. The solid curve is our the general equilibrium model presented in Sections 5.1–5.2. The dashed curve is a closed-economy version where households only spend on domestic varieties so  $\alpha_d^F = \alpha_c^F = 0$ .





*Notes*: This figure plots the share of the households' equilibrium spending response out of a \$500 check accounted for by each quartile of the distribution of labor income over the previous year (i.e., the basis of eligibility for checks). The red bars correspond to durables and the blue bars to non-durables.



Figure D.12: Sectoral output gaps with a supply shock

*Notes*: This figure plots the sectoral output gaps over time for various stimulus checks. The black and orange curves correspond to the same models as in Figure 5.3.

# **E** Alternative Models

This appendix describes the two alternative models that we discuss in the paper. Section E.1 presents a two-asset model of non-durables. Section E.2 presents a Calvo-Plus model of durables.

## E.1 Two-Asset Model of Non-Durables

In Sections 3–4, we compare the predictions of our model to those of a two-asset model of non-durables similar to Kaplan and Violante (2022). This appendix states the household's problem recursively and discusses the calibration. Households are indexed by three id-iosyncratic states: their holdings of illiquid financial asset (*b*); their holdings of liquid financial asset (*m*); and their idiosyncratic income (*y*). As before, we let  $\mathbf{x} \equiv (b, m, y)$  denote the vector of states.

*Continuation values*. The continuation values  $\{V_t(\cdot)\}$  can be characterized recursively as follows:<sup>92</sup>

1. Discrete choice. The household chooses whether to adjust its stock of illiquid asset.

<sup>&</sup>lt;sup>92</sup> Again, the terminal condition for  $V_{t+1}(\cdot)$  is the stationary value without stimulus checks.

The value associated to the discrete choice problem is

$$V_{t}(\mathbf{x}) \equiv \max\left\{V_{t}^{\mathrm{adjust}}\left(\mathcal{Y}_{t}(\mathbf{x};T_{t}) + \frac{b}{1+\pi_{t}},y\right) - \kappa,\right.$$
$$V_{t}^{\mathrm{not}}\left(\mathcal{Y}_{t}(\mathbf{x};T_{t}),\frac{b}{1+\pi_{t}},y\right)\right\}$$
(E.1)

where  $V^{\text{adjust}}$  is the continuation value when adjusting the stock of illiquid assets,  $V^{\text{not}}$  is the continuation value when not adjusting, and  $\kappa > 0$  is the adjustment cost. Cash-on-hand after interest payment and stimulus check is

$$\mathcal{Y}_{t}(\mathbf{x};T_{t}) \equiv \psi_{0,t} \left( yY_{t}^{\text{inc}} \right)^{1-\psi_{1}} + \frac{1+r_{t-1}^{m}}{1+\pi_{t}}m + \frac{r_{t-1}^{b}}{1+\pi_{t}}b + \frac{T_{t}}{1+\pi_{t}}$$
(E.2)

where  $Y_t^{\text{inc}}$  is real aggregate income,  $r_{t-1}^m$  is the return on the liquid asset,  $r_{t-1}^b$  is the return on the illiquid asset,  $\pi_t$  is inflation, and  $T_t$  are nominal stimulus checks.

2. **Illiquid asset adjustment.** If the household decides to adjust its stock of illiquid assets, it chooses its new stock of illiquid assets

$$V_t^{\text{adjust}}(m, y) \equiv \max_{b', m'} V_t^{\text{not}}(b', m', y)$$
s.t.  $b' + m' \le m$ ,  $b' \ge 0$ 
(E.3)

The continuation value  $V_t^{\text{not}}$  reflects the subsequent optimal consumption-saving choice that occurs in the same period.

3. **Consumption-saving.** If the household decides not to adjust its stock of illiquid assets, it chooses how much to consume and save in liquid asset

$$V_{t}^{\text{not}}(m, b, y) \equiv \max_{c, m'} u(c) + \beta \int V_{t+1}(b, m', y') \Gamma(dy'; y)$$
(E.4)  
s.t.  $c + m' \le m$ ,  $m' \ge 0$ 

*Calibration*. The calibration strategy follows Kaplan and Violante (2022) closely. We set  $u(c) = 1/(1-\sigma)c^{1-\sigma}$  with inverse elasticity of intertemporal substitution  $\sigma \to 1$ , as is usual in models of non-durable spending. We set the (real) return on cash to -2% per year and the spread on the illiquid asset to 6% per year.<sup>93</sup> We discipline internally

 $<sup>^{93}</sup>$  The real effective lower bound on the interest rate is -2% in our durables model (Section 5.2). In the two-asset model of non-durables, this would imply that the lower bound is binding even in steady state.

two parameters: the discount factor ( $\beta$ ); and the adjustment cost ( $\kappa$ ). We calibrate these parameters to match a share of total hand-to-mouth households of 41%, and a share of wealthy hand-to-mouth (with positive holdings of *b*) of 27% as in Kaplan and Violante (2022).

# E.2 Calvo-Plus Model

Distribution of net investment rates

In Section 3.5, we discussed a Calvo-Plus variant of our model. The figures below report several moments for this model.



#### Figure E.1: Calvo-Plus model

Conditional adjustment probability

*Notes*: The left panel plots the distribution of net investment rates (standardized) across two consecutive PSID waves between which households adjusted their stock of durables. The black curve is the data and the purple bars are the Calvo-Plus model. The right panel plots the adjustment probability conditional on a household not having adjusted so far. The confidence intervals are bootstrapped (10%).

Instead, we assume that monetary policy can decrease the interest rate by 3% in both model before it hits its effective lower bound. Indeed,  $r^m - \underline{r} = 3\%$  in our durables model.



Figure E.2: Size-dependence in the MPX in the Calvo-Plus model

*Notes*: This figure plots the MPX on durables and non-durables in the Calvo-Plus model discussed in Section 3.5, as a function of the size of stimulus checks. The left panel reports the quarterly MPX, while the right panel reports the annual MPX.

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