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Abstract

We examine the relationship between government debt and redistribution in Overlapping Generations Economies (OLG) with heterogeneous agents. The government uses capital and progressive labor taxes for spending, debt, and income redistribution. We show that increasing inequality leads to more progressivity, higher government debt, and higher capital taxation. To achieve optimal redistribution, debt increases substantially. We explore how borrowing limits hinder the government's ability to redistribute income. Calibrating the model to the U.S. (2000-10), we estimate that the optimal debt level should roughly double to respond to rising inequality.

JEL classification: H2. H6.

Keywords: Government debt. Capital taxation. Redistribution. Progressive taxation.

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1 Introduction

The widespread increase in public debt and rising inequality have triggered a heated debate about the need to adjust labor and capital income taxation. However, research addressing redistribution and debt is usually carried over by taking them as independent problems. When dealing with redistribution, the studies generally take the form of large-scale heterogeneous agent models, which complicates the interpretation of their results, either completely abstracting from public debt or setting it at a predetermined level. In turn, when analyzing government debt, the general approach is to resort to simpler representative agent models, which allow for sharper and transparent characterizations. The goal of this paper is to build a bridge between these two approaches, providing clear characterizations in an empirically meaningful framework.

We analyze in a tractable framework the implications of government debt and inequality, and their interaction, for taxation and redistribution; considering not only steady state prescriptions but *also transition paths*. We show that government debt and redistribution are closely linked. Abstracting from one has very important implications for the other, and vice versa. One particular situation in which the relevance of the link becomes evident is when the government must respond to rising labor income inequality. When public debt can be freely adjusted, it is optimal to increase not only the labor income tax progressivity, but also the tax on capital, AND, more importantly, government debt. When the debt's dynamic is muted, the response of the fiscal instruments is dampened, severely restricting the government's ability to redistribute.

The model's tractability provides important advantages. We can readily analyze the consequences of alternative scenarios at a minimal cost. For instance, how different is the policy reaction if the increase in inequality is transitory rather than permanent? How does the initial state of the economy shape optimal policies? How does the presence of large previous commitments (e.g., pension benefits) impact optimal policies? Yet, the tractability gain does not imply a relevant cost in terms of quantitative accuracy. Our setting is able to replicate the same levels of optimal taxation and redistribution as other much richer and complex frameworks in the literature, as long as we restrict the policy space analogously.

In order to clarify these channels, we build a heterogeneous agent Overlapping Generations Economy (OLG). Agents live for two periods and can only work in the first. They are born without financial assets but endowed with a unit of potential working time and log-normally distributed heterogeneous labor productivities. During their working lives, agents can accumulate financial assets, in either productive capital or government debt, that are consumed upon retirement. The government can tax savings (without bounds), labor income, and issue public debt (or assets). To redistribute income, the government has access to a labor income tax function á la Benabou (2002) and Heathcote et al. (2014), so that we can disentangle the progressivity from the average tax.

One of the key features of our economy is that agents are endowed with non-separable balance growth (CES) preferences for consumption and leisure. We do this for three reasons. First, from a theoretical point of view, they are the only preferences consistent with long-term growth. Second, from the empirical point of view, these preferences generate the important and well-documented retirement consumption puzzle, which is cumbersome to replicate with separable preferences.[1] Last but not least, they are instrumental in our characterization, greatly simplifying the problem.

We use the preferences' CES structure, together with the iso-elasticity of the income tax function, to show that the economy exhibits aggregation. This allows us to write each generation's average present value of utility as the simple product of two terms: 1) the utility of a fictitious representative agent, and 2) a factor capturing the planner's *taste for redistribution*, i.e., a transparent equity-efficiency trade-off. Nevertheless, due to the presence of inelastic savings from previous generations, the constrained (Ramsey) planner may be affected by time inconsistency problems, rendering the committed problem not recursive. To address this issue, we introduce the *promised consumption* to the retired population as a state variable. As a result, we can cast the Ramsey problem with commitment as a standard recursive problem. Armed with this simplified setting, we then appeal to standard tools to characterize the solution in a transparent fashion. An important implication of this approach is that the transitional dynamics arise naturally as a byproduct, so that there is no need to resort to special assumptions, complicated algorithms, or extreme computing power to obtain them.[2]

We sharply characterize the optimal labor income tax progressivity and the capital tax as functions of the (endogenous) implementability constraint's Lagrange multiplier. We do so

[1] See Laitner and Silverman (2005), Fisher et al. (2006) andAguiar and Hurst (2007) for evidence regarding the drop in consumption after retirement. In turn, Pijoan-Mas (2006) and Boppart et al. (2024) address the relevance of income-wealth effects on labor supply.

[2] We show that the problem's solution can be closely approximated using a standard tool such as Dynare. The Dynare-generated transitional dynamics do not differ much from the proper global solution, although it exhibits slightly faster convergence. One paper dealing with transitions is Dyrda and Pedroni (2022), which requires a complex numerical approach. For example, Le Grand and Ragot (2023) points to some theoretical issues related to their approach.

in steady state and along transition paths. We first show that our setting satisfies some properties previously found in the literature. As in Heathcote et al. (2017), as the heterogeneity vanishes, the economy converges to the first best. This is possible because the planner can use regressive labor income taxes to mimic the missing lump sum taxes, so that the capital tax is zero. Even when the heterogeneity is reintroduced, and as long as the planner has no limits on its debt decision, we show that with separable preferences, the capital tax is always zero. Thus, in settings with separable preferences, capital taxation arises only when an implicit or explicit constraint on debt accumulation is imposed (Proposition 3).[3].

In contrast, the nonseparable CES preferences have important implications for income tax progressivity, capital taxation, and government debt. It generates a tight link between all the fiscal variables. We first show that the labor income tax progressivity is characterized by the principal branch of a Lambert function of the Lagrange multiplier on the implementability constraint. The larger the multiplier, the lower the progressivity. Since larger spending needs, either due to government expenditures or outstanding debt, lead to a larger multiplier, there is a negative relationship between progressivity and previous fiscal commitments. In turn, this mapping is directly affected by the degree of heterogeneity and the Intertemporal Elasticity of Substitution (IES), which acts in the opposite direction of the Lagrange multiplier.

We show that more inequality leads to sharp increases in progressivity, capital income taxation, and government debt. The reason lies in the *complementarity* between consumption and leisure. When there is complementarity, a reduction in consumption reduces the marginal utility of leisure and thus increases the agent's willingness to work. Hence, when the IES is less than 1, a capital tax reduces both future and current consumption, stimulating labor supply.[4] This effect is particularly useful when the planner must distort the labor supply to raise revenue or redistribute. Increasing the tax on capital reduces the distortive effect of labor income taxation. Although this strategy can generate potential undesirable dynamic distortions, since the economy's total savings are affected, the planner **can always fully compensate for this drawback by appropriately accommodating the level of debt**. Any excess or lack of private savings can be compensated for by opposite movements in public savings.

We calibrate the economy to replicate standard moments usually targeted in the literature.

[3]See Krueger et al. (2021a), Conesa et al. (2009), Peterman (2013)

[4]In Bassetto and Benhabib (2006) and Straub and Werning (2020) the IES<1 also plays an important role, but for different reasons. In their paper, the low elasticity of investment to the tax rate makes any upper bound on taxation potentially binding at all periods. See Piguillem and Schneider (2013) for an extension of these results to an environment with heterogeneous labor productivity.

Our tractable setting replicates the optimal (steady-state) capital and progressive labor taxes previously found in works focusing purely on optimal taxation, e.g. Conesa et al. (2009), while allowing us to analyze the transition. Similarly, when we shutdown the government ability to issue debt, we found an optimal progressivity similar to Heathcote et al. (2017). This is an important feature of our environment. The tractability comes without loss of quantitative relevance, while allowing us to easily analyze a variety of transition paths that are of special relevance nowadays. How can fiscal policy adapt to pay for the current high level of debt? How to respond to the rising inequality? Is the response different if the change in inequality is transitory rather than permanent? To show the power and convenience of our simplified framework, we provide tentative answers to these questions.

We start by analyzing the steady-state relationship between inequality and government debt. In absence of restrictions on government debt, there is a strong positive association between inequality, capital taxation, and government debt. For example, when the Gini coefficient of the labor income is around 0.25 (low inequality), the optimal debt level is 0, while the optimal capital tax is around 10%. Instead, when the Gini coefficient reaches 0.45 (in line with the latest measure), the debt level is above 1.5 times GDP, which is associated with a capital tax above 50%. In this way, the planner can steeply increase the progressivity of the labor tax. It does so by leaving the median agent's marginal tax relatively unaffected, but steeply increasing the 90th percentile's marginal tax from 30% to 60%, and instead of taxing the 10th percentile at 20%, providing her with a marginal subsidy of 20%.

The relevance of the government's asset position begs the question of how different the result would be if increasing debt were not an option. One possibility is the presence of Fiscal Rules. The large sovereign debt positions currently observed in many countries have prompted a wide range of policy proposals aimed at either reducing debt levels or establishing credible bounds on them. For example, the European Union is in the final stages of redefining the Stability and Growth Pact, with the goal of limiting public debt to not more than 90% of GDP. Alternatively, one can think about modeling choices. When analyzing redistributive fiscal policy, it is customary to assume either a budget balance or to set debt at a fixed predetermined level. We find that all responses are dampened. The most affected instrument is capital taxation, which remains contained around 20%, when no debt is allowed, or at most 30%, when the debt limit is 60% of GDP. As a result, the increase in progressivity is also reduced, especially by reducing the average tax of the 90th percentile, which remains below 30%. This shows that achieving large redistribution and low government debt are two opposing objectives. The planner can choose to expand redistribution by accepting more debt or to reduce the level of debt, but then it must accept more inequality.

The previous results arise in steady state, which raises relevant questions: is it optimal to arrive to a steady state? How long would it take? Do the transition costs and benefits affect the results? For this reason, we also analyze the response to a permanent or temporary increase in inequality. We simulate a jump of the labor income Gini coefficient from 0.38 to 0.45, approximately the same as the change observed in the U.S. economy since the 1970s to the present (Heathcote et al., 2010). The economy converges to the new steady state slowly, in approximately six generations, with the new steady state coinciding with the aforementioned one. However, the convergence is non-monotone. There is an initial overshooting of the tax on capital, accompanied by an initial sharp drop in capital accumulation, which then recovers to remain below the new steady state. Importantly, the new equilibrium is achieved by drastically reducing promises to retirees, who see their consumption reduced by 20%. Last but not least, *government debt jumps in the first period to the new steady: upon learning of the shock, the economy immediately becomes a high debt country.*

1.1 Literature

This paper is related to the literature on government debt management with heterogeneous agents and its interaction with redistribution. The important role of debt in OLG economies and its interaction with taxation dates back to Diamond (1965) and Barro (1979) seminal papers. Recently, Lancia et al. (2024) revisited the role of government debt as an intergenerational insurance device, but focusing on its sustainability. Most of this literature was based on representative agent models. Aiyagari and McGrattan (1998) is the seminal paper that introduces heterogeneity as a determinant of public debt, although in their setting debt is instrumental in improving insurance provision. Recently, many contributions have relied on this type of settings to understand the interaction between government debt and income heterogeneity. As Dyrda and Pedroni (2022) who, given a steady state policy in Aiyagari (1995), analyzes the transition path of optimal taxation. In turn, Le Grand and Ragot (2023) provide conditions for the existence of such steady state and the implication for debt accumulation, but argue that the steady state may not exist. In a similar environment, but with a different approach, Auclert et al. (2024) show that under many parameterizations the steady state does not exist. In particular, they show that: "*the only versions of the model that lead to reasonable Ramsey steady states far from immiseration are those with non-balanced growth preferences and no wealth effects on labor supply*". Other recent papers also study government in a Ramsey equilibrium, but focusing on different aspects. Bassetto and Cui (2023) analyzes the role of debt in alleviating financial frictions, which we do not incorporate; and Bhandari et al. (2017b) and Bhandari et al. (2017a) analyze and provide a method to understand the implications of debt distribution, arguing that the optimal debt level is close to zero. We differ from this literature in that we abstract from idiosyncratic risk and focus on long-term type inequality. In addition we pay especial attention to non-separable preferences between consumption and leisure, which we believe are of empirical significance, although overlooked by the literature. Finally, although this type of preferences can be cumbersome, leading to the immiserisation result, the OLG structure allows us to overcome it.

An important reference for analyzing fiscal policy with heterogeneous agents is Werning (2007). He studies an infinite horizon economy with fixed types (as in our setting) and considering the possibility of non-separable preferences. This paper endows the planner with the possibility of using lump-sum taxation, which brings back the Ricardian equivalence. We do not allow lump-sum taxation and focus on an OLG economy which generates an important role for government debt. In addition, our tax function allows us to derive implications for the optimal progressivity of the labor income tax, not just its average values. Our setting is a simplified version of Conesa et al. (2009), but incorporates government debt as choice variable and provides an explicit parameterization of the tax progressivity.[5]

The relevance of government debt in heterogeneous agents economies without idiosyncratic risk for fiscal policy has been studied by Bassetto (2014), which focuses on how the distribution of political power changes the main source of tax revenue. He studies the smoothing properties of government debt in the spirit of Barro (1979), in an economy without capital. In Greulich et al. (2023), in a setting with heterogeneous agents but complete markets, government debt is the key to facilitating the transition to zero capital taxation in the long rung. In this economy, the upper limit on capital taxes plays an important role as in Bassetto and Benhabib (2006) and Straub and Werning (2020). In our setting, we do not impose a limit on capital taxation, but because of the preferences or limits to government debt, the tax on capital is always positive, even in the long run. Moreover, unlike most of the papers in the literature, we studied not only the level of labor taxation but also its progressivity by appealing to a nonlinear tax schedule.

[5]See also Krueger et al. (2021b) for some analytical characterizations and Peterman and Sager (2022) who computes the optimal level of debt in an economy similar to Conesa et al. (2009).

2 Environment

Time is discrete and indefinite $t = 0, 1, 2, ...$ In each period there are two types of agents, young agents indexed by superscript y and old agents indexed by o, from now on we refer to them as the young and old generation. In each period a measure $m_t(e)$ of agents is born, where e represents their endowment of labor. Alternatively, one can interpret e as the productivity of each agent. The density $m_t(e)$ is such that $m_t(e) \geq 0$ for all $e \geq 0$ and $\mathbb{E}_t(e) = 1$. In each period there is a population N_t, which is assumed to grow at a constant rate $1+n = N_t/N_{t-1}$. Thus, the densities must also satisfy $\int_e m_t(e)de = (1+n)\int_e m_{t-1}(e)de$. Agents can only work while young. Thus, if they reach old age, which occurs with probability η, their endowment of labor. While young, agents can consume c, save s, and provide labor l. Retired agents die with probability 1 after one period.

Since young agents are born without assets and die without leaving a bequest, in the second period old agents consume all their income:

As a result, each generation t solves the following problem:

$$c_t^o(e) = R_{t+1}s_t(e)$$

where R_t is the gross after-tax return on savings. When young, the agents internalize this and solve:

$$\begin{aligned} &\max_{s_t, l_t, c_t^y, c_t^o} \{u(c_t^y, l_t) + \beta\eta u(c_t^o)\} \\ s.t. \quad & c_t^y + s_t = w_t l_t e - T(w_t l_t e) + Tr_t \end{aligned} \tag{1}$$

where $T(w_t l_t e)$ denotes the income tax paid by the workers and Tr_t represents an initial transfer. Following Heathcote et al. (2014), we assume that after-tax labor income satisfies:

$$y_t(e) = w_t l_t e - T(w_t l_t e) = \chi(w_t e l_t)^{1-\rho}$$

Thus, from now on, we will work directly with the disposable income. Then, the household's first-order necessary condition is:

$$-u_l(c_t^y, l_t) = u_c(c_t^y, l_t)(1-\rho)\chi(w_t e)^{1-\rho} l_t^{-\rho} \tag{2}$$

$$u_c(c_t^y, l_t) = \beta\eta R_{t+1} u_c(c_t^o) \tag{3}$$

This condition determines the saving function, which is denoted $s_{t+1} = \psi(w_t, R_{t+1})$. As usual, $\psi_w(w, R) > 0$. But $\psi_R(w, R)$ is ambiguous. It is the point of Straub and Werning (2020).

Firms and returns. There is a constant return to scale production function $f(l, k)$ that uses capital and labor to produce the only good of the economy. Firms are profit maximizers. In a given period t, they hire young workers at a wage w_t and rent capital from old agents at a rental rate of R. The firm optimization problem implies:

$$\begin{aligned} f_l(k_t, l_t) &= w_t && (4) \\ f_k(k_t, l_t) &= 1 + r_t^f. && (5) \end{aligned}$$

where r_t^f is the net return on capital before taxes, k_t is the average stock of capital, and $\delta \in (0, 1]$ is the capital depreciation rate. Since capital can be taxed at rate τ_t the gross after tax return on capital is:[6]

$$R_t = 1 - \delta + (1 - \tau_t) r_t^f \tag{6}$$

Moreover, because both capital and government debt are risk-free assets, by arbitrage it must be that both assets have the same return: $R_t^b = R_t$ for all t.

Market clearing and feasibility. Any equilibrium allocation must satisfy four additional conditions: Asset's market clearing, good's market clearing, accidental bequests equal to inheritance, and the government budget constraint.

The government uses the revenue from capital and labor income, plus the issuance of new debt to pay previous debt, and allocates the remaining resources to an exogenous sequence of government spending $\{g_t\}_{t=0}^{\infty}$. The government budget constraint is satisfied if

$$g_t + R_t b_t = \tau_t R_t k_t + \int_e T(w_t l_t(e) e) m_t(e) de + b_{t+1}$$

All agents who die leave unintended bequests that are rebated to the new born, we denote these accidental bequests by Tr_t, which in equilibrium satisfy:

$$Tr_t = (1 - \eta) R_t \int_e m_t(e) s_t(e) de$$

[6] We consider alternative specifications where only the undepreciated returns on capital can be taxed: $R_t = 1 + (1 - \tau_t)(r_t^f - \delta)$.

Households allocate their savings to capital and government debt:

$$k_{t+1} + b_{t+1} = \int_e m_t(e) s_t(e) de \tag{7}$$

Finally, since the aggregate *effective* labor supply is $L_t = \int_e m(e) e l_t(e) de$ and the production function exhibits constant returns to scale it follows that the aggregate production is $f(L_t, k_t)$. Therefore, any feasible allocation must satisfy:

$$N_t c_t^y + \eta N_{t-1} c_{t-1}^o + k_{t+1} + g_t = f(k_t, L_t) + (1-\delta) k_t \tag{8}$$

The choice of tax function is instrumental in the characterization of the equilibrium. Since it is homothetic in income, it would preserve the homotheticity of the problem when preferences are also homothetic. To show this, we star with the following assumption that we then use to prove Proposition 1.

Assumption 1 *For all t and that preferences are:*

$$\begin{aligned} u(c^y, l) &= \frac{\left[(c^y)^\gamma (1-l)^{(1-\gamma)}\right]^{1-\sigma}}{1-\sigma} \\ u(c^o) &= \frac{(c^o)^{(1-\sigma)\gamma}}{1-\sigma} \end{aligned}$$

Assumption 2 *e is distributed Log-normal, i.e., $e \sim LN(\mu_e, \sigma_e)$, with standard deviation σ_e and $\mu_e = -\sigma_e/2$.*

The assumption $\mu_e = -\sigma_e^2/2$ makes sure that $\mathbb{E}(e) = 1$, and thus only enforces the normalization. Note that preferences when old are a special case of preferences when young if $l = 0$. In addition, the intertemporal elasticity of substitution is $\frac{1}{1-(1-\sigma)\gamma}$ rather than $\frac{1}{\sigma}$ as with standard CRRA preferences. The non-separability between consumption and leisure is important for three motives. First, it is well documented that upon retirement there is a sizable drop in consumption, which is difficult to rationalize with separable preferences. With Assumption 1 the drop in consumption arises naturally. Because there is a large increase in leisure after retirement, consumption must drop discontinuously to *smooth out* life-time utility. Second, we show in Proposition 2 that with CRRA separable preferences and unconstrained debt the tax on capital is zero in every period $t \geq 1$, while with non-separability the tax on capital can be either positive or negative. Finally, this assumption together with

Assumption 2 are instrumental allowing us to show that the economy exhibits aggregation, leading to the following proposition:

Proposition 1 *(Aggregation). Suppose Assumptions 1 and 2 hold and $\eta = n_t = 1$. Then, there exist market's weights $\omega(e, \rho)$ such that:*

a) For all t and all e:

$$l_t(e) = l_t = L_t; \qquad c_t^y(e) = \omega(e, \rho_t) c_t^y; \qquad c_t^o(e) = \omega(e, \rho_t) c_t^o$$

b) Individual life-time utilities are multiplicative in the average and individual components:

$$\begin{aligned} V(e, c_t^y, c_t^o, l_t) &= \omega(e, \rho_t)^{\gamma(1-\sigma)} \left[u(c_t^y, l_t) + \eta\beta u(c_t^o)\right] \\ &= \omega(e, \rho_t)^{\gamma(1-\sigma)} V(c_t^y, c_t^o, l_t) \end{aligned} \tag{9}$$

c) The market's weights satisfy:

$$\omega(e, \rho_t) = \frac{e^{1-\rho_t}}{\mathbb{E}[e^{1-\rho_t}]} \tag{10}$$

Proof: See Appendix A.

Proposition 1 is the key to reducing the dimensionality of the problem. Rather than solving for uncountable many allocations $\{c_t^y(e), c_t^o(e), l_t(e)\}_{\forall e}$ in each period, we only need to solve for the aggregate allocations $\{c_t^y, c_t^o, l_t\}$, which do not affect market weights. This does not mean that distribution of consumption is independent of fiscal policy; instead, equation (10) makes clear that $\omega(e, \rho)$ is fully determined by the progressivity of the labor income tax. When $\rho = 1$ progressivity is at its peak, which eliminates all equilibrium heterogeneity, since then $\omega(e, 1) = 1$ for all e. It is also interesting to compare with the situation in which $\rho = 0$. This is analogous to a linear labor income tax, also termed a *flat tax* in the policy literature. This implies that the optimal progressivity of the tax system would be determined mainly by equation (10). Notice that the aggregation results survive in the presence of heterogeneous returns. In particular, the individual labor supply still equals the aggregate $l_t(e, z) = l_t = L_t$. See Appendix E.2 for more details.

To analyze the optimal policy, we follow the classical Ramsey approach, which from now on we call the Ramsey problem. In a nutshell, instead of optimizing over the fiscal policy vector $\pi = \{\tau, b, \chi, \rho\}$, we look for allocations $\{c_t^y(\theta), c_t^o(\theta), b_{t+1}(\theta), k_{t+1}(\theta)\}$ that can be implemented as a competitive equilibrium with some fiscal policy vector π. We assume that the planner maximizes the present value of utility of the currently alive and all future generations. To do so, it discounts future generations at a rate θ. This intergenerational weighting factor does not need to coincide with the individual time discount factor β. However, we will occasionally assume that $\theta = \beta$, emphasizing that this is not important for our results.

Finally, assume that there is an initial distribution of assets $\{k_0^i, b_0^i\}$, with distribution $m_0(i)$, for each initially old agent i. Then, the problem is:

Lemma 1 *(Ramsey problem). Assumptions 1 and 2 hold. Then,*

Given $\{k_0^i, b_0^i\}$, the optimal fiscal policy solves:

$$\max_{\{k_{t+1}, l_t, c_t^y, c_t^o, \rho_t, \tau_0\}} \frac{\int u(c_{-1}^{o,i}) m_0(i) di}{\theta} + \sum_{t=0}^{\infty} \theta^t \int_0^{\infty} \omega(e, \rho_t)^{\gamma(1-\sigma)} V(c_t^y, c_t^o, l_t) m(e) de$$

$$s.t. \quad \gamma u(c_t, l_t) + \gamma \eta \beta u(c_t^o) = (1-\gamma) \frac{u(c_t, l) l_t}{(1-l_t)(1-\rho_t)}; \tag{IC}$$

$$c_t^y + \eta \frac{c_{t-1}^o}{1+n} + (1+n) k_{t+1} = f(k_t, l_t) + (1-\delta) k_t; \tag{FE}$$

$$c_{-1}^{o,i} = R(0)(k_0^i + b_0^i) \quad \forall i \tag{IC0}$$

Proof: See Appendix A.3.

The (IC) constraint is the standard *Implementability Constraint*, making sure that the budget constraint for each agent is satisfied, and thus the problem's solution can be implemented as a competitive equilibrium. For the initial generation, since labor and savings decisions have already been made, it reduces to (IC0). The second constraint (FE) is the *feasibility constraint* ensuring that the allocations are feasible. Because of Walras's Law, (IC) and (FE) imply that the government's budget constraint is also satisfied.

Notice that the planner can indirectly choose the initial old generation consumption $c_{-1}^o = \int c_{-1}^{o,i} m_0(i) di$. This is equivalent to endowing it with the possibility of defaulting on the inherited debt or to set an arbitrarily high tax on savings to pay for it, potentially rendering the debt problem irrelevant. Moreover, the planner is committed to future policies. It chooses a path of taxes and debt once and for all that can never be revised. As is clear comparing (IC) and (IC0), this creates a time inconsistency problem, since if the planner had the opportunity to re-optimize at period t it would do so incorporating that then c^o_{t-1} must satisfy (IC0) rather than (IC). As a result, the problem is not recursive.

Alternatively, we can assume that the planner must also commit to respect the expected consumption by the retired agents, even a period zero. This generates consistency across periods, but requires the introduction of an additional state variable. To simplify notation, let $x_t = c^o_{t-1}$ be the promised consumption to the previous generation, which will enter as an additional state variable that the planner must keep track of, and define: [7]

$$\Omega(\rho) = \int_0^\infty \omega(e,\rho)^{\gamma(1-\sigma)} m(e) de$$

In Lemma 1 we also show that by assuming a one-period commitment, the problem can be written as:

$$v(x,k) = \max_{\{l,c,\rho,k',x'\}} \left\{ \Omega(\rho)[u(c,l) + \eta\beta u(x')] + \theta v(x',k') \right\} \tag{PR}$$

subject to;

$$\begin{aligned} c + \eta\frac{x}{1+n} + (1+n)k' &\leq f(k,l) + (1-\delta)k \\ u(c,l) + \eta\beta u(x') &= \frac{(1-\gamma)l}{(1-l)\gamma}\frac{u(c,l)}{(1-\rho)} \end{aligned}$$

The last simplified version of the problem is instrumental in its characterization and highlights the main trade-offs. Note that the planner flow payoff has two components, 1) the utility of the average agent $u(c,l) + \eta\beta u(x')$ and 2) the welfare lost due to the remaining inequality $\Omega(\rho)$. We call this second component the planner's *taste for redistribution*, not because it reflects an intrinsic preference, but because it summarizes how inequality shapes the effective objective of the planner. When the planner chooses fiscal policy, it must balance the distortionary cost of taxation — captured by the utility of the representative agent— with the reduction in welfare losses from inequality, captured by its "taste for redistribution." Note also how ρ is helpful in relaxing the constraint (IC). For example, if $\Omega(\rho)$ were absent from the objective function, by the appropriate choice of ρ any allocation could be implementable, and thus the (IC) constraint would never be binding: the first-best allocation would be the solution. In the next section, we derive the set of conditions under which this is possible.

[7]See Marcet and Marimon (2019) for a reference on how the introduction of additional state variables is instrumental in rendering a problem recursive.

In addition, the presence of the promised consumption to the previous generation lends itself to interesting economic interpretations which we will develop in Section 5.2. As is clear from (PR), these promises reduce the amount or resources available to allocate. One can think of it as situations in which the economy has arrived at by previous borrowing decisions or perhaps due to generous pension systems.

Discussion on commitment and recursive setting. There are two elements that make it possible to write the problem in a recursive way: 1) the two-period OLG structure and 2) the fact that the planner can only choose the next-period capital tax. In this regard, the promised consumption to retired agents is the key. Given the expected capital tax, young agents' savings decisions imply an expected future consumption. In the next period, this consumption becomes a state variable for the planner, which enforces its implementation. As a result, it is as if the planner could fully commit to each future generations. If we were to incorporate more ages, we would need to incorporate additional state variables. For example, in a three-period OLG economy we would need to add a state variable keeping track of the promises to the middle age agents, in addition to the promises to old agents. Furthermore, since the planner cannot alter the consumption of savers, there is no incentive to manipulate asset prices or to confiscate through the labor taxation.

Finally, one may wonder about the time zero problem and the temptation to confiscate initial savings. There are two ways to interpret this under the prism of Problem (PR). To replicate exactly the outcome of the time-zero problem, using the recursive approach, we would need to include an ex ante maximization as follows:

$$\max_{\{\tau_0,l,c,x,\rho,k',x'\}} \left\{ \frac{\int_i u(c_{-1}^{o,i}) m_0(i) di}{\theta} + \Omega(\rho)[u(c,l) + \eta\beta u(x')] + \theta v(x',k') \right\}$$

subject to;

$$\begin{aligned} c + \eta\frac{x}{1+n} + (1+n)k' &\leq f(k,l) + (1-\delta)k \\ u(c,l) + \eta\beta u(x') &= \frac{(1-\gamma)l}{(1-l)\gamma}\frac{u(c,l)}{(1-\rho)} \\ x &= \int_i c_{-1}^{o,i} m_0(i) di \\ c_{-1}^{o,i} &= R(0)[k_0^i + b_0^i] \end{aligned}$$

Note that this time-zero problem consist in just choosing a initial state variable x. This choice would be determined not only by the average holdings of financial assets but also by its distribution, which does not need to coincide with m. From this point of view, the time zero problem is not different from (PR), it just provides some discipline to either calibrate it or interpret it. For this reason, from now on we would focus on Problem (PR), taking the initial x as given, but bearing in mind that it is affected by multiple factors, including the previously promised capital tax and the distribution of initial assets.

Asset's constraint. To be able to compare our results with the previous literature and understand how frictions on public borrowing can affect the results, we add exogenous limits to government debt.[8] We assume that in each period t the government debt must be contained in a set $[-\bar{b}, \bar{b}]f(k,l)$, for $\bar{b} \geq 0$. For example, setting $\bar{b} = 0$ generates results comparable to those in settings in which the government must balance the budget every period.

To operationalize these bounds in terms of allocations only, we use the fact that by equation (7) in equilibrium $s_t = k_{t+1} + b_{t+1}$. Hence, using the budget constraint and equation (2) together with Proposition 1, the government debt can be expressed as:

$$b_{t+1} = \int_0^\infty s_t(e)m(e)de - k_{t+1} = -k_{t+1} - c_t^y - \frac{u_l(c_t^y, l_t)}{u_c(c_t^y, l_t)} \frac{l_t}{(1-\rho_t)} \tag{11}$$

Equation (11) also shows how we pin down government debt when the equilibrium is decentralized. To solve a problem, we debt limits we just need to add to (PR) the constraint:

$$-\bar{b} \leq -k' - c - \frac{u_l(c,l)}{u_c(c,l)} \frac{l}{(1-\rho)} \leq \bar{b}$$

This equation highlights the tight link between debt limits and the progressivity of the labor income tax ρ. To shed light on the effect on capital taxation, note that in equilibrium the consumption of retired agents is $c_t^o = R_{t+1}(k_{t+1} + b_{t+1})$, thus substituting R_t by the Euler equation (3)

$$c_t^o = \frac{u_c(c_t^y, l_t)}{\eta \beta u_c(c_t^o)} [k_{t+1} + b_{t+1}]$$

[8]It is customary in the literature to analyze equilibria without government debt or with a constant level of it, see for instance Conesa et al. (2009), Krueger et al. (2021a), Ferriere et al. (2023).

Imposing that $-\bar{b}f(k_t, l_t) \leq b_{t+1} \leq \bar{b}f(k_t, l_t)$, the above equation generates:

$$-\bar{b} + (1+n)k_{t+1} \leq \eta\beta \frac{c_t^o u_c(c_t^o)}{u_c(c_t^y, l_t)} \leq \bar{b} + (1+n)k_{t+1} \tag{12}$$

Hence, debt limits impose constraints on the Euler equation that can eventually translate into capital taxation, and in turn, on the other fiscal choices.

3 Optimal progressivity and capital taxation.

In the next section, we solve the model numerically. Nevertheless, in this section we broadly characterize the solution in terms of Lagrange multipliers. We start by analyzing the case without debt limits and derive the main implications for capital taxation and labor income tax progressivity. We argue that the tax system does not need to be progressive, but it is so if the inequality is sufficiently high, and that there is a tight link between progressivity, capital taxation and government debt; concluding that **more inequality optimally leads not only to a more progressive labor income tax but also to a larger tax on capital and more government debt**.

Let $\mu = \mu(x, k)$ be the equilibrium multiplier attached to the (IC) constraint, then we have:

Proposition 2 *Suppose the assumptions of Proposition 1 are satisfied, then the optimal labor income tax progressivity and the marginal tax on capital income are characterized by:*

$$1 - \rho = \sqrt{\frac{W_0\left(-\frac{\mu}{2}\right)}{\frac{\sigma_e^2}{2}\gamma(1-\sigma)[\gamma(1-\sigma)-1]}} \tag{13}$$

$$\tau^{k\prime} = \frac{[1-\delta+f_k(k', l')]}{f_k(k', l')\eta}\left[\eta - \frac{\Omega(\rho)+\mu}{\Omega(\rho)+\mu\left(1-\frac{\Psi(l)}{(1-\rho)}\right)}\right] \tag{14}$$

where $W_0(\cdot)$ is the principal branch of the Lambert function, and $\Psi(l) = \frac{(1-\gamma)}{\gamma}\frac{l}{1-l}$.[9]

[9]The Lambert function is the implicitly solution to the following equation:

$$xe^x = y$$

When x is restricted to be a positive real number, the solution for each y is unique and denoted by $W_0(y)$.

Proof: See Appendix B.

Despite their dependence on endogenous variables, Equations (13) and (14) provide sharp insights about the economic mechanisms shaping the optimal policies. To see this, bear in mind that the function $W_0(\cdot)$ satisfies $W_0(0) = 0$ and is strictly increasing. From (13) this implies that whenever $\mu = 0$, it must be that $\rho = 1$. Intuitively, if labor taxes do not generate any distortion, or the (IC) constraint is not binding, it is optimal to fully redistribute income. Clearly, this would not be true in general, with the distortive impact preventing complete equalization of consumption. In turn, from (14) it follows that this policy would be accompanied by tax on capital proportional to $(\eta - 1)/\eta$. Thus, if $\eta = 1$, there is zero capital taxation, while whenever $\eta < 1$ it is optimal to subsidize savings. This happens because of the presence of an uninsured mortality risk. Agents do not internalize that their saved resources are valuable to the surviving population and thus tend to undersave. If agents had access to perfect annuity markets, this effect would disappear and the capital tax would be zero even when $\eta < 1$. This tendency to subsidize savings is an important force that remains throughout our analysis.

Note also from equation (13) that as $\sigma_e \to 0$, ρ appears to become a large negative number, approaching minus infinity if μ remains finite. In Appendix B.1 we show that in fact μ converges to zero and ρ remains finite but negative. In a representative agent economy, the planner uses a regressive labor income tax and avoids capital taxation (see equation (14)). The intuition for this result is the same as in Heathcote et al. (2017). When there is no heterogeneity, the equity-efficiency trade-off vanishes, then the planner appeals to a regressive tax system to mimic the missing lump-sum tax. As a result, it can implement the first-best allocation, even when some revenues must be raised to pay spending or outstanding debt. As in Diamond (1965), any discrepancy between the socially desired level of capital and the implied by the competitive equilibrium can be resolved appealing to the government debt. From equation (14) it follows that the tax on capital would again depend on the survival probability, leading to negative capital taxation when there is some uninsured survival risk.

The response of progressivity to an increase in inequality, σ_e, is not straightforward because μ could also be increasing in σ_e. It is evident from (13), though, that the first-order effect (keeping μ constant) of increasing inequality is an increase in the progressivity of the labor income tax system. This effect can be compensated for by changes in μ. However, in Section 5.1 (see Figure 1) we show numerically that this indirect effect is not strong enough to overcome the first-order impact and that as a result more inequality leads to more progressivity.

How inequality impacts capital taxation depends on the interaction between consumption and leisure. For simplicity, assume $\eta = 1$, bearing in mind that departures from it would lead to lower capital taxation. Note the factor $\Psi(l)$ involved in equation (14). This term appears because the preferences are non separable.[10] One can think about it as $\Psi(l) = 0$ when the utility function is separable in consumption and leisure. In that case, the tax on capital is always zero, for any level of inequality and in all states of the economy. This is also true for any discrepancy between the private and social discount factors β and θ. Indeed, in Appendix D we show that in steady state the optimal capital-labor ratio satisfies:

$$\frac{\theta}{1+n}\left[f_k'(k,l) + (1-\delta)\right] = 1 \tag{15}$$

stressing that the capital-labor ratio is determined by the growth-adjusted social discount factor $\theta/(1+n)$, which can be substantially different from private discounting. This discrepancy is often argued to be a driving force leading to capital taxation, as in Atkeson et al. (1999). How can the planner ensure that the private sector chooses the right level capital without resorting to capital taxation? The answer is straightforward: by choosing the appropriate level of government debt. Private agents are only concerned with their total savings, which by the market clearing condition (7) is composed of both physical capital and government bonds. Given any level of total savings, the planner pins down the optimal level of capital by choosing the right b. This also points to the relevance of debt management for optimal taxation in OLG economies. Any restriction on the position of government financial assets would translate into capital taxation.[11]

In contrast, when the utility is non-separable, the term $\Psi(l) > 0$ plays an important role, implying that the intertemporal allocations must be distorted. To better understand the relationship between τ and redistribution, in Appendix B we show that when $\eta = 1$ equation (14) can be rewritten as:

$$\tau^{k\prime} = \frac{[1-\delta + f_k(k', l')]}{f_k(k', l')}\left[\frac{2(1-\rho)\phi(\sigma_e^2)\Psi(l)}{1 - 2(1-\rho)\phi(\sigma_e^2))\left[1-\rho-\Psi(l)\right]}\right] \tag{16}$$

[10] Even in with our preferences, when $\sigma = 1$ this factor vanishes.

[11] Whether restrictions on public debt implies positive or negative capital taxation would depend on the parameters of the model economy. Even when $\beta = \theta$, the fact that the planner has a different planning horizon (infinite) from the private agents (finite) and that the agents have only so many periods to accumulate assets could imply more, the same, or less equilibrium capital accumulation than the desired by the planner. Thus, the planner could need positive or negative debt accumulation to achieve its desired outcome, and thus positive or negative capital taxation when its debt choice is restricted. See Section 3.2 for a detailed discussion.

where $\phi(\sigma_e^2) = \frac{\sigma_e^2}{2}\gamma(1-\sigma)[\gamma(1-\sigma)-1] \geq 0$ if $\sigma \geq 1$.

Some comments about the last equation are worth mentioning. First, the optimal policy prescribes a *future* tax on capital, there is no implication about the current one. This is due to the commitment assumption. The planner is not allowed to default on previous promises. The capital income tax in the current period is fully determined by past promises. In Appendix B we also show that the denominator of (16) is always positive. Hence, if $\sigma > 1$, and thus $\phi(\sigma_e^2) > 0$, **the capital income tax is in general strictly positive**.[12]

The reason for positive capital taxation lies in the complementarity between consumption and leisure. When $\sigma > 1$, a reduction in current consumption reduces the marginal utility of leisure, and thus, it increases the agents willingness to work. Because of the low intertemporal elasticity of substitution, a tax on capital reduces both future and current consumption, and so it also stimulates the labor supply. This effect is particularly useful when the planner must distort the labor supply to raise revenue or redistribute. By increasing the tax on capital, it reduces the distortive effect of the labor income taxation. Although this strategy can generate potential undesirable dynamic distortions, the planner **can always fully compensate for this effect by setting the right debt level**. As a result, all fiscal variables are closely linked.

Equation (16) also shows a clear relationship between labor and capital taxation. Since the relationship is shaped by other endogenous objects, it is difficult to characterize. Nevertheless, it provides some intuition to some results that we present in the quantitative analysis. Since more inequality (larger σ_e) can affect the value of ρ, its impact on capital taxation is unclear. On the one hand, holding everything else constant, more progressivity directly implies lower capital taxation and vice versa. This can be seen because keeping l and k', $\frac{\partial \tau^k}{\partial \rho} < 0$. *Whenever the tax on capital increases, the progressivity of the tax system must be reduced.* The intuition lies on the fact that both policies are driven by the need to distort the economy: μ. As μ increases in absolute value, there is a greater need for efficiency. Thus, redistribution becomes more "expensive", and therefore progressivity is reduced.

On the other hand, **an increase in inequality leads to more capital taxation**, and to compensate for its distortionary effect, **also to more government debt**. One can easily see from (16) that, again keeping everything else constant, $\frac{\partial \tau^k}{\partial \sigma_e} > 0$, an increase in inequality has a direct positive effect on capital taxation. This happens because the planner wants to

[12] In the numerical exercise we include the survival probability η, which creates the possibility of negative capital taxation. This happens because of the lack of annuity markets: the planner has an incentive to stimulate savings through subsidies to correct the uninsured survival risk.

raise more resources for redistribution purposes, so μ increases in absolute value. Of course, for the same reason, ρ also increases, counteracting the direct effect. However, in Section 5 we show numerically that the distortionary direct effect dominates and, thus, the capital income tax is increasing in inequality. In turn, to preserve the optimal level of capital, the planner must compensate for the desired lower private savings with its own savings decision; thus, the government debt also increases.

3.1 Relation of government debt with wedges

To emphasize the relationship between all the fiscal variables, we derive a steady state relationship between all optimal wedges (taxes) and government debt. To do this, we start by defining the average labor wedge. That is, the implicit average "tax" τ^L such that, in aggregate:

$$-u_l(c,l) = (1-\tau^L)u_c(c,l)f_l(k,l)$$

If we were analyzing a representative agent economy, τ^L would be analogous to a linear labor income tax. In our heterogeneous agent economy and as long as $\rho \neq 0$, every agent faces different marginal taxes. Hence, for comparison we define the average marginal distortion, which is determined by a combination of ρ and the location parameter χ in the tax function.[13] In Appendix B, equation 35, we provide a sharper characterization of it. Armed with this definition, we can state:

Lemma 2 *(Debt and Wedges). Suppose Assumptions 1 and 2 hold. Then, in steady state:*

$$\frac{b}{k} + 1 = \frac{\frac{1-\theta(1-\delta)}{\alpha}\left[1 - \bar{g} - \frac{(1-\tau^L)}{1-\rho}(1-\alpha)\right] - \theta\delta}{\eta(1-\tau^k(1-\theta)) - \theta} \tag{17}$$

$$\frac{1}{\tau^L} = \frac{-l(1-\sigma)(1-\gamma)}{\tau^k(1-\theta(1-\delta))\eta - (\eta-1)} + 1 \tag{18}$$

Proof: See Appendix B.3.

This lemma provides a structure for discussing the relationship among optimal policy choices. It is not a full characterization since it links endogenous variables with other endogenous variables. For this reason, to analyze how government debt responds to exogenous

[13] If $\rho = 0$ at the optimal, then $\chi = 1 - \tau^L$.

changes, we rely on the quantitative analysis of Section 5. Nevertheless, since equations (17) and (18) show clearly that when one fiscal tool moves the others must necessarily accommodate satisfying that restriction. In this sense, it provides valuable insights.

First, notice from equation (17) that it is not clear whether accumulation of debt or government assets ($b < 0$) is optimal. To understand its determinants, a useful benchmark is the homogeneous agent case without government spending ($\bar{g} = 0$). Since in this case all wedges are zero, equation (17) generates $\frac{b}{k} = \frac{1-\eta}{\eta-\theta}$. In economies in which the planner weights future generations at a faster pace than in the one in which agents survive to old age, it is optimal to accumulate assets. When the inequality is reversed, it is optimal for the government to issue debt. In an economy with heterogeneity or government spending, this tendency survives, but it is affected by the other policy choices.

Whenever the fiscal wedges are positive, they would interact among them and affect the debt level. In this regard, equation (18) is informative, generating an unambiguous positive monotone relationship between the capital tax and the average labor wedge, as long as $\sigma > 1$. If it is optimal to tax labor income, it is also optimal to tax capital income and vice versa. Then, both wedges would affect the level of debt.

To understand how, consider the case in which debt ($b > 0$) is optimal in equilibrium. This happens when θ is sufficiently small and requires both the numerator and denominator on the right-hand side of (17) to be positive. Then it is immediate that, keeping ρ constant, a larger tax on capital must be accompanied by more government debt (recall that τ^L must also be larger). Depending on the reasons leading to higher capital taxation, the progressivity of the labor tax system can go up or down. In particular, if the larger capital tax is due to an increase in inequality, ρ would also be larger, weakening the relationship. Nevertheless, in the quantitative section we show that, as long as $\theta < \eta$, the direct effect of τ^k and τ^L dominates, generating a clear positive relation between capital taxation and government debt.

3.2 The impact of debt constraints

The previous section shows that the ability to adjust government is an important element in implementing the optimal policy. In this section, we analyze the consequences of the constraints on government accumulation reintroducing (12). To this end, we show the results with the special case in which the utility is logarithmic. This is instructive because it is not only an special case of our preferences, but it also stresses that the impact of the constraints also holds with separable preferences. However, note that a priori it is not clear whether the upper or lower bound on b are binding. Depending on the calibration, any of the inequalities might bind. In the next section's baseline calibration, it is optimal to have a debt-to-GDP ratio of 60%. Hence, if we assume that $\bar{b} < 0.6$, the upper bound constraint becomes biding. Nevertheless, we present the result under any possibility.

Let $\underline{\nu}$ be the Lagrange multiplier on the left inequality of (12) and $\bar{\nu}$ the Lagrange multiplier on the right inequality, then we have:

Proposition 3 *(Constrained Debt). Suppose $\sigma = 1$, Assumption 2 hold and that debt is constrained by equation 12, then*

$$\tau^k = \left[\frac{\eta - 1}{\eta} + (\underline{\nu} - \bar{\nu})\frac{(1+\beta)}{\lambda}\right]\frac{(1+n)}{\theta}\frac{1}{f_k(k,l)} \tag{19}$$

Proof: See Appendix C.

To understand the implications of equation (19), it is instructive to consider alternative parameterizations. Suppose first that $\eta = 1$, so that agents survive for sure to old age and that the planner is not constrained in its debt choice, i.e $\underline{\nu} = \bar{\nu} = 0$. Then, the tax on capital is zero. Independently of the discrepancy between the social and private discount factors, it is not optimal tax capital accumulation. Instead, when $\eta < 1$, maintaining $\underline{\nu} = \bar{\nu} = 0$, equation (19) implies a *capital subsidy* for any level of heterogeneity. The reason for this is the same as in Section 3: due to the uninsured survival risk, the planner introduces a savings subsidy to replicate missing annuity markets.

When the accumulation constraints bind, the tax on capital can be affected in any direction depending on which constraint binds. Since only one of them can bind at the same time, it must be that either $\underline{\nu} > 0$ and $\bar{\nu} = 0$ or $\underline{\nu} = 0$ and $\bar{\nu} > 0$. When $\bar{\nu} > 0$, so that the government would like to issue more debt but is not allowed, the constraint translates into an even larger subsidy. This happens when the planner assesses that the private sector is not saving enough. Ideally, it would stimulate savings by issuing more debt, which in turn would become private savings. If it is not allowed to do so, the planner stimulates savings by directly subsidizing it. If instead $\underline{\nu} > 0$ it is optimal to eventually tax capital accumulation (depending on the value of η). In this case private savings are too high, hence the planner would like to accumulate assets (negative b), which in turn is a private liability. If it is limited in how much it can do so, the planner would directly tax savings.

The direction of these effects is true independently of the parameter values, as long as the constraints are binding. Of course, the calibration plays an important role in determining which constraint is binding. The larger θ with respect to β, the more likely $\underline{\nu}$ is positive and therefore the more likely that taxing capital is optimal. The opposite is true when θ is "small" with respect to β.

It is important to bear in mind these effects for two reasons. First, it could be that governments maybe be limited by exogenous reasons, for instance fiscal rules may limit the ability of governments to correct the level of private savings. If it were optimal to accumulate assets, but the constraint is on debt levels, there would be no impact on other fiscal variables. But if the government finds it optimal to accumulate large quantities of debt, it might have to adapt the tax policy. In our calibration, this is the case. Second, it is customary to analyze optimal redistribution through taxation abstracting from government debt, by assuming a period by period budget balanced. This is *de facto* a constraint in the choice set, which a priori is hard to evaluate if it is leading to lower or higher capital taxation. Depending on the calibration, the lower or the upper bound constraint could be binding.[14]

Finally, as we discussed in Section 3.1. The ability to redistribute income is closely linked to capital taxation. The distortions generated by increases in the progressivity of the labor income tax can be attenuated by larger capital taxation. However, if the tax on capital must be used to target potential discrepancies between socially desired and private savings, its flexibility is reduced, hindering the ability to tax labor income. As a result, a tighter debt constraint implies less progressive taxation.

4 Calibration

Table 1 presents the calibrated parameters. Some of them are borrowed from previous studies (Panel A), while others are calibrated to replicate relevant moments (Panel B). To match the target moments, we solve the competitive equilibrium given the observed tax policy. We follow Heathcote et al. (2010) and Heathcote et al. (2014) to calibrate, survival risk, income heterogeneity, and the shape of the labor income tax. In this way, we can replicate the observed Gini coefficient and the progressivity of the tax system. Similarly, following Conesa et al. (2009), we set the capital income tax to $\tau_0^k = 0.36$. Of course, in the next section, when we solve the Ramsey model, the whole tax system is a choice variable, and it may differ. Government spending is exogenous and is assumed to be equal to 15% of GDP. As is

[14]Assuming that the level of debt is different from zero, but still constant at a predetermined level has an analogous effect to a budget balanced. It is the possibility of adjusting the government debt that is beneficial for the planner, not its level.

standard in the literature, we set $\alpha = 0.36$, which is the average capital income share.

Given the previous parameters, we target the following moments: 1) average working hours of $l = 1/3$, 2) capital-to-output ratio of $\frac{k}{y} = 3$, 3) consumption drop after retirement at $\frac{c^o}{c^y} = 0.902 = \frac{x}{c}$, as in Bernheim et al. (2001), 4) the debt-to-GDP ratio $\frac{b}{y} = 0.6$, and 5) intertemporal elasticity of substitution of 1/2. Most targeted moments are standard and uncontroversial. The debt-to-GDP ratio might appear low compared to the levels reached after the great recession and the COVID pandemic, but since we are calibrating the economy to a "steady state" before them, it is in line with the observations through the 1970s-90s. We explore the case of low fertility, since the population growth is assumed to be 0 and low mortality, since the survival probability is assumed to be close to 1. Nevertheless, in Appendix F.1 we consider an alternative calibration incorporating only market debt and higher population growth, i.e. a low promises-high fertility scenario.

With this moments-targeting set we obtain $\{\beta, \gamma, \sigma, \delta, \theta\}$. To understand which parameters are the main determinants of the moments, we briefly discuss them here. First, note that level of capital is completely determined by the average hours worked and the capital to output ratio, since : $k = l/(\frac{f(k,l)}{k})^{1/(1-\alpha)}$. Thus, the return on capital becomes a function only of depreciation: $R = 1 - \delta + (1 - \tau_0)\, f_k\,(k, l) = 1 - \delta + (1 - \tau_0)\, \alpha \frac{f(k,l)}{k}$. Then, using the resource constraint we can obtain the young consumption to capital ratio, also as a function of depreciation only δ:

$$\frac{c}{k} = \left(\frac{f(k,l)(1-\bar{g})}{k} - \delta - n \right) / \left(1 + \frac{\eta}{1+n} \frac{x}{c} \right)$$

In turn, from the household budget constraint we can obtain the debt to capital ratio again as a function δ only:

$$\frac{b}{k} = \beta\eta\frac{c}{k} - (1+n) = \frac{x}{c}\frac{1}{R}\frac{c}{k} - (1+n)$$

Hence, multiplying the last by k/y, which is a targeted moment, we obtain the debt-to-GDP ratio as a function of δ only.

Next, notice that the intratemporal first-order necessary condition immediately generates a mapping from the targeted moments and the parameters previously set to the leisure share γ:

$$\frac{1-\gamma}{\gamma} = \frac{1}{cl^{\rho}}(1-\rho)\chi(w)^{1-\rho}E[e^{1-\rho_t}]$$

Once γ has been set, the intertemporal elasticity of substitution is straightforward since with the chosen preferences it is equal to $\frac{1}{1-\gamma(1-\sigma)}$. We choose σ to make this last ratio equal to 0.5. The individual discount factor β is calibrated to replicate the retirement consumption drop. Notice that by the Euler equation, given the after-tax return on savings, γ and σ, the discount factor β must satisfy:

$$\beta\eta = \left(\frac{x}{c}\right)^{1-\gamma(1-\sigma)} (1-l)^{(1-\gamma)(1-\sigma)} \frac{1}{R}$$

Finally, we must choose the social discount factor. This parameter is important in many papers to determine the capital tax, especially if there is an implicit or explicit bound on debt. As we show in Section 3, both the progressivity and the capital tax are independent of it when preferences are separable, as long as the government can freely choose the debt level. Thus, except when we analyze the limits on government borrowing, different θ's would only translate on different levels of debt and capital. We can think about the observed policies as arising from a political equilibrium, which would not be affected by an alternative tax system. Then, we set the intergenerational discount factor to be consistent with the chosen level of capital to output $\frac{\theta}{(1+n)}[1-\delta+f_k(k,l)] = 1$. Alternatively, we could set $\theta = \beta(1+n)$ as in Erosa and Gervais (2002). This approach does not change the optimal tax schedules, it only implies an unrealistic annualized capital to output ratio of 17.

5 Quantitative results

In this section, we compare the optimal policy with and without debt constraint across different steady states. More importantly, we study the responses of optimal policy to transitory and permanent shocks. We first characterize the optimal level of debt and tax instruments in steady state for the different values of the cross-sectional variance of labor productivity σ_e. Second, we study transitions towards the steady state from an arbitrary initial levels of debt and capital. Independently of the initial debt level, the economy converges to the unique steady state, which is characterized by a positive capital tax, high labor tax progressivity, and a positive debt level. The transition towards steady state is very slow. During the transition, policy instruments vary widely.

[15] The total federal liabilities is 240 % in 2021 and less than 100 % in 1995 `https://www.fiscal.treasury.gov/reports-statements/financial-report/mda-unsustainable-fiscal-path.html`

Table 1: Calibrated Parameters

Parameter	Target	Value	Source
Panel A: Directly Set Parameters			
Period length	25 to 60 years	35.00	Own choice
Population growth n	0.00	0.00	U.S. Census data
Survival probability η^y	0.996	0.87	Heathcote et al. (2014)
Heterogeneity σ^e	$GINI = 0.33$	0.60	Heathcote et al. (2010)
Progressivity ρ_0		0.18	Heathcote et al. (2014)
Capital tax τ_0		0.36	Conesa et al. (2009)
Government expenditure	$G_0/Y_0 = 0.20$	0.20	NIPA Tables
Cobb-Douglas α	0.36	0.36	NIPA Tables
Panel B: Calibrated Parameters			
Risk aversion σ	$IES = 0.50$	3.98	NIPA Tables
Consumption share γ	$l = 0.33$	0.34	NIPA Tables
Private discounting β	$c^o/c^y = 0.86$	0.70	Bernheim et al. (2001)
Social discounting θ	$K/Y = 4.4$	0.27	Flow of Funds
Risk aversion σ	$IES = 0.5$	3.98	Standard
Consumption share γ	$l = 0.33333$	0.34	Standard
Depreciation δ	$B_0/Y_0 = 0.6$	0.14	Flow of Funds
Tax shifter χ_0	government budget	0.59	Implied

Notes: All target values are annualized. The last column presents calibrated values. We start with directly set parameters. The length of a period is assumed to be 35 years, which corresponds to the length of the working life from 25 to 60 years. We assume low fertility ($n = 0$) and aging population with expected life expectancy of 90 years ($60 + \eta * 35 = 90$). The share of government expenditure 15% and the labor share of 0.36 are set to standard values (See for example Conesa et al. (2009) We set η to match the annualized probability of surviving of 0.996 from age 25 to age 60 for US men (Heathcote et al., 2014). We set the initial values for the labor tax function ρ_0 and χ_0 following Heathcote et al. (2014) and the initial value for capital tax following Conesa et al. (2009). We now turn to calibrated parameters. The individual discounting β is calibrated to target the drop of consumption in retirement $\frac{c^o}{c^y}$ is from Battistin et al. (2009), who find that, on average, their composite expenditure measure falls by 9.8 percent in retirement. Social discounting θ targets the capital to output ratio, which is assumed to be 3, The preference parameters σ and γ are calibrated to match the intertemporal elasticity of substitution, which is assumed to be 0.5, average hours, which is assumed to be 1/3. The depreciation is calibrated to match the debt to GDP ratio of 200%.[15]All target values are standard (See for example Conesa et al. (2009)).

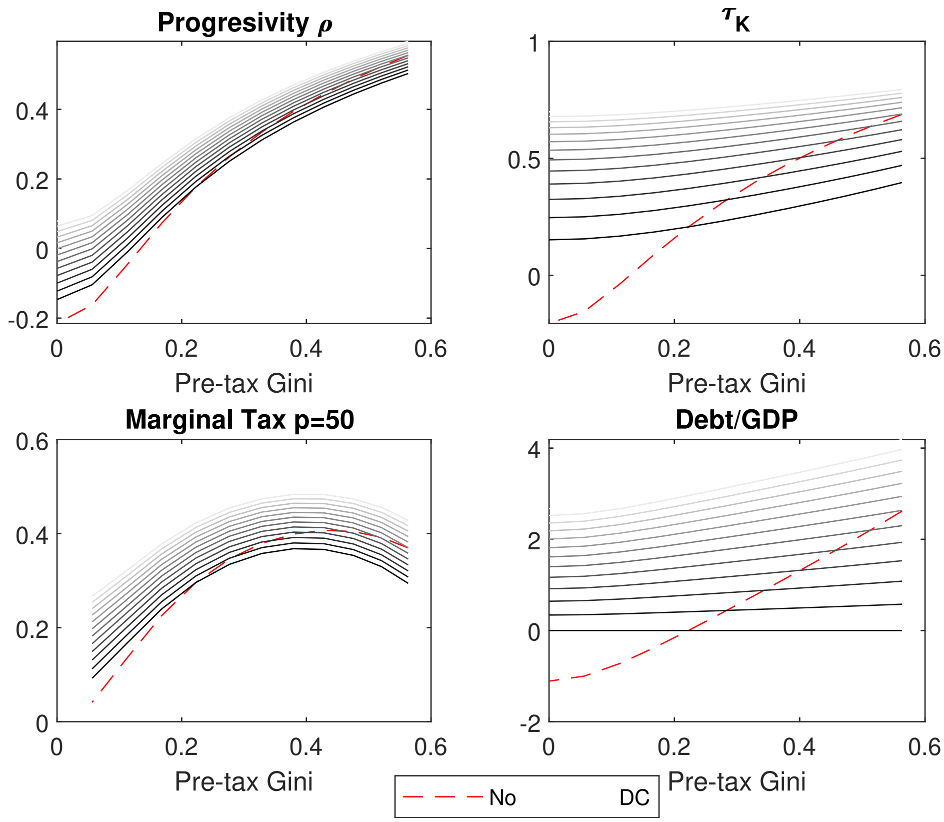
5.1 Steady state

We start by comparing the optimal policies at steady state for different values of the pre-tax labor earnings' Gini coefficient. In Appendix D we present the characterization of the solution for the steady-state outcome. We show how in steady state the system of equations characterizing the optimal allocations can be reduced to a system of two equations in two unknowns. Then, in Appendix E.1 we show, under special parameterization which is close to our baseline calibration ($\sigma = 2$ and $\gamma = 3$) that the steady state is unique. Finally, our numerical simulations show that, under all the initial conditions with which we experimented, the solutions converge to these theoretical characterizations[16].

Figure 1 plots four fiscal instruments: 1) labor tax progressivity ρ, 2) capital tax τ^k, 3) the marginal labor tax for the median worker, and, 4) the debt to capital ratio; all against pretax

[16]Note the these simulations do not require to assume the existence of such steady state, a problem present in Dyrda and Pedroni (2022).

Figure 1: Optimal Policies for Different Debt Limits



Notes: Red line is steady state without debt constraint. Gray lines represent different levels of debt limits.

Gini index. To understand the role of government debt, we also plot the analogous optimal policies that would arise if the planner were constrained to keep debt at a predetermined value. The dashed red lines correspond to the unrestricted policy discussed in Section 3, and the solid gray lines correspond to those that arise in a debt-constrained economy, discussed in Section 3.2. The lighter the gray line, the lower the level of the debt constraint.

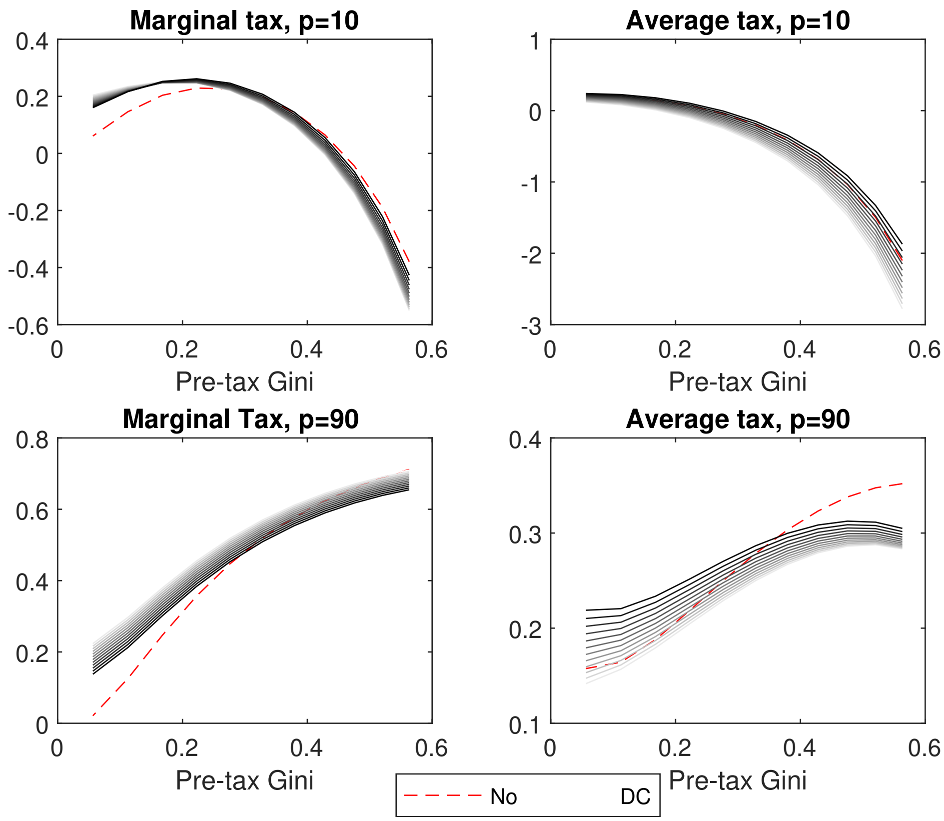
The baseline calibration corresponds to the red shaded line at a Gini coefficient equal to 0.33, which delivers a debt-to-GDP ratio of 0.8. There are several important patterns arising in Figure 1. First, in an economy without heterogeneity (Gini$=0$) the labor income tax is regressive, the government accumulates assets, and the capital tax is negative. The reason for the negative τ^k is the presence of uninsured survival risk. If we set $\eta = 1$, the capital tax would be zero. In this case, the economy achieves the first best by using the labor tax regressivity to replace the missing lump sum taxation.

Second, as inequality widens, all policy instruments increase sharply: progressivity, capital tax, and debt-to-capital ratio. With the baseline calibration, the capital tax is around 45%, the progressivity parameter ρ is above 0.3, higher than the observed 0.18. This progressivity implies a large marginal earnings tax for the median worker ($p = 50$ denotes the fiftieth percentile), around 40%. However, it is interesting that the relationship is not monotone. Further increases in inequality lead to reductions in this marginal tax rather than increases.

To understand the patterns in income taxation, Figure 2 explores what happens with marginal and average taxes on the upper and lower ends of the distribution. There we plot the average and marginal tax rates for tenth ($p = 10$) and ninetieth ($p = 90$) percentiles. As inequality increases, while the tax burden for the top earners is continuously increasing, reaching European levels around 50%, the tax burden for the bottom 10% is falling, and these agents become net recipients when the Gini coefficient is around 0.3. This is in line with recent estimates of transfers in the U.S. economy (Ferriere et al., 2023). We can see that the decreasing section of the marginal tax for the tenth percentile appears earlier than for the median worker. When inequality is at a low level, an increase in inequality requires larger marginal tax rates over most of the distribution. However, when the inequality is already at a high level, further increases in inequality call for larger marginal tax rates in the top of the distribution, larger transfers (rendering the average tax negative at the lower end), and even reductions on marginal tax rates at bottom of the distribution.

Finally, the important role played by the government debt becomes evident observing the gray solid lines. Here, two patterns are relevant: the flattening of optimal policies and the large impact on the optimal tax on capital. Recall the discussion in Section 3. The tax on capital is instrumental in reducing the distortionary cost of progressive labor income taxation. By using it, the planner may create unwanted distortion on capital accumulation, but this cost is easily overcome by accommodating its own savings (government debt) and, through the market clearing condition, pining down the right amount of capital. If the planner is not allowed to use the debt policy in this way is less willing to increase the distortionary effects of taxation, hence the curves flattening. Why does it translate mostly into different capital income tax rates? In OLG economies, individual savings decisions and planner savings decisions rarely coincide (even when $\beta = \theta$), which is due to different planning horizons and life cycle patterns. Again, as long as the government can freely choose its asset position, this is not problematic, but if this is not possible, the planner has no choice but to set the capital income tax to pin down the optimal savings' level, hence the large translations in the capital

Figure 2: Labor Taxes for Different Debt Limits



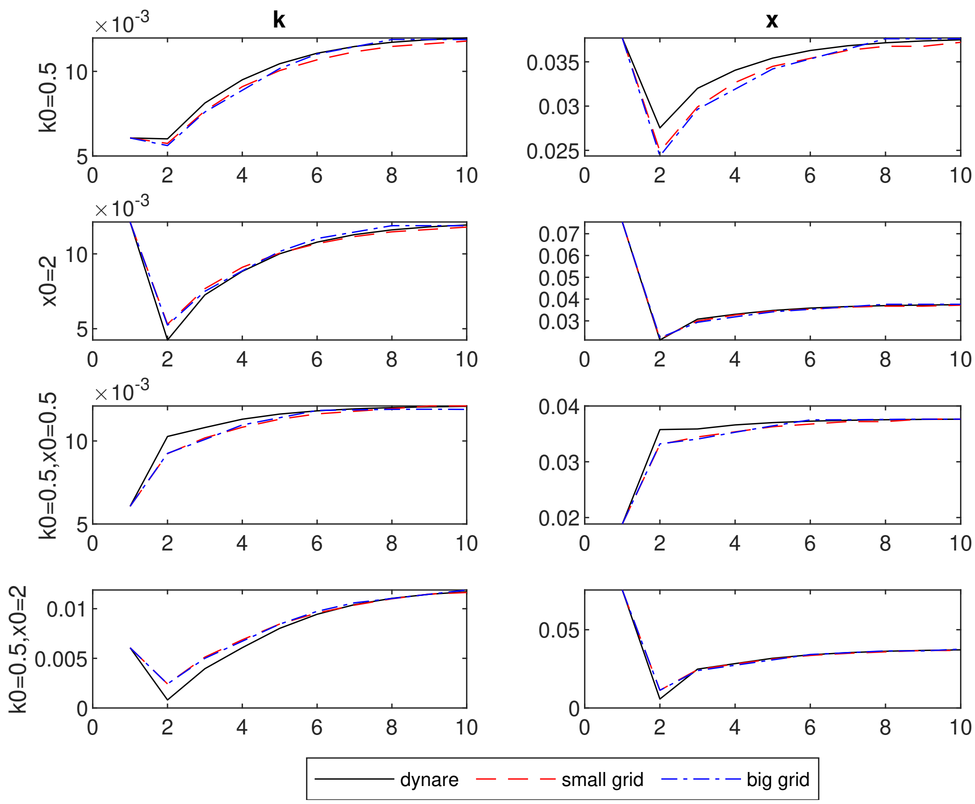
Notes: Red line is steady state without debt constraint. Black line represents the steady state for different debt limits.

tax curves.

5.2 Transitions

In this section, we analyze the transitional dynamics, focusing on the unconstrained economy. It is well known that in the infinite-horizon setup the steady-state Ramsey solution may rely on initial conditions, specifically the initial level of government debt (see, for instance, Chari and Kehoe (1999)). The purpose of this section is threefold. First, we show that in our OLG setting, the long-term values of government debt and taxes are well defined and independent of the initial debt level (and the initial capital stock). Second, we develop a global solution method (see Appendix G) that does not rely on the first-order necessary conditions. We do so by using value function iteration maximizing over grids. This allows us to verify that the analytical characterization, based on first-order conditions, is accurate and that the steady-state solution method is correct as well. Third, transitions are instrumental in providing answers about the speed and the way to converge from any initial condition to the long-term optimal policy. Finally, since the numerical method is specific to our environment, we compare the solutions with those arising from using the standard log-linearization method around the steady state.

Figure 3: Global and approximated solutions



Notes: Black solid line is the Dynare solution. The red dashed line represents the global solution on a small grid (80 points for capital and 90 points for the promised consumption x). The blue dashed dotted line represents the global solution on a larger grid (215 points for capital and 225 points for the promised consumption x). The figure contains two columns and four rows. The left column plots the values of capital (k), the right column plots the promised consumption of the current old (x). The rows represents four different initial conditions.

Figure 3 plots the global solution on small and large grids against the approximated solutions generated by Dynare.[17] The solid black line represents the Dynare solution. The

[17]Dynare (see Adjemian et al. (2022)) is in general unable to solve for the steady state equilibrium, which is necessary for the solution method to work. Thus, we feed the software with the presolved steady-state solutions.

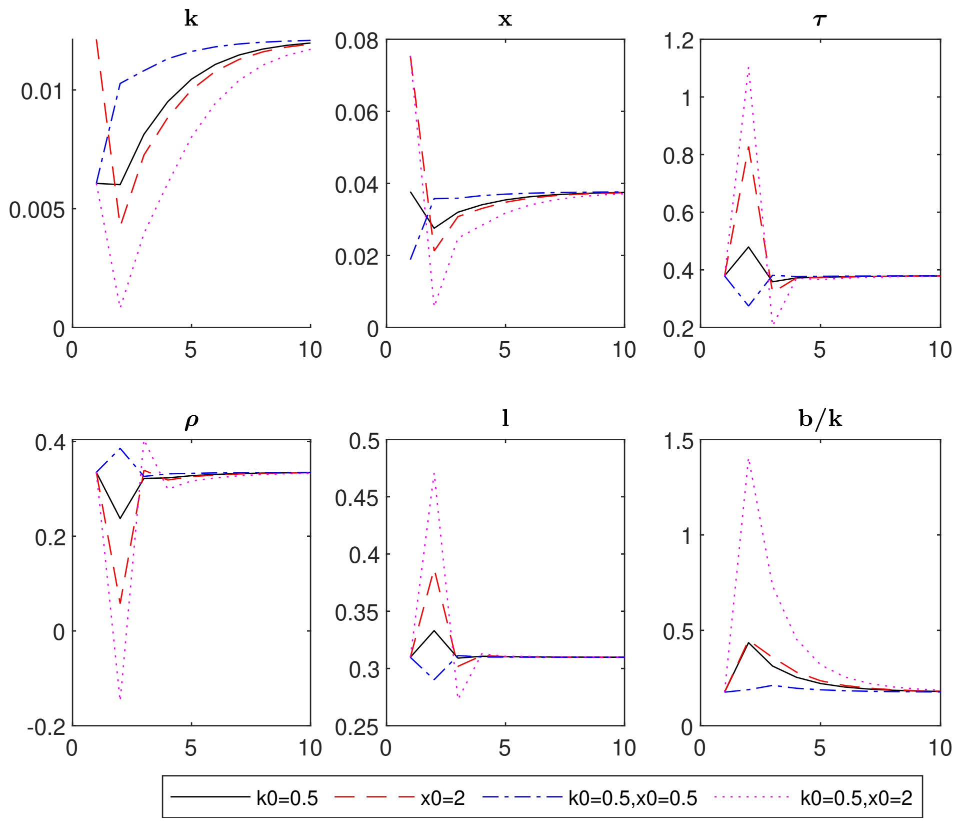
red dashed line represents the global solution on a small grid. The blue dashed dotted line represents the global solution on a finer grid. The figure contains two columns and four rows. The left column plots the values of capital (k), the right column the consumption promised to the current old (x). Each row shows that transition paths for sufficiently distant combination of the initial conditions, both above and below the steady state, to capture relevant effects due to nonlinearities.

There are two main takeaways from Figure 3. First, independently of the initial conditions, the economy always converges to the steady state characterized in Appendix D. Thus, although assume balanced growth preferences, the immiseration result pointed out by Auclert et al. (2024), among others, does not arise in our setting. The convergence is slow, taking between 8 and 10 generations to reach the steady-state equilibrium. Second, Dynare does a remarkable job reproducing the transitional dynamics, even when the initial states are far from its stationary point. This is important because it makes the handling of the model readily available to (and verifiable by) anyone with minimal knowledge on this platform. It also allows us to add complexity to the problem to analyze more complex issues, such as shocks to inequality, which we do in Sections 6.1 and 6.3.

Since both solution methods generate almost identical transition paths, from now on we present to Dynare solutions which can be easily replicated. Figure 4 plots the values of the capital-to-output ratio (k/y), the promised consumption of the current old (x), the capital tax (τ^k), progressivity (ρ), the labor supply (l) and the debt-to-capital ratio (b/k) under four scenarios. First, the solid black line represents the case of a low initial level of capital $k_0 = k_{ss}/2$. Second, the dashed red line captures the case of high promises (high debt or liabilities) $x_0 = 2 \times x_{ss}$. The dashed red line captures the case of high promises (high debt) $x_0 = 2 \times x_{ss}$. Third, the dashed dotted blue line presents the case of a low initial level of capital $k_0 = k_{ss}/2$ and small promises (low debt) $x_0 = x_{ss}/2$. Fourth, the dotted magenta line captures the case of a low initial level of capital $k_0 = k_{ss}/2$ and large promises (high debt) $x_0 = 2 \times x_{ss}$. We chose these large deviations (although not implausible) from the steady state to emphasize the convergence to the unique steady state of Section 5.1, and to appreciate the wide implied variations on optimal policies.

As noted in the previous figure, transitions toward steady state are not fast. It takes up to 8 generations for the main state variables to settle down. In all of the cases, the dynamic system converges to the same steady state. Along the transition paths, there are large movements on all the variables, sometimes nonmonotonically, except when both promises and

Figure 4: Convergence to Steady States



Notes: The solid black line represents the case of a low initial level of capital $k_0 = k_{ss}/2$. The dashed red line captures the case of high promises (high debt) $x_0 = 2 * x_{ss}$. The dashed dotted blue line presents the case of a low initial level of capital $k_0 = k_{ss}/2$ and small promises (low debt) $x_0 = 2 * x_{ss}$. The dotted magenta line captures the case of a low initial level of capital $k_0 = k_{ss}/2$ and large promises (high debt) $x_0 = 2 * x_{ss}$. k/y is capital to output ratio. x is the consumption of the old. τ is the capital tax. ρ is the progressivity of the labor tax. l labor choice. b/k is the debt to capital ratio.

capital are low. A case of particular interest is when the promises to previous generations are high. We can think of it as an initial situation in which the economy has large market debt and/or liabilities, and the planner must decide how to pay for it until it converges to the new optimal steady state. Looking at the red dashed (the capital stock is already at steady state level) and the dotted magenta line (the economy is also below the stationary level of capital), we can see that **the reduction in government liabilities is paid with a substantial increase in capital taxation and a steep decline in labor tax progressivity**.

This promise-paying policy is costly in terms of output. It achieves the goal of initially stimulating the labor supply to generate resources, but due to the high (transitory) levels of the tax on capital and government debt, the economy transitions by a long period of substantially reduced capital stock. Note also the nonmonotonicities. Although the government debt is monotonically decreasing until the new steady state, the changes on the progressivity and tax on capital are reverted at the transition's final stages: ρ converges from above (more progressivity than steady state) and τ converges from below (lower capital tax than in steady state). This pattern is reflected in a sudden and sharp drop in future promises, from substantially above steady state to sharply below. Although this pattern appears harsh, it is indicative of a dilemma that many developed economies are facing nowadays: they find themselves with sizable past promises, a stagnated population, and the economy below trend.

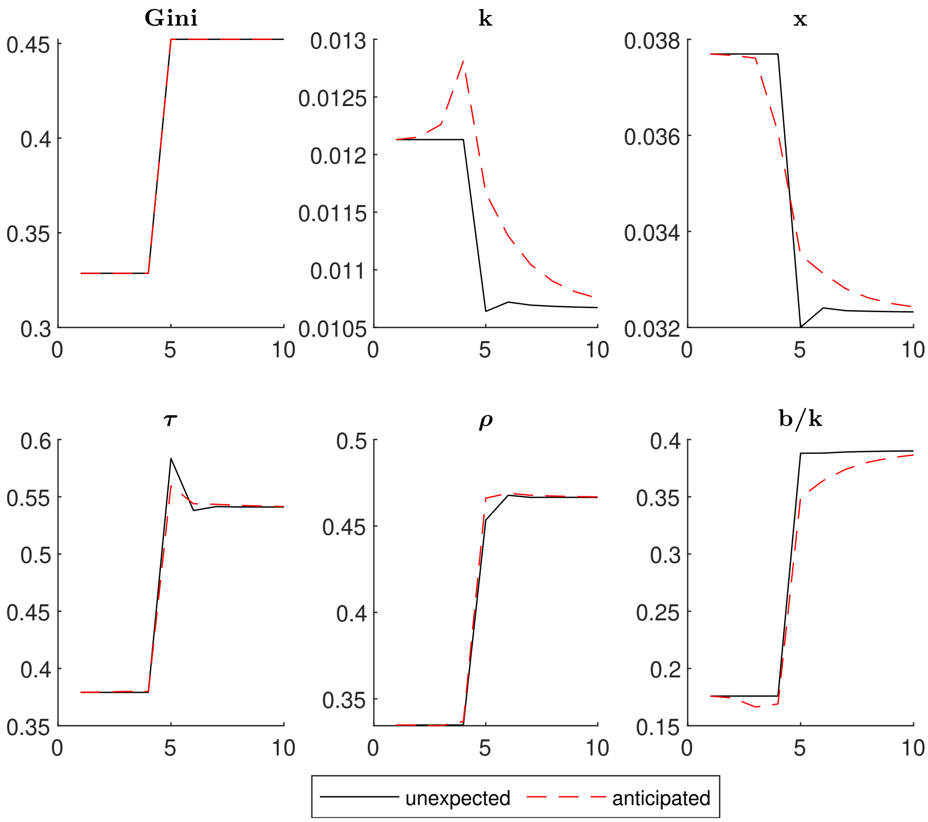
6 Optimal Policy with Rising Inequality

6.1 A permanent change in inequality

In this subsection, we analyze the impulse responses of a shock to inequality. In the last decade or so, there has been an important debate about the consequences of widening inequality and the implications for fiscal policy. There is a large and growing literature addressing this issue, which we think is lacking a comprehensive analysis in a setting in which all the policy instruments can change simultaneously. In addition, it is relevant to analyze the potential differential responses depending on the characteristics of the shock. Does it matter if the shock is permanent or transitory? Are there relevant ex-ante responses when the shock is anticipated? In this regard, the equivalence between our global solution and the linearized approach is instrumental because it allows us to readily appeal to pre-built and well-understood tools to analyze these issues. As in the previous section, we use the built-in routines in Dynare.

We start by analyzing the impulse response to a permanent increase in the Gini coefficient. To this end, we shock the labor earnings variance to replicate an increase in the Gini coefficient from 0.33 to 0.45, which corresponds to the observed change in the U.S. economy from the 1970s to the 2010s (Heathcote et al., 2010). We compare two cases: an unanticipated permanent shock with an anticipated one four periods ahead. Figure 5 presents the actual shock to Gini and the responses of capital k, the promised consumption to the old x, the tax on capital τ^k, labor tax progressivity ρ, and the debt-to-capital ratio b/k. In response to a permanent increase of inequality, the tax on capital, the progressivity and the debt level rises permanently. The increase is striking, with the capital tax reaching almost 55%, the progressivity almost doubling, implying a marginal tax rate in the 90th percentile of around 60%

Figure 5: Unanticipated vs anticipated permanent shock



Notes: We compare unanticipated to anticipated shock which permanently shifts the Gini from 0.32 to 0.45: unanticipated (the solid black line); anticipated (the dashed red line). Gini is Gini coefficient of labor income, which corresponds to the cross-sectional variance of labor productivity σ_e; k is capital; x is the consumption of the old; τ is the capital tax; ρ is the progressivity of the labor tax; b/k is the debt-to-capital ratio.

(see Figure 2). More importantly, this policy is accompanied by drastic increase in government debt, again doubling its previous level and an substantial reduction on the promises to retired agents. This finding is consistent with policies in many countries, especially European ones, that have exhibit a large increase in government debt and started reforms reducing the benefits to al future generation. This form of intergenerational insurance is consistent with Lancia et al. (2024).

If the shock is anticipated, the patterns are similar, with small differences. We can see that anticipation helps smooth out the effect of the shock. The main mechanism operates through a reduction in promises to the old agents. Immediately after learning about the (future) arrival of the shock, the planner begins to reduce the promised consumption to retirees. As a result, when the shock arrives, the decline in consumption for the old is less pronounced, and convergence to the new steady state takes much longer—up to ten generations. This is achieved through a less pronounced increase in capital taxation and an earlier buildup of government debt, leading to a non-monotonic but smoother path for capital accumulation. This exercise is instructive, as it provides insights into possible strategies for addressing widening inequality, which is expected to continue increasing in the future.

6.2 The role of government debt

The previous results rely heavily on a sharp increase in government debt. What happens when government debt cannot be adjusted? A natural starting point is to assume that the government maintains a balanced budget. This could represent, in reality, countries that, for institutional or cultural reasons, are averse to debt accumulation. It is also useful for comparison with other studies in the literature, where budget balance is either implicitly or explicitly assumed. Another alternative is to assume that, although some outstanding debt exists, the government cannot increase it further. For instance, one might consider that an underlying (and not explicitly modeled) default risk prevents the planner from accumulating additional debt.

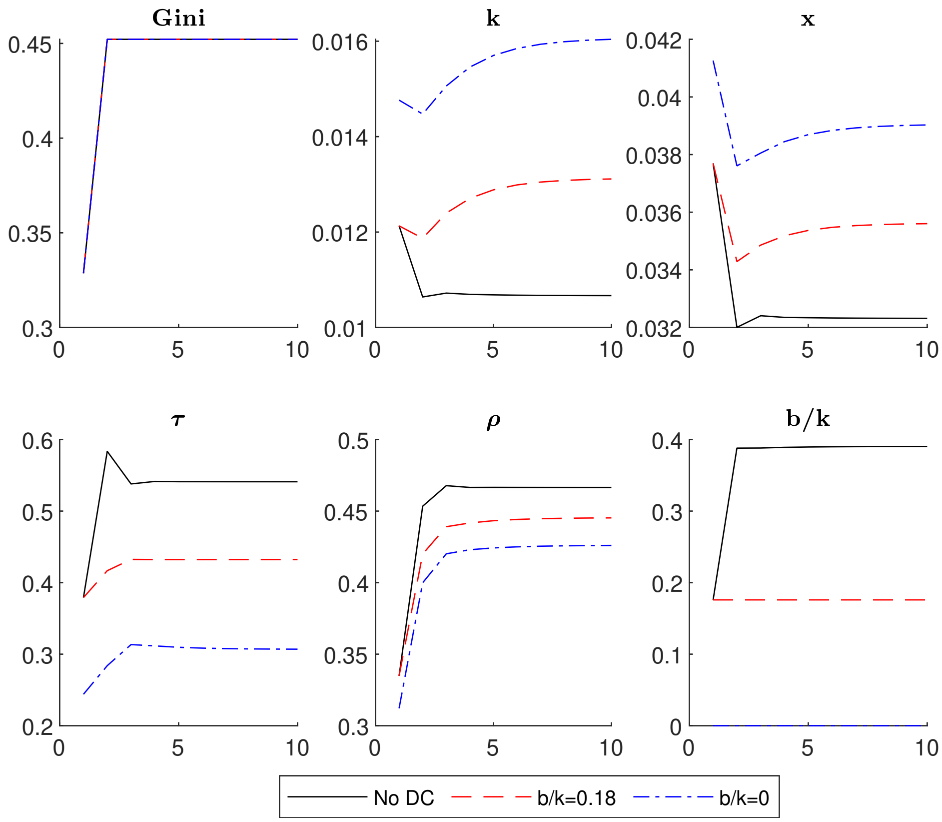
In Figure 6, we compare outcomes under three cases: no debt constraint (black continuous line), debt level fixed at the pre-shock value (red dashed line), and balanced budget (blue dashed-dotted line).

The first notable observation is that when debt is constrained, the response of promised future consumption is milder. In all cases, there is a non-monotonic trajectory for x, with an initial decline followed by a later recovery, though it never reaches the pre-shock level.

An important difference between the unconstrained and constrained cases is that the constrained policy prescribes a larger long-term capital stock. This results from a significantly milder reaction of the capital income tax, which also follows a non-monotonic pattern. Since the increased progressivity of the labor income tax reduces labor supply, the additional leisure lowers the marginal utility of young-age consumption. Because the capital tax does not change significantly, savings increase, and with constant government debt, this excess savings can only be channeled into capital accumulation.

There is also a level effect when imposing a balanced budget, as the model economy is solved under this constraint, which masks relative magnitudes. However, fixing the debt level at the baseline calibration clearly illustrates the significantly milder response of tax

Figure 6: Permanent shock with debt constraints



Notes: We compare unanticipated shock which permanently shifts the GINI from 0.33 to 0.45 under three scenarios: without debt constraint(solid black line); debt constraint, which is set to the debt-to-capital value in pre-shock steady state: $\frac{b}{k} = 0.18$ (dashed red line); balance budget debt constraint equals to zero (dashed-dotted blue line). Gini is Gini coefficient of labor income, which corresponds to the cross-sectional variance of labor productivity σ_e; k is capital; x is the consumption of the old; τ is the capital tax; ρ is the progressivity of the labor tax; b/k is the debt-to-capital ratio.

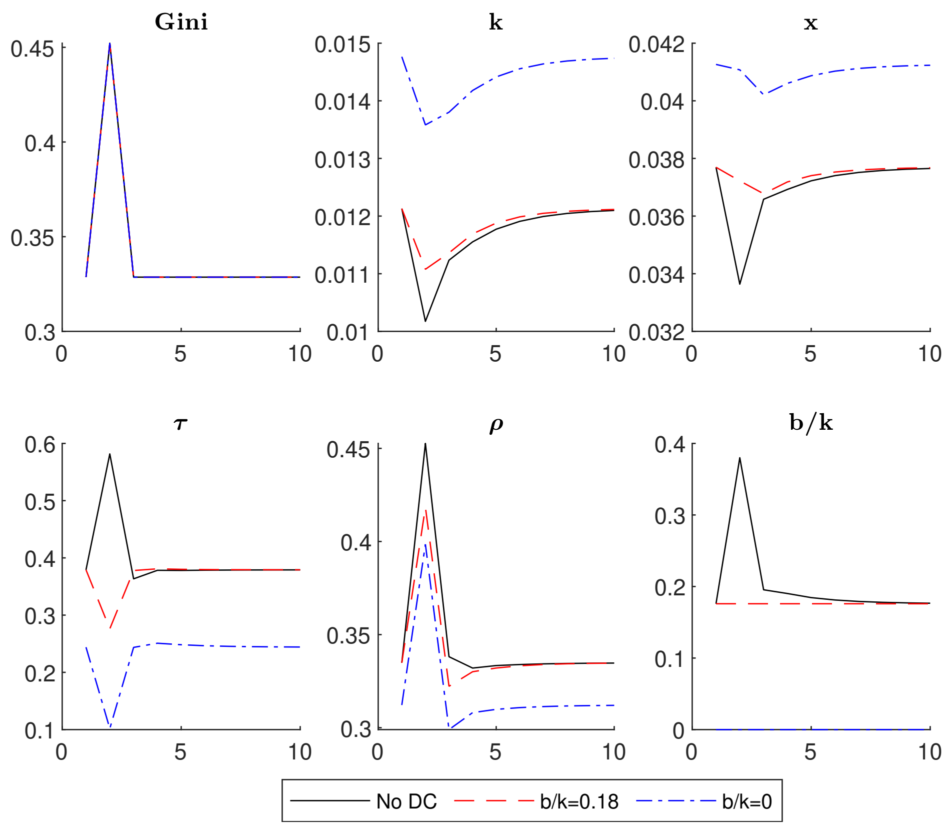
progressivity—about two-thirds of the unconstrained policy prescription—and the capital tax, which remains almost unchanged. Notably, the balanced budget scenario delivers capital taxation and progressivity levels that are remarkably similar to those in Conesa et al. (2009) and Heathcote et al. (2014), who explicitly assume a balanced budget government constraint.

6.3 When inequality increases temporarily

A natural question is whether the policy prescriptions change when the increase in inequality is temporary rather than permanent. This type of analysis is generally absent in the literature, which mostly focuses on comparisons between steady states. Does the persistence of the shock matter?

To this end, we now assume that the shock is *i.i.d.* over time. In Figure 7, we present the counterpart to Figure 6, but under a transitory shock. The notation and labeling remain the same. Comparing both figures, we find two main takeaways. First, when the policy is not constrained, the patterns resemble those of the permanent shock, but the movements are less pronounced. Second, the constraint on government debt does not significantly affect most patterns, except for capital taxation, where the prediction is reversed.

Figure 7: Temporary shock with debt constraints



Notes: We compare unanticipated temporary shock which increases the GINI from 0.33 to 0.45 for one period under three scenarios: without debt constraint(solid black line); debt constraint, which is set to the debt-to-capital value in pre-shock steady state: $\frac{b}{k} = 0.18$ (dashed red line); balance budget debt constraint equals to zero (dashed-dotted blue line). Gini is Gini coefficient of labor income, which corresponds to the cross-sectional variance of labor productivity σ_e; k is capital; x is the consumption of the old; τ is the capital tax; ρ is the progressivity of the labor tax; b/k is the debt-to-capital ratio.

Table 2: Summary of all experiments

Experiment	Initial	Max	Min	Long Run
Panel A: Capital Tax Rate τ				
Permanent, unanticipated	0.38	0.58	0.38	0.54
Permanent, anticipated	0.38	0.56	0.38	0.54
Temporary	0.38	0.58	0.36	0.38
Temporary with DC	0.38	0.38	0.28	0.38
Temporary with DC=0	0.24	0.25	0.10	0.24
Panel B: Progressivity ρ				
Permanent, unanticipated	0.33	0.47	0.33	0.47
Permanent, anticipated	0.33	0.47	0.33	0.47
Temporary	0.33	0.45	0.33	0.33
Temporary with DC	0.33	0.42	0.32	0.33
Temporary with DC=0	0.31	0.40	0.30	0.31
Panel C: Debt to Capital b/k				
Permanent, unanticipated	0.18	0.39	0.18	0.39
Permanent, anticipated	0.18	0.39	0.17	0.39
Temporary	0.18	0.38	0.18	0.18
Temporary with DC	0.18	0.18	0.18	0.18
Temporary with DC=0	0.00	0.00	0.00	0.00

Notes: Table summarizes the responses of the policy instruments to the changes in GINI from 0.329 to 0.45.

6.4 Summary of the all experiments

Table 2 presents the responses of key policy instruments to an increase in the Gini coefficient from 0.33 to 0.45. It reports the initial, maximum, minimum, and long-run values for three policy variables: the capital tax rate (τ), tax progressivity (ρ), and the debt-to-capital ratio (b/k). The table distinguishes between permanent and temporary changes in inequality, as well as the presence of debt constraints, such as debt limits where debt is fixed at the pre-shock level (DC) or balanced budget requirements ($DC = 0$).

Panel A focuses on the capital tax rate (τ). The results indicate that under permanent shocks, whether anticipated or unanticipated, the capital tax rate rises in the long run to 54% from an initial level of 38%. Temporary shocks, however, lead to fluctuations, with the tax rate returning to its initial value if no constraints are imposed. When a debt constraint is introduced, it limits the planner's ability to increase the capital tax, so the minimum tax rate is lower, but it eventually reverts to its initial level. If the debt constraint is set to zero (balanced budget, $DC = 0$), a 38% capital tax is no longer optimal. The capital tax rate starts at a lower level and remains subdued throughout. Regardless of whether the shock is permanent or temporary, the capital tax rate increases to at least 58% for one generation.

Panel B examines labor tax progressivity (ρ), showing that permanent increases in inequality lead to a sustained rise in tax progressivity to 0.47 from an initial level of 0.33, which corresponds to an increase in the marginal labor tax rate for the top 10% from 51% to 65%. Anticipation does not significantly affect the dynamics of progressivity. Temporary shocks generate smaller adjustments, particularly when a debt constraint is imposed. The minimum values in each scenario suggest that tax progressivity does not decrease significantly in the short run.

Panel C presents the behavior of the debt-to-capital ratio (b/k). Under permanent shocks, the debt-to-capital ratio increases significantly before stabilizing at more than double its initial level. For temporary shocks, the ratio temporarily more than doubles but eventually returns to its original level. In the case of a balanced budget requirement, the debt-to-capital ratio is forced to remain at zero.

Overall, the table highlights the differences between permanent and temporary shocks, as well as the role of debt constraints in shaping policy responses. The findings suggest that temporary shocks lead to greater variability in tax rates and progressivity, while permanent shocks result in lasting policy changes. Additionally, debt constraints, particularly a balanced budget requirement, significantly limit the government's ability to respond to inequality shocks.

7 Conclusions

Government debt and income inequality are usually considered two independent phenomena. The recent substantial increase in both has triggered a heated debate regarding the appropriate fiscal response to address them. However, the proposed solutions generally do not take their interaction into account. We have shown that greater inequality leads to substantially higher levels of redistribution, but also to significantly more government debt.

This interaction can be understood from two perspectives. On the one hand, from the perspective of redistributive policies, by accumulating debt that then becomes private sector savings, the government can mitigate the dynamic distortions caused by the necessary increase in redistribution. On the other hand, from the perspective of the debt burden, repaying obligations necessarily comes at the cost of lower redistribution and higher capital taxation. In short, more redistribution and more debt go hand in hand and represent an optimal response to rising inequality. Hence, limiting the government's borrowing capacity simply because it appears "excessive" would severely impair society's ability to expand much-needed redistributive policies.

We have deliberately considered a simplified environment that allows us to present transparent results, including a sharp characterization of fiscal instruments and transition paths that are easy to compute and evaluate. Our methodology can be extended to settings with heterogeneous returns to capital and additional age profiles. However, we have abstracted from two important factors: endogenous human capital and uncertain income profiles. These elements may introduce technical difficulties requiring a more quantitative approach, but we consider them important avenues for future research.

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