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Fiscal Rules and Discretion with Risk of Default*

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Abstract

We study optimal fiscal rules when present-biased governments can default on their debt. Beyond their tendency to overaccumulate debt, they exhibit biases in default incentives. Depending on the severity of the bias and the cost of default, governments may either over-default, inefficiently defaulting at low debt, or under-default, retaining excessively high debt. The latter arises from a “clean-slate externality”: governments fail to internalize the future benefits of offloading debt. Balancing the commitment to not overspend with the flexibility to react to shocks, it is optimal to complement spending rules with default rules that either forbid or force default.

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1 Introduction

Over the past few decades, sovereign debt has increased substantially in both developing and advanced economies. As a result, fiscal rules have become increasingly prevalent (Yared, 2019). These rules typically take the form of spending and deficit limits, which may or may not depend on the economy’s indebtedness. The driving force behind this wave of rules is the concern about debt sustainability and the implied risk of default. However, the implemented rules are generally ad hoc and not based on sound theories—and, when they are, they mostly abstract from the interaction with default risk.¹ How do fiscal rules change when a sovereign can default? Should there also be “default rules”—specific interventions in government default?

Without a formal framework to evaluate the issue, a common initial intuition is that “myopic” governments tend to overaccumulate debt and, thus, default too often or too early. In this paper, we formally analyze the problem and find that this initial intuition is partial and misleading. First, we show that myopic governments can indeed default too often but, surprisingly, also too rarely or too late. The direction of default bias hinges on how the benefits and costs of improving fiscal position through default are distributed over time. Consequently, depending on the economic environment, it may be optimal to either forbid or force default. Second, we examine how spending rules should be adjusted to counteract default risk and bias. It is desirable to lift or even eliminate spending limits in some situations, whereas in others they must be tightened. Finally, we perform a quantitative exploration and find that optimal default rules are substantially more important than spending rules: the welfare gains of regulating default can be an order of magnitude larger than the gains from curbing spending.

When analyzing fiscal rules, a question naturally arises: what is the underlying friction generating the need to impose rules? A commonly accepted reason is rooted in political economy. Political turnover, together with political polarization, creates incentives for an incumbent to overspend at the expense of future governments, leading to a *spending bias*. We formalize this bias in a continuous-time economy subject to random spending needs. These genuine needs represent the real social value of spending. The economy is run by an incumbent government. At every instant, a stochastic change of government may occur: the incumbent is replaced by a new one who draws a new spending need. Governments are forward-looking, but they value decisions made by other governments less. To be precise, the incumbent discounts allocations chosen by any future government by a factor $\beta \leq 1$, which captures the extent of political polarization. If $\beta < 1$, political turnover generates a

¹See Eyraud et al. (2020) for a comprehensive discussion of design of fiscal rules.

present bias resembling hyperbolic discounting, i.e., governments are myopic. Although the needs are genuine, the incumbent’s bias generates incentives to over-state them, leading to overspending and excessive debt accumulation.

Governments can save and borrow in non-contingent short-term bonds, which are supplied by competitive international lenders. When in debt, the government can renege and default on all its outstanding debt. The interest rate charged on the loans, therefore, reflects this default risk. After default, the economy is excluded from financial markets, as in [Eaton and Gersovitz \(1981\)](#), and suffers a proportional loss in resources, as in [Arellano \(2008\)](#). This status does not need to be permanent. In the future, the country may regain market access, the resource loss vanishes, and all previous debt is erased.

Beyond the tendency to overaccumulate debt, governments are also biased in their default incentives. The incumbent government weighs the current and future costs of defaulting against the benefits. Its bias in discounting the future, both costs and benefits, gives rise to two competing forces. First, after default, the *lack of insurance* due to financial exclusion is a cost backloaded to the future, when the potential shocks will materialize. Since the incumbent does not fully internalize this cost imposed on future governments, it tends to over-default, i.e., opting to default at inefficiently low debt levels. In contrast, when financial exclusion is only temporary, the lost insurance value decreases and a new force comes into play, which we term *clean-slate externality*.² Upon reentry, the country does so without any debt obligations. This is a backloaded benefit that the incumbent does not fully internalize when making the default decision. Contrary to the initial intuition, myopia may lead governments to under-default and hold onto excessive debt.

The direction of default bias is tightly linked to default patterns. When financial exclusion is permanent, governments with higher spending needs are less concerned about future insurance value and more reluctant to give up current spending to pay back debt and, thus, tend to default first and at lower debt levels. Instead, when financial exclusion is sufficiently short lived, the clean-slate externality starts to dominate. Types with lower current spending needs are more willing to deal with the immediate output loss implied by defaulting and find the prospect of a clean slate more appealing. As a result, the default pattern reverses: governments with lower spending needs prefer to default first. From this point of view, observing the default patterns may reveal the dominant force of inefficiency.

To correct these biases, a rule-maker that could perfectly observe and contract on the spending needs could implement rules that precisely limit the governments’ ability to spend,

²The use of the terminology “externality” may sound unusual to some readers. Externalities arise when the actions of one agent affect the welfare of another. In our context, we regard an incumbent government as a separate agent from a future one, which rationalizes our word choice.

or to default, or both. However, spending needs, which represent real needs, are affected by random events. It may be impossible for the rule-maker to fully and perfectly characterize them, rendering them *non-contractible*. This creates a trade-off between discretion, allowing governments to respond to shocks, and rules, to prevent inefficient spending and default.

Formally, we study a principal-agent problem, where the rule-maker (principal) delegates the decisions to governments (agents). We take the perspective of a noncommitted, benevolent planner who chooses spending and default allocations subject to the truthful revelation of spending needs. Unlike incumbent governments, the planner makes no distinction between governments, evaluating allocations as if $\beta = 1$. Moreover, the planner adopts an *ex ante* welfare criteria, choosing state-contingent allocations before knowing the realization of the spending needs. We show that the optimal incentive-compatible allocation can be implemented as a Markov equilibrium between current and future governments, when they are subject to not only *spending rules* but also *default rules*. Our approach has two appealing features. First, we do not constrain the set of instruments available to the rule-writer. The necessity for both spending and default rules emerge endogenously. Second, the planner's lack of commitment ensures that rules are *sustainable*: the rule-writer does not find herself in a situation in which she wishes to rewrite the rules in the future.

In addition to governments' default biases, the planner's objective introduces an *interest-rate externality*. When choosing the default strategy *ex ante*, the planner weights the welfare of all possible types. In contrast, individual governments, after learning the realization of their own needs, do not take into account how their default decisions affect the risk premium that non-defaulting types in the same debt level must pay, a force generating over-default. This externality is always present, even when present bias is absent.

We characterize the default rules that restore constrained efficiency. Consistent with our previous insights, the required rules crucially depend on the severity of present bias and the duration of financial market exclusion. When the exclusion is permanent, for any $\beta > 0$, only the insurance channel and interest-rate externality must be corrected. We characterize a debt threshold, such that, for debt positions above this threshold, default is not allowed; but when debt exceeds this level, the sovereign can default at its discretion. Loosely speaking, defaulting is forbidden if the debt is not sufficiently high.

In the opposite scenario, when financial exclusion is short-lived and the present bias is severe, the rule-maker now aims to correct the dominating clean-slate externality and governments' tendency to hold onto excessive debt. In other words, from the regulator's perspective, *the market fails to impose adequate discipline: it is willing to extend too much debt*. To resolve this inefficiency, the planner imposes a more stringent debt limit and commits to default beyond this limit. Naturally, lenders respond to this rule, and the market-imposed

debt limit tightens. The optimality of forced default can be interpreted as a hard debt limit—tighter than the market one—combined with immediate convergence through default whenever the limit is surpassed.

We also characterize how default risk shapes the spending rules. The optimal spending rule is a threshold rule, or a minimum-saving rule, as in [Amador, Werning and Angeletos \(2006\)](#). That is, there exists a threshold type, such that all governments reporting needs below the threshold are granted the discretion to spend as they wish, while those reporting needs above the threshold are bunched to the threshold type. This rule can easily be implemented with a spending cap or a deficit limit.

When debt is low and safe, the spending threshold type is independent of debt. However, as debt increases and default risk arises, the threshold type becomes *debt dependent*. Again, this rule can be implemented with either a spending cap or a deficit limit, but it needs to be tightened or loosened as debt accumulates. There are two main channels shaping the necessary modification to spending rules: 1) the change in the pool of borrowers, which we call a *selection effect*, and 2) the change in the interest rate.

The interest-rate channel reveals elements reminiscent of [Halac and Yared \(2018\)](#), who study coordinated fiscal rules in a world economy with endogenous interest rates. While a higher interest rate has a positive disciplining effect that curbs spending, unlike in their setting, in ours it also has a negative income effect, since interest payments are transfers to foreign lenders. Furthermore, we introduce a new *default-manipulation effect*, as the stringency of the spending rule affects default incentives. Thus, a priori, the overall effect can be ambiguous. However, in economies with monotonic default patterns, the two additional effects due to default have unambiguous directional effects. For example, if financial exclusion is permanent, governments with higher spending needs default first and at lower debt levels. In this case, as debt increases, only those with lower spending needs continue servicing the debt. Thus, the selection effect improves the pool of borrowers, leading to a relaxation of the spending rule. Moreover, since governments exhibit an over-default bias, relaxing the spending rule is desirable as it incentivizes them to avoid default.

Our theoretical results highlight the close interaction between spending and default rules. Spending rules affect default choices. Similarly, default rules also change the spending decisions. One may wonder, which type of rules is more relevant? Absent default rules, is it possible to approach efficiency using only spending rules? To answer these questions, we explore quantitatively the welfare contribution of each type of rules. We focus on two distinctive cases: 1) an economy with permanent financial exclusion and mild present bias, leading to a debt region where governments over-default and forbidding default is desirable, and 2) an economy with short-lived exclusion and severe present bias, with forced default.

In both cases, we find that as debt accumulates and becomes risky, default rules generate welfare gains that are at least an order of magnitude larger than those from spending rules alone. In the absence of default rules, spending limits can be adjusted to manipulate default choices towards efficiency; however, their contribution by itself is minimal. Imposing appropriate default rules can generate substantial welfare gains.

Literature review. We relate to several strands of literature: on time-inconsistent preferences and their relation to political economy; on the optimal trade-off between commitment and flexibility; and the rich, growing body of work on sovereign default.

The literature closest to our work was originated by [Amador, Werning and Angeletos \(2006\)](#), who analyze the optimal trade-off between commitment and flexibility when agents discount the future quasi-hyperbolically. The central premise is that agents, or governments, are tempted to overspend, making it optimal to limit their ability to accumulate debt. What makes the problem nontrivial is that agents are subject to random spending needs, which are either unobservable or non-contractable. Hence, it is desirable to grant them some discretion to respond to shocks. The optimal rule features threshold policies and bunching above. More recently, this approach has been extended by [Halac and Yared \(2014, 2018\)](#), in settings with persistent shocks or endogenous interest rates. In relation to their work, we incorporate sovereign default and characterize biases in government default decisions. We show that the optimal fiscal rules are generally debt-dependent and complemented by default rules.

We also draw on the rich literature analyzing environments with quasi-hyperbolic discounting, originated by [Strotz \(1955\)](#) and augmented by [Laibson \(1997\)](#). These environments present numerous technical challenges in characterizing agent behavior (see, e.g., [Cao and Werning, 2018](#); [Krusell and Smith, 2003](#)). Notably, as [Chatterjee and Eyigungor \(2016\)](#) pointed out, when the agents are subject to borrowing limits, all Markov equilibria exhibit discontinuous decision rules. Since default decisions imply de facto borrowing limits, we leverage the insights of [Piguillem and Shi \(2025\)](#), which establishes that a present-biased agent is behaviorally equivalent to an agent with optimistically biased expectations and provide conditions for equilibrium uniqueness. Moreover, we adopt a continuous-time setting, as do [Harris and Laibson \(2013\)](#) and [Maxted, Laibson and Moll \(2024\)](#), who consider a limiting behavior which they termed “instantaneous gratification,” and [Cao and Werning \(2016\)](#), who analyze the savings behavior for a more general “disagreement index.” In this paper, the continuous-time setting enables us to obtain sharp characterization.

Combining these two approaches, we bridge the theoretical work and the quantitative analysis and provide some insights on what types of rules should be implemented. By enriching the structure in the friction generating debt over-accumulation, the model allows for

a meaningful mapping to the data. To be precise, it maps neatly to the standard political economy models à la Persson and Svensson (1989), Alesina and Tabellini (1990), and Battaglini and Coate (2008). Thus, we can decompose the hyperbolic discounting factor into *political turnover* and *polarization*, which shape the optimal rules differently. This decomposition is along the line of recent quantitative work by, for example, Azzimonti, Battaglini and Coate (2016), who assess the effect of imposing a balanced budget.

Finally, we contribute to the theoretical and quantitative literature on sovereign default. We build on the seminal contributions of Eaton and Gersovitz (1981) and Arellano (2008), in continuous-time environment as in Bornstein (2020). Relative to the large body of positive, predominantly quantitative work on sovereign default, studies addressing normative issues are limited.³ These normative studies have examined the time inconsistency inherent in long-term debt issuance. For instance, Hatchondo, Martinez and Roch (2022) explore how committing to future decisions via fiscal rules could improve current outcomes.⁴ However, Aguiar and Amador (2019) show that economies with short-term debt are efficient and there is no role for fiscal rules. In contrast, we study the time inconsistency due to present bias. The under-default bias we uncover aligns with the insight of Amador (2012): the political-economy friction responsible for overspending can make the sovereign more inclined to repay, thereby enlarging debt capacity. To the best of our knowledge, Adam and Grill (2017) is the only paper that considers the possibility of default rules. They study a Ramsey problem with perfect information and geometric discounting find that, when default is costly, it is only optimal to default in response to “rare disaster” shocks. Finally, Alfaro and Kanczuk (2017) quantitatively evaluate some selected fiscal rules involving debt limits, whereas we theoretically characterize the optimal rules across different environments.

The remainder of the paper is organized as follows. Section 2 describes the model environment. Section 3 characterizes the rules-free equilibrium, and Section 4 emphasizes the biases in government spending and default decisions. Section 5 presents the optimal spending and default rules. Section 6 performs the quantitative exploration. Section 7 concludes.

2 Model

This section develops a model that incorporates political-economy frictions leading to present-biased governments in a sovereign default framework. We first describe the model environment and then discuss the forces that determine market interest rates.

³For positive work including political economy considerations see Cuadra and Sapriza (2008), Önder and Sunel (2020), and Cotoc, Johri and Sosa-Padilla (2025), among others.

⁴See also Chatterjee and Eyigungor (2015), Hatchondo, Martinez and Sosa-Padilla (2016), and Dovis (2018) for normative prescriptions to improve lending contracts.

2.1 Environment

Time is continuous and infinite, $t \in [0, \infty)$. At every instant t , the economy is governed by an incumbent government. A *political turnover* event occurs with Poisson arrival rate λ : the incumbent government loses power and is replaced by a new one. Each incumbent government receives an exogenous source of tax revenue τ and faces a spending choice g . Different governments attribute different values to their spending needs. This value is determined by their “taste” type θ . The type θ is *i.i.d.* and drawn from a bounded set $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\cdot)$, with an expected value $\mathbb{E}[\theta] = 1$. The change in preferences can be interpreted as arising from the underlying constituency’s opinions on the social value of spending, which change over time and determine the alteration of the country’s stance on fiscal policy.⁵ The preferences for spending flows are

$$\theta u(g),$$

where $u(\cdot)$ is strictly increasing, strictly concave, and twice differentiable for $g \in (0, \infty)$, so that $u'(g) > 0$ and $u''(g) < 0$. Thus, high types derive a higher marginal utility from spending than low types.

Present bias. All governments, whether incumbent or opposition, discount the future exponentially at rate ρ . However, incumbent governments attribute a lower value to the spending by future governments. Specifically, they discount the utility derived from future spending, when they are out of power, by an addition factor $\beta \in [0, 1)$. We refer to β as the present bias parameter.

One interpretation of the bias term β is that it captures, in a reduced-form way, the disagreement over the composition rather than the level of public spending within a country. This disagreement leads to a situation where one unit of spending transforms into one unit of consumption for a government in power, but it yields fewer units of consumption for a government out of office. Therefore, the bias β effectively reflects the degree of *political polarization*. In Section 3.2, we discuss further alternative interpretations.

Borrowing and default. The governments can borrow from a continuum of competitive risk-neutral lenders by issuing non-contingent short-term bonds. The lenders have access

⁵One possible interpretation is that preferences vary in response to the business cycle. For instance, Amador, Werning and Angeletos (2006) show that if utility is exponential, taste shocks are equivalent to income shocks. Another interpretation is that demographic changes in the constituency’s composition or power struggles between different parties can lead to preference shocks. See, for example, the entrepreneur-worker conflict in Azzimonti, de Francisco and Quadrini (2014).

to risk-free rate $r_f \leq \rho$. Let b denote the amount of outstanding government debt. At any instant, the incumbent government can default on its debt obligations, upon which it is excluded from financial markets, as in [Eaton and Gersovitz \(1981\)](#). While in default, tax revenues are reduced to $\kappa\tau$, with $\kappa \in (0, 1]$. This captures output losses due to financial exclusion, as in [Arellano \(2008\)](#). Financial access is regained at Poisson rate $\phi \geq 0$: upon regaining access, previously defaulted debt is fully discharged, and the government returns to the market with a zero-debt position.

As is customary in both the hyperbolic preference and sovereign default literatures, we focus on solution concepts that depend only on the payoff-relevant states. This Markovian restriction applies to the equilibrium and the optimal problem. Let $g(\theta, b)$ represent the spending of a government of type θ with debt position b . Further, let $\delta(\theta, b) \in \{0, 1\}$ indicate the default decision, which takes a value 1 if the government defaults and 0 otherwise. Then,

Definition 1 (Allocation). An allocation $\mathcal{A} = \{g(\theta, b), \delta(\theta, b)\}_{\theta \in \Theta, b \in \mathbb{R}}$ specifies the government spending and default for all types at all debt levels.

2.2 Market interest rates

We begin with several immediate observations regarding the determination of market interest rates. These observations rely on lending market only and thus hold regardless of whether any rules are imposed in the economy.

First, lenders could in principle charge an interest rate $r(\theta, b)$ based not only on the debt level but also on the government's type, reflecting their assessment of default risk. Bear in mind that along the equilibrium path, default may only be triggered by a discontinuous jump in the government's type, due to political turnover. To understand this dynamic, it is crucial to recognize that time is continuous and government debt is short-term in nature, leading to a smooth process of debt accumulation, which allows lenders to instantaneously adjust interest rates. For a government that has accrued substantial debt and is on the verge of default, lenders would refrain from extending any additional funds, anticipating an immediate default. Equivalently, lenders would impose an infinitely high interest rate in compensation, effectively precluding any further borrowing. Therefore, at the default threshold, further debt accumulation does not occur. This suggests that default is precipitated only by a sudden jump in θ .⁶ Moreover, as government types are *i.i.d.*, conditional on the incumbent not defaulting, the default risk is independent of the current type, implying that the interest rate depends *only* on the debt level b not on the type θ .

⁶To refine the model's business-cycle properties, we can introduce fluctuations in the tax revenue τ . For instance, if the tax revenue follows an Ornstein-Uhlenbeck process, which moves smoothly, it does not have a direct impact on default. Hence, we develop our main results without cycles.

Summarizing, we formally characterize the interest rate as follows:

$$r(\theta, b) = \begin{cases} r_f + \lambda \mathbb{E}[\delta(\theta, b)], & \text{if } \delta(\theta, b) = 0 \\ \infty, & \text{if } \delta(\theta, b) = 1. \end{cases}$$

When a government is not in default, the risk premium is computed according to the turnover rate λ multiplied by the expected default rate of incoming types, $\mathbb{E}[\delta(\theta, b)]$. By charging the appropriate interest rate, the lenders can make sure that the current government will not default, but they remain afraid that the future, still unknown, government will default on the debt it inherited. This default probability is intrinsically linked to the debt level. As the current type accumulates more debt, it enlarges the set of future types who would default.

While the full characterization of interest rates has been instrumental in our understanding of the debt market outcomes, it is nevertheless without loss of generality to restrict our attention to the interest rate schedule when the economy is in the non-default region, where $\delta(\theta, b) = 0$. For ease of notation, we rewrite the interest rate schedule in this region as:

$$r(b) = r_f + \lambda \mathbb{E}[\delta(\theta, b)]. \quad (1)$$

Correspondingly, the debt accumulation process is determined by the primary deficit, $g(\theta, b) - \tau$, and the cost of servicing existing debt, $r(b)b$:

$$\dot{b}(\theta, b) = r(b)b + g(\theta, b) - \tau. \quad (2)$$

3 Rules-free equilibrium

Before proceeding to study optimal rules, it is instructive to first understand the equilibrium outcome in the absence of rules. This section characterizes the rules-free equilibrium.

3.1 Governments' problem

To describe the governments' problem, we first introduce some notations. When the economy is not in default, let $w^n(\theta, b)$ be the value function of an incumbent with spending needs θ and debt b . Let $v^n(\theta, b)$ be the analogous value function from the perspective of an unbiased subject who values all future spending equally, as if $\beta = 1$. Similarly, when the economy is in default, let $w^d(\theta)$ and $v^d(\theta)$ be the incumbent's value function and the unbiased value function, respectively.

Following the discussion in Section 2.2, we observe that, when a new government θ comes

into power and inherits debt b , it decides whether to default. It will do so by comparing the values derived from the two options:

$$\delta^A(\theta, b) = \begin{cases} 0, & \text{if } w^n(\theta, b) \geq w^d(\theta) \\ 1, & \text{if } w^n(\theta, b) < w^d(\theta). \end{cases} \quad (3)$$

Taking into account potential default, the unbiased value function is given by:

$$v(\theta, b) = (1 - \delta^A(\theta, b))v^n(\theta, b) + \delta^A(\theta, b)v^d(\theta). \quad (4)$$

When the economy is not in default, the non-default value functions solve the following system of Hamilton-Jacobi-Bellman (HJB) equations:

$$\rho w^n(\theta, b) = \max_g \{ \theta u(g) + (r(b)b + g - \tau)w_b^n(\theta, b) + \lambda (\beta \mathbb{E}[v(\theta', b)] - w^n(\theta, b)) \} \quad (5)$$

$$\rho v^n(\theta, b) = \theta u(g^A(\theta, b)) + (r(b)b + g^A(\theta, b) - \tau)v_b^n(\theta, b) + \lambda (\mathbb{E}[v(\theta', b)] - v^n(\theta, b)), \quad (6)$$

where the term $\mathbb{E}[v(\theta', b)]$ also embodies the expectation over new government types and the potential default associated with government change.

To understand these equations, it is useful to start by assuming that default is not possible, so that $\mathbb{E}[v(\theta', b)] = \mathbb{E}[v^n(\theta', b)]$. As long as $\lambda = 0$, equation (5) represents a standard consumption-savings problem. When $\lambda > 0$, the political friction starts to play a role. The last term in (5) captures the effect of turnover. With arrival rate λ , the incumbent loses power, which implies a loss in value of $w^n(\theta, b)$, and it is replaced by a new government, generating a value $\beta \mathbb{E}[v(\theta', b)]$. Two components of this term are noteworthy. First, the future government's spending needs are not yet known, thus requiring an expectation over future types θ' . Second, and more importantly, the incumbent discounts the continuation value by the additional factor β . While in power, the incumbent can decide its spending level. We denote this discretionary spending decision by $g^A(\theta, b)$.

The unbiased continuation value $v^n(\theta, b)$ follows the companion HJB equation (6), where the bias factor related to political turnover is absent. It is evident that after the incumbent loses power, it does not care who will be in power, as long as it is not itself. All future alternative governments are uniformly discounted by β . Equation (6) also makes clear that the incumbent takes as given that future governments will spend (and borrow) following their own optimal choices $g^A(\theta, b)$. The incumbent correctly assesses that any future government, when in power, would exhibit the same bias and maximize its own value.

In the absence of default, equations (5) and (6) would suffice to describe governments'

problem. However, when default is possible, it remains to describe the value functions when in default. The equations analogous to (5) and (6) in this default scenario are:

$$\rho w^d(\theta) = \theta u(\kappa\tau) + \phi(w^n(\theta, 0) - w^d(\theta)) + \lambda(\beta \mathbb{E}[v^d(\theta')] - w^d(\theta)) \quad (7)$$

$$\rho v^d(\theta) = \theta u(\kappa\tau) + \phi(v^n(\theta, 0) - v^d(\theta)) + \lambda(\mathbb{E}[v^d(\theta')] - v^d(\theta)). \quad (8)$$

Equations (7)-(8) differ from (5)-(6) in that optimization is no longer possible when in default. Since the country is excluded from financial markets, it is no longer able to smooth out its spending. It also suffers a loss of resources, leaving only $\kappa\tau$ available for spending. Further, when a government has the chance to reaccess financial markets, it will do so for sure, since it can return with zero debt as the terms $w^n(\theta, 0)$ and $v^n(\theta, 0)$ make clear. Finally, notice that the bias β enters in the default status in the same way as in (5)-(6). The bias distorts not only the non-default value but also the default one. When a country could regain access to financial markets with a “clean slate,” the benefit might be only enjoyed by future governments, and thus, is heavily discounted by the incumbent. We refer to this effect as the “clean-slate externality,” which will have important implication for efficiency.

3.2 Equilibrium concept

We now define a Markov equilibrium.

Definition 2 (Markov equilibrium). A Markov equilibrium is a collection of value functions $\{w^n(\theta, b), w^d(\theta), v(\theta, b), v^n(\theta, b), v^d(\theta)\}$, decision functions $\{g^A(\theta, b), \delta^A(\theta, b)\}$, and interest rates $r(b)$ such that equations (1) and (3)-(8) are satisfied, $\forall \theta \in \Theta, b \in \mathbb{R}$.

Alternative interpretations. We have followed the political economy interpretation, where λ is the rate of political turnover and β reflects the degree of political polarization. However, this framework lends itself to multiple interpretations.

Our main interpretation draws from the seminal paper by [Alesina and Tabellini \(1990\)](#). Suppose the parties disagree (are polarized) on the desirable attributes of the public good. The party in power, by selecting attributes that align with its preferences, can derive a higher utility from a given level of spending than the opposition party. In this case, β would capture the loss in utility due to the suboptimal allocation from the perspective of the opposition party. Alternatively, consider a political environment with legislative bargaining, as in [Battaglini and Coate \(2008\)](#), where members of the governing coalition have access to “pork,” while those not in the coalition do not. With some probability, the current legislators in power remain in the governing coalition after a change of government and continue to have

access to pork. This probability, in turn, influences the effective present bias.⁷

We could also appeal to the extensive literature on quasi-hyperbolic discounting. Following [Strotz \(1955\)](#) and [Laibson \(1997\)](#), it is customary to model myopic individuals lacking self-control as exhibiting preferences that, in addition to the usual geometric discounting, place an additional factor β to further discount the future over the present. One could interpret this environment as the continuous-time equivalent of a present-biased agent, except that the bias arises randomly rather than deterministically, similar to the approach by [Cao and Werning \(2016\)](#). Indeed, when $\lambda \rightarrow \infty$ and $\underline{\theta} \rightarrow \bar{\theta}$, our model converges to the instantaneous gratification limit of [Harris and Laibson \(2013\)](#). From this perspective, the results of this paper can be extended to the optimal regulation of individual borrowing decisions involving strategic default, such as consumer credit.

Definition of discretion. It is opportune to provide a precise definition to “rules versus discretion,” along with the optimality conditions associated with the terminology. The incumbent government’s “discretionary” spending satisfies the first-order condition with respect to spending in (5):

$$\theta u'(g^A(\theta, b)) = -w_b^n(\theta, b), \quad (9)$$

which states that the marginal utility of spending is equal to the incumbent’s marginal cost of debt. Given that the government’s non-default value $w^n(\theta, b)$ is strictly decreasing in b , its default decision characterized by equation (3) implies that there is a unique “discretionary” default threshold $b^A(\theta)$, defined as:

$$w^n(\theta, b^A(\theta)) = w^d(\theta). \quad (10)$$

When debt exceeds the threshold, i.e., $b > b^A(\theta)$, a government of type θ finds it desirable to default. Furthermore, we specify a *lower* bound of debt, where no type would default and thus debt is safe, and an *upper* bound of debt, where all types prefer to default and thus default happens with certainty:

$$\underline{b}^A \equiv \inf_{\theta \in \Theta} b^A(\theta) \quad \text{and} \quad \bar{b}^A \equiv \sup_{\theta \in \Theta} b^A(\theta).$$

The conditions above are useful in Section 5, when we consider a planner who intervenes in the government decisions. In state (θ, b) , if the planner chooses to respect equation (9), we say that it is granting the government *discretion to spend*; if the planner respects equation

⁷Another interpretation related to political economy is that the preferences arise naturally from the aggregation of time-consistent agents with heterogeneous discount rates. See [Jackson and Yariv \(2014\)](#).

(3), it is allowing *discretion to default*. Of course, in the rules-free equilibrium, governments have full discretion in all decisions in all states, i.e., $g(\theta, b) = g^A(\theta, b)$ and $\delta(\theta, b) = \delta^A(\theta, b)$.

3.3 Spending and default patterns

We introduce the following assumption on the distribution of spending needs.

Assumption 1 (Type distribution). *The distribution function $F(\theta)$ admits a differentiable density $f(\theta)$ that satisfies:*

(i) *The elasticity of the density function with respect to the spending need is bounded below:*

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta}, \quad \forall \theta \in \Theta.$$

(ii) *The highest type $\bar{\theta}$ is finite, and its density $0 < f(\bar{\theta}) < \frac{\rho}{2\lambda\bar{\theta}}$.*

Assumption 1 states that the elasticity of the density function with respect to the type, $\theta f'(\theta)/f(\theta)$, is bounded below, limiting the speed at which the density can decrease. It also states that the density of the highest type is strictly positive but not too high. This assumption serves two purposes. First, it ensures that there exists a unique continuous Markov equilibrium, as established in [Piguillem and Shi \(2025\)](#). Their approach leverages an equivalence result: a present-biased agent is behaviorally equivalent to an agent with optimistically biased “expectation” about her future spending needs. In Proposition A1 in Appendix A, we show that this analogous mapping extends to the continuous-time limit here with default. Assumption 1 guarantees that the manipulation effect of future governments is not so strong as to generate multiplicity of equilibria. Second, it serves as a sufficient condition to ensure that the spending rule in the economy without default takes the form of a (unique) threshold type. In this regard, it aligns with the assumption described in [Amador, Werning and Angeletos \(2006\)](#). Thus, although Assumption 1 could be relaxed and still preserve the uniqueness of the equilibrium, as shown in [Piguillem and Shi \(2025\)](#), alternative specifications might complicate the characterization of optimal fiscal rules.

Finally, we focus on equilibria where $w^n(\theta, b)$ is concave.⁸ This property immediately implies that $g_b(\theta, b) < 0$. Without concavity none of these intuitive properties would hold. In Section 6 we present several numerical cases, all of them display concavity.

We now characterize some specific spending and default patterns that are instrumental for understanding the results in the subsequent section. In the next three lemmas, we show that,

⁸In [Chatterjee and Eyigungor \(2016\)](#), lotteries are essential to ensure the concavity of the value function and refine to continuous equilibria. In our setting, the discontinuity problem does not arise due to the smoothing properties of the θ shock.

in addition to the forces present in economies without bias, the bias significantly alters the outcomes. In these cases, we highlight the complications and interesting features introduced by the bias. We start by analyzing possible default patterns.

Characterizing default patterns requires understanding the forces affecting government's incentives, some of which are immediate and others realized in the future. In the short run, by defaulting the government spares the debt repayment burden and suffers an output loss. The future consequences involve the lack of insurance during financial exclusion, and the potential benefit of starting with a clean slate upon reentry. As a result, many different patterns can arise. One possibility is that all types default at the same debt level, $b^A(\theta) = b^A$, $\forall \theta \in \Theta$, making the default-risk region degenerate. When the default-risk region is non-degenerate, the default threshold $b^A(\theta)$ can be monotonically increasing, monotonically decreasing, or non-monotonic. We show cases where we can theoretically characterize the pattern.

The market also play an important role when pricing the debt. In particular, as we establish in Lemma A2 in the appendix, if the turnover rate is sufficiently high, the equilibrium features an endogenous borrowing limit. Since default can only occur in the event of a political change, a high risk premium is attributed to fast turnover, which is in line with the empirical findings of Cuadra and Sapriza (2008). The risk premium can be so high such that, even if the incumbent were to endure zero spending, it could not afford to cover the interest payment and has to default. This result implies that in environments with instantaneous gratification as in, for instance, Harris and Laibson (2013), where $\lambda \rightarrow \infty$, default will not occur in equilibrium, making it unsuitable for a general analysis. In what follows, we focus on economies with a moderate political turnover rate $\lambda < \bar{\lambda}$.

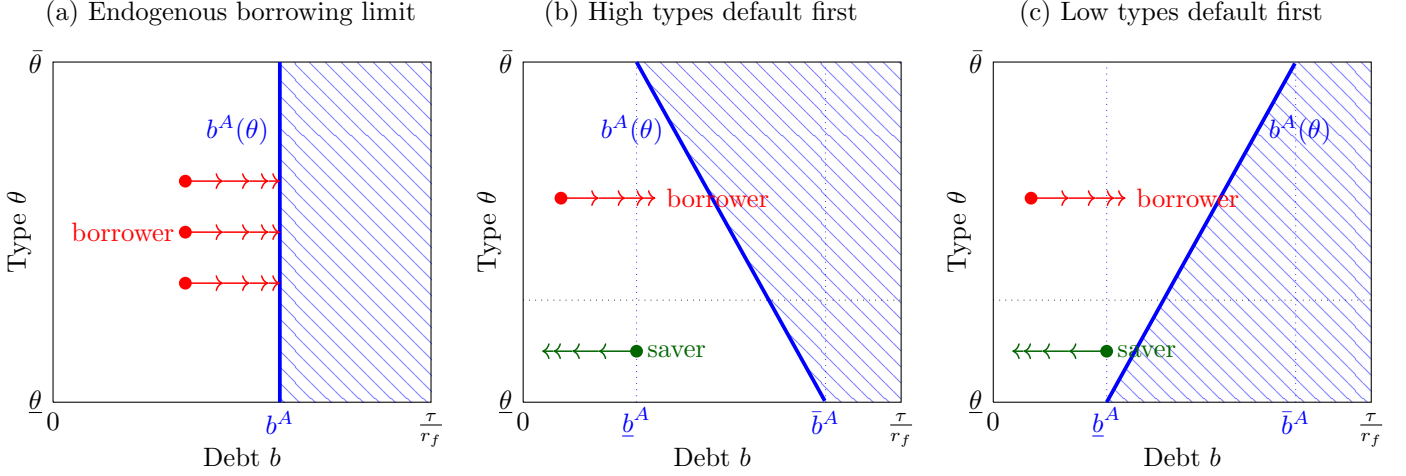
Even with moderate political turnover, the equilibrium may still exhibit a degenerate default threshold akin to a borrowing limit. This situation arises when the present bias is extreme, with $\beta = 0$, leading the incumbent government to completely disregard future governments' spending priorities. Regardless of their spending needs, all governments would like to borrow and spend the same amount, thereby accumulating debt at the same rate. As they approach their default threshold, market stops extending more debt, knowing that default would happen with certainty beyond this limit. That is, the market imposes an endogenous borrowing limit, as we summarize below.

Lemma 1 (Endogenous borrowing limit). *When present bias is extreme $\beta = 0$, the market endogenously imposes a borrowing limit, $b^A(\theta) = b^A$. Consider $\kappa < 1$, then:*

- (i) (Permanent exclusion). *If $\phi = 0$, the borrowing limit $b^A = \frac{1}{r_f}(1 - \kappa)\tau$.*
- (ii) (Temporary exclusion). *If $\phi > 0$, the borrowing limit $b^A < \frac{1}{r_f}(1 - \kappa)\tau$.*

Proof. See Appendix B.1. □

Figure 1: Spending and default patterns



Note: Panel (a) illustrates the default pattern in Lemma 1, and panel (b) presents the pattern in Lemma 2.

The borrowing limit, as characterized in Lemma 1, depends on the cost associated with default. If default results in permanent financial autarky, the consequent output loss serves as a lasting punishment with spending fixed to $\kappa\tau$. At the borrowing limit, after paying the interest payments, the government also faces a permanent constant streams of spending $\tau - r_f b^A$. The two must equate so that the government is indifferent between defaulting or not, which generates the borrowing limit $\frac{1}{r_f}(1 - \kappa)\tau$. However, as reentry into financial markets becomes easier, the punitive consequences of default diminish, effectively reducing the borrowing limit. Bear in mind that these market-imposed borrowing constraints may not be optimal from a planner's perspective, which we will revisit in Proposition 4.

When present bias is less extreme, the default threshold is generally non-degenerate. While a general characterization is demanding, when default results in permanent financial exclusion, higher θ types are unambiguously more tempted to default sooner. In this case, the clean-slate benefit is no longer present, while the loss of insurance becomes permanent. Aside from the budget effects of debt burden versus output costs, the default decision accounts for the loss of future insurance. Intuitively, governments with higher current spending needs are less concerned about the future insurance and, thus, are more prone to default first.

Lemma 2 (High types default first). *When exclusion is permanent, i.e., $\phi = 0$, the default threshold $b^A(\theta)$ is monotonically decreasing in spending needs. Furthermore, if there exists savers, i.e., $\dot{b}(\underline{\theta}, b^A(\underline{\theta})) < 0$, the default threshold is strictly decreasing: $\frac{\partial b^A(\theta)}{\partial \theta} < 0, \forall \theta \in \Theta$.*

Proof. See Appendix B.2. □

As intuitive as Lemma 2 may appear, the fact that we have shut down the benefit

stemming from restarting with a clean slate plays a crucial role in this outcome. When exclusion is temporary, $\phi > 0$, an additional force comes into play. First, the possibility of market reentry makes the lack of insurance only temporary, thereby diminishing the associated costs. Second, once the country reaccess the financial markets, it does so with a clean slate, all financial obligations are erased. As a result, it may no longer be the case the more tempted types are more tempted to default. When financial exclusion is sufficiently short lived, the clean-slate benefit starts to dominate. Since the least tempted types are less concerned about the current output cost and value future outcomes more, they default sooner at a lower level of debt.

Figure 1 illustrates various possibilities that may arise in equilibrium. Panel (a) shows an economy with extreme present bias, as characterized in Lemma 1. Panel (b) presents a case where the clean-slate benefit is absent while the insurance cost is maximized, as in Lemma 2. Finally, panel (c) depicts a scenario where the clean-slate effect plays a dominant role.

The default patterns have a relevant impact on the equilibrium spending patterns. In order to understand this, it is useful to analyze the Euler equation.

Lemma 3 (Euler equation). *Suppose preferences are CRRA with parameter γ . Then, $\forall \theta \in \Theta, b \geq b^A(\theta)$,*

$$\frac{\dot{g}^A(\theta, b)}{g^A(\theta, b)} = \frac{1}{\gamma} \left(\frac{\partial(r(b)b)}{\partial b} - \rho - \lambda + \lambda\beta \frac{-\frac{\partial}{\partial b} \mathbb{E}[v(\theta', b)]}{\theta u'(g(\theta, b))} \right). \quad (11)$$

Proof. See Appendix B.3. □

While equation (11) closely resembles that of a standard consumption-savings problem, there are two key differences. First, due to the presence of default risk, the interest rate is endogenous and reflects the default risk premium, hence the term $\frac{\partial(r(b)b)}{\partial b}$, as opposed to the usual exogenous risk-free rate r_f . The second key difference is the term $\lambda\beta \frac{-\frac{\partial}{\partial b} \mathbb{E}[v(\theta', b)]}{\theta u'(g(\theta, b))}$. Without default choice, or if the default threshold were independent of type, we would have $\frac{\partial}{\partial b} \mathbb{E}[v(\theta', b)] = \mathbb{E}[v_b^n(\theta', b)]$. When $\beta = 1$, it is straightforward that $\theta u'(g(\theta, b)) = -w_b^n(\theta, b) = -v_b^n(\theta, b) > 0$. Thus, this term would capture the incumbent's precautionary savings motive, since the ratio determines the expected future marginal cost of debt relative to the current one. The larger this relative cost, the lower the spending growth.⁹

When $\beta < 1$ and possibility of default, some considerations complicate the analysis. To start with, the intuitive precautionary savings property may be lost. This arises due to the fact that the incumbent could disagree with the default choice of the future government.

⁹In Appendix C.1 we show that, if the lowest type $\underline{\theta}$ is not too small, the saving levels are bounded, which guarantees that the problem is bounded.

Hence, the last term in equation (11) would also capture a future default manipulation effect, making it possible that $-\frac{\partial}{\partial b}\mathbb{E}[v(\theta', b)] < 0$. When this term is positive, the default manipulation effect is still present, but the precautionary savings motive interpretation remains. Still, in this case, two potential distortions arise with respect to an unbiased agent. First, there is a direct effect because of the lower weight $\beta < 1$ multiplying the marginal value of wealth. Second, there is also a dynamic indirect effect because the unbiased agent would choose spending according to its biased marginal cost of debt, as in its optimality condition (9), rather than $\theta u'(g(\theta, b)) = -v_b^n(\theta, b)$.

4 Biases

The presence of overspending biases in environments with hyperbolic discounting and exogenous interest rates is well understood. However, as anticipated in Lemma 3, the spending bias when debt can be repudiated is unclear. Furthermore, the potential bias in default choices has not been analyzed. In this section, we show that such a bias indeed exists and can go in either direction: there can be excessive or insufficient default. Moreover, the overspending bias does not necessarily persist when default is possible, but we provide conditions under which it remains.

4.1 Default bias

To characterize the default bias, it is informative to analyze what an unbiased observer, or a planner, would do if she had perfect information about the current spending needs θ . The unbiased observer who values allocations irrespective of the identity of the government in power would assess the outcomes using the unbiased value functions $v(\cdot)$ rather than $w(\cdot)$. Thus, it is crucial to understand how these two differ. To proceed, we establish the following relation which we uncover in Appendix A1: $\forall \theta \in \Theta, b \geq b^A(\theta)$,

$$\beta [v^n(\theta, b) - v^d(\theta)] = [w^n(\theta, b) - w^d(\theta)] - (1 - \beta) \theta [w_\theta^n(\theta, b) - w_\theta^d(\theta)]. \quad (12)$$

It is clear from equation (12) that as $\beta \rightarrow 1$, the unbiased observer aligns their default decisions with those of the incumbent government. In contrast, when $\beta < 1$, a discrepancy emerges between the decisions. To illustrate, consider a type- θ government who is at its default threshold $b^A(\theta)$ such that $w^n(\theta, b^A(\theta)) = w^d(\theta)$. The discrepancy is determined by the sign of the second term on the right-hand side, $w_\theta^n(\theta, b^A(\theta)) - w_\theta^d(\theta)$, which indicates whether the default threshold $b^A(\theta)$ is locally increasing or decreasing. If the default threshold is decreasing, this term is negative. It follows then that the unbiased observer values

relatively more the non-default path than the defaulting one. Conversely, if the default threshold is increasing, this term is positive, suggesting that the unbiased observer values more the default choice than the biased agent. We say that in the first case, the type- θ government over-defaults, while in the second case, it under-defaults. The following proposition summarizes the two contrasting biases:

Proposition 1 (Default bias). *Suppose $\beta \in (0, 1)$. Consider any type $\theta \in \Theta$,*

- (i) *If $\frac{\partial b^A(\theta)}{\partial \theta} < 0$, then $v^n(\theta, b^A(\theta)) > v^d(\theta)$, indicating that type θ over-defaults at $b^A(\theta)$.*
- (ii) *If $\frac{\partial b^A(\theta)}{\partial \theta} > 0$, then $v^n(\theta, b^A(\theta)) < v^d(\theta)$, indicating that type θ under-defaults at $b^A(\theta)$.*

Proof. See Appendix B.4. □

Why is the default bias tied to the slope of the default threshold $b^A(\theta)$? What does the default pattern inform us about biases? In a nutshell, it captures the dominating concern for default: future insurance loss or a future clean slate. Consider first the case where default leads to permanent financial exclusion $\phi = 0$, so that the clean-slate benefit is absent while the insurance cost is maximized. Recall that the resulting default threshold, as described in Lemma 2, is monotonically decreasing. That is, the more tempted incumbent values less the cost of lost insurance and default first. However, due to its present bias, the incumbent does not fully internalize the insurance value for future governments: there is an *insurance externality* leading to over-default.

When financial exclusion is sufficiently short-lived, the clean-slate effect dominates. Now the less tempted incumbent government, less concerned about the immediate output cost and valuing more the future clean-slate benefit, will default first. Again, due to its present bias, the incumbent does not fully internalize the value of future debt reduction: the *clean-slate externality* leads to under-default. This under-default bias aligns with the insights of Amador (2012), who shows that the political-economy friction responsible for overspending can make the sovereign more inclined to repay, thereby enlarging its debt capacity.

The under-default bias is quantitatively relevant and significant. As stressed by Uribe and Schmitt-Grohé (2017) (see Chapter 13), in the workhorse Eaton and Gersovitz (1981) framework, standard calibrations imply that the exclusion risk is quantitatively negligible. Instead, the main factor determining default choices is not the insurance cost but rather the output cost. Under the prism of our theory, this means that most countries would be defaulting too little, not too much. Incumbent governments, wary of the immediate consequences of a recession, do not properly internalize that defaulting could significantly improve the country's future fiscal position. As these benefits accrue to subsequent governments, incumbents are unwilling to bear the immediate costs and therefore refrain from defaulting.

4.2 Spending bias

To understand the spending bias, we differentiate equation (12) with respect to the debt position: $\forall \theta \in \Theta, b \leq b^A(\theta)$,

$$\beta v_b^n(\theta, b) = w_b^n(\theta, b) - (1 - \beta) \theta w_{\theta b}^n(\theta, b). \quad (13)$$

Recall that agents choose spending such that $\theta u'(g^A(\theta, b)) = -w_b^n(\theta, b)$. Instead, the unbiased planner, if it could observe θ , it would prefer to choose a spending level $g^P(\theta, b)$ such that $\theta u'(g^P(\theta, b)) = -v_b^n(\theta, b)$. We say there is an overspending bias whenever the agent spends more than the unbiased level, $g^A(\theta, b) > g^P(\theta, b)$, which occurs if and only if $-w_b^n(\theta, b) < -v_b^n(\theta, b)$. When $\beta \in (0, 1)$, it is clear from equation (13) that the bias arises and depends on the value of the cross-partial $w_{\theta b}^n(\theta, b)$. In particular, it follows that there is a spending bias if and only if $\theta w_{\theta b}^n(\theta, b) > w_b^n(\theta, b)$. Absent the possibility of default and with exogenous interest rates, it is in general the case that agent overspends. However, when default is possible, the forces become more complex. We have:

Proposition 2 (Spending bias). *Suppose $\beta \in (0, 1)$, then*

- (i) $g^A(\theta, b) > g^P(\theta, b)$ if and only if $g_\theta^A(\theta, b) > 0$.
- (ii) If $\frac{\partial b^A(\theta)}{\partial \theta} < 0$, then there is a spending bias, $g^A(\theta, b) > g^P(\theta, b)$.

Proof. See Appendix B.5. □

The proposition first establishes an equivalence result between the spending bias and the monotonicity of the spending function. This equivalence is intuitive, if more tempted types want to spend more, due to the bias $\beta < 1$, they would be overspending. Indeed, in settings with exogenous interest rates, this result is straightforward. Hence, the reader may wonder why the condition in part (ii)? Is it possible that the agents may not display a spending bias but instead a savings bias?

The answer is less straightforward due to potential default manipulation through spending whenever the default decision is not optimal. Bear in mind that the agents, although biased, are sophisticated. They want to succumb to temptation today but would like future agents to make the right choices. When there is over-default, agents tend to restrain themselves to prevent a future default. When the current spending needs are low, this restraining effect is larger. From the perspective of an unbiased planner, the same mechanism operates but stronger. Therefore, the spending pattern $g^A(\theta, b)$ is not affected relative to $g^P(\theta, b)$, and the spending bias is preserved.

When there is under-default, the manipulation effect acts in the opposite direction. Now agents have an extra incentive to spend to increase the likelihood of a future default. This effect is larger when the current spending needs are low and could potentially revert the agent’s spending pattern, with $g_{\theta}^A(\theta, b) \leq 0$. From the unbiased perspective, again the same mechanism operates, but since the manipulation effect is stronger, the planner could prefer to spend even more. Thus, it is unclear whether there is a spending bias or a savings bias.

5 Optimal rules

In this section, we characterize the constrained efficient allocation and show that it can be implemented by spending and default rules that restrict the government’s actions. We then establish the specific forms of rules for some combinations of model parameters. In this analysis, we focus on economies that exhibit an overspending bias, while the default bias can go in either direction.

5.1 Delegation problem

We consider a planner, or rule-writer, who maximizes the ex ante social welfare—before the information about θ is realized—for each financial position. We follow [Piguillem and Shi \(2025\)](#) who, in a discrete-time environment without default, define an overlapping planner’s problem, where in every period the incumbent must abide by the rules imposed by the previous government and can choose history-independent rules that would constrain the next one. They show that the best an incumbent-planner can do in that setting is to maximize $\mathbb{E}[V(\theta, b)]$. This result clearly extends to the continuous-time limit. This implies that the planner weights all future governments decisions equally seeking correct their biases. Instead of allowing for future full discretion to choose spending and default, the planner chooses the allocation $\mathcal{A} = \{g(\theta, b), \delta(\theta, b)\}_{\theta \in \Theta, b \in \mathbb{R}}$. Specifically, it chooses $\mathcal{A}(b) = \{g(\theta, b), \delta(\theta, b)\}_{\theta \in \Theta}$ to maximize $\mathbb{E}[v(\theta, b; \mathcal{A})]$ at every debt level b .

We assume that the spending needs, while observable, are nevertheless *non-contractible*.¹⁰ For example, one may think that it would be politically infeasible to write a rule that constrains a specific political party. If spending needs were contractible, the planner would be able to choose its desired contingent allocation and force governments to follow it. The non-

¹⁰Alternatively, it is commonly assumed in the literature that types are unobservable (see [Amador, Werning and Angeletos, 2006](#)). This assumption is also compelling, given that the government is likely to be better informed about its own spending needs. In many settings, this is equivalent to our non-contractibility assumption. With the possibility of default, we opt for non-contractibility to avoid potential technical complications associated with lender’s learning.

contractibility renders it impossible to use this information and, instead, the planner must rely on self reports. Thus, the optimal rules solve a principal-agent problem: the principal chooses an allocation contingent on $\{\theta, b\}$, subject to the truthful revelation. Hence, we refer to the allocation as a mechanism. Furthermore, we rule out monetary transfers, making the problem effectively a delegation problem, with a meaningful trade-off between discretion, to smooth out the shocks, and rules to correct for the biases.

To formalize the problem, some modifications to the previous value functions are required. By reporting $\tilde{\theta}$, a government of type θ with debt b obtains a value $w(\tilde{\theta}, \theta, b; \mathcal{A})$ given by:

$$w(x, \theta, b; \mathcal{A}) = (1 - \delta(\tilde{\theta}, b))w^n(\tilde{\theta}, \theta, b; \mathcal{A}) + \delta(\tilde{\theta}, b)w^d(\tilde{\theta}, \theta; \mathcal{A}),$$

where $w^n(\tilde{\theta}, \theta, b; \mathcal{A})$ and $w^d(\tilde{\theta}, \theta; \mathcal{A})$ satisfy the following HJB equations:

$$(\rho + \lambda)w^n(\tilde{\theta}, \theta, b; \mathcal{A}) = \theta u(g(\tilde{\theta}, b)) + \dot{b}(\tilde{\theta}, b)w_b^n(\tilde{\theta}, \theta, b; \mathcal{A}) + \lambda\beta\mathbb{E}[v(\theta', b; \mathcal{A})] \quad (14)$$

$$(\rho + \lambda + \phi)w^d(\tilde{\theta}, \theta; \mathcal{A}) = \theta u(\kappa\tau) + \phi w^n(\tilde{\theta}, \theta, 0; \mathcal{A}) + \lambda\beta\mathbb{E}[v^d(\theta'; \mathcal{A})]. \quad (15)$$

Equations (14) and (15) are slight modifications of (5) and (7) with two notable differences. First, the value functions are evaluated at the planner's proposed allocation rather than the governments' discretionary choices. Second, each government can misrepresent its type as an alternative $\tilde{\theta}$. Hence, we introduce the report, $\hat{\theta}$, as an additional argument in the value functions. Notice that the incumbent takes the truthful reporting of future governments as given. For this reason, when a future government comes into power, the continuation values, $\mathbb{E}[v(\theta', b; \mathcal{A})]$ and $\mathbb{E}[v^d(\theta'; \mathcal{A})]$, remain unaltered with respect to (5) and (7), except that they are now evaluated at choices imposed by the planner rather than the government's discretionary choices. To be precise, the value functions are conditional on the mechanism, $w(\tilde{\theta}, \theta, b; \mathcal{A})$, $v(\theta, b; \mathcal{A})$. We sometimes omit the term $\mathcal{A}$ for notational ease, but readers should bear in mind that these values are determined by the entire mechanism.

We now formally state the planner's problem. At each debt level b , the planner chooses the allocation $\mathcal{A}(b)$ to maximize the ex ante expected social value:

$$\max_{\mathcal{A}(b)} \mathbb{E}[v(\theta, b; \mathcal{A})] \quad (16)$$

subject to

$$w(\theta, \theta, b; \mathcal{A}) \geq w(\tilde{\theta}, \theta, b; \mathcal{A}), \forall \theta, \tilde{\theta} \in \Theta \quad (17)$$

$$r(b; \mathcal{A}) \text{ given by equation (1)}. \quad (18)$$

Equation (17) is the incentive-compatibility condition: type θ prefers to reveal its true type

rather than imitating any other type. For convenience, after imposing truth-telling, we adopt the notation $w^i(\theta, b; \mathcal{A}) = w^i(\theta, \theta, b; \mathcal{A})$, for $i \in \{n, d\}$. Condition (18) captures that the planner accounts for market interest-rate responses to its choices.

Remarks on planner’s problem. Two features of problem (16) are worth mentioning.

First, since the *planner maximizes the ex ante social value*, it is implicitly allowed to transfer the ex post utility across types by manipulating the interest rate. This happens due to market incompleteness. When a type θ defaults, it does not internalize that its action increases the interest rate that other types must pay at the same debt level. This gives rise to an *interest-rate externality*. Recall that in Section 4.1, from an ex post perspective, when there is no present bias the default decision is efficient state by state, as in Aguiar and Amador (2019). In contrast, from an ex ante perspective, even when there is no present bias, the planner may find it desirable to intervene, as we summarize in the following lemma.

Lemma 4 (Interest-rate externality). *If the principal had perfect information, it would default when debt exceeds $b^P(\theta)$, which satisfies:*

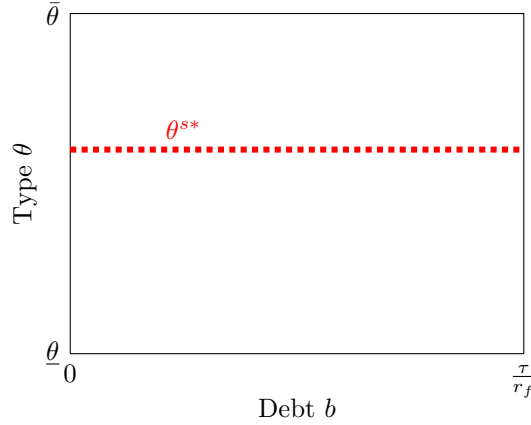
$$v^n(\theta, b^P(\theta)) = v^d(\theta) + \lambda \mathbb{E} \left[\frac{\partial v^n(\theta', b^P(\theta))}{\partial r(b^P(\theta))} (1 - \delta^P(\theta', b^P(\theta))) \right]. \quad (19)$$

Proof. See Appendix B.6. □

Even if there were no bias and θ were contractible, the planner’s default decision would differ from the government, $\delta^P(\theta, b) \neq \delta^A(\theta, b)$ and $b^P(\theta) \neq b^A(\theta)$. To see this, note that when $\beta = 1$, $v^n(\theta, b) = w^n(\theta, b)$ and $v^d(\theta) = w^d(\theta)$. Since a government would default when $w^d(\theta) > w^n(\theta, b)$ or, equivalently, $v^d(\theta) > v^n(\theta, b)$, the calculation in equation (19) differs by taking into account the interest-rate effect: if one type defaults, it increases the interest rate that lenders charge to all other types that do not default.

Second, the history independence of the mechanism renders it static. This restriction is potentially without loss of generality in some settings. Indeed, as established by Halac and Yared (2014), when default is not a possibility, and with a logarithmic utility function, the optimal mechanism is static if the spending needs θ is *i.i.d* over time. As long as $\lambda < \infty$, this equivalence may not hold in our setting. Moreover, the default choice brings out the possibility of price manipulation. By committing to future allocations, which may turn out to be suboptimal ex post, the planner could manipulate future interest rates in ways that are beneficial from the initial perspective. See for instance Adam and Grill (2017) and Hatchondo, Martinez and Roch (2022). We leave it to future research to investigate whether the optimal mechanism remains static when default is possible.

Figure 2: Spending rule in the no-default benchmark



Although history dependence is appealing in some regards, it raises many questions about the sustainability of such policies that could render them impractical. In addition, the impact of a price-manipulation strategy could be minimal depending on the utility function.¹¹ Our static mechanism has the advantage that the planner does not have incentives to alter it ex post and, thus, it is *time-consistent* and with straightforward implementation.

No-default benchmark. Before proceeding, it is instructive to consider a well-understood benchmark economy in which there is no default risk. We do so by making the default cost prohibitively high, i.e., $\kappa = 0$, such that no government would ever find it desirable to default. In this case, the government can now borrow up to the natural borrowing limit $\frac{\tau}{r_f}$. The following lemma presents the benchmark spending rule illustrated in Figure 2.

Lemma 5 (No-default benchmark: AWA rule). *When default is not possible, for all debt levels $b < \frac{\tau}{r_f}$, there exists a debt-independent spending threshold θ^{s*} such that discretion is granted below the threshold and rules are imposed above.*

- (i) *If the present bias is severe $\beta \leq \underline{\theta}$, all discretion is taken away.*
- (ii) *If the present bias is mild $\beta > \underline{\theta}$, the spending threshold is characterized by:*

$$\theta^{s*} = \beta \mathbb{E}[\theta | \theta \geq \theta^{s*}]. \quad (20)$$

Proof. See Appendix B.7. □

¹¹For instance, with the logarithmic utility function, the income and substitution effects of interest-rate changes could cancel out, leaving allocations unaffected.

The threshold in equation (20) is identical to the one in Amador, Werning and Angeletos (2006), hereafter referred to as the AWA rule. Assumption 1 ensures that it is uniquely defined. The threshold is increasing in β : the less present-biased the governments are, the more discretion is allowed. In one extreme, with a severe present bias, i.e., $\beta \leq \underline{\theta}$, the rule-maker takes away all discretion. In the other extreme, absent a present bias, i.e., $\beta = 1$, full discretion is allowed.

It may appear puzzling that the formula in (20) does not involve the turnover rate λ . After all, political turnover is at the root of the friction, leading to the need for rules. A higher turnover rate λ increases the frequency at which the political friction β impacts the economy, calling for stricter rules, but it also increases the frequency of the genuine spending shocks, requiring more insurance and thus looser rules. The two effects cancel out, rendering the spending threshold independent of the turnover rate.

5.2 Optimal allocation

We now present the key proposition of the key paper, which fully characterizes the optimal allocation that solves problem (16). It provides three thresholds that delineate the state of economy in which discretion is granted and other states in which rules—spending or default rules—must be imposed.

Proposition 3 (Optimal allocation). *There exists a lower bound $\underline{b}$ and an upper bound $\bar{b}$, with $\underline{b}^A \leq \underline{b} \leq \bar{b} \leq \bar{b}^A$, and a debt-dependent spending threshold $\theta^s(b) \in \Theta$ for all $b \leq \bar{b}$.*

- (i) *The spending rule $g(\theta, b)$ allows for discretion below the spending threshold and imposes rules above: $\forall b \leq \bar{b}$,*

$$g(\theta, b) = \begin{cases} g^A(\theta, b), & \forall \theta \leq \theta^s(b) \\ g^A(\theta^s(b), b), & \forall \theta \geq \theta^s(b); \end{cases} \quad (21)$$

- (ii) *The default rule forbids default at low debt levels, forces default at high debt levels, and allows for discretionary default at intermediate debt levels: $\forall \theta \in \Theta$,*

$$\delta(\theta, b) = \begin{cases} 0, & \forall b \leq \underline{b} \\ \delta^A(\theta, b), & \forall b \in (\underline{b}, \bar{b}] \\ 1, & \forall b > \bar{b}, \end{cases} \quad (22)$$

where the lower bound $\underline{b}$ is the highest debt level that satisfies:

$$\mathbb{E}[v^n(\theta, \underline{b})] = \mathbb{E}[v^n(\theta, \underline{b})(1 - \delta^A(\theta, \underline{b})) + v^d(\theta)\delta^A(\theta, \underline{b})], \quad (23)$$

and the upper bound $\bar{b}$ is the lowest debt level that satisfies:

$$\mathbb{E} [v^d(\theta)] = \mathbb{E} [v^n(\theta, \bar{b}) (1 - \delta^A(\theta, \bar{b})) + v^d(\theta) \delta^A(\theta, \bar{b})]. \quad (24)$$

Proof. See Appendix B.8. □

Proposition 3 is simple and powerful. It states that all that the planner can do to improve outcomes is to determine the state of the economy in which the governments are free to follow their preferred choices, while in the remaining states, they must abide by the imposed rules.

Part (i) divides the state of government types into two regions: a discretion region and a rules region. Incumbents claiming sufficiently low spending needs are granted discretion: they can choose their desired spending, i.e., $g(\theta, b) = g^A(\theta, b)$. However, for incumbents claiming spending needs that exceed the threshold, they are bound by a predetermined spending limit. This holds regardless of whether debt is safe or risky, thus nesting the non-default benchmark characterized in Lemma 5. However, when default risk arises, the threshold depends on the country's indebtedness; hence, the planner could tighten or loosen the degree of discretion granted as debt accumulates. We analyze these possibilities in Section 5.4, but a priori, the adjustment can go in either direction.

Similarly part (ii) divides regions of debt into those where default is restricted and those where governments can default discretionarily. At debt levels where the planner would like some types to default but not all, i.e., $b \in (\underline{b}, \bar{b}]$, the only incentive-compatible default rule is to allow for discretionary default, $\delta(\theta, b) = \delta^A(\theta, b)$. This is due to the binary nature of the default choice, making it impossible for the principal to alter the agent's default behavior, as types that prefer to default can always pretend to be a default type and vice versa. Thus, the rule can only forbid or force default altogether. Combining this with the insights of Proposition 1, the optimal default policy generally features three debt regions, although some regions may be degenerate: forbidding default at low debt levels, forcing default at high debt levels, and allowing the government to default at its own discretion at intermediate debt levels. This pattern holds true independently of the spending rules in place, which can affect government default incentives at the margin but not the nature of the default rules.

The debt bounds for the default intervention regions characterized in equations (23) and (24) are intuitive. When the government over-defaults at low levels of debt, forbidding debt is desirable, until we reach a debt level such that allowing for discretionary default starts to deliver the same expected social value as forbidding default, as stated in equation (23), before it outweighs. When the government under-defaults at high levels of debt, forcing default is desirable as soon as debt reaches a level such that forcing default yields the same expected social value as discretionary default, as stated in equation (24).

The existence of the “default-forbidden” region is not inconsequential. Unlike the rules-free equilibrium, such an intervention generates a discontinuous jump in interest rate, which in turn generates the government’s discretionary spending decisions. Specifically, as long as debt remains below $\underline{b}$ the risk premium is zero, but once debt surpasses $\underline{b}$, a positive measure of types default and the interest rate jumps above the risk-free rate, so $\lim_{b \downarrow \underline{b}} r(b) > r_f$. In the steps solving problem (16), we have appealed to the first-order approach to characterize the spending policy. This approach requires the spending function $g(\theta, b)$ to be differentiable. As we formalize in Lemma B2 in Appendix B.11, the discontinuous jump occurs only at one specific level of debt, ensuring that $g(\theta, b)$ remains differentiable almost everywhere. The value functions, despite exhibiting a kink at $\underline{b}$, remain continuous.

Implementation. The implementation of the allocation and rules characterized in Proposition 3 warrants a discussion. The implementation of the spending rules in part (i) is straightforward and can be achieved through commonly observed fiscal rules. For example, a debt-contingent spending cap or a debt-contingent deficit limit will do. Keeping the rules-free equilibrium of Section 3 in mind, adding the constraint $g(\theta, b) \leq g^A(\theta^s(b), b)$, $\forall \theta \in \Theta, b \leq \bar{b}$, to the agent’s maximization in equation (5) would achieve the desired outcome.¹² Low types would be unconstrained, choosing their preferred spending, while high types would be bound by the constraint and only spend $g^A(\theta^s(b), b)$.

The implementation of the default rules perhaps raises more questions. Taken literally, it implies that either the constitution or a special law, requiring a supermajority to be overturned, forbids default when the debt level $b \leq \underline{b}$, or forces it when $b > \bar{b}$. These kinds of rules are rare, if not completely absent, in the currently observed set of “fiscal rules.” Their unusual existence should not be a deterrent to future implementation. Moreover, the rule-writer could use indirect mechanisms without explicitly forbidding or mandating default but nevertheless achieve the desired outcomes.

For example, to prevent default, a fiscal rule could require that whenever the debt level is below, say, 60% of GDP, the payments of debt services must have priority in the budget before any funds can be allocated to spending. Once this debt threshold is exceeded, the government could freely allocate spending, including the possibility of not paying the debt obligations. Such rules are, for example, observed in Bulgaria’s Public Finance Act, which states that any interest and principal payables related to government debt shall constitute a priority liability for the state budget.¹³ Alternatively, an indirect implementation is to

¹²The same fiscal rule can be implemented with a deficit limit imposing $\dot{b}(\theta, b) \leq \dot{b}^A(\theta^s(b), b)$, $\forall \theta \in \Theta, b \leq \bar{b}$.

¹³Bulgaria’s Public Finance Act states that “any interest and principal payables related to government debt shall constitute a priority liability for the state budget” and that “any interest and principal payables related to municipal debt shall constitute a priority liability for the municipal budget.” See Chapter Two:

relax spending rules to incentivize the sovereign not to default when it enters the default risk region. We revisit this alternative implementation in Sections 5.4 and 6.3.

A rule that forces default resembles a debt limit. For instance, this rule can be established by a special law stating that any debt above a certain limit, say 125% of GDP, would not be recognized as a legitimate obligation and would be considered outright illegal. This creates an off-equilibrium path “threat” that any debt exceeding this limit would be defaulted on. Lenders would then respond by refraining from extending additional funding, thereby creating a de facto hard debt limit. This limit would be tighter than the limit that would be imposed absent the rule. Again, there is an indirect implementation involving a draconian spending or deficit limit that leaves the incumbent no choice but to default.

Proposition 3 also implies an *optimal convergence path* if the initial debt happens to be outside the “desired” range. As an example, the European Fiscal Compact states that the debt-to-GDP ratio of the member countries cannot exceed 60% of GDP and that the deficit should be no more than 3% of GDP. When a country exceeds the 60% threshold, a “Debt-Brake Rule” is triggered, which essentially tightens the deficit limit to a 1% surplus. Since many European countries are currently above the 60% mark, the agreement has triggered a growing literature that, taking the target as given, studies the optimal debt path toward it. Such an approach is faulty or partial at best. It is impossible to study the optimal convergency path without incorporating into the framework the reason that gives rise to the target. They go hand in hand.

Finally, through the lens of our theory, the European Fiscal Compact is “incomplete” in the sense that it does not provide guidelines regarding the course of action when facing default decisions. Is it implicit in the rule that governments with a debt level below 60% of GDP cannot default? How far above the 60% mark is tolerable? Our theory dictates that beyond a certain level of indebtedness, instead of bailing out the sovereign, it should be forced to default to offload the debt and immediately converge to a sound financial position.

5.3 Default rules

We now further characterize the default rules of Proposition 3. Recall that in the rules-free equilibrium, one possible outcome is a market-imposed endogenous borrowing limit. This arises when government present bias is extreme as described in Lemma 1. In this case, Lemma 5 applies. The planner removes all discretion to spend and imposes a uniform spending plan for all types, which in turn equalizes the default incentive across types. Hence, the market ends up imposing an endogenous borrowing limit b^A . This limit may not be optimal from the

Fiscal Rules, articles 38 and 40, at https://www.minfin.bg/upload/12587/Public_Finance_Act-EN.pdf.

planner's perspective, in which case fiscal rules should aim to adjust the debt capacity. To do so, the planner imposes a *debt limit rule*. The rule states that a government can borrow only up to the limit and never default before reaching this limit. If the government happens to start with a debt level exceeding the limit, it must default. In parallel to Lemma 1, the following proposition describes conditions under which a debt limit is optimal.

Proposition 4 (Optimal debt limit). *When present bias is extreme $\beta = 0$, the optimal default rule is a debt limit b^P . Then:*

- (i) (Permanent exclusion). *If $\phi = 0$, the optimal debt limit coincides with the one imposed by the market, $b^P = b^A = \frac{1}{r_f}(1 - \kappa)\tau$.*
- (ii) (Temporary exclusion). *If $\phi > 0$, the market sets an excessively high borrowing limit. The planner imposes a tighter limit, $b^P < b^A < \frac{1}{r_f}(1 - \kappa)\tau$.*

Proof. See Appendix B.9. □

To understand the optimality of the market-imposed limit, recall that the discrepancy in the default incentives between the planner and the agent manifests in two ways. When assessing the dynamic effects of default, which are backloaded to the future, the incumbent undervalues the cost associated with lost insurance during financial exclusion, but it also undervalues the potential benefit of starting with a clean slate upon reentering the market.

When default results in permanent financial exclusion, case (i), which eliminates the clean-slate benefit, it turns out that the default incentives between the planner and the agent are aligned. In terms of the budget effects of default, the planner's assessment coincides with the agent's. Once in default, the economy falls into permanent autarky with spending fixed to $\kappa\tau$. At the borrowing limit, the economy also faces a “permanent” constant spending $\tau - r_fb^P$. Comparing these two constant streams of consumption implies an optimal debt capacity of $\frac{1}{r_f}(1 - \kappa)\tau$, which aligns with the market outcome.

However, when exclusion is temporary, case (ii), the agent defaults too late. This discrepancy is driven by the clean-slate externality. A myopic government is reluctant to incur the immediate costs of default, since the benefits of starting with a clean slate are likely to accrue to a future one. As a result, the government holds on to too much debt, and the market is willing to extend it. To mitigate this bias, the fiscal rule forces the government to default when debt surpasses the desirable limit. Naturally, this rule would tighten the debt capacity in the first place, as the market responds by extending less debt.

The debt limit result is informative about the directions towards which the planner wishes to intervene, but it abstracts from the risk-premium effect. In reality, the risk premium is

prevalent and the main subject of policymakers' concern. We now examine cases in which the present bias is milder and default happens on the equilibrium path.

Proposition 5 (Forbid or force default). *Suppose $\beta \in (\underline{\theta}, 1)$, then it is optimal to:*

- (i) *Forbid default. If $b^A(\theta)$ is monotonically decreasing, the debt bounds are $\underline{b} > \underline{b}^A$ and $\bar{b} = \bar{b}^A$. For all $b \in [\underline{b}^A, \underline{b})$, discretionary default generates $\mathbb{E}[v(\theta, b)] < \mathbb{E}[v^n(\theta, b)]$.*
- (ii) *Force default. If $b^A(\theta)$ is monotonically increasing, the debt bounds are $\underline{b} = \underline{b}^A$ and $\bar{b} < \bar{b}^A$. For all $b \in (\bar{b}, \bar{b}^A]$, discretionary default generates $\mathbb{E}[v(\theta, b)] < \mathbb{E}[v^d(\theta)]$.*

Proof. See Appendix B.10. □

This proposition presents directions of default intervention for two distinctive cases, in parallel to Proposition 1. Figure 3 illustrates these cases. The blue line depicts the desired discretionary default threshold by agent, the dashed black line what the planner would like it to be, and the black continuous line the optimal default rule. For completeness, we also plot there the optimal spending rule, with red dotted lines, that we discuss in the next section.

Case (i) is illustrated in panel (a). When high need types default first, the government's default incentive is predominantly driven by the insurance effect. Due to its present bias, the incumbent does not fully internalize the insurance value for future governments, leading to over-default. This misalignment gives rise to a debt region where forbidding default dominates the outcome with discretionary default. The forces reverse in case (ii), illustrated in panel (b). When low need types default first, the clean-slate effect dominates. In this case, the incumbent does not fully internalize the clean-slate benefit for future governments, leading to under-default. Hence, there is a debt region where forcing default dominates the outcome with discretionary default.

5.4 Spending rules

With default risk, the characterization of the spending threshold becomes more involved compared to the benchmark in Lemma 5, precluding the possibility of a closed-form solution. Nevertheless, we present an implicit characterization. We describe the formulas for economies with monotonic default patterns, although the result can be extended to non-monotonic cases. The following proposition considers the case with a monotonically decreasing default threshold $b^A(\theta)$.¹⁴ When the default threshold is monotonically decreasing, there exists a default threshold type $\theta^d(b)$ such that types $\theta \in [\underline{\theta}, \theta^d(b)]$ prefer not to default at b . For notational ease, in the proposition, we abbreviate the thresholds as θ^d and θ^s , but readers should bear in mind that they are debt dependent.

¹⁴Proposition B1 in the appendix addresses the case where default is monotonically increasing.

Proposition 6 (Spending threshold). *Suppose the default threshold $b^d(\theta)$ is monotonically decreasing. Then, $\forall b \geq \underline{b}$, the spending threshold is characterized by:*

$$\begin{aligned}
0 = & \chi(b) \left(\underbrace{\beta \mathbb{E} [\theta | \theta^s \leq \theta \leq \theta^d]}_{\text{Selection effect} < 0} - \theta^s \right) \\
& + \underbrace{\left\{ \int_{\underline{\theta}}^{\theta^s} [v_b^n(\theta, b) - w_b^n(\theta, b)] \frac{\partial g(\theta, b)}{\partial r(b)} dF(\theta) \right\}}_{\text{Discipline effect} > 0} + \underbrace{\left\{ \int_{\underline{\theta}}^{\theta^s} [g(\theta, b) - g(\theta^s, b)] \frac{\partial v_b^n(\theta, b)}{\partial r(b)} dF(\theta) \right\}}_{\text{Insurance effect} < 0} \\
& + \underbrace{b \int_{\underline{\theta}}^{\theta^d} v_b^n(\theta, b) dF(\theta)}_{\text{Income effect} < 0} + \underbrace{\frac{1}{\lambda} \left[\dot{b}(\theta^s, b) v_b^n(\theta^d, b) - (\rho + \lambda) (v^n(\theta^d, b) - v^d(\theta^d)) \right]}_{\text{Default manipulation}} \underbrace{\left\} \frac{\partial r(b)}{\partial \theta^s}}_{< 0},
\end{aligned} \tag{25}$$

where $\chi(b) \geq 0$.

Proof. See Appendix B.11. □

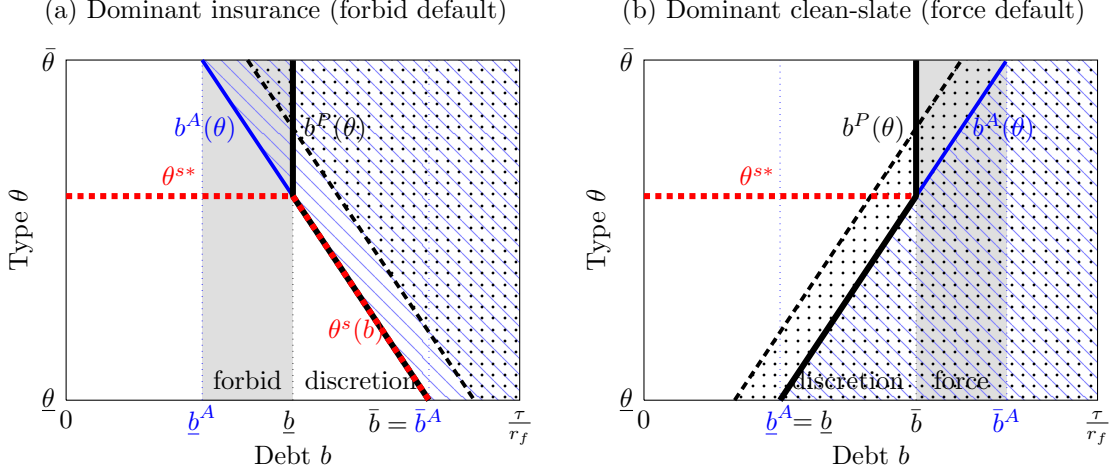
Equation (25) describes five components determining the spending threshold. This spending threshold is a “modified” AWA threshold plus additional terms that only appear when there is default risk. At debt levels $b < \underline{b}$, the risk of default is zero. Thus, the formula simplifies to the one in equation (20). In addition, for the spending rule to be meaningful, it must apply to governments that have a spending choice to make. A corner solution $\theta^s(b) \geq \theta^d(b)$ would render the rule innocuous, since all constrained types have defaulted.

The first component is akin to equation (20), but now the conditional expectation is over the non-default types instead of all types. This selection effect alone would generate a threshold smaller than θ^{s*} . Since the planner knows that only less tempted types remain not defaulting, it can tighten the spending limit.

The second and third components are similar to those in Halac and Yared (2018) and have an ambiguous combined effect. The disciplining effect tends to tighten the spending rule. A tighter spending rule pushes more types to default, which in turn increases the interest rate and reduces overspending. The *insurance effect* works in the opposite direction: relaxing the spending rule leads to better insurance.

The fourth one is what we term the income effect, while Halac and Yared (2018) call it the redistribution effect. In their environment, in a global economy, changes in the interest rate have redistributive effects across countries. Since their bond is in zero net supply, there is no “aggregate” debt effect. In our environment, instead, relaxing spending rules help to reduce the risk premium, and in turn the debt burden for the country, who must transfer resources to foreign lenders. This effect pushes for lifting the spending rule.

Figure 3: Illustration of spending rules



Last but not least, the final component is default manipulation. Although it is difficult to determine its direction in general, when the default threshold is monotonic, we can do so. In this case with monotonically decreasing $b^A(\theta)$, since there is excessive default, it is desirable to relax the spending rule to incentivize governments not to default. As we uncover in Section 6, this effect appears to be most important quantitatively, pushing the spending rule to the corner.

When $b^A(b)$ is monotonically increasing, the formula presented in the companion Proposition B1 resembles the one here, but some of the forces are reversed. One difference is that the selection effect is instead the high types, so that the rule, as in AWA, bunches the top of the distribution. Importantly, the default manipulation effect pushes toward tightening the spending rule to incentive governments to default.

Figure 3 illustrates how the default manipulation effect affects the spending rule. Panel (a) presents a case where the discretionary default $b^A(\theta)$, shown by the blue solid line, is monotonically decreasing. The red dotted line corresponds to the optimal spending rule. Absent default risk, including in debt regions where default is forbidden, the standard AWA rule is optimal. When default risk starts to arise, all the effects discussed in Proposition 6 come into play. If the default manipulation effect is large enough, it can lead to a corner solution. Since the rule is consequential only for non-default types and there is over-default, the spending rule is fully relaxed to incentivize the government not to default, generating $\theta^s(b) = \theta^d(b)$. In panel (b), instead, there is under-default. Absent default risk, the optimal spending rule again coincides with the AWA rule. When the risk starts to operate, the default manipulation effect might suggest a more stringent spending rule to incentivize default, but a combination of opposing forces is at play.

6 Quantitative explorations

In this section, we aim to understand some quantitative implications of our theory and to explore alternative patterns for which we do not have full theoretical results. Our goal is general, not focused on a particular country. Thus, the calibration strategy is directed to generate special cases rather than to replicate any empirical regularity. In this context, we also analyze the relative relevance of fiscal and default rules in terms of their impact on quantities and welfare. Therefore, these results should be interpreted as guidelines for future research, rather than conclusive answers regarding any particular dimension of the problem.

6.1 Parameterization and regimes

We specify a constant relative risk aversion (CRRA) utility function for spending, with a risk aversion parameter γ . The spending needs follow a uniform distribution in the domain $[\underline{\theta}, \bar{\theta}]$, where $\bar{\theta} = 1 - \underline{\theta}$ and the expected value is normalized to 1. Given these functional forms, the model comprises of nine parameters, which we divide into two sets: a set of common parameters $\{\gamma, \rho, r_f, \tau, \lambda, \underline{\theta}\}$ and a set of case-specific parameters $\{\beta, \kappa, \phi\}$. As our theoretical characterization shows, the main element shaping equilibrium outcomes—both with and without rules—is the default pattern. This pattern can be monotonically decreasing, monotonically increasing, or non-monotonic. Accordingly, we consider these three regimes.

Table 1 reports the parameters, calibrated at an annual frequency. We set the risk aversion parameter γ to 1, allowing comparisons with closed-form solutions in an economy without default.¹⁵ As is customary, we set the risk-free rate r_f to match an annual risk-free rate of 4%. The discount rate ρ is calibrated to equal to the risk-free rate, so that in a deterministic economy the optimal debt level would remain constant. The tax revenue τ corresponds to the size of government revenue as a percentage of GDP. We choose $\tau = 0.45$ to represent European economies, although this parameter is inconsequential for the results. We use a turnover rate λ of $1/8$, which implies that a government lasts, on average, 8 years in power.¹⁶ Finally, we set $\underline{\theta} = 0.3$ so that there is substantial variation in spending needs.

6.2 Rules-free equilibria

The remaining three parameters, $\{\beta, \kappa, \phi\}$, are chosen to generate three distinct equilibrium configurations. In the first and simplest one, the insurance effect dominates.

¹⁵To understand the implications of risk aversion in a similar environment see Önder and Sunel (2020).

¹⁶This moderate turnover rate ensures that the outcome described in Lemma A2 does not occur.

Table 1: Parameters

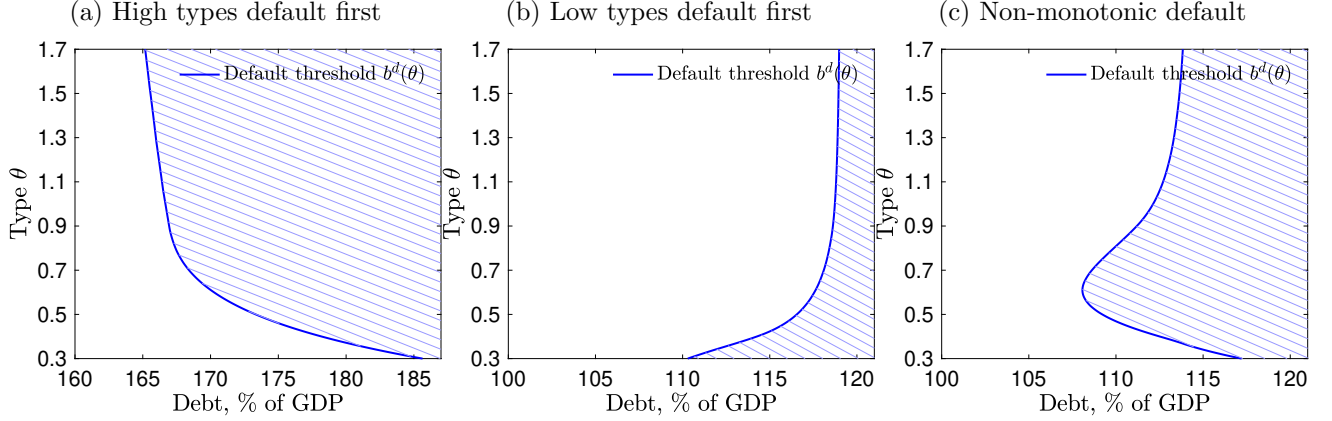
Parameter	Regime 1	Regime 2	Regime 3
	Mild bias & permanent exclusion	Severe bias & easy re-access	Mild bias & easy re-access
<i>Common parameters</i>			
CRRA γ		1	
Discount rate ρ		0.04	
Risk-free rate r_f		0.04	
Tax revenue τ		0.45	
Turnover rate λ		1/8	
Spending preference θ		$U(0.3, 1.7)$	
<i>Regime-specific parameters</i>			
Present bias β	0.8	0.4	0.8
Output loss κ	0.87	0.7	0.7
Re-access rate ϕ	0	0.1	0.1

Regime 1: Dominant insurance externality. We first consider a regime with mild present bias, $\beta = 0.8$, so that the default region is not degenerate. Furthermore, by assuming permanent exclusion, $\phi = 0$, and a mild output loss, $\kappa = 0.87$ (equivalent to a 13% output loss), the default cost is front-loaded, offering no future benefits.

The resulting equilibrium default pattern is displayed in Panel (a) of Figure 4. The blue continuous line depicts the default threshold: the level of debt at which each type θ would default. For instance, if the debt level is below 165% of GDP, no type would default. When debt reaches 165% of GDP, the type with the highest spending needs ($\theta = 1.7$) defaults and, as debt accumulates further, even types with lower spending needs start to default. At a debt level around 185% of GDP, default occurs with certainty, implying the maximum debt that the market is willing to extend. Notice that this happens in a monotonic fashion, first the most tempted types default and as debt increases additional types of lower needs default too: the default threshold $b^d(\theta)$ is monotonically decreasing in θ .

In this case, the insurance effect is dominant. With the calibrated parameters, when debt reaches 165% of GDP, the interest payments amount to $r_f b^A / \tau = 0.04 \times 1.65 / 0.45 \approx 15\%$ of budget. Instead, whenever a government defaults the available resources are reduced by $1 - \kappa = 13\%$. If the insurance effect were absent, as it would be for a hypothetical type with infinitely high spending needs, $\theta \rightarrow \infty$, this government would default as soon as the debt burden exceeds the output cost. In this calibration, the highest type is bounded, so it is still concerned about the lack of future insurance. For this reason, even the most tempted type is willing to bear a debt burden in excess of the output cost. The lower the spending needs,

Figure 4: Rules-free equilibrium



Note: The calibration in each panel corresponds to columns (1), (2), and (3) in Table 1.

the higher the debt burden a government is willing to bear.

The interest-rate channel contributes to the observed non-linearity of $b^d(\theta)$: as more types default, the risk premium rises, increasing the debt burden for types that continue to service the debt. Of course, when a type- θ government contemplates default, it only weights its own costs and benefits, not internalizing that the implied default risk increases the burden for other types. This “externality” is one of the inefficiencies the planner seeks to correct.

The pattern can drastically reverse when exclusion is temporary:

Regime 2: Dominant clean-slate externality. Suppose the present bias is substantial but not extreme, with $\beta = 0.4 > \underline{\theta}$, ensuring a non-degenerate default region. After default, governments can reaccess financial markets at rate $\phi = 0.1$.¹⁷ To generate a comparable level of sustainable debt, we assume a larger output loss upon default, setting $\kappa = 0.70$.

In panel (b) of Figure 4, the equilibrium default pattern is reversed relative to panel (a). The default threshold $b^d(\theta)$ remains monotonic but is now increasing in type: the government with lower needs wants to default first. This reversal is driven by the likelihood of regaining financial market access. The previous budgetary consideration comparing the debt burden with the output loss remains, but now a government must also consider the future benefit of returning to the market with a zero-debt position after defaulting. This benefit is captured by the term $\phi (w^n(\theta, 0) - w^d(\theta))$ in equation (7), which depends on the reaccess rate ϕ . We can make the same budgetary calculation as in the previous case comparing the debt burden with the cost of default. In panel (b), when debt reaches 110% of GDP, the interest payment

¹⁷Cruces and Trebesch (2013) document that the financial exclusion duration depends on the haircut size, with restructuring involving higher haircuts associated with longer periods of exclusion. They estimate that a haircut of 60% or above implies a probability of remaining excluded after 10 years is slightly over 50%.

burden is $r_f \bar{b}^A / \tau = 0.04 \times 1.1 / 0.45 \approx 10\%$ of budget, much lower than the 30% revenue loss with $\kappa = 0.7$. Absent the clean-slate benefit, this calculation would imply a much higher debt level for any type to be willing to default. When the clean-slate benefit is sufficiently large, the types with lower spending needs are more willing to pay the immediate cost of default to reap the future benefit of restarting with a sound financial position, generating an increasing threshold $b^d(\theta)$.

Looking again at equation (7), it is clear that the argument above abstracts from the last term, $\lambda (\beta \mathbb{E}[v^d(\theta')] - w^d(\theta))$. As long as β is small, the relevance of this term is reduced. However, as β increases, it can again reshape the equilibrium configuration:

Regime 3: Both externalities operating. Suppose the default cost parameters are identical to Regime 2, but the present bias parameter $\beta = 0.8$ as in Regime 1.

This last term in equation (7) captures the continuation value during the time the economy remains in exclusion. It reflects the cost to be endured by the government defaulting or by the successors. Thus, the larger β is, the larger the perceived cost. This additional cost borne by successors carries a larger weight for the lower types, as they care less about the reduction in their consumption during default. Therefore, at the lower end of the distribution, this impact is larger, reshaping their default behavior more in line with Regime 1. Instead, at the upper end of the distribution, the larger β has significantly smaller impact, since their decisions rely more on their already high spending needs. Hence, their pattern of default remains close to the one in Regime 2. In short, under this configuration, the intermediate types default first, and the insurance and clean-slate externalities can each dominate depending on the debt level.

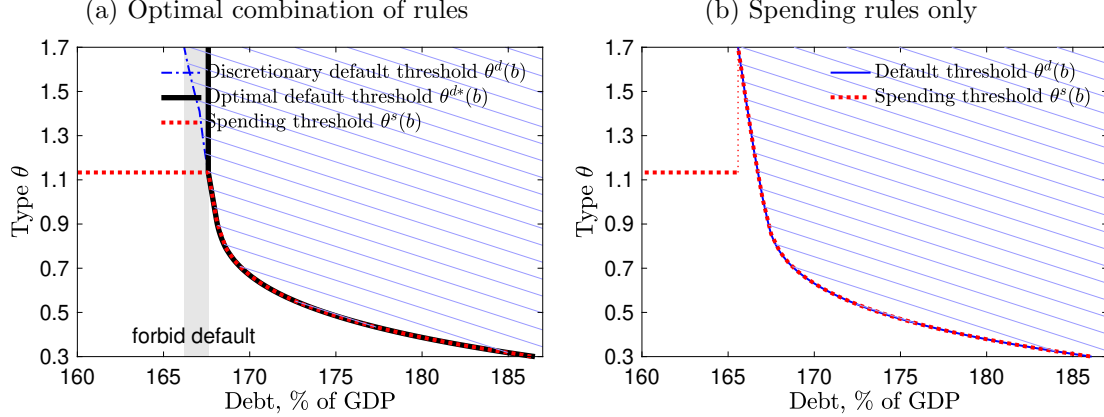
6.3 Spending and default rules

In this section, we analyze how fiscal and default rules can help restore efficiency in the economies described in the previous section. For simplicity, and to align with theoretical results, we focus exclusively on cases where the default pattern is monotonic.

Clearly, the type of intervention will depend on which externality dominates and the instruments available to the planner. To understand the contribution of each policy tool, we present not only the optimal combination of spending and default rules, but also the best spending rule, if default rules were not available. This exercise is relevant because default rules are scarce in reality. As a result, spending rules are usually directed to address the default risk as well.

We first consider Regime 1, in which exclusion from financial markets is permanent and the insurance externality dominates. Panel (a) of Figure 5 illustrates the optimal default

Figure 5: Optimal rules under dominant insurance externality



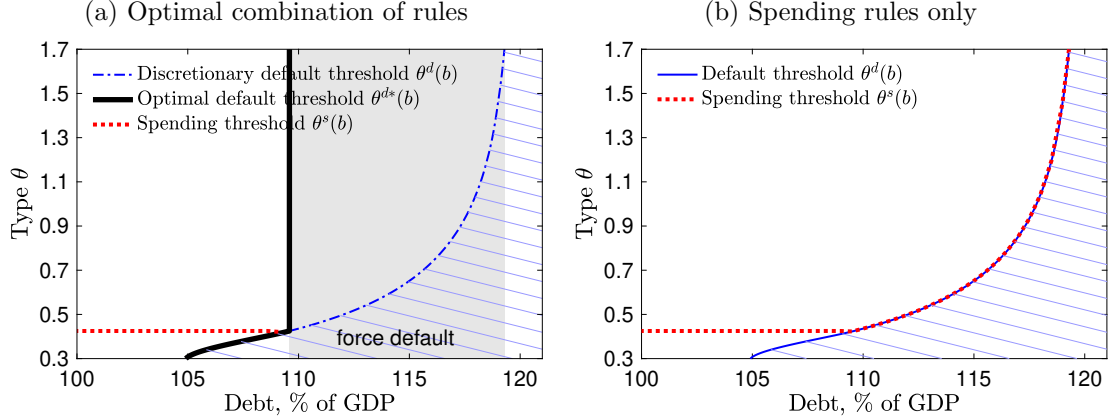
and fiscal rules. There the red dotted line depicts the optimal spending rule $\theta^s(b)$, the black solid line the optimal default threshold $\theta^{d*}(b)$, and the blue dashed-dotted line the discretionary default threshold if the agent were subject to the spending rule alone. There are two important features to notice. First, comparing the black line with the blue line, there is a gray shaded debt region where some government types would default if it were at their own discretion, while the planner finds it optimal not to default: forbidding default is thus optimal, in line with our theoretical characterization in case (i) of Proposition 5.

Second, when debt becomes risky, the default risk drastically affects the spending rules: the spending threshold jumps from the AWA rule to the same value as the default threshold, i.e., $\theta^s(b) = \theta^d(b)$. This implies that, de facto, all types that remain without defaulting are no longer subject to spending restrictions. According to the formula in Proposition 6, the spending threshold is at a corner. While a precise decomposition of the effects at play is challenging, the default manipulation effect is largely at work: at debt levels ranging from 168-186% of GDP, the marginal default type still over-defaults, and allowing them to spend as they wish conditional on repaying the debt helps incentivize them not to default.

Panel (b) of Figure 5 illustrates the optimal spending rule when the planner is unable to regulate default choices directly. In such circumstances, the planner can only attempt to manipulate the default decision by adjusting the spending rule. It does so by also removing the spending rules at debt levels in the 166-168% range. This adjustment is preferable to the spending rule in panel (a) for this debt region, as the latter would only induce more defaults due to restricted spending. In short, with or without default rules, it is optimal to eliminate all spending restrictions as soon as the risk premium is positive and leave it to the market to impose the necessary discipline.

We now turn to Regime 2, in which the clean-slate externality dominates. The optimal

Figure 6: Optimal rules under dominant clean-slate externality

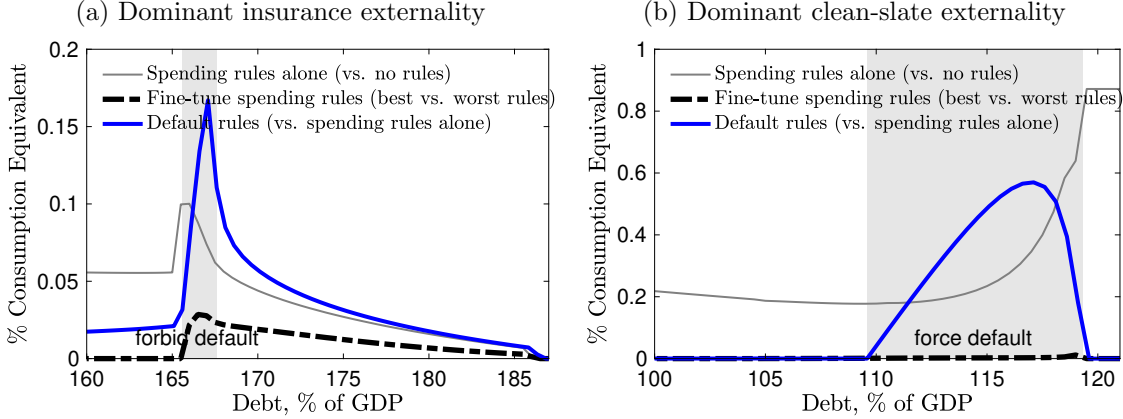


policy is illustrated in Figure 6. As in Proposition 5 case (ii), it is optimal to force default when debt is sufficiently high. In panel (a), this can be seen comparing the black solid line, representing the planner's default threshold, and the blue dashed-dotted line, which represents the government's discretionary choice. When debt reaches 109% of GDP, the two lines bifurcate, and the gray shaded area indicates the region where the government, not fully internalizing the benefit of offloading the debt, avoids a costly default. Deeming this outcome suboptimal, the planner then forces a default. From another perspective, the market fails to provide adequate discipline by extending too much debt. As a remedy, the rule effectively imposes a debt limit and tightens the economy's debt capacity.

In this case, prior to forcing default, the possibility of default has a minor impact on the spending rule, depicted by the red dotted line. When debt is safe, the spending threshold follows the AWA formula. As debt becomes risky, the spending threshold does not initially appear visibly altered, potentially due to a mixture of opposing forces.

In panel (b) of Figure 6, we show the optimal spending rule when default rules are not available. Compared with panel (a), the optimal spending threshold remains the same until we reach the debt level where forcing default would have been desirable. Beyond this level, the spending rule now compensates for the absence of a default rule. However, the default manipulation here operates in the opposite direction of Regime 1. To incentivize default, the planner sets the spending threshold at the lowest possible level and takes away all discretion. In the absence of default rules, it is optimal to impose the strongest possible austerity measures. It is important to note that this prescription has implicitly restricted the planner's spending choice set to only those types who are not defaulting, since for defaulters, the spending rule is irrelevant. Given that the solution is at the corner, one might wonder whether the outcome could be further improved. Indeed, unlike Regime 1, here the planner

Figure 7: Decomposition of welfare gains



Notes: The welfare gains from implementing the optimal spending rules alone are calculated relative to the no-rule equilibrium. The gains from fine-tuning spending rules involve a comparison between the best and the worst spending rules at debt levels with default risk. In the scenario where high types default first, the worst rule takes away all discretion. In the scenario where low types default, the worst rule imposes no rules. The additional gains from implementing default rules compare the outcomes under optimal rules to those under spending rules alone.

could design an even more stringent rule that would operate off-equilibrium. By further tightening the spending cap, the planner could make every type prefer to default rather than comply with the spending rule, thereby achieving the same outcome as with a default rule. This kind of “austerity measures,” which may appear excessively restrictive, would in fact be instrumental in implementing the optimal allocation.

6.4 Welfare decomposition

How important are default and fiscal rules? What are the welfare gains from implementing them? We answer these questions in several ways. First, it is interesting to quantify the welfare contribution of the spending rules, compared to a rules-free scenario. Second, one may wonder the importance of fine-tuning the spending rule to address default risk, as opposed to maintaining the standard AWA rule. Finally, how relevant are default rules? Do they generate a sizeable gain relative to spending rules alone?

Figure 7 presents these welfare decompositions, focusing on regions with default risk. Panel (a) corresponds to Regime 1 and panel (b) corresponds to Regime 2. Recall that we calibrate the economy in panel (a) with a high β , while the monotonic increasing threshold of panel (b) requires a lower β . In both panels, the gray solid line shows the welfare gains from imposing spending rules alone, compared to the rules-free equilibria. The blue solid line represents the additional welfare gain from implementing the default rules. Hence, the total welfare gains are sum of these two components. Finally, the black dashed line depicts

the welfare differences of fine tuning the spending rule, when it is used alone, relative to the AWA formula. Since in the default-risk region the AWA formula is not defined, we compare the best with the worst spending rule.

Three important patterns emerge. First, spending and default rules generate sizeable welfare gains, especially when the present bias is severe. Second, default rules generate, in most of the state space, larger welfare gains than spending rules, even when the latter are optimized to compensate for the absence of default rules. Third, fine-tuning spending rules has minimal impact on welfare, especially when the present bias is severe.

The finding that spending rules are more important when the present bias is severe is intuitive: the larger the friction, the greater the gains from correcting it. However, we also note that as debt increases, the welfare gains of the spending rules diminish in Regime 1 (panel (a)), while the gains increase in Regime 2 (panel (b)). This is due to the different selection effect of default. In the former regime, as debt grows, the pool of non-defaulters subject concentrates in the types with the lowest spending needs. Conversely, in the latter regime, types with lower spending needs default first, so only those with the large needs remain, which exacerbates the average on-path spending bias.

Although spending rules deliver considerable welfare gains, the simple AWA rule captures most of the gains. This is especially evident when the spending bias is severe. Recall Figure 6 (b): in the debt region where the planner does not regulate default, the optimal spending rule is almost identical to the AWA rule. Hence, additional modifications provide minimal improvements. In the under-default region, where the planner tightens the spending rule to induce default, the slight contribution of the best relative to the worst rule should be read as the impact of the optimal spending rule on the default being minimal. When the bias is mild, the fine-tuning does generate some relevant welfare gains. In this case, the fine-tuned rule is more effective in preventing default in the over-default region: when granted more discretion to spend, some types find it desirable to honor their debt.

Default rules appear to be largest contributors to the welfare gains. Again, there are clear differences depending on whether the insurance externality or clean-slate externality dominates. In panel (b), the blue line has an inverted-U shape, due to the interaction of two effects: the mass of agents that are forced to default and the average welfare gains from forcing them to do so. On one end, when debt is at 109% of GDP, a large mass of types are forced to default, but the average effect from those doing so is negligible. On the other end, when debt reaches 119%, there are large gains from forcing an agent to default, but there is only an infinitesimal mass of types who have not defaulted yet. As a result, the welfare gains are close to zero in both ends but peak in the middle of the region. At a debt level of around 115%, both effects are sizable: there is a sizable mass of types should be forced to

default, and the gains from each type are substantial.

Finally, in panel (a), the welfare gains depicted by the blue line peak at debt levels where default is forbidden. These gains stem from improved insurance and reduced interest rate by preventing default. As a result, there are dynamic effects that translate into welfare gains for all types, even those not directly impacted by the default rule. For this reason, there are positive gains at debt levels below 165%, where debt is safe from default risk, and at debt levels exceeding 168%, where the ban on default no longer applies. In the former region, the gains are associated mostly with types with high spending needs, who are borrowing and approaching default and can eventually benefit from lower interest rates. In the latter region, the gains arise from types with low spending needs, who are paying off the debt and moving towards debt levels with lower interest rates.

7 Conclusions

Sovereign debt accumulation has long been a controversial issue. While the ability to borrow is recognized as an essential tool for smoothing against adverse shocks, government debt is also widely regarded as being exploited by self-interested governments for their own benefits. Hence, fiscal rules appear as a fundamental component of every healthy democracy. This concern triggers hefty policy debates, especially when government debt is subject to default risk, highlighting the potential debt unsustainability and making the debate unavoidable. In contexts where financial markets are incomplete, defaulting on past obligations does not necessarily indicate inefficiency; it could also serve as a welfare-improving tool. Thus, to analyze the need and optimality of fiscal rules, one must incorporate these three key elements: the need to smooth spending, political bias, and risk of default. In this paper, we have approached the problem based on this premise.

We have shown that, although uncommon, default rules are essential for curbing myopic political behavior. Traditional fiscal rules, as spending and deficit limits, could be directed and effective in dealing with this issue, but most of the welfare gains arise from rules designed to affect default decision directly. We have characterized situations where governments should be left to default at their discretion, using the information available to them, and others where rules must be imposed. Perhaps consistent with common intuition, we find that governments may default too early, necessitating rules to prevent default. However, in this case rather than tightening spending rules, it is better to eliminate them. Moreover, contrary to the common intuition, we have shown that the default bias can go in the opposite direction: governments may be overly concerned about the cost of default borne by their own administration and hold on to excessive amount of debt, making it optimal to commit

to default and thereby reduce the economy's debt capacity.

We view this paper as a first step toward developing a theory that can provide precise quantitative prescriptions for real-world case studies. To derive clear theoretical prescriptions, we have deliberately omitted certain features of reality. Nevertheless, the framework can be enriched in many ways. Among others, incorporating debt maturity structure and endogenous political turnover stand out as key avenues in future work.

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Online Appendix

A Supporting results

We establish several results that are useful for subsequent proofs.

We first extend the equivalence between present bias and optimism bias, established by [Piguillem and Shi \(2025\)](#), to a continuous-time setting with default. To do so, we formulate the problem in sequential representation. When not in default, the value functions satisfy

$$\begin{aligned} w^n(\theta, b) &= \int_0^T e^{-\rho t} \theta u(g_t) dt + e^{-\rho T} \beta \mathbb{E}[v(\theta', b_T)] \\ v^n(\theta, b) &= \int_0^T e^{-\rho t} \theta u(g_t) dt + e^{-\rho T} \mathbb{E}[v(\theta', b_T)], \end{aligned}$$

where $\{g_t, b_t\}_{t \geq 0}$ denotes the path of the spending plan and the corresponding path of the debt position for the agent starting in state (θ, b) at time 0, and T denotes the time at which political turnover occurs for the first time.

Adjusting for the Poisson rate of the turnover shock, the value functions become

$$w^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} (\theta u(g_t) + \lambda \beta \mathbb{E}[v(\theta', b_t)]) dt \quad (\text{A1})$$

$$v^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} (\theta u(g_t) + \lambda \mathbb{E}[v(\theta', b_t)]) dt. \quad (\text{A2})$$

In the first lemma, we establish several useful relations between the value functions:

Lemma A1. *In a rules-free equilibrium or under any incentive-compatible spending plan, the value functions satisfy:*

$$\beta v^n(\theta, b) = w^n(\theta, b) - (1 - \beta) \theta w_\theta^n(\theta, b) \quad (\text{A3})$$

$$\beta v^d(\theta) = w^d(\theta) - (1 - \beta) \theta w_\theta^d(\theta). \quad (\text{A4})$$

Proof. From the sequential representation in equations (A1) and (A2), we obtain:

$$w^n(\theta, b) - \beta v^n(\theta, b) = (1 - \beta) \theta \int e^{-(\rho+\lambda)t} u(g_t) dt. \quad (\text{A5})$$

Applying the Envelope Theorem to equation (A1):

$$w_\theta^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} u(g_t) dt. \quad (\text{A6})$$

Note that equation (A5) holds with any arbitrary spending plan, while equation (A6) requires the agent's optimality condition. Hence the latter holds under the equilibrium spending. It also holds under any incentive-compatible spending plan. Combining equations (A5) and (A6), we obtain the relation in equation (A3).

When in default, according to the HJB equations (7) and (8):

$$w^d(\theta) - \beta v^d(\theta) = \frac{1}{\rho + \lambda + \phi} [(1 - \beta) \theta u(\kappa\tau) + \phi(w^n(\theta, 0) - \beta v^n(\theta, 0))].$$

Assessing equation (A3) at $b = 0$ and incorporating it in the equation above:

$$w^d(\theta) - \beta v^d(\theta) = (1 - \beta) \theta \frac{1}{\rho + \lambda + \phi} [u(\kappa\tau) + \phi w_\theta^n(\theta, 0)]. \quad (\text{A7})$$

Differentiating the HJB equation (7) with respect to θ :

$$w_\theta^d(\theta) = \frac{1}{\rho + \lambda + \phi} [u(\kappa\tau) + \phi w_\theta^n(\theta, 0)]. \quad (\text{A8})$$

Combining equations (A7) and (A8), we obtain the relation in equation (A4). $\square$

Lemma A1 allows us to transform the present-biased agent's problem into that of an equivalent "optimistically biased" agent. To facilitate this, let $w(\theta, b)$ denote the government's value that accounts for the default decision $\delta(\theta, b)$, in parallel with equation (4):

$$w(\theta, b) = (1 - \delta(\theta, b))w^n(\theta, b) + \delta(\theta, b)w^d(\theta). \quad (\text{A9})$$

Proposition A1. *In a rules-free equilibrium, the present-biased government's problem is equivalent to that of a hypothetical optimistically biased government:*

$$(\rho + \lambda) w^n(\theta, b) = \max_g \left\{ \theta u(g) + (r(b)b + g - \tau) w_b^n(\theta, b) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} w(\theta', b) d\tilde{F}(\theta') \right\} \quad (\text{A10})$$

$$(\rho + \lambda) w^d(\theta) = \theta u(\kappa\tau) + \phi(w^n(\theta, 0) - w^d(\theta)) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} w^d(\theta') d\tilde{F}(\theta'), \quad (\text{A11})$$

where the latter assigns the following cumulative weights to future spending needs:

$$\tilde{F}(\theta) = \begin{cases} F(\theta) + (1 - \beta) \theta f(\theta), & \underline{\theta} \leq \theta < \bar{\theta} \\ 1, & \theta = \bar{\theta}. \end{cases} \quad (\text{A12})$$

This adjusted weighting yields a perceived average need equal to $\int_{\underline{\theta}}^{\bar{\theta}} \theta d\tilde{F}(\theta) = \beta$.

Proof. We first show that the expected continuation value when the economy is not in default can be expressed as

$$\beta \mathbb{E} [v^n(\theta, b)] = \int_{\underline{\theta}}^{\bar{\theta}} w^n(\theta, b) d\tilde{F}(\theta). \quad (\text{A13})$$

To show this, we use the relation in equation (A3):

$$\beta \mathbb{E} [v^n(\theta, b)] = \int_{\underline{\theta}}^{\bar{\theta}} [w^n(\theta, b) - (1 - \beta) \theta w_\theta^n(\theta, b)] dF(\theta).$$

Applying integration by parts to the second term gives

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} \theta w_\theta^n(\theta, b) dF(\theta) &= \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) dw^n(\theta, b) \\ &= \bar{\theta} f(\bar{\theta}) w^n(\bar{\theta}, b) - \underline{\theta} f(\underline{\theta}) w^n(\underline{\theta}, b) - \int_{\underline{\theta}}^{\bar{\theta}} w^n(\theta, b) d[\theta f(\theta)]. \end{aligned}$$

Combining the two equations above we obtain equation (A13)

Similarly, using equation (A4), we can express the continuation value when the economy is in default as

$$\beta \mathbb{E} [v^d(\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} w^d(\theta) d\tilde{F}(\theta). \quad (\text{A14})$$

We can further extend the expression for the continuation value with any default decision. Using equations (4) and (A9) and relations (A3) and (A4):

$$\beta \mathbb{E} [v(\theta, b)] = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta, b) dF(\theta) - (1 - \beta) \int_{\underline{\theta}}^{\bar{\theta}} \theta [(1 - \delta(\theta, b)) w_\theta^n(\theta, b) + \delta(\theta, b) w_\theta^d(\theta)] dF(\theta).$$

Again, by integration by parts:

$$\begin{aligned} & \int_{\underline{\theta}}^{\bar{\theta}} \theta [(1 - \delta(\theta, b)) w_\theta^n(\theta, b) + \delta(\theta, b) w_\theta^d(\theta)] dF(\theta) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) (1 - \delta(\theta, b)) dw^n(\theta, b) + \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) \delta(\theta, b) dw^d(\theta) \\ &= \theta f(\theta) (1 - \delta(\theta, b)) w^n(\theta, b) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} w^n(\theta, b) d[(1 - \delta(\theta, b)) \theta f(\theta)] \\ & \quad + \theta f(\theta) \delta(\theta, b) w^d(\theta) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} w^d(\theta) d[\delta(\theta, b) \theta f(\theta)] \\ &= \theta f(\theta) w(\theta, b) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} w(\theta, b) d[\theta f(\theta)]. \end{aligned}$$

As a result:

$$\beta \mathbb{E}[v(\theta, b)] = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta, b) d\tilde{F}(\theta). \quad (\text{A15})$$

Substituting (A15) and (A14) in the HJB equations (5) and (7), we obtain the modified versions in (A10) and (A11).

The weighted average can be computed as:

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta d\tilde{F}(\theta) = \underline{\theta} \times (1 - \beta) \underline{\theta} f(\underline{\theta}) + \int_{\underline{\theta}_+}^{\bar{\theta}_-} \theta d\tilde{F}(\theta) - \bar{\theta} \times (1 - \beta) \bar{\theta} f(\bar{\theta}).$$

By integration by parts:

$$\int_{\underline{\theta}_+}^{\bar{\theta}_-} \theta d\tilde{F}(\theta) = 1 + (1 - \beta) \int_{\underline{\theta}_+}^{\bar{\theta}_-} \theta d[\theta f(\theta)] = 1 + (1 - \beta) \left(\theta^2 f(\theta) \Big|_{\underline{\theta}_+}^{\bar{\theta}_-} - \int_{\underline{\theta}_+}^{\bar{\theta}_-} \theta f(\theta) d\theta \right),$$

which makes the average equal to β . $\square$

We provide some remarks on Proposition A1. First, although the proposition establishes equivalence in a rule-free equilibrium, its proof relies only on the relation in Lemma A1; hence the same equivalence extends to any incentive-compatible spending plan.

Second, the adjusted weights implied by $\tilde{F}(\theta)$ are

$$d\tilde{F}(\theta) = \begin{cases} (1 - \beta) \underline{\theta} f(\underline{\theta}), & \text{for } \theta = \underline{\theta} \\ [(2 - \beta) f(\theta) - (1 - \beta) \theta f'(\theta)] d\theta, & \text{for } \theta \in (\underline{\theta}, \bar{\theta}) \\ -(1 - \beta) \bar{\theta} f(\bar{\theta}), & \text{for } \theta = \bar{\theta}. \end{cases} \quad (\text{A16})$$

Assumption 1 ensures that these weights are positive for the interior types $\theta \in (\underline{\theta}, \bar{\theta})$. However, there are potentially mass points at the ends of the distribution. In particular, the weight assigned to $\bar{\theta}$ is $-(1 - \beta) \bar{\theta} f(\bar{\theta})$, which is strictly negative since $f(\bar{\theta}) > 0$.

Finally, an immediate corollary follows from equations (A13)-(A15):

Corollary A1. *At debt level b , with discretionary default:*

$$\beta (\mathbb{E}[v(\theta, b)] - \mathbb{E}[v^d(\theta)]) = \int_{\underline{\theta}}^{\bar{\theta}} \max \{w^n(\theta, b) - w^d(\theta), 0\} d\tilde{F}(\theta) \quad (\text{A17})$$

$$\beta (\mathbb{E}[v(\theta, b)] - \mathbb{E}[v^n(\theta, b)]) = \int_{\underline{\theta}}^{\bar{\theta}} \max \{w^d(\theta) - w^n(\theta, b), 0\} d\tilde{F}(\theta). \quad (\text{A18})$$

Proof. Combining equations (A14) and (A15) gives

$$\beta (\mathbb{E} [v(\theta, b)] - \mathbb{E} [v^d(\theta)]) = \int_{\underline{\theta}}^{\bar{\theta}} (w(\theta, b) - w^d(\theta, b)) d\tilde{F}(\theta).$$

The agent's discretionary default decision (3) implies that $w(\theta, b) = \max \{w^n(\theta, b), w^d(\theta)\}$. Substituting this expression into the integral above immediately delivers (A17). Similarly, we can use equations (A13) and (A15) to derive (A18). $\square$

Lastly, we establish that in economies with high political turnover, an endogenous borrowing constraint arises in equilibrium. In this case, the indifference condition (10) for discretionary default does not hold with equality. We abstract away from such cases in our main analysis.

Lemma A2 (High political turnover). *If the political turnover rate exceeds a threshold, i.e., $\lambda \geq \bar{\lambda}$, the rules-free equilibrium features an endogenous borrowing limit.*

Proof. Consider debt level $\underline{b}^A$ at which default risk begins to arise. Suppose some types strictly prefer not to default at $\underline{b}_A$. Then there exists a type that remains willing to repay at the debt upper bound $\bar{b}^A > \underline{b}^A$. At that debt level, the market would charge an interest rate $r(\bar{b}^A) = r_f + \lambda$. If the risk premium rises so sharply that, as soon as debt exceeds $\underline{b}^A$, the required interest payment immediately surpasses tax revenue, then the government cannot service the debt even with zero spending:

$$(r_f + \lambda)\bar{b}^A > (r_f + \lambda)\underline{b}^A \geq \tau.$$

As a result, the government has no option but to default the instant debt exceeds $\underline{b}^A$, despite $w^n(\theta, \underline{b}^A) > w^d(\theta)$, because the value of not defaulting falls to negative infinity. Hence a borrowing limit emerges where $\underline{b}^A = \bar{b}^A = b^A$.

In the knife-edge case where the tax revenue exactly covers the interest payment, we obtain the cutoff for the turnover rate $\bar{\lambda}$ according to $(r_f + \bar{\lambda})b^A = \tau$. $\square$

B Proofs

B.1 Proof of Lemma 1

When $\beta = 0$, the government behaves like a time-consistent agent with a higher discount rate $\rho + \lambda > r_f$. Because the taste θ merely scales utility, all types borrow and spend the same amount, $g^A(\theta, b) = g^A(b)$, and they would default at the same debt level, leading to an

endogenous borrowing limit b^A . Accordingly, the government's value functions are linear in θ : $w^n(\theta, b) = \theta \tilde{w}^n(b)$ and $w^d(\theta) = \theta \tilde{w}^d$, where $\tilde{w}^n(b)$ and $\tilde{w}^d$ satisfy:

$$\begin{aligned} (\rho + \lambda) \tilde{w}^n(b) &= \max_g \{u(g) + (r_f b + g - \tau) \tilde{w}_b^n(b)\}, \quad \forall b \leq b^A \\ (\rho + \lambda) \tilde{w}^d &= u(\kappa \tau) + \phi (\tilde{w}^n(0) - \tilde{w}^d). \end{aligned}$$

The borrowing limit b^A is characterized by

$$(\rho + \lambda) (\tilde{w}^n(b^A) - \tilde{w}^d) = u(g^A(b^A)) - u(\kappa \tau) - \phi (\tilde{w}^n(0) - \tilde{w}^d) = 0. \quad (\text{B1})$$

Since the government borrows to spend until hitting the borrowing limit, its budget constraint gives $g^A(b^A) = \tau - r_f b^A$. When financial exclusion is permanent $\phi = 0$, equation (B1) implies that $g^A(b^A) = \kappa \tau$ and in turn the borrowing limit $b^A = \frac{1}{r_f} (1 - \kappa) \tau$. With temporary exclusion $\phi > 0$ and revenue loss in default $\kappa < 1$, the government's value in good standing at zero debt exceeds its value in default, that is, $\tilde{w}^n(0) > \tilde{w}^d$. Thus, the borrowing limit implied by (B1) tightens relative to permanent exclusion: $b^A < \frac{1}{r_f} (1 - \kappa) \tau$.

B.2 Proof of Lemma 2

To characterize the government's default pattern, we differentiate its indifference condition (10) with respect to θ and obtain:

$$\frac{\partial b^A(\theta)}{\partial \theta} b_\theta^A(\theta) = \frac{w_\theta^n(\theta, b^A(\theta)) - w_\theta^d(\theta)}{-w_b^n(\theta, b^A(\theta))}. \quad (\text{B2})$$

The default pattern is therefore governed by the sign of $w_\theta^n(\theta, b^A(\theta)) - w_\theta^d(\theta)$.

To analyze this term, we different the HJB equations (14) and (15) for $w^n(\theta, b)$ and $w^d(\theta)$ with respect to θ :

$$(\rho + \lambda) w_\theta^n(\theta, b) = u(g(\theta, b)) + (\theta u'(g(\theta, b)) + w_b(\theta, b)) g_\theta(\theta, b) + \dot{b}(\theta, b) w_{b\theta}^n(\theta, b) \quad (\text{B3})$$

$$(\rho + \lambda) w_\theta^d(\theta) = u(\kappa \tau) + \phi (w_\theta^n(\theta, 0) - w_\theta^d(\theta)). \quad (\text{B4})$$

In the rules-free equilibrium, the first-order condition (9) for spending holds. With the incentive-compatible spending, the IC condition (B12) holds. In either case, equation (B3) reduces to

$$(\rho + \lambda) w_\theta^n(\theta, b) = u(g(\theta, b)) + \dot{b}(\theta, b) w_{b\theta}^n(\theta, b). \quad (\text{B5})$$

These results allow us to obtain an upper bound for the term we aim to sign:

$$\begin{aligned}
& (\rho + \lambda)\theta (w_\theta^n(\theta, b^A(\theta)) - w_\theta^d(\theta)) \\
&= \theta (u(g(\theta, b^A(\theta))) - u(\kappa\tau)) + \dot{b}(\theta, b^A(\theta)) \theta w_{b\theta}^n(\theta, b^A(\theta)) - \phi \theta (w_\theta^n(\theta, 0) - w_\theta^d(\theta)) \\
&\leq \theta (u(g(\theta, b^A(\theta))) - u(\kappa\tau)) + \dot{b}(\theta, b^A(\theta)) w_b^n(\theta, b^A(\theta)) - \phi \theta (w_\theta^n(\theta, 0) - w_\theta^d(\theta)) \\
&= -\lambda\beta (\mathbb{E}[v(\theta', b^A(\theta))] - \mathbb{E}[v^d(\theta')]) + \phi [(w^n(\theta, 0) - w^d(\theta)) - \theta (w_\theta^n(\theta, 0) - w_\theta^d(\theta))] \\
&= -\lambda\beta (\mathbb{E}[v(\theta', b^A(\theta))] - \mathbb{E}[v^d(\theta')]) + \frac{\phi\beta}{1-\beta} [(w^n(\theta, 0) - v^d(\theta)) - (w^n(\theta, 0) - w^d(\theta))] .
\end{aligned}$$

In the block of derivations above, the second line is obtained by subtracting equation (B4) from (B5) and multiplying θ . Going to the third line, since we restrict our attention to economies with increasing spending $g_\theta(\theta, b) \geq 0$, the first-order condition (9) immediately implies $\theta w_{b\theta}^n(\theta, b) \geq w_b^n(\theta, b)$. Moreover, at the default threshold, $\dot{b}(\theta, b^A(\theta)) \leq 0$. Naturally, this line holds with equality if $\dot{b}(\theta, b^A(\theta)) = 0$. The fourth line follows from the indifference condition (10). The last line is obtained by assessing equation (12) at $b = 0$.

While the expression in the last line is generally difficult to sign, when $\phi = 0$, it reduces to only the first term, $-\lambda\beta (\mathbb{E}[v(\theta', b^A(\theta))] - \mathbb{E}[v^d(\theta')])$, the sign of which is unambiguous.

Permanent financial exclusion. Setting $\phi = 0$, we have

$$\frac{\partial b^A(\theta)}{\partial \theta} \leq \frac{-\lambda\beta (\mathbb{E}[v(\theta', b^A(\theta))] - \mathbb{E}[v^d(\theta')])}{-(\rho + \lambda)\theta w_b^n(\theta, b^A(\theta))}.$$

Consider any debt level that exceeds the default threshold of the highest type, $\forall b \geq b^A(\bar{\theta})$. Applying equation (A17) in Corollary A1:

$$\begin{aligned}
& \beta (\mathbb{E}[v(\theta, b)] - \mathbb{E}[v^d(\theta)]) = \int_{\underline{\theta}}^{\bar{\theta}} \max\{w^n(\theta, b) - w^d(\theta), 0\} d\tilde{F}(\theta) \\
&= \underbrace{\int_{\underline{\theta}}^{\bar{\theta}^-} \max\{w^n(\theta, b) - w^d(\theta), 0\} d\tilde{F}(\theta)}_{\geq 0} - \underbrace{\max\{w^n(\bar{\theta}, b) - w^d(\bar{\theta}), 0\}}_{=0} (1 - \beta)\bar{\theta}f(\bar{\theta}) \geq 0.
\end{aligned} \tag{B6}$$

To see the signs above, recall that Assumption 1 implies that $d\tilde{F}(\theta)$ only places negative weight at the top: that is, $d\tilde{F}(\theta) > 0$ for all $\theta < \bar{\theta}$, while $d\tilde{F}(\bar{\theta}) = -(1 - \beta)\bar{\theta}f(\bar{\theta}) < 0$. Further, since the highest type has defaulted, $w^n(\bar{\theta}, b) - w^d(\bar{\theta}) < 0$.

Therefore, $\frac{\partial b^A(\theta)}{\partial \theta} \leq 0$, for the highest type $\bar{\theta}$ and all types that default after $\bar{\theta}$. This implies that the highest type is the first one to default $b^A(\bar{\theta}) = \underline{b}^A$. It follows then $\frac{\partial b^A(\theta)}{\partial \theta} \leq 0, \forall \theta \in \Theta$. Further, the lowest type is the last to default $b^A(\underline{\theta}) = \bar{b}^A$.

When there exist savers, there must be a positive mass of types in the neighborhood of $\underline{\theta}$ saving at their default threshold, $\dot{b}(\theta, b^A(\theta)) < 0$ and $\theta w_{bb}^n(\theta, b^A(\theta)) > w_b^n(\theta, b^A(\theta))$, suggesting the inequality is strict: $\frac{\partial b^A(\theta)}{\partial \theta} < 0$ in that neighborhood. Revisiting (B6), we obtain that $\forall b \in [\underline{b}^A, \bar{b}^A)$,

$$\beta (\mathbb{E}[v(\theta, b)] - \mathbb{E}[v^d(\theta)]) = \int_{\underline{\theta}}^{\bar{\theta}^-} \max\{w^n(\theta, b) - w^d(\theta), 0\} d\tilde{F}(\theta) > 0.$$

This implies that the default threshold is strictly decreasing everywhere: $\frac{\partial b^A(\theta)}{\partial \theta} < 0, \forall \theta \in \Theta$.

B.3 Proof of Lemma 3

To understanding the government's spending pattern, we derive the Euler equation. To do so, we differentiate the HJB equation (5) with respect to b :

$$\begin{aligned} \left(\rho + \lambda - \frac{\partial(r(b)b)}{\partial b}\right) w_b^n(\theta, b) &= (\theta u'(g^A(\theta, b)) + w_b^n(\theta, b)) g_b^A(\theta, b) \\ &\quad + \dot{b}(\theta, b) w_{bb}^n(\theta, b) + \lambda \beta \frac{\partial \mathbb{E}[v(\theta', b)]}{\partial b}. \end{aligned}$$

Incorporating the first-order condition (9), the equation above becomes:

$$\left(\frac{\partial(r(b)b)}{\partial b} - \rho - \lambda\right) \theta u'(g^A(\theta, b)) = \dot{b}(\theta, b) w_{bb}^n(\theta, b) + \lambda \beta \frac{\partial \mathbb{E}[v(\theta', b)]}{\partial b}. \quad (\text{B7})$$

The spending path is inherently linked to the borrowing or saving pattern, according to

$$\dot{g}^A(\theta, b) \equiv \dot{b}(\theta, b) g_b^A(\theta, b). \quad (\text{B8})$$

For savers, as they pay off debt $\dot{b}(\theta, b) < 0$, their spending grows $\dot{g}(\theta, b) > 0$. For borrowers, as they accumulate debt $\dot{b}(\theta, b) > 0$, their spending declines $\dot{g}(\theta, b) < 0$.

We further differentiate the first-order condition (9) with respect to b and, by combining it with equation (B8), we obtain:

$$\dot{b}(\theta, b) w_{bb}^n(\theta, b) = -\theta u''(g^A(\theta, b)) \dot{g}^A(\theta, b),$$

which allows us to rewrite equation (B7) as:

$$\left(\frac{\partial(r(b)b)}{\partial b} - \rho - \lambda\right) \theta u'(g^A(\theta, b)) = -\theta u''(g^A(\theta, b)) \dot{g}^A(\theta, b) + \lambda \beta \frac{\partial \mathbb{E}[v(\theta', b)]}{\partial b}.$$

Reorganizing the equation above leads to:

$$\frac{\dot{g}^A(\theta, b)}{g^A(\theta, b)} = \frac{u'(g^A(\theta, b))}{-g^A(\theta, b) u''(g^A(\theta, b))} \left(\frac{\partial(r(b)b)}{\partial b} - \rho - \lambda + \lambda \beta \frac{-\frac{\partial}{\partial b} \mathbb{E}[v(\theta', b)]}{\theta u'(g^A(\theta, b))} \right).$$

Without loss of generality, we consider a CRRA utility with risk aversion parameter γ . Thus, $\frac{-gu''(g)}{u'(g)} = \gamma$. We obtain the Euler equation (11).

B.4 Proof of Proposition 1

Consider $\beta > 0$ so that there is separation in the default threshold. Recall that, as indicated by equation (B2), the default pattern is governed by the sign of $w_\theta^n(\theta, b^A(\theta)) - w_\theta^d(\theta)$.

The default threshold $\partial b^A(\theta)/\partial \theta < 0$ if and only if $w_\theta^n(\theta, b^A(\theta)) < w_\theta^d(\theta)$, which, according to equation (12), is equivalent to $v^n(\theta, b^A(\theta)) > v^d(\theta)$.

Similarly, the default threshold $\partial b^A(\theta)/\partial \theta > 0$ if and only if $w_\theta^n(\theta, b^A(\theta)) > w_\theta^d(\theta)$, which is equivalent to $v^n(\theta, b^A(\theta)) < v^d(\theta)$.

B.5 Proof of Proposition 2

Recall that $g^A(\theta, b) > g^P(\theta, b)$ if and only if $-w_b^n(\theta, b) < -v_b^n(\theta, b)$, which from equation (13) happens if and only if $\theta w_{\theta b}^n(\theta, b) > w_b^n(\theta, b)$.

To establish part (i), we differentiate the first-order condition (9) with respect to θ :

$$u'(g^A(\theta, b)) + \theta u''(g^A(\theta, b)) g_\theta^A(\theta, b) = -w_{\theta b}^n(\theta, b).$$

Hence, since $u(\cdot)$ is concave, $g_\theta^A(\theta, b) > 0$ if only if $-w_{\theta b}^n(\theta, b) < u'(g^A(\theta, b))$, which using (9) again happens if and only if $\theta w_{\theta b}^n(\theta, b) > w_b^n(\theta, b)$. This completes the proof.

For part (ii), we consider economies in which $b^A(\theta)$ is monotonically decreasing. We differentiate equation (A1) with respect to b :

$$w_b^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} \left(\theta u'(g_t) \frac{\partial g_t}{\partial b} + \lambda \beta \frac{\partial \mathbb{E}[v(\theta', b_t)]}{\partial b_t} \frac{\partial b_t}{\partial b} \right) dt. \quad (\text{B9})$$

Further, differentiating equation (A6) with respect to b generates

$$w_{\theta b}^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} u'(g_t) \frac{\partial g_t}{\partial b} dt.$$

Incorporating the equation above in (B9):

$$w_b^n(\theta, b) = \theta w_{\theta b}^n(\theta, b) + \lambda \beta \int_0^\infty e^{-(\rho+\lambda)t} \frac{\partial \mathbb{E}[v(\theta', b_t)]}{\partial b_t} \frac{\partial b_t}{\partial b} dt.$$

To prove the statement, it suffices to show that the last term in the above equation is strictly negative such that $\theta w_{\theta b}^n(\theta, b) \geq w_b^n(\theta, b)$. We first establish that $\frac{\partial b_t}{\partial b} > 0$, for all t , following the same argument based on agent optimality for Theorem 1 of Chatterjee and Eyigungor (2016). For brevity, we skip replicating the argument.

While it is clear that $v_b^n(\theta, b) < 0$, the sign of $\frac{\partial \mathbb{E}[v(\theta, b)]}{\partial b}$ is not immediately obvious. We demonstrate that it is negative in debt regions both with and without default risk. In the default-free region $b \leq \underline{b}^A$, it is straightforward that

$$\frac{\partial \mathbb{E}[v(\theta', b_t)]}{\partial b_t} = \mathbb{E}[v_b^n(\theta', b_t)] < 0.$$

In the default-risk region $b \in [\underline{b}^A, \bar{b}^A]$, let $\theta^A(b)$ be the inverse of $b^A(\theta)$. Since $\frac{\partial b^A(\theta)}{\partial \theta} < 0$, the inverse is uniquely defined and $\frac{\partial \theta^A(b)}{\partial b} < 0$. By Proposition 1, $v^n(\theta^A(b), b) > v^d(\theta^A(b))$. Then,

$$\frac{\partial \mathbb{E}[v(\theta', b)]}{\partial b} = \int_{\underline{\theta}}^{\theta^A(b)} v_b^n(\theta', b) dF(\theta') + [v^n(\theta^A(b), b) - v^d(\theta^A(b))] \frac{\partial \theta^A(b)}{\partial b} f(\theta^A(b)) < 0.$$

It follows that $\theta w_{\theta b}^n(\theta, b) \geq w_b^n(\theta, b)$ and in turn $g^A(\theta, b) > g^P(\theta, b)$.

B.6 Proof of Lemma 4

The principal's objective can be written as:

$$\mathbb{E}[v(\theta, b)] = \int_{\underline{\theta}}^{\bar{\theta}} (v^n(\theta, b) (1 - \delta(\theta, b)) + v^d(\theta) \delta(\theta, b)) dF(\theta).$$

In taste state θ at debt b , comparing not defaulting to defaulting, the objective changes by

$$\left[v^n(\theta, b) - v^d(\theta) - \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', b)}{\partial r(b)} (1 - \delta(\theta', b)) dF(\theta') \right] f(\theta). \quad (\text{B10})$$

The term $\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', b)}{\partial r(b)} (1 - \delta(\theta', b)) dF(\theta')$ captures the interest-rate effect of default decision of θ on other types, which is why it is multiplied by change in the interest rate $\lambda f(\theta)$. The expression in (B10) immediately implies equation (19).

B.7 Proof of Lemma 5

We start with an equivalence that facilitates the proof:

Lemma B1. *At any debt level b , for all $\theta \geq \theta^s(b)$, the value functions are affine in θ and satisfy the following relation:*

$$w^n(\theta, b) = \beta v^n\left(\frac{\theta}{\beta}, b\right).$$

Proof. For any given sequence, $\{g_t, b_t\}_{b_0=b}$, independent of θ , the fact that the value functions are affine follows directly from equations (A1) and (A2). In addition, simple algebraic calculations generate:

$$w^n(\theta, b; \{g_t, b_t\}) = \beta v^n\left(\frac{\theta}{\beta}, b; \{g_t, b_t\}\right).$$

Since for all $\theta \geq \theta^s(b)$, it must be that $g_t(\theta, b) = g_t(\theta^s(b), b)$ and $b_t(\theta, b) = b_t(\theta^s(b), b)$, the statement of the lemma follows immediately. $\square$

Consider an economy in which the default punishment is prohibitively high, $\kappa = 0$. The agents would never default, and thus they can borrow up to the natural debt limit $\frac{\tau}{r_f}$. In problem (16), we only need to choose the spending policy $\mathcal{A} = \{g(\theta, b)\}_{\theta \in \Theta, b < \frac{\tau}{r_f}}$ or, equivalently, the spending threshold $\theta^s(b)$. Then, $\forall b < \frac{\tau}{r_f}$, the planner chooses θ^s to maximize

$$\mathbb{E}[v^n(\theta, b)] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, b) dF(\theta) + \int_{\theta^s}^{\bar{\theta}} v^n(\theta, b) dF(\theta).$$

The optimality condition with respect to θ^s :

$$\frac{\partial \mathbb{E}[v^n(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0. \quad (\text{B11})$$

We first examine types $\theta \leq \theta^s$. Differentiating the HJB equations with respect to θ^s and incorporating agent's first-order condition (9) and the fact that optimality implies $\partial \mathbb{E}[v^n(\theta, b)] / \partial \theta^s = 0$:

$$\begin{aligned} (\rho + \lambda) \frac{\partial w^n(\theta, b)}{\partial \theta^s} &= \dot{b}(\theta, b) \frac{\partial w_b^n(\theta, b)}{\partial \theta^s} \\ (\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} &= (\theta u'(g(\theta, b)) + v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} + \dot{b}(\theta, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s}. \end{aligned}$$

From the former of the equations above, because of the Envelope theorem that states that only direct effects matter, we guess and verify that $\frac{\partial w^n(\theta, b)}{\partial \theta^s} = \frac{\partial w_b^n(\theta, b)}{\partial \theta^s} = 0$. This implies

that $\frac{\partial g(\theta, b)}{\partial \theta^s} = 0$. Further, from the latter equation, we also guess and verify that $\frac{\partial v^n(\theta, b)}{\partial \theta^s} = \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} = 0$. Thus, the optimality condition (B11) simplifies to:

$$\frac{\partial \mathbb{E}[v^n(\theta, b)]}{\partial \theta^s} = \int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0.$$

We now examine types $\theta > \theta^s$. Differentiating the HJB equation with respect to θ^s :

$$(\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} = (\theta u'(g(\theta^s, b)) + v_b^n(\theta, b)) g_\theta(\theta^s, b) + \dot{b}(\theta^s, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s}.$$

The optimality condition (B11) further turns into

$$g_\theta(\theta^s, b) \int_{\theta^s}^{\bar{\theta}} (\theta u'(g(\theta^s, b)) + v_b^n(\theta, b)) dF(\theta) + \dot{b}(\theta^s, b) \int_{\theta^s}^{\bar{\theta}} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0.$$

Since $g_\theta(\theta^s, b) > 0$ and $\int_{\theta^s}^{\bar{\theta}} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0$, we obtain:

$$\int_{\theta^s}^{\bar{\theta}} (\theta u'(g(\theta^s, b)) + v_b^n(\theta, b)) dF(\theta) = 0.$$

The first-order condition (9) at θ^s implies $\theta u'(g(\theta^s, b)) = -\frac{\theta}{\theta^s} w_b^n(\theta^s, b)$. Since for $\theta \geq \theta^s(b)$ the value functions are affine by Lemma B1, we can write $v^n(\theta, b) = \theta v^1(b) + v^2(b)$. Moreover, using the relation between v^n and w^n , $\theta u'(g(\theta^s, b)) = -\frac{\theta}{\theta^s} w_b^n(\theta^s, b) = -\frac{\theta}{\theta^s} \beta v_b^n(\theta^s/\beta, b) = -(\frac{\theta}{\theta^s} \beta v_b^2(b) + \theta v_b^1(b))$ and $v_b^n(\theta, b) = v_b^2(b) + \theta v_b^1(b)$. Replacing this in the equation above:

$$\int_{\theta^s}^{\bar{\theta}} (\beta \theta - \theta^s) dF(\theta) = 0,$$

which can be rewritten as equation (20). The spending threshold is thus independent of b .

Assumption 1 ensures that $\beta \mathbb{E}[\theta | \theta > \theta^s] - \theta^s$ is strictly decreasing in θ^s . If $\beta > \underline{\theta}$, combined with $\beta \bar{\theta} - \bar{\theta} \leq 0$, equation (20) has a unique interior solution. However, if $\beta \leq \underline{\theta}$, a corner solution is obtained.

B.8 Proof of Proposition 3

We characterize the default and spending rules by analyzing each while holding the other fixed. We show that the standard rules-versus-discretion result applies here to both rules.

Default rule. First, we show that at debt levels where the planner would like to let some types default but not all types, the only incentive-compatible default rule is to allow for

discretionary default, $\delta(\theta, b) = \delta^A(\theta, b)$, $\forall b \in (\underline{b}, \bar{b}]$. At a given debt level b , we partition the set of reports into two subsets: a non-default set and a default set,

$$\Theta^n(b) = \{\theta \in \Theta : \delta(\theta, b) = 0\} \quad \text{and} \quad \Theta^d(b) = \{\theta \in \Theta : \delta(\theta, b) = 1\}.$$

For any $b \in (\underline{b}, \bar{b}]$, the sets $\Theta^n(b)$ and $\Theta^d(b)$ are both nonempty. The agent will report in the non-default set if and only if

$$\max_{\hat{\theta} \in \Theta^n(b)} w^n(\hat{\theta}, \theta, b; \mathcal{A}) > \max_{\hat{\theta} \in \Theta^d(b)} w^d(\hat{\theta}, \theta; \mathcal{A}).$$

Note that the binary nature of the default choice makes it impossible for the principal to directly alter the agent's default behavior, fixing the spending rule, although the spending rule can affect agent's default behavior indirectly. Agent types that can obtain a higher value defaulting would pretend to be in the default set Θ^d . Likewise, types that can obtain a higher value not defaulting would pretend to be a type that maximizes their value in the non-default set Θ^n .

Next, we show that the optimal default rule $\delta(\theta, b)$ must be increasing in b . This can be obtained from the discretionary default $\delta^A(\theta, b)$ increasing in b . This in turn implies that the default probability $\mathbb{E}[\delta(\theta, b)]$ is increasing in b , so is the interest rate $r(b)$.

Further, when $b = 0$, the principal would never default, and at $b = \frac{\tau}{r_f}$, the principal would always default. Then, there exists a lower bound $\underline{b} \geq 0$ such that below this debt level, it is desirable to preclude any type from defaulting: $\delta(\theta, b) = 0, \forall \theta$, for all $b \leq \underline{b}$. There also exists an upper bound $\bar{b} < \frac{\tau}{r_f}$ such that exceeding this level all types should default: $\delta(\theta, b) = 1, \forall \theta$, for all $b > \bar{b}$. In combination, the following relations hold: $0 \leq \underline{b} \leq \bar{b} < \frac{\tau}{r_f}$. In the intermediate debt level, the principal allows for discretionary default: $\delta(\theta, b) = \delta^A(\theta, b)$, $\forall \theta$, for all $b \in (\underline{b}, \bar{b}]$.

Spending rule. We analyze economies featuring increasing spending in the rules-free equilibrium in Proposition 2. Without loss of generality, we consider spending $g(\theta, b)$ that is piecewise differentiable.

Conditional on not defaulting, $\delta(\hat{\theta}, b) = 0$, the agent's optimality condition for report:

$$w_{\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}) = 0.$$

Differentiating the condition above with respect to b , we must have $w_{b\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}) = 0$.

Differentiating the HJB equation (14) with respect to $\hat{\theta}$ and $\hat{\theta} < \bar{\theta}$:

$$(\rho + \lambda) w_{\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}) = \left(\theta u' \left(g(\hat{\theta}, b) \right) + w_b^n(\hat{\theta}, \theta, b; \mathcal{A}) \right) g_{\hat{\theta}}(\hat{\theta}, b) + \dot{b}(\hat{\theta}, b) w_{b\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}).$$

Since $w_{\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}) = 0$ and $w_{b\hat{\theta}}^n(\hat{\theta}, \theta, b; \mathcal{A}) = 0$, and imposing truth-telling, $\hat{\theta} = \theta$, the equation above becomes

$$(\theta u'(g(\theta, b)) + w_b^n(\theta, b)) g_{\theta}(\theta, b) = 0. \quad (\text{B12})$$

We can see from equation (B12) that there is limited scope for intervention: the first term is the same as without intervention, and the second term implies that intervention can only constrain changes in spending. Thus, as in the economy without default, the optimal intervention either grants flexibility, allowing agents to spend at their discretion according to their first-order condition (9), or imposes a rule where $g_{\theta}(\theta, b) = 0$ holds.

Since spending is increasing in type, the optimal allocation features a spending threshold, which we denote by $\theta^s(b)$. All types with $\theta \leq \theta^s(b)$ have flexibility and all types $\theta > \theta^s(b)$ are bunched with type $\theta^s(b)$.

Conditional on defaulting, $\delta(\hat{\theta}, b) = 1$, differentiating the HJB equation (15) with respect to $\hat{\theta}$:

$$w_{\hat{\theta}}^d(\hat{\theta}, \theta; \mathcal{A}) = \frac{\phi}{\rho + \phi + \lambda} w_{\hat{\theta}}^n(\hat{\theta}, \theta, 0; \mathcal{A}) = 0.$$

Recall that when the debt position is zero, the agent will never default, nor will the planner wish the agent to default. As long as the agent reports truthfully when its debt position is zero, the agent will report truthfully when in default.

B.9 Proof of Proposition 4

With present bias is extreme $\beta = 0$, Appendix B.1 shows that in the rules-free equilibrium value functions are linear in θ : $w^n(\theta, b) = \theta \tilde{w}^n(b)$ and $w^d(\theta) = \theta \tilde{w}^d$. The linearity also holds when all discretion for spending is taken away, so all types incur the same spending, $g(\theta, b) = g(b)$.

From the principal's perspective, the expected values $\mathbb{E}[v^n(\theta, b)] = \tilde{v}^n(b)$ and $\mathbb{E}[v^d(\theta)] = \tilde{v}^d$, where $\tilde{v}^n(b)$ and $\tilde{v}^d$ are the equivalent values of an unbiased agent with discount rate ρ :

$$\begin{aligned} \rho \tilde{v}^n(b) &= \max_g \{ u(g) + (r_f b + g - \tau) \tilde{v}_b^n(b) \} \\ \rho \tilde{v}^d &= u(\kappa \tau) + \phi (\tilde{v}^n(0) - \tilde{v}^d). \end{aligned}$$

The principal imposes a spending rule $g(b)$ for all types that solves the problem above.

If the government is granted discretionary default, it is subject to a market-imposed borrowing limit: $b \leq b^A$. Given the risk-free rate $r_f \leq \rho$, the spending rule would generate borrowing until hitting the borrowing limit, that is $\dot{b}(b^A) = 0$. As in Appendix B.1, the market-imposed borrowing limit b^A solves

$$(\rho + \lambda) (\tilde{w}^n(b^A) - \tilde{w}^d) = 0 \iff u(g(b^A)) - u(\kappa\tau) - \phi(\tilde{w}^n(0) - \tilde{w}^d) = 0.$$

From the principal's perspective, at this market-imposed borrowing limit,

$$\rho(\mathbb{E}[v^n(\theta, b^A)] - \mathbb{E}[v^d(\theta)]) = \rho(\tilde{v}^n(b^A) - \tilde{v}^d) = u(g(b^A)) - u(\kappa\tau) - \phi(\tilde{v}^n(0) - \tilde{v}^d),$$

while the principal wishes to implement a debt limit b^P such that $\mathbb{E}[v^n(\theta, b^P)] = \mathbb{E}[v^d(\theta)]$.

When financial exclusion is permanent $\phi = 0$, the default incentives between the agent and the principal are exactly aligned, $\mathbb{E}[v^n(\theta, b^A)] = \mathbb{E}[v^d(\theta)]$. Thus, $b^P = b^A = \frac{1}{r_f}(\kappa - 1)\tau$.

However, with temporary exclusion $\phi > 0$ and revenue loss in default $\kappa < 1$, the gap in the reaccess values, $\tilde{v}^n(0) - \tilde{v}^d$ versus $\tilde{w}^n(0) - \tilde{w}^d$, will lead to a discrepancy in the default incentives. To understand this gap, notice that at $b = 0$,

$$\begin{aligned} (\rho + \phi)(\tilde{v}^n(0) - \tilde{v}^d) &= u(g(0)) - u(\kappa\tau) + (g(0) - \tau)\tilde{v}_b^n(0) \\ (\rho + \lambda + \phi)(\tilde{w}^n(0) - \tilde{w}^d) &= u(g(0)) - u(\kappa\tau) + (g(0) - \tau)\tilde{w}_b^n(0). \end{aligned}$$

We obtain that

$$(\rho + \phi)[(\tilde{v}^n(0) - \tilde{v}^d) - (\tilde{w}^n(0) - \tilde{w}^d)] = \underbrace{\dot{b}(0)}_{<0} \underbrace{(\tilde{v}_b^n(0) - \tilde{w}_b^n(0))}_{<0} + \lambda \underbrace{(\tilde{w}^n(0) - \tilde{w}^d)}_{>0} > 0.$$

It follows then $\tilde{v}^n(0) - \tilde{v}^d > \tilde{w}^n(0) - \tilde{w}^d$, implying that $\mathbb{E}[v^n(\theta, b^A)] < \mathbb{E}[v^d(\theta)]$. Thus, the principal tightens the debt limit, $b^P < b^A$.

B.10 Proof of Proposition 5

Recall the discretionary default threshold defined in equation (10). When default is monotonic, again let $\theta^A(b)$ be the inverse of $b^A(\theta)$: $\forall b \in [\underline{b}^A, \bar{b}^A]$,

$$w^n(\theta^d(b), b) = w^d(\theta^d(b)).$$

In the steps that follow, we apply equations (A17) and (A18) from Corollary A1.

Forbid default. Suppose the threshold is *monotonically decreasing*, with high types defaulting first. Then types below the threshold prefer not to default, i.e., $w^n(\theta, b) > w^d(\theta)$, $\forall \theta < \theta^d(b)$, and those above it prefer to default, i.e., $w^n(\theta, b) < w^d(\theta)$, $\forall \theta > \theta^d(b)$. Then, the discretionary default outcome dominates the outcome when all types default: $\forall b \in (\underline{b}^A, \bar{b}^A)$,

$$\beta (\mathbb{E} [v(\theta, b)] - \mathbb{E} [v^d(\theta)]) = \int_{\underline{\theta}}^{\theta^d(b)} (w^n(\theta, b) - w^d(\theta)) d\tilde{F}(\theta) > 0.$$

However, comparing the discretionary default outcome to the outcome when no type defaults:

$$\beta (\mathbb{E} [v(\theta, b)] - \mathbb{E} [v^n(\theta, b)]) = \int_{\theta^d(b)}^{\bar{\theta}} (w^d(\theta) - w^n(\theta, b)) d\tilde{F}(\theta).$$

We show that over-default can occur: given the negative weight at $\bar{\theta}$, the expression above can be negative at debt levels where default risk just emerged. Note that at $b = \underline{b}^A$, the expression is zero, and its derivative is negative:

$$\frac{\partial}{\partial b} (\beta (\mathbb{E} [v(\theta, \underline{b}^A)] - \mathbb{E} [v^n(\theta, \underline{b}^A)])) = w_b^n(\bar{\theta}, \underline{b}^A) (1 - \beta) \bar{\theta} f(\bar{\theta}) < 0.$$

By continuity, there exists a cutoff $\tilde{b}$ such that in the range $(\underline{b}^A, \tilde{b})$, the expression is strictly negative. It is therefore desirable to forbid default in this debt region.

Force default. Now suppose the threshold is *monotonically increasing*, i.e., low types default first. Then types below the threshold prefer to default, so that $w^n(\theta, b) < w^d(\theta)$, $\forall \theta < \theta^d(b)$, and those above it prefer not to default, i.e., $w^n(\theta, b) > w^d(\theta)$, $\forall \theta > \theta^d(b)$. Then, comparing the discretionary default outcome to when all types default:

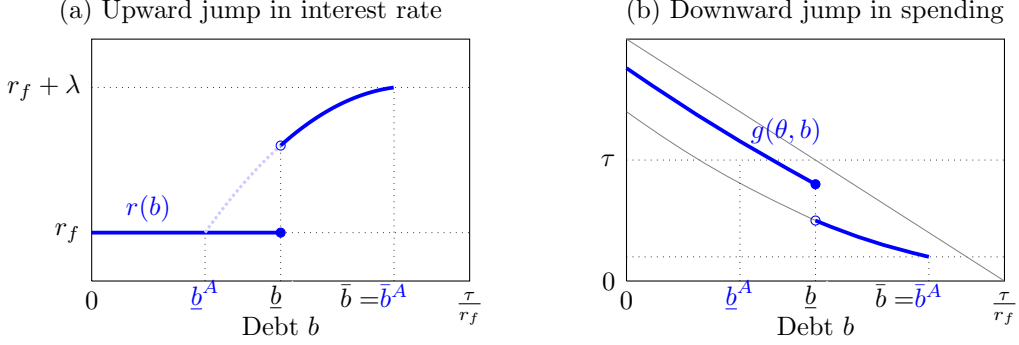
$$\beta (\mathbb{E} [v(\theta, b)] - \mathbb{E} [v^d(\theta)]) = \int_{\theta^d(b)}^{\bar{\theta}} (w^n(\theta, b) - w^d(\theta)) d\tilde{F}(\theta).$$

We show that under-default can occur: given the negative weight at $\bar{\theta}$, the expression above can be negative when the economy is very heavy in debt. Note that at $b = \bar{b}^A$, the expression is zero, and its derivative is positive:

$$\frac{\partial}{\partial b} (\beta (\mathbb{E} [v(\theta, \bar{b}^A)] - \mathbb{E} [v^d(\theta)])) = -w_b^n(\bar{\theta}, \bar{b}^A) (1 - \beta) \bar{\theta} f(\bar{\theta}) > 0.$$

By continuity, there exists a cutoff $\tilde{b}$ such that in the range $(\tilde{b}, \bar{b}^A)$, the expression is strictly negative. It is therefore desirable to force default in this debt region.

Figure B1: Coexistence of default-forbidden and discretionary default regions



In comparison, discretionary default outcome dominates no type defaulting: $\forall b \in (\underline{b}^A, \bar{b}^A)$,

$$\beta (\mathbb{E} [v(\theta, b)] - \mathbb{E} [v^n(\theta, b)]) = \int_{\underline{\theta}}^{\theta^d(b)} (w^d(\theta) - w^n(\theta, b)) d\tilde{F}(\theta) > 0.$$

B.11 Proof of Proposition 6

We first discuss whether the main object of interest is differentiable. The next lemma states that when the principal forbids default for low levels of debt, the spending function develops a point of discontinuity, but it remains differentiable almost everywhere. The continuity of the value functions is preserved. The pattern is illustrated in Figure B1.

Lemma B2 (Differentiability). *If the optimal default rule features a default-forbidden region and a discretionary default region, i.e., $\underline{b}^A < \underline{b} < \bar{b}$, when debt surpasses $\underline{b}$:*

- i) Interest rate $r(b)$ jumps upward: $r(\underline{b}) < \lim_{b \downarrow \underline{b}} r(b)$.
- ii) Discretionary spending $g^A(\theta, b)$ jumps downward: $g^A(\theta, \underline{b}) > \lim_{b \downarrow \underline{b}} g^A(\theta, b)$.
- iii) Value functions display a kink and are piecewise differentiable:

$$w_b^n(\theta, \underline{b}) > \lim_{b \downarrow \underline{b}} w_b^n(\theta, b) \quad \text{and} \quad v_b^n(\theta, \underline{b}) > \lim_{b \downarrow \underline{b}} v_b^n(\theta, b).$$

Proof. The proof is straightforward and thus omitted. $\square$

At any debt level $b \geq \bar{b}$, all types default. As a result, there is no spending rule to be made in autarky. At any debt level $b < \bar{b}$, no type defaults, and the interest rate is $r(b) = r_f$. In this case, equation (25) reduces to the formula in (20) and holds trivially.

We now consider an intermediate debt level $b \in [\underline{b}, \bar{b})$. The steps closely follow those in Appendix B.7, but here we must account for how the spending rule affects default incentives

and the resulting interest rate. Specifically, the interest-rate equation (1) can be written as $r(b) = r_f + \lambda(1 - F(\theta^d))$. The spending threshold θ^s affects interest rate through default:

$$\frac{\partial r(b)}{\partial \theta^s} = -\lambda f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s}. \quad (\text{B13})$$

Without loss of generality, we consider a spending threshold in the non-default area, $\theta^s \leq \theta^d$. Since a local perturbation in the spending threshold doesn't affect the reaccess value $v^n(\theta, 0)$, it also does not affect the default value: $\frac{\partial v^d(\theta)}{\partial \theta^s} = 0$.

The maximization objective in (16) can be rewritten as:

$$\mathbb{E}[v(\theta, b)] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, b) dF(\theta) + \int_{\theta^s}^{\theta^d} v^n(\theta, b) dF(\theta) + \int_{\theta^d}^{\bar{\theta}} v^d(\theta) dF(\theta).$$

The optimality condition with respect to θ^s :

$$\frac{\partial \mathbb{E}[v(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\theta^d} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + (v^n(\theta^d, b) - v^d(\theta^d)) f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s} = 0.$$

Note that the term $\frac{\partial v^n(\theta, b)}{\partial \theta^s}$ captures the total effects, including the direct effect of the spending rule and the indirect effects from the response in default and thus the interest rate. Substituting (B13) in the optimality condition above:

$$\frac{\partial \mathbb{E}[v(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\theta^d} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) - \frac{1}{\lambda} (v^n(\theta^d, b) - v^d(\theta^d)) \frac{\partial r(b)}{\partial \theta^s} = 0. \quad (\text{B14})$$

Since the principal is assumed to have no commitment and optimizes for every b , (B14) must hold for all b , so the cross-partial is also equal to zero:

$$\frac{\partial \mathbb{E}[v(\theta, b)]}{\partial \theta^s \partial b} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\theta^d} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) - \frac{1}{\lambda} v_b^n(\theta^d, b) \frac{\partial r(b)}{\partial \theta^s} = 0. \quad (\text{B15})$$

Differentiating the HJB equation for $v^n(\theta, b)$ with respect to θ^s and using the fact that optimality implies $\partial \mathbb{E}[v(\theta, b)] / \partial \theta^s = 0$:

$$\begin{aligned} (\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} &= (\theta u'(g(\theta, b)) + v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} + \dot{b}(\theta, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} + b v_b^n(\theta, b) \frac{\partial r(b)}{\partial \theta^s}, \forall \theta \leq \theta^s \\ (\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} &= (\theta u'(g(\theta^s, b)) + v_b^n(\theta, b)) g_\theta(\theta^s, b) + \dot{b}(\theta^s, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} + b v_b^n(\theta, b) \frac{\partial r(b)}{\partial \theta^s}, \forall \theta > \theta^s. \end{aligned}$$

Replacing the terms above in equation (B14):

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta^s} \left[(\theta u'(g(\theta, b)) + v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} + \dot{b}(\theta, b) \frac{\partial v_b(\theta, b)}{\partial \theta^s} + b v_b^n(\theta, b) \frac{\partial r(b)}{\partial \theta^s} \right] dF(\theta) \\ & + \int_{\theta^s}^{\theta^d} \left[(\theta u'(g(\theta^s, b)) + v_b^n(\theta, b)) g_{\theta}(\theta^s, b) + \dot{b}(\theta^s, b) \frac{\partial v_b(\theta, b)}{\partial \theta^s} + b v_b^n(\theta, b) \frac{\partial r(b)}{\partial \theta^s} \right] dF(\theta) \\ & - \frac{\rho + \lambda}{\lambda} (v^n(\theta^d, b) - v^d(\theta^d)) \frac{\partial r(b)}{\partial \theta^s} = 0. \end{aligned}$$

We make a few transformations. Notice that from the cross-partial in (B15), we have:

$$\int_{\theta^s}^{\theta^d} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) = \frac{1}{\lambda} v_b^n(\theta^d, b) \frac{\partial r(b)}{\partial \theta^s} - \int_{\underline{\theta}}^{\theta^s} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta),$$

which, together with $\dot{b}(\theta, b) - \dot{b}(\theta^s, b) = g(\theta, b) - g(\theta^s, b)$ according to the budget (2), implies:

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta^s} \dot{b}(\theta, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + \dot{b}(\theta^s, b) \int_{\theta^s}^{\theta^d} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) \\ & = \int_{\underline{\theta}}^{\theta^s} (g(\theta, b) - g(\theta^s, b)) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + \frac{1}{\lambda} \dot{b}(\theta^s, b) v_b^n(\theta^d, b) \frac{\partial r(b)}{\partial \theta^s}. \end{aligned}$$

Using $\theta u'(g(\theta^s, b)) = -\frac{\theta}{\theta^s} w_b^n(\theta^s, b)$ and the result in Lemma B1, we have:

$$\int_{\theta^s}^{\theta^d} (\theta u'(g(\theta^s, b)) + v_b^n(\theta^s, b)) dF(\theta) g_{\theta}(\theta^s, b) = \chi(b) (\beta \mathbb{E}[\theta | \theta^s \leq \theta \leq \theta^d] - \theta^s),$$

where $\chi(b) \equiv \frac{w_b^n(\theta^s, b) - v_b^n(\theta^s, b)}{(1-\beta)\theta^s} g_{\theta}(\theta^s, b) (F(\theta^d) - F(\theta^s)) \geq 0$.

Incorporating the two transformed expressions above and the first-order condition of unconstrained agents $\theta u'(g(\theta, b)) = -w_b^n(\theta, b)$:

$$\begin{aligned} & \chi(b) (\beta \mathbb{E}[\theta | \theta^s \leq \theta \leq \theta^d] - \theta^s) + \int_{\underline{\theta}}^{\theta^s} (v_b^n(\theta, b) - w_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} dF(\theta) \\ & + \int_{\underline{\theta}}^{\theta^s} (g(\theta, b) - g(\theta^s, b)) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + b \int_{\underline{\theta}}^{\theta^d} v_b^n(\theta, b) dF(\theta) \frac{\partial r(b)}{\partial \theta^s} \\ & + \frac{1}{\lambda} [\dot{b}(\theta^s, b) v_b^n(\theta^d, b) - (\rho + \lambda) (v^n(\theta^d, b) - v^d(\theta^d))] \frac{\partial r(b)}{\partial \theta^s} = 0 \end{aligned}$$

Rearranging the equation above, we get the implicit expression for θ^s in equation (25).

From Proposition 1, when default is monotonically decreasing, we have $v^n(\theta^d(b), b) > v^d(\theta^d(b))$, which tends to push the default manipulation term toward negative.

The companion version of Proposition 6.

Proposition B1 (Spending threshold). *Suppose the default threshold $b^d(\theta)$ is monotonically increasing. Then, $\forall b \geq \underline{b}$, the spending threshold is characterized by:*

$$\begin{aligned}
0 = & \chi(b) \left(\underbrace{\beta \mathbb{E} [\theta | \theta \geq \theta^s \geq \theta^d]}_{\text{Selection effect} > 0} - \theta^s \right) \\
& + \left\{ \underbrace{\int_{\theta^d}^{\theta^s} [v_b^n(\theta, b) - w_b^n(\theta, b)] \frac{\partial g(\theta, b)}{\partial r(b)} dF(\theta)}_{\text{Discipline effect} > 0} + \underbrace{\int_{\theta^d}^{\theta^s} [g(\theta, b) - g(\theta^s, b)] \frac{\partial v_b^n(\theta, b)}{\partial r(b)} dF(\theta)}_{\text{Insurance effect} < 0} \right. \\
& \left. + \underbrace{b \int_{\theta^d}^{\bar{\theta}} v_b^n(\theta, b) dF(\theta)}_{\text{Income effect} < 0} + \underbrace{\frac{1}{\lambda} [\dot{b}(\theta^s, b) v_b^n(\theta^d, b) - (\rho + \lambda) (v^n(\theta^d, b) - v^d(\theta^d))]}_{\text{Default manipulation} > 0} \right\} \underbrace{\frac{\partial r(b)}{\partial \theta^s}}_{< 0},
\end{aligned} \tag{B16}$$

where $\chi(b) \equiv \frac{w_b^n(\theta^s, b) - v_b^n(\theta^s, b)}{(1 - \beta)\theta^s} g_\theta(\theta^s, b) (1 - F(\theta^s)) \geq 0$.

Proof. The proof follows the same steps as the proof for Proposition 6 and is therefore omitted. The sign of the default manipulation effect follows from Proposition 1: when default is monotonically decreasing, we have $v^n(\theta^d, b) < v^d(\theta^d)$. $\square$

C Supplementary results

C.1 Bounded savings

To derive conditions ensuring that the government's maximum saving is bounded, we focus on saving regions. In this case, $\frac{\partial(r(b)b)}{\partial b} = r_f$. Using the result in equation (A13), we have

$$-\beta \mathbb{E} [v_b^n(\theta, b)] = - \int_{\underline{\theta}}^{\bar{\theta}} w_b^n(\theta, b) d\tilde{F}(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \theta u'(g^A(\theta, b)) d\tilde{F}(\theta).$$

The Euler equation (11) becomes

$$\frac{\dot{g}^A(\theta, b)}{g^A(\theta, b)} = \frac{1}{\gamma} \left(r_f - \rho - \lambda + \lambda \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta' u'(g^A(\theta', b)) d\tilde{F}(\theta')}{\theta u'(g^A(\theta, b))} \right).$$

At debt levels $b < \underline{b}^A$, the highest type $\bar{\theta}$ borrows and accumulates debt. The question is whether the lowest type $\underline{\theta}$ is a saver or a borrower. When $b \rightarrow -\infty$, spending for all types $g(\theta, b) \rightarrow \infty$, and their marginal utilities converge to zero at the same speed. We have:

$$\lim_{b \rightarrow -\infty} \frac{\dot{g}^A(\underline{\theta}, b)}{g^A(\underline{\theta}, b)} = \frac{1}{\gamma} \left(r_f - \rho - \lambda + \lambda \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta d\tilde{F}(\theta)}{\underline{\theta}} \right) = \frac{1}{\gamma} \left(r_f - \rho - \lambda + \lambda \frac{\beta}{\underline{\theta}} \right).$$

Thus, if and only if $\underline{\theta} > \frac{\lambda}{\lambda + \rho - r_f} \beta$, the lowest type borrows too when its savings are large, $\lim_{b \rightarrow -\infty} \dot{b}(\underline{\theta}, b) > 0$, guaranteeing an upper bound on savings.

C.2 Quantitative results

Figure C1 displays the market interest rates for Regimes 1 and 2 explored in Section 6.

Figure C1: Interest rates

