



EIEF Working Paper 25/10

November 2025

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April 2025

Abstract

This paper studies the relationship between fiscal rules and intergovernmental transfers within a federation. We analyze an environment where present-biased governments must insure against future shocks. The present bias generates a reason for fiscal rules to exist, while risk sharing motives bring out the need for transfers. A central authority designs the optimal combination of state contingent transfers and fiscal rules that maximize the federation's welfare. We show that independently of the present bias, it is optimal to achieve the first-best pattern of aggregate spending. However, how it is implemented depends on the intensity of the bias. When the bias is mild, a mechanism akin to an emergency fund with tight fiscal rules arises, while when the bias is severe, it is optimal to provide loans with contingent payments and to loosen up fiscal rules. Moreover, there is always a degree of bias for which a fiscal union is not optimal.

JEL classification: E6, H6, H77.

Keywords: Fiscal rules. Federalism. Risk sharing. Transfers.

*We thank Manuel Amador, Claudio Michelacci, seminar participants at EIEF, Theory and Methods Macro (T2M) conference and EIEF-BdI macro workshop. E-mails: jorlandi@redphd.it. and facundo.piguillem@gmail.com.

1 Introduction

Federations and confederations are two common forms of political organization.¹ Regardless of the specific arrangement, two central questions arise: how much risk should be shared among members and how deep should fiscal integration be? Under ideal conditions, extensive risk sharing would be the norm. However, concerns about fiscal responsibility and mistrust among members often create resistance, limiting the extent of such agreements. To mitigate the risks of fiscal irresponsibility, the past two decades have seen a surge in the adoption of fiscal rules, not only at the national level but also at the supranational and subnational levels.² This raises another set of relevant questions. How do fiscal rules and risk-sharing mechanisms interact? Are fiscal unions welfare improving in this context? We argue that even when the lack of commitment to remain in the federation is not an issue, a fiscal union may not be optimal. The optimal federation would be just a set of fiscal rules that each member must respect.

We analyze an environment in which a group of present-biased governments can form a federation with a fiscal union. The present bias captures in reduced form political economy frictions, as in Alesina and Tabellini (1990), or a time-inconsistent social welfare function due to aggregation, as in Jackson and Yariv (2015). Within the federation, member states can pool resources in a common budget and design a system of transfers and contributions. In addition, members are subject to idiosyncratic spending shocks. Absent the federation, they can only insure against the shocks through a single risk-free non-state contingent bond. Thus, by providing state contingent transfers across member states, the fiscal union can provide the missing insurance. Without the present bias, the environment would be the same as Atkeson and Lucas (1992).

A natural benchmark is the optimal fiscal design outside the federation, which also coincides with the optimal design of a federation without a fiscal union. In this case, the best a country can do is to self-insure issuing debt (or accumulating assets) and to impose a fiscal rule as in Amador et al. (2006). This rule solves the optimal trade-off between commitment, to control the spending bias, and flexibility allowing governments to accommodate spending to the realization of the shock. The solution is a threshold for the spending shock. If the government claims to be experiencing spending needs below the threshold, it can freely choose spending. If it claims needs above the threshold, it can only spend as much as the type that could have experienced a shock equal to the threshold. An appealing feature of this solution

¹We understand by federation a union of states where the members relinquish the possibility of exiting. In contrast, in a confederation the states retain full self-determination. Although in practice the distinction is not so stark, we use these terms in this sense. Federations are more prevalent inside countries, while the confederative approach is usually preferred for associations among countries.

²See the IMF Fiscal Rules Dataset (2013) and Budina et al. (2012).

is that it can easily be implemented with a spending or deficit limit: a fiscal rule. From now on, we refer to this solution as the AWA rule.

Creating a fiscal union in this context has clear benefits; by pooling resources, members with good luck can transfer resources to members with bad luck, overcoming the lack of state contingent bonds. However, this can further fuel the spending bias, generating additional incentives for members to claim high spending needs. This may require stronger fiscal rules, but could also make the fiscal union undesirable. With a transfer system in place, another benefit appears; by choosing the right slope on the marginal transfers, the federation could generate incentives to alleviate the spending bias that would not be possible outside it. This could lead to potentially looser fiscal rules. Overall, the impact and consequences of a federation are ambiguous.

We take the Ramsey approach. There is a planner that maximizes the average ex-ante utility of the federation. This coincides with the ex-ante expected value of each member. The planner can choose a transfer system with a lump-sum common component plus transfers contingent on the spending level. Each component can either be positive or negative. We interpret this transfer system as a **loan contract** when the lump sum component is positive and subsequent transfers negative; and as a joint **contingency fund** when the lump sum component is negative and ex-post transfers are positive.

To understand the subsequent results, it is worth bearing in mind the types of inefficiencies present in our setting. There are two frictions that counterbalance each other. A spending bias that leads to too much debt accumulation, and excess precautionary savings due to market incompleteness.³ From the perspective of each individual country, as in AWA, the second friction does not play any role because only intertemporal smoothing is possible. In the context of a federation that allows both intertemporal and intratemporal reallocation of resources, the second friction is fully internalized.

We first show a result that shapes most of the uncovered patterns. For any intensity of spending bias and for any distribution of shocks, it is optimal to respect what we call dynamic aggregate efficiency. In the sense that aggregate savings and spending are the same as the ones that would arise in the first-best allocation (by a planner with full information and unrestricted instruments). The dynamic aggregate efficiency must be respected under all parametric specifications and under any transfer system. This does not mean that the federation achieves the first-best solution, since the distribution of spending is never the same as the one that a planner with perfect information and unrestricted instruments would choose.

³This effect is similar to Aiyagari (1994) when individuals are subject to uninsured idiosyncratic income risk. It is present as long as the preferences exhibit a positive coefficient of prudence.

Then, we start by characterizing the optimal affine transfer system. This system is simpler to analyze than the nonlinear and delivers the same patterns and intuition as the more general nonlinear system. We first show that a fiscal union may not be necessary. When the spending bias is neither mild nor severe, the two frictions affecting debt choices cancel each other. Hence, any transfer among member states would upset this delicate balance, leading to either too little or too much debt. Fiscal rules are still imposed on the members of the federation: they coincide with the ones that each member state would have imposed on themselves, had they not been participating in the federation. That is, a federation with no fiscal integration but making sure that fiscal rules are respected appears as an optimal agreement.

A fiscal union is optimal when the spending bias is close to the extremes. If it is mild, the excess precautionary savings motive dominates. In this case, a *common fund* emerges as optimal. All governments contribute the same amount to the fund and withdraw resources contingent on the realization of the shock. The larger the needs and spending, the larger the withdrawal. Since it generates a net transfer of resources from governments with low spending needs to those with high needs, we call this a **redistributive federation**. One may be concerned that the implicit subsidy on spending may exacerbate the spending bias, which indeed happens, but the economy is already in a situation with excessive precautionary savings, hence the additional spending contributes to increasing welfare. Of course, the spending can still spiral out of control if member states report excessively high spending needs. To avoid this problem, fiscal rules must be tightened up with respect to a federation without a fiscal union.

When the spending bias is severe, it becomes the dominant force, overcoming the precautionary savings motive and leading to aggregate overaccumulation of debt. In this case, a loan contract with contingent payments emerges as optimal. Each member receives a loan of the same amount and repays contingent on their spending choice. The more spending, the larger the repayment. This configuration generates total net transfers from high-need governments to low-need ones; for this reason, we call it a **punitive federation**. The debt repayment acts as a tax on spending which keeps total spending under control, making it possible to relax the fiscal rules with respect to the federation without a fiscal union. The relaxation of the rules also helps with insurance concerns, since then more countries find themselves with the ability to accommodate spending to their needs.

When we allow for a nonlinear transfer system, the aforementioned patterns remain, affecting only marginal transfers. There is still a spending bias for which the two frictions cancel each other, rendering a fiscal union unnecessary. However, in the punitive and redistributive federations, the central authority may decide to reinforce or moderate the marginal

incentives to spend depending on the distribution's shape. Whether it does it in one direction or the other depends on the relative occurrence of the high-needs shock vs. the low-needs one. Consider, for instance, the redistributive federation. For insurance purposes, it would be optimal to make the marginal transfer increasing in the size of the shock or, equivalently, on the observed spending. The problem is that this policy can be costly, since resources must be raised from types with low needs. We show that when the distribution of shocks has a thin upper tail, so that there is a relative scarcity of high-spenders, it is optimal to make the marginal transfer increasing (progressive) on spending. In contrast, if the tail is thick, decreasing (regressive) marginal transfers are optimal. Analogous considerations apply on the punitive federation.

Finally, we analyze the consequences of endowing member states with the possibility of exiting the federation. In this regard, it is important to bear in mind that members might be tempted to exit for two reasons: 1) whenever they are net contributors inside the fiscal union, or 2) when the fiscal rules are binding and they cannot spend according to their perceived needs. The first reason is standard in the literature, while the second is special to our environment. As expected, one of the implications of the lack of commitment to the federation is that the depth of the fiscal union is limited, but it is so in an asymmetric fashion. Although the redistributive federation is slightly affected, the punitive one is severely limited, to the point that it closely resembles a federation without fiscal union.

A takeaway from these findings is that no fiscal union is the optimal arrangement when the spending bias is moderate or severe. Another implication is that, when the participation constraints are binding, the fiscal rules must be relaxed. This happens independently of whether the constraint is binding because the marginal participant is a net contributor or because the fiscal rule is too harsh for its spending needs. As a result, in a punitive federation the fiscal rule can be substantially looser than in the autarky benchmark, while in the redistributive federation the tightening implied by progressive system interacts with the relation due to the lack commitment, leaving the final outcome ambiguous.

The remainder of the paper is organized as follows. Section 2 briefly discusses the literature. Section 3.1 describes the theoretical and institutional environment. 4 characterizes the the solution when member states cannot exit the federation. Section 5 analyzes the consequences of the possibility of leaving the federation. Section 6 concludes.

2 Literature review

This paper relates to two strands of literature, 1) fiscal federalism and risk sharing and 2) the optimal delegation literature with time-inconsistent (present biased) agents. Our

main contribution is to analyze the intersection of the two. Fiscal federalism has a long tradition in economics, dating back to the seminal work of Buchanan (1950) and Oates (1972). This literature emphasizes, among other things, the important role that risk sharing and intergovernmental transfers play. A detailed review in this short section would be extensive and necessarily unfair, the interested reader can, however, see the recent surveys by Oates (1999) and Agrawal et al. (2024).

Due to the relevance of these issues for many international associations, recent studies have paid especial attention to the effect of lack of commitment to the federation on the design of a fiscal union and other risk sharing mechanisms; see, for instance, Farhi and Werning (2017), Ferrari et al. (2021), Abrahám et al. (2025) and Abrahám et al. (2023). This more recent approach, as the previous, analyzes standard preferences with geometric discounting. Our approach resembles quasi-hyperbolic discounting which can be easily microfounded as the result of a dynamic political game. We believe that this element is essential on the design of federations or confederations with democratic members.

The delegation literature has studied the optimal design of mechanisms to improve outcomes when agents are present bias.⁴ In addition to Amador et al. (2006), Alonso and Matouschek (2008) and Amador and Bagwell (2013), Halac and Yared (2014, 2018, 2022) have made important contributions drawing insights instrumental in designing optimal fiscal rules. We depart from this literature in two dimensions. First, we allow for the possibility of transfers among agents and participation constraints, which are absent in all previous studies. In this dimension, Halac and Yared (2018) finds some mechanisms similar to ours. They analyze the design of an optimal coordinated fiscal rule when a group of countries share the same borrowing-lending market. There, the planner internalizes that fiscal rules affect the equilibrium interest rate, which generates indirect transfers between borrowers and lenders.⁵

The second dimension in which we differ is that we use the Ramsey approach to optimal policy, rather than mechanism design. This last approach has the advantage that it does not impose any restrictions on the set of instruments that can be used; they arise endogenously as a result of the maximization. An influential paper taking this approach is Atkeson and Lucas (1992), although the environments are very similar, they do not consider the case of present bias agents.⁶ When this bias is present, the approach can be difficult to work with. To obtain sharp characterizations, one must resort to special cases, which may significantly

⁴See Azzimonti et al. (2016) and Pigullem and Riboni (2020) for studies that analyze fiscal rules with a microfounded present bias.

⁵Other works analyzing the trade off between commitment and flexibility Athey et al. (2005), and concerning policy coordination between countries Chari and Kehoe (1990), Persson and Tabellini (1995), Azzimonti and Quadrini (2024) and Berriel et al. (2024).

⁶In the context of the village insurance problem our results can be interpreted as that the optimal insurance is no insurance at all when agents are present bias and, thus, the immiseration result may not hold.

narrow down the set of applications. In particular, some results are known when preferences are quasi-linear, as in Galperti (2015), or when the planner can burn utility, as in Sublet (2023). In the first case, the insurance problem washes away, while in the second case, the mapping to a transfer system is unclear. To avoid these complications, we take a step backward in one direction to move forward in the other, and we assume a tractable parametric functional form for the transfer function.

3 Theoretical and institutional framework

3.1 Environment

We study a simple fiscal policy model involving a continuum of governments, each with the ability to make its own spending and borrowing decisions. Time is discrete and runs for two periods $t = 1, 2$. There is a unit mass of ex ante identical governments. Ex-post, at the beginning of the first period, each government experiences a shock to its spending needs, denoted by θ , which is drawn randomly from a compact set $\Theta = [\underline{\theta}, \bar{\theta}]$ with distribution function $F(\theta)$. We normalized the distribution such that $\mathbb{E}[\theta] = 1$. After the realization of the shock, each government chooses the first period spending g and second period spending k subject to a budget constraint:

$$g + \frac{k}{1+r} = W + T(g) \quad (1)$$

where $W = W_1 + \frac{W_2}{1+r}$ represents the present value of the government's tax revenue in the two periods, $T(g)$ are potential intergovernmental transfers, which can be negative, and r is an exogenous interest rate.

Governments have time-inconsistent preferences. Ex ante, prior to the realization of θ , governments order allocations according to:

$$\mathbb{E}[\theta u(g) + \delta \omega(k)] \quad (2)$$

where $u'(\cdot) > 0, \omega'(\cdot) > 0$ and $u''(\cdot) < 0, \omega''(\cdot) < 0$ and $\delta \in [0, 1]$ is a standard time discount factor. However, ex-post, after the government has observed the shock's realization, it orders allocations according to:

$$\theta u(g) + \delta \beta \omega(k) \quad (3)$$

The key difference between the last equation and (2) is the presence of the additional discounting $\beta \in [0, 1]$. If $\beta = 1$, and g could be chosen contingent on the realization of θ ,

a government seeking to maximize either of the preferences would arrive at the same result. However, whenever $\beta < 1$, the government would discount the future too much and thus borrow too much from the ex ante perspective. Note that in both cases, a higher value of θ increases the marginal utility of first-period spending, so it represents real higher spending needs. Hence, even from an ex ante perspective, it would be desirable that spending could respond to θ . This introduces a value for flexibility.

The factor $\beta < 1$ captures in a reduced form a tendency of governments to over-spend due to political turnover and preference misalignment as in Alesina and Tabellini (1990). As shown, among others, by Piguillem and Riboni (2020) such friction is isomorphic to hyperbolic discounting, which it would be the literal interpretation of these preferences. For this reason, and to simplify the exposition it is convenient to operate directly with the reduced form approach, as Aguiar and Amador (2011) and Halac and Yared (2014), but keeping in mind an underlying political decision process.

To understand its implications, we define two useful and common benchmarks. First, assume without loss of generality that $\delta(1 + r) = 1$ and $r = 0$. This implies that if full insurance were possible, spending would be constant over time. For future reference, we start by analyzing the case in which intergovernmental transfers are not possible, so that each government can only self-insure. In this case, each government would like, ex ante, to choose state contingent first and second period spending $\{g^{ea}(\theta), k^{ea}(\theta)\}$ satisfying:⁷

$$\theta u'(g^{ea}(\theta)) = \omega'(k^{ea}(\theta)) \quad (4)$$

For different levels of θ , governments would prefer to have different combinations of first and second period spending. However, after the realization of the shock, if a government is free to choose spending, it would choose $\{g^f(\theta), k^f(\theta)\}$ satisfying:

$$\theta u'(g^f(\theta)) = \beta \omega'(k^f(\theta)) \quad (5)$$

Thus, due to the concavity of both utility functions, and whenever $\beta < 1$, it is straightforward that the government would overborrow with respect to (4). We refer to this allocation as the flexibility allocation. This tendency itself would not be a problem if θ were observable and contractible. The government could write ex ante a state contingent “fiscal rule” stating how much can be spent in period 1 depending on the realization of the shock.

However, arguing that shocks to spending needs are either observable or contractible is difficult. For instance, whether an earthquake has happened or not is fairly straightforward to determine, but how much spending is (optimal) necessary to deal with its consequences

⁷This maximization is subject to the budget constraint (1) with $T = 0$.

is clearly more problematic. Given a shock θ , even if a fiscal rule were in place stating that spending should be determined by condition (4), an incumbent government can always argue that actual needs are better represented by shock $\tilde{\theta}$ such that $g^{ea}(\tilde{\theta}) = g^f(\theta)$.

To avoid these complications, it is natural to assume that governments are privately informed about the realization of the shock, rendering the ex-ante full commitment contract not feasible. As a result, the ex-ante government faces a trade-off between commitment and flexibility. Amador et al. (2006) studies this problem and finds that under some conditions on $F(\theta)$, the optimal contract takes the form of a θ threshold decision rule: all governments claiming spending needs below the threshold are free to choose spending and do so following (5), while all those claiming needs above this threshold, say θ^{NT} , the government must spend $g^f(\theta^{NT})$. Since this allocation is easily implemented with either a spending or deficit limit, from now on we will call it a fiscal rule. AWA also sharply characterize this threshold; their result holds when the density function satisfies certain conditions. Since we will derive analogous results, we impose the same assumption:

Assumption 1 *The density function $f(\theta)$ is differentiable and satisfies:*

a) *Bounded elasticity:*

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta}, \quad \forall \theta \in \Theta.$$

b) *$\bar{\theta}$ is finite and $f(\bar{\theta}) > 0$.*

For future reference we reproduce the AWA result here without a proof:

Definition 1 (AWA rule) *Absent intergovernmental transfers the optimal threshold $\theta^{NT} \in [\underline{\theta}, \bar{\theta}]$ satisfies:*

$$\mathbb{E}[\theta | \theta \geq \theta^{NT}] = \frac{\theta^{NT}}{\beta} \tag{6}$$

This is a natural benchmark that will play an important role in our analysis. The threshold provides the optimal balance between discretion, for those with $\theta \leq \theta^{NT}$, and commitment, for those with shocks above the threshold who cannot adjust spending to their needs. Assumption 1 makes sure that θ^{NT} is unique and below the upper bound of the distribution.⁸ There are two important assumptions that lead to its derivation. The first is that transfers across time are not possible, hence the NT superscript. Second, governments must remain in the contract. Since our goal is to analyze fiscal rules and risk sharing within the framework of

⁸As us, AWA also assume that θ is *i.i.d.* over time. Halac and Yared (2014) extend their results to environments with persistence shocks.

fiscal unions, accounting for the possibility of secession, we introduce both intergovernmental transfers and the potential for exiting the contract.

Another useful benchmark is the first best (full information) allocation, which when transfers are possible and unrestricted, is implementable:

Lemma 1 *First best allocation $\{g^{fb}(\theta), k^{fb}(\theta)\}$ satisfy:*

i) *There is full insurance.*

$$\theta u'(g^{fb}(\theta)) = \tilde{\theta} u'(g^{fb}(\tilde{\theta})) = \omega'(k^{fb}(\theta)) \quad \forall \theta, \tilde{\theta} \in \Theta \quad (7)$$

ii) *Dynamic aggregate efficiency: if $u(\cdot) = \log(\cdot)$ then $\int g^{fb}(\theta) dF(\theta) = \int k^{fb}(\theta) dF(\theta)$.*

Proof: See Appendix A.1.

This result is standard. If shocks were observable, a planner would design a transfer schedule such that not only the condition for the ex ante optimum is satisfied, but also the marginal utilities between different states of nature are equalized. The dynamic aggregate efficiency is an important feature that we will revisit later. Since $\delta(1+r) = 1$, with logarithmic utility it is the simple prescription that aggregate spending should be constant over time. In this two-period framework, this is not necessarily true with different preferences because in the first period there is risk, while in the second there is none. In an infinite horizon model with risk in every period the constancy of aggregate spending would be true for any concave preferences.

3.2 The federation design

To draw insights regarding federations and fiscal unions, we allow the possibility of transfers across types. We interpret each type θ not just as a realization of the shock but as a different realization for each member of a continuum of self-governed entities. These can be countries or subnational entities, such as states or provinces. These entities can form a “Federation” and share the risk. Since they are aware of their overspending tendency, at the time of the formation of the union, before any uncertainty is realized, the Federation designs a system of optimal transfers and fiscal rules limiting the spending behavior. We also assume that the local governments do not have access to state contingent bonds and the only source of insurance against shocks is through this risk-sharing agreement. We will also consider the possibility that any participant in this contract can ex post, after learning their type θ , exit the agreement.

Transfers and fiscal rules. A standard approach in this literature is to follow the mechanism design approach, as in Mirrlees (1971), and to assume that θ is not observable or contractible and then to solve for the optimal incentive compatible allocations. That is, one looks for the allocations that maximize ex ante welfare, in a set in which it is optimal for the countries to reveal the true realization θ . This approach has the advantage that it does not impose any restrictions on the set of instruments that can be used; they arise endogenously as a result of the maximization. However, the approach can also be difficult to work with when transfers are possible. To obtain sharp characterizations, one must resort to special cases, which may significantly narrow down the set of applications. In particular, some results are known when preferences are quasi-linear, as in Galperti (2015), or when the planner can burn utility, as in Sublet (2023). In the first case, the insurance problem washes away, while in the second case, the mapping to a transfer system is unclear. To avoid these complications, we take a step backward in one direction to move forward in the other.⁹ Following the Ramsey approach, we assume a functional form for the transfer function, $T(g)$, which depends on the amount spent and is restricted to be in the following set:

Assumption 2 *Intergovernmental transfers are homothetic in first-period spending.*

$$T(g) = g - (1 - \tau)g^{1-\rho} + T_0$$

with

$$\int_{\underline{\theta}}^{\bar{\theta}} T(g(\theta)) dF(\theta) = 0$$

This function, similar to Bénabou (2002), is instrumental in providing analytical results when preferences are homothetic, making it a popular choice in the optimal taxation literature.¹⁰ Its homothetic functional form is flexible enough to allow for some curvature of the transfer schedule, with the parameter ρ determining the *progressivity* of the federal transfer system. When $\rho > 0$, marginal transfers exceed average transfers, so the transfer system is progressive. Conversely, the transfer system is regressive when $\rho < 0$. The case $\rho = 0$ implies that marginal and average transfers are equal: this is the case for the affine transfer system. T_0 represents a fixed lump-sum component of the transfer, not present in Bénabou (2002), that all governments pay or receive regardless of their spending choice and are taken as given. Nevertheless, the federation has only access to resources from inside the union, hence the second part of the assumption making sure that the budget balances.

⁹See Beshears et al. (2020) for a similar line of argumentation and taking a similar approach to us.

¹⁰In the optimal taxation literature the lump sum component is in general absent, with the exception of Ferriere and Navarro (2024), but without providing analytical solutions. The lump sum component is essential in our analysis as it allows us to map the policy instrument to a debt contract.

Although Assumption 2 can be interpreted as a tax function, we interpret it as a common fund or a loan contract. To see this, consider the case with $\rho = 0$ such that $T(g) = \tau g + T_0$. It is straightforward from the second part of Assumption 2 that whenever $\tau > 0$ it must be that $T_0 < 0$. This case can be implemented as an *federal fund* with withdrawals proportional to spending needs. Before the shock is realized, all countries make an equal contribution T_0 to the common fund and can pay a fixed proportion of their spending with the resources extracted from the fund. Analogously, when $\tau < 0$ it must be that $T_0 > 0$, which can be implemented as a *loan* from the federation, of size T_0 , with *contingent payments* $\tau g(\theta)$. When $\rho \neq 0$, the interpretation is the same, with the only difference that withdrawals from the fund or the loan payments can be nonlinear, either increasing or decreasing with the amount spent. For this reason, in what follows we refer to them as loan- and fund-type contracts.

Note that governments, taking as given the transfer schedule $T(g)$, will now choose a flexible allocation that satisfies:

$$\theta u'(g^f(\theta)) = \beta \omega'(k^f(\theta)) [1 - T'(g^f(\theta))] \quad (8)$$

Whenever the transfer schedule $T(g)$ is increasing, it provides local incentives to over-borrow. On the other hand, whenever the marginal transfer is decreasing, it disincentivizes first-period spending. Hence, given the tendency to overspend, one may think it would always be optimal to implement a transfer contract that indirectly “taxes” current spending. For instance, we can define T_{pigou} as any transfer schedule that satisfies:

$$1 - T'_{pigou}(g) = \frac{1}{\beta}$$

Such a transfer schedule would counterbalance the governments’ present bias, providing enough incentives to achieve intertemporal smoothing as in the ex-ante optimum condition (4). However, the presence of contingent transfers makes condition (7) the right target. Such a policy would achieve the second equality in (7), but it does not help with the first equality targeting the equality among federation’s members. On the contrary, this policy would have a negative effect through the budget constraint: resources would be transferred from countries with high spending needs to countries with low spending needs, exacerbating inequality in the federation in terms of first period marginal utilities.

Moreover, analogously to the pecuniary externality in Aiyagari (1994) economies, the market incompleteness generates excessive savings (see Appendix B.1 for a formal proof with CRRA utility), counterbalancing the impact of the present-bias. The interaction between these two effects plays a key role shaping the results of this paper.

Due to the triggered shift in incentives, the introduction of transfers has also potential

implications for fiscal rules, with respect to the fiscal regulations prior to the federation's creation. Transfers from other countries could generate another incentive for governments to pretend to be affected by adverse shocks beyond the underlying present bias. If some fiscal rules were in place, they might be modified, and if there were none, it might be necessary to introduce new ones. This could lead to tighter fiscal rules. On the other hand, the federation could leverage the incentives due to the taxing component of the transfers and set up a mechanism, ameliorating the spending bias and to relax or eliminate any fiscal rules in place.

Hence, we assume that the federation can also impose deficit or spending limits.¹¹ This is the same type of mechanism that arises endogenously in AWA, when transfers are not possible. Since the transfer function in Assumption 2 preserves the monotonicity of decisions on the realization of θ , we can also study the problem from the perspective of the types θ that are constrained and those who are free to choose. Then we have:

Definition 2 (Fiscal rule) *A fiscal rule is a threshold $\theta_p \in [\underline{\theta}, \bar{\theta}]$, with associated spending $\{g_p, k_p\}$ determined by (8), such that:*

- i) Governments with $\theta \leq \theta_p$ have flexibility to choose spending according to (8).*
- ii) Governments with $\theta \geq \theta_p$ must choose the allocation $\{g_p, k_p\}$.*

As a result, the federation must choose ex ante the quadruple $\{\tau, \rho, T_0, \theta_p\}$ that maximizes the expected present value of utility of each member.

Participation constraints. The possibility of exiting is a key element when analyzing any type of fiscal, monetary, or political union; for example Ferrari et al. (2021). However, except for Halac and Yared (2022), the literature on fiscal rules has generally overlooked the issue of participation constraints. This is also a crucial issue for us, especially when applying the theory to a federation or a union of sovereign countries that retain the possibility of exiting. We assume that in the federation formation step, the designer internalizes this possibility, designing a contract such that it is not optimal for any member to leave, no matter the contingency. Notice that this is a simplifying assumption: one could design a federation that leaves room for exiting in case some extreme contingencies are realized, such that keeping a country in the federation is more costly than to let it leave. However, countries leaving unions or federations have always been a rare contingency in history, suggesting that the federation will design a mechanism robust to exits.

¹¹Due to the budget constraint this is also equivalent to a debt limit (lower bound on k , in our model). In an infinite horizon model it would be akin to a renegotiated debt limit.

We consider interim participation constraints: once the type is revealed, each local government decides whether to stay in or to exit the federation. Hence, the federation chooses the optimal allocation $\{g(\theta), k(\theta)\}$ subject to the constraint:

$$\theta u(g(\theta)) + \beta \omega(k(\theta)) \geq \theta u(g^{ex}(\theta, \phi)) + \beta \omega(k^{ex}(\theta, \phi)) \quad \forall \theta \in \Theta \quad (9)$$

where $\{g^{ex}(\theta, \phi), k^{ex}(\theta, \phi)\}$ is the allocation that each government would obtain if it were to exit the agreement, given the exogenous cost of leaving ϕ . We assume that once is out of the federation, the government applies no fiscal rule, receives or makes no transfers, and chooses spending levels subject to the budget constraint:

$$g + k = (1 - \phi)W \quad (10)$$

We interpret the cost of leaving as a reduced-form way to incorporate all the benefits of a federation that are not explicitly modeled, like removal of trade barriers, common foreign policy, international reputation, etc. The larger these benefits are, the less binding the participation constraints are. Whether we model the cost of leaving the federation as an output cost or a utility cost depends on the specific consequences of secession. However, this decision does not conceptually alter our approach. Our results remain robust under either of the modeling assumptions.

Two important elements that make our participation constraint different from the usual problem in the federations literature are the presence of the present bias and the potential existence of common fiscal rules. Absent these two components, it is usually the rich member that is reluctant to make transfers and tempted to exit. The temptation for the rich member remains when the transfer system is redistributive but, in addition with the present bias and a potential fiscal rule in place, members with high spending needs may also be tempted to exit. As a result, the participation constraint could be binding anywhere across the distribution, the top, the bottom, the middle, or a combination of these possibilities.

4 Federation without possibility of exit

We start the analysis disregarding the possibility of exiting, one can think, for instance, that $\phi = 1$, so that no country would ever leave. We first pose the federation maximization problem and then transform it into a simplified Ramsey that is easier to handle. The federation

designer, or the planner, solves the following problem:

$$\begin{aligned}
& \max_{\{g(\theta), k(\theta), T_0, \theta_p, \rho\}} \left\{ \int_{\underline{\theta}}^{\theta_p} [\theta u(g(\theta)) + \omega(k(\theta))] dF(\theta) + \int_{\theta_p}^{\bar{\theta}} [\theta u(g(\theta_p)) + \omega(k(\theta_p))] dF(\theta) \right\} \quad (11) \\
& s.t. \quad \int g(\theta) f(\theta) + \int k(\theta) f(\theta) \leq W \\
& \quad \theta g u'(g) + \beta k \omega'(k) = (W + T_0) \beta \omega'(k), \quad \forall \theta \leq \theta_p \\
& \quad \frac{\theta u'(g(\theta))}{\omega'(k(\theta))} = \left[\frac{g(\theta)}{g(\bar{\theta})} \right]^{-\rho} \frac{\bar{\theta} u'(g(\bar{\theta}))}{\omega'(k(\bar{\theta}))}; \quad \forall \theta, \bar{\theta} \leq \theta_p
\end{aligned}$$

The objective of the federation is to maximize the ex ante present value of all its members. For this reason, the parameter β does not appear in the objective welfare function. Note also that by the law of large numbers, this objective function also coincides with the ex ante expected value of each member. The first constraint is simply feasibility; the federation cannot allocate more resources than the total pooled assets of all members. The second and third are implementability constraints, making sure that the allocation can be implemented with the instruments in Assumption 2.

It is clear that the implementability constraints significantly complicated the solution. For tractability, we make the following assumption:

Assumption 3 *The utilities of the first and second periods are logarithmic.*

$$u(g) = \log(g), \quad \omega(k) = \log(k)$$

This assumption implies that the allocations are proportional to each other. Thus, it is possible to express them as a proportion of g_p and k_p specified in Definition 2. To do so, define:

$$\chi(\theta; \theta_p, \rho) = \frac{\theta_p + \beta(1 - \rho)}{\theta + \beta(1 - \rho)} \quad (12)$$

In Appendix A.2 we show that:

$$k(\theta) = \chi(\theta; \theta_p, \rho) k_p \quad (13)$$

$$g(\theta) = \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} g_p \quad (14)$$

Note that $g(\theta_p) = g_p$ and $k(\theta) = k_p$. The linearity of the allocations also allows us to write the aggregates as functions of g_p and k_p :

$$G(g_p, \theta_p) = g_p \left[\int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} dF(\theta) + (1 - F(\theta_p)) \right] \quad (15)$$

$$K(k_p, \theta_p) = k_p \left[\int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho) dF(\theta) + (1 - F(\theta_p)) \right] \quad (16)$$

In Appendix A.2 we also show that there is a one-to-one mapping from τ and T_0 to g_p and k_p . Hence, we can abstract from the choice $\{\tau, T_0\}$ and directly choose $\{g_p, k_p\}$, bearing in mind that there are some implicit policy parameters that implement those allocations. As a result, under Assumption 3 the optimal federation design is characterized by the solution of the following problem:

$$\begin{aligned} \max_{\{g_p, k_p, \theta_p, \rho\}} \int_{\underline{\theta}}^{\theta_p} \left(\theta \log \left[\chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} g_p \right] + \log [\chi(\theta; \theta_p, \rho) k_p] \right) dF(\theta) + \\ \int_{\theta_p}^{\bar{\theta}} \{ \theta \log[g_p] + \log[k_p] \} dF(\theta) \\ s.t. \quad G(g_p, \theta_p) + K(k_p, \theta_p) \leq W \end{aligned}$$

This problem is substantially simpler than (11). By using the utility's functional forms and characterizing the optimal choices, we can eliminate the implementability constraints, with only the feasibility constraint remaining. We start by stating a key result that is important for understanding the features of the optimal federation design:

Proposition 1 *Suppose Assumptions 2 and 3 hold, then the optimal allocation achieves dynamic aggregate efficiency, in the sense that:*

$$G(g_p, \theta_p) = K(k_p, \theta_p) = \frac{W}{2}$$

Proof: Let λ be the multiplier on the feasibility constraint, then the first order conditions with respect to g_p and k_p yield:

$$\frac{1}{g_p} = \lambda \left\{ \int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} dF(\theta) + (1 - F(\theta_p)) \right\} = \lambda \frac{G(g_p, \theta_p)}{g_p} \quad (17)$$

$$\frac{1}{k_p} = \lambda \left\{ \int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho) dF(\theta) + (1 - F(\theta_p)) \right\} = \lambda \frac{K(k_p, \theta_p)}{k_p} \quad (18)$$

The second equality in equations (17) and (18) follow from equations (15) and (16). Then,

since $\lambda > 0$ and the feasibility constraint holds with equality, the statement of the proposition is immediate. ■

We call it dynamic aggregate efficiency because, as we show in Lemma 1, the equality between aggregate spending of the first and second periods (constancy over time) is a characteristic of the first-best allocation. In the first-best allocations, a planner would like to perfectly smooth the aggregate spending over time. A constrained planner designing the federation would retain this feature independently of the severity of the spending bias and the extent of the spending needs' risk.

This result is by no means obvious. As we discussed in Section 3.2, there are two frictions acting in opposite directions. A precautionary savings motive that leads to too little spending and the present bias pushing toward excessive spending. Thus, one might think that depending on the intensity of the insurance needs vs. the temptation to overspend, the planner would be willing to accept some inefficiency on the aggregate dynamics, with perhaps too little or too much aggregate spending in the first period. Instead, with logarithmic pay-offs, aggregate dynamics takes precedence over the other concerns. The reason is that then, due to the cancellation of income and substitution effects, the objective function is separable between the static and dynamic components. As dynamic efficiency is achievable with any instrument that affects savings decisions, the planner takes care of it. Without Assumption 3, the perfect smoothing may not hold, but as long as the marginal utility is convex, all the qualitative findings of this paper still hold.

Proposition 1 is key to the design of optimal transfers and fiscal rules within the Federation. In particular, the marginal transfers would have to adjust to make sure that this condition is always true, which in turn would have an impact on the fiscal rule. To analyze how, we have the following.

Proposition 2 *The optimal fiscal rule inside the federation exists and satisfies:*

$$\mathbb{E}[\theta | \theta \geq \theta_p] = \frac{\theta_p}{\beta} + T'(g_p) \frac{2g_p}{W} \quad (19)$$

Proof: See Appendix A.3.

Equation (19) illustrates in a straightforward manner how the marginal transfer and the fiscal rule interact with each other. Recall that the optimal fiscal rule is determined by the point at which the marginal benefit of enhanced flexibility to respond to shocks equals the marginal cost due to excessive spending. Absent transfers, as shown in equation (6), the first

two terms in equation (19) suffice, coinciding with the AWA rule of Definition 1. However, introducing marginal transfers at this equilibrium threshold changes the balance.

Due to Assumption 1 the difference $\mathbb{E}[\theta|\theta \geq \theta_p] - \frac{\theta_p}{\beta}$ is decreasing in θ_p , if $T'(\theta_p)$ is positive, the equilibrium θ_p must be smaller than the nontransfer threshold θ^{NT} . In this case, the federation provides more insurance to governments with flexibility: the larger their needs, the larger the transfer. The increased insurance is costly, as it provides stronger incentives to overborrow. As a result, it becomes optimal to tighten the fiscal rule, reducing the set of governments with flexibility. In contrast, a negative $T'(\theta_p)$ would lead to a relaxation of the optimal fiscal rule with respect to AWA. In this case, the negative marginal transfer acts as a Pigouvian tax on spending, which reduces the incentives to overspend. This reduces the insurance component of the transfer to governments with flexibility, but allows the planner to compensate by reducing the set of constrained governments. The only scenario in which the fiscal rule remains unaffected is in which the marginal transfer at the threshold equals zero.

Lemma 2 *Suppose $\rho \leq \frac{2\theta}{\beta}$, there exists at least one interior solution to Equation (19), in the sense that for any $\beta \in (0, 1]$, there is one solution $\theta_p \in (\underline{\theta}, \bar{\theta})$. Moreover, whenever $\rho = 0$, this solution increases monotonically in β .*

Proof: See Appendix A.4.

The implications of Lemma 2 are twofold. It clearly states that a solution exists and, thus, analyzing equation (19) is meaningful. But it also states that the solution of interest is interior; neither no flexibility at all nor full flexibility is ever optimal. This is in stark contrast with the setting without transfers, where depending on the degree of present bias taking all flexibility away could be optimal (if β is sufficiently low), or granting full flexibility could be desirable (when $\beta \rightarrow 1$).

However, the uniqueness of is not guaranteed; we also show in Appendix A.4 that $\theta_p = \underline{\theta}$ is a solution to equation(19) as well, but this solution corresponds to a local minimum. All the remaining solutions are interior. To ensure that the interior solution is unique, we need the implicit function defined in (19) to be have at most one inflection point for all β and ρ . We can prove that this is true in general when either θ_p is close to $\underline{\theta}$, or for all θ_p when β is small enough (see Appendix A.6). With additional assumptions, it is possible to provide sharper results. For instance, in Appendix B.3 we show that if the distribution of shocks is uniform and we constrain the set of contracts to be linear, $\rho = 0$, there is a unique monotone solution to equation (19). Nevertheless, we present many numerical simulations for alternative distributions and for arbitrary ρ which confirm that the solution is, as far as we tested it, unique and monotone in β .

The following corollary is a consequence of Lemma 2:

Corollary 1 *There exists at least one present-bias $\beta^{NT} \in (0, 1)$ such that $\theta_p = \theta^{NT}$.*

Proof: The equations characterizing $\theta_p(\beta)$ and $\theta^{NT}(\beta)$ are continuous in β . In addition, due to Assumption 1, $\theta^{NT}(\beta)$ is increasing in β with $\theta^{NT}(0) = \underline{\theta}$ and $\theta^{NT}(1) = \bar{\theta}$. Hence, it follows from Lemma 2 that there exists $\beta^{NT} \in (\underline{\theta}, 1)$ such that $\theta^{NT}(\beta^{NT}) = \theta_p(\beta^{NT})$. ■

We can strengthen the previous results by imposing additional structure on the setting. By assuming that the distribution is uniform and restricting the set of instruments to be affine, we can provide a sharper characterization that it is also instrumental in explaining the intuition behind the main mechanisms.

Corollary 2 *Assume $\rho = 0$ and $f(\theta) \sim U[\underline{\theta}, \bar{\theta}]$. There exists a unique β^{NT} such that $\theta^{NT}(\beta^{NT}) = \theta_p(\beta^{NT})$. Moreover,*

a) $\theta_p < \theta^{NT}$ and $\tau > 0$ whenever $\beta \in (\beta^{NT}, 1]$.

b) $\theta_p > \theta^{NT}$ and $\tau < 0$ whenever $\beta \in (0, \beta^{NT})$.

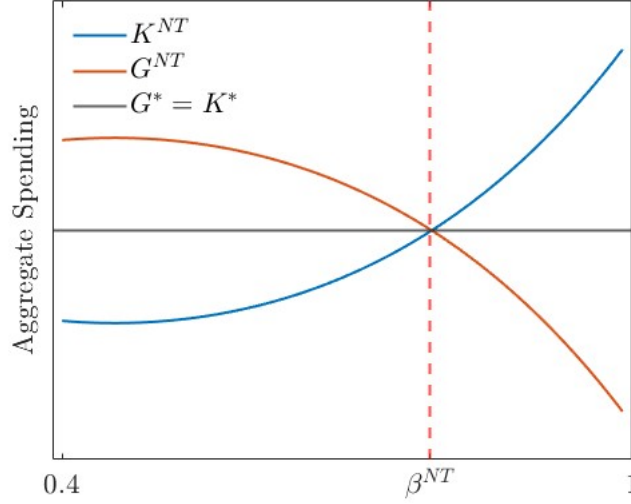
Proof: The uniqueness of β^{NT} follows from the uniqueness of the threshold, as shown in Appendix B.3. Since both fiscal rules are monotone increasing in β and $\theta_p(1) < \bar{\theta} = \theta^{NT}(1)$ (with the reverse inequality when $\beta \leq \underline{\theta}$), the inequalities regarding θ_p and θ^{NT} follow. In order to see the implications for the marginal distortion, notice that when $\rho = 0$ equation (19) becomes

$$\mathbb{E}[\theta | \theta \geq \theta_p] = \frac{\theta_p}{\beta} + \tau \frac{2g_p}{W} \quad (20)$$

Due to Assumption 1, $\mathbb{E}[\theta | \theta \geq \theta_p] - \frac{\theta_p}{\beta}$ is decreasing in θ_p and θ^{NT} is defined such that $\mathbb{E}[\theta | \theta \geq \theta^{NT}] - \frac{\theta^{NT}}{\beta^{NT}} = 0$, the implications for τ are immediate. ■

To understand the intuition for these results, we plot in Figures 1 and 2 the results of Corollary 2. Figure 1 shows how aggregate first and second periods spending changes with β , comparing the environments with and without transfers. The black continuous line depicts the optimal aggregate spending for both the first and second periods. As stated in Proposition 1, the aggregate dynamic efficiency requires that spending in the first and second periods is equalized, hence the single line $G^* = K^*$. The red decreasing curve is the aggregate first-period spending generated by the AWA rule, while the blue increasing line is the implied second-period spending. It is clear from the figure that, in general, aggregate

Figure 1: Aggregate spending: AWA vs federation



Note: the figure is computed assuming a $U[0.15, 1.85]$ and $\rho = 0$. To verify the first order necessary conditions, the optimal is computed maximizing over a grid of θ_p and τ .

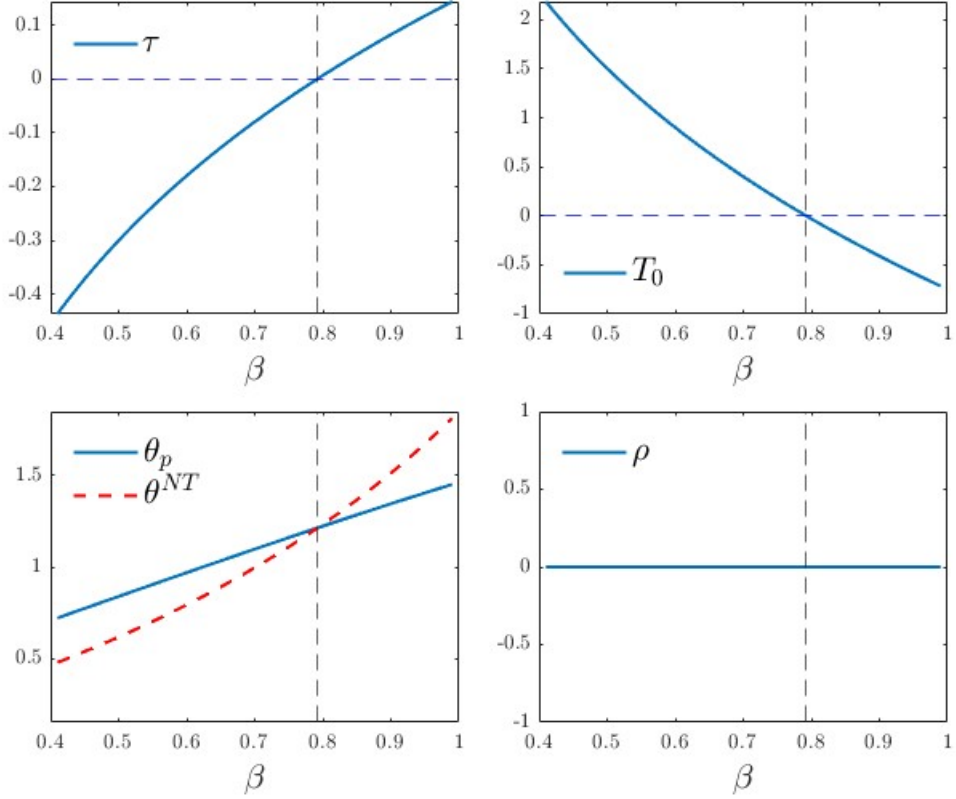
spending patterns do not coincide with the optimal first-best spending patterns, except when $\beta = \beta^{NT}$.

These results are shaped by the interaction of two “frictions”: the spending bias and the excess precautionary savings due to market incompleteness. The first leads to too much spending, while the second leads to too little. For example, when $\beta = 1$ the spending bias is absent, and thus if transfers are not possible, it is optimal not to impose fiscal rules. This generates excessive savings $K^{NT} > G^{NT}$. As β decreases, it is optimal to start to impose fiscal rules, which become tighter as β falls. The present bias by itself reduces first-period spending, eventually leading to too much first-period spending. The AWA fiscal rule slows down this process, but it does not stop it.¹² Under the right parametrization the interaction of these two forces can restore, at least, aggregate dynamic efficiency, but it happens only by chance. Instead, when transfers are possible, aggregate efficiency takes prevalence and for any intensity of the spending bias, dynamic aggregate efficiency is restored.

Figure 2 illustrates the implications for policy instruments. The lower right figure simply states that this solution corresponds to the predictions of Corollary 2, assuming that only linear ($\rho = 0$) contracts are possible. The lower left figure compares the fiscal rule in the fiscal union with the AWA fiscal rule. There, it can be seen that both increase monotonically in β and intersect at β^{NT} . This is the same value of β^{NT} depicted in Figure 1. As stated in

¹²Even in the absence of fiscal rules or any other intervention, there would be a $\beta \in (0, 1)$ such that aggregate spending in the first and second period would be equal.

Figure 2: Optimal linear contract



Note: the figure is computed assuming a $U[0.15, 1.85]$ and $\rho = 0$. To verify the first order necessary conditions, the optimal is computed maximizing over a grid of θ_p and τ .

the previous results, θ^{NT} spans from $\beta^{NT} = \underline{\theta}$ when $\beta \leq \underline{\theta}$ to $\beta^{NT} = \bar{\theta}$ when $\beta = 1$. Instead, θ_p , although monotone increasing, remains above $\underline{\theta}$ for all β and never reaches $\bar{\theta}$. Inside a fiscal union, fiscal rules are tighter than outside it, when the present bias is small, and the rules are looser when the present bias is large.

To understand the implications for transfers, the upper panel shows the marginal transfer on the left and the lump sum component on the right. At $\beta = \beta^{NT}$, the marginal transfer is zero $\tau = 0$. This follows from the fact that any positive or negative marginal transfer would create a distortion that would make the federation lose the dynamic efficiency. Due to the restriction in the contract, this also implies that the lump sum component and the totals transfers are zero. As a result, fiscal unions are not necessary. A confederation that imposes **only** fiscal rules on its members would be the optimal arrangement.

Fiscal unions become optimal when the present bias is either small or large. If the present bias is sufficiently mild, $\beta > \beta^{NT}$, the marginal transfer τ is positive. Under such a system, countries pay a lump sum contribution and receive a contingent payment depending on their spent amount. This kind of mechanism could appear faulty since incentivizes countries to

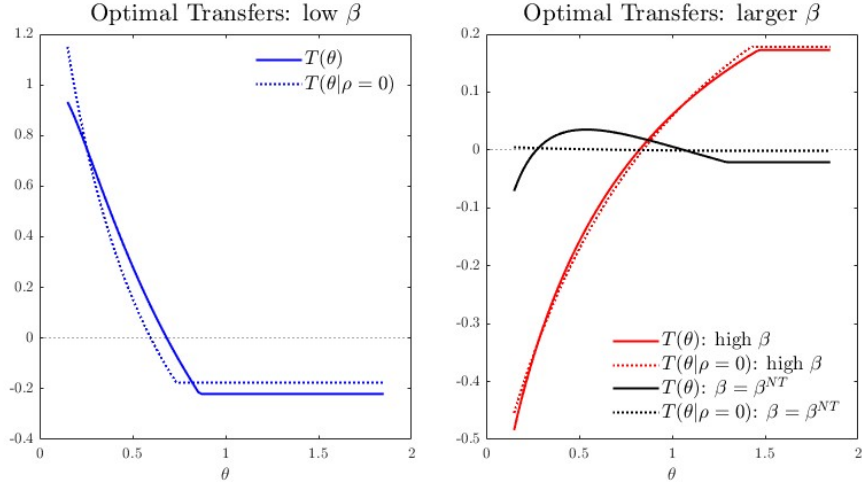
spend and overstate their true spending needs. However, in these economies, the precautionary savings motive is dominant, and thus the spending subsidy helps to attenuate the savings bias. The subsidy could have a disproportionate impact on those countries with extreme spending needs, but the presence of a fiscal rule ensures that it remains under control. We interpret this type of contract as a common **contingency fund**. All countries contribute to the common fund the same amount T_0 , then the larger the government's need, and hence its spending in the first period, the more resources it can withdraw from the Fund.

Instead, when $\beta < \beta^{NT}$ the present bias is the dominant force. In this case, the marginal transfer is negative $\tau < 0$ and the lump sum is positive $T_0 > 0$. Since the tendency to overspend is so problematic that it can lead to excessive aggregate first-period spending, it is optimal to “tax” spending. Each government received in compensation a transfer from the federation. This policy can be implemented with a contract resembling a **loan with state contingent repayments**. All governments receive the same loan T_0 and must repay the federation proportionally to the amount they spend in the first period. The more the spending, the larger the loan payment. Since this taxing policy attenuates the spending bias, specially at the top of the distribution of θ , it is possible to relax the fiscal rule with respect to the one outside a fiscal union.

An important question is who are the winners and losers of the transfer system. How much intergovernmental insurance is possible? Figure 3 illustrates the patterns of total net transfers for each type θ . That is, the function $T(g)$ from Assumption 2 evaluated in the optimal $g(\theta)$, which we denote by $T(\theta)$. We do so for alternative levels of present bias; a high present bias environment with $\beta < \beta^{NT}$ (left panel) and moderated and low present bias environments with $\beta = \beta^{NT}$ (in black) and $\beta > \beta^{NT}$ (in red), respectively (right panel). In all cases the dotted lines correspond to the solutions under the optimal linear contract (Corollary 2), while the continuous lines depict the analogous transfers under the optimal nonlinear contract.

The red lines on the right panel (low spending bias) illustrate the typical risk-sharing pattern inside a federation. Governments with low spending needs make a net contribution, while governments that experience large spending need shocks receive a net transfers from the federation. This is a standard **redistributive federation**: a group of states that establish a fiscal union where members with low needs (or abundant resources) make indirect transfers to those with high needs (or scarce resources). The black curves, still on the right panel, exhibit a **federation without fiscal union**: the federation imposes fiscal rule on its members, but the intergovernmental transfers are negligible. Due to the canceling effects of the excess precautionary savings motive with the spending bias, engaging in redistributive policies is no longer optimal.

Figure 3: Total net transfers

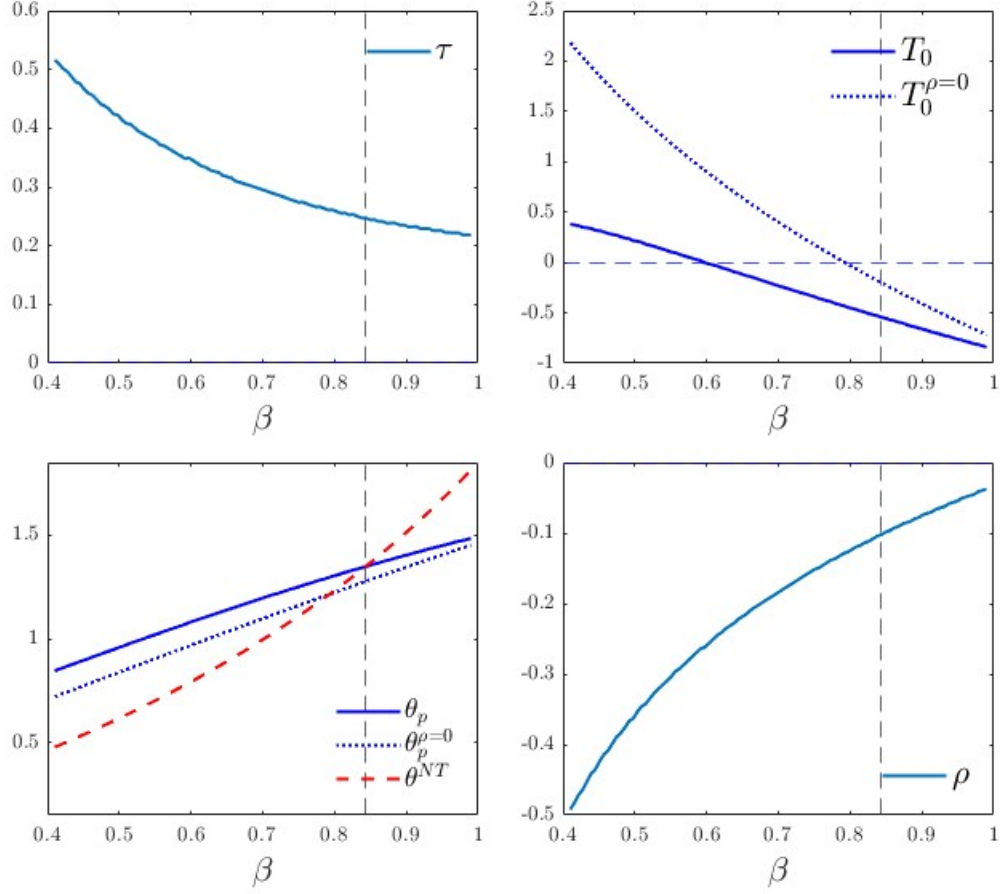


Note: the figure is computed assuming a $U[0.15, 1.85]$ and $\rho = 0$ for the dotted lines, and the optimal ρ for the continuous lines. β^{NT} corresponds to the setting with only a linear contract. To verify the first order necessary conditions, the optimal is computed maximizing over a grid of θ_p and τ . For the left panel, $\beta = 0.47$ while for the right panel $\beta = 0.97$

Finally, the left panel depicts the case with severe spending bias. In this case, the pattern of net transfers is inverted. The high-need governments are net contributors, while the low-need governments are net receptors. We call this case a **punitive federation**. The federation now includes a fiscal union, but its purpose is to tax the high spenders to punish them for overspending and use the proceeds to reward the responsible low spenders with additional resources.

In the next section we discuss in more detail the implications of allowing for a nonlinear contract, however, it is clear from Figure 3, that these results are not due to the restriction to the linear contract. When we allow the planner to optimize over ρ a remarkable similar pattern emerges. The incentives and patterns are akin to those in Halac and Yared (2018). In their setting, changes in the interest rate play the role of the marginal subsidy in our setting. When the interest rate increases, the savers benefit, and the borrowers are punished, generating an implicit transfer. Analogously, they find that depending on the value of β , the planner may prefer to loosen the fiscals, which increases the interest rate, or tighten the rules up, which decreases it. They do not have, however, a result analogous to our Proposition 1 stressing the prevalence of dynamic aggregate efficiency.

Figure 4: Optimal non-linear contract

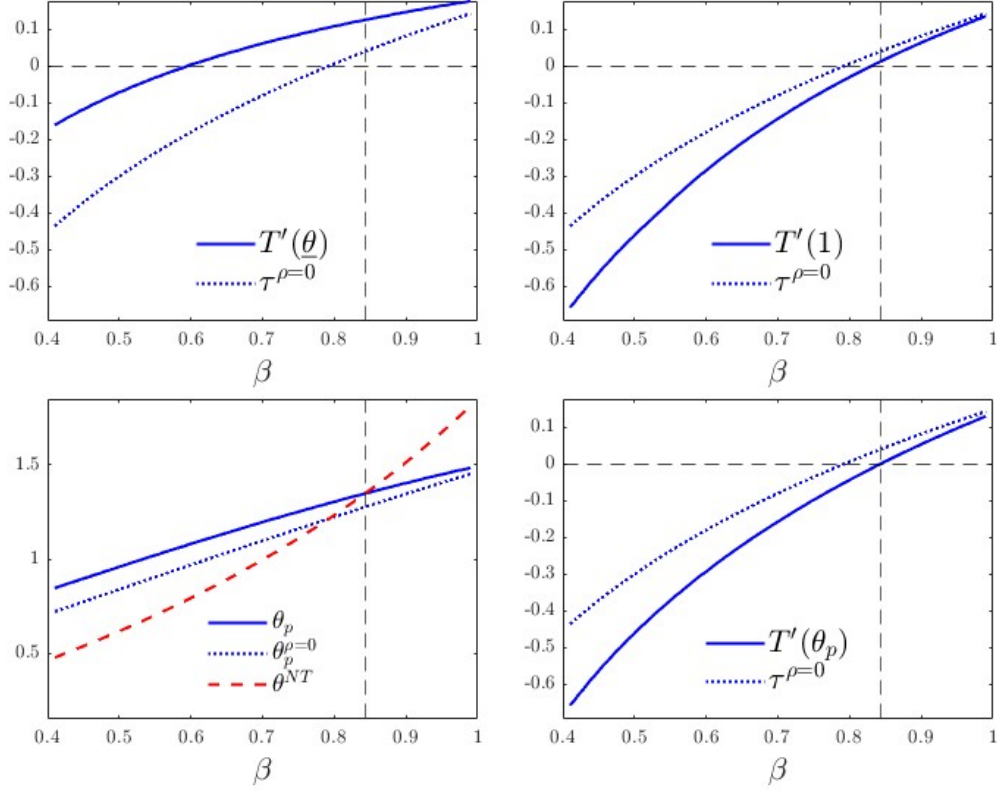


4.1 Non-linear transfer system

In the previous section we focus on the linear contract, since Corollary 2 provides sharp theoretical characterization. In this section, we show that the patterns and intuition developed before remain unaltered when the planner can freely choose ρ . Figure 4 is analogous to Figure 2. Starting from the lower right panel, we can see that in this case, assuming a uniform distribution, the optimal ρ is always negative. This implies that $T(g)$ is concave, so that the marginal transfer decreases with spending. The more the country spends, the lower the marginal transfer or the higher the tax. This provides additional incentives, with respect to the linear case, to control the spending bias. Hence, it is possible to relax the fiscal rule, which always lies above the fiscal rule under a linear contract (lower left panel of Figure 4). Again, as stated in Corollary 1 the fiscal rule never reaches the corners, as β varies and intersects the AWA fiscal rule. Note that the intersection between the optimal rule now happens at a different point from the linear contract. Consistent with equation (19) these two fiscal rules crossed when $T'(g_p) = 0$.

To understand the patterns of transfers, it is useful to analyze the implications of the

Figure 5: Marginal transfers



nonlinear contract for marginal transfers $T'(g(\theta)) = T'(\theta)$. Figure 5 assumes a uniform distribution of shocks and shows marginal transfers as a function of β at three points on the distribution of θ : $\underline{\theta}$, $\theta = 1$, and θ_p . Looking at the lower right panel, it is clear that $T'(\theta_p)$ resembles the pattern of τ under the linear contract. It is negative for low β values, positive for large β , and is exactly zero at $\beta = \beta^{NT}$, the point where the optimal rule crosses the AWA rule. For comparison, in each figure we also plot the optimal τ under the linear contract assumption. By choosing the optimal ρ the planner is able to provide better incentives. For all values of β , the types with more temptation to spend receive smaller marginal transfers, and the types with less temptation (upper left corner) receive larger marginal transfers with respect to the linear contract.

To complement this information, in Appendix B.2, Figure 9 we show marginal transfers as a function θ , for alternative values of β . It depicts the implications of $\rho < 0$, for any value of β the optimal transfer system features decreasing marginal transfers, the more the country spends, the lower the marginal transfer. That is, with a uniform distribution of shocks, the incentive motive dominates over the redistributive motive for all degrees of spending bias. As β approaches 1, the slope of the marginal transfer function reduces (ρ approaches zero), but never reaches zero. One might be tempted to conclude that this is a general feature of

the problem. However, this is not the case. In Appendix B.2 we show the solutions under a truncated exponential distribution. In that case, for values of β sufficiently close to 1, ρ is positive (see Figure 12) generating an increasing marginal transfer function (Figure 10). This implies that for sufficiently low temptation to spend, the redistributive motive may dominate, and thus the planner allocates more resources to types who need it the most. The key difference between the uniform and exponential distributions is that the last has a thin tail. There are a few types with very high spending needs, so it is not too costly to provide insurance. In contrast, if types are uniformly distributed, there are just too many governments that would be claiming large spending needs and too few with low needs to finance them. Hence, redistributive policies are not feasible.

The distribution of types also affects the areas where the loan or the common fund is optimal. In the upper right panel of Figure 4, the common fund is optimal for a substantially larger set of economies when a nonlinear transfer system is possible if high spending needs are very likely (uniform distribution). In contrast, if large spending needs are relatively less likely, as with the uniform distribution of types, the loan contract is the optimal choice for most levels of temptation to spend (see Figure 12).

5 Federation with possibility of exit

This section relaxes the full commitment assumption by introducing the possibility for member states to exit the federation after the realization of the shock. Leaving the federation implies that the country is no longer constrained by the fiscal rule, but that it also must forfeit any transfer rights or obligations associated with the fiscal union.

There are two primary reasons why a country might consider exiting: (i) the government is constrained by fiscal rules and wishes to increase its spending, or (ii) the federation's transfer system imposes a net fiscal burden. Although the transfer system generally enhances welfare, its scale can be significantly limited by the threat of potential member exit. We show that both the transfer schedule and the fiscal rule are affected by the possibility of exit. Although the introduction of participation constraints (PCs) makes both types of federation more difficult to implement, punitive federations are more affected by the risk of exit.

Allowing the output cost of leaving the federation to take any value $\phi \in [0, 1]$, the federation designer now solves the following problem:

$$\begin{aligned}
& \max_{\{g(\theta), k(\theta), T_0, \theta_p, \rho\}} \left\{ \int_{\underline{\theta}}^{\theta_p} [\theta u(g(\theta)) + \omega(k(\theta))] dF(\theta) + \int_{\theta_p}^{\bar{\theta}} [\theta u(g(\theta_p)) + \omega(k(\theta_p))] dF(\theta) \right\} \quad (21) \\
& s.t. \quad \int g(\theta) f(\theta) + \int k(\theta) f(\theta) \leq W \\
& \quad \theta g u'(g) + \beta k \omega'(k) = (W + T_0) \beta \omega'(k), \quad \forall \theta \leq \theta_p \\
& \quad \frac{\theta u'(g(\theta))}{\omega'(k(\theta))} = \left[\frac{g(\theta)}{g(\bar{\theta})} \right]^{-\rho} \frac{\bar{\theta} u'(g(\bar{\theta}))}{\omega'(k(\bar{\theta}))}; \quad \forall \theta, \bar{\theta} \leq \theta_p \\
& \quad \theta u(g(\theta)) + \beta \omega(k(\theta)) \geq \theta u(g^{ex}(\theta, \phi)) + \beta \omega(k^{ex}(\theta, \phi)) \quad \forall \theta \in \Theta
\end{aligned}$$

Problem (21) is the same as (11) in Section 4, with the addition of the participation constraints for each government. The right-hand side of each constraint represents the governments' outside option given the exit cost ϕ . First, it is important to state a technical lemma that greatly simplifies the analysis.

Lemma 3 *An allocation $\{g(\theta), k(\theta)\}_{\theta \in \Theta}$ satisfies the participation constraints if and only if it holds for the extreme types $\underline{\theta}$ and $\bar{\theta}$.*

Proof: See Appendix B.5.

Lemma 3 allows us to simplify the problem. A sufficient condition for the participation constraint to hold for every $\theta \in \Theta$ is that it holds at the extremes of the distribution. Intuitively, the local governments drawing $\bar{\theta}$ or $\underline{\theta}$ are the most tempted to exit. For example, in a redistributive federation, these can be, respectively, the government most constrained by the fiscal rule and the one that potentially contributes the most to the common fund. In a punitive federation, the $\underline{\theta}$ -government receives positive transfers, and hence its participation constraint will never bind. In this case $\bar{\theta}$ -government may face both a negative transfer and a tight fiscal rule, either of which could make the participation constraint binding. In general, it does not need to be the case that only one extreme type is constrained. It is straightforward to provide examples, by varying β , in which the participation constraints in both extremes are binding.

Using Lemma 3, we can solve problem (21) by considering only the participation constraints at the extremes of the distribution. We now proceed to show how the key results from the previous section are affected by the introduction of the possibility of exit.

Proposition 3 *Let $\{\bar{\mu}, \underline{\mu}\}$ be the Lagrange multipliers of the participation constraint at $\bar{\theta}$ and $\underline{\theta}$, respectively and suppose Assumptions 2 and 3 holds, then dynamic aggregate efficiency is not generally achieved in presence of participation constraints and:*

$$\begin{cases} G(g_p, \theta_p) > K(k_p, \theta_p), & \text{if } \bar{\mu}(\bar{\theta} - \beta) > \underline{\mu}(\beta - \underline{\theta}) \\ G(g_p, \theta_p) < K(k_p, \theta_p), & \text{if } \bar{\mu}(\bar{\theta} - \beta) < \underline{\mu}(\beta - \underline{\theta}) \\ G(g_p, \theta_p) = K(k_p, \theta_p), & \text{if } \bar{\mu}(\bar{\theta} - \beta) = \underline{\mu}(\beta - \underline{\theta}) \end{cases}$$

Proof: See Appendix A.7.

When the cost of leaving the federation is high enough such that the participation constraints are never binding ($\underline{\mu} = \bar{\mu} = 0$), the problem is equivalent to that without exit possibility; hence, dynamic aggregate efficiency is achieved. When participation constraints are binding, the result of Proposition 1 is preserved only under restrictive conditions. When only $PC(\underline{\theta})$ binds, the ability of the federation to subsidize first-period spending is limited by the threat of exit from low-need countries. As a result, at the optimum, there is excess aggregate savings. This would happen in the redistributive federation where $\underline{\theta}$ is a net contributor to the common fund.

When only $PC(\bar{\theta})$ binds, there are two cases: (i) the federation is limited in the ability to implement a punitive federation, unable to optimally tax first period spending, (ii) the federation is implementing a redistributive federation but the unconstrained optimal fiscal rule is not implementable and relaxing it, or increasing further the transfers, leads to excess aggregate spending. The only case in which dynamic aggregate efficiency is preserved requires both PCs to be binding and the respective effect on aggregate spending G and aggregate savings K to cancel out. Participation constraints hence have a direct impact on the implementable optimal transfer system, and the following corollary formalizes this idea. If low-need governments are tempted to exit, marginal transfers must fall. If high-need governments are at risk of exiting, the federation must increase transfers to retain them.

In order to derive more intuitive characterizations, in the remainder of this section we assume $\rho = 0$ and focus on the linear contract. Nevertheless, in Appendix B.6 we present numerical simulations with $\rho \neq 0$, stressing that the qualitative findings are the same when the linearity assumption is dropped. Then we have,

Corollary 3 *Define τ^* and τ^{PC} as optimal marginal transfers when $\phi = 1$ and when $\phi < 1$, respectively. Then, given a fiscal rule θ_p ,*

$$\begin{cases} \tau^{PC} > \tau^*, & \text{if } \bar{\mu}(\bar{\theta} - \beta) > \underline{\mu}(\beta - \underline{\theta}) \\ \tau^{PC} < \tau^*, & \text{if } \bar{\mu}(\bar{\theta} - \beta) < \underline{\mu}(\beta - \underline{\theta}) \\ \tau^{PC} = \tau^*, & \text{if } \bar{\mu}(\bar{\theta} - \beta) = \underline{\mu}(\beta - \underline{\theta}) \end{cases}$$

Proof: See Appendix A.8.

The optimal linear contract illustrated in Corollary 3 follows directly from the previous discussion. With binding PCs, the federation will be either less punitive, when the high-need government is tempted to leave, or less redistributive, when the low-need government is the one tempted. In Section 4 we uncovered a tight relationship between optimal transfers and the fiscal rule. There, larger marginal transfers led to tighter fiscal rules, while lower marginal transfers allowed the federation to loosen up these limits. Although this relationship is still present, it is less straightforward when participation constraints might be binding.

Proposition 4 *Under Assumption 2 and 3, the optimal fiscal rule satisfies the following condition:*

$$\mathbb{E}[\theta|\theta \geq \theta_p^{PC}] = \frac{\theta_p^{PC}}{\beta} + \lambda\tau(\bar{\mu}, \underline{\mu})g_p - \frac{\bar{\mu}}{1 - F(\theta_p^{PC})}(\bar{\theta} - \theta_p^{PC}) \quad (22)$$

Proof: The proof is analogous to that in Proposition 2 incorporating the Lagrange multiplier in the participation constraint.

Proposition 4 shows that the relationship between transfers and fiscal rules of Proposition 3 is preserved. Other than that, even disregarding the fact that τ , g_p , λ , $\bar{\mu}$ and $\underline{\mu}$ are endogenous, it is not possible to draw any straightforward effect. The following Lemma provides additional intuition on the interaction between PCs, fiscal rules and transfers.

Lemma 4 *Define θ_p^* as the optimal fiscal rule when $\phi = 1$. Then:*

a) *when only $PC(\underline{\theta})$ is binding, then $\theta_p^{PC} \geq \theta_p^*$*

b) *when $PC(\bar{\theta})$ is binding, then $\theta_p^{PC} \geq \theta_p^*$ if*

$$\bar{\theta} \frac{A'(\theta_p^{PC})}{A(\theta_p^{PC})} + \beta \frac{B'(\theta_p^{PC})}{B(\theta_p^{PC})} \leq 0 \quad (23)$$

where $A(\theta_p^{PC}) = \frac{G}{g_p}$ and $B(\theta_p^{PC}) = \frac{K}{k_p}$

Proof: See Appendix A.9

Lemma 4 sheds light on how participation constraints (PC) affect the fiscal rule compared to the results of the previous section. Part (a) states that when low-need governments are tempted to exit, as in the case of a redistributive federation, the fiscal rule is relaxed by the federation. This may appear counterintuitive, since one would expected that if it is the less

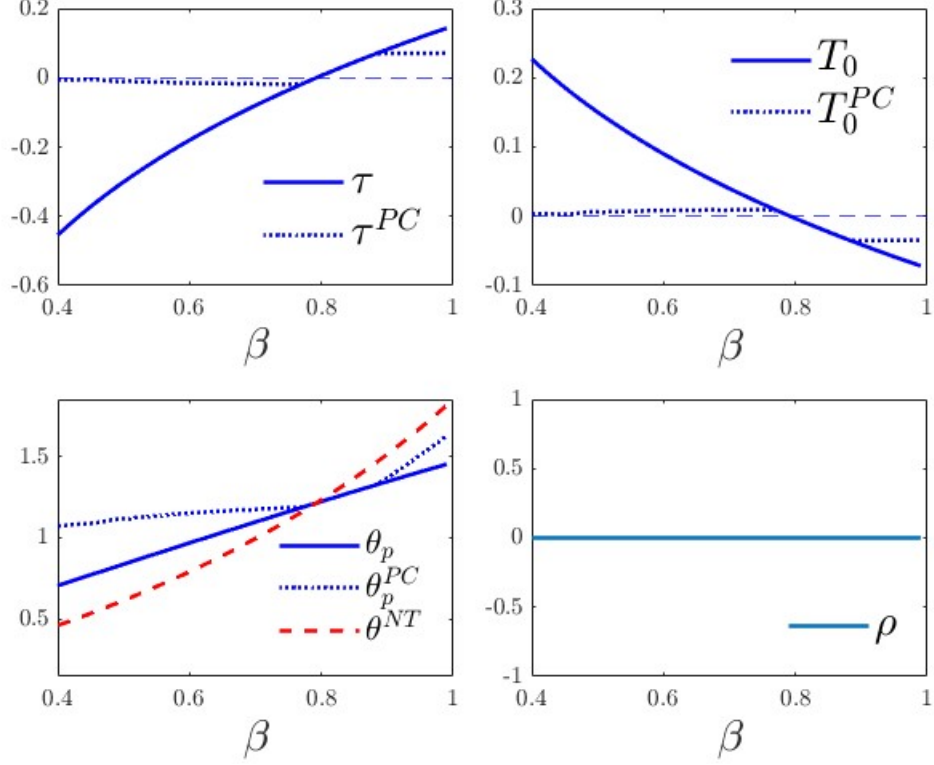
present biased agent the one who wants to exit, it will remain inside by forcing tighter fiscal rules on the other members. However, when PCs are binding for low-need governments, the marginal transfers are reduced (as shown in Corollary 3). As the members are then less tempted to overspend, it is possible to loosen the fiscal rule. Part (b) states that when high-need governments are tempted to leave, the effect on the fiscal rule becomes ambiguous. On the one hand, the same mechanism as in part (a), now with $\tau < 0$ rather than $\tau > 0$, would suggest a tighter rule. On the other hand, the direct effect of a high temptation to spend pushes in the opposite direction. If condition (23) holds, the second force dominates, and whenever PCs are binding, the optimal fiscal rule becomes looser.

Note that whether condition (23) is satisfied depends only on the distribution of shocks and β . Since $A' < 0$ and $B' > 0$ the condition is more likely to be satisfied when $\bar{\theta}$ is large and β is small, which is fairly intuitive. In fact, in our numerical simulations, this is always true at the optimal allocations. Nevertheless, in Appendix A.9, we show that with a uniform distribution of shocks, the condition is always true if β is sufficiently small. A high degree of present bias drives the optimal contract toward a punitive federation, which imposes higher costs on high-need governments. Therefore, if the participation constraint for the highest-need type is binding only under a punitive federation, condition (23) is more likely to be satisfied, making a looser fiscal rule more likely to emerge.

Figure 6 illustrates the implications for policy instruments of introducing the possibility of exit, assuming an output cost of leaving the federation of $\phi = 2.5\%$. As in Figure 2, the lower left panel simply reflects the assumption that $\rho = 0$. The lower right panel compares the fiscal rule under three scenarios: with and without participation constraints (PCs), and under the AWA fiscal rule. Since this economy satisfies condition (23), when PC at $\underline{\theta}$ is binding, the optimal fiscal rule with the possibility of exit is always looser than the benchmark, regardless of the degree of present bias. When governments are tempted to exit, whether they have high or low spending needs, the optimal response is to relax fiscal rules.

The implications for transfers are aligned with the predictions of Corollary 3. When β is close to one and a redistributive fiscal union is optimal, participation constraints become binding for low-need governments. As a result, marginal transfers must be reduced to prevent the exit of net contributors. As β decreases, a punitive federation would become optimal. Then, the participation constraints for high-need governments begin to bind, forcing the federation to substantially scale back the redistributive features. While for $\beta > \beta^{NT}$ the redistributive fiscal union remains viable, albeit weakened, the punitive federation becomes negligible in equilibrium. This result reconciles our model with the empirical observation that punitive federations are rarely, if ever, implemented. Both types of fiscal union are constrained by the possibility of exit, but attempts to implement a punitive federation face

Figure 6: Optimal linear contract with exit possibility



Note: the figure is computed assuming a $U[0.15, 1.85]$, $\rho = 0$ and $\phi = 0.025$. To verify the first order necessary conditions, the optimal is computed maximizing over a grid of θ_p and τ .

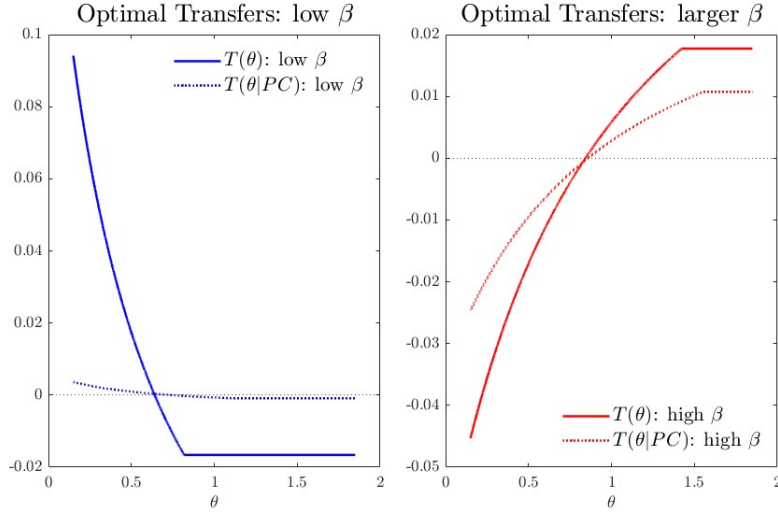
a particularly strong obstacle: a high-need government, in this context, can hit two birds with one stone. Although such a federation may be optimal ex ante due to strong present-bias frictions, it becomes unfeasible ex post, once governments have the option to exit and retain both their policy flexibility and the resources they would otherwise transfer to other countries.

This effect is more clearly illustrated in Figure 7: when β is low, the fiscal union is not implementable once participation constraints are introduced. When β is high, the redistributive nature of the fiscal union is weakened but is still preserved. In Appendix B.6, we present additional simulations showing how the optimal contract varies with different levels of exit costs and allowing for non-linear contracts with $\rho \neq 0$.

6 Concluding Remarks

How much fiscal integration is optimal inside a federation, between countries, or within a country is a fundamental issue in public finance. There are two key elements shaping these decisions: how much insurance can be achieved through the integration and how some

Figure 7: Total net transfers



Note: the figure is computed assuming a $U[0.15, 1.85]$, $\rho = 0$ and $\phi = 0.025$. To verify the first order necessary conditions, the optimal is computed maximizing over a grid of θ_p and τ . For the left panel, $\beta = 0.47$ while for the right panel $\beta = 0.97$

members could profit from it overspending resources originated in other jurisdictions. In this paper, we revisit this issue in a context in which the decision makers in the member states are made by present biased agents. This captures the fact that in modern democracies, the decision makers are usually elected policy makers who, due to the political process, may act as present bias agents.

We show that even when the commitment to a federation is not an issue, the possibility of insurance is limited, so much so that a fiscal union may not be optimal. In this case, the best a federation can achieve is to design a set of fiscal rules that would apply to and must be respected by all members. Within the federation, each member acts as an independent fiscal entity. Whenever a fiscal union is advisable, the optimal fiscal arrangement depends on whether the goal is to provide insurance or to control fiscal irresponsibility. If the goal is to provide insurance, the optimal arrangement can be implemented with a common fund with contingent withdrawals (increasing in spending) and tighter fiscal rules. If the goal is to control fiscal irresponsibility, a loan contract with contingent payments and looser fiscal rules appears as an appropriate mechanism. In this case, the commitment to remain within the union becomes more relevant as members in need become net contributors.

We have purposely streamlined the environment to focus the attention on two elements that we believe are fundamental for the existence of a fiscal union. However, many important questions remain unanswered. What is the optimal size of the federation? How should the fiscal union deal with aggregate risk? How the fiscal union changes in the presence of public goods? etc. All these are important issues that we leave for future research.

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A Appendix

A.1 Proof of Lemma 1

Suppose that an unrestricted planner maximizes:

$$\begin{aligned}
 & \max_{\{g(\theta), k(\theta)\}} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} [\theta u(g(\theta)) + \delta u(k(\theta))] f(\theta) \right\} \\
 s.t. \quad & \int_{\underline{\theta}}^{\bar{\theta}} g(\theta) f(\theta) + \frac{1}{(1+r)} \int_{\underline{\theta}}^{\bar{\theta}} k(\theta) f(\theta) \leq W
 \end{aligned}$$

The first-order necessary conditions are:

$$\theta u'(g(\theta)) = \lambda; \quad \delta u'(k(\theta)) = \frac{\lambda}{1+r}; \quad \forall \theta$$

With the log utility, the first-order necessary conditions imply $g(\theta) = \theta/\lambda$ and $k(\theta) = \delta(1+r)/\lambda$. Integrating both, it is clear that aggregate dynamic efficiency requires $\int_{\underline{\theta}}^{\bar{\theta}} k(\theta) f(\theta) = \delta(1+r)/\lambda$ and $\int_{\underline{\theta}}^{\bar{\theta}} g(\theta) f(\theta) = 1/\lambda$, which are equal whenever $\delta(1+r) = 1$. Since the objective function is concave and the constraint set is convex, these conditions are also sufficient.

A.2 Setting the Ramsey problem

Replacing Assumption 2 in the budget constraint (1), generates $(1-\tau)g^{1-\rho} + k - T_0 \leq W$. Thus, (8), can be written as $\theta u'(g) = \beta(1-\tau)(1-\rho)g^{-\rho}\omega'(k)$ and using the log is generates:

$$\theta \frac{1}{g(\theta)} = \beta(1-\tau)(1-\rho)g^{-\rho} \frac{1}{k(\theta)} \quad (24)$$

Solving for $(1-\tau)g^{1-\rho}$ and replacing it inside the budget constraint:

$$k(\theta) = (W + T_0) \frac{\beta(1-\rho)}{\theta + \beta(1-\rho)}$$

From which we can isolate T_0 :

$$T_0 = \frac{\theta + \beta(1-\rho)}{\beta(1-\rho)} k(\theta) - W$$

which allows us to write second period spending of all governments in terms of second period spending for the threshold country hit by shock θ_p :

$$k(\theta) = \frac{\theta_p + \beta(1-\rho)}{\theta + \beta(1-\rho)} k_p \quad (25)$$

Performing the analogous exercise of solving (24) for $k(\theta)$ and replacing it into the budget constraint, we obtain:

$$g(\theta) = \left(\frac{\theta_p + \beta(1-\rho)}{\theta + \beta(1-\rho)} \right)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} g_p \quad (26)$$

Where g_p and k_p satisfy:

$$k(\theta_p) = (W + T_0) \frac{\beta(1-\rho)}{\theta_p + \beta(1-\rho)} \quad (27)$$

$$g_p(\theta_p) = k(\theta_p)^{\frac{1}{1-\rho}} \left[\frac{\theta_p}{\beta(1-\tau)(1-\rho)} \right]^{\frac{1}{1-\rho}} \quad (28)$$

Introducing these equations into the welfare function and using the definition of χ we arrive at the simplified problem of Section 4. Moreover, from equations (27) and (27) is straightforward that there is a one to one mapping from τ and T_0 to g_p and k_p .

Lemma 5 *Welfare is not concave at $\theta_p = \underline{\theta}$ and so it cannot be a maximum. It implies that some flexibility is always optimal and $\theta_p > \underline{\theta}$ even with $\beta \rightarrow 0$.*

Proof: The Hessian of the welfare function is not negative semidefinite at $\theta_p = \underline{\theta}$. We can rewrite welfare as $W(\theta_p) = \int_{\underline{\theta}}^{\theta_p} m(\theta; \theta_p) f(\theta)$, where

$$m(\theta; \theta_p) = \left[\frac{\theta}{1-\rho} + 1 \right] \log(\chi(\theta, \rho)) + \frac{\theta}{1-\rho} [\log(\theta) - \log(\theta_p)]$$

Thus,

$$\frac{\partial W}{\partial \theta_p} = m(\theta_p; \theta_p) + \int_{\underline{\theta}}^{\theta_p} m'(\theta; \theta_p) f(\theta) = \int_{\underline{\theta}}^{\theta_p} m'(\theta; \theta_p) f(\theta)$$

given that $m(\theta_p; \theta_p) = 0$. Hence, the second derivative becomes:

$$\frac{\partial^2 W}{\partial \theta_p^2} = m'(\theta_p; \theta_p) f(\theta_p) + \int_{\underline{\theta}}^{\theta_p} m''(\theta; \theta_p) f(\theta)$$

Since $m'(\theta_p; \theta_p) = \frac{1-\beta}{\theta_p+\beta} \geq 0$ and $m''(\theta; \theta_p) = -\frac{\theta+1}{(\theta_p+\beta)^2} - \frac{\theta}{\theta_p^2} < 0$ it follows that the Hessian is not negative semidefinite at $\theta_p = \underline{\theta}$ given that the integral vanishes and only the positive term remains. The nondiagonal term of the Hessian are nil and the other diagonal elements are, respectively $-1/(g_p)^2$ and $-1/(g_p)^2$. Hence, by the principal minors condition, the matrix is negative semidefinite only if $\frac{\partial^2 W}{\partial \theta_p^2} < 0$. Thus, if $\theta_p = \underline{\theta}$ cannot be a local maximum. ■

A.3 Proof of Proposition 2

We want to write the first-order necessary conditions with respect to θ_p . Since the algebra is tedious, we first compute some useful derivatives. Let $\tilde{\beta} = \beta(1-\rho)$, then

$$\begin{aligned} \frac{\partial \left(\chi(\theta, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} \right)}{\partial \theta_p} &= \frac{1}{1-\rho} \left(\chi(\theta, \rho)^{\frac{\rho}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{\rho}{1-\rho}} \right) \left[\frac{\theta}{(\theta + \tilde{\beta})\theta_p} - \frac{\theta_p + \tilde{\beta}}{\theta + \tilde{\beta}} \frac{\theta}{\theta_p^2} \right] \\ &= -\frac{\beta}{\theta_p} \left(\chi(\theta, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} \right) \frac{1}{\theta_p + \tilde{\beta}} \end{aligned}$$

With,

$$\frac{\partial \chi(\theta, \rho)}{\partial \theta_p} = \frac{1}{\theta + \tilde{\beta}} \quad \text{and} \quad \frac{\partial \log(\chi(\theta, \rho))}{\partial \theta_p} = \frac{1}{\theta_p + \tilde{\beta}}$$

Let $U(\theta_p, \beta)$ be the objective function. Note that:

$$U_1(\theta_p, \beta) = \int_{\underline{\theta}}^{\theta_p} \left[-\frac{\beta\theta}{\theta_p(\theta_p + \tilde{\beta})} + \frac{1}{\theta_p + \tilde{\beta}} \right] f(\theta) d\theta$$

$$= \frac{1}{\theta_p + \tilde{\beta}} \left[-\frac{\beta}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta f(\theta) d\theta + F(\theta_p) \right] \quad (29)$$

Then the first-order necessary condition with respect to θ_p is:

$$U_1(\theta_p, \beta) - \lambda \int_{\underline{\theta}}^{\theta_p} \left\{ -\frac{\beta g_p}{\theta_p} \left[\left(\chi(\theta, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} \right) \frac{1}{\theta_p + \tilde{\beta}} \right] + k_p \frac{1}{\theta_p + \tilde{\beta}} \right\} f(\theta) d\theta = 0$$

$$- \frac{\beta}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta f(\theta) d\theta + F(\theta_p) =$$

$$\lambda k_p \int_{\underline{\theta}}^{\theta_p} \chi(\theta, \rho) f(\theta) d\theta - \frac{\beta}{\theta_p} \lambda g_p \int_{\underline{\theta}}^{\theta_p} \left(\chi(\theta, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} \right) f(\theta) d\theta$$

Now using the fact that $\mathbb{E}(\theta) = 1$ and plugging in (17) and (18) we obtain:

$$-\frac{\beta}{\theta_p} \left[1 - \int_{\theta_p}^{\bar{\theta}} \theta f(\theta) d\theta \right] + F(\theta_p) = 1 - \lambda k_p (1 - F(\theta_p)) - \frac{\beta}{\theta_p} (1 - \lambda g_p (1 - F(\theta_p)))$$

Simplifying the terms and dividing by $1 - F(\theta_p)$ we obtain the fiscal rule.

$$\mathbb{E}[\theta | \theta \geq \theta_p] = \frac{\theta_p}{\beta} + \lambda \left(g_p - \frac{\theta_p}{\beta} k_p \right) \quad (30)$$

Note that (24) evaluated at θ_p implies:

$$\theta_p \frac{1}{g(\theta_p)} = \beta(1 - \tau)(1 - \rho)g(\theta_p)^{-\rho} \frac{1}{k(\theta_p)} = \beta[1 - T'(g_p)] \frac{1}{k(\theta_p)}$$

Thus, replacing (17) and (18) in the above:

$$\theta_p \lambda \frac{G(\theta_p, \rho)}{g(\theta_p)} = \beta[1 - T'(g_p)] \lambda \frac{K(\theta_p, \rho)}{k(\theta_p)}$$

$$\Rightarrow 1 - T'(g_p) = \frac{\theta_p k(\theta_p)}{\beta g(\theta_p)}$$

Where we have used Proposition 1 to cancel the aggregates. Replacing the last in (30):

$$\mathbb{E}[\theta|\theta \geq \theta_p] = \frac{\theta_p}{\beta} + \lambda g_p T'(g_p)$$

Since equation (17) implies $\lambda = G(\theta_p, g_p)^{-1} = \frac{2}{W}$, the proposition's statement follows. ■

A.4 Existence of interior θ_p (proof of Lemma 2)

Here we prove the existence of a θ_p such that (30) holds and that $\theta_p < \bar{\theta}$, under the assumption that $\rho = 0$. We start by using (17) to define:

$$h(\theta_p) = \mathbb{E}[\theta|\theta \geq \theta_p] - \frac{\theta_p}{\beta} - \frac{1}{A(\theta_p)} + \frac{\theta_p}{\beta B(\theta_p)} \quad (31)$$

where

$$A(\theta_p) = \int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}} dF(\theta) + (1 - F(\theta_p))$$

$$B(\theta_p) = \int_{\underline{\theta}}^{\theta_p} \chi(\theta; \theta_p, \rho) dF(\theta) + (1 - F(\theta_p))$$

First of all it is easy to show that $h(\underline{\theta}) = 0$. Note that $A(\underline{\theta}) = 1$ and $B(\underline{\theta}) = 1$ are both equal to 1, since integrals would be defined over a point and be equal to zero. Then we know that $\theta_p = \underline{\theta}$ is always going to be a solution to (30). We will later show that it represents a local minimum. However, we show that $h(\theta_p) > 0$ when $\theta_p \rightarrow \underline{\theta}^+$. In order to show that, we take the derivative of $h(\theta_p)$:

$$h'(\theta_p) = \frac{\partial \mathbb{E}[\theta|\theta \geq \theta_p]}{\partial \theta_p} - \frac{1}{\beta} + \frac{A'(\theta_p)}{A^2(\theta_p)} + \frac{1}{\beta B(\theta_p)} - \frac{\theta_p B'(\theta_p)}{\beta B^2(\theta_p)}$$

The first term is always positive. We are interested in $h'(\underline{\theta})$. Note that

$$A'(\theta_p) = \int_{\underline{\theta}}^{\theta_p} \frac{\partial \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}}}{\partial \theta_p} dF(\theta) + f(\theta_p) - f(\theta_p)$$

which is equal to 0 when $\theta_p = \underline{\theta}$. With similar manipulations, it is straightforward that $B'(\underline{\theta}) = 0$. This implies that:

$$h'(\underline{\theta}) = \frac{\partial \mathbb{E}[\theta|\theta \geq \theta_p]}{\partial \theta_p} \Big|_{\theta_p = \underline{\theta}} > 0$$

which follows from differentiation of the equation.

The next step is to show that $h(\bar{\theta}) < 0$. At the upper bound it must be that

$$h(\bar{\theta}) = \bar{\theta}(1 - \frac{1}{\beta}) - \frac{1}{A(\bar{\theta})} + \frac{\bar{\theta}}{\beta B(\bar{\theta})} = \left(\bar{\theta} - \frac{1}{A(\bar{\theta})} \right) + \frac{\bar{\theta}}{\beta} \left[\frac{1}{B(\bar{\theta})} - 1 \right]$$

Recall that $\chi(\theta, \theta_p, \rho) = \frac{\theta_p + \beta(1-\rho)}{\theta + \beta(1-\rho)} > 1$, for all $\theta < \theta_p \leq \bar{\theta}$. It follows that $B(\bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \chi(\theta; \bar{\theta}, \rho) dF(\theta) > 1$. Therefore, $\frac{1}{B(\bar{\theta})} - 1 < 0$. Hence, since the last term is negative and $\beta \leq 1$, it must be that

$$h(\bar{\theta}) \leq \left(\bar{\theta} - \frac{1}{A(\bar{\theta})} \right) + \bar{\theta} \left[\frac{1}{B(\bar{\theta})} - 1 \right] = \frac{\bar{\theta}}{B(\bar{\theta})} - \frac{1}{A(\bar{\theta})}$$

Thus, we need to show that $\bar{\theta}A(\bar{\theta}) < B(\bar{\theta})$. To do so, notice that $\chi(\theta; \bar{\theta}, \rho)$ is strictly convex in θ and that $[\chi(\theta; \bar{\theta}, \rho)\theta]^{\frac{1}{1-\rho}}$ is concave if the assumption of the Lemma holds. Then, from Jensen's inequality it follows that:¹³

$$\bar{\theta}A(\bar{\theta}) = \bar{\theta}^{\frac{-\rho}{1-\rho}} \mathbb{E} \left[[\chi(\theta; \bar{\theta}, \rho)\theta]^{\frac{1}{1-\rho}} \right] \leq \bar{\theta}^{\frac{-\rho}{1-\rho}} \chi(1; \bar{\theta}, \rho)^{\frac{1}{1-\rho}} \leq \chi(1; \bar{\theta}, \rho) < \mathbb{E} [\chi(\theta; \bar{\theta}, \rho)] = B(\bar{\theta})$$

where the strict inequalities follow from strict convexity and that $\mathbb{E}[\theta] = 1$. The middle inequality holds for each $\rho < 1$ given that:

$$\begin{aligned} \bar{\theta}^{\frac{-\rho}{1-\rho}} \chi(1; \bar{\theta}, \rho)^{\frac{1}{1-\rho}} \leq \chi(1; \bar{\theta}, \rho) &\Rightarrow \bar{\theta}^{\frac{-\rho}{1-\rho}} \leq [\chi(1; \bar{\theta}, \rho)]^{\frac{-\rho}{1-\rho}} \\ \frac{1}{\bar{\theta}^{\frac{\rho}{1-\rho}}} &\leq \frac{1}{[\chi(1; \bar{\theta}, \rho)]^{\frac{\rho}{1-\rho}}} \Rightarrow \chi(1; \bar{\theta}, \rho) \leq \bar{\theta} \end{aligned}$$

which is always true given that $\bar{\theta} \geq 1$ and $\rho < 1$. The proof that θ_p is increasing in β when $\rho = 0$ can be found in Appendix A.5. ■

A.5 θ_p is increasing in β with linear contract. (Used in Lemma 2)

In this appendix we show that the interior solution of Lemma 2 is monotone increasing in β when $\rho = 0$. To do so, notice that the first-order necessary condition can be written in a compact way as:

$$U_1(\theta_p, \beta) = \lambda [g_p A'(\theta_p; \beta) + k_p B'(\theta_p; \beta)]$$

¹³To see this, bear in mind that the concavity is with respect to θ only. Thus, we can abstract from the constant terms and look at the properties of the function $m(\theta) = \left[\frac{\theta}{\theta + \beta(1-\rho)} \right]^{\frac{1}{1-\rho}}$. Differentiating it is possible to show that the first derivative is positive, and the second derivative is negative whenever $\rho \leq \frac{2\theta}{\beta}$.

where $U_1 > 0$ is given by equation (29). Substitution inside the conditions for g_p and k_p , it becomes

$$U_1(\theta_p, \beta) = \frac{A'(\theta_p; \beta)}{A(\theta_p; \beta)} + \frac{B'(\theta_p; \beta)}{B(\theta_p; \beta)} = H(\theta_p; \beta)$$

Using the implicit function theorem, the change of θ_p with respect to β is given by:

$$\frac{d\theta_p}{d\beta} = -\frac{U_{1,2}(\theta_p; \beta) - H_2(\theta_p; \beta)}{U_{1,1}(\theta_p; \beta) - H_1(\theta_p; \beta)}$$

where the subindex captures the variable with respect to which the derivative is taken. I.e, $A_1 = \frac{\partial A(\theta_p; \beta)}{\partial \theta_p}$ and $U_{1,2} = \frac{\partial^2 U}{\partial \theta_p \partial \beta}$. From equation (29) evaluated at $\rho = 0$ we have that

$$U_{1,2} = -\frac{1}{\theta_p + \beta} \left[U_1 + \frac{1}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \right] = -\frac{1}{\theta_p + \beta} \left[\frac{A_1}{A} + \frac{B_1}{B} + \frac{1}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \right] \quad (32)$$

where we have substituted U_1 with $H(\cdot)$ given by the first order condition. Moreover, from the definitions of $A(\cdot)$, $B(\cdot)$ (see Appendix A.6), it is straightforward that

$$B_2 = \frac{\partial B}{\partial \beta} = \int_{\underline{\theta}}^{\theta_p} \frac{\theta - \theta_p}{(\theta + \beta)^2} dF(\theta) < 0 \quad (33)$$

$$B_{1,2} = \frac{\partial^2 B}{\partial \theta_p \partial \beta} = - \int_{\underline{\theta}}^{\theta_p} \frac{1}{(\theta + \beta)^2} dF(\theta) < 0 \quad (34)$$

$$\frac{\partial^2 A}{\partial \theta_p \partial \beta} = \frac{-1}{\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta}{\theta + \beta} \right) dF(\theta) + \frac{\beta}{\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{\theta}{(\theta + \beta)^2} dF(\theta) = \frac{-1}{\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta}{\theta + \beta} \right)^2 dF(\theta)$$

Note that for θ_p to be a maximizer it must be the case that $U_{1,1} - H_1 \leq 0$.¹⁴ Thus, the sign of $\frac{d\theta_p}{d\beta}$, at the optimal choice, is the same as the sign of $U_{1,2} - H_2$, which is also equal to the sign of $(\theta_p + \beta)(U_{1,2} - H_2)$.

Then, since

$$H_2 = \frac{\frac{\partial^2 A}{\partial \theta_p \partial \beta} A - A_1 A_2}{A^2} + \frac{\frac{\partial^2 B}{\partial \theta_p \partial \beta} B - B_1 B_2}{B^2}$$

Using equation (32), the sign of the derivative is determined by:

$$-\frac{A_1}{A} - \frac{B_1}{B} - (\theta_p + \beta) \left[\frac{A_{1,2}}{A} - \frac{A_1 A_2}{A^2} + \frac{B_{1,2}}{B} - \frac{B_1 B_2}{B^2} \right] - \frac{1}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \quad (35)$$

¹⁴That is, although the function $h(\cdot)$ in the previous section has two solutions, only the second one where $h(\theta_p)$ is decreasing, is maximizer.

Although the expression appears difficult, note that

$$B_1 + (\theta_p + \beta)B_{1,2} = \int_{\underline{\theta}}^{\theta_p} \frac{1}{(\theta + \beta)} dF(\theta) - \int_{\underline{\theta}}^{\theta_p} \frac{(\theta_p + \beta)}{(\theta + \beta)^2} dF(\theta) = \int_{\underline{\theta}}^{\theta_p} \frac{\theta - \theta_p}{(\theta + \beta)^2} dF(\theta) = B_2$$

As a result:

$$-\frac{B_1}{B} - (\theta_p + \beta) \frac{B_{1,2}}{B} + (\theta_p + \beta) \frac{B_1 B_2}{B^2} = -\frac{B_2}{B} \left[1 - \frac{B_1(\theta_p + \beta)}{B} \right] = -\frac{B_2}{B^2} [1 - F(\theta_p)]$$

where the last equality follows from the fact that $B = (\theta_p + \beta)B_1 + (1 - F(\theta_p))$. Since from equation (33) follows that $B_2 < 0$, all the terms involving B add to a positive number. Moreover, since $A_1 < 0$, $A_2 < 0$ and $A_{1,2} < 0$, all terms that involve A in expression (35) also add to a positive number.

There is still the last negative term in the expression (35). This negative term is not enough to overcome the summation of positive terms due to A because:

$$\begin{aligned} -\frac{A_1}{A} - (\theta_p + \beta) \frac{A_{1,2}}{A} &= \frac{1}{A} \left[\frac{\beta}{\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{\theta}{\theta + \beta} dF(\theta) + \frac{(\theta_p + \beta)}{\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta}{\theta + \beta} \right)^2 dF(\theta) \right] \\ &= \frac{1}{A\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{2\theta\beta + \beta^2 + \theta\theta_p}{(\theta + \beta)^2} dF(\theta) \geq \frac{1}{A\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta + \beta}{\theta + \beta} \right)^2 dF(\theta) = \frac{F(\theta_p)}{A\theta_p^2} \end{aligned} \quad (36)$$

Then we have

$$\frac{F(\theta_p)}{A\theta_p^2} - \frac{1}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \geq \frac{1}{\theta_p^2} \left[F(\theta_p) - \theta_p \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \right] = \frac{1}{\theta_p^2} \left[F(\theta_p) - \theta_p \left(1 - \int_{\theta_p}^{\bar{\theta}} \theta dF(\theta) \right) \right]$$

The inequality is due to the fact that $A \leq 1$ and the last equality arises from $E(\theta) = 1$. Moreover:

$$\frac{1}{\theta_p^2} \left[F(\theta_p) - \theta_p \left(1 - \int_{\theta_p}^{\bar{\theta}} \theta dF(\theta) \right) \right] \geq \frac{1}{\theta_p^2} [F(\theta_p) - \theta_p + \theta_p^2]$$

This is true for all $\theta_p \geq 1$ and depending on the distributions, also for smaller θ_p . For example, if θ is distributed $U([0, 2])$, it is true for all the support. To extend the result for $\theta_p < 1$, note that from equation (36) we can avoid the inequality and write:

$$\frac{1}{A\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{2\theta\beta + \beta^2 + \theta\theta_p}{(\theta + \beta)^2} dF(\theta) = \frac{1}{A\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{(\theta + \beta)^2 + \theta(\theta_p - \theta)}{(\theta + \beta)^2} dF(\theta)$$

Furthermore,

$$\frac{1}{A\theta_p^2} \int_{\underline{\theta}}^{\theta_p} \frac{(\theta + \beta)^2 + \theta(\theta_p - \theta)}{(\theta + \beta)^2} dF(\theta) - \frac{1}{\theta_p} \int_{\underline{\theta}}^{\theta_p} \theta dF(\theta) \geq$$

$$\frac{1}{\theta_p^2} \left[\int_{\underline{\theta}}^{\theta_p} \frac{(\theta + \beta)^2 (1 - \theta_p \theta) + \theta(\theta_p - \theta)}{(\theta + \beta)^2} dF(\theta) \right] \geq 0$$

The first inequality is due to $A \leq 1$ and the second is always true since $\theta_p \leq 1$ and the integration is over $\theta \leq \theta_p$.

A.6 Characterization of $h(\theta_p)$

The optimal θ_p solves $h(\theta_p)$. We have already shown that this equation is initially increasing and that close to the upper bound is strictly negative. This implies that $h'(\theta_p)$ becomes negative for some level of $\theta_p \in (\underline{\theta}, \bar{\theta})$. It follows from the Intermediate Value Theorem that the number of zeros of the $h(\cdot)$ function must be odd, which means that it either has 1 or more than 3 zeros. To have three zeros, the function must have either two local minima and one local maximum or two local maxima and one local minimum. This implies that the second derivative changes sign at least twice (from negative to positive and from positive to negative or viceversa). It follows that in order to show that $h(\theta_p) = 0$ has a unique solution, it is enough to show that the function has at most one inflection point. For example, if the function is strictly concave in the entire support, than the solution is unique. But strict concavity is a sufficient condition, not necessary. To see this consider the derivatives:

$$h'(\theta_p) = \frac{\partial \mathbb{E}[\theta | \theta \geq \theta_p]}{\partial \theta_p} - \frac{1}{\beta} + \frac{A'(\theta_p)}{A^2(\theta_p)} + \frac{1}{\beta B(\theta_p)} - \frac{\theta_p B'(\theta_p)}{\beta B^2(\theta_p)}$$

$$h''(\theta_p) = \frac{\partial^2 \mathbb{E}[\theta | \theta \geq \theta_p]}{\partial^2 \theta_p} + \frac{A''(\theta_p)A(\theta_p) - 2[A'(\theta_p)]^2}{A^3(\theta_p)} - \frac{2B'(\theta_p)}{\beta B^2(\theta_p)} - \frac{\theta_p}{\beta} \frac{B''(\theta_p)B(\theta_p) - 2[B'(\theta_p)]^2}{B^3(\theta_p)}$$

where

$$A'(\theta_p) = \int_{\underline{\theta}}^{\theta_p} \frac{\partial \chi(\theta; \theta_p, \rho)^{\frac{1}{1-\rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1-\rho}}}{\partial \theta_p} dF(\theta) = \frac{-\beta}{\theta_p^2} \left[\frac{\theta_p + \tilde{\beta}}{\theta_p} \right]^{\frac{\rho}{1-\rho}} \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta}{\theta + \tilde{\beta}} \right)^{\frac{1}{1-\rho}} dF(\theta) \leq 0$$

$$A''(\theta_p) = \frac{\beta}{\theta_p^2} \left[\frac{\theta_p + \tilde{\beta}}{\theta_p} \right]^{\frac{\rho}{1-\rho}} \left[\left(2 + \frac{\beta \rho}{\theta_p + \tilde{\beta}} \right) \int_{\underline{\theta}}^{\theta_p} \left(\frac{\theta}{\theta + \tilde{\beta}} \right)^{\frac{1}{1-\rho}} dF(\theta) - \frac{\theta_p f(\theta_p)}{\theta_p + \tilde{\beta}} \right] \quad (37)$$

$$B'(\theta_p) = \int_{\underline{\theta}}^{\theta_p} \frac{\partial \chi(\theta; \theta_p, \rho)}{\partial \theta_p} dF(\theta) = \int_{\underline{\theta}}^{\theta_p} \frac{1}{\theta + \tilde{\beta}} dF(\theta) \geq 0 \quad (38)$$

$$B''(\theta_p) = \frac{f(\theta_p)}{\theta_p + \tilde{\beta}} \geq 0$$

where $\tilde{\beta} = \beta(1 - \rho)$. Notice that all the terms involving B in $h''(\theta_p)$ add to a negative number because:

$$\theta_p \frac{B'(\theta_p)}{B(\theta_p)} - 1 = \theta_p \frac{\int_{\underline{\theta}}^{\theta_p} \frac{1}{\theta + \tilde{\beta}} dF(\theta)}{\int_{\underline{\theta}}^{\theta_p} \frac{\theta_p + \tilde{\beta}}{\theta + \tilde{\beta}} dF(\theta) + 1 - F(\theta_p)} - 1 \leq 0$$

since $F(\theta_p) \leq 1$ and $\theta_p < \theta_p + \tilde{\beta}$ for all $\beta > 0$. Since $\frac{\partial^2 \mathbb{E}[\theta | \theta \geq \theta_p]}{\partial^2 \theta_p}$ is concave by Assumption 1, a **sufficient** condition for $h(\cdot)$ to have only one inflection point (or only one θ_p such that $h''(\theta_p) = 0$) is to have $A''(\underline{\theta}) < 0$ and $A''(\cdot)$ to be monotonically nondecreasing. This ensures that even if $A''(\cdot)$ turns positive and possibly makes $h''(\cdot)$ positive, the latter will remain positive for the entire support, generating only one inflection point.

A.7 Proof of Proposition 3

We can write the federation's problem as follows:

$$\begin{aligned} \max_{\{c_p, k_p, \theta_p, \rho\}} \int_{\underline{\theta}}^{\theta_p} \left\{ \theta \log \left[\left(\frac{\theta_p + \beta(1 - \rho)}{\theta + \beta(1 - \rho)} \right)^{\frac{1}{1 - \rho}} \left[\frac{\theta}{\theta_p} \right]^{\frac{1}{1 - \rho}} g_p \right] + \log \left[\frac{\theta_p + \beta(1 - \rho)}{\theta + \beta(1 - \rho)} k_p \right] \right\} f(\theta) d\theta + \\ \int_{\theta_p}^{\bar{\theta}} \{ \theta \log[g_p] + \log[k_p] \} f(\theta) d\theta \\ s.t. \quad G(g_p, \theta_p) + K(k_p, \theta_p) \leq W \\ \bar{\theta} \log g_p + \beta \log k_p \geq \bar{\theta} u(g^{ex}(\bar{\theta}, \phi)) + \beta u(k^{ex}(\bar{\theta}, \phi)) \\ \underline{\theta} \log \left[g_p \chi(\underline{\theta}, \theta_p, \rho) \frac{\theta}{\theta_p} \right] + \beta \log [k_p \chi(\underline{\theta}, \theta_p, \rho)] \geq \underline{\theta} u(g^{ex}(\underline{\theta}, \phi)) + \beta u(k^{ex}(\underline{\theta}, \phi)) \end{aligned}$$

The FOCs with respect to g_p and k_p :

$$\frac{1}{g_p} (1 + \bar{\theta} \bar{\mu} + \underline{\theta} \underline{\mu}) = \lambda \left[1 - F(\theta_p) + \int_{\underline{\theta}}^{\theta_p} \left(\chi(\theta, \theta_p) \left[\frac{\theta}{\theta_p} \right] \right) f(\theta) d\theta \right] \quad (39)$$

$$\frac{1}{k_p} [1 + \beta(\bar{\mu} + \underline{\mu})] = \lambda \left[1 - F(\theta_p) + \int_{\underline{\theta}}^{\theta_p} (\chi(\theta, 0)) f(\theta) d\theta \right] \quad (40)$$

Taking the ratio between (39) and (40) it follows that $\frac{G}{K} = \frac{1 + \bar{\theta} \bar{\mu} + \underline{\theta} \underline{\mu}}{1 + \beta \bar{\mu} + \beta \underline{\mu}}$. ■

A.8 Proof of Corollary 3

From (39) and (40) we obtain $\frac{k_p}{g_p} = \frac{1 + \beta \bar{\mu} + \beta \underline{\mu}}{1 + \bar{\theta} \bar{\mu} + \underline{\theta} \underline{\mu}} \frac{A(\theta_p)}{B(\theta_p)}$, Recall that, from the implementation we have that

$$\tau = 1 - \frac{\theta_p k_p}{\beta g_p} = 1 - \frac{\theta_p}{\beta} \frac{1 + \beta \bar{\mu} + \beta \underline{\mu}}{1 + \bar{\theta} \bar{\mu} + \underline{\theta} \underline{\mu}} \frac{A(\theta_p)}{B(\theta_p)}$$

Subtracting from both sides the optimal transfer absent PCs $\tau^* = 1 - \frac{\theta_p}{\beta} \frac{A(\theta_p)}{B(\theta_p)}$, we find that for a given fiscal rule θ_p , the optimal marginal transfers satisfy:

$$\tau^{PC} - \tau^* = \frac{\theta_p}{\beta} \frac{A(\theta_p)}{B(\theta_p)} \left[\frac{\bar{\mu}(\bar{\theta} - \beta) - \underline{\mu}(\beta - \underline{\theta})}{1 + \bar{\theta}\bar{\mu} + \underline{\theta}\underline{\mu}} \right]$$

A.9 Proof of Proposition 4

As we did to obtain (31), we can rewrite the optimality condition from Proposition 4 as $\tilde{h}(\theta_p^{PC}) = 0$, where:

$$\tilde{h}(\theta_p) = \mathbb{E}[\theta | \theta \geq \theta_p] - \frac{\theta_p}{\beta} - \frac{1}{A(\theta_p)} + \frac{\theta_p}{\beta B(\theta_p)} - \bar{\mu} \left[\frac{\bar{\theta}}{A(\theta_p)} - \frac{\theta_p}{B(\theta_p)} \right] + \underline{\mu} \left[\frac{\theta_p}{B(\theta_p)} - \frac{\underline{\theta}}{A(\theta_p)} \right] + \underline{\mu} \left[\frac{\bar{\theta} - \theta_p}{1 - F(\theta_p)} \right]$$

or, relating to equation (31):

$$\tilde{h}(\theta_p) = h(\theta_p) - \bar{\mu} \left[\frac{\bar{\theta}}{A(\theta_p)} - \frac{\theta_p}{B(\theta_p)} \right] + \underline{\mu} \left[\frac{\theta_p}{B(\theta_p)} - \frac{\underline{\theta}}{A(\theta_p)} \right] + \underline{\mu} \left[\frac{\bar{\theta} - \theta_p}{1 - F(\theta_p)} \right]$$

Recall that without PCs, the optimal rule satisfies $h(\theta_p) = 0$, while at the constrained optimum θ_p^{PC} we have:

$$h(\theta_p^{PC}) = \bar{\mu} \left[\frac{\bar{\theta}}{A(\theta_p^{PC})} - \frac{\theta_p^{PC}}{B(\theta_p^{PC})} \right] - \underline{\mu} \left[\frac{\theta_p^{PC}}{B(\theta_p^{PC})} - \frac{\underline{\theta}}{A(\theta_p^{PC})} \right] - \bar{\mu} \left[\frac{\bar{\theta} - \theta_p^{PC}}{1 - F(\theta_p^{PC})} \right] \quad (41)$$

To prove the proposition, given that we already showed that under the uniform $h'(\theta_p) < 0$ for $\theta_p \geq \theta_p^*$ where θ_p^* is the optimal rule without PCs, it is enough to show that the RHS is always weakly negative to show that $\theta_p^{PC} \geq \theta_p^*$. Starting from the second term of the RHS, we can reformulate the term inside the square brackets as:

$$\begin{aligned} \frac{\theta_p A - \underline{\theta} B}{AB} &= (\theta_p + \beta) \left[\int_{\underline{\theta}}^{\theta_p} \frac{\theta}{\theta + \beta} dF(\theta) - \int_{\underline{\theta}}^{\theta_p} \frac{\underline{\theta}}{\theta + \beta} dF(\theta) \right] + (1 - F(\theta_p))(\theta_p - \underline{\theta}) \\ &= (\theta_p + \beta) \left[\int_{\underline{\theta}}^{\theta_p} \frac{\theta - \underline{\theta}}{\theta + \beta} dF(\theta) \right] + (1 - F(\theta_p))(\theta_p - \underline{\theta}) \geq 0 \end{aligned}$$

when only the constraint at the bottom is binding, $\bar{\mu} = 0, \underline{\mu} > 0$, then $\theta_p^{PC} \geq \theta_p^*$. This is generally true whenever $\rho = 0$ and does not require any distributional assumption, which proves the first part of the proposition. In order to prove the second part, we need to add the first and the second terms of (41) and study under which condition it also takes a negative

sign. Collecting for $\bar{\mu}$, the term is negative if and only if:

$$\frac{\bar{\theta}}{A(\theta_p^{PC})} - \frac{\theta_p^{PC}}{B(\theta_p^{PC})} \leq \frac{\bar{\theta} - \theta_p^{PC}}{1 - F(\theta_p^{PC})}$$

which we can rearrange and obtain:

$$\frac{\bar{\theta}}{\theta_p} \geq \frac{A(\theta_p^{PC})}{B(\theta_p^{PC})} \frac{B(\theta_p^{PC}) - (1 - F(\theta_p^{PC}))}{A(\theta_p^{PC}) - (1 - F(\theta_p^{PC}))}$$

Now recall that, by the definition of $A(\theta_p^{PC})$ and $B(\theta_p^{PC})$, we have that:

$$\begin{aligned} A(\theta_p^{PC}) - (1 - F(\theta_p^{PC})) &= \frac{\theta_p^{PC} + \beta}{\theta_p^{PC}} \int_{\underline{\theta}}^{\theta_p^{PC}} \frac{\theta}{\theta + \beta} dF(\theta) \\ B(\theta_p^{PC}) - (1 - F(\theta_p^{PC})) &= \theta_p^{PC} + \beta \int_{\underline{\theta}}^{\theta_p^{PC}} \frac{1}{\theta + \beta} dF(\theta) \end{aligned}$$

Using the definitions of A' and B' from Appendix A.6 the previous condition becomes condition (23).