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Abstract

We analyze the optimal macroprudential policy under the presence of news shocks. News are shocks to the growth rate that convey information about future growth. In this context, crises are characterized by long periods with positive shocks (and good news) that eventually revert, rendering the collateral constraint binding and triggering deleveraging. In this environment it is optimal to tax borrowing during good times, and let agents act freely leaving the allocations undistorted, including borrowing and lending, when the economy reverts to a bad state. We contrast our findings to the case of standard, shocks to the level of income, where it is optimal to tax debt in bad times, when agents need to borrow the most for precautionary savings motives. Also, taxes are used much less often and are around one-tenth of those under level shocks.

Keywords: Macroprudential Policy; Financial Crises; Pecuniary Externality
JEL Classification: E32; E44; G18

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1. Introduction

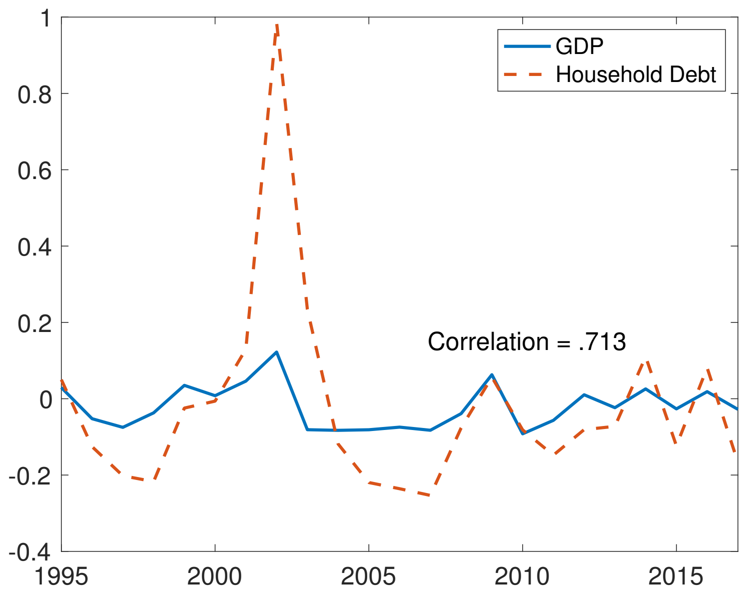
A large literature has examined the consequences of systemic externalities with endogenous borrowing constraints, and the optimal policy to prevent "over-borrowing". But, as Schmitt-Grohé and Uribe [13] have highlighted, all these models share one unappealing feature: when the economy is subject to temporary income shocks, households over-borrow in bad times resulting in counter-cyclical macroprudential policy, so that regulation is looser in booms and tighter in recessions. This result is at odds with standard views on the cyclicality of macroprudential policy, calling for pro-cyclical taxes to curb over-borrowing in booms. In this paper we show that the assumption of a stationary income process is crucial to obtain this result. By introducing a trend to the endowment and focusing on shocks to its growth rate, we show that both the timing and size of the optimal policy is affected.

To do so, we analyze a benchmark model that allows us to assess the optimal macroprudential regulation in the presence of shocks to the trend of income. We build upon the work by Bianchi [5], in which a systemic externality sets the stage for the analysis of constrained optimal (Ramsey) policy. In a nutshell, the model is a standard small open economy, where a continuum of identical agents must decide how to allocate their income to consume a tradable and a non-tradable good. The agents can move income through time using a one period non-state contingent financial asset measured in units of the tradable good. Importantly, agents are constrained on how negative (in debt) they can be. They can borrow only up to a fraction of the value of their endowments: they are collateral constrained. This constraint is important because the agents' income (endowment) is subject to random shocks. We show that the nature of the shock is a key determinant of the underlying reason to borrow, and therefore has very important implications for regulation.

To see this, consider first the cyclical implications of the Bianchi [5] benchmark. There the endowments follow a persistent, but stationary, stochastic process affecting their level (henceforth, "level shocks"). In this case, after a negative shock, and in order to smooth consumption, agents dig into their savings anticipating that the endowment will (mean-) revert in the future. Thus, agents borrow in bad times and save in good times. Because markets are incomplete, the individual borrowing cost does not fully internalize the social cost of debt. This implies that the regulator needs to tax borrowing strongly in bad times, in order to make agents internalize the systemic risk implied by their borrowing decisions.

In contrast, consider the environment with shocks to the growth rate rather than to the level. Now, a negative shock not only affects the whole stream of current and future income, but, if the shock is persistent, it also signals (provides "news") that the future could be even worse. Thus, agents drastically reduce their consumption and increase their savings to insure against it: agents save in bad times. In this case, the incentives of the agents and the planner

Figure 1: Annual Growth Rates, GDP and Total Household Debt for Argentina



Notes: Annual data for Argentina, 1995-2017. Real GDP is from the World Bank. Household Debt contains all loans and debt securities and is from the International Monetary Fund's Global Debt Database. Both series are expressed in annual growth rates. Correlation coefficient=0.713.

are aligned. A positive shock has the opposite effect: when it happens agents receive more income, but they also believe that the future will be even brighter and sharply increase their borrowing to bring resources from the future to the present. Crucially, when each individual is riding this wave of "optimism", they do not internalize that when the process eventually reverts the large level of accumulated debt will affect everyone in the economy by tightening the collateral constraint. Again, because markets are incomplete, a social planner who wants to align the private and social incentives to borrow would tax (or regulate more) in good times. To sum up, we establish that under persistent trend shocks macroprudential policy is pro-cyclical, whereas under level shocks macroprudential policy is counter-cyclical.

Our results underline the importance of singling out the right motive for debt accumulation in order to analyze macroprudential policy. But, do agents borrow in good or in bad times? There is ample evidence in the literature that shocks providing information about the future state of the economy are an important source of business cycle fluctuations (e.g. Beaudry and Portier [4] and Jaimovich and Rebelo [11].) A related point was made in the context of developing countries by Aguiar and Gopinath [1], stressing the permanent income channel to explain the counter-cyclical behavior of the current account using trend shocks. In sum, a large body of work points towards the possibility that, in several contexts, agents may borrow in good times rather than in bad times.

As an illustration, the view that agents borrow in good times is supported by Figure 1, which shows the annual growth rates of GDP and total household debt for Argentina over the period 1995-2017.[1] The two series in the plot exhibit a clear positive relationship. In order to capture this feature in our model, we use a tractable way of modeling the idea that growth today is usually followed by growth *in the future*, leading to higher long-run income. Using an identification approach developed in Blanchard et al. [7], we establish this is an accurate representation of GDP dynamics for the Argentinian economy and that the persistence of the growth rate is around 0.79. This allows for *current* growth to signal *future growth* in expectation. A forward-looking consumer thus will optimally increase his current consumption in good times, leading to debt accumulation.

The high persistence in the permanent component connects our paper closely to the large literature on"news shocks", which broadly posits that advance information about future income plays a sizable role in business cycle dynamics.[2] The literature has employed different approaches to model news shocks. Pioneered by Beaudry and Portier [4], and followed by the important work of Jaimovich and Rebelo [11], Schmitt-Grohe and Uribe [12], and others, one approach is to model news as perfect signals about future TFP shocks. Another approach, employed by Blanchard et al. [7], Barsky and Sims [3], and Cao and L'Huillier [9], among others, models advance information with signals about *persistent* permanent shocks.[3] It is only in this last sense that we call our our process "news shocks", to emphasize the relevance played by news component. Anyhow, in what follows we will refer to it as growth shocks, trend shocks and news shocks, interchangeably.

We also estimate the stationary component of Argentinian GDP and use it in what we call the "level shocks" economy and use it as a benchmark to compare our results. To minimize departures from the literature, we calibrate the benchmark economy using exactly the same parameters as Bianchi [5]. Since with the same parameterization the trend economy generates a higher crisis frequency, we adjust the discount factor, while keeping all other parameters constant, to obtain the same frequency of crisis in both economies. Our main findings are 1) as anticipated by our previous intuition, the optimal tax on debt is highly pro-cyclical, while it is highly counter-cyclical in the level shocks economy, and 2) the level of taxation is around one-tenth of that in the level shocks economy.

[1] We use Argentinean data to perform our quantitative exercises.

[2] Our shocks are also dubbed permanent or trend shocks in the literature. See Section 1.1 for more details emphasizing the differences with other approaches.

[3] The signal about future growth in our environment is not jammed by noise, and thus there is no signal extraction problem as in other related works. This does not mean however that there is no advance information. Because of the high persistence of the process, there is.

To understand these results it is important to keep in mind that the agents and the planner in the trend (news) shocks economy are a great deal more patient. This is because to achieve the same crisis probability, the level shocks economy requires a high degree of impatience on the part of households ($\beta = .91$) to incentivize borrowing and overcome the precautionary savings motive.[4] Since agents have a strong incentive to borrow against the future when positive income arrives, the news economy can achieve the same crisis probability and a similar level of indebtedness with a standard, higher value for the discount factor ($\beta = .95$). Thus, in principle, since the optimal tax on debt is forward-looking, the tax in the news economy should be higher, not smaller. However, a lower shadow value of borrowing when a crisis occurs introduces opposing force leading to lower taxes. In the trend shocks economy, crises occur after a succession of positive shocks revert into negative shocks. With trend shocks agents do not want to borrow, but to save in response to a negative shock since negative trend growth today implies a lower future endowment and a stronger desire to keep resources in the future, rather than bringing them to the present. By saving in bad times, agents move away from the constraint rather than towards it, and thus private and social incentives are better aligned. In addition, even though the constraint could bind in the future under some realizations of the shock, the shadow value of an extra unit of debt is smaller because agents have less desire to borrow. All in all, the optimal tax on borrowing is drastically reduced in the news economy compared to the level shocks economy.

Another important difference between level and trend shocks regards the frequency of optimal debt taxation. With trend shocks, our economy features an unconditional probability of strictly positive taxation of 62%, whereas this probability is 88% in the level shocks case. Thus, the persistent trend shocks view of the household dynamics of debt assigns a different role to macroprudential policy in which it should be used much less often.

1.1. Related literature

Our paper is closely related to several recent papers following the seminal model of Bianchi [5]. Two recent papers study the interaction between "news" and macroprudential policy. Bianchi et al. [6] study noisy news about future economic fundamentals as well as regime changes in the world interest rate. Like us, in their paper shocks today give some information about future endowments and borrowing needs. In their model, agents receive signals about the future level of the endowment. Differently, we study shocks to the growth rate of the endowment that are persistent, providing "news" to agents in the spirit of Blan-

[4] This is standard in the literature on precautionary savings. Because agents try to avoid being constrained, they tend to accumulate assets and only borrow when necessary. To replicate the observed high levels of debt one must parameterize the economy with unusually low levels of time discounting.

chard et al. [7]. They also find pro-cyclical taxes (Figure 5 in their paper). However, the planner *also* taxes debt under bad news, presumably due to the standard precautionary motive under mean-reverting shocks (absent in our model, which features permanent shocks only). Akıncı and Chahrour [2] consider a model with endogenous production and investment, and study the effect of news about future labor productivity. Their paper, however, does not analyze the constrained optimal macroprudential policy, which is our focus.

Seoane and Yurdagul [15] build a model of sudden stops with level and trend shocks to households' endowments. Though their model is very similar to ours, our goals differ substantially. Specifically, the aim of their paper is to understand the persistent responses of aggregate variables following episodes of financial crises, whereas we are interested mostly in the implications for the timing and size of macroprudential policy over the business cycle. In addition, our estimation strategy is quite different from theirs, as they perform a full-information Bayesian estimation whereas we use the methodology of Blanchard et al. [7]. Though our methods differ, our estimated persistence of the process driving the endowment growth rate is very close to theirs. Finally, we show that the model with trend shocks can match the same frequency of crisis as the temporary shocks model with a much higher discount factor. We therefore view our contribution as complimentary to theirs.

The paper proceeds as follows. Section 2 presents the model used for quantitative purposes. Section 3 presents the results. Section 4 concludes.

2. Model

We study the effect of persistent shocks to the growth of the endowments. Following Blanchard et al. [7], we will use the fact that a persistent shock to the growth rate not only changes the whole stream of current and future endowments, but also predicts future analogous changes. A particularly appealing feature of this approach is that it will immediately allow us to compare our findings to what is obtained in the case of shocks to the endowment level (level shocks), which is the standard assumption in most of the quantitative macroprudential literature to date. Actually, as explained in detail in this section, trend shocks induce dramatically different debt behavior than do level shocks, and thus have drastically contrasting implications for macroprudential policy.

2.1. Recursive formulation

We consider a small open economy with an infinitely lived representative household with period utility

$$\frac{c_t^{1-\sigma}}{1-\sigma}$$

where the parameter $\sigma > 0$ represents the inverse of the intratemporal eEasticity of sSubtitution (IES). Total consumption c_t is a composite of tradable and non-tradable goods which are aggregated using a standard Constant Elasticity of Substitution (CES) function:

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}$$

where the $\eta \in \Re$ captures the elasticity of substitution between tradable and non-tradable goods. The household is subject to a budget constraint

$$c_t^T + p_t c_t^N + (1-\tau_t)b_{t+1} = e_t^T + p_t e_t^N + (1+r)b_t + T_t \tag{1}$$

where p_t is the relative price of non-tradables, τ_t is the tax on the one-period non-contingent debt b_{t+1} taken out in period t, e_t^T and e_t^N are the endowments of tradable and non-tradable goods, r is the exogenous interest rate, and T_t is a lump-sum transfer. Notice that debt is measured in terms of the tradable good.

However, households are also subject to a per-period borrowing constraint given by:

$$b_{t+1} \geq -\kappa(e_t^T + p_t e_t^N) \tag{2}$$

where κ is the parameter designating the proportion of the value of each endowment that can be pledged as collateral. This is the key element of the economy that generates the reason for financial regulation. When households borrow, or deleverage, they affect the relative price of the non-tradable good, which in turn affects the collateral constraint of all households in the economy. Because financial markets are incomplete, it creates a pecuniary externality that could lead to inefficient borrowing. Thus, a social planner who internalizes this effect could improve the equilibrium outcomes through either regulation or taxation.

Households use borrowing and lending to transfer resources across time and thus smooth consumption. How this is done depends crucially on the structure of the income process. The endowments are subject to shocks following Aguiar and Gopinath [1][5]:

$$e_t^T = e_{t-1}^T \exp(g_t) \tag{3}$$

$$e_t^N = \gamma e_t^T \tag{4}$$

A key feature of the growth rate is that it follows an AR(1) process:

$$g_t = \rho g_{t-1} + \varepsilon_t$$

[5] For simplicity, we have assumed that the two endowments are perfectly correlated, though this is not strictly necessary to transform our model.

As a result, when a positive shock arrives, not only does it increase the amount of resources today and in every subsequent period, but it also implies that more positive shocks may follow in the future. This creates the incentive to borrow to start enjoying the expected future prosperity in the present.

Finally, the resource constraints are given by

$$c_t^T = e_t^T + (1+r)b_t - b_{t+1} \tag{5}$$

$$c_t^N = e_t^N \tag{6}$$

We are interested in deriving the optimal allocation in this environment. To do so, we follow the approach of the classic public finance literature and solve the Ramsey social planner's problem. This amounts to maximizing the ex-ante present value of households' utility subject to the planner's solution being able to be implemented as a competitive equilibrium. An important element of this problem is the equilibrium price p_t (which determines the real exchange rate), which can be obtained in closed form as a function of tradable consumption and the endowment of non-tradables

$$p_t = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{e_t^N}\right)^{\eta+1} \tag{7}$$

It can be shown that the recursive Ramsey planner's problem solves:

$$V_t(b_t, g_t) = \max_{\{c_t^T, c_t^N, b_{t+1}\}} \; u(c_t) + \beta \mathbb{E}\left[V_{t+1}(b_{t+1}, g_{t+1})\right]$$

subject to

$$c_t = [\omega (c_t^T)^{-\eta} + (1-\omega)(e_t^N)^{-\eta}]^{-1/\eta}$$

$$b_{t+1} \geq -\kappa (e_t^T + \frac{1-\omega}{\omega}\left(\frac{c_t^T}{e_t^N}\right)^{\eta+1} e_t^N)$$

and

$$c_t^T = e_t^T + (1+r)b_t - b_{t+1}$$

where we have substituted for the price p_t by (7). Notice that the household's budget constraint is absent, because one can always adjust the (implied) lump sum tax to make it hold in the equilibrium of any implementation. More importantly, notice that the planner is subject to the same collateral constraint (2) as the individuals. Unlike households, the planner internalizes that different consumption allocations, and hence borrowing, have an impact on the equilibrium price, while the households take the latter as given.

2.2. Transformed Model

We solve the Ramsey problem numerically by value function iteration. In order to do so we first need to transform the model to achieve bounded choice sets in the discretized model. The precise issue with using the non-transformed model is the following. As usual, we approximate the solution of the problem by defining grids for choice variables. Since the process for the (logarithm of the) endowment e_t^T has a unit root, the space of realizations of the endowment is unbounded, and choice variables are highly likely to attain values beyond the borders of the grid. Thus, the discretized solution provides a poor approximation of the actual solution of the non-transformed planner's problem. The transformation of the model below takes care of this problem.

In order to do so, we define three variables $\tilde{c}_t^T$, $\tilde{c}_t^N$, and $\tilde{b}_t$ as follows:

$$\tilde{c}_t^T \equiv \frac{c_t^T}{e_{t-1}^T}; \quad \tilde{c}_t^N \equiv \frac{c_t^N}{e_{t-1}^T}; \quad \tilde{b}_t \equiv \frac{b_t}{e_{t-1}^T}$$

Period consumption can be written in terms of the transformed variables as

$$\tilde{c}_t = [\omega(\tilde{c}_t^T)^{-\eta} + (1-\omega)(\tilde{c}_t^N)^{-\eta}]^{-1/\eta} \tag{8}$$

Dividing both sides of the budget constraint by e_{t-1} and after some rearrangement we obtain the transformed budget constraint:

$$\tilde{c}_t^T + p_t\tilde{c}_t^N + (1-\tau_t)\tilde{b}_{t+1}\exp(g_t) = (1+\gamma p_t)\exp(g_t) + (1+r)\tilde{b}_t + \tilde{T}_t \tag{9}$$

Inspection of equation (9) reveals that a positive growth shock g_t proportionally increases the transformed debt and thereby relaxes the period t budget constraint. When the current growth rate of the economy is high, agents can borrow more today while facing a smaller debt burden since future income will be higher. That is, a positive growth shock g_t has the effect of increasing the value of debt in units of consumption in the current period from $\tilde{b}_{t+1}$ to $\exp(g_t)\tilde{b}_{t+1}$, but next period interest is paid only on $\tilde{b}_{t+1}$.

The collateral constraint, written in terms of the transformed variables, is

$$\tilde{b}_{t+1} \geq -\kappa(1+\gamma p_t) \tag{10}$$

where

$$p_t = \frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\gamma\exp(g_t)}\right)^{\eta+1}$$

Notice, then, that (8), (9), and (10) define the transformed model in terms of variables $\tilde{c}_t^T$, $\tilde{c}_t^N$, and $\tilde{b}_{t+1}$. It remains to define period utility $u(c_t)$ in terms of transformed variables, which is given by:

$$u(c_t) = (e^T_{t-1})^{1-\sigma}\frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}$$

The present value of utility is:

$$V_t = E_t\sum_{s=t}^{\infty}\beta^{s-t}(e^T_{s-1})^{1-\sigma}\frac{\tilde{c}_s^{1-\sigma}}{1-\sigma}$$

but by definition

$$e^T_{s-1} = e^T_{t-1}\prod_{i=t}^{s-1}\exp(g_i); \quad \text{with} \quad \prod_{i=t}^{t-1}\exp(g_i) = 1$$

Thus the present value of utility is:

$$V_t = \beta^{-1}(e^T_{t-1})^{1-\sigma}E_t\sum_{s=t}^{\infty}\prod_{i=t}^{s-1}\tilde{\beta}_i\frac{\tilde{c}_s^{1-\sigma}}{1-\sigma}$$

where $\tilde{\beta}_i = \beta\exp(g_i)^{1-\sigma}$. For the present value of utility to be well defined we therefore assume $\beta\exp(g_s)^{1-\sigma} < 1$, for all s. The transformed problem for the household can be written as:

$$V(\tilde{b}, g) = \max_{\tilde{b}',\tilde{c}}\left\{u(\tilde{c}) + \beta\exp(g)^{1-\sigma}EV(\tilde{b}', g')\right\} \tag{11}$$

subject to (8), (9), and (10).

2.3. Implementation with a tax on debt

We now show that the constrained efficient allocation chosen by the planner can be implemented in the decentralized equilibrium with a state-contingent tax on debt, rebated to households as a lump sum transfer. Following Bianchi [5] (Proposition 2), we first consider the constrained efficient equilibrium. The constrained efficient allocations are characterized by $\{\tilde{c}^T_t, \tilde{c}^N_t, \tilde{b}_{t+1}, p_t, \mu^P_t\}_{t=0}^{\infty}$, given initial debt $b_0 = 0$ such that the planner's euler equation is satisfied, the lagrange multiplier on the collateral constraint $\mu^P_t \geq 0$, and the resource constraints and market clearing price hold for each t.

The planner's euler equation is given by

$$u_T(\tilde{c}_t)\exp(g_t) = \mu^P_t\big(1 - \varphi_t\exp(g_t)\big) + \exp(g_t)^{1-\sigma}\beta(1+r)E\big[u_T(\tilde{c}_{t+1}) + \mu^P_{t+1}\varphi_{t+1}\big]$$

where $\varphi_t = \kappa\gamma(1+\eta)\frac{1-\omega}{\omega}\frac{(\tilde{c}_t)^{\eta}}{(\gamma\exp(g_t))^{\eta+1}}$ and u_T is the marginal utility of tradable consumption.

The decentralized equilibrium allocations are characterized by $\{\tilde{c}^T_t, \tilde{c}^N_t, \tilde{b}_{t+1}, p_t, \mu_t, \tau_t,\}_{t=0}^{\infty}$ given $b_0 = 0$ and $\tau_0 = 0$, such that the representative household optimizes, the lagrange multiplier on the collateral constraint $\mu_t \geq 0$, and the budget and resource constraints are satisfied.

The household's euler equation is

$$(1-\tau_t)u_T(\tilde{c}_t)\exp(g_t) = \mu_t + \exp(g_t)^{1-\sigma}\beta(1+r)E\big[u_T(\tilde{c}_{t+1})\big]$$

Evaluating all variables at the planner's allocations and borrowing choices leads us to the following expression for taxes in period t:[6]

$$\tau_t = \frac{E\big[\mu^P_{t+1}\varphi_{t+1}\big]}{E\big[u_T(\tilde{c}_{t+1})\big]}$$

and the corresponding lump sum transfer is $T_t = \tau_t \exp(g_t)\tilde{b}_{t+1}$. As can be seen from the above expression, the tax is increasing in the right hand side of the planner's euler equation, which is increasing in the probability that the constraint binds in period $t+1$.

As in Schmitt-Grohé and Uribe [13], we find it useful to derive the tax from the household's euler equation, plugging in the planner's borrowing choices. When the borrowing constraint does not bind for the household, we have

$$\tau_t = 1 - \frac{\exp(g_t)^{-\sigma}\beta(1+r)E\big[u_T(\tilde{c}^P_{t+1})\big]}{u_T(\tilde{c}^P_t)}$$

where $\tilde{c}^P$ indicates consumption evaluated at the constrained optimal choice.[7]

3. Calibration and Quantitative Results

Our primary goal in the calibration is to study the behavior of optimal policy for a standard parametrization of the model. The set of parameters to calibrate is given by $\{\beta, r, \kappa, \omega, \eta\}$ and the parameters of the process for the growth shocks g_t. To ease the comparison, we use the same values for r, κ, ω, and η as Bianchi [5]. Due to a strong precautionary motive, in economies with uninsurable risk it is usually cumbersome to obtain borrowing in equilibrium, because households instead have incentives to accumulate positive assets. As a solution, the literature resorts to low discount factors, thereby inducing a strong

[6] As discussed in Schmitt-Grohé and Uribe [13], the value of τ_t in periods in which the collateral constraint binds is indeterminate; we thus focus on cases in which $\mu^P_t = 0$.

[7] Schmitt-Grohé and Uribe [13] derive the tax under the assumption $\mu_t = 0$ because "the policymaker can pick the capital control policy in such a way that when the collateral constraint binds, individual agents feel that they would make the same debt choice whether they were constrained by the collateral restriction or not." (pp 8-9).

Table 1: *Calibrated Parameters, Trend Shocks*

Parameter	Value
Interest rate	$r = 0.04$
Discount factor	$\beta = 0.95$
Credit coefficient	$\kappa = 0.32$
Share of non-tradables	$\omega = 0.31$
Risk aversion	$\sigma = 2$
Elasticity of substitution	$1/(1+\eta) = 0.83$

preference for the present. In our environment, however, the desire for borrowing is stronger, which would generate unusually high levels of debt and frequency of crisis if we were to use similar discount factors. With temporary shocks, setting $\beta = .91$ as in Bianchi [5] implies an annual crisis probability of 7.5%.[8,9] In the trend shock economy, we choose the discount factor to match this probability, giving us $\beta = .95$, a standard value.

The model is simulated at an annual frequency. The interest rate is set to 4 percent.[10] The credit coefficient entering (10), κ, is set to 0.32 and the share of tradables $\omega = .31$. The intratemporal elasticity of substitution in the consumption aggregator, $1/(1+\eta)$, is set to 0.83. Table 1 summarizes the calibration of the set of parameters $\{\beta, r, \kappa, \omega, \eta\}$. Since all the parameters are the same as in Bianchi [5], except the necessary change in β to render the crisis frequency consistent across models, the differences between models arise only from the different stochastic processes. As we will explain below, the difference in β only reduces the quantitative impact of our results.[11]

Growth shocks follow an AR(1) process

$$g_t = \rho g_{t-1} + \varepsilon_t \tag{12}$$

where ρ is a persistence parameter in $[0, 1)$, and ε_t is an i.i.d. growth shock drawn from a Normal distribution with mean zero and variance σ_ε^2. Because of the unit root in the logarithm of e_t^T in (3), the growth shock g_t is a permanent shock. We estimate this process

[8]Table 3 lists the parameters used in the temporary shocks model to which we compare our results.

[9]We could have alternatively re-calibrated our model to match the targets in Bianchi [5]. Results are very similar to those reported here and are shown in Appendix Appendix C.

[10]We check that $\beta(1+r) < 1$ in order to make sure that financial assets are contained in a bounded set. See, for instance, Chamberlain and Wilson [10] for further details.

[11]In terms of our conclusions, setting such a high discount factor should work against us, as more patient households weight the future more heavily, leading to higher tax rates. However, as we discuss in the introduction and below, households' and the planner's incentives are more aligned with trend shocks, leading to a lower tax rate.

Table 2: *Calibration: Stochastic Process for Growth Rate Shocks*

Parameter	Description	Value
ρ	Persistence, g_t	0.7897
σ_ε	Standard Deviation, ε_t	0.0124

via maximum likelihood using annual data from Argentina between 1960 and 2017.[12,13] The estimated parameters for the process are shown in Table 2.[14]

3.1. Results: Inspecting the Mechanism

We begin by plotting the Ramsey planner's policy functions in three different states, which, for simplicity, we will call the High state, the Middle State, and the Low state, shown in Figure 2a. The High state corresponds to a growth state equal to two standard deviations above the zero-mean. The Middle state corresponds to the state of the economy when growth is at its mean. The Low state corresponds to the state when growth is two standard deviations below zero. Each of these functions maps a current level of (transformed) asset holdings $\tilde{b}_t$ into next period's asset holdings $\tilde{b}_{t+1}$. We also plotted the 45 degree line. All the points above it represent debt reductions (or asset accumulation), while the points below it represent debt accumulation.

There are three main features of these policy functions to notice. First, and most importantly, the higher the state, the lower the position of the policy function. That is, the top policy function corresponds to the Low state, the middle to the Middle state, and the bottom to the High state. This directly implies that the higher the state, the *more* the Ramsey planner borrows. Second, each of these functions is "V"-shaped, the upward-sloping region corresponding to unconstrained values of $\tilde{b}_{t+1}$, and the downward-sloping region corresponding to constrained $\tilde{b}_{t+1}$[15]. Third, the middle and bottom policy functions (Middle and High states) cross the 45 degree line on the upward sloping region and for negative values of $\tilde{b}_{t+1}$ (the agent taking on debt into period $t+1$). The top policy function (Low state) does not

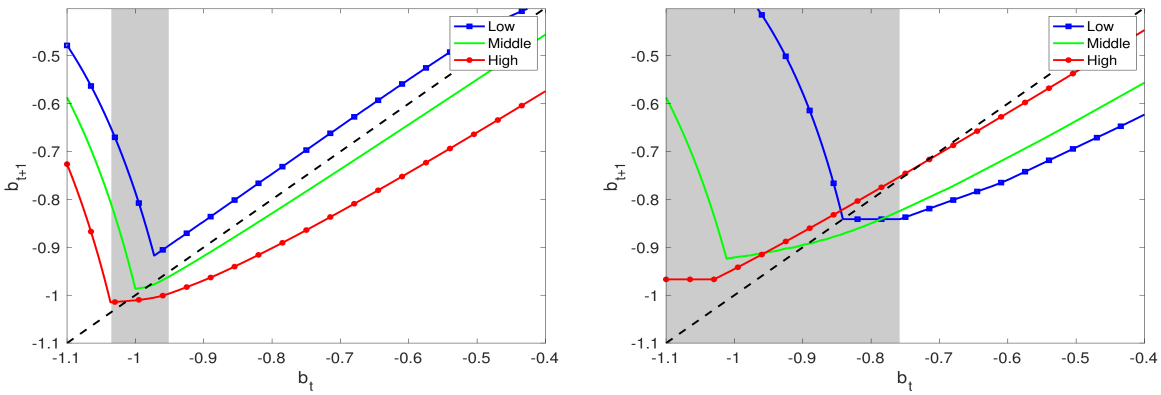
[12]Specifically, we use per-capita GDP and consumption data from the World Bank. We follow closely an identification procedure for the permanent process used by Blanchard et al. [7]. This procedure is based on a former influential paper by Blundell and Preston [8], which proposed to use consumption data to make inferences about long-run income in dynamic models. For more details, see Blanchard et al. [7].

[13]Seoane and Yurdagul [15] also estimate the process for growth rate shocks for Argentina using a full information Bayesian strategy. Despite using a different estimation procedure, our value for ρ is close to theirs, 0.67.

[14]In order to compute the value of σ_ε, we multiply $1-\rho$ by the standard deviation of the GDP growth rate, .0590, see Blanchard et al. [7] for details.

[15]See Appendix B for a discussion on the possibility of multiple equilibria in the binding region of debt.

Figure 2: *Tax Regions and Policy Functions of the Constrained Planner*



(a) Trend Shocks (b) Level Shocks

Notes: Policy functions of the constrained planner for $\tilde{b}_{t+1}$ as a function of $\tilde{b}_t$ when the shock is to the trend (left) and level (right). The policy function labeled "Middle" corresponds to the value of the shock equal to its mean, "Low" and "High" correspond to values of the shock two standard deviations below and above the mean, respectively. Shaded regions indicate at least one state in which there is a strictly positive tax for a given value of $\tilde{b}_t$.

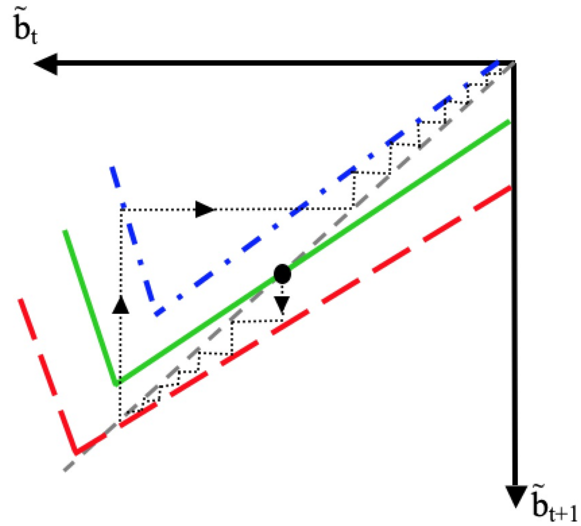
cross the 45 degree line over the grid used for this simulation ($\tilde{b}_t \in [-1.1, -0.4]$), but may do so for very large values of $\tilde{b}_t$.

The shape of these policy functions contains rich information on the dynamics of debt and on the implicit taxes that decentralize the planner's solution. In order to see this, consider the schematic chart shown in Figure 3. In that example, the dynamics begin at the fixed point of the middle policy function (denoted by the dot in the middle of the chart, where the Middle policy function crosses the 45 degree line). Then, the economy transits to the High state, and the planner begins accumulating debt. The reason is the effect of a trend shock. This effect can be understood by recognizing that, in the High state, the logarithm of endowment is a supermartingale so long as the state is persistent:

$$E\left[\log(e_{t+1}^T)|\log(e_t^T), e_t^T = H\right] > \log(e_t^T)$$

Thus, the future endowment is expected to be high, and consumption smoothing pushes the planner to borrow.

Figure 3: *Schematic Dynamics of Debt*



In the example, the economy stays in the High state for a while, and there is convergence of debt accumulation to the fixed point in the High state. At some point the economy transits to the Low state, and the economy enters a crisis. Net capital flows out $(-((1+r)\tilde{b}_t - \tilde{b}_{t+1}\exp(g_t)) > 0)$ and the crisis immediately implies deleveraging. However, so long as the economy stays in the Low state for some time, the trend shock effect implies that deleveraging will be sustained. The planner gradually repays his debt and can even accumulate assets $(\tilde{b}_{t+1} \geq 0)$, unless the economy transits again to a higher state.

The optimal dynamics of debt described previously have sharp implications for the taxes that implement the planner's solution. First, because in the Low state the policy function is above the 45 degree line, saving is optimal. Intuitively, this suggests that the incentives for the household in the competitive equilibrium and for the planner are aligned, and there is little or no need for financial regulation. This can be seen from the expression for the optimal tax which is

$$\tau_t = \frac{E\left[\mu^P_{t+1}\varphi_{t+1}\right]}{E\left[u_T(\tilde{c}_{t+1})\right]}$$

This expression shows that if the probability of a crisis in the next period is zero (and thus $E[\mu^P_{t+1}] = 0$), then the optimal tax is zero. In the Low state, this is indeed the case. Second, in the High state and when the amount of debt entering the period $\tilde{b}_t$ is rather small, there are incentives to borrow and in principle need for regulation. However, a key insight from this analysis is that because the kinks of the policy functions nearly line up one on top of the other, the set of debt levels $\tilde{b}_t$ that can trigger a crisis is rather small. This interval, or 'tax region', is shown by the shaded portion of Figure 2a. Thus, even though there are

Table 3: *Calibrated Parameters, Level Shocks*

Parameter	Value
Interest rate	$r = 0.04$
Discount factor	$\beta = 0.91$
Credit coefficient	$\kappa = 0.32$
Share of non-tradables	$\omega = 0.31$
Risk aversion	$\sigma = 2$
Elasticity of substitution	$1/(1+\eta) = 0.83$
Persistence, g_t	$\rho = 0.7897$
Standard Deviation, ε_t	$\sigma_\varepsilon = 0.0524$

strong incentives to borrow in the High state, the region where crises can occur is small, and therefore a government intervention is necessary only when the stock of debt is very high.

To emphasize the novelty of these results, we contrast our findings to the case of level shocks, which is the standard case analyzed by Bianchi [5], among others. To this end, we consider a variation of the model above with the process $e_t^T = \exp(g_t)$ instead of (3), where the process for g_t is estimated simultaneously with the growth rate shocks described in the previous section.[16] Notice that now the endowment no longer has a unit root (ε_t in stochastic process (12) is thus a level shock). To stay as close to the existing literature as possible, we calibrate $\beta = .91$, the same value used in Bianchi [5]. The parameters of the level shocks model are summarized in Table 3.

Figure 2b shows the planner's policy functions in this case, using the corresponding definition of the High, Middle, and Low states (the High state corresponding to two-standard-deviations in the log-level of the endowment above zero, the Middle state corresponding to zero, and the Low state corresponding to two-standard-deviations below zero.) The key feature of these policy functions is their position, which is the opposite of what we obtained in the trend shocks case. That is, in the economy with level shocks, the policy function in the High state is positioned at the top and he policy function in the Low state is positioned at the bottom. Moreover, the kinks of the policy functions are no longer aligned vertically, which as we will explain, has direct implications for the frequency at which the planner taxes in order to implement the constrained efficient outcome.

Figure 2b shows the stark difference in the order of the policy functions for the Low, Middle, and High states. Intuitively, under level shocks, precautionary savings pushes the

[16] Specifically, $g_t^L = \rho g_{t-1}^L + \varepsilon_t^L$, with $\rho = 0.7897$ and $\varepsilon^L \sim N(0, \sigma_{\varepsilon^L})$, where $\sigma_{\varepsilon^L} = \sqrt{.7897} \times 0.0590$. See Blanchard et al. [7] for details.

planner to save in good times and borrow in bad times. However, since in the Low state the value of collateral is low, the constraint binds for lower levels of debt, causing the order of the policy functions to reverse. We want to emphasize that the reversion in the policy functions' order only happens when the persistence of the growth shock (ρ) is high, not just because it is a permanent shock. When ρ is large enough, a positive shock to the trend today signals (provides "news") that more positive shocks may follow: the economy is entering a booming state. Anticipating better times, agents borrow to start enjoying the expected future income today. To stress this point, in Appendix C we compare our calibration to a trend shock economy with $\rho = 0.1$, where the reversion does not happen.

Furthermore, the shaded region in Figure 2b shows that the tax region in the level shocks model is far larger than in the trend shocks case. With level shocks, there are strong incentives to borrow in the Low state, and therefore the region where financial crises can occur is large, leading to a large range of outstanding debt for which the constraint may bind in the future. This suggests that the unconditional probability of the tax being positive is higher than under trend shocks. This is indeed the case, with the economy featuring strictly positive taxes 88% of the time, compared to 62% under trend shocks.[17]

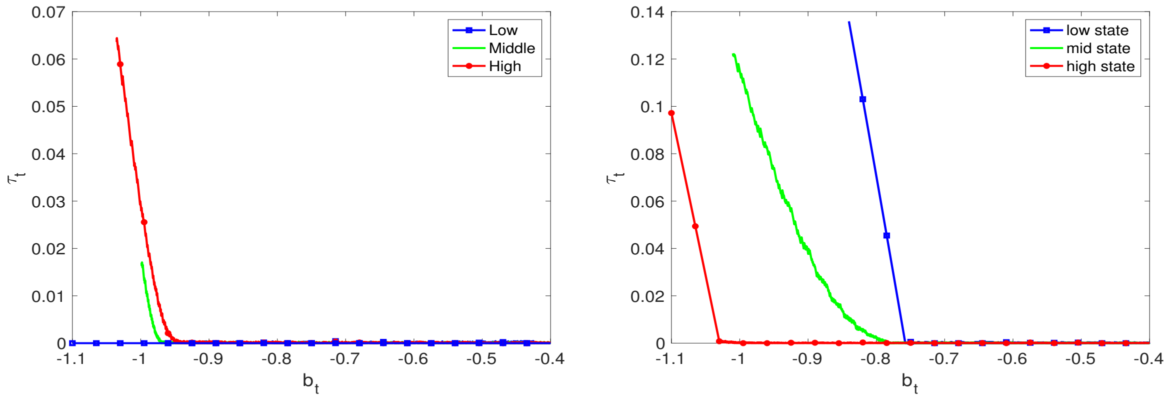
3.2. Results: Cyclicality and Size of Taxes

Figure 4 shows the policy functions for taxes in the three states corresponding to the policy functions in the previous section. Following Schmitt-Grohé and Uribe [13], we set taxes to be undefined (NaN) when the collateral constraint binds for the planner, since the tax rate is indeterminate in these states. Comparing the two panels, for a given endowment shock, taxes are increasing in the level of outstanding debt. However, the level of outstanding debt and state in which taxes are imposed is vastly different in the two cases. With trend shocks, shown in panel (a), taxes are highest in the High state, slightly lower in the Middle state, and zero in the Low state. Conversely, in the level shocks case in panel (b), taxes are highest in the Low state and lowest in the High state. As can be seen here and in Figure 2, the region for which there are positive taxes is far larger in the case of level shocks. Further, taxes under level shocks can reach almost 14%, compared to just under 7% in the case of trend shocks.

To illustrate the contrasting borrowing/saving behavior under trend/level shocks, we replicate two figures from Schmitt-Grohé and Uribe [13], shown in Figures 5 and 6. After simulating the model for 1,000,000 periods and discarding the first 10,000, we identify each

[17] At this point, the reader may wonder if the trend shocks case also differs strongly from the level shocks case in terms of ergodic distributions of debt. It does not. Even though the annual ergodic mean debt per unit of endowment of the consumption good b_t/e^T_{t-1} is a bit lower in the case of level shocks (-0.905 instead of -0.904), the shape of both distributions is similar.

Figure 4: *Policy Function: Constrained Optimal Tax Rates*



(a) Trend Shocks (b) Level Shocks

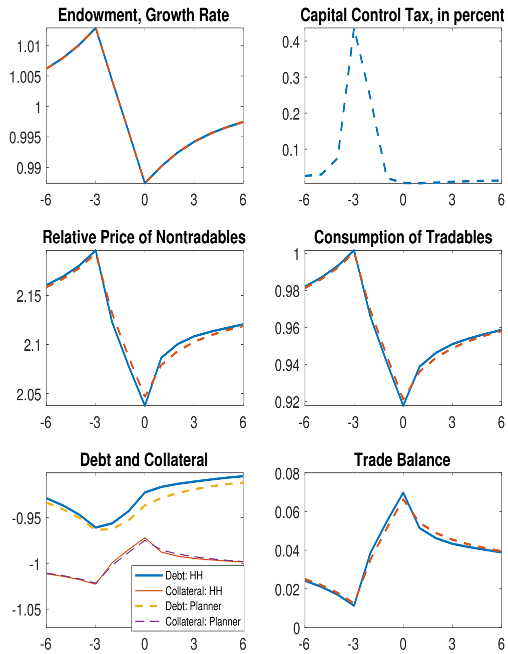
Notes: Policy functions of the constrained efficient tax τ_t as a function of $\tilde{b}_t$ when the shock is to the trend (left) and level (right).

boom-bust cycle around a period in which output went from above to below trend over 3 years. Similarly, we identify financial crises as those 10-year episodes in which the collateral constraint binds in the decentralized economy in at least one year. Figure 5 shows averages of key variables over each boom-bust cycle for the household and the planner, shown with solid and dashed lines, respectively, and Figure 6 shows the same variables during a crisis.[18]

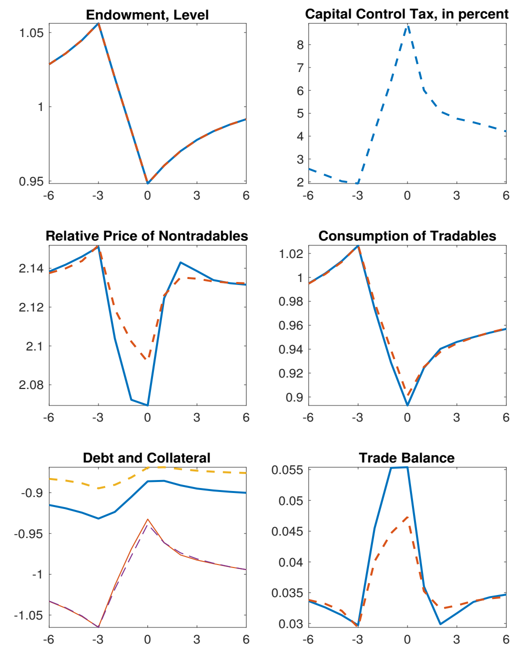
Beginning with the typical boom-bust cycle, Figure 5a shows the patterns in the trend shock case and Figure 5b in the level shock economy. In each panel, we plot the period from 3 years before the peak to 6 years after the trough. In our simulations, there are on average 8 non-overlapping boom-bust cycles per century. The relative price and capital control tax are directly comparable across panels (a) and (b), but because of the non-stationary nature of our model, panel (a) plots transformed consumption, debt, collateral, and the trade balance, while panel (b) plots their un-transformed levels.

[18] Appendix C contains impulse response functions for a one-time, one standard deviation shock for the same variables contained in Figures 5 and 6.

Figure 5: *Boom-Bust Cycles under Trend and Level Shocks*



(a) Trend Shocks



(b) Level Shocks

Notes: Each panel shows the average over all boom-bust cycles in simulations of 1,000,000 periods for the decentralized economy (solid line) and Ramsey planner (dashed). Due to a high level of skewness, the top right panel shows the median tax rate in percent.

The first row of panel (a) plots the endowment's path and the median tax rate necessary to align the incentives of the households with the planner.[19] In the trend shock case, the average expansion (contraction) experiences growth 1.3% above (below) average. Most importantly, taxes on debt are pro-cyclical, reaching almost 0.5% in the boom and falling to nearly zero in the contraction. Because households borrow more than is socially optimal in good times, this is when the planner has the strongest incentive to tax. Instead, the tax falls to zero during and after the contraction because debt increases to a level at which the probability that the constraint binds is very low, giving little reason for the planner to tax.

In the second row, consumption of tradables falls as the endowments decrease, causing a drop in the relative price. In the third row, the drop in endowments and relative price leads to a decrease in the collateral value, shown by the thin lines increasing (becoming less negative), and a simultaneous decrease in debt, shown by the thick lines. This panel

[19]As in Schmitt-Grohé and Uribe [13], we plot the median rather than the mean tax due to the skewness of the distribution of taxes.

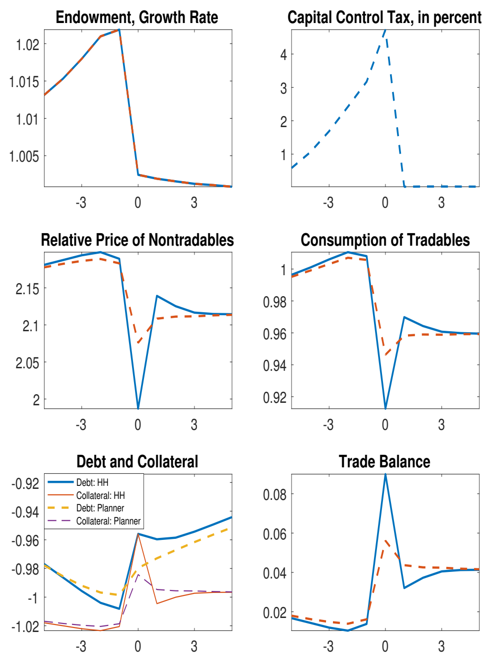
shows that the choice of debt, relative to the present endowment slightly falls during the contraction, though not significantly due to the relatively small size of the cycle in the case of trend shocks. Finally, the trade balance rises: consumption falls by more than output because trend shocks have large future implications about the path of endowments and therefore consumption.

Turning to Figure 5b, qualitatively, the path of each variable, excluding the tax, is similar to the trend shocks case discussed above. However, the intuition is slightly different: here, the endowment's level falls significantly, while consumption only slightly falls in relative terms because shocks are to the level and agents have the incentive to smooth over time. Since the endowment falls by more than consumption of tradables, the relative price falls and trade balance rises by less than in the trend shocks case. Although the debt dynamics are similar, the contrast in the path of taxes is stark: taxes are strongly countercyclical, because households borrow to smooth consumption in bad times. Thus, in the trend shocks case, taxes increase in booms and fall in recessions, while in the level shocks case, the opposite is true. Moreover, taxes in the level shocks economy are an order of magnitude larger than in the trend shocks case.

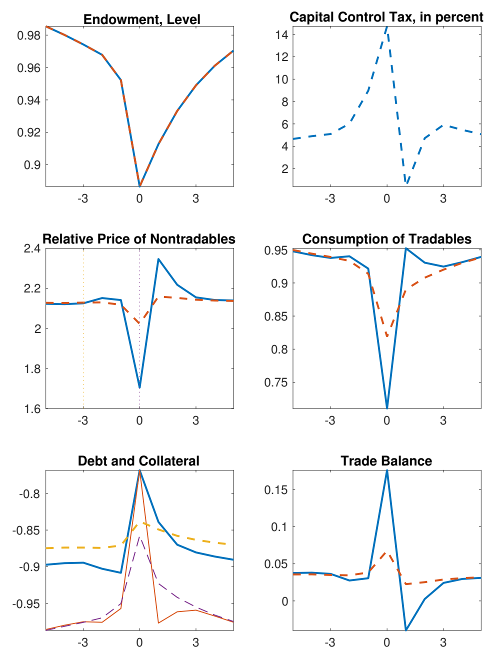
We explore this further in Figure 6, in which we plot the typical financial crisis. As discussed above, the two models are calibrated to achieve the same crisis probability, with a financial crisis occurring once every 16 years in our simulations. In this figure, year 0 indicates the year in which the constraint first binds in the decentralized economy. We center each 11-year window around this date, and again plot the averages for each panel (median for the tax rate). The financial crisis leads to sharp drops in the endowment, consumption and prices, collateral, and debt, and a sharp increase in the trade balance in the decentralized economy. Because the planner borrows less in the years preceding the crisis, the constraint does not bind for the planner, and prices, consumption, debt, and the value of collateral all fall by less, and the trade balance increases by less when the crisis occurs.

With trend shocks, in panel (a), the sharp drop in the growth rate (almost 2% during the crisis) of the endowment is sustained as the steady state growth rate is 1. Conversely, in the level shocks case, the level of the endowment is below its steady state value even 5 years before the crisis, falls by roughly 10%, and then reverts quickly towards its mean. As in the previous figure, the qualitative behavior of tradable consumption, relative prices, debt, and the trade balance is similar in the two panels.

Figure 6: *Financial Crises under Trend and Level Shocks*



(a) Trend Shocks



(b) Level Shocks

Notes: Each panel shows the average over all financial crises in simulations of 1,000,000 periods for the decentralized economy (solid line) and Ramsey planner (dashed). Due to a high level of skewness, the top right panel shows the median tax rate in percent.

Again, we stress the behavior of the tax in the two economies. With trend shocks, the path of the tax in Figure 6a is increasing in the boom and falls one period after the crisis hits. The reason for this lag is because taxes are undefined in states in which the constraint binds for the planner, thus this tax rate takes into account only those simulations for which the planner does not bind but the household does, leading to taxes of almost 4.5% to bring the decentralized economy out of the binding region. After the crisis period, taxes fall to nearly zero (roughly 0.2%) since debt decreases enough to bring the crisis probability close to zero. In the level shocks case shown in Figure 6b, taxes rise significantly, reaching over 14% in the crisis period. Notice that taxes are increasing as the endowment falls in the years preceding the crisis in this case, whereas with trend shocks the opposite is true. When the constraint binds, debt falls, leading to a decline in the probability of a future crisis and therefore a sharp decline in the tax rate. Even in crisis periods, the difference in the cyclicality and size of taxes in the two economies is stark.

Table 4 shows the percentage deviations of key variables during the financial crisis from

Table 4: *Severity of Financial Crises*

	Decentralized	Planner
Consumption/GDP	-1.6%	-0.5%
Current Account/GDP	3.5%	0.1%
Real Exchange Rate Depreciation	14.1%	5.1%

The financial crisis is defined as the period in which the collateral constraint binds. Consumption and the Real Exchange Rate are reported as percentage deviations from the ergodic mean. Current Account/GDP is reported as the deviation from the ergodic mean. Decentralized denotes the simulations using the policy functions corresponding to the decentralized economy, and Planner denotes the simulations using the Ramsey Planner's policy functions.

their ergodic means.[20] The current account-GDP ratio in the model with trend shocks is given by $(1+r)\tilde{b}_t/\exp(g_t) - \tilde{b}_{t+1}$. The results in the table, similarly to those in Table 2 in Bianchi [5], show that the consumption-GDP ratio, the current account-GDP ratio, and the real exchange rate all react far more in the decentralized economy than under the planner's allocation.[21] However, the fall in consumption and the current account are far less than what is found in Bianchi [5], and the real exchange rate also depreciates by about 5 percentage points less. This is because small changes in the growth rate can lead to financial crises, and thus the responses of key variables are also muted.

Finally, Table 5 shows important second moments in our model compared to the data. Like Bianchi [5], the model is successful at replicating many of the unconditional second moments in the data, computed using the 1,000,000-period simulations described above. Again, the externality induces more volatility in consumption, the real exchange rate, the current account, and the trade balance, though less so than in the model with level shocks, shown in Table 3 of Bianchi [5]. Because the planner's borrowing choices are closer to the decentralized economy, the differences in the behavior of key variables are small. Further, the model matches well the comovement in these variables with GDP. Here, unlike in Bianchi [5], both the decentralized and constrained optimal equilibria can account for the strong negative correlation in the current account and trade balance ratios to GDP, for the same reason that the second moments coming from the two equilibria are similar.

To sum up, we have obtained three main results. First, unless the stock of debt is so high that a crisis can occur, the constrained planner strongly increases borrowing in higher states of the world. The reason is an income effect which pushes the planner to allow for the benefits of consumption smoothing when expectations about future income are rosy. The

[20] We define the real exchange rate as $\left[\omega^{1/(1+\eta)} + (1-\omega)^{1/(1+\eta)}(p_t)^{\eta/(1+\eta)}\right]^{-(1+\eta)/\eta}$.

[21] We report consumption/GDP rather than the level of consumption due to the trend growth in our model.

Table 5: *Second Moments*

	Decentralized	Planner	Data
Standard Deviations			
Consumption/GDP	1.4	1.2	2.6
Real Exchange Rate	5.5	4.8	8.2
Current Account/GDP	3.7	3.3	3.6
Trade Balance/GDP	4.2	3.8	2.4
Correlation with GDP in units of tradables			
Consumption/GDP	.76	.82	.70
Real Exchange Rate	.75	.83	.41
Current Account/GDP	-.78	-.82	-.63
Trade Balance/GDP	-.76	-.82	-.84

Decentralized denotes the simulations using the policy functions corresponding to the decentralized economy, and Planner denotes the simulations using the Ramsey Planner's policy functions. The annual Consumption/GDP ratio is computed using Argentinian data from the World Bank, 1960-2007. Remaining data is taken from Bianchi [5], Table 3.

planner decides to do this even though he approaches the region where crises can occur. In this region, the optimal debt accumulation is small or zero. Second, under trend shocks, the planner taxes in good times, because then the incentives to borrow are strong. However, since the economy is growing strongly, the magnitude of the optimal tax is small. The opposite happens under level shocks: the planner taxes at high rates in bad times, because a precautionary motive provides incentives to borrow. Third, the probability of taxation under trend shocks is quite small. The reason is that crises occur for very high levels of debt accumulation.

4. Conclusions

In this paper we have analyzed the problem of macroprudential regulation in the presence of growth shocks to income. Positive shocks to the growth lead to optimally allowing for more borrowing. However, when the cumulated amount of borrowing is high enough, taxation of debt is optimal in order to make agents internalize the systemic externality of their decisions. Moreover, taxation of borrowing is pro-cyclical because there is little or no need of regulation in the case of negative trend shocks. This is in contrast to the case usually analyzed in this literature so far of contemporaneous (level) shocks to income, where optimal taxation of borrowing is counter-cyclical.

The main policy implication of our theory is that regulators should pay special attention to debt accumulation in booming times and it is then, and only then, when borrowing should be regulated. Booms sometimes could be induced by economic reforms. For instance, many Latin American countries experienced large increases in indebtedness after economic reforms that were expected to be successful, which ended up in subsequent debt crises. Our findings suggest that financial markets should be liberalized at slower pace and that some controls should remain.

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Appendix A. Solution Method

We solve the decentralized equilibrium using policy function iteration and the Ramsey planner's problem using value function iteration. Both economies are solved on a grid of 2,000 evenly spaced points between -1.1 and -0.4 for transformed debt, $\tilde{b}_t$. We discretize the growth rate shocks into 41 states using the Tauchen Method to approximate our estimated process, with parameters shown in Table 2, and spanning two standard deviations around the mean, $g = 1$. All remaining parameters are identical to [5] and shown in Table 1.

Following Schmitt-Grohé and Uribe [13] we set the optimal tax on debt equal to NaN in all states in which the collateral constraint binds for the Ramsey planner. Given the planner's optimal policy, we solve for the optimal tax on debt as

$$1 - \frac{\exp(g_t)^{-\sigma}\beta(1+r)E\left[u_T(\tilde{c}^P_{t+1})\right]}{u_T(\tilde{c}^P_t)}$$

Appendix B. Multiple Solutions for Constrained Debt

In this section we show that the functional forms we use, following Bianchi [5], lead to two solutions for debt when the collateral constraint binds. We then discuss our choice of the solution used in the quantitative analysis. For a thorough discussion of this issue, see Schmitt-Grohé and Uribe [14].

The borrowing constraint in the transformed model is given by:

$$\tilde{b}_{t+1} \geq -\kappa(1+\gamma p_t) \tag{B.1}$$

where

$$p_t = \frac{1-\omega}{\omega}\left(\frac{\tilde{c}^T_t}{\gamma\exp(g_t)}\right)^{\eta+1}$$

and

$$\tilde{c}^T_t = \exp(g_t) + (1+r)\tilde{b}_t - \exp(g_t)\tilde{b}_{t+1}$$

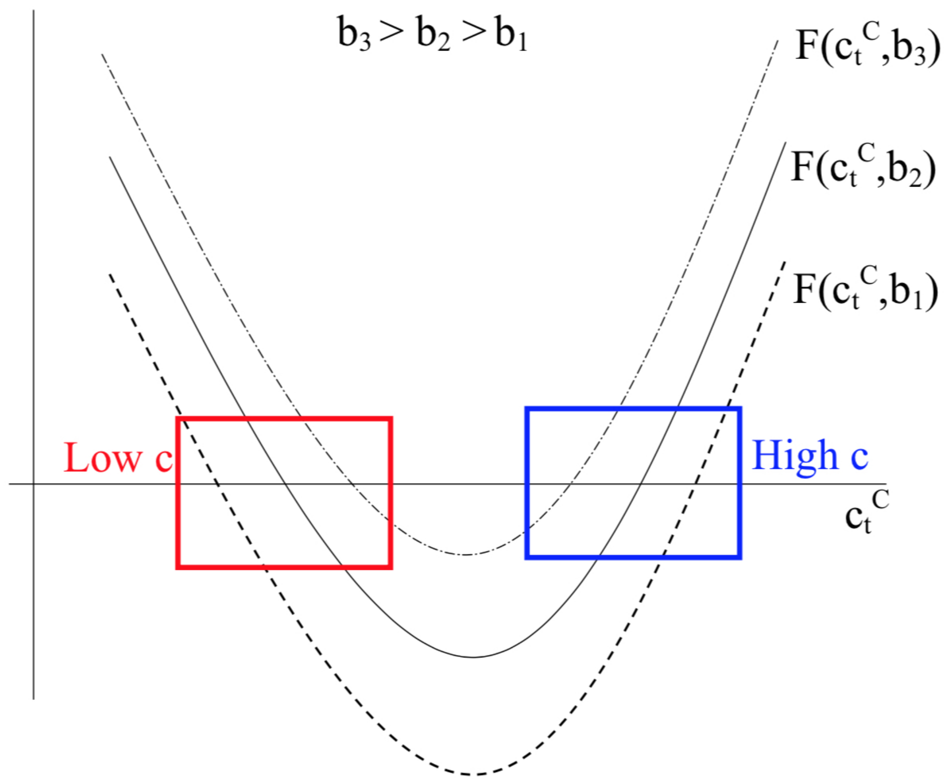
The trend shocks g_t follow an AR(1) process:

$$g_t = \rho g_{t-1} + \varepsilon_t$$

Rearranging (B.1) and plugging in for p_t and $\tilde{b}_{t+1}$, we have:

$$\exp(g_t) + (1+r)\tilde{b}_t - \tilde{c}^T_t \geq -\kappa\left[1+\gamma\left(\frac{1-\omega}{\omega}\left(\frac{\tilde{c}^T_t}{\gamma\exp(g_t)}\right)^{\eta+1}\right)\right]$$

Figure B.7: Solutions for Tradable Consumption: Binding Collateral Constraint



Define

$$F(\tilde{c}_t^T, \tilde{b}_t, g_t) = \exp(g_t) + (1+r)\tilde{b}_t - \tilde{c}_t^T + \kappa\left[1 + \gamma\left(\frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\gamma\exp(g_t)}\right)^{\eta+1}\right)\right]$$

Then the borrowing constraint can be written as $F(\tilde{c}_t^T, \tilde{b}_t, g_t) \geq 0$. When the constraint binds, denote the solution for consumption of tradables $c_t^C \equiv \tilde{c}^T(\tilde{b}_t, g_t)$, where $F(c_t^C, \tilde{b}_t, g_t) = 0$.

It is easy to see that when $\eta = 1$, we have a quadratic equation in $\tilde{c}_t^T$, which, given the state of the economy leads to two solutions for $\tilde{b}_{t+1}$. Our numerical simulations suggest that this is also the case in the parametrization shown in Table 1. Figure B.7 shows a schematic depiction of this result.

Notice that in the "Low c" case, when the solution for consumption is low, as outstanding debt increases (becomes more negative) from b_3 to b_1, consumption falls, leading to a decrease in the relative price and a tightening of the collateral constraint. This results in a policy function similar to Bianchi [5] and the rest of the literature, with an upward and followed by a downward sloping region of the policy function as outstanding debt increases. Differently, in the "High c" case, as outstanding debt increases, consumption increases and the value of collateral increases, allowing agents to borrow more as a function of outstanding debt when the constraint binds. This is the opposite result of the literature, and we therefore focus only on the "Low c" solution.

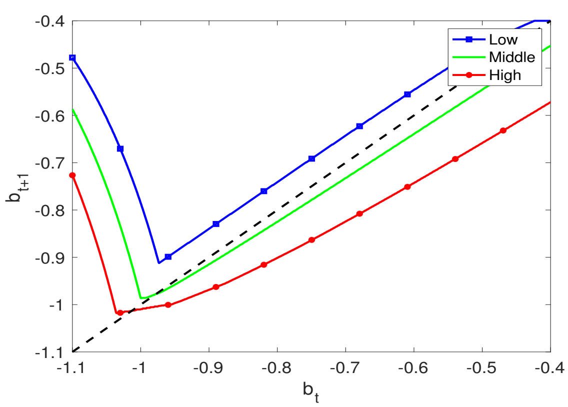
Appendix C. Supplemental Figures and Tables

In this section we plot additional figures comparing our model to 1) a low persistence economy with trend shocks, and 2) the level shocks benchmark.

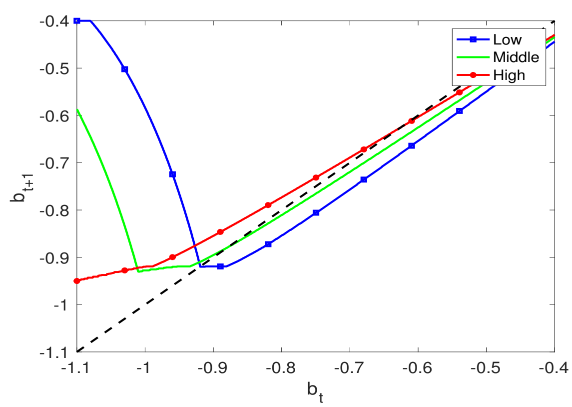
Appendix C.1. Effect of Persistence

First, we highlight the importance of persistence in the trend shock process for the qualitative and quantitative implications of our model. Recall that growth rate shocks follow an AR(1) process: $g_t = \rho g_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. Figure C.8 plots policy functions for the Ramsey planner for two levels of ρ, holding the standard deviation σ_ε fixed. As one can see, for very low levels of persistence ($\rho = 0.1$ in the right panel), the planner wants to borrow most in the low state, *even* for permanent (trend) shocks. Instead, for high persistence ($\rho = .7897$ in the left panel, the value we estimate and use in the paper), the order of the policy functions is reversed. Therefore, the persistence of trend shocks is crucial to obtain our results.

Figure C.8: Policy Functions of the Ramsey Planner



(a) High Persistence



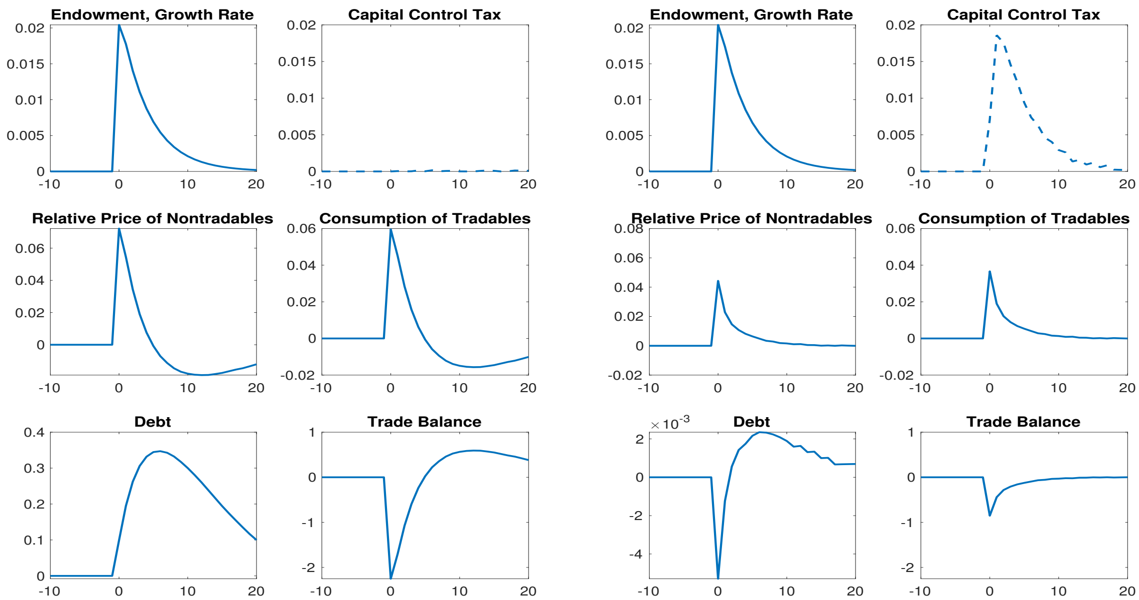
(b) Low Persistence

Notes: Policy functions of the constrained planner for $\tilde{b}_{t+1}$ as a function of $\tilde{b}_t$ when the persistence of shocks is high (0.78, left panel) and low (0.1, right panel). The policy function labeled "Middle" corresponds to the value of the shock equal to its mean, "Low" and "High" correspond to values of the shock two standard deviations below and above the mean, respectively.

Appendix C.2. Impulse Response Functions

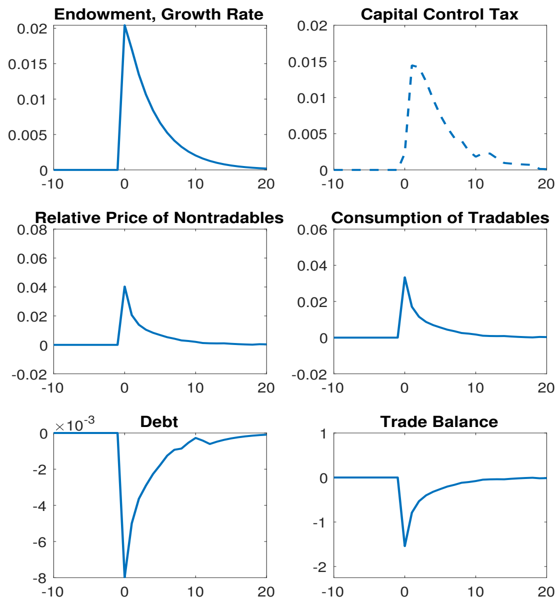
We report state-dependent impulse response functions to illustrate the behavior of consumption, prices, debt, and taxes in response to a positive one-standard deviation shock to output. We vary the initial level of the growth rate shock, and simulate the model, fixing this shock, for 3,000 periods to arrive at the ergodic distribution. Then, we shock the economy one time with a one-standard deviation increase in the growth rate shock, after which it returns to its initial level. Figure C.9 shows the results, in percentage deviations from the pre-shock level of each variable.

Figure C.9: Impulse Responses to a 1 SD Positive Endowment Shock



(a) Initial Shock 1 SD Below Mean

(b) Initial Shock at Mean



(c) Initial Shock 1 SD Above Mean

Appendix C.3. Alternative Calibration

In this section we present an alternative calibration of the permanent and temporary shocks models. In Section 3 we stay as close to the literature as possible by using the parameters from Bianchi [5]. Here we show that we could have also chosen κ and β in our model to match the 5.5% crisis probability and 29% average debt to GDP, the targets in Bianchi [5], and obtained nearly identical results. As can be seen in tables C.6 and C.7, under the stochastic processes we estimate, to match the same moments requires a small increase in households' patience as well as in the fraction of income that limits borrowing.

Figures C.10 to C.12 plot the policy functions and tax regions, typical boom bust cycle, and typical financial crisis, corresponding to Figures 2, 5, and 6. As can be seen from the figures, both the order of the policy functions and shape of the tax regions is similar to Figure 2. Most importantly, the size and cyclicality of the taxes in Figures C.12 and C.12 are only slightly lower than in Figures 5 and 6. Finally, Tables C.8 and C.9 show results analogous to those contained in Tables 4 and 5. By increasing the debt limit, the severity of crises increases slightly for both the decentralized economy and the planner. Similarly, volatility of the key moments shown in Table 5 also increases somewhat, though the correlations for both the decentralized economy and planner are nearly unchanged.

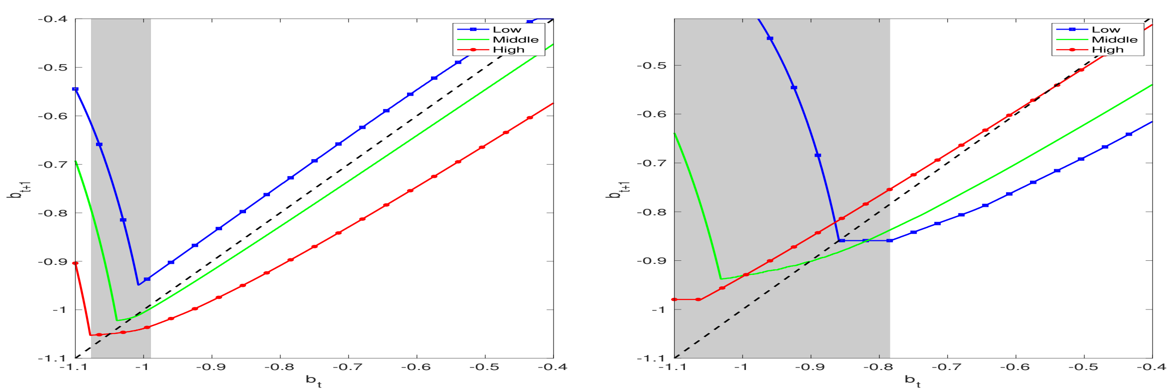
Table C.6: *Calibrated Parameters, Trend Shocks*

Parameter	Value
Interest rate	$r = 0.04$
Discount factor	$\beta = 0.952$
Credit coefficient	$\kappa = 0.333$
Share of non-tradables	$\omega = 0.31$
Risk aversion	$\sigma = 2$
Elasticity of substitution	$1/(1+\eta) = 0.83$

Table C.7: *Calibrated Parameters, Trend Shocks*

Parameter	Value
Interest rate	$r = 0.04$
Discount factor	$\beta = 0.92$
Credit coefficient	$\kappa = 0.327$
Share of non-tradables	$\omega = 0.31$
Risk aversion	$\sigma = 2$
Elasticity of substitution	$1/(1+\eta) = 0.83$

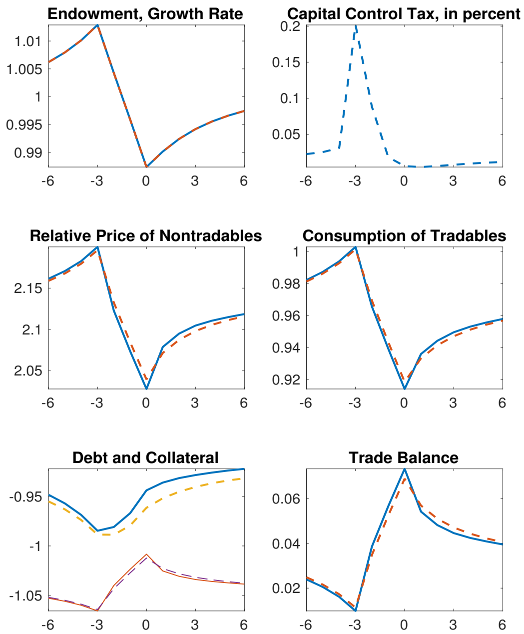
Figure C.10: *Tax Regions and Policy Functions of the Constrained Planner*



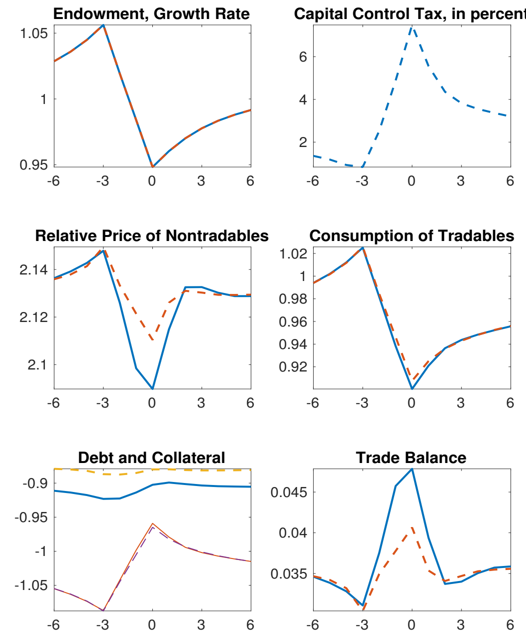
(a) Trend Shocks (b) Level Shocks

Notes: Alternative calibration shown in Tables C.6 and C.7. Policy functions of the constrained planner for $\tilde{b}_{t+1}$ as a function of $\tilde{b}_t$ when the shock is to the trend (left) and level (right). The policy function labeled "Middle" corresponds to the value of the shock equal to its mean, "Low" and "High" correspond to values of the shock two standard deviations below and above the mean, respectively. Shaded regions indicate at least one state in which there is a strictly positive tax for a given value of $\tilde{b}_t$.

Figure C.11: *Boom-Bust Cycles under Trend and Level Shocks*



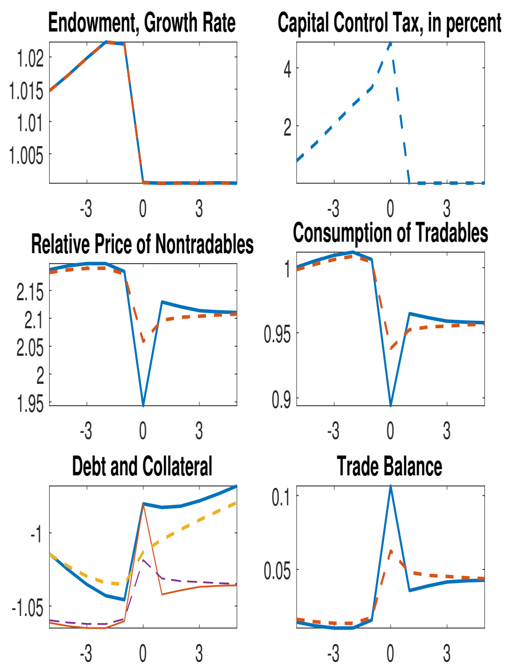
(a) Trend Shocks



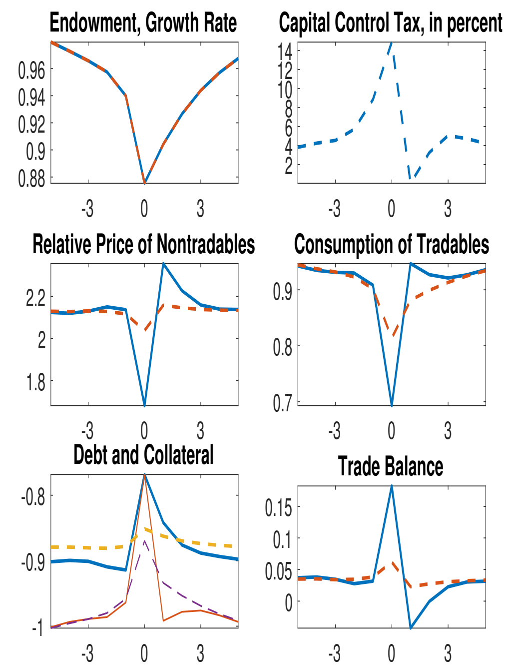
(b) Level Shocks

Notes: Alternative calibration shown in Tables C.6 and C.7. Each panel shows the average over all boom-bust cycles in simulations of 1,000,000 periods for the decentralized economy (solid line) and Ramsey planner (dashed). Due to a high level of skewness, the top right panel shows the median tax rate in percent.

Figure C.12: *Financial Crises under Trend and Level Shocks*



(a) Trend Shocks



(b) Level Shocks

Notes: Alternative calibration shown in Tables C.6 and C.7. Each panel shows the average over all financial crises in simulations of 1,000,000 periods for the decentralized economy (solid line) and Ramsey planner (dashed). Due to a high level of skewness, the top right panel shows the median tax rate in percent.

Table C.8: *Severity of Financial Crises*

	Decentralized	Planner
Consumption/GDP	-2.3%	-0.8%
Current Account/GDP	4.8%	0.5%
Real Exchange Rate Depreciation	18.9%	6.6%

Alternative calibration shown in Tables C.6 and C.7. The financial crisis is defined as the period in which the collateral constraint binds. Consumption and the Real Exchange Rate are reported as percentage deviations from the ergodic mean. Current Account/GDP is reported as the deviation from the ergodic mean. Decentralized denotes the simulations using the policy functions corresponding to the decentralized economy, and Planner denotes the simulations using the Ramsey Planner's policy functions.

Table C.9: *Second Moments*

	Decentralized	Planner	Data
Standard Deviations			
Consumption/GDP	1.5	1.3	2.6
Real Exchange Rate	6.0	5.2	8.2
Current Account/GDP	3.9	3.6	3.6
Trade Balance/GDP	4.6	4.1	2.4
Correlation with GDP in units of tradables			
Consumption/GDP	.76	.83	.70
Real Exchange Rate	.74	.83	.41
Current Account/GDP	-.78	-.82	-.63
Trade Balance/GDP	-.77	-.82	-.84

Alternative calibration shown in Tables C.6 and C.7. Decentralized denotes the simulations using the policy functions corresponding to the decentralized economy, and Planner denotes the simulations using the Ramsey Planner's policy functions. The annual Consumption/GDP ratio is computed using Argentinian data from the World Bank, 1960-2007. Remaining data is taken from Bianchi [5], Table 3.