Pricing and Constructing International Government Bond Portfolios *

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Abstract

In developed government bond markets, even simple diversification strategies are shown to offer significant benefits due to imperfectly correlated term-structure dynamics. We derive a stochastic discount factor to price this asset class by projecting returns onto the unconditional mean-variance efficient portfolio. The resulting market price of risk varies substantially over time, peaking during crises and periods of inflation rate dispersion. International bond returns exhibit a strong factor structure, but common sources of return variation show little connection to priced risks. Hedging unpriced risks from naive or factor-based strategies enhances Sharpe ratios significantly, even when portfolio weight limits are imposed.

JEL codes: G11, G12, G15.

Keywords: international government bond portfolios, bond risk premia, stochastic discount factor.

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1 Introduction

Sovereign bond markets represent one of the largest and most liquid asset class globally, second in size only to public equity markets. Yet there is little existing academic work that analyzes how internationally diversified government bond portfolios are priced, what their risk-return characteristics are, or how efficient alternative portfolio construction and trading strategies are. In fact, models of sovereign bond pricing are almost exclusively cast in single-country settings. Similarly, the existing literature on active government bond portfolio management largely focuses on the optimal choice of duration or on the relative mispricing within a fixed-income market, while allocation across bond markets is usually not discussed (see, for example, the relevant chapters in Bodie, Kane, and Marcus, 2020). In contrast to the single-country focus of the academic literature, the availability of globally diversified government bond funds and exchange-traded funds (ETFs) has increased over the past years and the demand for these products is on the rise.[1]

In this paper, we depart from the traditional single-country focus and take a multi-market perspective to investigate the risk factors driving multi-country currency-hedged government bond portfolios.[2] We derive several key results. First, to our knowledge, we are first to provide stylized facts on internationally diversified developed markets government bond portfolios. Our main sample comprises ten major developed sovereign debt markets, denoted as G10*, collectively representing approximately 70% of the global market capitalization for local-currency debt. In particular, we show that substantial diversification benefits arise

[1]The ratio of total net assets of global versus domestic fixed-income funds in the U.S. has approximately doubled over the past three decades. Using data from Morningstar, Figure A1 in the Appendix plots this ratio and shows that it increased from about 7.5% in 1995 to almost 15% in 2022. This ratio is only a lower bound for the actual international diversification of U.S. bond investors, as they are likely to also achieve diversification by combining domestic bond funds from different countries in their own portfolios.

[2]The exact definition of currency-hedged bond returns is provided in Section 3.1.

even from naive diversification strategies, such as GDP-weighted portfolios, simply due to imperfectly correlated term-structure movements in the different markets.

Second, we construct the unconditional mean-variance efficient (UMVE) portfolio from the G10* markets, thus providing a stochastic discount factor (SDF). We find that the UMVE portfolio can be obtained using expected return forecasts based on forward spreads and real yields and a time-varying covariance matrix, estimated with a shrinkage method. The resulting SDF is shown to price the individual G10* bond markets as well as those of a sample of additional markets and various dynamic trading strategies.

Third, we analyze the properties of the UMVE portfolio. We find that optimally investing in international government bonds improves the Sharpe ratio substantially, from an average of 0.46 for individual markets to a value that exceeds 1. We also show that the expected Sharpe ratio of the UMVE exhibits substantial time-variation. Specifically, it increases around the financial crisis in 2008, during the European sovereign debt crisis between 2010 and 2012 and during the COVID-19 crisis. The highest expected Sharpe ratio is observed in the fourth quarter of 2022, when concerns about inflation and tightening monetary policies raised investors' marginal utility of wealth. These are all periods during which the SDF implies a high market price of risk in international government bond markets. Importantly, we document that the expected Sharpe ratio of the UMVE portfolio is significantly related to the dispersion of inflation rates across developed sovereign bond markets: the higher the cross-sectional standard deviation of inflation rates, the higher is the expected Sharpe ratio of the UMVE portfolio. In addition, the Sharpe ratio is negatively related to financial intermediaries' capital ratios and to an index of global economic policy uncertainty.

Fourth, the analysis reveals that there is very little relation between common sources of variation in international bond market returns and priced risks. To this end, we perform a principal component analysis (PCA) and first show that the G10* government bond returns display a strong factor structure, with the first 3 principal components (PCs) explaining 86% of the variance. However, these common sources of variation represent mostly unpriced risks, since the first 10 PCs only explain around 22% of the UMVE return variation.

Finally, we provide relevant insights for bond investors. We find that strategies such as naive diversification or various carry, value, and momentum strategies all exhibit large amounts of unpriced risks. By constructing portfolios that hedge out such unpriced components from bond strategies, Sharpe ratios improve substantially. For instance, applying such a hedging strategy to a naively diversified (1/N) portfolio, the Sharpe ratio almost doubles. We observe similar improvements for other popular factor strategies, such as carry or value. Furthermore, we find that, for practical implementation, the estimation of expected returns is essential, while estimation of variances matters to a lesser degree.

Implementing bond portfolio strategies that hedge out unpriced risks is associated with substantial time-variation and large absolute values of the portfolio weights for different markets. To shed light on the feasibility of such strategies, we provide sensitivity analyses and impose restrictions on the absolute values of portfolio weights. We find that portfolio performance is remarkably robust to imposing such limits. Only when we impose a long-only restriction on the portfolio strategies, some of them drop in performance. But this appears excessively restrictive, since bond futures exist in all markets in our sample except for one, which makes shorting a particular market easy for professional market participants.

Our paper is related to several strands of literature. First, it intersects with the work on return prediction in single domestic government bond markets. A large literature has emerged since Fama and Bliss (1987), who document the empirical failure of the expectations hypothesis, as returns are predictable using maturity-specific forward rates. Cochrane and Piazzesi (2005) make fuller use of the forward curve for return prediction. They provide evidence that a common risk factor prices bonds of various maturities. Ludvigson and Ng (2009) show that macroeconomic and financial variables substantially improve the predictability of bond risk premia. Cieslak and Povala (2015) emphasize the importance of inflation to predict government bond returns. In their empirical specification, cycles of yields in excess of trend inflation contain information on future returns.

Second, a small number of papers analyze international bond return predictability and yield curve fluctuation across different markets (e.g., Jotikasthira, Le, and Lundblad, 2015; Bekaert and Ermolov, 2023). Ilmanen (1995) analyzes bond predictability in six countries and finds that expected bond returns are high when the term spread and the real bond yield are high. Dahlquist and Hasseltoft (2013) construct local and global Cochrane-Piazzesi factors and find that both predict international bond returns. Brooks and Moskowitz (2019) instead find that carry and value predict excess returns on government bonds. More recently, Lustig, Stathopoulos, and Verdelhan (2019) find that the predictability of long-term foreign bonds of developed countries decreases as the bonds' maturity increases. They conclude that, at the long end of the yield curve, local currency term premia and currency premia move in different directions and, therefore, offset each other.

Third, our paper relates to the literature that derives the SDF from the mean variance efficient portfolio, as developed by Hansen and Richard (1987), Ferson and Siegel (2001), Ferson and Siegel (2003), Ferson and Siegel (2009), and, more recently, Chernov, Lochstoer, and Lundeby (2022), Maurer, Tô, and Tran (2021) and Chernov, Dahlquist, and Lochstoer (2023). Our analysis also derives the unconditional mean variance efficient portfolio, but we analyze a previously unexplored yet important class of assets, namely international government bonds.

In summary, our paper is related to strands of literature that either take a single-country perspective on government bond markets, or focus on other asset classes, such as the stock market or currency markets. We differ from all these papers, since we analyze the pricing of developed countries' government bonds in a portfolio context. We derive the UMVE portfolio for this asset class out-of-sample, provide insights about its average realized Sharpe ratio, the time-series properties of the resulting expected risk premia, and on how common sources of return variation in international bond portfolios are related to priced risks.

The paper proceeds as follows. Section 2 discusses the theoretical foundations. Section 3 describes the empirical strategy and provides stylized evidence. Section 4 presents all the empirical findings. Section 5 explores the implications for bond portfolio strategies. Section 6 includes robustness checks. Section 7 concludes.

2 A Linear Factor Model

This section derives the theoretical foundations for the empirical strategy that will be implemented in the remainder of the paper. Specifically, we derive an SDF that prices international bond markets both conditionally and unconditionally via the UMVE portfolio. Moreover, the UMVE portfolio derived below prices not only individual markets, but also all admissible dynamic trading strategies constructed from these individual markets.

The foundations of the role of conditioning information in dynamic asset pricing models have been described by Hansen and Richard (1987), Ferson and Siegel (2001), and more recently by Chernov et al. (2022), on which we draw in the following description of the approach.

The UMVE portfolio is defined by a dynamic trading strategy in the N assets with excess returns $rx^U_{t+1} = (\mathbf{w}^U_t)^\top \mathbf{rx}_{t+1}$. Its weights $\mathbf{w}^U_t$ are a function of the time-t covariance matrix $\mathbb{V}_t(\mathbf{rx}_{t+1})$ and expected excess returns $\mathbb{E}_t(\mathbf{rx}_{t+1})$:

$$\mathbf{w}^U_t = \frac{\mathbb{V}^{-1}_t(\mathbf{rx}_{t+1})\,\mathbb{E}_t(\mathbf{rx}_{t+1})}{1 + \mathbb{E}_t(\mathbf{rx}_{t+1})^\top\,\mathbb{V}^{-1}_t(\mathbf{rx}_{t+1})\,\mathbb{E}_t(\mathbf{rx}_{t+1})}. \tag{1}$$

While any portfolio that has weights proportional to $\mathbb{V}^{-1}_t(\mathbf{rx}_{t+1})\,\mathbb{E}_t(\mathbf{rx}_{t+1})$ is conditionally mean-variance efficient, only scaling by the term $(1+\mathbb{E}_t(\mathbf{rx}_{t+1})^\top\,\mathbb{V}^{-1}_t(\mathbf{rx}_{t+1})\,\mathbb{E}_t(\mathbf{rx}_{t+1}))^{-1}$ provides the UMVE portfolio. As has been shown by Hansen and Richard (1987), the unconditionally efficient portfolio must be conditionally efficient, while the converse needs not be true.

The UMVE prices all individual assets and any combination of them, including dynamic trading strategies. Specifically, we can write both conditional and unconditional linear factor pricing models in terms of the UMVE. The unconditional relationship is:

$$\mathbb{E}(rx^p_{t+1}) = \beta^p\,\mathbb{E}(rx^U_{t+1})\,, \tag{2}$$

where the unconditional beta $\beta^p = Cov(rx^p_{t+1}, rx^U_{t+1})\,\mathbb{V}^{-1}(rx^U_{t+1})$. This relationship holds for any asset or strategy p.

Similarly, the conditional version of the asset pricing model is given by

$$\mathbb{E}_t(rx^p_{t+1}) = \beta^p_t\,\mathbb{E}_t(rx^U_{t+1})\,, \tag{3}$$

with a conditional beta $\beta^p_t = Cov_t(rx^p_{t+1}, rx^U_{t+1})\,\mathbb{V}^{-1}_t(rx^U_{t+1})$.

Finally, the UMVE-implied SDF is

$$M_{t+1} = 1 - \left(rx^U_{t+1} - \mathbb{E}(rx^U_{t+1}) \right) \, , \tag{4}$$

and it satisfies

$$\mathbb{E}_t(M_{t+1} rx^p_{t+1}) = 0 \tag{5}$$

for all dynamic trading strategies p in the basis assets. It is important to point out that this SDF M_{t+1}, defined in Equation (4), is estimated from the UMVE portfolio returns. Thus, it can be interpreted as an unconditional linear projection of the true SDF onto the payoff space of all possible trading strategies that can be constructed from the N basis assets.

3 Empirical Strategy and Stylized Facts

Empirical implementation of the approach described in Section 2 to construct the UMVE portfolio requires the estimation of i) expected returns and ii) the conditional variance. To determine the weights of the UMVE portfolio at any time t, we will use only information available at that point in time, while returns are observed out-of-sample, in the subsequent period. The estimation of conditional covariances faces substantial challenges in a high-dimensional asset space, such as in the equity market. Indeed, the estimated covariance matrix might not have full rank or even exhibit negative eigenvalues. Therefore, the low-dimensional asset space resulting from our focus on a small cross-section of 10 major government bond markets provides an important advantage when estimating the variance-covariance matrix.

The remainder of this section explains how bond excess returns are defined and presents stylized evidence for the markets in our sample. Furthermore, it describes how expected excess returns and their conditional covariances are estimated and reports the data sources.

3.1 Bond Excess Returns

To analyze the pricing of internationally diversified bond portfolios, we consider a portfolio strategy, where each month investments in local currency bonds are financed via short-term local-currency loans. We hereby represent each country's bond market by a synthetic 10-year zero-coupon bond, obtained from the appropriate zero-coupon bond curve.[3] The corresponding holding period excess return, rx^i_{t+1}, is therefore given by

$$rx^i_{t+1} := rx^{(10Y)}_{i,t \to t+1M} = \left(\frac{P^{(10Y-1M)}_{i,t+1M}}{P^{(10Y)}_{i,t}} - \frac{1}{P^{(1M)}_{i,t}} \right) \frac{S_{i,t+1M}}{S_{i,t}} \,, \tag{6}$$

where $P^{(10Y)}_{i,t}$ and $P^{(1M)}_{i,t}$ are the local currency prices of a 10-year government bond and of a 1-month government bill at time t for country i, respectively, and $S_{i,t}$ is the exchange rate of country i, expressed as the price of the foreign currency in units of the home currency, which we interpret to be the U.S. dollar. We calculate time series of excess returns as defined in Equation (6) at monthly frequency, using data from the last day of each month.

Note that the excess return rx^i_{t+1}, defined in Equation (6), can also be interpreted as the dynacy-hedged return resulting from a long position in the foreign dynacy bond plus a short position in a 1-month currency forward.[4] This strategy is therefore also consistent with the definition of hedged bond returns used by Koijen, Moskowitz, Pedersen, and Vrugt (2018) and Du and Huber (2023).[5] Campbell, Serfaty-de Medeiros, and Viceira (2010) find that

[3]This maturity is frequently used by practitioners and in the academic literature to represent government bond markets (see, e.g., Jordà, Knoll, Kuvshinov, Schularick, and Taylor, 2019; Lustig et al., 2019).

[4]For a detailed derivation for the equivalence of our returns and forward-hedged returns, we refer to the Appendix, Table B1.

[5]We show in Table B2 in the Appendix that one-month changes in exchange rates do not explain excess returns rx^i_{t+1} as defined in Equation (6). Adjusted R^2's of regressions of excess returns on one-month

investors holding international bonds should hedge their currency risk. Consistent with this conclusion, Sialm and Zhu (2022) provide empirical evidence analyzing mutual fund data and find that around 90% of U.S. international fixed-income funds use currency forwards to manage their foreign exchange exposure (although they tend to hedge only partially). They also show that hedging out foreign exchange risk does not decrease the funds' abnormal returns, but reduces their return variability.

In our main empirical analysis we focus on government bond markets of the following countries, which we refer to as G10*: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the United States. The currencies of these countries are the ones which are frequently considered in the foreign exchange literature, which refers to them as "G10 currencies" (see, for example, Lustig et al. (2019) and Chernov et al. (2023)). Government bonds of these countries are typically issued in local markets and denominated in local currencies (Chen, Ganum, Liu, Martinez, and Peria, 2019). In fact, 99.8% of the outstanding long-term central government debt of the countries in our main sample is denominated in domestic currency, according to the Bank for International Settlements (BIS).[6] We therefore focus on local currency bonds in our analysis. We represent the Eurozone by the German government bond market, which is arguably its safest and most liquid national bond market. All countries in the main sample have liquid government bond markets and they cover about 68% of the global local-currency long-term debt in the BIS statistic, as of the end of 2020. As we use only German bonds to represent the Eurozone,

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changes in exchange rates are essentially equal to zero. This interpretation of the excess returns rx^i_{t+1} mirrors a standard approach used by international bond fund managers when hedging foreign exchange fluctuations. For example, the Bloomberg Global Aggregate Index publishes hedged returns, where rolling positions in one-month forward contracts are reset at the end of each month, and no adjustments to the hedge is made during the month.

[6]Data source: BIS, http://www.bis.org/statistics/c4.pdf. The percentage does not include New Zealand, where data are unavailable in the BIS statistic.

and since Euro area government bonds tend to be highly correlated, our implicit coverage of world government bond markets is plausibly even larger. A further advantage of our sample is that the issuing countries are very unlikely to default, such that the government bonds in our sample can be considered as practically default-risk free assets.

3.2 Data

To calculate bond excess returns, as defined in Equation (6), we collect daily data from Bloomberg on long-term and short-term zero-coupon yields for the period from January 1995 to December 2022 for the G10* countries.[7] For some analyses, we expand the set of countries and thus we also obtain data on zero-coupon yields and exchange rates for additional markets. For this extended sample, we select all OECD member countries which do not belong to the G10* and for which data availability for the zero-yield curve from Bloomberg is adequate. This process defines a sample of 20 countries. In addition we consider Hong Kong and Singapore for which sufficiently long time series of zero yield curves are also available and which have liquid and sizable government bond and foreign exchange markets. The final list of extended countries is composed of: Austria, Belgium, Finland, France, Greece, Ireland, Italy, the Netherlands, Portugal, Slovakia, Slovenia and Spain, which belong to the Euro area as of 2022, and Chile, Denmark, Colombia, Czech Republic, Hong Kong, Hungary, Israel, Mexico, Singapore, and Turkey, which are countries outside the Euro area.

We also use several macro data sources. From Datastream, we collect monthly data on consumer price indices (CPIs), unemployment rates, and quarterly data on gross domestic product (GDP) for each country in our sample.[8] We then compute the cross-sectional

[7]Since Bloomberg does not provide data on the 1-month yield for all countries in our sample, we approximate it using the 3-month yield, which is consistently available.

[8]For the GDP data, to move from quarterly to monthly frequency, we assign the same value for all three

standard deviation of these variables as a measure of dispersion across macroeconomic dimensions. As proxies for the return and volatility of the stock market, we collect data for the MSCI World Index and for the VIX Index from Bloomberg. The Global Economic Policy Uncertainty Index (EPU) is obtained from the website of Scott Baker, Nick Bloom, and Steven Davis. This index is based on Baker, Bloom, and Davis (2016) and measures global policy-related economic uncertainty. We also obtain data on the intermediary capital ratio, defined by He, Kelly, and Manela (2017) as the aggregate value of market equity divided by the aggregate value of total assets (i.e, aggregate market equity plus book debt) computed for the largest financial intermediaries in the U.S. These data are available from the authors' websites.[9]

3.3 Stylized Facts

We first provide empirical facts about the risk-return characteristics of government bond markets and dynamic government bond portfolio strategies. Using Equation (6), we calculate annualized average excess returns, standard deviations, and Sharpe ratios for the G10* countries. Table 1 provides descriptive statistics for annualized one-month holding period excess returns of 10-year government bonds by country of issuance. Although all countries can be considered as very close to default free, investors earned attractive returns over the sample period. In fact, Sharpe ratios are above 0.4 in all markets except New Zealand. The high average returns over the sample period partially reflect a worldwide downwards trend in bond yields, which persisted until March 2022.

Table 2 compares risk-return characteristics from individual bond markets with those that can be obtained from naively diversified static portfolios and from simple dynamic portfo-

months of each quarter.

[9]Table C2 in Appendix C provides descriptive statistics for the above macro and market variables.

lio strategies. Comparing Panel A of Table 2, which denotes the benchmark case with no diversification, to Panel B, which displays simple diversification strategies, it becomes clear that international diversification matters for bond investors. By definition, naive ($1/N$) diversification across markets achieves an average excess return identical to the cross-sectional average of the single-countries in the sample, which equals 3.3% p.a. The individual countries' average standard deviation equals 7.4%. By diversifying, investors could have lowered the standard deviation of their portfolio considerably by almost 20% to 6.1%, thereby achieving a substantially higher Sharpe ratio: While the individual countries' average Sharpe ratio equals 0.46 in our sample, naive diversification across the 10 markets increases the Sharpe ratio by almost 20% to 0.54. A GDP-weighted strategy yields a marginally worse risk-return trade-off, with a Sharpe ratio of 0.53. Panel C of Table 2 explores whether simple portfolio strategies already realize most of the benefits of internationally diversified bond portfolios, or whether conditional strategies, which should be easily accessible to bond portfolio managers, lead to further improvements. To this end, we investigate long-short portfolios sorted on carry, value and momentum. Returns to these strategies have been shown to contribute significantly to portfolio performance in different asset classes, such as equity, currencies, and fixed income (e.g., Fama and French, 2012; Asness, Moskowitz, and Pedersen, 2013; Koijen et al., 2018). We therefore use the following characteristics to construct simple dynamic portfolio strategies: the slope of the yield curve, i.e. the difference between the 10-year yield and the local 1-month yield (*carry*), the real bond yield, i.e. the difference between the 10-year yield and inflation (*value*), and momentum, i.e. the cumulative return over 12 months skipping the most recent one (*momentum*). We document that all long-short bond portfolios achieve positive performance. The Sharpe ratios for carry, value and momentum are 0.81, 0.55, and 0.28, respectively. Interestingly, the carry strategy outperforms all others substantially. This evidence is consistent with the literature highlighting that returns from carry generate positive alpha for a host of asset classes (Koijen et al., 2018).

Figure 1 illustrates time variation in returns and risk for the bond portfolio strategies. Specifically, Panels A and B display the extent to which average excess returns fluctuate over time by plotting moving averages over 36 months for the returns of the naive and the GDP-weighted portfolios, as well as the factor portfolios carry, value, and momentum from Table 2 (Panel C). The considerable heterogeneity in the performance dynamics of these bond portfolio strategies further supports the conjecture that investors might be able to benefit from conditional strategies. Figure 1, Panels C and D instead show the time series of the standard deviations of these five portfolios, where the estimation procedure used to compute the covariances is explained in detail in Section 3.4.2. While it can be seen that all portfolios show considerable fluctuations in their standard deviations over time, the factor strategies appear to exhibit lower risk levels on average. Motivated by these findings, we proceed to analyze the properties of the UMVE portfolio, using the parsimonious theoretical framework described in Section 2. To this end, we first explain how the two inputs for its estimation, namely expected returns and conditional variance are derived.

3.4 Estimation of Expected Bond Returns and Covariances

3.4.1 Expected Returns

To forecast expected excess returns $\mathbb{E}_t(rx^i_{t+1})$ one month ahead, we rely on forward spread and value signals. These are two well established predictive variables, for which there is a large empirical literature, both as potential predictors of excess returns in local bond markets as well as of other asset classes. Specifically, we run the following pooled ordinary least squares (OLS) regression, using expanding time windows, with standard errors clustered at the country level:

$$\mathbb{E}_t(rx^i_{t+1}) = \gamma_t \; ForwardSpread_{i,t} + \delta_t \; Value_{i,t} \,, \tag{7}$$

where we define the explanatory variables as follows:

$$ForwardSpread_{i,t} = \frac{P^{(10Y-1M)}_{i,t}}{P^{(10Y)}_{i,t}} - \frac{1}{P^{(1M)}_{i,t}} \,. \tag{8}$$

Many papers, starting with Fama and Bliss (1987) and Campbell and Shiller (1991), document the empirical failure of the expectations hypothesis and that forward rates predict future bond returns: steep yield curves signal high subsequent holding period returns. More recently, Koijen et al. (2018) provide evidence that similar carry measures predict returns cross-sectionally and in the time series for different asset classes. Their definition of global bond carry is a slightly modified version of the Fama and Bliss (1987) forward spread. Linking carry concepts from the currency and bond markets, Andrews, Colacito, Croce, and Gavazzoni (2023) show that sorting countries on the slope of their yield curves constitutes a profitable trading strategy, in particular since 2008 when returns from the traditional currency carry have weakened.

Next we define value by the real bond yield:

$$Value_{i,t} = Y^{(10Y)}_{i,t} - \pi_{i,t} \,, \tag{9}$$

where $\pi_{i,t} = \ln(CPI_{i,t}/CPI_{i,t-12M})$ and CPI is a country's consumer price index. Inflation has been considered an important driver of bond holding period returns, both in academic research as well as among practitioners. Campbell and Ammer (1993) show that unexpected excess bond returns must be associated with changes in expected inflation rates, future real interest rates, or future excess bond returns. Because bond payoffs are fixed, changes in expected inflation rates impact nominal bond returns even if expected real returns are constant. More recently, Cieslak and Povala (2015) find that a cycle factor that captures deviations of observed bond yields from those fitted on the basis of current and past inflation predicts excess returns of long-term yields.

It is plausible that sophisticated investors were aware of the relation between the above predictive variables and bond returns throughout our sample period. As discussed above, the shape of the yield curve has already been identified as a determinant of bond risk premia by Fama and Bliss (1987) and Campbell and Ammer (1993), who split the bond yield into expected real rates and other components, thus emphasizing the role of value measures as defined in Equation (9) above. Also, Ilmanen (1995) considers both the term spread and the real bond yield as instruments to explain bond excess returns. To ensure that we use only information up to time t to form expectations about returns ending in period $t+1$, we re-estimate the coefficients in Equation (7) every month using an expanding sample. For the regression, we require a minimum of 60 months of data, and thus we obtain expected returns starting with January 2001.

Figure 2 shows the time series of the estimated coefficients $\hat{\gamma}_t$ and $\hat{\delta}_t$, while Table 3 reports their descriptive statistics. Even though there is some time variation in the estimates, they appear to be quite stable. Consistent with the literature, both the forward spread and bond value have positive coefficients, but only the former has, on average, a statistically significant relation with expected returns. In Table 3, the average t-stats are 2.65 and 1.35 for the forward spread and bond value, respectively. Despite weak statistical significance of value for forecasting bond returns, it will turn out to be, jointly with the forward spread, an economically relevant signal in constructing the UMVE portfolio, as reported in Section 4.2.

3.4.2 Covariances

We estimate the covariance matrix using a three-step procedure. First, we compute monthly bond returns at daily frequency. We require a high frequency of observations in this stage as this allows us to estimate a covariance matrix $\hat{\Sigma}_t$ for each month, thus capturing time variation. Second, given the low number of observations within a single month, we apply the quadratic shrinkage method of Ledoit and Wolf (2022), obtaining $\tilde{\Sigma}_t$. Finally, we calculate an exponentially weighted average of the shrunk covariance matrices, to arrive at the estimate of the covariance matrix $\mathbb{V}_t$:

$$\mathbb{V}_t = (1-\lambda)\tilde{\Sigma}_t + \lambda\, \mathbb{V}_{t-1}\,, \tag{10}$$

where we set λ=0.94.[10]

3.4.3 Unbiasedness

As a sanity test, we first check if the main objects that we use for constructing the UMVE portfolio are unbiased. In particular, we regress (i) realized excess returns rx^i_{t+1} on expected excess returns $\mathbb{E}_t(rx^i_{t+1})$, (ii) squared returns in excess of expected values, $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$, on estimated variances, $\sigma^2_t(rx^i_{t+1})$, which are the diagonal elements of the matrix $\mathbb{V}_t$, and (iii) products of returns $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariances $cov_t(rx^i_{t+1}, rx^j_{t+1})$, which are the off-diagonal elements of $\mathbb{V}_t$.

Table 4 shows that the regression of realized returns on predicted returns produces a coefficient close to 1 (the estimated coefficient is equal to 0.90). The conditional variance also

[10] We set $\mathbb{V}_1$ equal to $\tilde{\Sigma}_1$.

appears consistently estimated, as indicated by the coefficient of 0.95. Similar conclusions obtain for the estimated covariance, where the regression coefficient almost exactly equals 1.00. Thus, according to Table 4, the predictive regressions are unbiased, as in none of the cases the coefficients are statistically significantly different from 1.[11]

4 Empirical Evidence

After defining the set of the test assets, this section provides the main results of the asset pricing tests to evaluate the linear factor model, discusses the implications, and analyzes the properties of the UMVE portfolio returns.

4.1 Test Assets

Following Hansen and Richard (1987), one can evaluate the conditional linear beta pricing relationship from Equation (3) by testing the unconditional linear beta pricing relation specified in Equation (2) for a comprehensive set of dynamic trading strategies as test assets. Therefore, in addition to individual countries' bond returns, we include 14 trading strategies in our set of test assets. Specifically, we define the return of a dynamic portfolio strategy P, rx_{t+1}^{P}, as

$$rx_{t+1}^{P} = \sum_{i=1}^{N_t} w_{i,t}^{P} rx_{t+1}^{i}\,, \tag{11}$$

where $w_{i,t}^{P}$ is the weight of country i in strategy P and N_t is the number of countries at time t. In addition to the naive $1/N$ and the GDP-weighted portfolios, we use bond carry,

[11] This is also true when predicting returns and covariances of trading strategies, as shown in Table D1 in the Appendix.

value and momentum as trading signals, which we define as follows:

Carry	difference between the 10-year yield and the local 1-month yield $Carry_{i,t} = Y_{i,t}^{(10Y)} - Y_{i,t}^{(1M)}$,
Value	real bond yield, as defined in Equation (9) $Value_{i,t} = Y_{i,t}^{(10Y)} - \pi_{i,t}$,
Momentum	cumulative return over 12 months, skipping the last month $Mom12_{i,t} = \prod_{k=1}^{11} \left(\frac{P_{i,t-k}^{(10Y-1M)}}{P_{i,t-k-1}^{(10Y)}} \frac{S_{i,t-k}}{S_{i,t-k-1}} \right) - 1$,
ShortTermMom	return over the last month $Mom1_{i,t} = \frac{P_{i,t}^{(10Y-1M)}}{P_{i,t-1}^{(10Y)}} \frac{S_{i,t}}{S_{i,t-1}} - 1$.

Table 5 lists the different trading strategies and provides the details of the respective portfolio constructions.

4.2 Regression Tests of the UMVE Portfolio

To analyze whether the UMVE portfolio prices international government bonds, we test Equation (2) and run traditional regression tests of the following form:

$$rx_{t+1}^{i} = \alpha_i + \beta_i rx_{t+1}^{U} + \epsilon_{i,t+1}, \tag{12}$$

where rx_{t+1}^{i} denotes the excess returns of test asset i (where i denotes the country or the trading strategy, respectively) and rx_{t+1}^{U} is the UMVE portfolio's excess return as defined in Section 2.

Table 6 shows the results of the hypothesis test for the G10* markets. Specifically, it shows for each country the annualized Sharpe ratio (SR), the mean excess returns ($Mean$), and the t-test of the mean excess returns (t_mean). It further reports the alphas ($Alpha$) from country regressions defined in Equation (12), their t-statistics (t_alpha), and the associated adjusted R^2 ($adj.R^2$). While Sharpe ratios range from 0.26 to 0.70 and mean annualized excess returns range between 2.20% and 4.01%, alphas are small and not significantly different from zero. The alpha for Germany has the largest t-statistic, with a value of 1.28. Japan exhibits the largest adjusted R^2: the UMVE portfolio excess returns explain 16% of Japanese bond excess returns. Overall, results confirm the validity of the UMVE portfolio, as it is able to individually price government bond markets without bias. However, the relatively low R^2s suggest that only a part of bond return variations is priced. This observation is consistent with the finding of Chernov et al. (2023) for currencies.

Table 7 reports the results for portfolio strategies. The Sharpe ratios of the 14 portfolio strategies range from -0.34 to 0.79. All alphas with respect to the UMVE portfolio are statistically insignificant. The UMVE portfolio prices a relatively large share of the variation in the cross-sectional carry and time series value strategies, as expressed by the R^2 of 15%. The other R^2s are relatively low, similar in magnitude to those of Table 6. This suggests again that a substantial fraction of the variation of the returns is unpriced.

Table 8 reports the results of the Gibbons, Ross, and Shanken (1989) (GRS) test. The first column shows the Sharpe ratio of the UMVE portfolio, constructed from the full estimation model defined in Equation (7) (top row), and of restricted UMVE portfolios, which use the forward spread only (middle row) or value only (bottom row) as predictor variable. As can be seen, the UMVE portfolio based on the full model prices bond markets substantially better than the UMVE portfolio obtained from the restricted models. This indicates that, while the bond value variable exhibits poor statistical significance when we look at its effect in the forecasting regression (see Table 3), it is an important variable, jointly with the forward spread, when it comes to pricing the cross-section of bond returns. We also observe that the Sharpe ratio of the optimal UMVE portfolio is much higher than the Sharpe ratios of the individual bond markets and those of bond strategies. In Table 8, the column headed *All strategies (GRS Tests)* shows the key result, namely that the p-value of the test whether the alphas from regressing each portfolio strategy on the UMVE portfolio are jointly equal to zero. The GRS test fails to reject the null with a p-value equal to 0.402. Overall, we conclude that the full estimation model represents a valid SDF. In the spirit of Chernov et al. (2023), we also apply the GRS test to UMVE candidate portfolios constructed on the basis of the forward spread or value only as predictor variable. We test these versions of the UMVE portfolio on different strategies. Results are shown in the middle and bottom rows of Table 8. Although these restricted models price some trading strategies, they lack the ability to price all strategies jointly.

4.3 Pricing Other Countries

Since a global SDF should not only price the assets from which it has been constructed, we go beyond the original set of the G10* markets, and implement tests using a wider universe of bond markets. Specifically, we test if the UMVE portfolio prices single-country bonds and strategies based on an extended set of countries, which are not used to construct the UMVE portfolio. We therefore run regression tests, as specified in Section 4.2, using the excess returns of other countries' bond markets and the trading strategies associated with most of these markets[12] as the dependent variable in Equation (12).

[12] To guarantee a balanced panel and data availability from January 2001 (starting date of the UMVE) for the trading strategies, we use only data on the following markets to construct the alternative countries' portfolio strategies: Austria, Belgium, Denmark, Finland, France, Hong Kong, Ireland, Italy, Netherlands,

We report the results in Table 9. While, on average, alphas are larger in magnitude compared to the results for the main markets (see Table 6), all but one are not statistically significant at the 5% level. This is confirmed both for markets pegged to the Euro and, importantly, also for other markets. The only exception is Italy, which exhibits a t-value of 1.99. This might reflect the different behavior of Italian bonds during and after the European sovereign debt crisis, when their status as a safe asset was in doubt. The picture does not materially change when we consider bond strategies constructed from this set of bonds. Table 10 shows that most alphas are insignificant (at the 5% level), with the exception of cross-sectional carry and cross-sectional 12-month momentum. In summary, there is little evidence of market segmentation, as the UMVE portfolio constructed from the main bond markets does not generally lead to large pricing errors even in a broader asset universe.

4.4 Unpriced Return Components

Our results so far confirm that the UMVE portfolio prices both individual countries' bond excess returns and the excess returns obtained from portfolio strategies. However, the share of variation in returns explained by the UMVE, as expressed by the regressions R^2s, is relatively low. We therefore construct bond portfolios which hedge out the unpriced sources of common variation in returns. The recent literature explores the properties of hedged and unhedged portfolios and their asset pricing implications (e.g., Daniel, Mota, Rottke, and Santos, 2020; Lopez-Lira and Roussanov, 2023). These papers find that Sharpe ratios improve substantially when hedging unpriced components of risk. We compute for each strategy the conditional beta β_t^P with respect to the UMVE portfolio as follows:

Portugal, Singapore, and Spain.

$$\beta_t^P = \frac{(\mathbf{w}_t^P)^\top \, \mathbb{V}_t(\mathbf{rx}_{t+1})\mathbf{w}_t^U}{(\mathbf{w}_t^U)^\top \, \mathbb{V}_t(\mathbf{rx}_{t+1})\mathbf{w}_t^U}\,, \tag{13}$$

where $\mathbf{w}_t^P$ is a vector of strategy weights, as defined in Table 5, and $\mathbf{w}_t^U$ is the vector with the weights of the UMVE portfolio.

We define the hedged portfolio, HDP, as a portfolio with excess returns given by $\beta_t^P rx_{t+1}^U$, i.e. as a portfolio which only has systematic factor exposure. The unpriced component is captured by the hedging portfolio, HGP, with returns equal to the return difference between the original strategy portfolio, P, and the hedged portfolio, HDP. In particular:

$$rx_{t+1}^{HGP} = rx_{t+1}^P - \beta_t^P rx_{t+1}^U. \tag{14}$$

The weights of the hedged portfolios are given by $\mathbf{w}_t^{HDP} = \beta_t^P \cdot \mathbf{w}_t^U$. Figures E1, E2 and E3 in Appendix E plot the time series of portfolio weights of the respective hedged trading strategy.

Figure 3 displays the annualized Sharpe ratios of the original portfolios, of the hedging portfolios and of the hedged portfolios. The Sharpe ratios of the hedging portfolios are close to zero, and always lower than those of the original strategies. This accords well with the notion that risks which are not related to the SDF should not be associated with risk premia. We also observe that for most strategies the Sharpe ratios of the hedged portfolios are all significantly larger than those of the hedging and the original portfolio strategies, respectively. This is consistent with Daniel et al. (2020), who conclude that the risk-adjusted portfolio return can be improved by eliminating unpriced risks. Indeed, while the risk-adjusted portfolio returns for the original strategies are in the range of -0.34 to 0.79, those of the hedged portfolios range between 0.15 and 1.07.

To gain additional insight into the structure of bond returns and their relation to the UMVE portfolio, we perform a Principal Components Analysis (PCA) of bond excess returns. Figure 4 displays the loadings of the first three principal components (PCs). Consistent with the literature on interest rates, the first PC is a level factor, as all countries load positively on it. The second PC seems to be an Australasia vs. rest of the world factor. Australia and New Zealand have indeed positive loadings, while the remaining countries have negative ones. With respect to the third PC, we cannot see a clear, intuitive interpretation.

Figure 5 documents how much of the variation in the UMVE portfolio returns is explained by the PCs. Panel B of Figure 5 reports the R^2 of these regressions. The first bar shows the results from regressing the UMVE on the first PC only, the second one uses the first two PCs as regressors and similar interpretations apply for the other bars. Interestingly, the first three PCs explain less than 10% of the variation in the UMVE excess returns. This is despite the fact that these three PCs jointly explain around 86% of the variation in bond returns, as documented in Panel A of Figure 5. Even when all PCs are considered, the R^2 only increases to 22%. This implies that the UMVE is not spanned by the PCs, since a significant fraction of its variation remains unexplained. Panel C of Figure 5 shows the alphas from a regression of bond returns on the PCs, as well as their confidence intervals. It can be seen that the resulting alphas are all positive and statistically significant. Thus, PCs extracted from bond returns do not adequately explain the UMVE portfolio returns.

Overall, this evidence suggests that there is an important timing component in the UMVE portfolio, which is not reflected in the PCs of bond returns. The latter are, indeed, essentially a linear combination of individual bond returns weighted by constant loadings, and as such cannot capture the dynamic nature of the UMVE portfolio. Such a timing component is likely to be even more strongly reflected in trading strategies. We therefore turn our attention to dynamic trading strategies next.

Figure 6 shows the PC loadings of the (dynamic) bond trading strategies. These loadings do not imply a straightforward economic interpretation of the PCs, except for the first PC, which again seems to reflect a level factor. As displayed in Panel A of Figure 7, the first three PCs explain 82% of the variation in portfolio strategies. However, even though these three PCs capture the time variation of strategy returns quite well, they only explain a small share of priced risks, i.e. 11%, as documented in Panel B of Figure 7. If we use all PCs, the R^2 increases to 34%. As was the case for individual bonds, we document that the factors explaining the variation in the returns on trading strategies are not important drivers of the variation in the UMVE portfolio returns. However, consistent with the fact that trading strategies better capture the dynamic nature of the UMVE, the principal components obtained from these portfolios explain a higher fraction of the UMVE portfolio returns than the ones obtained from the set of individual bond markets.

4.5 Sharpe Ratio Decomposition

Section 4.4 documents that the Sharpe ratios of almost all traditional trading strategies can be substantially improved by hedging out risk components unrelated to UMVE portfolio returns. This section analyzes whether the hedging performance of the UMVE-based SDF is mainly due to capturing the cross-sectional and time series variation of the various markets' expected returns, or mainly due to correctly capturing the time-varying return dependence, or whether both are of similar importance.

To this end, we successively shut off elements in the estimation of the UMVE portfolio (i.e., expected returns, variances, correlations) and only let the others vary. Comparing hedging strategies based on the restricted UMVE portfolios helps to identify the sources of the Sharpe ratio improvements. Specifically, our approach considers the following alternatives: First, we fix expected returns at the historical average across all markets over the estimation window, but use the estimates for the conditional variance from the full model (as described in Section 3.4.2). Thus, we shut off any cross-sectional variation in conditional expected returns when constructing the UMVE portfolio. Second, we use estimates for expected returns for each market from the full model (as described in Section 3.4.1), but employ a covariance matrix $\mathbb{V}_t^{\bar{\sigma}}$ where we set each market's variance equal to the cross-sectional average. Thus, here we shut off any cross-sectional variation in conditional variances when constructing the UMVE portfolio. Third, we again estimate expected returns based on the full model (as described in Section 3.4.1), but use a covariance matrix $\mathbb{V}_t^{\bar{\rho}}$ where we set all correlations equal to the average estimated pairwise correlation. Thus, here we shut off any cross-sectional variation in conditional correlations. Note that all three described estimates are out-of-sample since the values at which a component of the estimation is fixed only depends on past returns. We describe our approach in more detail in Appendix F.

We refer to the hedged portfolios resulting from the three restricted SDF estimations as *Hedged: Constant Expected Returns*, *Hedged: Constant Sigma* and *Hedged: Constant Correlation*. Figure 8 shows the annualized Sharpe ratios for these portfolios. To facilitate easier interpretation, the first bar reports the Sharpe ratios for the hedged portfolios when using the unrestricted UMVE portfolio returns, which are the same as those in Figure 3. As can be seen from Figure 8, only the portfolios *Hedged: Constant Correlation* have Sharpe ratios close to the ones obtained from the hedged strategies that make full use of the estimated parameters, as is done in Figure 3. In contrast, Sharpe ratios are much lower when using an SDF that is estimated from the UMVE portfolios that assume constant expected returns or constant standard deviations across markets. When comparing the second (green) and third (turquoise) bars in the charts for the different strategies, we see that modelling the cross-section of expected returns is more important in defining the UMVE portfolio than estimating the cross-section of variances. Indeed, the hedged portfolios when using UMVE portfolio estimations based on constant variances have higher Sharpe ratios on average than in the case where UMVE portfolios are constructed based on constant expected returns, indicating that shutting off cross-sectional variation in expected returns worsens the performance of the resulting SDF more than shutting off cross-sectional variation in variances.[13]

4.6 UMVE Portfolio Characteristics

This section reports important properties of the UMVE portfolio shown to price bond markets. In particular, we provide descriptive statistics and analyze its time-varying risk-return characteristics. To this purpose, we scale the UMVE portfolio, so that its returns have the same average volatility as the $1/N$ portfolio, i.e. a portfolio that allocates capital equally across all bond markets.

Table 11 summarizes descriptive statistics. Specifically, it shows that the UMVE portfolio returns an impressive average annualized excess return of 6.9% with a volatility of 6.1%. This implies a Sharpe ratio of 1.1. The maximum drawdown is 8.6% over the entire period from January 2001 to December 2022. The second set of rows shows the optimal portfolio weights

[13] The analysis in this subsection eliminates various sources of cross-sectional differences in the parameters that determine the UMVE portfolio. For completeness, the Appendix G reports results of an alternative exercise, which eliminates various sources of time series differences instead. Specifically, we i) use the in-sample country-specific average excess returns, but let the conditional variance be estimated as in Section 3.4.2 and ii) we let expected returns be estimated as in Section 3.4.1, but use the in-sample estimated covariance matrix, by computing the arithmetic average of all monthly shrunk covariance matrices. The detailed description of this approach as well as the results are available in the Appendix G. Overall, the outcome is consistent with the conclusions of this subsection in the sense that capturing time series differences in expected returns is more important than capturing time series differences in covariances.

averaged across time. It can be seen that individual positions are quite large: the average absolute weight is about four times that of the $1/N$ portfolio. The dynamics of the portfolio composition are illustrated in Figure 9, which plots the weights of the constituents of the UMVE portfolio over time. There is considerable time-variation in the portfolio composition, which is consistent with Green and Hollifield (1992), who argue that the presence of a dominant factor can lead to extreme weights of assets in mean-variance efficient portfolios. The sensitivity of results with respect to restrictions on portfolio turnover is discussed in Section 5.

Figure 10 plots the UMVE's annualized expected Sharpe ratio over time. The Figure illustrates that the market price of risk exhibits significant time-variation. The highest expected Sharpe ratio is approximately 3 and it reaches a minimum of approximately 0.66. We observe relatively high Sharpe ratios in periods of crises, as reflected by the peaks around the financial crisis 2008, the European sovereign debt crisis from 2010 to 2012, and the recent COVID-19 crisis. The highest values correspond to the fourth quarter of 2022. During that year, average inflation rates reached a record high, triggered also by Russia's invasion of Ukraine in late February 2022. This suggests that the unprecedented sharp increase in yields led by aggressive monetary policy tightening and increases in inflation expectations raised investors' marginal value of wealth. In accordance with this observation, the difference in the average expected Sharpe ratio between recession and non-recession periods, as defined by the NBER for the U.S. and the Eurostat Business Cycle Clock for the Euro area, is positive and statistically significant.[14] Overall, this suggests that the estimated price of risk is high when the current macroeconomic situation is bleak, consistent with the intuition

[14] We regress the UMVE expected Sharpe ratio on a constant, plus a recession dummy (equal to 1 for recession periods as defined by the NBER and the Eurostat Business Cycle Clock, 0 otherwise) and document a positive coefficient of 0.260 which is statistically significant at the 5% level, using Newey and West (1994) standard errors.

that investors have high marginal utility in times of crisis and, as a result, require more compensation for foregoing consumption by investing in a risky bond portfolio.

Figure 10 suggests that the market price of risk in international government bond portfolios is related to the macro economy. This is consistent with a large literature that links risk premia in single government bond markets to macro variables.[15] Important macro variables that have been found to affect bond risk premia include output growth, unemployment and inflation (e.g., Ludvigson and Ng (2009)). We therefore wish to shed light on how the potential benefits from internationally diversified government bond portfolios depend on these macro variables. Of course, if changes in the macro-economy are exactly the same across the G10* countries, then macro-driven term-structure changes should also be identical and allocating assets across different countries' bond markets becomes irrelevant. Diversification matters only when there is heterogeneity in the macro-economic dynamics. We therefore include the cross-sectional standard deviation of the macro variables (*GDP SD*, *Unempl SD*, *Infla SD*) to explain the dynamics of the UMVE portfolio's expected Sharpe ratio. In addition, we control for global equity returns (*MSCIRet*) and a measure of overall risk, captured by the *VIX* index, which has been shown by Miranda-Agrippino and Rey (2020) to correlate with the global cycle. We also include the risk bearing capacity of large U.S. institutions, measured by their capital ratio (*CapitalRatio*) as defined in He et al. (2017). Finally, we control for global economic policy uncertainty (*GlobEPU*), based on Baker et al. (2016). The measure is computed as the GDP-weighted average of countries' Economic Policy Uncertainty (EPU) indices.

Table 12 reports the regression results. It can be seen that the dispersion in inflation is the most important economic variable, which robustly explains a significant fraction in the

[15] For recent examples, see Moench and Soofi-Siavash (2022), Fang, Liu, and Roussanov (2022), or Cieslak and Pflueger (2023).

market price of risk in bond markets. Specifically, higher cross-sectional standard deviation of inflation is associated with a higher expected Sharpe ratio. This highlights the significant role of inflation across markets, which affects both interest rate movements and risk factors. The MSCI World Index and the VIX also contribute positively to the dependent variable, although to a lesser extent. When the global stock market realizes positive returns, bond returns may decline contemporaneously, but risk-adjusted expected bond returns increase. We also find that increases in the VIX, which usually coincide with bad times, lead to increases in the price of risk. The intermediary capital ratio has a negative and statistically significant effect (only at the 10% level) on the UMVE expected Sharpe ratio. As the capital ratio is a sensible proxy for financial institutions' risk-taking capacity, a low value may indicate lower ability and willingness to make risky investments and thus a higher market price of risk. It is also consistent with investors being less optimistic about macro-economic conditions, as indicated by the large and negative correlation of -0.62 between the capital ratio measure and the recession dummy. Table 12 also shows that the Global EPU Index is negatively and statistically significantly related to the UMVE expected Sharpe ratio. At first sight, this result may appear surprising, especially in light of the higher expected Sharpe ratios in recessions, as described above. However, securities prices are forward looking and reflect investors' expectations about future macro-economic developments. When an economy is already known to be in a recession, investors are likely to expect the macro-economic situation to improve in the future. Thus, to transfer wealth from the poor current recession state to a possibly improved future via a risky bond investment may require a high Sharpe ratio. By contrast, the Global EPU Index is not a recession proxy, and can instead be interpreted as a forward looking measure of uncertainty (Brexit referendum, Trump election, etc.). When this index is high, investors may find government bond investments more attractive than investments in other risky assets, such as stocks, consistent with a flight to quality and the negative regression coefficient reported in Table 12.

5 Implications for Bond Portfolio Strategies

In this section we discuss implications of our findings for bond portfolio management. We start by illustrating the time-varying expected risk premia of traditional bond portfolio strategies. We then provide information about the time-varying composition of the UMVE portfolio, and to what extent extreme long and short positions, as shown in Table 11, are required to obtain the attractive risk-return relation of the UMVE.

Once the portfolio projection of the SDF is known, one can calculate the expected risk premium of various bond strategies in real time. Specifically, for each bond strategy considered by an investor, she can obtain the expected Sharpe ratio based on estimates of expected excess returns and the covariance matrix. Figure 11 illustrates the cross-sectional and the time variation of expected Sharpe ratios for the trading strategies described in Section 4.1. Positive (negative) values are shown in green (red) color; darker areas indicate larger absolute values. First, Figure 11 reveals considerable heterogeneity: simple diversification (naive and GDP-weighted), and carry strategies have almost always positive expected Sharpe ratios, while momentum strategies show notable fluctuations between positive and negative values. The attractiveness of value strategies appears to vary at lower frequency.

While a real-time assessment of the risk-return properties of a given trading strategy, as illustrated in Figure 11, may be useful for investors, it does not speak to the overall optimal portfolio. We thus turn to the analysis of the practical implementability of the UMVE portfolio. We have shown that hedged portfolio strategies lead to economically attractive risk-return ratios which, in most cases, beat the original trading strategies by a wide margin. It is therefore natural to ask whether these strategies would be implementable in practice. Such a practical implementation may face two challenges. First, the hedged portfolio strategies may require extreme position weights to achieve their high performance. To see how sensitive the results are with regard to a portfolio manager's ability to take extreme market positions, we impose minimum and maximum weights for the hedged portfolio strategies and compare the resulting Sharpe ratios with the ones of the original strategies. A second potential concern relates to the feasibility of taking short positions. In principle, short positions should be easily achievable, since long-term government bond futures are available in all markets of our main sample with the exception of Norway.[16] However, many asset managers, such as conventional mutual fund managers, face legal or investor-imposed short-selling constraints. We therefore also analyze strategies with minimum weights of zero in each market as a robustness check. Specifically, we implement the following two lower and upper bounds on portfolio weights: First, we restrict portfolio weights for each country between 0% and 20% and second, we consider a restricted strategy with portfolio weights in the range between -20% to +20%. We report the results in Figure 12. Remarkably, even when we apply such relatively conservative weight restrictions, the hedged strategies beat the original trading strategies with Sharpe ratios close to those achieved by the unrestricted portfolios. However, it appears that a considerable part of the performance comes from short positions: While 11 out of 14 long-only strategies still outperform their unhedged counterparts, their

[16] Koijen et al. (2018) include the same ten bond markets as our paper but calculate synthetic bond futures returns because of liquidity concerns and data availability. For markets where they have access to bond futures prices, they find correlations above 0.95 between actual and synthetic futures returns. Similarly, we find correlations above 0.97 between excess returns that we calculate from yield curves and futures excess returns for Australia, Canada, Germany, Sweden, Switzerland, the UK, and the U.S.A.. We find somewhat lower correlations for futures markets that are less liquid (New Zealand 0.81, Japan 0.92). There is no long-term government bond futures contract traded for Norway.

Sharpe ratios tend to be far lower than those of portfolios that allow at least limited short positions. The only exceptions are the momentum portfolios. Here, restricting investors from taking short positions even turns out to be beneficial. Overall, the hedged portfolios improve Sharpe ratios considerably, even when large portfolio weights are not allowed.

Finally, Table 11 shows that implementation of the UMVE requires high portfolio turnover which might raise concerns about trading costs. Yet, this is not likely to be an important issue for major government bond markets since they all feature derivatives markets. Trading costs for bond futures are very low, typically in the order of magnitude of 1 basis point. Transactions in the cash market tend to be slightly more expensive. Favero, Pagano, and von Thadden (2010) quantify the median cost of trading German government bonds with 10 year maturity with 3 basis points. Using a range from 1 to 3 basis points, trading costs will lower the return of the UMVE by only about 0.2% to 0.6% p.a.

6 Robustness

6.1 Estimation of Expected Returns: Alternative Specifications

In this subsection, we test alternative specifications of Equation (7). Specifically, we i) add a constant, ii) add country fixed effects, and use alternative definitions of iii) the forward spread and iv) the bond value variables.

Specifically, for the alternative specification (iii) we compute the forward spread variable defined over 1 year, as follows:

$$ForwardSpread_t^* = \frac{P_{i,t}^{(9Y)}}{P_{i,t}^{(10Y)}} - \frac{1}{P_{i,t}^{(1Y)}}\,, \tag{15}$$

and not over 1 month, as defined in Equation (8).

The alternative bond value variable used for the alternative specification (iv) is based on trend inflation, as defined by Cieslak and Povala (2015), instead of historical inflation. Specifically, we compute:

$$
\begin{aligned}
Value^{*}_{t} &= Y^{(10Y)}_{i,t} - \tau^{CPI}_{t} \\
\tau^{CPI}_{t} &= \frac{(1-v)}{(1-v^{M})} \sum_{i=0}^{M-1} v^{i} \pi_{t-i} \,,
\end{aligned}
\tag{16}
$$

where $\pi_t = \ln(CPI_t/CPI_{t-12M})$ is the annual inflation in month t, $M = 120$ is the number of months considered for measuring inflation, and v is a parameter calibrated to inflation survey data which we set equal to 0.987, as in Cieslak and Povala (2015).[17] Trend inflation, as defined above, has been shown to forecast future inflation and to predict bond excess returns (see Cieslak and Povala (2015) and the literature cited there).

Results from these alternative specifications are summarized in Appendix D, Table D2. Overall, we find a slight weakening of evidence for the validity of the resulting alternative SDFs, thus providing further support for the original model specification.

6.2 Estimation of Expected Returns: Including Additional Predictive Variables

We explore the robustness of our findings with respect to the model of expected returns. In particular, we consider momentum and currency trend variables and credit risk as additional

[17] We obtain time series for All Items CPI from the OECD Main Economic Indicators database, available via Archival Fred at the website of the Federal Reserve Bank of St. Louis. We use vintage data to construct inflation rates in order to analyze portfolios constructed with information available at the time. To further allow for a publication lag, we lag inflation rates by two months except for Australia and New Zealand, for which we apply additional lags to account for the quarterly frequency of the data.

candidate explanatory variables.

6.2.1 Momentum and Currency Trend Signals

In addition to the variables used in Equation (7), we include long-term and short-term momentum, as well as currency trend signals when modelling expected returns. Specifically, we estimate the following out-of-sample regressions:

$$\mathbb{E}_t(rx^i_{t+1}) = \psi_{1,t} ForwardSpread^i_t + \psi_{2,t} Value^i_t + \psi_{3,t} LTMom^i_t + \psi_{4,t} STMom^i_t + \psi_{4,t} FXTrend^i_t \,, \tag{17}$$

where the definitions of $ForwardSpread$ and $Value$ are unchanged from Section 3.4.1. $LTMom$ is defined as the cumulative return from $t-12$ to $t-1$. $STMom$ is defined as the most recent return. $FXTrend$ is defined, similarly to Chernov et al. (2023), as a simple currency trend signal: $S_{i,t}/S_{i,t-12M} - 1$. We find that including these variables weakens the results when compared to the model with carry and value only. Specifically, this alternative model leads to noisier conditional expectations. More importantly, while a UMVE portfolio constructed from three or more signals still seems to correctly price all trading strategies, its Sharpe Ratio is lower than the one resulting from the UMVE that only relies on two signals. We provide detailed results in the Appendix D (Panels A to D of Table D3 and Table D4).

6.2.2 Credit Risk

The G10* countries defined in Section 3.1 of this paper all have developed economies and are economically and politically stable. We therefore interpret their bond markets as essentially default-free. However, the literature discusses credit risk as a potentially important component of the risk premium even for such countries (Longstaff, Pan, Pedersen, and Singleton, 2011; Augustin, Sokolovski, Subrahmanyam, and Tomio, 2022). Therefore, as an additional robustness check, we include sovereign credit risk, which we measure using sovereign CDS spreads, to estimate expected returns.

Specifically, we run out-of-sample regressions of the following form:

$$\mathbb{E}_t(rx^i_{t+1}) = \theta_{1,t} ForwardSpread^i_t + \theta_{2,t} Value^i_t + \theta_{3,t} \Delta CreditRisk^i_t \,, \tag{18}$$

where $\Delta CreditRisk^i_t$ is computed as $CDS_{i,t} - CDS_{i,t-1M}$, with $CDS_{i,t}$ and $CDS_{i,t-1M}$ being the monthly average CDS spread levels (in basis points) at time t and $t-1M$, respectively. We use data on 5-year USD-denominated sovereign CDS spreads from Markit and follow Della Corte, Sarno, Schmeling, and Wagner (2022) for an accurate selection of the CDS restructuring clauses.[18] To obtain a balanced panel of liquid CDS contracts, we use data starting from the end of 2003. However, we lack liquid data for Switzerland (up to 2005) and the United Kingdom (up to 2008). To proxy for the missing CDS data, we use contracts of comparable countries based on credit ratings and levels of investor protection.[19] We provide results in Panel E of Table D3 and in Table D4, fifth row. We find that adding CDS does not improve the prediction of bond expected returns or variances.

[18]We focus on contracts denominated in USD and with 5-year maturity, as they are the most liquid contracts in the sovereign CDS market.

[19]To identify comparable countries, we match credit ratings, using Thomson Reuters as data source for Fitch Long Term Default Ratings. To match investor protection, we use La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1998) and select countries of the same legal origin and with a low number of deviations in all sub-categories of shareholder rights, creditor rights, and rule of law. For these countries, we compute the average CDS spreads and use them for the period for which the respective matched country has no liquid CDS data. To avoid potential jumps between the two time series, we apply an adjustment factor to the matched average CDS spread.

6.3 Estimation of Expected Returns: Fixed Estimation Window

We further estimate Equation (7) using constant 5-year rolling windows, instead of expanding windows. As expected, the time series of the estimated coefficients $\hat{\gamma}_t$ and $\hat{\delta}_t$ exhibit more fluctuation, as shown in Figure D1. However, we still obtain valid and unbiased estimates of the expected returns, as documented in Table D5. Furthermore, results from the regression tests of the UMVE portfolio on the test assets (see Tables D6 and D7) document the robustness of our findings.

6.4 Regression Test of the UMVE Portfolio: Other Maturities

We have so far provided evidence that the UMVE derived in Sections 2 and 3 prices individual bond markets and trading strategies well. These tests relied on holding-period returns from positions in the countries' 10-year bonds. To analyze whether the UMVE also prices international bonds in different maturity buckets, we now test its validity using holding-period returns from alternative long-term bond maturities. Specifically, we analyze whether the UMVE also prices 5-year and 15-year maturities.

Results are reported in Table D8. One can see that none of the regressions yields a statistically significant alpha at the conventional 5% level. This is true even at the shorter 5-year maturity (except for Switzerland), for which one may expect that countries' monetary policies in the wake of the Great Financial Crisis and COVID-19 may have led to some distortions or structural breaks, not captured by the 10-year return-based UMVE. Overall, this evidence provides further support for the validity of the UMVE.

6.5 Time Series of Expected UMVE Sharpe Ratio: Including Credit risk

As a robustness check, we include a proxy of global sovereign credit risk to explain the time series of the UMVE expected Sharpe Ratio. Specifically, we use *GlobCDS*, which we construct as the GDP-weighted average of countries' 5-year CDS spread changes. CDS spread changes are constructed as described in detailed in Section 6.2.2. Table D9 shows the regression output. Results confirm the importance of dispersion in inflation as a driver of the price of risk. The *GlobCDS* variable is not statistically significant, which is in line with our argument that credit risk does not play a significant role in the G10* markets.

6.6 Alternative Benchmark Portfolios

6.6.1 The Naive Portfolio

As shown above, the SDF projection on the UMVE portfolio is supported by standard validation tests. However, one may ask whether a naive SDF, such as the one derived from a projection on the $1/N$ portfolio, also leads to similar results as the projection on the UMVE portfolio. In this respect, we perform regression tests as in Tables 6 and 7, but using the naive $1/N$ portfolio returns to define the SDF. Results are reported in Tables D10 and D11. Although the $1/N$ portfolio is able to correctly price some individual government bond and portfolio strategy returns, we find that, on average, the t-statistics of the alphas are much higher than those obtained when using the SDF from a projection on the UMVE portfolio (see Tables 6 and 7). Importantly, for some test assets, the alphas resulting from the $1/N$ portfolio are statistically significantly different from zero. We therefore conclude that the SDF based on the naive portfolio strategy is dominated by the SDF derived in Section 2.

6.6.2 The Minimum Entropy Approach

In our main analysis, we obtain real-time estimates of the UMVE-based SDF. This approach derives a projected SDF from the unconditional mean-variance efficient portfolio. An alternative approach is based on minimum entropy, which takes into account higher moments of returns. Following Sandulescu, Trojani, and Vedolin (2021), we provide an in-sample estimate of the minimum entropy SDF for the international government bond markets. Specifically, we obtain the minimum entropy SDF, $\tilde{M}$, as

$$\tilde{M}_t = R^{-1}_{\hat{\lambda}^*,t}\,, \tag{19}$$

where $R_{\hat{\lambda}^*,t}$ denotes the gross returns of the optimal growth portfolio. The latter is defined as

$$R_{\hat{\lambda}^*,t} = \sum_{i=1}^{N} \hat{\lambda}_i^* rx_t^i + R_{US}^{(1M)}\,, \tag{20}$$

where $\hat{\lambda}_i^*$ is the optimal portfolio weight for country i and $R_{US}^{(1M)}$ is the gross return of 1-month U.S. T-bills. We estimate the optimal portfolio weights from the solution to the following set of empirical moment conditions:[20]

$$\hat{\mathbb{E}}[R^{-1}_{\hat{\lambda}^*}\mathbf{rx}] = \mathbf{0}\,, \tag{21}$$

where **rx** is the vector of excess returns of our test assets, i.e. the individual bond markets. Following Sandulescu et al. (2021), we obtain the vector of portfolio weights $\hat{\lambda}^*$, i.e. the parameters to be estimated from Equation (21), using the method of moments.

[20] For details, see Equation (17) from Sandulescu et al. (2021).

We report the results of the hypothesis test specified in Equation (12) in Tables D12 and D13, while the results from the GRS test are reported in the bottom row of Table D4. Overall, the optimal growth portfolio, which is constructed in-sample, explains a larger fraction of the variation in bond returns and portfolio strategies than the UMVE portfolio, which is constructed out-of-sample. However, the UMVE portfolio dominates with respect to pricing international government bonds. As reported in Tables D12 and D13, with the exception of cross-sectional carry and cross-sectional value, alphas of individual country and strategy regressions are statistically indistinguishable from zero. However, using the GRS test we reject the hypothesis that alphas are jointly equal to 0 when using the minimum entropy SDF.

7 Conclusion

This paper investigates the pricing of currency-hedged government bonds and presents several key findings. We document substantial diversification benefits for investors due to imperfectly correlated term-structure movements in the different markets, even when only considering developed markets and simple strategies, such as GDP-weighted or equally weighted portfolios. We find that the unconditional mean-variance efficient portfolio of $G10^*$ government bonds can be obtained using expected return forecasts based on forward spreads and real yields and a time-varying covariance matrix, estimated with a shrinkage method. The resulting SDF is shown to price the individual $G10^*$ bond markets as well as additional markets and various dynamic trading strategies.

While the SDF derived in this paper is based on portfolios of bonds with 10-year maturities, we find that it also prices bonds in other maturity buckets well. To rule out that credit risks have significant effects on risk premia even though we focus on developed countries, we extend the analysis to include CDS spreads as a robustness test. We find that adding CDS does not further improve the prediction of expected returns or variances.

The UMVE portfolio is shown to have several interesting properties. First, we find that optimally investing in international government bonds achieves a large improvement in the Sharpe ratio, from the average individual market's Sharpe ratio of approximately 0.46 to a value greater than 1. However, the UMVE exhibits substantial time-variation in the implied market price of risk. Specifically, it increases around recessions and crises, with the highest expected Sharpe ratio being observed in the fourth quarter of 2022, when concerns about inflation and tightening monetary policies raised investors' marginal utility of wealth. The paper documents that the expected Sharpe ratio of the UMVE portfolio is strongly positively related to the dispersion of inflation rates across developed sovereign bond markets. In addition, expected Sharpe ratios are negatively related to intermediaries' capital ratios and to an index of global economic policy uncertainty.

While bond returns exhibit a strong factor structure, with 86% of the variance being explained by the first 3 principle components, there is very little relation between common sources of variation in international bond market returns and priced risk. The first 10 principle components only explain around 22% of the UMVE portfolio return variation.

Consistent with this finding, portfolio strategies, such as equally weighting or various factor-based portfolio strategies, such as carry, value, and momentum strategies, all exhibit large amounts of unpriced risks. By constructing portfolios that hedge out such unpriced components from bond strategies, Sharpe ratios improve substantially. Our analysis also suggests that, for successful practical implementation, the estimation of expected returns is essential, while estimation of variances matters to a lesser degree.

To shed light on the feasibility of strategies that hedge out unpriced risks, we provide sensitivity analyses and impose restrictions on the absolute values of portfolio weights. We document that portfolio performance is remarkably robust to imposing such limits. Only when long-only restrictions are imposed, the portfolio performance drops. However, bond futures exist in almost all of the G10* markets, which should make shorting a particular market easy for professional market participants.

Recognizing that the existing finance literature provides little theoretical and empirical foundation on which international government bond portfolio strategies can be built, this paper provides a first step to fill this gap. However, given the growing size and economic importance of this asset class, this is likely to remain an active area of research. One obvious extension is to analyze pricing kernels for a broader universe of bond markets which also includes emerging markets, where default risks are likely to play a role.

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Table 1: Descriptive Statistics of Bond Returns

	Mean	Median	Std. Dev.	SR
Australia	3.906	3.501	9.007	0.434
Canada	4.364	3.240	7.498	0.582
Germany	4.105	6.814	6.861	0.598
Japan	3.646	2.796	5.103	0.715
New Zealand	2.791	4.571	8.515	0.328
Norway	3.226	3.135	7.352	0.439
Sweden	5.313	5.612	8.100	0.656
Switzerland	3.486	4.521	5.717	0.610
United Kingdom	3.401	3.158	8.136	0.418
United States	4.310	2.214	9.013	0.478

This table shows descriptive statistics of monthly 10-year government bond excess returns (in % p.a.) and annualized Sharpe Ratios (SR) by country of issuance. Statistics refer to the G10* countries. Bond returns are in monthly frequency. Data are from January 1995 to December 2022 and measured in USD.

Table 2: Stylized Evidence

	Mean	Median	Std. Dev.	SR
A. No diversification				
Individual countries' average	3.307	3.145	7.379	0.463
B. Simple strategies				
Naive ($1/N$)	3.307	2.309	6.077	0.544
GDP-weighted	3.505	2.862	6.637	0.528
C. Conditional strategies				
Carry	3.262	2.684	4.011	0.813
Value	2.170	1.291	3.913	0.555
Momentum	1.098	0.961	3.860	0.284

This table shows the mean return in % p.a., median return in % p.a., standard deviation in % p.a., and annualized Sharpe Ratio (SR) of individual countries' average as well as global bond portfolio strategies. The sample consists of Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Panel A shows results with no diversification. Panel B shows results of simple portfolio strategies. Panel C presents conditional strategies, where portfolios are constructed using rank-weights following Asness et al. (2013) and focusing on the following signals respectively: bond carry, defined as the difference between 10-year yield and the local 1-month yield; bond value, defined as the difference between the 10-year yield and inflation; and bond momentum, defined as the cumulative return over 12 months skipping the last month. Data are from December 1995 to December 2022.

Table 3: Estimated Coefficients from Predictive Regressions

Statistic	Mean	Median	Std. Dev.	Pctl(10)	Pctl(90)
$\hat{\gamma}_t$	1.489	1.568	0.269	1.115	1.800
t-stat $\hat{\gamma}_t$	2.655	2.643	0.692	1.702	3.546
$\hat{\delta}_t$	0.058	0.054	0.014	0.043	0.077
t-stat $\hat{\delta}_t$	1.355	1.318	0.243	1.106	1.640

This table reports descriptive statistics of the estimated coefficients from the pooled regressions, as specified in Equation (7). $\hat{\gamma}_t$, $\hat{\delta}_t$ are the estimated coefficients of the forward spread and value, respectively; t-stat is the t-statistic of these variables.

Table 4: Predictive Ability of Conditional Expectations, Variance and Covariance for Individual Countries

	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.903		
	(0.315)		
$\sigma^2_t(rx^i_{t+1})$		0.952	
		(0.086)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			0.999
			(0.115)
Observations	2,640	2,640	11,880
Adjusted R^2	0.010	0.057	0.055

This table shows estimates from regressions of realized returns, rx^i_{t+1}, on expected returns, $\mathbb{E}_t(rx^i_{t+1})$, column (1), of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$ on estimated variance, $\sigma^2_t(rx^i_{t+1})$, column (2), and of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariance, column (3). The estimation of the conditional means and the conditional covariances follows Sections 3.4.1 and 3.4.2, respectively. Standard errors are clustered at the month level. Under the null hypothesis that coefficients are different from 1, ***,**,* denote that estimates are statistically significant at the 1, 5, and 10 percent levels.

Table 5: Trading Strategies

Acronym	Name	Description
$1/N$	Naive	equally-weighted average of countries' bond returns
GDP-weighted	GDP-weighted	GDP-weighted average of countries' bond returns
Average Carry	Average Carry	long (short) in all countries when the average bond carry is positive (negative)
CS-Carry	Cross-Sectional Carry	rank weights (Asness et al., 2013) based on bond carry
TS-Carry	Time Series Carry	long (short) in countries with a positive (negative) bond carry
Average Value	Average Value	long (short) in all countries when the average bond value is positive (negative)
CS-Value	Cross-Sectional Value	rank weights based on bond value
TS-Value	Time Series Value	long (short) in countries with a positive (negative) bond value
Average Mom12	Average Momentum 12 Months	long (short) in all countries when the average long-term bond momentum is positive (negative)
CS-Mom12	Cross-Sectional Momentum 12 Months	rank weights based on long-term bond momentum
TS-Mom12	Time Series Momentum 12 Months	long (short) in countries with a positive (negative) long-term bond momentum
Average Mom1	Average Momentum 1 Month	long (short) in all countries when the average short-term bond momentum is positive (negative)
CS-Mom1	Cross-Sectional Momentum 1 Month	rank weights based on short-term bond momentum
TS-Mom1	Time Series Momentum 1 Month	long (short) in countries with a positive (negative) short-term bond momentum

Table 6: Testing the UMVE using Individual Country Bond Returns

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
Australia	0.263	2.200	1.238	1.043	0.558	0.010
Canada	0.463	3.209	2.179	1.028	0.714	0.075
Germany	0.502	3.480	2.363	1.654	1.282	0.049
Japan	0.704	2.351	3.316	0.758	1.099	0.158
New Zealand	0.285	2.293	1.341	0.975	0.558	0.019
Norway	0.383	2.746	1.805	1.573	1.041	0.017
Sweden	0.453	3.265	2.135	1.544	1.076	0.042
Switzerland	0.509	2.913	2.396	1.384	1.168	0.051
United Kingdom	0.313	2.533	1.474	0.948	0.577	0.026
United States	0.440	4.012	2.073	1.206	0.657	0.072

This table shows for individual countries the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing individual countries' bond returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). The t-statistics are heteroskedasticity robust. Data are from January 2001 to December 2022.

Table 7: Testing the UMVE using Portfolio Strategies

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
$1/N$	0.477	2.900	2.246	1.211	0.996	0.056
GDP-weighted	0.498	3.349	2.343	1.151	0.876	0.079
Average Carry	0.617	3.730	2.906	1.581	1.317	0.096
CS-Carry	0.786	3.038	3.701	1.309	1.644	0.148
TS-Carry	0.518	2.782	2.437	1.078	1.011	0.074
Average Value	0.617	3.726	2.903	1.424	1.105	0.109
CS-Value	0.517	1.716	2.435	0.804	1.159	0.058
TS-Value	0.600	3.029	2.825	0.800	0.763	0.149
Average Mom12	0.507	3.080	2.388	1.181	0.926	0.074
CS-Mom12	0.248	0.819	1.167	0.622	0.799	-0.001
TS-Mom12	0.403	6.310	1.898	2.473	0.708	0.044
Average Mom1	0.070	0.428	0.329	0.934	0.728	0.002
CS-Mom1	-0.344	-1.180	-1.618	-1.501	-1.945	0.004
TS-Mom1	-0.001	-0.005	-0.005	0.456	0.466	0.003

This table shows for global bond portfolio strategies, as defined in Table 5, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing portfolio returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). The t-statistics are heteroskedasticity robust. Data are from January 2001 to December 2022.

Table 8: Sharpe Ratio and GRS Test for the UMVE Portfolio Using Portfolio Strategies

Model	SR UMVE	GRS Tests – p-values			
		All strategies	All Carry	All Value	All Carry and Value
Optimal	1.135	0.402	0.205	0.392	0.256
Forward spread only	0.870	0.036	0.057	0.010	0.012
Value only	0.635	0.010	0.000	0.163	0.001

This table shows in column (1) the Sharpe ratio (SR) of the UMVE portfolio constructed using the full estimation model, i.e. based on the forward spread and value as predictor variables (top row), and of restricted UMVE portfolios constructed using the forward spread only (middle row) or using the value only (bottom row) as predictor variable. Column (2) shows the p-value of the GRS test if the alphas from regressing all global portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (3) shows the p-value of the GRS test if the alphas from regressing global carry portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (4) shows the p-value of the GRS test if the alphas from regressing global value portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (5) shows the p-value of the GRS test if the alphas from regressing global carry and value portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Data are from January 2001 until December 2022.

Table 9: Testing the UMVE Portfolio Using Alternative Countries' Bond Returns

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
Euro area						
Austria	0.528	3.793	2.487	2.032	1.487	0.041
Belgium	0.540	4.178	2.541	2.279	1.526	0.042
Finland	0.495	3.544	2.331	1.742	1.273	0.045
France	0.549	3.955	2.587	2.341	1.686	0.034
Greece*	0.275	10.513	1.296	13.221	1.531	0.000
Ireland	0.465	5.145	2.188	3.607	1.403	0.011
Italy	0.533	5.162	2.510	4.058	1.991	0.006
Netherlands	0.513	3.670	2.415	1.888	1.402	0.044
Portugal	0.436	5.758	2.054	4.817	1.642	-0.000
Slovakia*	0.431	3.516	1.769	2.702	1.275	0.004
Slovenia*	0.322	3.267	1.372	2.820	1.111	-0.003
Spain	0.548	4.905	2.581	3.382	1.732	0.018
Other countries						
Chile*	0.286	2.989	1.188	2.943	1.122	-0.005
Denmark	0.480	3.507	2.262	1.917	1.350	0.033
Colombia*	0.251	3.628	1.030	0.823	0.226	0.030
Czech Republic*	0.431	3.980	2.029	2.202	1.109	0.017
Hong Kong	0.449	4.427	2.113	2.016	0.963	0.038
Hungary*	0.231	3.536	1.080	2.337	0.693	0.001
Israel*	0.577	5.262	2.444	3.748	1.685	0.023
Mexico*	0.305	3.822	1.348	1.541	0.549	0.024
Poland	0.330	3.806	1.552	2.501	0.992	0.002
Singapore	0.461	3.456	2.173	1.030	0.645	0.067
Turkey*	0.348	12.138	1.468	5.287	0.642	0.035

This table shows for other countries, which we do not use for the estimation of the UMVE portfolio, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing other countries' bond returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 until December 2022. *For these countries data are available for a shorter time period. Specifically, data availability starts as follows: Chile: September 2005, Colombia: April 2006, Czech Republic: December 2000, Greece: December 2000, Hungary: March 2001, Israel: March 2005, Mexico: August 2003, Slovakia: April 2006, Slovenia: December 2004, Turkey: April 2005.

Table 10: Testing the UMVE Portfolio Using Alternative Countries' Portfolio Strategies

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
$1/N$	0.622	4.292	2.930	2.592	1.885	0.041
GDP-weighted	0.612	4.342	2.884	2.776	1.952	0.033
Average Carry	0.622	4.292	2.930	2.592	1.885	0.041
CS-Carry	0.652	4.701	3.068	4.941	2.822	-0.002
TS-Carry	0.616	4.077	2.899	2.505	1.853	0.038
Average Value	0.699	4.798	3.289	1.657	1.131	0.155
CS-Value	0.409	2.805	1.927	2.722	1.666	-0.004
TS-Value	0.821	4.548	3.865	2.095	1.801	0.145
Average Mom12	0.726	4.978	3.418	2.688	1.850	0.089
CS-Mom12	0.544	3.376	2.563	4.169	2.823	0.011
TS-Mom12	0.533	4.903	2.508	1.744	0.866	0.089
Average Mom1	0.327	2.279	1.538	2.456	1.669	-0.003
CS-Mom1	-0.088	-0.492	-0.415	-0.152	-0.116	-0.002
TS-Mom1	0.201	1.204	0.947	1.389	1.149	-0.003

This table shows for global bond portfolio strategies, as defined in Table 5, of other countries, which we do not use for the estimation of the UMVE portfolio, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing portfolio strategies of other countries' bond returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 until December 2022.

Table 11: Descriptive Statistics for the UMVE Portfolio

	UMVE
Mean (in %)	6.914
Median (in %)	6.429
Std. Dev. (in %)	6.092
Sharpe Ratio	1.135
Maximum Drawdown (in %)	8.674
$\mathbf{w}_{i,t}^{U,+}$ (in %)	45.544
$\mathbf{w}_{i,t}^{U,-}$ (in %)	-38.008
$\lvert\mathbf{w}_{i,t}^{U}\rvert$ (in %)	42.170
$\sum \lvert\mathbf{w}_{i,t}^{U} - \mathbf{w}_{i,t-1}^{U}\rvert$	1.711

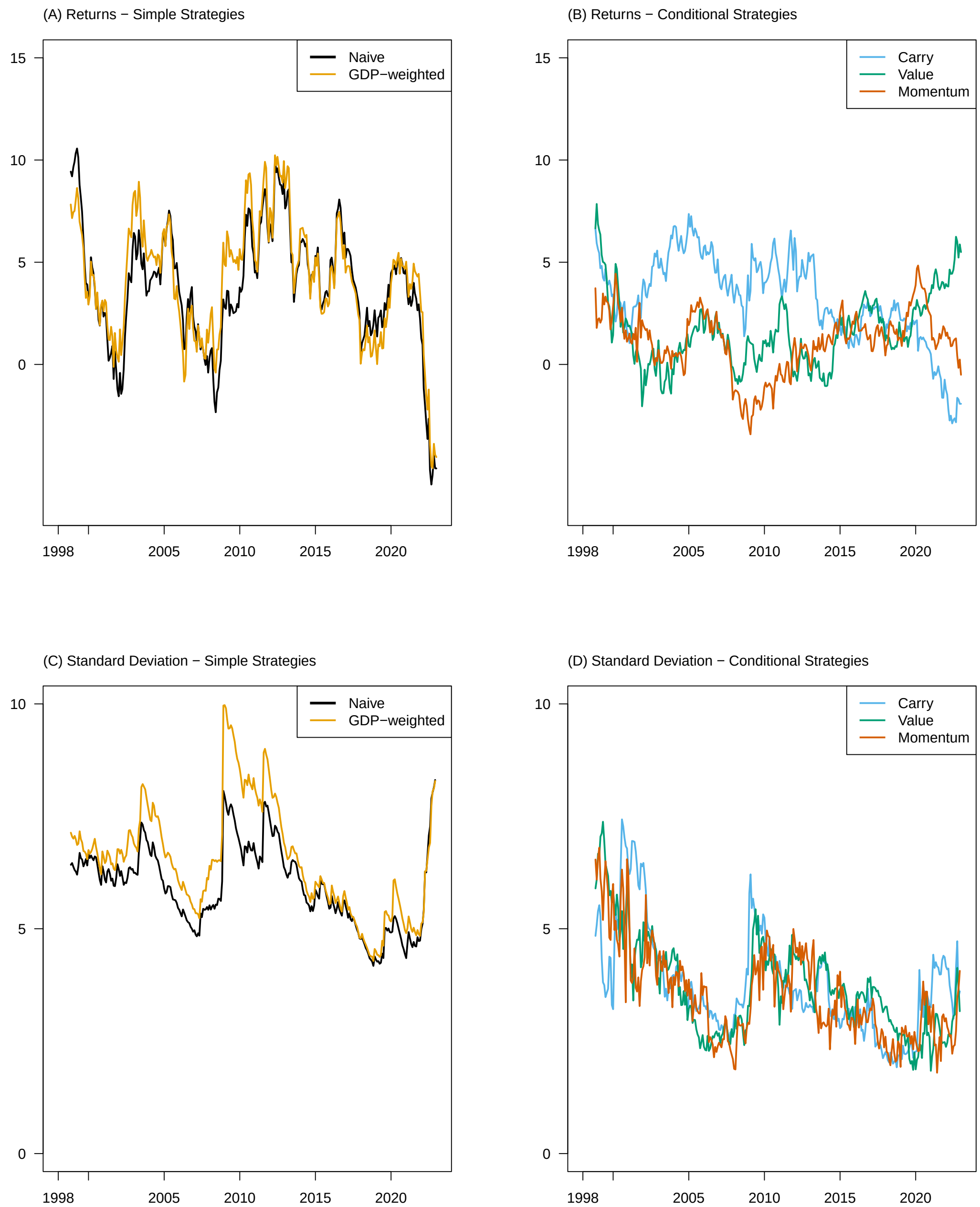
This table shows descriptive statistics for the UMVE portfolio. Returns are scaled to have the same volatility as the naive portfolio. Specifically, it shows the mean excess return (in % p.a.), median excess return (in % p.a.), standard deviation (in % p.a.), Sharpe ratio (annualized), and maximum drawdown (in %). The second set of rows includes statistics of the UMVE portfolio weights averaged across time and markets. It reports the average absolute portfolio weight (in %), the average of positive weights (in %), the average of negative weights (in %), and the average portfolio turnover. Data are from January 2001 to December 2022.

Table 12: Time Series Expected UMVE Sharpe Ratio and Macro and Market Variables

	UMVE Expected Sharpe Ratio		
	(1)	(2)	(3)
GDP SD	0.005	0.009	0.043
	(0.023)	(0.024)	(0.031)
Unempl SD	−0.001	−0.007	−0.004
	(0.005)	(0.006)	(0.006)
Infla SD	0.651***	0.603***	0.624***
	(0.094)	(0.105)	(0.103)
MSCIRet		0.722**	0.682*
		(0.316)	(0.360)
VIX Index		0.009**	0.008*
		(0.004)	(0.005)
CapitalRatio			−2.692*
			(1.626)
GlobEPU			−0.001**
			(0.0005)
Constant	0.554*	0.448	0.323
	(0.335)	(0.344)	(0.492)
Observations	264	264	264
Adjusted R^2	0.519	0.537	0.567

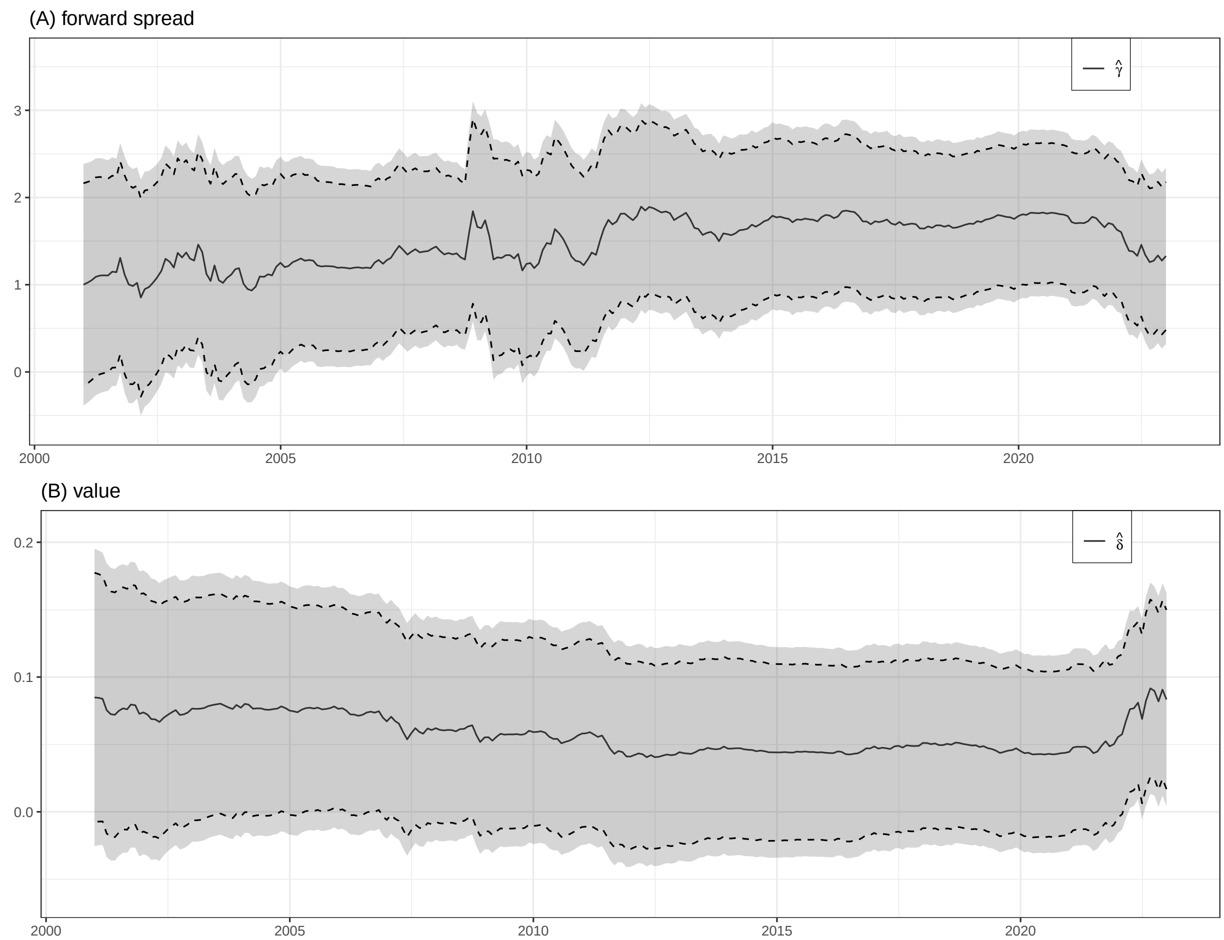
This table shows estimates from regressions of the UMVE expected Sharpe ratio on macro and market variables. *GDP SD*, *Unempl SD*, and *Infla SD* are the cross-sectional standard deviations of countries' GDP, Unemployment, Inflation, respectively. *MSCIRet* is the return on the MSCI World Index. *VIX Index* is an index of the implied volatility of the S&P 500 and is calculated from S&P 500 index options. *CapitalRatio* is the intermediary capital ratio as defined by He et al. (2017) and *GlobEPU* is the Global Economic Policy Uncertainty Index based on Baker et al. (2016). Newey-West standard errors with 6 lags are reported in parenthesis (Newey and West, 1994). Data are from January 2001 to December 2022. ***,**,* denote that estimates are statistically significant at the 1, 5 and 10 percent levels.

Figure 1: Time Variation in Returns and Standard Deviations



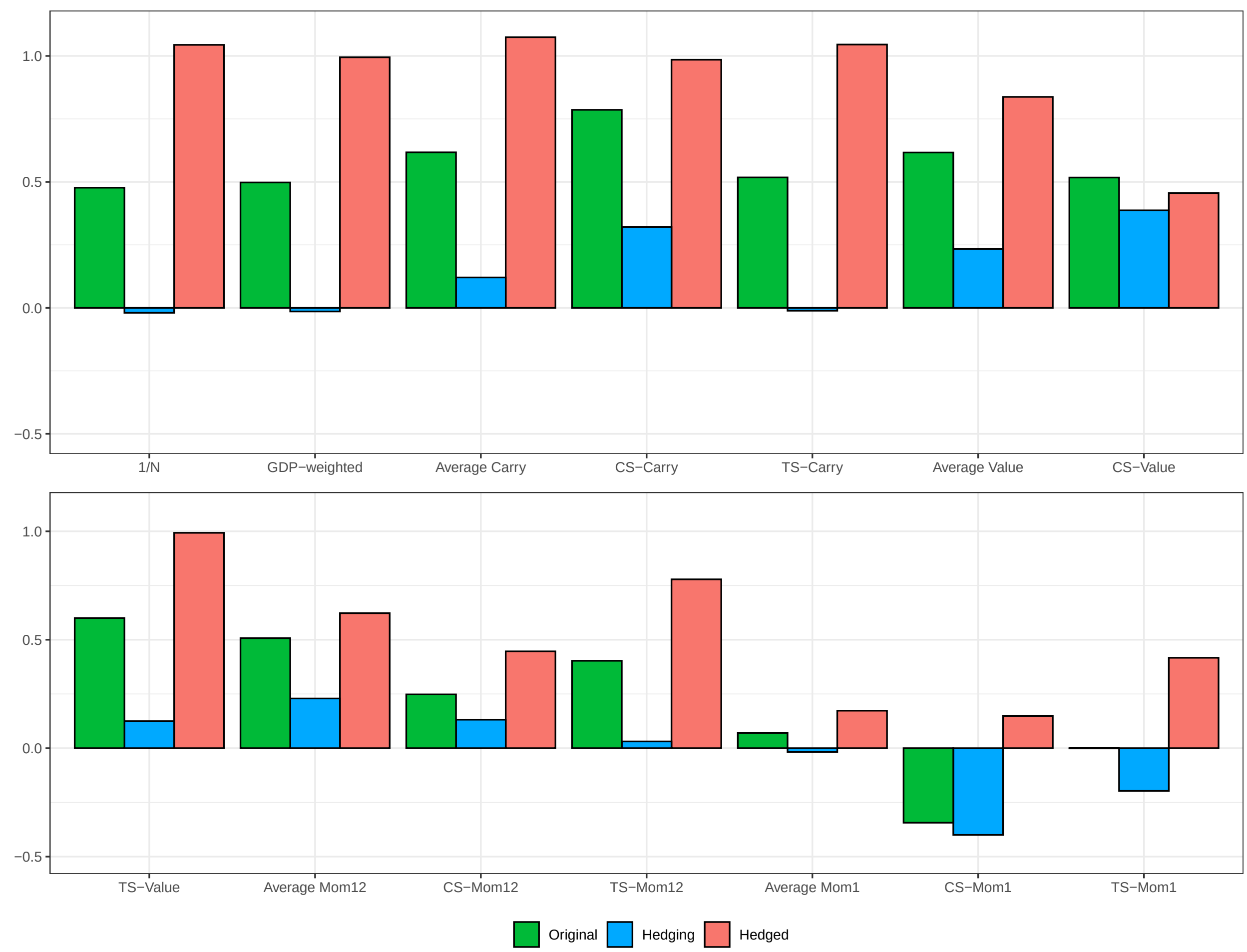
Panel A shows annualized moving average returns over 36 months for the naive ($1/N$) and the GDP-weighted portfolios. Panel B shows moving average returns over 36 months for the rank-weighted conditional portfolios (Asness et al., 2013). These are constructed using bond carry, value, and momentum as signals. Bond carry is defined as the difference between 10-year yield and the local 1-month yield. Bond value is defined as the difference between the 10-year yield and inflation. Bond momentum is defined as the cumulative return over 12 months skipping the last month. Panels C and D plot standard deviations for the simple and conditional strategies, respectively. Details on the covariance matrix estimation, based on Ledoit and Wolf (2022), which is used for the standard deviations, are reported in Section 3.4.2. Bond data belong to the following countries: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States. Data are from October 1998 to December 2022.

Figure 2: Time Series of Estimated Coefficients from Predictive Regression



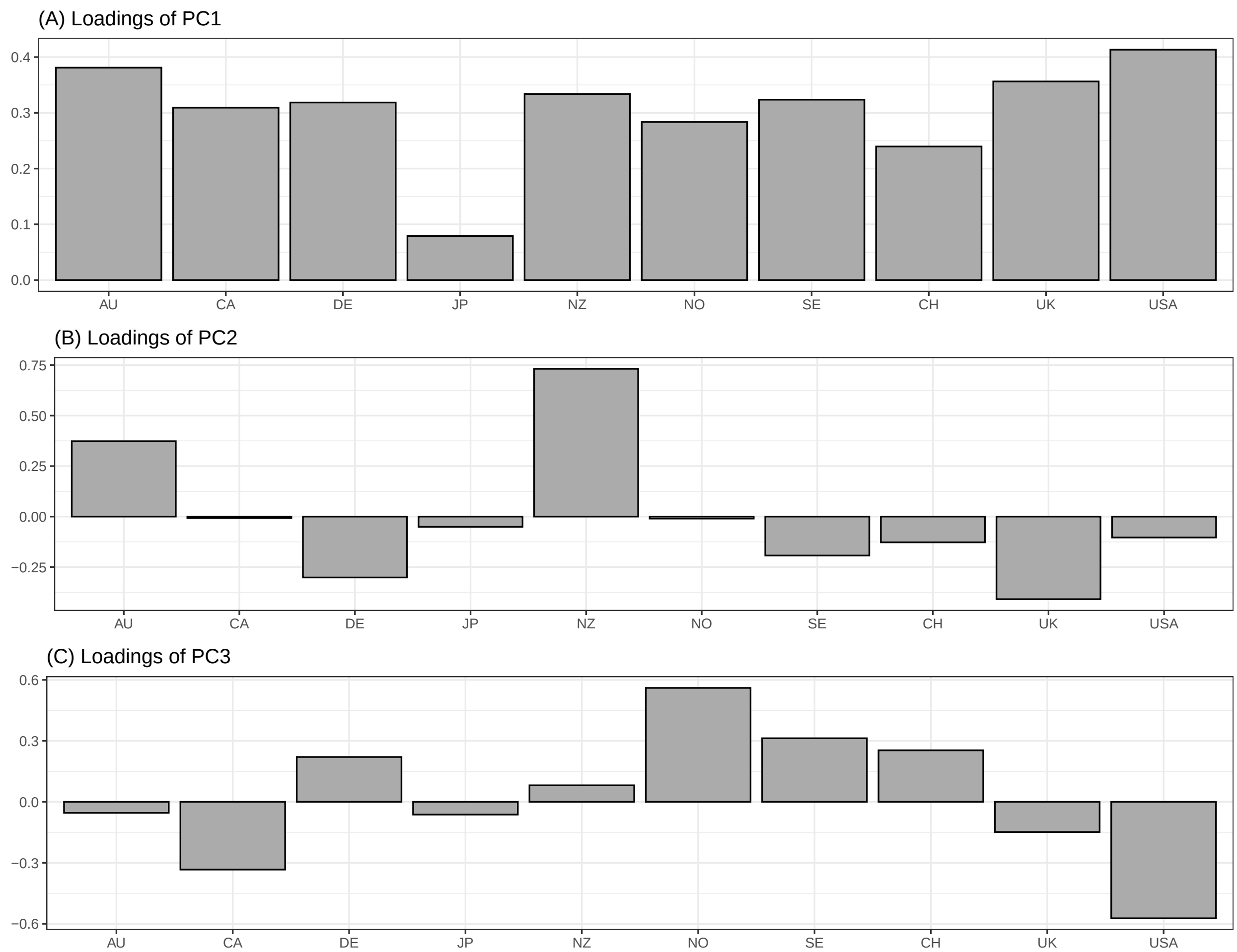
This figure shows the time series of the estimated coefficients from pooled regressions of the forward spread ($\hat{\gamma}$) and bond value ($\hat{\delta}$) on bond excess returns using expanding rolling windows. Panel A displays the estimated coefficient of the forward spread variable and Panel B of the bond value variable. The dotted lines and the shaded areas show the 90% and 95% confidence intervals, respectively. Standard errors are clustered at the country level.

Figure 3: Sharpe Ratio of Original, Hedging, and Hedged Strategies



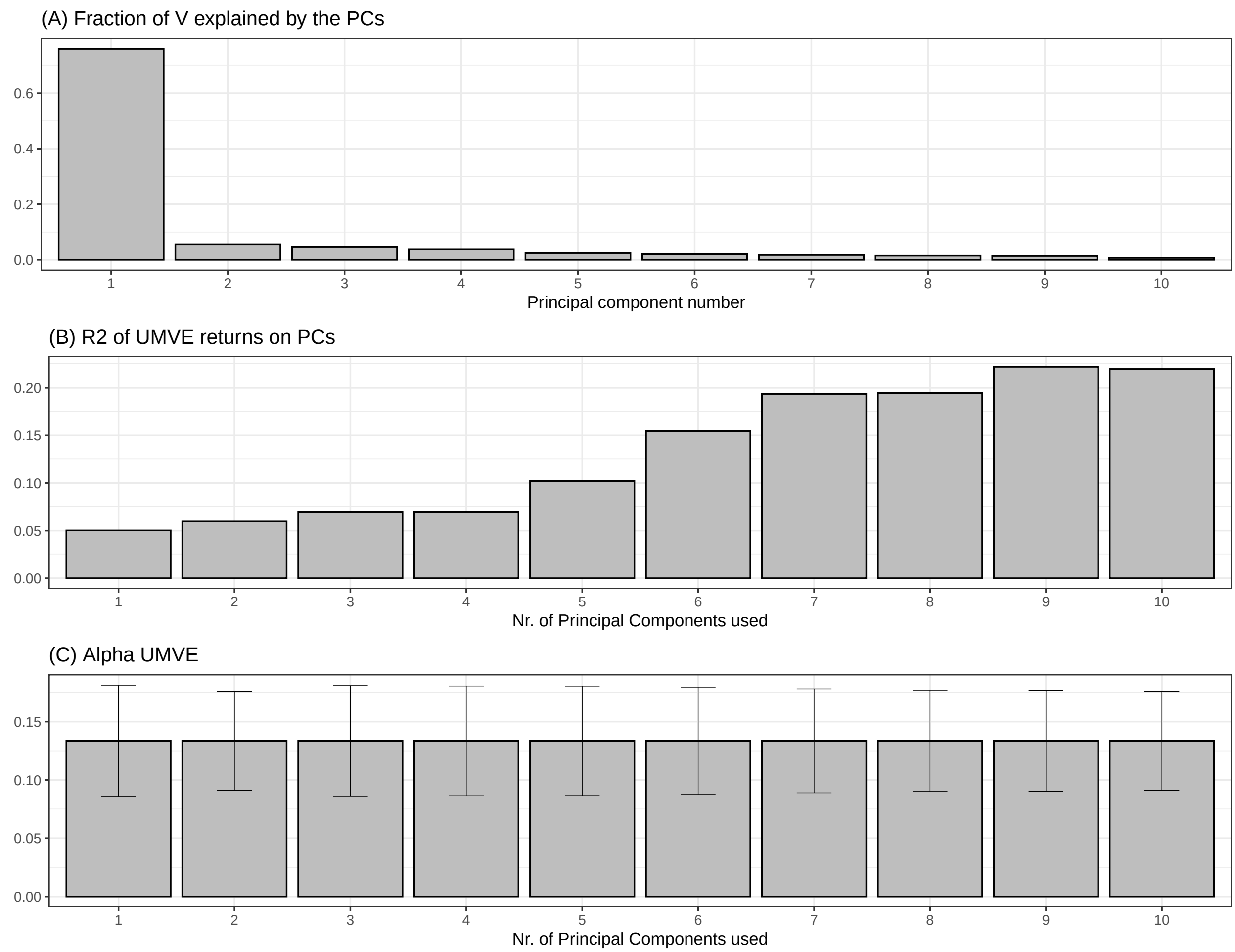
This figure shows the annualized Sharpe ratio of the original, hedging and hedged portfolio strategies. The original portfolio strategies are defined in Table 5. The hedged portfolio is the portfolio with systematic factor exposure, as defined in Section 4.4, while the hedging portfolio captures the unpriced source of common variation in bond returns. Data are from January 2001 to December 2022.

Figure 4: Principal Component Analysis of Returns: Loadings



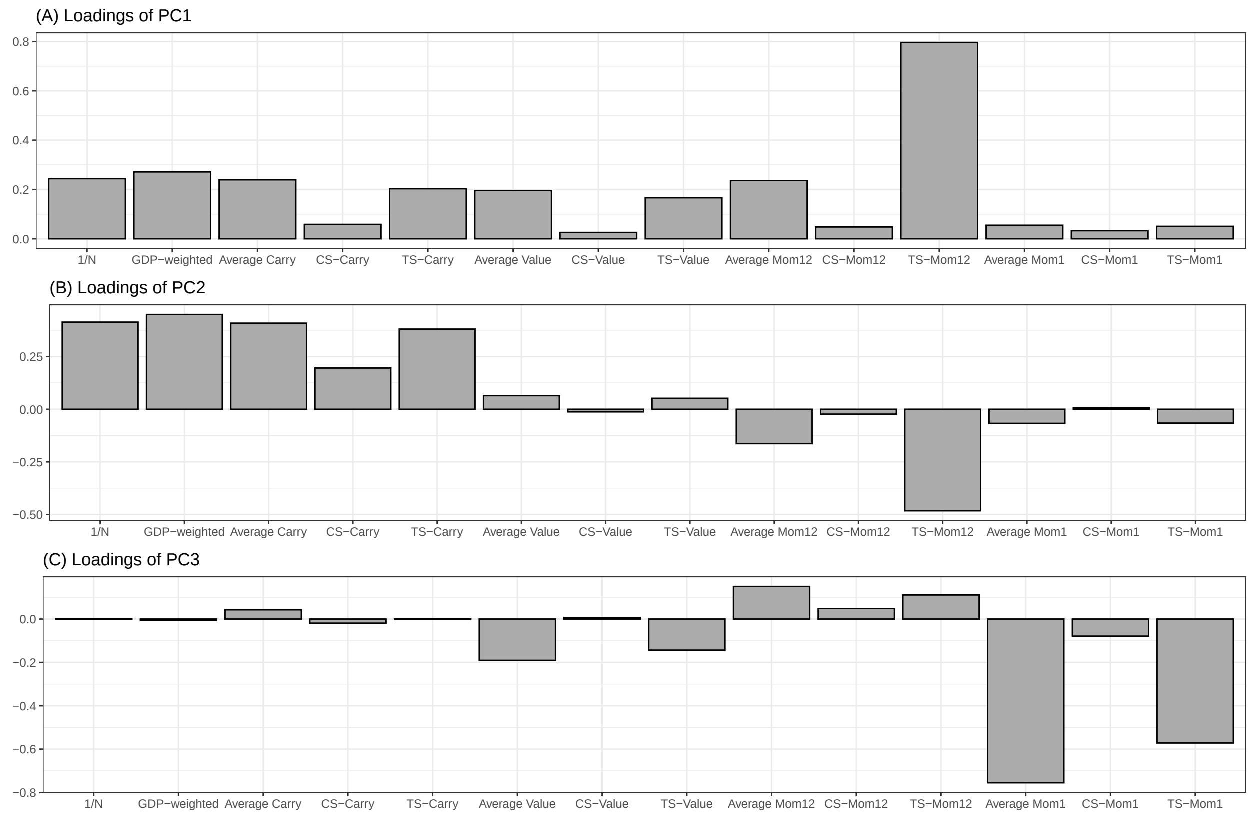
This figure shows loadings from Principal Component Analysis of bond returns. Bond returns refer to the G10* countries. Panel A reports loadings for PC1, Panel B for PC2, and Panel C for PC3. Data are from January 2001 to December 2022.

Figure 5: Principal Component Analysis of Bond Returns and UMVE Returns



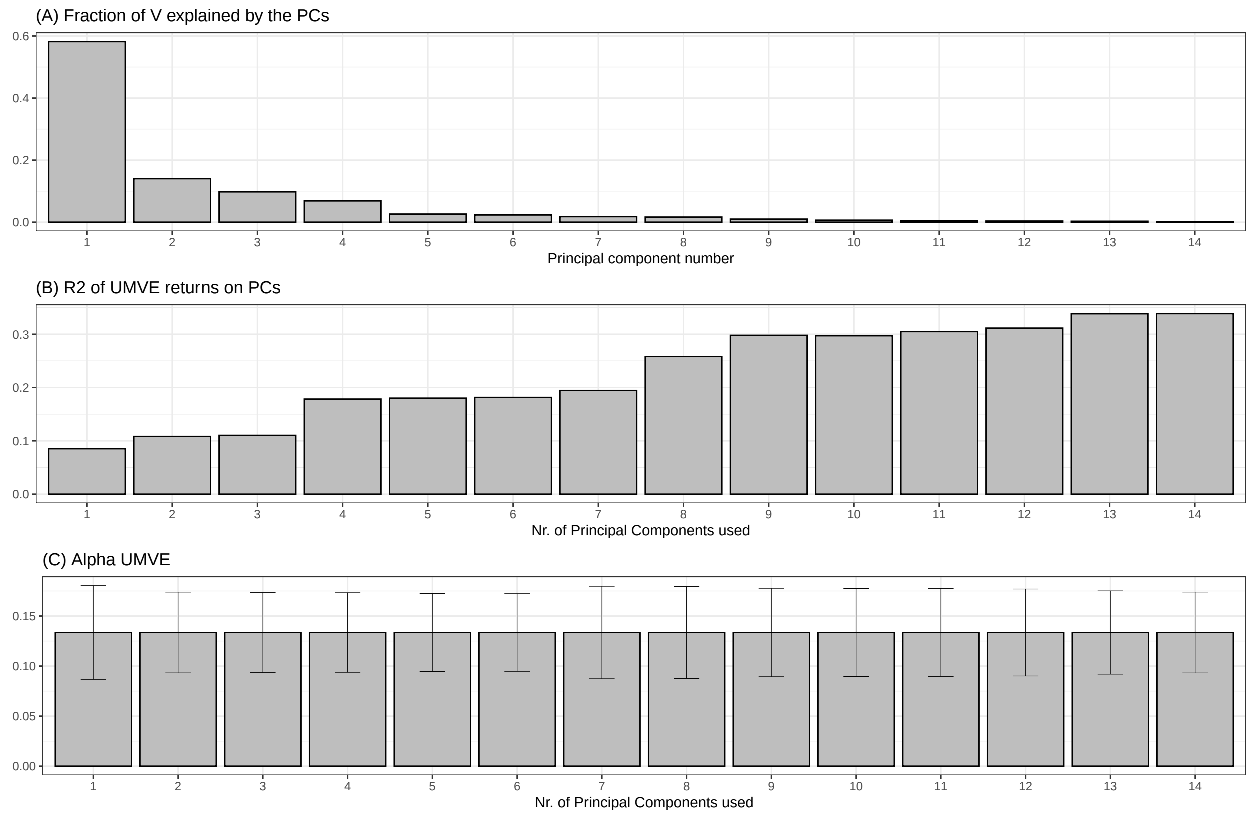
This figure shows in Panel A the fraction of variance explained by the PCs of bond returns. Bond returns refer to the G10* countries. Panel B shows the adjusted R^2 from regressing the UMVE returns on the PCs of bond returns. Panel C shows the alpha from these regressions, the black lines are the 95% confidence intervals. Data are from January 2001 to December 2022.

Figure 6: Principal Component Analysis of Strategies: Loadings



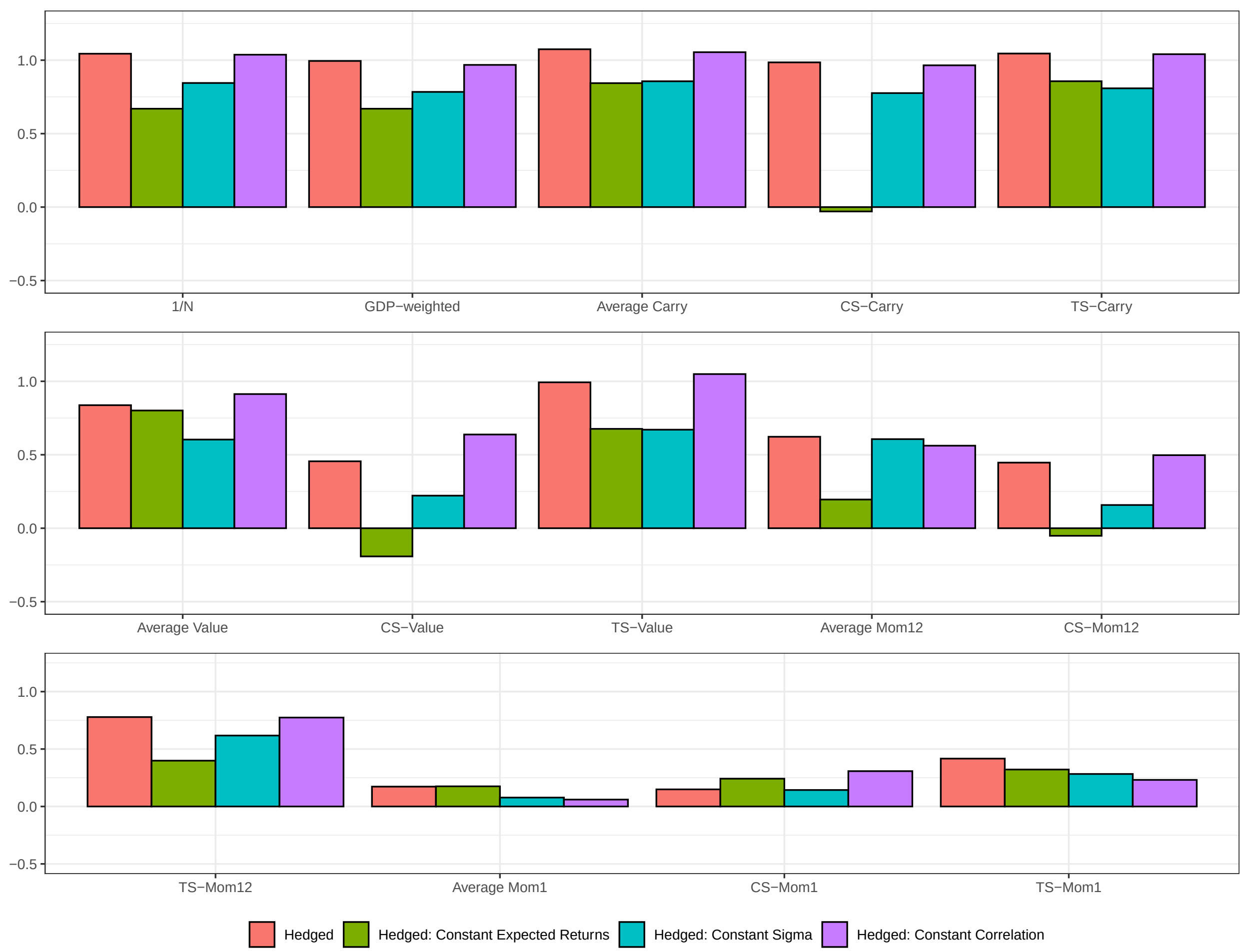
This figure shows loadings from Principal Component Analysis of global bond strategy returns. The portfolio strategies are defined in Table 5. Panel A reports loadings for PC1, Panel B for PC2, and Panel C for PC3. Data are from January 2001 to December 2022.

Figure 7: Principal Component Analysis of Strategies and UMVE Returns



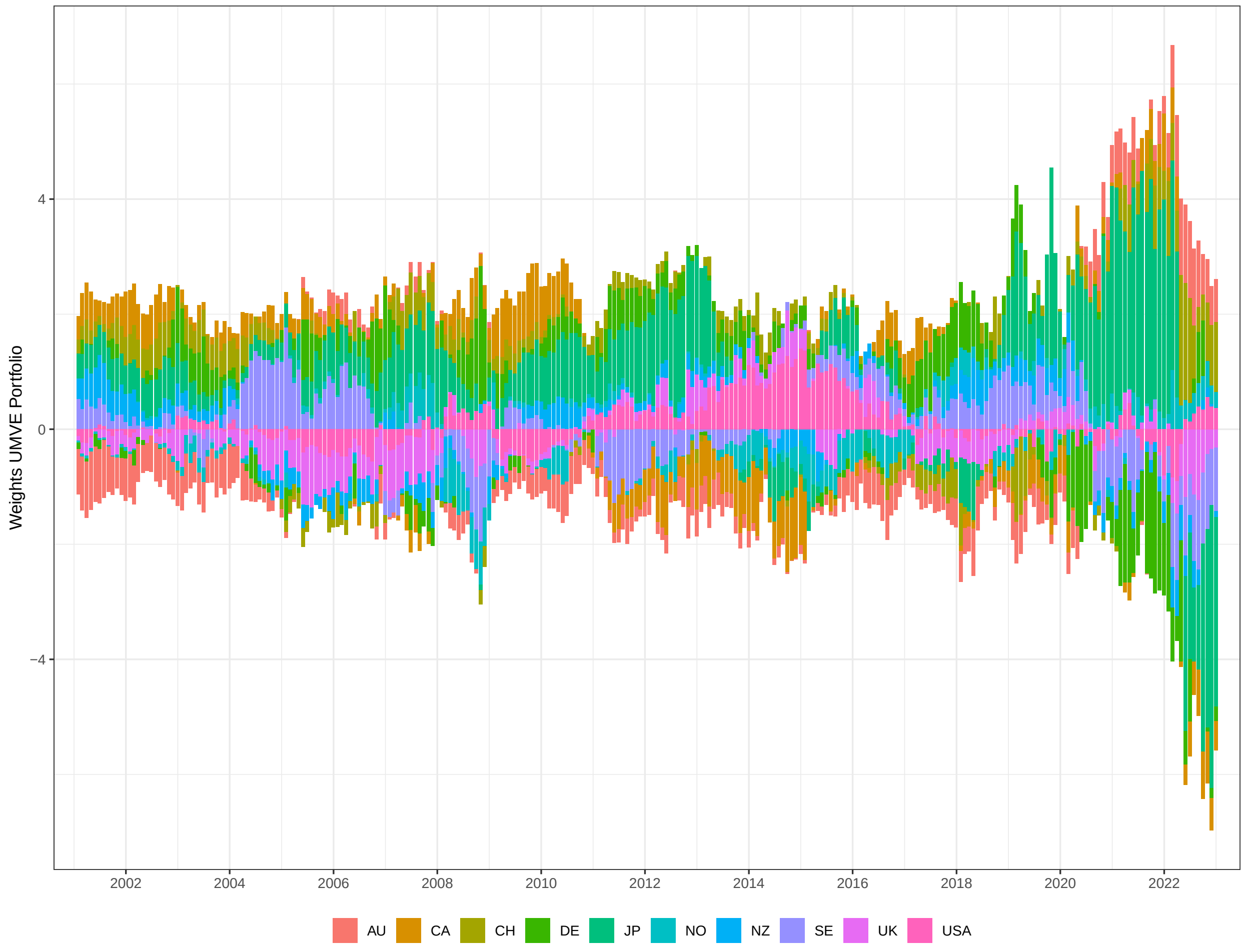
This figure shows in Panel A the fraction of variance explained by the PCs of global bond strategy returns. The portfolio strategies are defined in Table 5. Panel B shows the adjusted R^2 from regressing the UMVE returns on the PCs. Panel C shows the alpha from these regressions, the black lines are the 95% confidence intervals. Data are from January 2001 to December 2022.

Figure 8: Sharpe Ratio Hedged Strategies Decomposition



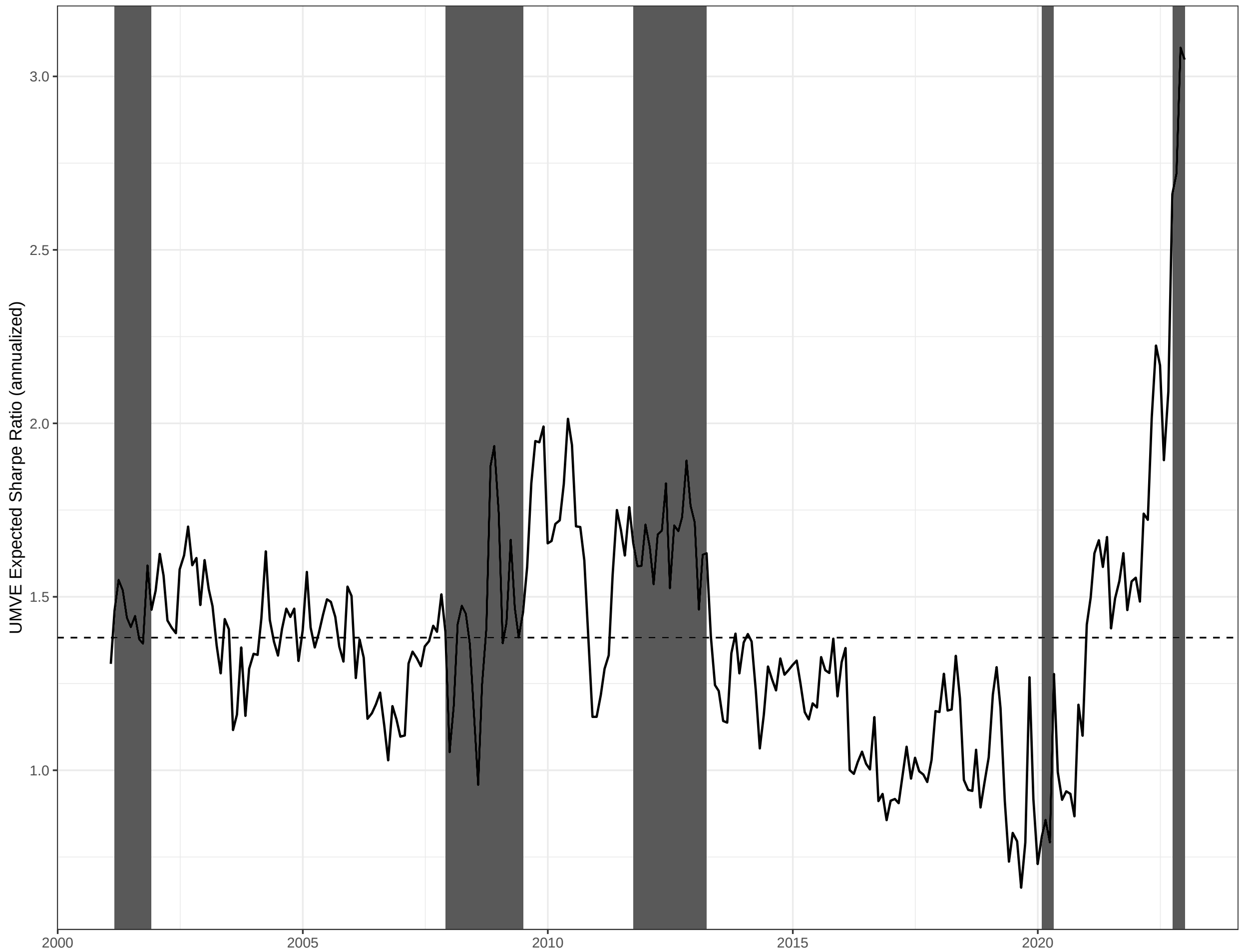
This figure shows, for each strategy, the annualized Sharpe ratio of the hedged portfolio (first bar), as defined in Section 4.4, and three alternative hedged portfolios constructed from restricted UMVE versions. We construct the restricted UMVE versions by successively shutting off one element in the estimation (i.e., expected returns, variances, correlations) and letting the others be estimated from the full model (as described in Sections 3.4.2 and 3.4.1). Specifically, the restricted UMVE portfolios shut off any cross-sectional variation in conditional expected returns, or in conditional variances, or in conditional correlations (as described in Section 4.5). The hedged portfolios resulting from these restricted UMVE portfolios correspond to the hedged portfolio with constant expected returns (*Hedged: Constant Expected Returns*), the hedged portfolio with constant sigma (*Hedged: Constant Sigma*) and the hedged portfolio with constant correlation (*Hedged: Constant Correlation*). Data are from January 2001 to December 2022.

Figure 9: Times Series of Portfolio Weights



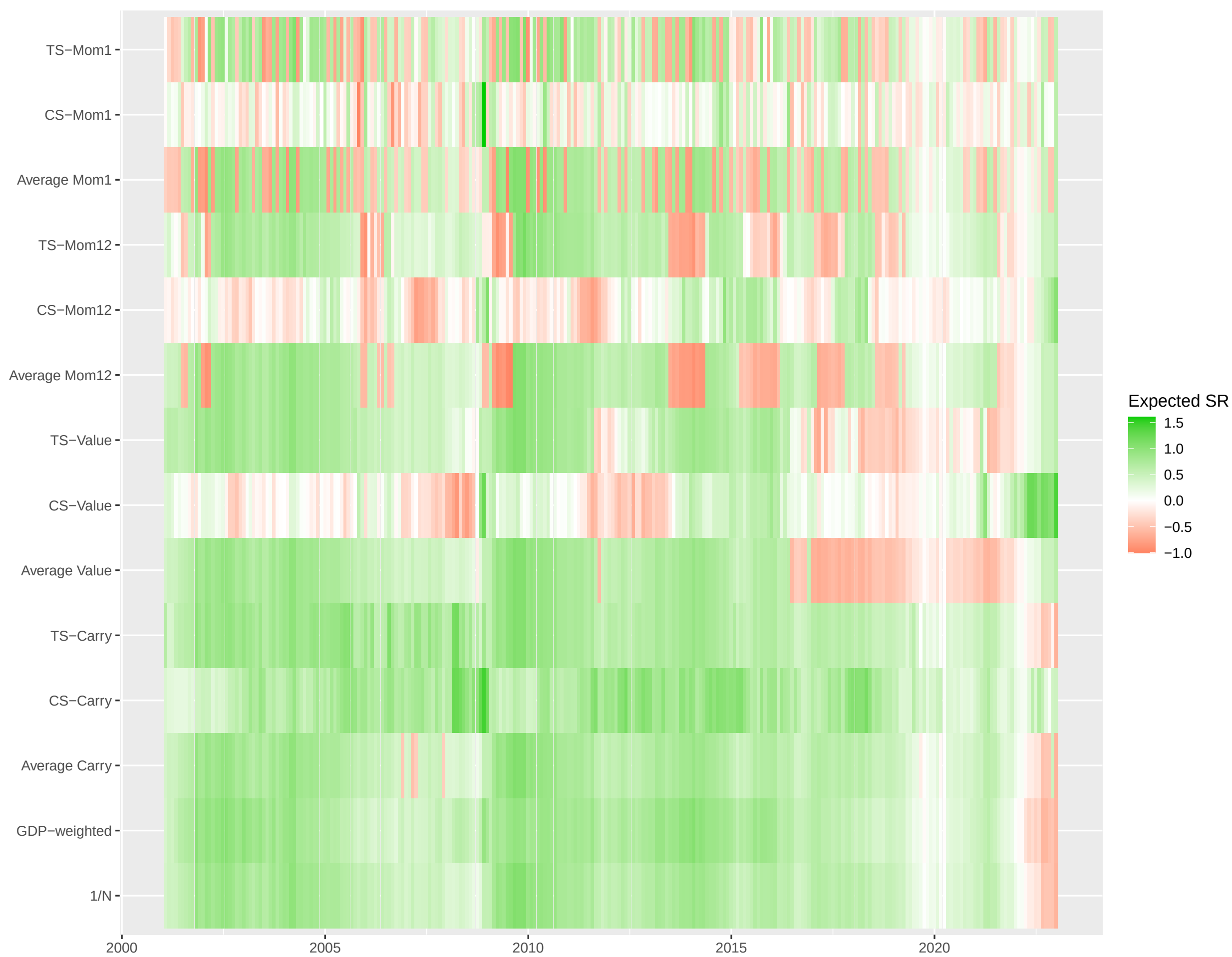
This figure shows the time series of weights for the UMVE portfolio. The UMVE portfolio returns are scaled to have the same volatility as that of the naive portfolio. Data are from January 2001 to December 2022.

Figure 10: Properties of the UMVE Portfolio



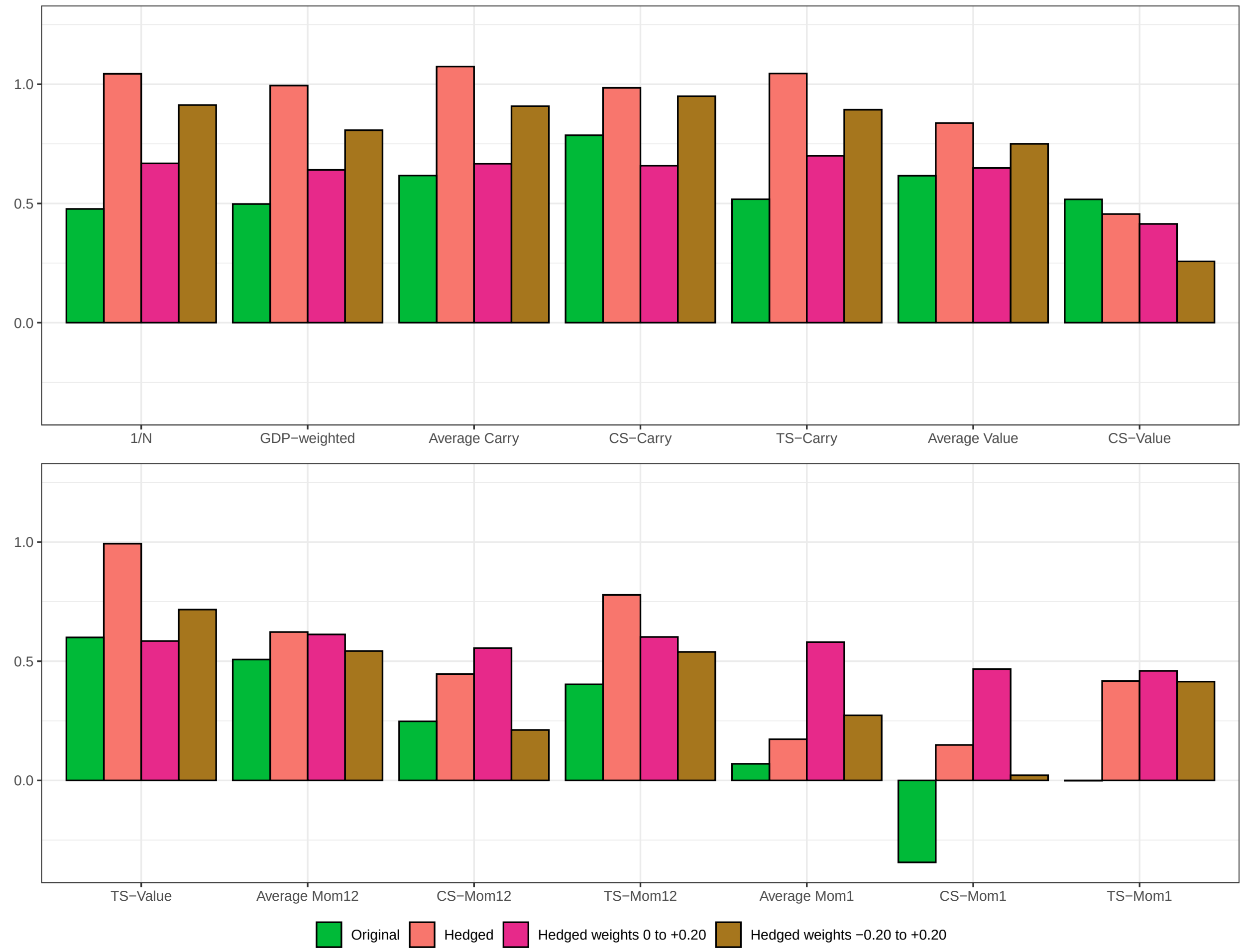
The figure shows the time series of the expected Sharpe ratio of the UMVE portfolio (annualized in %). The shaded areas correspond to recession periods, as defined by the NBER for the USA and the Eurostat Business Cycle Clock for the Euro area. The dotted-dash line indicates its mean value. Data are from January 2001 to December 2022.

Figure 11: Portfolio Strategies Heatmap



This figure shows a heatmap of the portfolio strategies' expected Sharpe ratio over time. The estimation of the conditional means and the conditional covariances follows Sections 3.4.1 and 3.4.2. Detailed description of the trading strategies is available in Table 5. Data are from January 2001 to December 2022.

Figure 12: Portfolio Strategies with Weights Cutoffs



This figure shows the annualized Sharpe ratio of the original, hedged portfolio strategies, and hedged portfolio strategies with weight restrictions. Hedged weights 0 to +0.20 means that we force the weights of the hedged portfolio to be positive (or zero) and the maximum country's weight cannot excess 20% of the total portfolio. Hedged weights -0.20 to +0.20 means that we force the weights of the hedged portfolio to be in the range between -20% and +20%.

Appendix for
"Pricing and Constructing International Government Bond Portfolios"

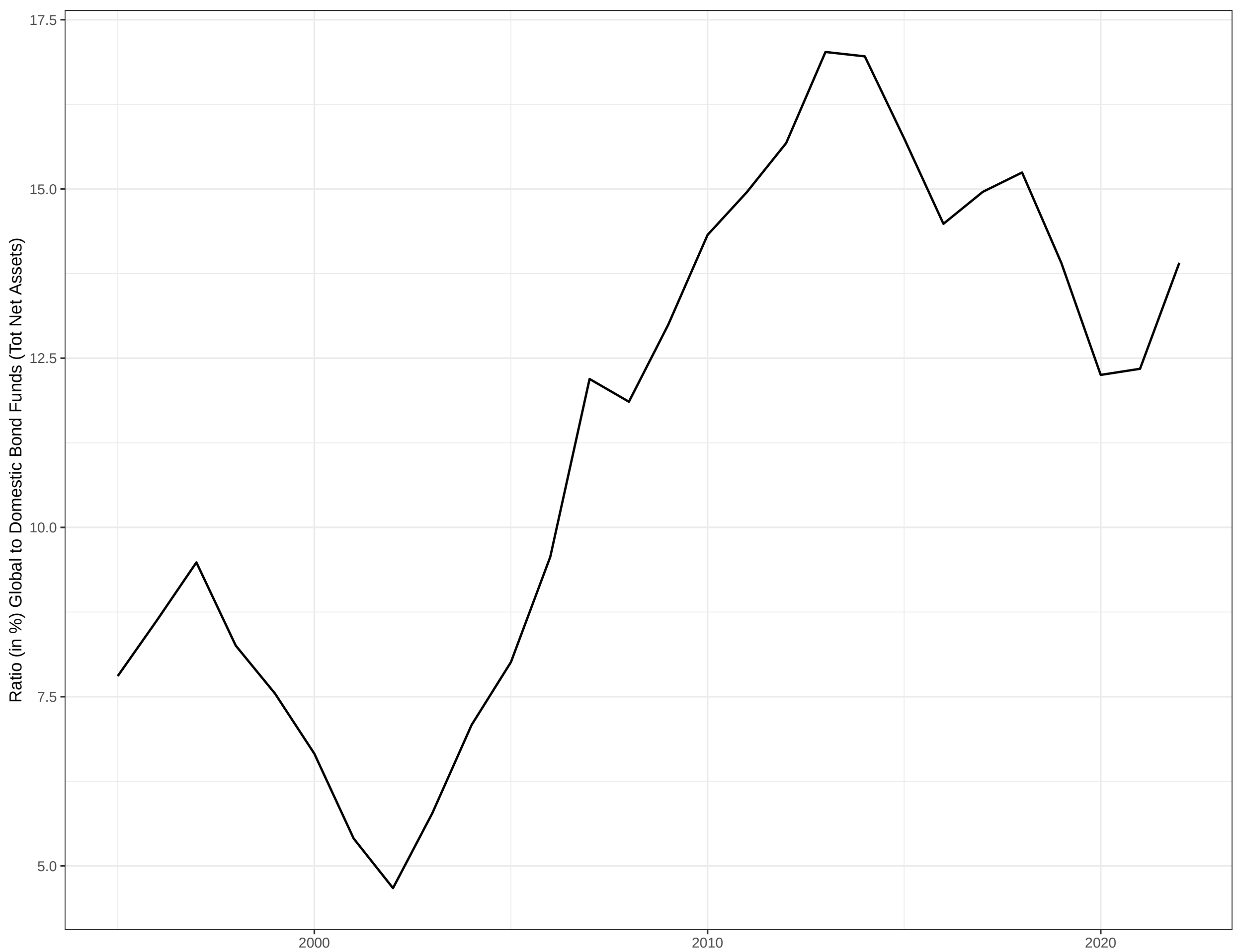
Otto Randl, Giorgia Simion, and Josef Zechner

The Appendix contains:

- a figure showing the evolution of the ratio between total net assets of global bond funds and domestic bond funds over time (Section A);
- derivation of the relationship between bond returns that are currency-hedged with forward rates and with a long and short position in the same foreign market (Section B);
- descriptive statistics of the macro and market variables (Section C);
- additional robustness checks and validation tests of the UMVE portfolio (Section D);
- evolution over time of the hedged portfolio weights (Section E);
- description of the Sharpe ratio decomposition when eliminating cross-sectional variation in expected returns, variances, and correlations (Section F);
- description of the Sharpe ratio decomposition when eliminating time-series variation in expected returns, and conditional variances (Section G).

A Global versus Domestic Bond Funds

Figure A1: Global versus Domestic Bond Funds



The figure shows the ratio (in %) between the total net assets of global bond funds and the total net assets of domestic bond funds over time. Global bond funds are fixed-income funds that satisfy one of the following criteria: i) the Morningstar Category includes "Global", ii) the Prospectus Objective indicates a global investment focus, iii) the Investment Area is "Global". Domestic bond funds are fixed-income funds whose Investment Area is "United States of America". Data are yearly from 1995 to 2022. Data source is Morningstar.

B Bond Returns from Currency-Hedged Positions

Table B1: Bond Returns with Forwards

Transaction		Cash Flow in USD	
		t	$t+1M$
1	Buy $\frac{1}{P_{i,t}^{(10Y)} S_{i,t}}$ 10-year foreign bond	-1	$\frac{P_{i,t+1M}^{(10Y-1M)} S_{i,t+1M}}{P_{i,t}^{(10Y)} S_{i,t}}$
2	Sell $\frac{1}{P_{i,t}^{(USD,1M)}}$ 1-month US T-bill	+1	$-\frac{1}{P_{i,t}^{(USD,1M)}}$
3	Sell forward $\frac{1}{S_{i,t} \times P_{i,t}^{(1M)}}$ units of FC	0	$\frac{F_{i,t} - S_{i,t+1M}}{S_{i,t} P_{i,t}^{(1M)}}$
	Net cash flow	0	$\frac{P_{i,t+1M}^{(10Y-1M)} S_{i,t+1M}}{P_{i,t}^{(10Y)} S_{i,t}} - \frac{1}{P_{i,t}^{(USD,1M)}} + \frac{F_{i,t} - S_{i,t+1M}}{S_{i,t} P_{i,t}^{(1M)}}$

In Table B1, the long position (1) in the foreign-currency 10-year bond leads to a cash outflow of USD 1, which is financed by a short position (2) in USD-denominated T-bills. Currency risk is hedged by selling forward foreign currency (3); the size of the forward contract is the expected value of the bond position at $t+1M$ under the expectation hypothesis (the original foreign-currency amount increased by the foreign risk-free rate). As the combined cash flows of (1), (2), and (3) sum up to 0 at time t, the total cash flow at $t+1M$ can be interpreted as the excess return of the currency-hedged position, which we denote as $rx_{i,t \to t+1M}^{(10Y,FWD-HGD)}$.

In Equation (B1), we apply Covered Interest Rate Parity (CIP) to show that the excess return $rx_{i,t \to t+1M}^{(10Y,FWD-HGD)}$ exactly matches the excess return from Equation (6).

$$
\begin{aligned}
rx_{i,t\to t+1M}^{(10Y,FWD-HGD)} &= \frac{P_{i,t+1M}^{(10Y-1M)} S_{i,t+1M}}{P_{i,t}^{(10Y)} S_{i,t}} - \frac{1}{P_{i,t}^{(USD,1M)}} + \frac{F_{i,t} - S_{i,t+1M}}{S_{i,t} P_{i,t}^{(1M)}} \\
&= \frac{P_{i,t+1M}^{(10Y-1M)} S_{i,t+1M}}{P_{i,t}^{(10Y)} S_{i,t}} - \frac{1}{P_{i,t}^{(USD,1M)}} + \frac{S_{i,t} \frac{P_{i,t}^{(1M)}}{P_{i,t}^{(USD,1M)}} - S_{i,t+1M}}{S_{i,t} P_{i,t}^{(1M)}} \\
&= \left(\frac{P_{i,t+1M}^{(10Y-1M)}}{P_{i,t}^{(10Y)}} - \frac{1}{P_{i,t}^{(1M)}} \right) \frac{S_{i,t+1M}}{S_{i,t}} \\
&= rx_{t+1}^{i}
\end{aligned} \tag{B1}
$$

Note that rx_{t+1}^{i} can be decomposed into $rx_{i,t+1}^{(10Y,LC)}\left(1 + r_{i,t+1}^{FX}\right) = rx_{i,t+1}^{(10Y,LC)} + rx_{i,t+1}^{(10Y,LC)} r_{i,t+1}^{FX}$, where $rx_{i,t+1}^{(10Y,LC)}$ is the local-currency excess return of the 10-year bond and $r_{i,t+1}^{FX}$ is the percentage change in the exchange rate. The decomposition highlights that the remaining influence of currency movements is small: the product of the two terms, $rx_{i,t+1}^{(10Y,LC)}$ and $r_{i,t+1}^{FX}$, is typically close to zero.[1] Table B2 shows empirically that this is indeed the case in our sample, as currency-hedged returns can not be explained by changes in the exchange rate.

[1]The remaining quantity risk arises from the fact that at time t the price of the bond at $t + 1M$ is not known. As pointed out by Driessen, Melenberg, and Nijman (2003) and shown by our own analysis, this quantity risk is negligible for currency-hedged excess returns.

Table B2: Adjusted R^2 from Regressing Bond Returns on one-month Changes in the Exchange Rate

	adj.R^2 $(rx_{t+1,Tbill})$	adj.R^2 (rx_{t+1})
Australia	0.602	0.021
Canada	0.494	0.022
Germany	0.615	0.009
Japan	0.829	-0.003
New Zealand	0.665	0.000
Norway	0.672	0.007
Sweden	0.619	0.020
Switzerland	0.760	-0.001
United Kingdom	0.634	0.010
Mean	0.638	0.020

This table shows the adjusted R^2 from regressing countries' excess bond returns on one-month change in exchange rate. Bond returns, rx_{t+1}, are defined as in Equation (6). Bond returns, $rx_{t+1,Tbill}$, are computed borrowing in the local currency (USD) and investing in the foreign long-term bond, i.e. $\left(P_{i,t+1M}^{(10Y-1M)}/P_{i,t}^{(10Y)} \times S_{i,t+1M}/S_{i,t}\right) - \left(1/P_{i,t}^{(USD,1M)}\right)$.

C Additional Descriptive Statistics

Table C1: Descriptive Statistics of the Forward Spread and Value Characteristics

	Mean	Median	Std. Dev.	Pctl(10)	Pctl(90)
A. Forward spread					
Australia	1.262	1.280	0.274	-0.013	2.531
Canada	1.947	1.911	0.377	0.232	3.652
Germany	2.147	2.162	0.309	0.644	3.611
Japan	1.994	2.009	0.308	0.663	3.292
New Zealand	0.935	1.137	0.471	-1.433	2.795
Norway	1.264	1.366	0.394	-0.623	2.983
Sweden	1.998	1.985	0.299	0.714	3.446
Switzerland	1.900	1.753	0.259	0.729	3.128
United Kingdom	1.343	1.390	0.573	-1.279	4.050
United States	2.445	1.988	0.523	0.304	5.079
B. Value					
Australia	2.193	2.056	2.211	-0.259	5.149
Canada	1.733	1.711	2.335	-0.544	4.610
Germany	1.290	1.702	2.574	-1.714	4.226
Japan	0.936	1.187	1.543	-0.859	2.714
New Zealand	2.785	3.060	2.348	-0.400	5.615
Norway	1.587	1.613	2.364	-1.518	4.739
Sweden	1.919	2.114	3.185	-1.436	5.583
Switzerland	1.243	1.273	1.456	-0.897	3.030
United Kingdom	1.503	1.833	2.788	-1.372	4.707
United States	1.321	1.514	2.199	-0.842	3.944

This table shows descriptive statistics of bond forward spread and bond value characteristics (in % p.a.). Forward spread is defined in Section 3.4.1 as $P_{i,t}^{(10Y-1M)}/P_{i,t}^{(10Y)} - 1/P_{i,t}^{(1M)}$, while bond value is the real bond yield, i.e. $Y_{i,t}^{(10Y)} - \pi_{i,t}$. Data are from January 1995 to December 2022.

Table C2: Descriptive Statistics of Macro and Market Variables

Statistic	N	Mean	Std. Dev.	Pctl(10)	Pctl(90)
GDP SD	264	11.965	1.399	9.986	13.981
Unempl SD	264	11.411	6.195	5.924	18.324
Infla SD	264	1.187	0.383	0.747	1.638
MSCIRet	264	0.006	0.046	-0.056	0.057
VIX Index	264	20.158	8.265	12.092	30.612
CapitalRatio	264	0.065	0.018	0.044	0.087
GlobEPU	264	147.377	76.413	68.987	267.881

This table shows descriptive statistics in monthly frequency of macro and market variables. *GDP SD*, *Unempl SD*, and *Infla SD* are the cross-sectional standard deviations of countries' GDP, Unemployment, Inflation, respectively. *MSCIRet* is the return on the MSCI World Index. *VIX Index* is an index of the implied volatility of the S&P 500 and is calculated from S&P 500 index options. *CapitalRatio* is the intermediary capital ratio as defined by He et al. (2017) and *GlobEPU* is the Global Economic Policy Uncertainty Index based on Baker et al. (2016). Data are from January 2001 to December 2022.

D Alternative Specifications and Tests of the UMVE Portfolio

Table D1: Predictive Ability of Conditional Expectations, Variance and Covariance for Portfolio Strategies

	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	1.031		
	(0.444)		
$\sigma^2_t(rx^i_{t+1})$		0.852	
		(0.155)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			0.854
			(0.130)
Observations	3,696	3,696	24,024
Adjusted R^2	0.027	0.284	0.242

This table shows estimates from regressions of realized return strategies, rx^i_{t+1}, on expected return strategies, $\mathbb{E}_t(rx^i_{t+1})$, column (1), of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$ on estimated variance for strategies, $\sigma^2_t(rx^i_{t+1})$, column (2), and of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariance, column (3). The estimation of the conditional means and the conditional covariances follows Sections 3.4.1 and 3.4.2. A detailed description of the trading strategies is available in Table 5. Standard errors are clustered at the month level. Under the null hypothesis that coefficients are different from 1, ***,**,* denote that estimates are statistically significant at the 1, 5, and 10 percent levels.

Table D2: Predictive Ability of Conditional Expectations and Variance: Alternative Specification of the Predictive Regression

Panel A: Constant			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.801		
	(0.321)		
$\sigma^2_t(rx^i_{t+1})$		0.958	
		(0.087)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.008
			(0.117)
Observations	2,640	2,640	11,880
Adjusted R^2	0.005	0.056	0.053

Panel B: Country Fixed Effects			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.596		
	(0.263)		
$\sigma^2_t(rx^i_{t+1})$		0.969	
		(0.087)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.018
			(0.117)
Observations	2,640	2,640	11,880
Adjusted R^2	0.002	0.057	0.054

Panel C: Fama-Bliss 1Y Forward Spread			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.904		
	(0.320)		
$\sigma^2_t(rx^i_{t+1})$		0.953	
		(0.086)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.000
			(0.115)
Observations	2,640	2,640	11,880
Adjusted R^2	0.009	0.057	0.055

Continuation of Table D2

Panel D: Trend Inflation			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.907		
	(0.313)		
$\sigma^2_t(rx^i_{t+1})$		0.949	
		(0.086)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			0.995
			(0.115)
Observations	2,640	2,640	11,880
Adjusted R^2	0.012	0.056	0.054

This table shows estimates from regressions of realized returns, rx^i_{t+1}, on expected returns, $\mathbb{E}_t(rx^i_{t+1})$, column (1), of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$ on estimated variance, $\sigma^2_t(rx^i_{t+1})$, column (2), and of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariance, column (3). In Panel A expected returns are computed adding the constant to Equation (7). In Panel B expected returns are computed adding country fixed effects to Equation (7). In Panel C expected returns are computed following Equation (7) but using an alternative definition of forward spread: $P^{(9Y)}_{i,t}/P^{(10Y)}_{i,t} - 1/P^{(1Y)}_{i,t}$. In Panel D expected returns are computed following Equation (7) but using an alternative definition of bond value: $Y^{(10Y)}_{i,t} - \tau^{CPI}_t$, where τ^{CPI}_t is trend inflation, as defined in Cieslak and Povala (2015). Standard errors are clustered at the month level. Under the null hypothesis that coefficients are different from 1, ***,**,* denote that estimates are statistically significant at the 1, 5, and 10 percent levels.

Table D3: Predictive Ability of Conditional Expectations, Variance and Covariance: Alternative Predictable Variables

	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
Panel A: Including Long-Term Momentum			
$\mathbb{E}_t(rx^i_{t+1})$	0.791		
	(0.297)		
$\sigma^2_t(rx^i_{t+1})$		0.959	
		(0.087)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.008
			(0.117)
Observations	2,640	2,640	11,880
Adjusted R^2	0.004	0.058	0.055
Panel B: Including Short-Term Momentum			
$\mathbb{E}_t(rx^i_{t+1})$	0.818		
	(0.086)		
$\sigma^2_t(rx^i_{t+1})$		1.007	
		(0.116)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.014
			(0.116)
Observations	2,640	2,640	11,880
Adjusted R^2	0.005	0.056	0.053
Panel C: Including Currency Trend Signal			
$\mathbb{E}_t(rx^i_{t+1})$	0.790		
	(0.329)		
$\sigma^2_t(rx^i_{t+1})$		0.959	
		(0.086)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.008
			(0.116)
Observations	2,640	2,640	11,880
Adjusted R^2	0.005	0.058	0.056

Continuation of Table D3

Panel D: Including Long-Term Momentum, Short-Term Momentum and Currency Trend Signal			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.595		
	(0.306)		
$\sigma^2_t(rx^i_{t+1})$		0.969	
		(0.089)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.022
			(0.120)
Observations	2,640	2,640	11,880
Adjusted R^2	0.001	0.056	0.054

Panel E: Including Credit Risk			
	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.523*		
	(0.239)		
$\sigma^2_t(rx^i_{t+1})$		0.957	
		(0.080)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			0.984
			(0.137)
Observations	1,680	1,680	7,560
Adjusted R^2	0.002	0.075	0.073

This table shows estimates from regressions of realized returns, rx^i_{t+1}, on expected returns, $\mathbb{E}_t(rx^i_{t+1})$, column (1), of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$ on estimated variance, $\sigma^2_t(rx^i_{t+1})$, column (2), and of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariance, column (3). Panel A uses forward spread, value and long-term momentum as predictors for the estimation of expected returns. Panel B uses forward spread, value, and short-term momentum as predictors for the estimation of expected returns. Panel C uses forward spread, value and currency trend signals as predictors for the estimation of expected returns. Currency trend signal is defined as $S_{i,t}/S_{i,t-12M} - 1$. Panel D uses forward spread, value, long-term and short-term momentum, and currency trend signals as predictors for the estimation of expected returns, as defined in Equation (17). Panel E uses forward spread, value, and credit risk as predictors for the estimation of expected returns, as defined in Equation (18). Credit risk is defined as monthly changes in sovereign CDS spreads, as detailed in Section 6.2.2. Standard errors are clustered at the month level. Under the null hypothesis that coefficients are different from 1, ***,**,* denote that estimates are statistically significant at the 1, 5, and 10 percent levels.

Table D4: Sharpe Ratio and GRS Test for Alternative UMVE Portfolios Using Portfolio Strategies

Model	SR UMVE	GRS Tests – p-values			
		All strategies	All Carry	All Value	All Carry and Value
Optimal	1.135	0.402	0.205	0.392	0.256
Forward spread Value LTMom	0.988	0.458	0.670	0.200	0.375
Forward spread Value FX Trend	0.945	0.455	0.656	0.185	0.343
Forward spread Value STMom	0.913	0.417	0.642	0.154	0.298
Forward spread Value Credit Risk	0.811	0.294	0.551	0.056	0.138
Forward spread Value LTMom STMom FX Trend	0.717	0.240	0.420	0.078	0.129
Minimum Entropy Approach	0.861	0.002	0.016	0.010	0.001

This table shows in column (1) the Sharpe ratio (SR) for the optimal UMVE portfolio, which uses two signals, i.e. forward spread and value as predictors, and for alternative UMVE portfolios. Specifically, the alternative UMVE portfolios use the following signals: forward spread, value, long-term momentum (second row); forward spread, value, currency trend signals (third row); forward spread, value, short-term momentum (fourth row); forward spread, value, credit risk (fifth row); forward spread, value, long-term momentum, short-term momentum, currency trend signals (sixth row). The bottom row shows the results for the optimal growth portfolio based on the minimum entropy approach. Column (2) shows the p-value of the GRS test if the alphas from regressing all global portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (3) shows the p-value of the GRS test if the alphas from regressing global carry portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (4) shows the p-value of the GRS test if the alphas from regressing global value portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Column (5) shows the p-value of the GRS test if the alphas from regressing global carry and value portfolio strategies on the respective UMVE portfolio are jointly equal to zero. Data are from January 2001 until December 2022 (with the exception of regressions involving credit risk that start in 2009).

Table D5: Predictive Ability of Conditional Expectations and Variance: Fixed-Width Estimation Window for Expected Returns

	rx^i_{t+1}	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$	$(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$
$\mathbb{E}_t(rx^i_{t+1})$	0.659		
	(0.317)		
$\sigma^2_t(rx^i_{t+1})$		0.971	
		(0.091)	
$cov_t(rx^i_{t+1}, rx^j_{t+1})$			1.020
			(0.122)
Observations	2,640	2,640	11,880
Adjusted R^2	0.008	0.065	0.061

This table shows estimates from regressions of realized returns, rx^i_{t+1}, on expected returns, $\mathbb{E}_t(rx^i_{t+1})$, column (1), of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1}))^2$ on estimated variance, $\sigma^2_t(rx^i_{t+1})$, column (2), and of $(rx^i_{t+1} - \mathbb{E}_t(rx^i_{t+1})) \cdot (rx^j_{t+1} - \mathbb{E}_t(rx^j_{t+1}))$ on estimated covariance, column (3). Expected returns are estimated using 5-year rolling windows. Standard errors are clustered at the month level. Under the null hypothesis that coefficients are different from 1, ***,**,* denote that estimates are statistically significant at the 1, 5, and 10 percent levels.

Table D6: Testing the UMVE Using Individual Country Bond Returns: Fixed-Width Estimation Window for Expected Returns

	SR	Mean	t_mean	alpha	t_alpha	adj.R^2
Australia	0.263	2.200	1.238	1.172	0.624	0.009
Canada	0.463	3.209	2.179	1.300	0.900	0.067
Germany	0.502	3.480	2.363	1.803	1.397	0.049
Japan	0.704	2.351	3.316	1.058	1.533	0.118
New Zealand	0.285	2.293	1.341	1.580	0.906	0.004
Norway	0.383	2.746	1.805	1.894	1.240	0.009
Sweden	0.453	3.265	2.135	1.827	1.248	0.034
Switzerland	0.509	2.913	2.396	1.745	1.448	0.034
United Kingdom	0.313	2.533	1.474	0.796	0.492	0.038
United States	0.440	4.012	2.073	1.280	0.700	0.080

This table shows for individual countries the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing individual countries' bond returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Expected returns used for the construction of the UMVE portfolio are estimated using 5-year rolling windows. Data are from January 2001 to December 2022.

Table D7: Testing the UMVE Using Portfolio Strategies: Fixed-Width Estimation Window for Expected Returns

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
$1/N$	0.477	2.900	2.246	1.446	1.182	0.048
GDP-weighted	0.498	3.349	2.343	1.281	0.973	0.082
Average Carry	0.617	3.730	2.906	1.992	1.640	0.073
CS-Carry	0.786	3.038	3.701	1.088	1.483	0.223
TS-Carry	0.518	2.782	2.437	1.295	1.209	0.066
Average Value	0.617	3.726	2.903	1.679	1.308	0.101
CS-Value	0.517	1.716	2.435	0.970	1.429	0.045
TS-Value	0.600	3.029	2.825	1.089	1.032	0.132
Average Mom12	0.507	3.080	2.388	1.686	1.309	0.045
CS-Mom12	0.248	0.819	1.167	0.730	0.945	-0.003
TS-Mom12	0.403	6.310	1.898	3.811	1.081	0.020
Average Mom1	0.070	0.428	0.329	0.980	0.765	0.004
CS-Mom1	-0.344	-1.180	-1.618	-1.409	-1.798	0.001
TS-Mom1	-0.001	-0.005	-0.005	0.446	0.455	0.004

This table shows for global bond portfolio strategies, as defined in Table 5, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing individual countries' bond returns on the UMVE portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Expected returns used for the construction of the UMVE portfolio are estimated using 5-year rolling windows. Data are from January 2001 to December 2022.

Table D8: Testing the UMVE Using Individual Country Bond Returns for Other Maturities

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
5-year maturity						
Australia	0.293	1.224	1.379	1.014	1.070	-0.003
Canada	0.524	1.985	2.467	0.961	1.216	0.050
Germany	0.522	1.857	2.457	1.299	1.874	0.016
Japan	0.690	0.941	3.247	0.415	1.423	0.099
New Zealand	0.343	1.316	1.614	0.698	0.842	0.014
Norway	0.462	1.692	2.174	1.224	1.488	0.008
Sweden	0.533	1.886	2.510	1.270	1.747	0.018
Switzerland	0.596	1.633	2.805	1.176	1.980	0.017
United Kingdom	0.386	1.566	1.818	1.199	1.472	0.002
United States	0.546	2.542	2.571	1.372	1.456	0.043
15-year maturity						
Australia	0.254	3.230	1.195	1.161	0.414	0.016
Canada	0.467	4.306	2.200	1.224	0.647	0.087
Germany	0.445	4.806	2.097	1.860	0.938	0.055
Japan	0.509	2.851	2.397	0.370	0.329	0.131
New Zealand	0.236	2.920	1.113	0.756	0.283	0.022
Norway	0.308	3.345	1.450	1.888	0.818	0.010
Sweden	0.417	4.566	1.961	1.902	0.869	0.045
Switzerland	0.464	4.123	2.185	1.217	0.698	0.080
United Kingdom	0.278	3.088	1.308	1.109	0.484	0.021
United States	0.350	4.571	1.649	0.307	0.115	0.081

This table shows for individual countries the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing individual countries' bond returns on the UMVE portfolio returns (alpha, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Individual countries' returns are computed from the 5-year and 15-year zero curves. Data are from January 2001 to December 2022.

Table D9: Time Series Expected UMVE Sharpe Ratio and Macro and Market Variables: Including Credit Risk

	Dependent variable:
	UMVE Expected Sharpe Ratio
GDP SD	-0.043^{*}
	(0.023)
Unempl SD	-0.001
	(0.006)
Infla SD	0.737^{***}
	(0.098)
MSCIRet	0.266
	(0.401)
VIX Index	0.007
	(0.005)
GlobCDS	-0.121
	(0.127)
Constant	0.950^{***}
	(0.304)
Observations	228
Adjusted R^2	0.622

This table shows estimates from regressions of the UMVE expected Sharpe ratio on macro and market variables. *GDP SD*, *Unempl SD*, and *Infla SD* are the cross-sectional standard deviations of countries' GDP, Unemployment, Inflation, respectively. *MSCIRet* is the return on the MSCI World Index. *VIX Index* is an index of the implied volatility of the S&P 500 and is calculated from S&P 500 index options. *GlobCDS* is the GDP-weighted average of countries' CDS spread changes. Newey-West standard errors with 6 lags are reported in parenthesis (Newey and West (1994)). Data are from January 2004 to December 2022. ***,**,* denote that estimates are statistically significant at the 1, 5 and 10 percent levels.

Table D10: Testing the Naive Portfolio Using Individual Country Bond Returns

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
Australia	0.263	2.200	1.238	-1.411	-1.862	0.818
Canada	0.463	3.209	2.179	0.270	0.398	0.789
Germany	0.502	3.480	2.363	0.429	0.743	0.851
Japan	0.704	2.351	3.316	1.549	2.465	0.251
New Zealand	0.285	2.293	1.341	-0.877	-0.881	0.681
Norway	0.383	2.746	1.805	0.023	0.026	0.633
Sweden	0.453	3.265	2.135	0.167	0.243	0.813
Switzerland	0.509	2.913	2.396	0.593	0.923	0.721
United Kingdom	0.313	2.533	1.474	-0.853	-0.966	0.769
United States	0.440	4.012	2.073	0.110	0.126	0.806

This table shows for individual countries the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (*t_mean*). It further shows the alpha from regressing individual countries' bond returns on the naive portfolio returns (*Alpha*, in % p.a.), the t-statistics for the alpha (*t_alpha*), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 to December 2022.

Table D11: Testing the Naive Portfolio Using Portfolio Strategies

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
GDP-weighted	0.498	3.349	2.343	0.269	0.653	0.920
Average Carry	0.617	3.730	2.906	1.002	1.924	0.895
CS-Carry	0.786	3.038	3.701	2.253	2.994	0.178
TS-Carry	0.518	2.782	2.437	0.366	0.938	0.888
Average Value	0.617	3.726	2.903	2.506	2.035	0.176
CS-Value	0.517	1.716	2.435	1.598	2.184	0.002
TS-Value	0.600	3.029	2.825	1.993	1.914	0.182
Average Mom12	0.507	3.080	2.388	1.974	1.599	0.142
CS-Mom12	0.248	0.819	1.167	0.547	0.792	0.026
TS-Mom12	0.403	6.310	1.898	2.248	0.803	0.293
Average Mom1	0.070	0.428	0.329	0.168	0.131	0.004
CS-Mom1	-0.344	-1.180	-1.618	-1.400	-1.941	0.014
TS-Mom1	-0.001	-0.005	-0.005	-0.203	-0.201	0.004

This table shows for global bond portfolio strategies, as defined in Table 5, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing portfolio returns on the naive portfolio returns ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 to December 2022.

Table D12: Testing the Growth Optimal Portfolio Using Individual Country Bond Returns

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
Australia	0.259	2.176	1.219	-0.280	-0.159	0.112
Canada	0.466	3.237	2.190	-0.409	-0.327	0.369
Germany	0.500	3.471	2.348	-0.361	-0.304	0.408
Japan	0.684	2.272	3.212	0.146	0.272	0.550
New Zealand	0.290	2.341	1.364	-0.360	-0.214	0.148
Norway	0.384	2.754	1.803	-0.441	-0.335	0.264
Sweden	0.456	3.287	2.142	-0.372	-0.276	0.345
Switzerland	0.508	2.911	2.386	-0.259	-0.264	0.410
United Kingdom	0.313	2.539	1.472	-0.394	-0.238	0.174
United States	0.443	4.046	2.083	-0.516	-0.315	0.335

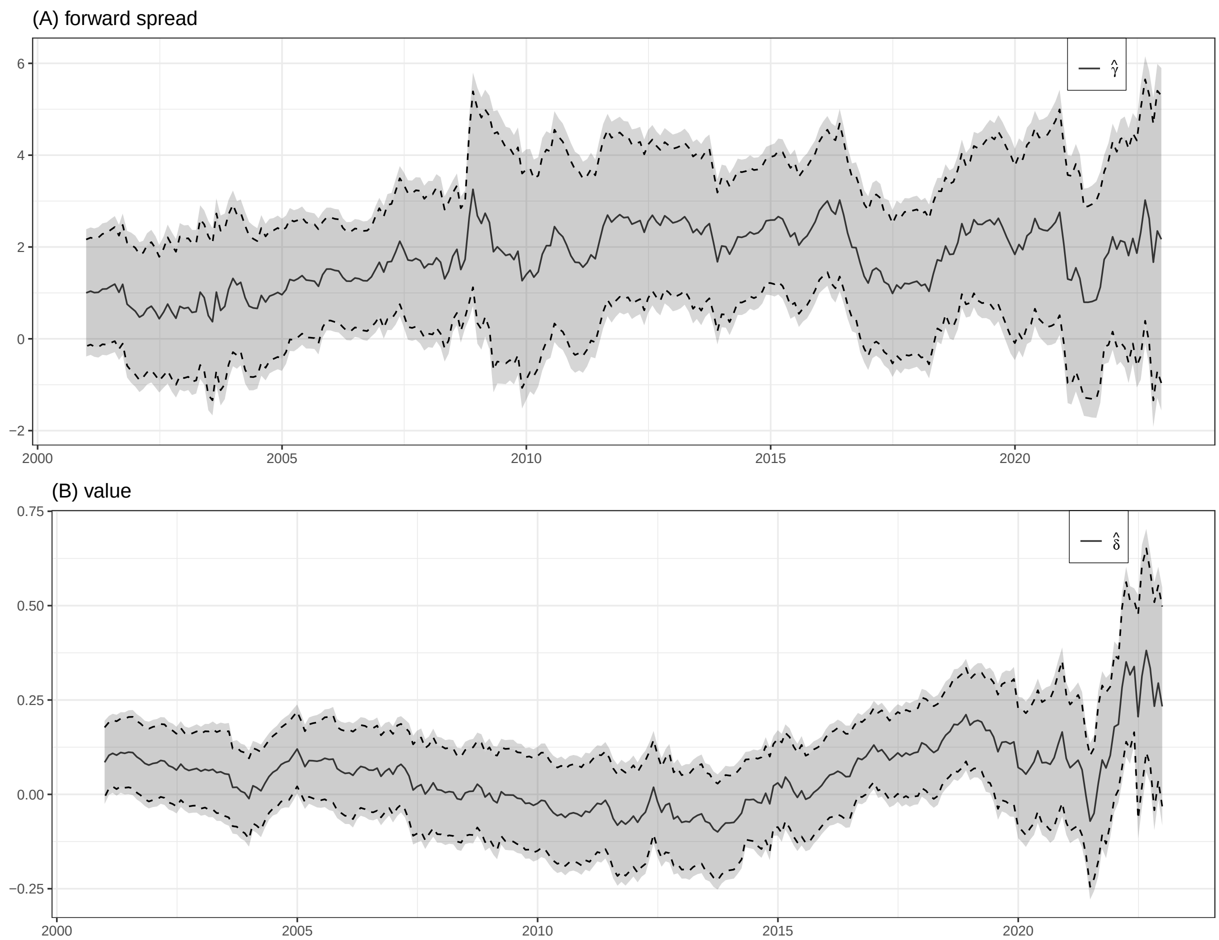
This table shows for individual countries the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (*t_mean*). It further shows the alpha from regressing individual countries' bond returns on the growth optimal portfolio obtained using the minimum entropy approach (*Alpha*, in % p.a.), the t-statistics for the alpha (*t_alpha*), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 to December 2022.

Table D13: Testing the Growth Optimal Portfolio Using Portfolio Strategies

	SR	Mean	t_mean	Alpha	t_alpha	adj.R^2
$1/N$	0.477	2.900	2.246	-0.325	-0.301	0.376
GDP-weighted	0.498	3.349	2.343	-0.365	-0.320	0.407
Average Carry	0.617	3.730	2.906	0.773	0.676	0.324
CS-Carry	0.786	3.038	3.701	1.956	2.302	0.098
TS-Carry	0.518	2.782	2.437	0.130	0.132	0.325
Average Value	0.617	3.726	2.903	2.229	1.761	0.080
CS-Value	0.517	1.716	2.435	2.196	3.165	0.022
TS-Value	0.600	3.029	2.825	1.687	1.574	0.093
Average Mom12	0.507	3.080	2.388	1.710	1.354	0.067
CS-Mom12	0.248	0.819	1.167	0.911	1.257	-0.003
TS-Mom12	0.403	6.310	1.898	1.456	0.458	0.128
Average Mom1	0.070	0.428	0.329	0.329	0.254	-0.004
CS-Mom1	-0.344	-1.180	-1.618	-1.355	-1.745	0.001
TS-Mom1	-0.001	-0.005	-0.005	-0.197	-0.194	-0.001

This table shows for global bond portfolio strategies, as defined in Table 5, the annualized Sharpe ratio (SR), mean bond returns ($Mean$, in % p.a.), and the t-statistic for the mean bond returns (t_mean). It further shows the alpha from regressing portfolio returns on the growth optimal portfolio obtained using the minimum entropy approach ($Alpha$, in % p.a.), the t-statistics for the alpha (t_alpha), and the adjusted R^2 from this regression ($adj.R^2$). Data are from January 2001 to December 2022.

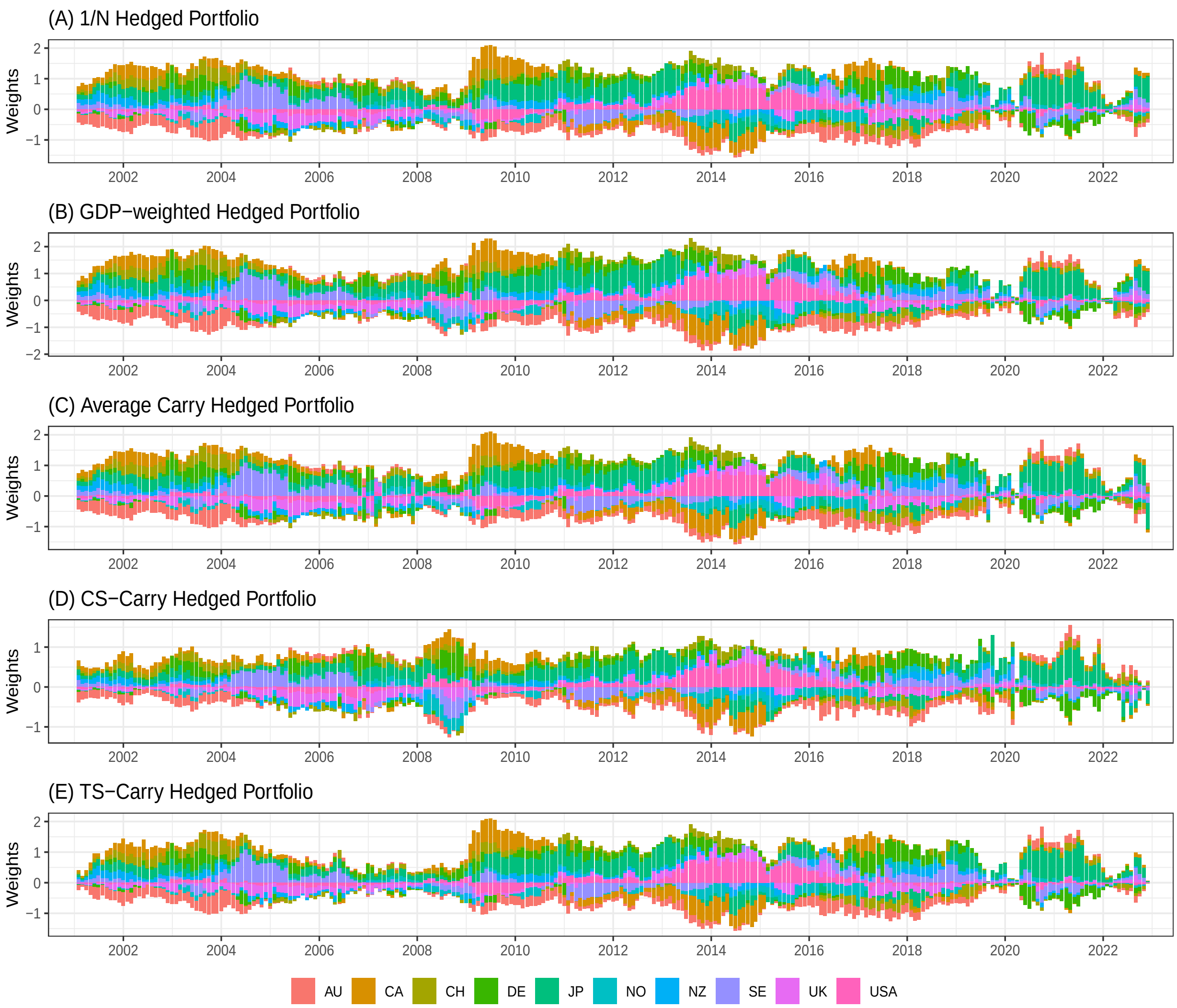
Figure D1: Time Series of Estimated Coefficients from Predictive Regression: Fixed-Width Estimation Window for Expected Returns



This figure shows the time series of the estimated coefficients from pooled regressions of the forward spread ($\hat{\gamma}$) and bond value ($\hat{\delta}$) on bond excess returns using 5-year rolling windows. Panel A displays the estimated coefficient of the forward spread variable and Panel B of the bond value variable. The range between the dotted lines and the shaded areas represents the 90% and 95% confidence intervals of the variables, respectively. Standard errors are clustered at the country level.

E Time Series of the Hedged Portfolio Weights

Figure E1: Time Series of Weights for the Hedged Strategies: Panel (A)

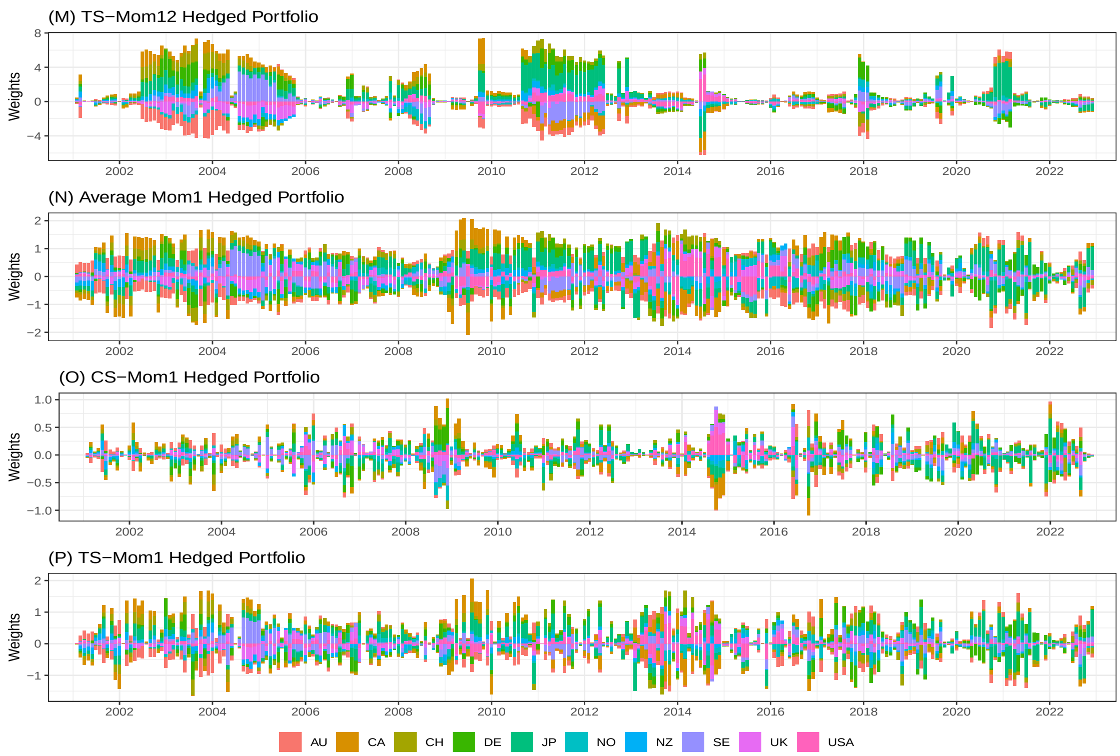


This graph shows the time series of the weights of the hedged strategies, as defined in Section 4.4.

Figure E2: Time Series of Weights for the Hedged Strategies: Panel (B)

This graph shows the time series of the weights of the hedged strategies, as defined in Section 4.4.

Figure E3: Time Series of Weights for the Hedged Strategies: Panel (C)



This graph shows the time series of the weights of the hedged strategies, as defined in Section 4.4.

F Sharpe Ratio Decomposition Eliminating Cross-Sectional Variation

F.1 Constant Expected Returns

Constant expected returns are computed as the average of excess returns across all markets over the estimation window. Specifically, we regress excess returns on a constant only, using expanding time windows. Using this approach, expected returns are only constant across markets but not time invariant.

F.2 Constant Variances

The conditional covariance matrix $\mathbb{V}_t^{\bar{\sigma}}$ has no cross-sectional variation in variances. To achieve this, we first take the final $\mathbb{V}_t$ matrix, constructed, as in Section 3.4.2, at each point in time, and split it up into the two components as follows:

$$\begin{bmatrix} \sigma_i & & & \\ & \sigma_j & & \\ & & \ddots & \\ & & & \sigma_N \end{bmatrix} \cdot \begin{bmatrix} 1 & \rho_{i,j} & \cdots & \rho_{i,N} \\ \rho_{j,i} & 1 & & \\ \ddots & & \ddots & \\ \rho_{N,j} & & & 1 \end{bmatrix} \cdot \begin{bmatrix} \sigma_i & & & \\ & \sigma_j & & \\ & & \ddots & \\ & & & \sigma_N \end{bmatrix} \tag{F1}$$

where σ_i is the bond standard deviation of market i. $\rho_{i,j}$ is the bond correlation between country i and j. Second, we calculate the cross-sectional average of the sigmas $\bar{\sigma} = (\sigma_i + ... \sigma_N)/N$ and use it to reconstruct a new matrix with constant standard deviations $\mathbb{V}_t^{\bar{\sigma}}$. Note that the standard deviations are only constant across markets but not time invariant.[2]

[2]In order to simplify the notation, we suppress the time subscripts for standard deviations and correlations.

F.3 Constant Correlations

The conditional covariance matrix $\mathbb{V}_t^{\bar{\rho}}$ has no cross-sectional variation in correlations. To achieve this, we use the above matrix decomposition F1, but now we compute the average correlation $\bar{\rho}$. For this, we take all pairwise correlations $\rho_{i,j}$ where $i \neq j$ (the upper half of the matrix, without the diagonal). As a last step, we replace all pairwise correlations by this average and recompute a new matrix $\mathbb{V}_t^{\bar{\rho}}$. In this new covariance matrix, only standard deviations are different across countries, but correlations are constant.

G Sharpe Ratio Decomposition Eliminating Time Series Variation

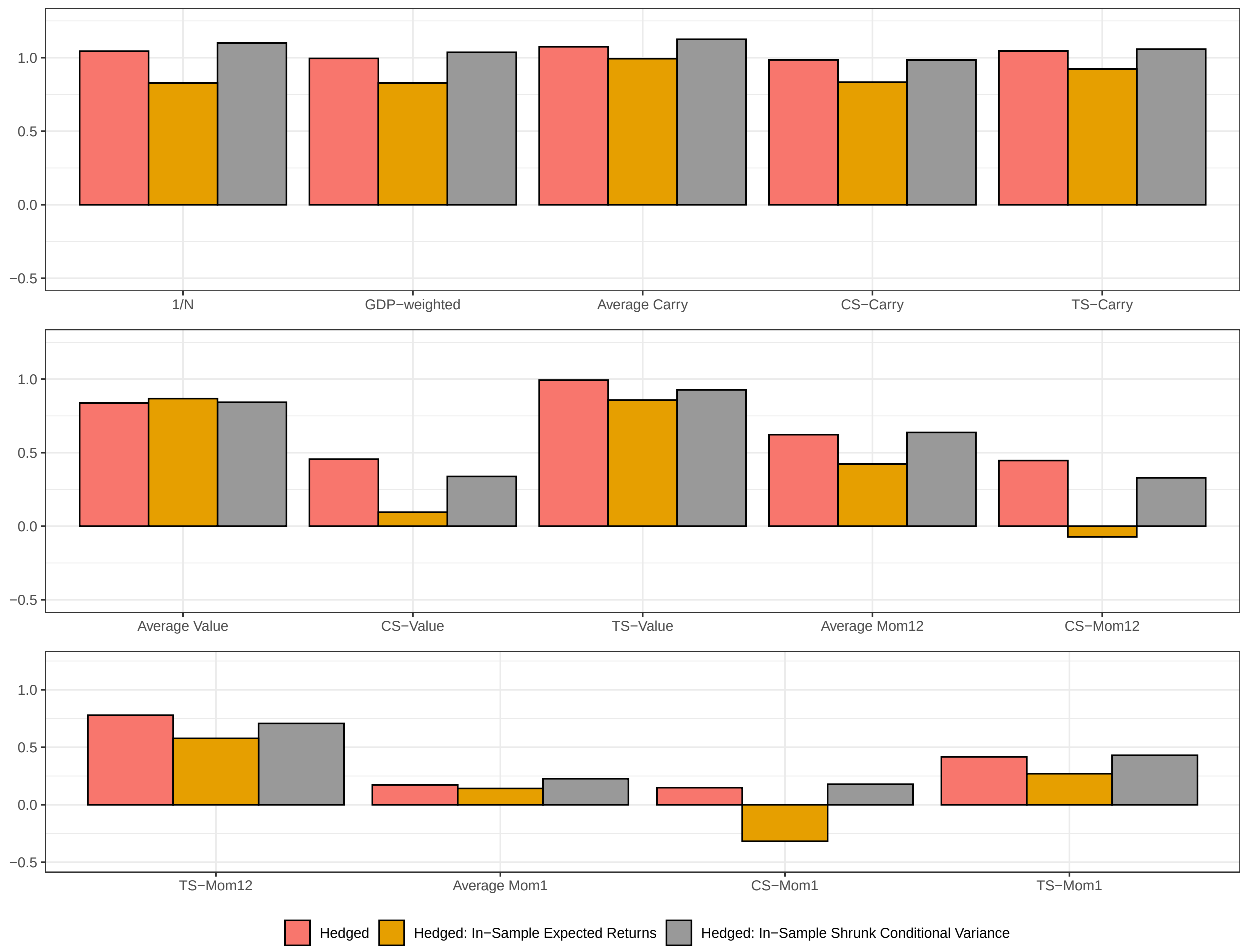
G.1 In-sample Expected Returns

The in-sample expected returns are computed for each country as the arithmetic average across time of all monthly excess holding period returns. As a result, there is no time series variation in returns.

G.2 In-sample Shrunk Conditional Variance

The in-sample shrunk covariance matrix $\bar{\tilde{\Sigma}}$ is computed as the arithmetic average across time of all the monthly shrunk covariance matrices $\tilde{\Sigma}_t$, as defined in Section 3.4.2. As a result, there is no time series variation in variances and correlations.

Figure G1: Sharpe Ratio Hedged Strategies Decomposition when Fixing One Element in the Time series



This figure shows, for each strategy, the annualized Sharpe ratio of the hedged portfolio (first bar), as defined in Section 4.4, and two alternative hedged portfolios constructed from restricted UMVE versions. Specifically, the restricted UMVE portfolios shut off any time series variation in expected returns, or in conditional variance (as described in detailed in Appendix G). The hedged portfolios resulting from these restricted UMVE portfolios correspond to the hedged portfolio with in-sample expected returns (*Hedged: In-Sample Expected Returns*), and the hedged portfolio with in-sample shrunk covariance matrix (*Hedged: In-Sample Shrunk Conditional Variance*).