Optimal Capital Taxation with Borrowing Constraints and Entrepreneurial Heterogeneity*

Preliminary draft

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Abstract

This paper analyzes the optimal taxation of capital, labor, wealth and capital income in a model with entrepreneurial heterogeneity and financial frictions as in Itskhoki and Moll (2019a). We consider both a closed economy, populated by workers that buy corporate bonds, and a small open economy with hand-to-mouth workers where entrepreneurs finance capital by using their wealth and by borrowing abroad. Taxes are levied to finance unproductive government spending. The optimal fiscal policy differs depending on whether we consider the closed economy or the small open economy. In the former the taxes on labor and pure capital should be positive, while the tax on capital income should be zero. The wealth tax levied on workers and entrepreneurs and the pure capital tax levied on firms are equivalent fiscal instruments. When we consider a small open economy, we find that the pure capital tax should be set to zero but both the capital income tax and the labor tax should be positive. In this case the capital income tax and the wealth tax are perfect substitutes.

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1 Introduction

Since the seminal work of Judd (1985) and Chamley (1986), the debate on optimal taxation in dynamic economies has centered on whether the tax on capital, in the long run, should be zero. Over the years, a large literature has analyzed the robustness of this classical result to the introduction of externalities, tax restrictions, various demographic structures, market imperfections, etc. While some contributions support the original Chamley (1986) and Judd (1985) result,[1] others find that the optimal tax on capital is different from zero.[2]

In models of the Chamley-Judd tradition the return on investment is the same across agents and a capital income tax is in general equivalent to a wealth tax. Recently however, following a contribution by Guvenen et al. (2023), the debate has been enriched by the observation that, in models with only one asset, these two forms of taxation diverge when rates of returns on wealth are heterogeneous. The reason is that the tax base of the wealth tax is given by wealth (or income from wealth), while the capital income tax is based both on profits and income from wealth. Guvenen et al. (2023) study these two types of taxation in a two sector OLG economy where entrepreneurs who are subject to productivity shocks use only capital to produce, and workers are hand to mouth. Their analysis shows that a wealth tax is preferable since it shifts the tax burden onto entrepreneurs with low productivity, reducing the efficiency costs for entrepreneurs with high productivity, while a tax on capital income distorts production efficiency penalizing more productive entrepreneurs. In contrast, Boar and Midrigan (2023) reach the opposite results in the context of an infinite horizon model with two types of entrepreneurs: one type characterized by decreasing returns to scale and liquidity constraints, and one type characterized by constant returns to scale and no liquidity constrains. Although they admit that a capital income tax implies an efficiency loss, they find that is nevertheless preferable to a wealth tax on distributional grounds.

Our paper intends to contribute to this debate by proposing a model that, besides providing some clean analytical results, takes into account some additional elements that deserve attention. First, we think that, in evaluating the optimality of taxes, it is important to con-

[1] See, e.g., Jones et al. (1997), Atkeson et al. (1999), Chari and Kehoe (1999), Judd (1999) and Chari et al. (2020). More recently Greulich et al. (2023) show that when agents differ in their labor productivity and markets are complete, the zero capital tax result is confirmed, although capital taxes should remain high for a long time before reaching the limit.

[2] See, e.g., Aiyagari (1995), Lansing (1999), Chamley (2001), Bassetto and Benhabib (2006). More recently Straub and Werning (2020) have shown that once the dynamic properties of the Judd (1985) and Chamley (1986) models are taken into account, the optimal capital tax is positive in the long run if the intertemporal elasticity of substitution is below one, while it slowly converges to zero with other values of such an elasticity. Using different environments also Benhabib and Szőke (2021), and Bassetto and Cui (2023) find that positive capital taxation may be optimal in the long run.

sider that workers contribute to the economy not only by supplying labor, but also through their savings. Hence a wealth tax, beside redistributing income among entrepreneurs, also redistributes income between entrepreneurs and workers. Second, we observe that capital is not only wealth, but also a factor of production whose user's cost can be affected by taxation. Hence, we introduce, beside a wealth tax and a capital income tax, a tax on capital directly paid by firms that we call a pure capital tax. In a heterogeneous agents economy, this tax may differ from a wealth tax as it enters the capital income tax base. Third, we wish to understand the crucial role of interest rate adjustments in determining the optimality of the various taxes. To this purpose we develop our model both as a closed economy and as a small open economy where, because of perfect international capital mobility, the interest rate is fixed at the world level. In this open economy, a further element emerges, since taxing capital as an input involves not only taxing wealth of entrepreneurs but also taxing the corporate bonds held by foreigners.

We study the question of wealth and capital taxation versus income capital taxation through the lens of the heterogeneous agents model recently proposed by Itskhoki and Moll (2019a) that, in our opinion, is well suited to capture the working of capitalist economies and to address the tax issues previously outlined.[3] In our economy which, similarly to Judd (1985), has both entrepreneurs and workers, heterogeneity comes from the entrepreneurial side: depending on the realization of a random productivity shock, producers decide each time whether to engage in production or to exit the market.[4] Entrepreneurs face borrowing constraints that limit the amount of capital they can employ and differ in their wealth and productivity. Because of heterogeneity and borrowing constraints, entrepreneurs make positive profits which, in this environment, are a crucial driver of economic activity; without them, entrepreneurs would not have any incentive to undertake production.

We start with a quantitative differential tax analysis of the closed economy. Two revenue-neutral experiments are studied: one in which a rise in the wealth tax is compensated by a reduction in the labor tax, and one in which a cut in the capital income tax is offset by a heavier wealth tax. In the former case, we find that workers' consumption, the capital stock, entrepreneurial wealth, and aggregate output fall. The wage rate remains fairly invariant

[3]As will be apparent later on, the focus of our paper and the types of analyses we perform are quite different from the one conducted by Itskhoki and Moll (2019a).

[4]The source of heterogeneity we consider is quite different from the one analyzed by relevant papers in the literature. Aiyagari (1995), İmrohoroğlu (1998), Chamley (2001) and Conesa et al. (2009) analyze economies in which borrowing constraints prevent individuals from insuring against idiosyncratic shocks and agents have a precautionary motive for savings. Werning (2007) and Greulich et al. (2023) study environments in which workers differ in their productivity, while in Bassetto and Benhabib (2006), and Benhabib and Szőőke (2021) agents differ in their initial wealth.

from the initial equilibrium to the new one. The fraction of active entrepreneurs is reduced by the change in the tax mix. This experiment suggests that taxing capital is more distortionary than taxing labor. Interestingly, the effects are greater, the greater is entrepreneurs' heterogeneity.

When capital taxation is raised at the scope of cutting the capital income tax, instead, while the consumption of workers and aggregate capital are curtailed, entrepreneurial wealth, aggregate production, and workers' wages are increased in the new steady state. What is worth emphasizing is that in this second experiment, the deadweight loss of taxation is lower than in the previous case as aggregate production and income from labor are increased.

We then proceed to the normative analysis of our economy. In all the cases we consider, we assume that the social welfare function is given by the utility integral of workers. The planner chooses the optimal linear taxes on factors of production (labor and capital), the tax on wealth and the tax on capital income to finance a given flow of unproductive government spending. We find that, in the long-run, the optimal taxes on capital and labor are positive, while the optimal capital income tax is zero. Given that the optimal tax on capital income is zero, the wealth tax paid by workers and entrepreneurs and the pure capital tax paid by firms are equivalent fiscal tools.

The optimal tax structure obtained for the closed economy has two complementary explanations. First, a positive pure capital tax (which is equivalent to a wealth tax) is optimal because it is borne by entrepreneurs (both active and inactive ones) and by workers. As such, it is useful to distribute the tax burden more broadly across different economic agents, while a capital income tax falls primarily on active entrepreneurs, distorting significantly the economy. The redistribution of the tax burden among entrepreneurs echoes the analysis presented by Guvenen et al. (2023), which we extend by highlighting the broader sharing of the burden between entrepreneurs and workers.

Second, in an economy characterized by financial frictions, the demand for capital is constrained by the entrepreneurs' borrowing constraints, which depend on their wealth. As a result, active entrepreneurs earn positive profits. These profits represent pure rents that only generate income effects. However, as these rents are essential to sustain the entrepreneurial activity, they cannot be fully taxed without adverse effects. This constraint on rent taxation leads to the conclusion that a positive pure capital tax is optimal, being a way to tax entrepreneurs with a reduced impact on production.

Next, we study a small open economy with hand-to-mouth workers and perfect international capital mobility. The major difference with respect to the closed economy case is that, while in a closed economy the effect of pure capital taxes is dampened by a decrease in the interest rate, in a small open economy the interest rate does not react, being fixed at the world level. Another key distinction is that the supply of capital is now determined by the wealth of entrepreneurs and the corporate bonds held by foreign residents, rather than by the wealth of entrepreneurs and the corporate bonds purchased by domestic workers-savers.

Surprisingly, the results obtained for the closed economy no longer hold. The optimal capital income tax is positive, whereas the pure capital tax must be zero. The labor tax is then used to finance the portion of government spending not covered by capital income taxation. In the small economy, differently for what happened in the closed economy case, a capital income tax and a wealth tax are perfectly substitutable fiscal instruments. This is because both taxes affect only the disposable income of entrepreneurs and, hence, a wealth tax can be optimally designed to have an equivalent effect of the capital income tax, being the interest rate given at world level.

The reason why in the small open economy the optimal capital tax is zero, while the tax on capital income and labor are positive, lies on the much bigger impact that a tax on capital has on the cost of capital and aggregate output, due to the exogenous interest rate. Because of this, a capital tax enters directly total factor productivity, not only at the steady state, but also along the transition path. A tax on capital income, by comparison, does not produce the same direct negative distortionary effects and is therefore preferred. It is important to notice that the burden of financing government spending should be distributed between workers and entrepreneurs.

The paper is structured as follows. Section 2 presents the closed economy setup and investigates the implications of a change in the tax mix. Section 3 deals with the optimal tax structure of a closed economy. Section 4 analyzes the case of a small open economy. Section 5 concludes.

2 Closed economy

2.1 The model

Consider a closed economy with an infinite horizon populated with workers, entrepreneurs and the government, as in Itskhoki and Moll (2019a,b). Workers, who are homogeneous, work, consume and save by accumulating bonds issued by entrepreneurs and government bonds. Entrepreneurs, which are subject to borrowing constraints, are heterogeneous in productivity and wealth; they produce by using capital and labor, consume and accumulate wealth. The government levies different types of taxes: a wealth tax paid by both workers and entrepreneurs at the same rate, and taxes on labor, capital, and capital income (defined as the sum of profits and income from wealth) that are paid by entrepreneurs.

The representative worker maximizes the following intertemporal utility function

$$\int_0^\infty e^{-\rho t} u(c, \ell) \mathrm{d}t, \tag{1}$$

subject to the flow budget constraint

$$c + \dot{b} + \dot{d} = (r - \tau_a)(b + d) + w\ell + T, \tag{2}$$

where c is worker's consumption, ℓ labor hours supplied, b entrepreneurial bonds, d government bonds, r the real interest rate, τ_a a specific wealth tax, w the real wage, and T government lump-sum transfers. $u(\cdot)$ denotes the instantaneous utility function, that satisfies the neoclassical properties of regularity, and ρ is the subjective discount rate.

The worker's optimality conditions are

$$-\frac{u_\ell}{u_c} = w, \tag{3a}$$

$$\frac{\dot{u}_c}{u_c} = \rho - r + \tau_a, \tag{3b}$$

and the proper transversality condition.

At each date, an entrepreneur, who owns a private firm, uses labor, n, and capital, k, to produce an output according to a Cobb-Douglas technology, given by $A\,(zk)^\alpha\, n^{1-\alpha}$, with $\alpha \in (0, 1)$ and $A > 0$ that represent technological parameters. As entrepreneurs are heterogeneous, the joint distribution of their wealth, a, and productivity, z, is given by $G(a, z)$. As in Itskhoki and Moll (2019a), we assume that the idiosyncratic productivity z, independent of the wealth distribution, is drawn from a Pareto distribution, $G_z(z) = 1 - z^{-\eta}$. The shape parameter, $\eta > 1$, measures the degree of entrepreneurial heterogeneity. Productivity shocks are iid across entrepreneurs and time.

Assuming a logarithmic instantaneous utility function, an entrepreneur maximizes

$$\mathbb{E}_0 \int_0^\infty e^{-\delta t} \ln c^e \mathrm{d}t, \tag{4}$$

subject to the flow budget constraint

$$c^e + \dot{a} = (1 - \tau_\kappa)\left[\pi(a, z) + ra\right] - \tau_a a, \tag{5}$$

where c^e denotes entrepreneur's consumption, τ_κ a proportional tax rate on entrepreneurial capital income, $\pi(a, z)$ maximum profit, and δ the subjective discount rate (with $\delta > \rho$). Entrepreneurs are subject to the same specific wealth tax τ_a on their wealth as workers.

Financial markets are incomplete, in that entrepreneurs face a collateral constraint given by

$$k \leq \lambda a, \tag{6}$$

where $\lambda \geq 1$ measures the efficiency of financial markets.

The maximum profit of an entrepreneur is given by

$$\pi(a,z) = \max_{n,k \leq \lambda a} \left[A\,(zk)^{\alpha}\, n^{1-\alpha} - (1+\tau_{\ell})wn - (1+\tau_{\kappa\kappa})rk \right], \tag{7}$$

where τ_{ℓ} and $\tau_{\kappa\kappa}$ are proportional tax rates on labor and capital, respectively.[5]

Entrepreneurs choose capital and labor to maximize profits. At the same time, they decide on consumption and saving by using the maximum level of profits (that depends on a and z) obtained from factors' demands. Profit maximization entails

$$k(a,z) = \lambda a\, \mathbb{I}(z \geq \underline{z}), \tag{8a}$$

$$n(a,z) = \left[\frac{(1-\alpha)A}{w(1+\tau_{\ell})} \right]^{\frac{1}{\alpha}} zk(a,z). \tag{8b}$$

Using (8a) and (8b) in (7), the maximum profit of an entrepreneur can be expressed as

$$\pi(a,z) = \left(\frac{z}{\underline{z}} - 1 \right) \lambda a(1+\tau_{\kappa\kappa})r, \tag{9}$$

where $\underline{z}$ represents a productivity cutoff (obtained when entrepreneurial profits are zero), given by

$$\underline{z} = \frac{(1+\tau_{\kappa\kappa})r}{\alpha \left[\frac{(1-\alpha)}{w(1+\tau_{\ell})} \right]^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}}. \tag{10}$$

The entrepreneur's profit function (9) is linear in a. Entrepreneurs with $z \geq \underline{z}$ find it profitable to produce, while entrepreneurs with $z < \underline{z}$ do not produce and exit the market as they earn negative profits.

After using (9) in (5), the optimal entrepreneurial decisions on consumption and saving imply

$$c^{e} = \delta a, \tag{11a}$$

[5] Notice that differently from Itskhoki and Moll (2019a), we consider a payroll tax instead of a wage tax paid by workers. This is not relevant for our results, as the labor market is perfectly competitive.

$$\dot{a} = (1-\tau_\kappa)\left[\left(\frac{z}{\underline{z}}-1\right)\lambda(1+\tau_{\kappa\kappa})+1\right]ra - \tau_a a - \delta a. \tag{11b}$$

Let $x = \int a \mathrm{d}G(a,z)$ denote aggregate wealth of entrepreneurs. The aggregate levels of capital, κ, and labor, ℓ, are then given by

$$\kappa = \int\int_{z\geq\underline{z}} \lambda a \mathrm{d}G_z(z)\mathrm{d}G_a(a) = \lambda x \underline{z}^{-\eta}, \tag{12a}$$

$$\ell = \left[\frac{(1-\alpha)A}{w(1+\tau_\ell)}\right]^{\frac{1}{\alpha}} \int\int_{z\geq\underline{z}} zk(a,z)\mathrm{d}G_z(z)\mathrm{d}G_a(a) = \left[\frac{(1-\alpha)A}{w(1+\tau_\ell)}\right]^{\frac{1}{\alpha}} \frac{\lambda\eta}{\eta-1} x\underline{z}^{1-\eta}, \tag{12b}$$

where $\underline{z}^{-\eta}$ represents the fraction of active entrepreneurs.

Aggregating outputs of active entrepreneurs yields

$$y = A\left(\frac{\eta}{\eta-1}\right)^{\alpha} \underline{z}^{\alpha}\kappa^{\alpha}\ell^{1-\alpha}. \tag{13}$$

Plugging (12a) into (13) for $\underline{z}$, the aggregate production function becomes

$$y = A\left(\frac{\eta}{\eta-1}\right)^{\alpha} \lambda^{\frac{\alpha}{\eta}} x^{\frac{\alpha}{\eta}} \kappa^{\frac{\alpha(\eta-1)}{\eta}} \ell^{1-\alpha}. \tag{14}$$

The joint consideration of (12a) and (12b) gives the factor shares

$$w\ell = \frac{(1-\alpha)}{(1+\tau_\ell)}y, \tag{15a}$$

$$r\kappa = \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}y. \tag{15b}$$

Aggregating (9) and using (12a) together with (15b), we obtain the profit share

$$\Pi = \frac{\alpha}{\eta}y. \tag{15c}$$

The dynamics of entrepreneurs' wealth, obtained by aggregating (11b), is:

$$\dot{x} = (1-\tau_\kappa)\left(\frac{\alpha}{\eta}y + rx\right) - \tau_a x - \delta x. \tag{16}$$

Note that the average after-tax return on entrepreneurial wealth is $(1-\tau_\kappa)\left(\frac{\alpha}{\eta}\frac{y}{x}+r\right)-\tau_a$, while the return on wealth received by workers is $r-\tau_a$.

Entrepreneurs finance capital employed in production by using their wealth and by issuing bonds bought by workers. Therefore, the amount of aggregate capital supplied is

$$\kappa = x + b. \tag{17}$$

The government finances the budget deficit by issuing public debt. Government expenditures are given by an exogenous flow of unproductive spending, g, workers' lump-sum transfers, and the service of public debt. At each date, the government budget constraint is

$$\dot{d} = g + T + rd - \left[\tau_{\kappa\kappa} r\kappa + \tau_{\ell} w\ell + \tau_a(b + d + x) + \tau_{\kappa}\left(\frac{\alpha}{\eta}y + rx\right)\right]. \tag{18}$$

Finally, combining the budget constraints of workers and entrepreneurs with the government budget constraint yields the resource constraint of the economy:

$$y = c + \delta x + \dot{\kappa} + g. \tag{19}$$

Decentralized equilibrium Given the initial aggregate levels of workers' wealth b_0, entrepreneurial wealth x_0, and government debt d_0, a decentralized equilibrium is a sequence of aggregate variables $\{c, \ell, b, c^e, y, x, \kappa\}$, government policies $\{d, T, \tau_a, \tau_\kappa, \tau_{\kappa\kappa}, \tau_\ell\}$, and factor prices $\{w, r\}$ such that: (i) workers maximize (1) subject to (2), taking prices as given; (ii) entrepreneurs maximize (4) subject to (5) and (9), taking prices as given; (iii) entrepreneurial wealth evolves according to (16); (iv) the government satisfies (18); and (v) the market clearing conditions hold.

2.2 Positive analysis of changes in the tax mix

In this section, we provide a brief description the key features of the model and then we examine how macroeconomic variables quantitatively respond to some tax policy experiments. The numerical analysis is preparatory for the study of Ramsey policy.

Before we proceed, it is important to return to the distinction among a pure capital tax, a capital income tax, and a wealth tax. Taxing pure capital implies indirectly taxing both the income from entrepreneurial wealth and the income from workers' wealth via the interest rate. In contrast, a capital income tax directly taxes both the returns on entrepreneurial wealth and pure profits at the same rate. Wealth taxation instead is levied on assets of both entrepreneurs and workers uniformly. This tax appears redundant, as it essentially affects the economy as a pure capital tax. However, since a pure capital tax enters the base for a capital income tax, it can, in principle, differ from a wealth tax, provided that the capital income tax is greater than zero. As we will see, these differences in tax bases play a critical role in determining the optimality of these various forms of taxation.

For the analysis of the model and the quantitative investigation, we assume that the representative worker has the following instantaneous utility,[6]

$$u(c, \ell) = \ln c - \psi \frac{\ell^{1+\phi}}{1+\phi}. \tag{20}$$

The full short-run model, analytically described in Appendix A, satisfies the properties of dynamic stability. The core dynamic system, obtained by substituting out the static relationships, once linearized around the long-run equilibrium, involves three dynamic variables: c, x, and κ. The long-run equilibrium is a saddlepoint with one eigenvalue having a positive real part, associated with c (the variable free to move on impact), and with two eigenvalues having negative real parts, associated with x and κ (the predetermined variables). The local dynamic properties of the model are studied numerically.

Steady state Since in the steady state $\overline{r} = \rho + \tau_a$, the capital stock is given by

$$\overline{\kappa} = \frac{\alpha(\eta - 1)\overline{y}}{\eta(1 + \tau_{\kappa\kappa})(\rho + \tau_a)}, \tag{21}$$

where overbar variables denote long-run values.

Plugging (21) into the aggregate production function (14), we get

$$\overline{y} = \Theta[(1 + \tau_{\kappa\kappa})(\rho + \tau_a)]^{-\gamma(\eta-1)} \overline{x}^{\gamma} \overline{\ell}^{1-\gamma}, \tag{22}$$

where

$$\Theta \equiv A^{\frac{\eta\gamma}{\alpha}} \left(\frac{\eta\lambda}{\eta - 1}\right)^{\gamma} \alpha^{\gamma(\eta-1)} \text{ and } \gamma \equiv \frac{\alpha}{\alpha + \eta(1 - \alpha)}.$$

The core steady-state model can be reduced to the following system[7]

$$\overline{c} = \frac{\eta[\delta - (1 - \tau_\kappa)\rho + \tau_\kappa \tau_a)]}{\alpha(1 - \tau_\kappa)} \overline{x} - \delta\overline{x} - g, \tag{23a}$$

$$\overline{c} = \frac{(1 - \alpha)}{\psi(1 + \tau_\ell)} \Theta^{\frac{1+\phi}{1-\gamma}} \left\{ \frac{\alpha(1 - \tau_\kappa)}{\eta[\delta - (1 - \tau_\kappa)\rho + \tau_\kappa \tau_a)]} \right\}^{\frac{\gamma+\phi}{1-\gamma}} [(1 + \tau_{\kappa\kappa})(\rho + \tau_a)]^{\frac{-\alpha(\eta-1)(1+\phi)}{\eta(1-\alpha)}} \overline{x}^{-\phi}. \tag{23b}$$

[6]Notice that the results of the normative analysis presented in Section 3 are independent of the functional form taken by $u(c, \ell)$, as long as such a utility function satisfies the standard properties of regularity. In the quantitative analysis, the of use (20) is intended solely to simplify the analysis.

[7]See Appendix B.

Equation (23a) derives from the feasibility constraint (19) and the entrepreneurs' budget constraint (16), while equation (22) is obtained combining the workers' consumption-labor trade-off with the aggregate long-run production function (2) and the entrepreneurs' budget constraint. These two equations jointly determine the long-run values of workers' consumption, $\overline{c}$, and entrepreneurial wealth, $\overline{x}$.

It is worth noting that τ_a is qualitatively equivalent to $\tau_{\kappa\kappa}$ when τ_κ=0. When τ_κ differs from zero, instead, τ_a affects the economy also through (23a).

In the long run, the fraction of active entrepreneurs is decreasing in $\tau_{\kappa\kappa}$ and τ_a, increasing in τ_κ, and independent of τ_ℓ; that is,

$$\overline{\underline{z}}^{-\eta} = \frac{(\eta-1)[\delta-(1-\tau_\kappa)\rho+\tau_\kappa\tau_a)]}{\lambda(1+\tau_{\kappa\kappa})(\rho+\tau_a)(1-\tau_\kappa)}. \tag{24}$$

Quantitative analysis We now analyze two permanent, revenue-neutral changes in the tax mix. Since the optimal capital income tax rate is zero, for expositional purposes our quantitative analysis will be limited to experiments involving labor, pure capital, and capital income taxation. Specifically, we will look at the macroeconomic impact of replacing labor taxes with capital taxes, and increasing capital taxes while reducing entrepreneurial income taxes.

These two differential tax experiments serve as a foundation for optimal tax analysis, as they help to understand how different taxes influence the economy. By examining two tax rates together and isolating the effects of other taxes, these experiments offer insights into the comparative impact of taxation. Furthermore, they illuminate how tax systems operate under a balanced budget constraint, providing valuable information about the relationship between tax rates and the macroeconomic equilibrium. Lastly, unlike the optimal tax analysis, these experiments do not require specific assumptions about the social welfare function.

We start by computing the response of the main macroeconomic variables to a 1% permanent unexpected increase of the capital tax rate accompanied by a revenue-neutral change in the labor tax rate. Parameter values are reported in Table 1, while in Figure 1 the responses are presented as deviations in percentage points from the steady state. For the sake of simplicity, we set $\tau_a = T = d = 0$. To emphasize the role of entrepreneurial heterogeneity on the response of aggregate variables, each panel displays three structural responses: one for a low value of η, one for an intermediate level of η, and one for a high value of η.[8]

When $\tau_{\kappa\kappa}$ increases (and τ_ℓ decreases in order to keep tax revenues invariant), the interest

[8]Notice also that since that the support of the Pareto distribution is the interval $(1,\infty)$, $\underline{z}$ is given by the maximum between 1 and the RHS of (10).

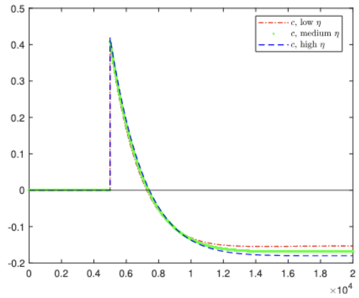
rate suddenly jumps down, overshooting its long-run value given by the discount rate ρ. Because of perfect foresight, workers' consumption jumps up on impact and then converges over time to a lower steady-state value. At the same time, labor hours initially drop because of the increase in consumption, but then they settle to a higher steady-state value. A higher $\tau_{\kappa\kappa}$ also implies a fall in capital, causing a fall in production, despite the increase in the amount of labor. This in turn leads to a fall in aggregate profits of entrepreneurs. The reduction in the entrepreneurs' disposable income causes a decrease in their wealth x, as they dissave. This reinforces the fall in the level of capital, because of the borrowing constraint. It is interesting to notice that the effects of the increase in $\tau_{\kappa\kappa}$ on consumption, labor, output and wealth are increasing in η (i.e., they decrease with entrepreneurial heterogeneity).

Parameter	**Value**	**Description**	**Source**
ρ	0.03	workers' discount rate	Itskhoki and Moll (2019a)
δ	0.10	entrepreneurs' discount rate	Itskhoki and Moll (2019a)
α	0.33	capital share	standard value
ϕ	1.22	inverse Frisch elasticity	Chetty et al. (2011)
ψ	1	marginal disutility of work	Itskhoki and Moll (2019a)
λ	4	financial constraint	
$\tau_{\kappa\kappa 0}$	0.10	initial pure capital tax	Acemoglu et al. (2020)
$\tau_{\kappa 0}$	0.10	initial capital income tax	Acemoglu et al. (2020)
$\tau_{\ell 0}$	0.26	initial labor tax	Acemoglu et al. (2020)
η_L	2.20	Pareto shape parameter	
η_M	2.50	Pareto shape parameter	
η_H	2.80	Pareto shape parameter	

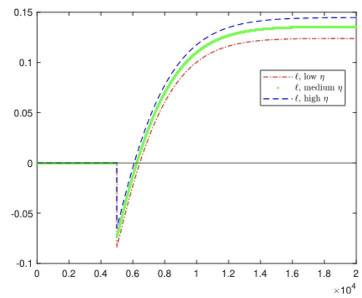
Table 1: Parameter values

The temporary decline in the interest rate induces workers to lower their savings and hence the amount of bonds they acquire. Wages, after an impact increase, return asymptotically almost to their initial value. The wage-bill increases both on impact and in the long run. On the entrepreneurial side, it is worth emphasizing that the rise in $\tau_{\kappa\kappa}$ induces a reduction in the fraction of active entrepreneurs, as $\underline{z}$ goes up. The instantaneous social welfare jumps up on impact and then declines monotonically toward the steady-state value. This fact plays a relevant role for the long-run contraction of capital, output, and consumption of workers and entrepreneurs.

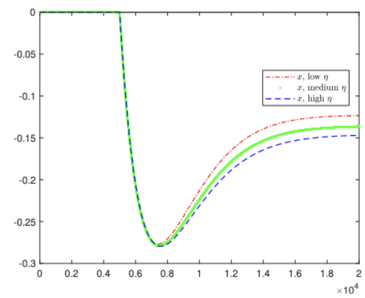
Figure 2 presents the results of replacing the entrepreneurs' income tax rate with $\tau_{\kappa\kappa}$ when the labor income tax rate remains constant. As before, we assume that $\tau_a = T = d = 0$. Relieving entrepreneurial income from taxation fosters entrepreneurial wealth formation, thus driving consumption of entrepreneurs up. This produces positive steady-state effects on aggregate output, the wage rate, and the wage-bill. Consumption of workers asymptotically declines, after the initial increase, while labor hours, that fall on impact, are increased over time. The fraction of active entrepreneurs



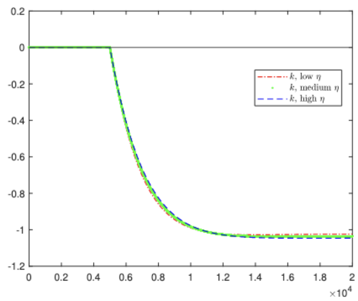
(a) c



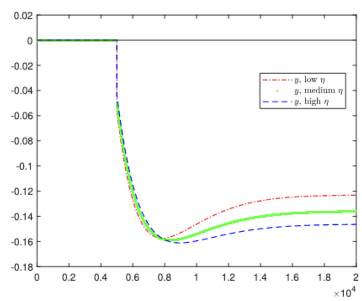
(b) ℓ



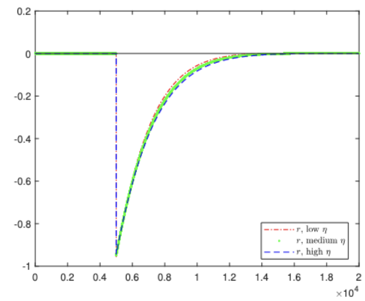
(c) x



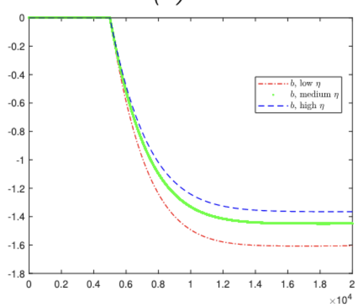
(d) κ



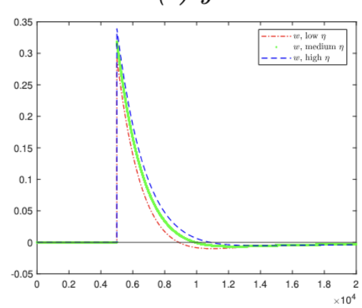
(e) y



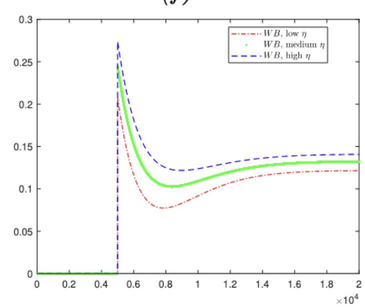
(f) r



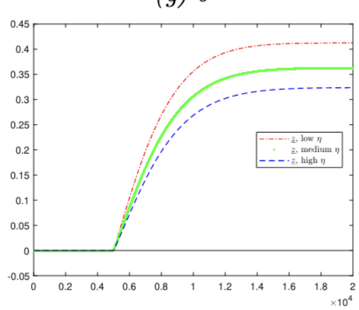
(g) b



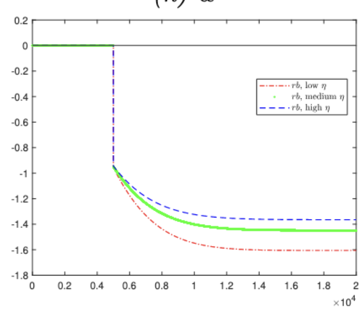
(h) w



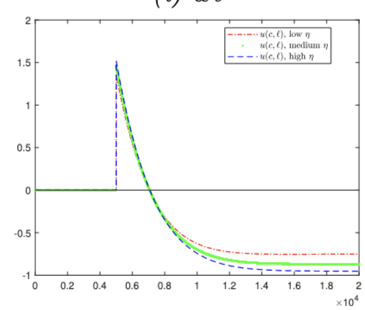
(i) $w\ell$



(j) $\underline{z}$

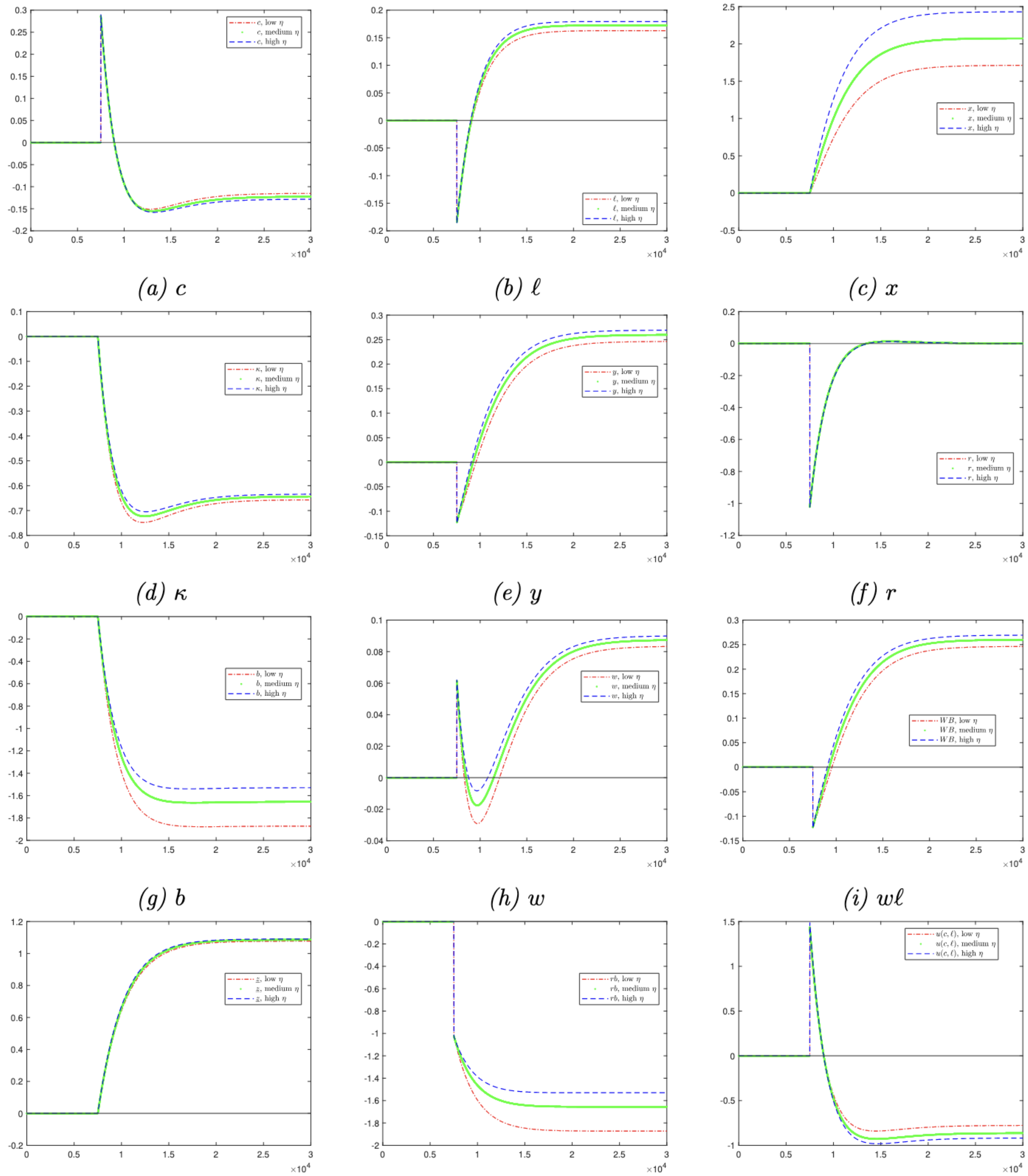


(k) rb



(l) $u(c, \ell)$

Figure 1: Dynamic responses to a 1% increase in $\tau_{\kappa\kappa}$ *when* τ_ℓ *adjusts accordingly in order to keep the tax revenue constant and* τ_κ *is kept invariant.*



(a) c *(b)* ℓ *(c)* x

(d) κ *(e)* y *(f)* r

(g) b *(h)* w *(i)* $w\ell$

(j) $\underline{z}$ *(k)* rb *(l)* $u(c, \ell)$

Figure 2: Dynamic responses to a 1% increase in $\tau_{\kappa\kappa}$ *when* τ_κ *adjusts accordingly in order to leave the tax revenue constant and* τ_ℓ *is kept invariant.*

3 Closed economy: optimal taxation

The normative analysis is based on the assumption that the social planner is benevolent toward workers and maximizes their utility integral, given by (1).[9] We investigate the optimal policy plan when a given stream of public expenditures must be financed.

When taxes are collected to finance a given flow of government spending, the optimal tax policy is found by solving the Ramsey problem through the 'primal approach' (Lucas and Stokey, 1983). The objective of the Ramsey planner is to maximize the social welfare function subject to: (i) the implementability constraint, (ii) the aggregate budget constraint of entrepreneurs, (iii) the resource constraint, and (iv) the Euler equation of workers, once the demand for capital and the aggregate production function are taken into account.[10]

Integrating (2) forward and using the no-Ponzi game condition for workers, we get

$$\int_0^{\infty}(c - w\ell)e^{-\int_0^t (r-\tau_a)ds}\mathrm{d}t = (b_0 + d_0)\,. \tag{25}$$

In turn, integrating the Euler equation of workers (3b), we obtain

$$\frac{u_c e^{-\rho t}}{u_{c_0}} = e^{-\int_0^t (r-\tau_a)rds}, \tag{26}$$

where u_{c_0} denotes the marginal utility of consumption at $t = 0$. Substituting (26) into (25), after using the optimal consumption-leisure choice of workers, we get the implementability constraint

$$\int_0^{\infty}(cu_c + \ell u_\ell)e^{-\rho t}\mathrm{d}t = u_{c_0}(b_0 + d_0)\,. \tag{27}$$

The planner chooses c, ℓ, κ, x, $\tau_{\kappa\kappa}$, τ_κ and τ_a to maximize (1) subject to (3b), after incorporating (15b), (19) and (27), with y given by (14). We assume that $\tau_\kappa \geq 0$. The initial values $b_0 > 0$, $d_0 > 0$ and $x_0 > 0$ are given.[11]

[9] A more general social welfare function would be given by a weighted average of the worker intertemporal utility function and the entrepreneur's one. Since $\delta > \rho$ (i.e., workers are less impatient than entrepreneurs), our qualitative results below will remain asymptotically unaffected when the planner gives a positive weight to the welfare of entrepreneurs as such a component vanishes in the steady state. See Itskhoki and Moll (2019b) for a discussion of this case.

[10] Notice that, in general, the consumers' Euler equation is incorporated into the implementability constraint. In our case, despite we use the implementability constraint, such an Euler equation must be considered explicitly since the aggregate budget constraint of entrepreneurs includes the interest rate, obtained from the demand for capital.

[11] Appendix C provides a detailed derivation of the results proposed in this paragraph.

The first-order conditions of this normative problem with respect to κ, x, and τ_κ, and the Kuhn-Tucker conditions are[12]

$$\kappa: \qquad -(\dot{\mu\omega}) + \mu\omega\rho = \mu\theta(1-\tau_\kappa)\frac{\alpha}{\eta}\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa} + \mu\omega\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa}, \tag{28a}$$

$$x: \qquad -(\dot{\mu\theta}) + \mu\theta\rho = \mu\theta(1-\tau_\kappa)\left[\left(\frac{\alpha}{\eta}\right)^2\frac{y}{x} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \tau_a - \delta\right] + \mu\omega\left(\frac{\alpha}{\eta}\frac{y}{x} - \delta\right), \tag{28b}$$

$$\tau_\kappa: \qquad \theta\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{k}\right]y = \zeta, \tag{28c}$$

$$\text{KT}: \qquad \tau_\kappa \geq 0, \qquad \zeta \geq 0, \qquad \tau_\kappa\zeta = 0. \tag{28d}$$

where $\mu\theta$, $\mu\omega$, and $\mu\zeta$ are the Lagrange multipliers associated with the budget constraint of entrepreneurs, the feasibility constraint, and the constraints for τ_κ, respectively.

From the Kuhn-Tucker conditions, two configurations are possible: one is given by $\tau_\kappa > 0$ and $\zeta = 0$, and one given by $\tau_\kappa = 0$ and $\zeta > 0$. By using (28c), the former configuration implies that $\theta = 0$; this is impossible as it would mean that the aggregate budget constraint of entrepreneurs is irrelevant for the social planner, a misspecification of the Ramsey problem. The other possibility is $\tau_\kappa = 0$ and $\zeta > 0$; this case is consistent with the correct specification of the normative problem and the associated Ramsey policy, implying $\theta > 0$. Therefore, $\widetilde{\tau}_\kappa$=0.

Notice that since the optimal condition for $\tau_{\kappa\kappa}$ coincides with that for τ_a (as $\widetilde{\tau}_\kappa$=0), the system of first-order conditions is indeterminate. This indeterminacy arises because the two tax instruments, $\tau_{\kappa\kappa}$ and τ_a, are equivalent in achieving the Ramsey optimum. Therefore, one of the two tax rates can be fixed to eliminate this indeterminacy. In the following analysis, we fix τ_a and solve for the optimal $\tau_{\kappa\kappa}$.

Combining (28b) and (28a), evaluated at the steady state, and setting parametrically $\tau_a = 0$,[13] we get the optimal capital tax rate

$$\widetilde{\tau}_{\kappa\kappa} = \frac{\alpha\rho}{(\eta-\alpha)\delta} > 0. \tag{29}$$

Notice that $\widetilde{\tau}_{\kappa\kappa}$ is independent of government spending.

Considering the instantaneous utility function (20), the optimal labor tax rate, obtained

[12]Notice that the optimality conditions incorporate the planner's optimal choice for $\tau_{\kappa\kappa}$. See Appendix C.

[13]The relationship between the optimal value of $\tau_{\kappa\kappa}$ and τ_a is described Appendix C.

by combining the planner's optimality conditions with respect to c and ℓ, is[14]

$$\widetilde{\tau}_\ell = \frac{1}{(\eta-\alpha)\delta}\left\{\alpha\rho\overline{c} + \eta\rho^2\overline{x} + \overline{\mu}(1+\phi)\left[\overline{c}[(\eta-\alpha)\delta + \alpha\rho] + \eta\rho^2\overline{x}\right]\right\} > 0, \tag{30}$$

where the Lagrange multiplier associated with the implementability constraint, $\overline{\mu}$, is strictly positive because of distortionary labor taxation.[15]

We can summarize these results with the following proposition:

Proposition 1

At the steady state, when the tax on wealth is zero, the optimal taxes on pure capital and labor are positive, while the optimal tax on capital income is zero. The optimal pure capital tax rate is increasing in the profit rate α/η. When the planner optimally chooses also the wealth tax rate, an implementation indeterminacy emerges since wealth taxation is equivalent to pure capital taxation.

The normative results obtained here can be explained as follows. First, a tax on capital income is a tax on productive entrepreneurs (those having a productivity above the cutoff), penalizing more the most productive ones. In contrast, pure capital taxation (or wealth taxation, which is equivalent in this context) is levied on accumulated wealth rather than income or productivity. It applies to both entrepreneurial and workers' wealth, thereby broadening the tax base and shifting the burden toward less productive entrepreneurs. As a result, such a tax reduces economic distortions and encourages a more efficient resource allocation, thus promoting growth and increasing social welfare. For these reasons, eliminating the tax on capital income while focusing taxation on pure capital and labor represents a desirable tax policy result.

Second, we must recognize that, in this economy, productivity differences among entrepreneurs and borrowing constraints generate positive profits, which are pure rents. By entering the disposable income of entrepreneurs, profits affect their savings and hence spur the accumulation of entrepreneurial wealth. As entrepreneurs can borrow more by providing more collateral, entrepreneurial wealth increases the stock of capital increases and output. In this economy profits are not only Ricardian rents that produce only income effects, but

[14]See Appendix C.

[15]This multiplier is given by

$$\overline{\mu} = \frac{1}{(1+\phi)}\left\{\frac{(\eta-\alpha)\delta}{\overline{c}[(\eta-\alpha)\delta+\alpha\rho]+\eta\rho^2\overline{x}}\frac{(1-\alpha)y}{\psi\ell^{1+\phi}} - 1\right\} > 0.$$

represent a fundamental engine of growth. Hence they cannot be taxed at a confiscatory rate as prescribed by the principles of optimal taxation.

With a confiscatory profit tax ruled out, the key question becomes why the planner should exempt entrepreneurial income from taxation and instead rely exclusively on capital and labor taxes to fund government spending. A tax on capital income, which is the sum of profits and the entrepreneurs' income from wealth, directly affects pledgeable wealth, and therefore has a direct impact on the demand for capital, that is different from the standard neoclassical one. Notice, from equation (14), that entrepreneurs' wealth enters directly the production function. A tax on capital is better, from a second best perspective, because has a smaller effect on entrepreneurial income, being indirectly levied through the interest rate on both on workers and entrepreneurs. Moreover, it is partly compensated by a decrease in the interest rate, which represents the before-tax cost of capital to firms. Hence it has a much smaller impact on aggregate output; this makes it preferable to a tax on entrepreneurial income.

The impossibility to fully tax profits can be considered as a form of tax restriction, which is a consequence of the structure of our economy. Therefore, the principles of Diamond and Mirrlees (1971), and Atkinson and Stiglitz (1972), which in general support the optimality of a zero capital tax in standard neoclassical growth models with no restrictions, are violated.[16]

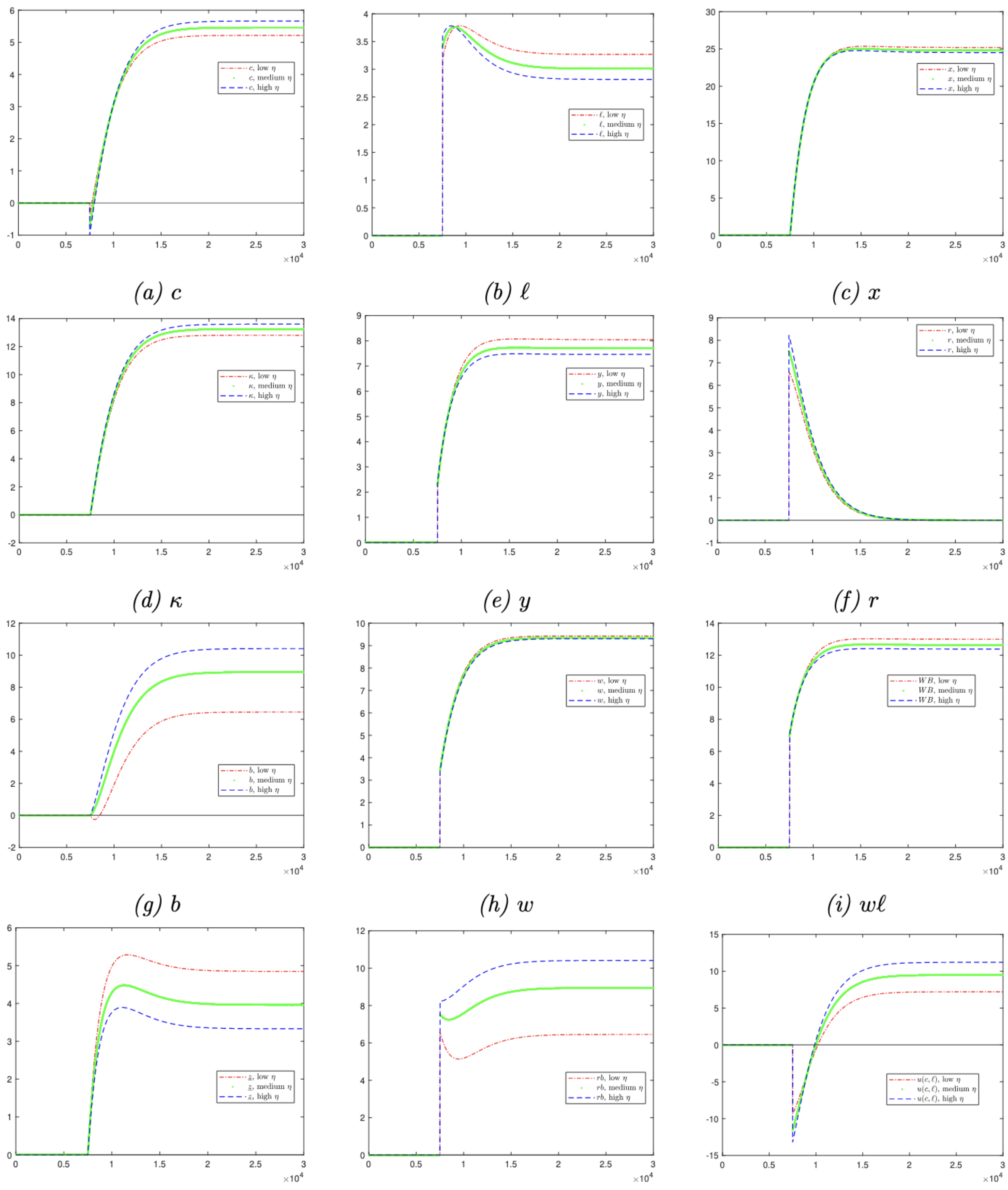
Furthermore, our capital tax result is consistent with the observations in Jones et al. (1997) who argue that the second-best capital tax usually differs from zero when the capital stock enters the implementability constraint or the planner takes into account constraints, not considered by consumers/savers, that contain the capital stock. In our case, the element that supports the Jones et al. (1997) observation is given by the entrepreneurs' budget constraint, in which profits, that cannot be fiscally eliminated, depend on entrepreneurial wealth and the capital stock.

Notice finally that the optimal capital income tax rate is a function of the profit rate α/η. The larger η, the smaller the degree of heterogeneity in our economy; as η increases, entrepreneurs become more and more homogeneous, and the optimal capital tax decreases accordingly.

We analyze the economy's dynamic response to a fiscal reform involving a permanent and unexpected adoption of the long-run optimal tax rates prescribed by the Ramsey plan. Figure 3 numerically evaluates the effects of setting the tax rates at their optimal second-best values. The initial levels of tax rates are those reported in Table 1. This tax reform involves lowering

[16]For the implications of profits and tax restrictions on production efficiency, see Stiglitz and Dasgupta (1971), and Munk (1980), respectively.

the entrepreneurial income tax to zero, reducing the capital tax to 0.046, and adjusting the labor tax according to its long-run value. In this experiment, the level of public debt is kept constant, while lump-sum transfers are adjusted continuously in order to balance the government budget. As in the previous numerical exercises, each panel displays the dynamic responses under three different levels of entrepreneurial heterogeneity, η. With the implementation of second-best tax rates, the interest rate initially rises due to the reduction in the capital tax rate. Since consumption increases in the long run, the rise in the interest rate causes an immediate drop in consumption, which is necessary to generate expectations of future increases, and leads to an increase in labor hours. As a result, workers' instantaneous utility declines instantaneously, implying a welfare loss. The reduction of the entrepreneurial income tax rate to zero stimulates the accumulation of entrepreneurial wealth, and thus of capital. This accumulation of capital is corroborated by the reduction in the capital tax rate. Along the transition path, output, the wage-bill, and workers' consumption increase, while the interest rate declines monotonically, converging toward its long-run value, which is determined by the workers' discount rate. Additionally, the Ramsey policy strengthens entrepreneurial selection by raising the productivity cutoff and, consequently, reducing the fraction of active entrepreneurs. In the steady state, this tax reform is expansionary for the economy and improves social welfare.



(a) c (b) ℓ (c) x

(d) κ (e) y (f) r

(g) b (h) w (i) $w\ell$

(j) $\underline{z}$ (k) rb (l) $u(c, \ell)$

Figure 3: Dynamic effects of a tax reform based on the adoption of the long-run optimal tax rates

4 Small open economy

We now consider a small open economy that faces perfect international capital mobility and is populated by hand-to-mouth workers.[17] In this economy, entrepreneurs finance domestic capital by using their wealth and issuing bonds that are held by foreign residents; that is, domestic capital is partly financed through foreign debt.[18] The budget constraint of workers is obtained from (2) by setting $b = d = T = 0$, while the aggregate budget constraint of entrepreneurs is given by

$$\dot{x} = (1 - \tau_\kappa) \left(\frac{\alpha}{\eta} y + r^* x \right) - \tau_a x - \delta x, \tag{31}$$

where r^* represents the exogenous world interest rate.

The supply of capital is now given by

$$\kappa = x + f, \tag{32}$$

where f denotes bonds issued by entrepreneurs and held by foreign residents. As the before-tax user's cost of capital is exogenously determined, the demand for capital is

$$\kappa = \frac{\alpha(\eta - 1)y}{\eta(1 + \tau_{\kappa\kappa})r^*}. \tag{33}$$

Plugging (33) into (14), we get the aggregate production function of this small open economy

$$y = \Theta^* (1 + \tau_{\kappa\kappa})^{-\gamma(\eta-1)} x^\gamma \ell^{1-\gamma}, \tag{34}$$

where $\Theta^* \equiv \left(\frac{\alpha}{r^*}\right)^{\gamma(\eta-1)} A^{\frac{\gamma\eta}{\alpha}} \left(\frac{\eta\lambda}{\eta-1}\right)^\gamma$ and $\gamma \equiv \frac{\alpha}{\alpha+\eta(1-\alpha)}$. The term $\Theta^*(1 + \tau_{\kappa\kappa})^{-\gamma(\eta-1)}$ can be regarded as "aggregate" total factor productivity of this small open economy. The capital tax rate enters the aggregate production function as the constant cost of capital for firms (due to the exogenous interest rate) fixes the average productivity of capital.

[17] In a small open economy model with infinite lives, a fixed discount rate and no financial transaction costs, the assumption of hand-to-mouth workers allows us to avoid the so-called *zero root problem*, which implies that the steady state depends on initial conditions and that the equilibrium dynamics is characterized by random walk elements. The *zero root problem* is implied by the arbitrary equality between the exogenous world interest rate and the fixed discount rate, needed to ensure that a stationary state is reached. See Giavazzi and Wyplosz (1985), and Schmitt-Grohé and Uribe (2003).

[18] The hypothesis of foreign financing of domestic capital is considered, for example, by Hayashi and Prescott (2008).

The government budget constraint is the same as (18), once the world interest rate, r^*, is taken into account and $d = T = 0$. Finally, the resource constraint is given by

$$\dot{\kappa} - \dot{f} = y - r^* f - c - \delta x - g. \tag{35}$$

In the open economy, the fraction of active entrepreneurs is given by

$$\underline{z}^{-\eta} = \frac{(\eta - 1)(\delta + \tau_a)}{(1 + \tau_{\kappa\kappa})(1 - \tau_\kappa) r^* \lambda}. \tag{36}$$

$\underline{z}^{-\eta}$ is decreasing in $\tau_{\kappa\kappa}$, and increasing τ_a and τ_κ. A wealth tax and a capital income tax qualitatively affect the fraction of active entrepreneurs in the same way.

4.1 Optimal taxation: optimal taxation

As in the closed economy, we investigate an optimal Ramsey tax plan.

The Ramsey planner maximizes (1) subject to three constraints. The first constraint is given by the budget constraint of workers combined with (3a). Given that workers are hand-to-mouth, this constraint can be be seen as a sort of static implementability constraint, given by

$$c u_c + \ell u_\ell = 0. \tag{37a}$$

The second constraint is given by (31).

The third constraint is the feasibility constraint, that can be rewritten, after using the entrepreneurs' budget constraint, as

$$y = c + \left[(1 - \tau_\kappa) \frac{\alpha}{\eta} + \frac{\alpha(\eta - 1)}{\eta(1 + \tau_{\kappa\kappa})} \right] y + g - (\tau_\kappa r^* + \delta) x. \tag{37b}$$

We can now state the following

Proposition 2

In a small open economy, at the steady state, the optimal tax policy entails a zero pure capital tax, and positive taxes on capital income and labor when the wealth tax is zero. The capital income tax rate is given by

$$\tau_\kappa^* = \frac{\rho}{\delta + \rho} > 0.$$

When also the wealth tax is optimally chosen, we have an implementation indeterminacy since the wealth tax and the capital income tax are perfect substitutes.

In this case, we depart substantially from the closed economy results. In a small open economy, a capital tax is highly distortionary. Since the before tax interest rate is fixed at the world level, it cannot adjust to compensate the capital tax, and it affects directly total factor productivity. In this open economy, a positive capital income tax, instead, is less distortionary and is needed, together with a labor tax, to finance the given stream of government expenditure. The labor tax rate is obtained from the government budget constraint, once the expression for τ_{κ}^{*} is employed.

Although a capital income tax is distortionary, since it primarily impacts more productive entrepreneurs, a pure capital tax is even more distortionary, as it negatively affects total factor productivity. Therefore, a capital income tax is preferable due to its relatively lower distortionary effects. Moreover, since a capital income tax only impacts the disposable income of entrepreneurs, an optimal wealth tax can be designed by equivalently affecting their after-tax disposable income.

Therefore, in a small open economy where workers do not save, the entrepreneurial income tax must be combined with a labor tax to reduce the distortions associated with the need to collect a prescribed amount of revenues.

5 Conclusion

In this paper we studied a model that emphasizes the role of entrepreneurs that are heterogeneous in productivity and wealth, and are subject to borrowing constraints. We think that these features of the economy are crucial to capture some essential elements of capitalism that are not taken into account by the neoclassical growth model and to open a new perspective on the analysis of optimal taxation. We focus on pure capital, labor, wealth and capital income taxes, that are levied to finance unproductive government spending. We consider two cases: a closed economy, in which entrepreneurs finance capital both through their wealth and by issuing corporate bonds, and a small open economy, populated by hand-to-mouth workers, in which domestic capital is financed through entrepreneurial wealth and capital imported from abroad i.e., corporate bonds bought by foreign residents.

We found that the structure of optimal taxation should differ between a closed economy, where the interest rate is allowed to react to capital taxation, and a small open economy where the interest rate is fixed at the world level. In the former economy taxes on pure capital and labor should be positive, while the tax on capital income should be zero. When the capital income tax is optimally chosen, the wealth tax and the tax on pure capital turn out to be equivalent. In a small open economy the optimal fiscal policy is totally different: the tax on capital income, together with the labor tax, should be positive, while the tax on pure capital should be zero. In this case the wealth tax and the capital income tax are perfect substitutes.

The classic Chamley (1986) and Judd (1985) result on capital taxation arises from a narrow perspective on the role of capital in a market economy. When the analysis is extended to include heterogeneous entrepreneurs facing financial constraints, the structure of the economy shifts, enlarging the capital's role and the application of tax instruments. In this more comprehensive framework, the implications of capital taxation differ depending on whether it is levied on the cost of capital, capital income, or financial wealth, as well as on the types of taxpayers. Consequently, the normative conclusions regarding capital taxation obtained in this environment diverge from those established in the earlier literature.

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Appendix

A Closed economy: The short-run model

Consider the model presented in Paragraph 2.2. Assuming $u(c,\ell) = \ln c - \psi \frac{\ell^{1+\phi}}{1+\phi}$, the full short-run model is characterized by the following equations:

$$-\frac{u_\ell}{u_c} = \psi c \ell^\phi = w, \tag{A.1}$$

$$w = \frac{(1-\alpha)}{(1+\tau_\ell)} \frac{y}{\ell}, \tag{A.2}$$

$$\dot{c} = c(r - \rho - \tau_a), \tag{A.3}$$

$$\dot{b} + \dot{d} = wl + (r - \tau_a)(b+d) + T - c, \tag{A.4}$$

$$\kappa = x + b, \tag{A.5}$$

$$r = \frac{\alpha(\eta-1)}{(1+\tau_{\kappa\kappa})\eta} \frac{y}{\kappa} \tag{A.6}$$

$$\dot{x} = (1-\tau_\kappa)\left(\frac{\alpha}{\eta} y + rx\right) - \tau_a x - \delta x, \tag{A.7}$$

$$\dot{\kappa} = y - c - \delta x - g, \tag{A.8}$$

$$y = \Theta x^{\frac{\alpha}{\eta}} \kappa^{\frac{\alpha(\eta-1)}{\eta}} \ell^{1-\alpha}. \tag{A.9}$$

The system (A.1)-(A.9) determines the following endogenous variables: ℓ, κ, x, b, c, w, y, r, and d.[19]

B Closed economy: The core steady-state model

The core steady-state model is given by the following reduced system:

$$\psi \bar{c} \bar{\ell}^\phi = \frac{(1-\alpha)}{(1+\tau_\ell)} \frac{\bar{y}}{\bar{\ell}}, \tag{B.1a}$$

$$\frac{\alpha}{\eta} \bar{y} = \frac{[\delta - (1-\tau_\kappa)\rho + \tau_\kappa \tau_a)]}{(1-\tau_\kappa)} \bar{x}, \tag{B.1b}$$

$$\bar{y} = \bar{c} + \delta \bar{x} + g, \tag{B.1c}$$

$$\bar{y} = \Theta[(1+\tau_{\kappa\kappa})(\rho+\tau_a)]^{-\gamma(\eta-1)} \bar{x}^\gamma \bar{\ell}^{1-\gamma}, \tag{B.1d}$$

where $\Theta \equiv A^{\frac{\eta\gamma}{\alpha}} \left(\frac{\eta\lambda}{\eta-1}\right)^\gamma \alpha^{\gamma(\eta-1)}$ and $\gamma \equiv \frac{\alpha}{\alpha+\eta(1-\alpha)}$.

[19] Notice that the government budget constraint (18) is redundant by the Walras law.

C Closed economy: Optimal taxation

Consider the second-best analysis of Paragraph 3. The Ramsey problem can be formulated as follows

$$\max_{\{c,\ell,\kappa,x,\tau_{\kappa\kappa},\tau_a,\tau_\kappa\}} \int_0^\infty e^{-\rho t} u(c,\ell)\mathrm{d}t,$$

subject to

$$\int_0^\infty (cu_c + \ell u_\ell)e^{-\rho t}\mathrm{d}t = u_{c0}\left(b_0 + d_0\right), \qquad [\mu], \quad \text{(C.1)}$$

$$\dot{x} = (1-\tau_\kappa)\left[\frac{\alpha}{\eta}y + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa}x\right] - (\tau_a + \delta)x, \qquad [\mu\theta], \quad \text{(C.2)}$$

$$\dot{\kappa} = y - c - \delta x - g, \qquad [\mu\omega], \quad \text{(C.3)}$$

$$\frac{\dot{u}_c}{u_c} = \left[\rho - \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} + \tau_a\right], \qquad [\mu\xi], \quad \text{(C.4)}$$

$$\tau_\kappa \geq 0, \qquad [\mu\zeta], \quad \text{(C.5)}$$

where $u_c = u_c(c,\ell)$ and $y = A\left(\frac{\eta}{\eta-1}\right)^\alpha \lambda^{\frac{\alpha}{\eta}} x^{\frac{\alpha}{\eta}} \kappa^{\frac{\alpha(\eta-1)}{\eta}} \ell^{1-\alpha}$. $b_0 > 0$, $d_0 > 0$ and $x_0 > 0$ are given. The Lagrange multipliers associated with the different constraints appear in square brackets.

Assume that the instantaneous utility function is given by $U = \ln c - \psi\frac{\ell^{1+\phi}}{1+\phi}$.[20]

The current-value Hamiltonian is

$$\mathcal{H} = \ln c - \psi\frac{\ell^{1+\phi}}{1+\phi} + \mu\left(1-\psi\ell^{1+\phi}\right) + \mu\theta\left\{(1-\tau_\kappa)\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{\kappa}\right]y - (\tau_a+\delta)x\right\} +$$
$$+ \mu\omega\left(y - c - \delta x - g\right) + \mu\xi c\left[\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \rho - \tau_a\right] + \mu\zeta\tau_\kappa. \quad \text{(C.6)}$$

The first-order conditions of this normative problem are

[20] Such a functional form has no implications for the qualitative results of the normative analysis. Moreover, the steady state solutions $\tau_{\kappa\kappa}$, τ_a, and τ_κ are analytically unaffected.

$$c: -(\dot{\mu\xi}) + \mu\xi\rho = c^{-1} - \mu\omega + \mu\xi\left[\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \rho - \tau_a\right], \tag{C.7}$$

$$\ell: \psi\ell^{\phi}\left[1+\mu(1+\phi)\right] = \left\{\theta(1-\tau_\kappa)\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{\kappa}\right] + \omega + \xi c\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})\kappa}\right\}\mu(1-\alpha)\frac{y}{\ell}, \tag{C.8}$$

$$\kappa: -(\dot{\mu\omega}) + \mu\omega\rho = \mu\theta(1-\tau_\kappa)\left\{\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{\kappa}\right]\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa} - \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{xy}{\kappa^2}\right\} + \tag{C.9}$$

$$+\mu\omega\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa} + \mu\xi c\left[\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})\kappa} - \frac{1}{(1+\tau_{\kappa\kappa})\kappa}\right]\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa},$$

$$x: -(\dot{\mu\theta}) + \mu\theta\rho = \mu\theta\left\{(1-\tau_\kappa)\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{\kappa}\right]\frac{\alpha}{\eta}\frac{y}{x} + (1-\tau_\kappa)\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \tau_a - \delta\right\} +$$

$$+\mu\omega\left(\frac{\alpha}{\eta}\frac{y}{x} - \delta\right) + \mu\xi c\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})\kappa}\frac{\alpha}{\eta}\frac{y}{x}, \tag{C.10}$$

$$\tau_{\kappa\kappa}: \theta(1-\tau_\kappa)x = -\xi c, \tag{C.11}$$

$$\tau_a: \theta x = -\xi c, \tag{C.12}$$

$$\tau_\kappa: \theta\left[\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{x}{\kappa}\right]y = \zeta, \tag{C.13}$$

$$\text{KT}: \tau_\kappa \geq 0, \qquad \zeta \geq 0, \qquad \tau_\kappa\zeta = 0. \tag{C.14}$$

From the Kuhn-Tucker conditions, two configurations are possible: one is given by $\tau_\kappa > 0$ and $\zeta = 0$, and one is given by $\tau_\kappa = 0$ and $\zeta > 0$. By using (C.11), the former configuration implies that $\theta = 0$; this is impossible as it would mean that the aggregate budget constraint of entrepreneurs is irrelevant for the social planner, a misspecification of the Ramsey problem. The other possibility is $\tau_\kappa = 0$ and $\zeta > 0$; this solution is consistent with the solution of the Ramsey problem, implying $\theta > 0$. Therefore, τ_κ =0.

Using (C.11) and τ_κ =0, the optimality conditions of the planner's problem become

$$c : (\dot{\mu})\theta x + \left(\frac{\dot{\theta x}}{c}\right)\mu c - \mu\rho\theta x = 1 - \mu\omega c - \mu\theta x \left[\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \rho - \tau_a\right], \tag{C.15}$$

$$\ell : \psi\ell^{\phi}\left[1+\mu(1+\phi)\right] = \mu\left(\theta\frac{\alpha}{\eta} + \omega\right)(1-\alpha)\frac{y}{\ell}, \tag{C.16}$$

$$\kappa : \; -(\dot{\mu\omega}) + \mu\omega\rho = \mu\left(\theta\frac{\alpha}{\eta} + \omega\right)\frac{\alpha(\eta-1)}{\eta}\frac{y}{\kappa} \tag{C.17}$$

$$x : \; -(\dot{\mu\theta}) + \mu\theta\rho = \mu\theta\left[\left(\frac{\alpha}{\eta}\right)^2\frac{y}{x} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\frac{y}{\kappa} - \tau_a - \delta\right] + \mu\omega\left(\frac{\alpha}{\eta}\frac{y}{x} - \delta\right). \tag{C.18}$$

Steady state tax results Since the optimal condition for $\tau_{\kappa\kappa}$ coincides with that for τ_a (as τ_κ=0), the system of the first-order conditions is indeterminate. This indeterminacy arises because the two tax instruments, $\tau_{\kappa\kappa}$ and τ_a, are equivalent in achieving the Ramsey optimum. Therefore, one of the two tax rates can be fixed to eliminate this indeterminacy. Let us fix τ_a and solve the optimal tax structure for the optimal $\tau_{\kappa\kappa}$.

As in the steady state $\dfrac{\alpha(\eta-1)}{\eta(1+\overline{\tau_{\kappa\kappa}})}\dfrac{y}{\kappa} = \rho + \tau_a$ and $\frac{\alpha}{\eta}\frac{\overline{y}}{\overline{x}} = \delta - \rho$, the planner's first-order conditions with respect to κ and x can be written as

$$-\overline{\omega}[\tau_a + \overline{\tau_{\kappa\kappa}}(1+\tau_w)]\overline{\tau_{\kappa\kappa}} = \overline{\theta}\frac{\alpha}{\eta}(1+\overline{\tau_{\kappa\kappa}})(\rho+\tau_w), \tag{C.19}$$

$$\overline{\omega}\rho = \overline{\theta}\left[\frac{\alpha}{\eta}(\delta-\rho) - \delta\right]. \tag{C.20}$$

From these two equations, the following optimal capital tax rate is obtained:

$$\overline{\tau_{\kappa\kappa}} = \frac{\frac{\alpha}{\eta}\rho^2 - (1-\frac{\alpha}{\eta})\delta\tau_a}{(1-\frac{\alpha}{\eta})\delta(\rho+\tau_a)} > 0. \tag{C.21}$$

$\overline{\tau_{\kappa\kappa}} > 0$ if $\tau_a \in [0, \dfrac{\frac{\alpha}{\eta}\rho^2}{(1-\frac{\alpha}{\eta})\delta})$. In such an interval, $\overline{\tau_{\kappa\kappa}}$ is a monotonically decreasing function of τ_a. When $\tau_a = 0$, then $\widetilde{\tau}_{\kappa\kappa} = \frac{\alpha\rho}{(\eta-\alpha)\delta} > 0$.

The long-run planner's optimal condition with respect to c is

$$\overline{\mu\omega c} = 1 + \overline{\mu}\rho\overline{\theta}\overline{x}. \tag{C.22}$$

Substituting (C.20) into (C.22), we obtain

$$\overline{\mu\theta} = -\frac{\eta\rho}{c[(\eta-\alpha)\delta+\alpha\rho]+\eta x\rho^2}, \tag{C.23}$$

$$\overline{\mu\omega} = \frac{\delta(\eta-\alpha)+\alpha\rho}{c[(\eta-\alpha)\delta+\alpha\rho]+\eta x\rho^2}. \tag{C.24}$$

Plugging (C.23) and (C.24) into (C.16), and contrasting the obtained relationship with the decentralized optimal condition with respect to ℓ yields

$$\widetilde{\tau}_\ell = \frac{1}{(\eta-\alpha)\delta}\left\{\alpha\rho\overline{c}+\eta\rho^2\overline{x}+\overline{\mu}(1+\phi)\left[\overline{c}[(\eta-\alpha)\delta+\alpha\rho]+\eta\rho^2\overline{x}\right]\right\} > 0. \tag{C.25}$$

where

$$\overline{\mu} = \frac{1}{(1+\phi)}\left\{\frac{(\eta-\alpha)\delta}{\overline{c}[(\eta-\alpha)\delta+\alpha\rho]+\eta\rho^2\overline{x}}\frac{(1-\alpha)\overline{y}}{\psi\ell^{1+\phi}}-1\right\} > 0.$$

D Small open economy: Optimal taxation

Consider the second-best tax policy in a small open economy. Combining (35) with the entrepreneurs' budget constraint and using (33) together with the relationship $f = \kappa - x$ (obtained from the supply of capital), we get

$$y = c + \left[(1-\tau_\kappa)\frac{\alpha}{\eta}+\frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\right]y + g - (\tau_\kappa r^* + \delta)x. \tag{D.1}$$

This is the resource constraint, once x is substituted out by using the entrepreneurs' budget constraint.

Plugging the optimal labor-leisure trade-off of workers in their budget constraint yields

$$cu_c + \ell u_\ell = 0. \tag{D.2}$$

Now, the optimal second-best problem is

$$\max_{\{c,\ell,x,\tau_{\kappa\kappa},\tau_\kappa,\tau_a\}} \int_0^\infty e^{-\rho t} u(c,\ell)\mathrm{d}t,$$

subject to

$$cu_c + \ell u_\ell = 0, \qquad [\mu], \tag{D.3}$$

$$\dot{x} = (1-\tau_\kappa)\left(\frac{\alpha}{\eta}y + r^* x\right) - (\delta+\tau_a)x, \qquad [\mu\theta], \tag{D.4}$$

$$y = c + \left[(1-\tau_\kappa)\frac{\alpha}{\eta} + \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\right] y + g - (\tau_\kappa r^* + \tau_a)x, \qquad [\mu\omega], \tag{D.5}$$

and $x_0 > 0$ given. Output is given by the production function $y = \Theta^*(1+\tau_k)^{-\gamma(\eta-1)} x^\gamma \ell^{1-\gamma}$. Square bracket contains the Lagrange multipliers of the optimal problem.

The current-value Hamiltonian of this Ramsey problem is

$$\mathcal{H} = u(c,\ell) + \mu\left(cu_c + \ell u_\ell\right) + \mu\theta\left[(1-\tau_\kappa)\left(\frac{\alpha}{\eta}y + r^* x\right) - (\delta+\tau_a)x\right] + \\ +\mu\omega\left\{\left[1 - (1-\tau_\kappa)\frac{\alpha}{\eta} - \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\right] y - c - g + (\tau_\kappa r^* + \tau_a)x\right\}. \tag{D.6}$$

The first-order conditions of the Ramsey problem are

$$c: u_c + \mu(u_c + cu_{cc} + \ell u_{c\ell}) = \mu\omega, \tag{D.7}$$

$$\ell: u_\ell + \mu(cu_{c\ell} + u_\ell + \ell u_{c\ell}) = \mu\left\{\theta(1-\tau_\kappa)\frac{\alpha}{\eta} + \omega\left[1 - (1-\tau_\kappa)\frac{\alpha}{\eta} - \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\right]\right\}\frac{(1-\gamma)y}{\ell}, \tag{D.8}$$

$$x: -\dot{(\mu\theta)} + \mu\theta\rho = \mu\theta\left[(1-\tau_\kappa)\left(\frac{\alpha}{\eta}\frac{\gamma y}{x} + r^*\right) - (\delta+\tau_a)\right] + \tag{D.9}$$

$$+\mu\omega\left\{\left[1 - (1-\tau_\kappa)\frac{\alpha}{\eta} - \frac{\alpha(\eta-1)}{\eta(1+\tau_{\kappa\kappa})}\right]\frac{\gamma y}{x} + (\tau_\kappa r^* + \tau_a)\right\}, \tag{D.10}$$

$$\tau_{\kappa\kappa}: (\theta-\omega)\left[(1-\tau_\kappa)\frac{\alpha}{\eta}y_{\tau_{\kappa\kappa}}\right] + \omega\left[y_{\tau_{\kappa\kappa}} - \frac{\alpha(\eta-1)}{\eta}\frac{y_{\tau_{\kappa\kappa}}(1+\tau_{\kappa\kappa}) - y}{(1+\tau_{\kappa\kappa})^2}\right] = 0, \tag{D.11}$$

$$\tau_\kappa: (\theta-\omega)\left(\frac{\alpha}{\eta}y + r^* x\right) = 0 \tag{D.12}$$

$$\tau_a: (\theta-\omega)x = 0. \tag{D.13}$$

where $y_{\tau_{\kappa\kappa}} = \dfrac{\partial y}{\partial \tau_{\kappa\kappa}} = -\dfrac{\gamma(\eta-1)y}{(1+\tau_{\kappa\kappa})}$.

Condition (D.12) implies $\omega = \theta$. Using this condition and the expression $\gamma \equiv \frac{\alpha}{\alpha+\eta(1-\alpha)}$, (D.11) implies $\tau^*_{\kappa\kappa}$=0.

The optimal conditions for τ_κ and τ_a are equivalent. Also in the open economy, choosing τ_κ and τ_a together implies an implementation indeterminacy as these two tax rates are perfectly substitutable. This indeterminacy can be eliminated by fixing one these two tax rates. We choose to fix τ_a and obtain the solution for $\tau_{\kappa\kappa}$ conditional on such a variable.

Considering together (D.10) and (D.4), both evaluated at the steady state, and taking into account $\tau_{\kappa\kappa}^*$=0 yields

$$\tau_\kappa^* = \frac{\rho - \tau_a}{\rho + \delta}. \tag{D.14}$$

$\tau_\kappa^* > 0$ if $\tau_a \in [0,\rho)$. In such an interval, τ_κ is a linear decreasing function of τ_a. When $\tau_a = 0$, $\tau_\kappa^* = \dfrac{\rho}{\delta + \rho} > 0$.