Delayed Childbearing and Urban Revival: A Structural Approach*

Ana Moreno-Maldonado† Clara Santamaria‡

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Abstract

Since 1980, college graduates have increasingly sorted into the downtowns of U.S. cities. This led to urban revival, a process that involves fast growth in income and housing prices downtown. Motivated by the observation that young childless households concentrate downtown, we link urban revival to delayed childbearing. As college graduates postpone parenthood, more of them are childless when young and locate downtown. Estimating a dynamic model of fertility timing and within-city location choices, we find delayed childbearing accounts for 52% of urban revival. The impact of changes in fertility choices is amplified by the response of housing prices and amenities.

Keywords: Residential choice, Fertility Timing, Amenities, Welfare Inequality.
JEL Codes: J13, R21, R23.

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†University of Mannheim. Email: ana.moreno@uni-mannheim.de
‡Sciences Po and CEPR. Email: clara.santamaria@sciencespo.fr

1 Introduction

Residential location choices in the United States are tightly linked to fertility. Households with children are typically overrepresented in the suburbs, where school quality is higher and houses are larger. In contrast, individuals without children tend to concentrate downtown, where there is a high density of consumption amenities such as bars or theaters. This observation suggests that fertility trends may have important implications for the structure of the city and vice versa.

This paper focuses on the link between two trends that have sparked widespread interest. The first, urban revival, refers to the faster increase in housing prices and average income downtown compared with the suburbs that has taken place in U.S. cities since 1980 (Couture and Handbury, 2023). The second trend is the delay in childbearing. It is characterized by the increase in the age at which women have their first child. This trend has been more pronounced among highly educated women (Bailey, Guldi and Hershbein, 2014). Delayed childbearing can be expected to contribute to urban revival if young households with no children value living downtown more than those with children, as suggested by their location choices. In this case, the delay in childbearing, by increasing the share of the young without children, leads to a higher valuation of downtown by the population. Since high-skilled households delay childbearing more, their desire to live downtown rises relative to low-skilled households. Finally, their increased presence downtown generates urban revival.

The goal of this paper is to quantify for the first time to what extent the delay in childbearing contributes to urban revival, accounting for the general equilibrium effects of amenities and housing markets, and to measure its implications for welfare inequality. We find that delayed childbearing accounts for 52% of the observed urban revival and that endogenous amenities and housing prices played an important role. This quantification advances our understanding of potential winners and losers of the observed change in urban structure. Whereas households benefit from improvements in amenities generated by urban revival, increases in housing rents may undermine these gains. The valuation of different aspects of amenities and the cost of housing depends on each household's characteristics (Almagro and Dominguez-Iino, 2019). Thus, understanding the drivers behind urban revival is crucial for establishing the net welfare effect for different demographic groups. Moreover, we show that connecting these two trends is key for evaluating the welfare consequences of changes in the incentives to delay fertility. We find that ignoring the effect of delayed childbearing on the urban structure would lead to underestimating welfare inequality.

To quantify how much urban revival can be accounted for by the delay in childbearing, we proceed in two steps. First, we provide evidence of the link between fertility decisions and urban revival. We document trends in fertility and residential choices and perform a decomposition of urban revival to isolate the effect of changes in fertility. Second, we propose and quantify a structural dynamic model of fertility timing and within-city location choice. We use the model to evaluate how much urban revival can be explained by changes in the incentives to delay childbearing, incorporating general equilibrium effects through housing prices and endogenous amenities. Finally, the model allows us to evaluate the welfare implications of delayed childbearing.

We start by documenting some facts on the link between fertility and residential locations in the United States. First, we show that families are around 35% less likely to reside in downtown neighborhoods than households with no children and that this has been a persistent aspect of U.S. cities since 1970.[1] Second, we show that younger households have a higher propensity to live downtown: Close to 14% of households between 20 and 24 years old lived downtown in 1980 compared with only around 8% of households between 35 and 44 years old. Third, women have increasingly delayed motherhood (Bailey, Guldi and Hershbein, 2014): In 1970, only 10% of women had their first child after 30 years old; the proportion increased to 25% by 2010 for all women and to 45% for college-educated women.

Next, we perform a back-of-the-envelope decomposition of the increase in the ratio of average income downtown relative to the suburbs. We isolate the effect of changes in the fraction of individuals with children, conditional on age and skill. We find that in this decomposition, the composition of fertility can account for 20% of the increase in the average income ratio, consistent with Couture and Handbury

[1] Downtown is defined as containing 10% of the city's population. Around 12.5% of households with no children reside downtown, compared with only around 8.5% of households with children.

(2020)'s findings.

Motivated by this reduced-form evidence, we propose and estimate a structural dynamic model of fertility timing and within-city location choice. We use the model to quantify both the direct impact of delayed childbearing on urban revival and the indirect impact via the endogenous response of housing prices and neighborhood amenities. One novelty of our analysis is that we quantify a new feedback effect that arises from the interaction of location and fertility choices. As more young individuals without kids remain in the center, downtown neighborhoods become more attractive to young childless individuals, which creates additional incentives to delay parenthood and reinforces the process of urban revival.

The model features a city with two types of locations, downtown and suburbs, that differ in amenities and housing supply. The city is a single labor market, so household income does not change with location.[2] Households are heterogeneous in their skill level and their preferences for locations and children. They decide whether and when to have children and where to live.

Households face a trade-off when choosing the timing of parenthood. On the one hand, waiting implies that they may not be able to have children due to declining fertility with age. On the other hand, the income penalty for having children is reduced when households delay. The lower child penalty associated with postponing, labeled the *delay premium*, is an important driver of delayed childbearing (Caucutt, Guner and Knowles 2002; Buckles 2008; Adda, Dustmann and Stevens 2017).

Lastly, we model an endogenous supply of amenities that reacts to changes in the demographic composition of neighborhoods. We include a response of amenities to the ratio of high-skilled to low-skilled households, as has been previously done in the literature.[3] In addition, we allow the supply of amenities to react to the ratio of households with no kids, relative to households with kids.

We estimate the model using individual-level census data for the United States from 1990 to 2010. First, to estimate income penalties for having kids that vary by period and fertility timing, we follow Kleven (2023). Consistent with previous

[2]For simplicity, we abstract from commuting decisions. In the estimation, commuting costs associated with a location will be captured by the location's amenities.

[3]See for instance, Guerrieri, Hartley and Hurst (2013); Almagro and Dominguez-Iino (2019); Hoelzlein (2019); Curci and Yusaf (2020); Su (2022).

findings, our estimates reveal the existence of a delay premium, because women who delay experience a lower penalty than those who have kids earlier.[4] Then, we estimate the Fréchet parameter for the idiosyncratic preference for children by regressing fertility choices on model-based economic incentives that impact fertility. Finally, we propose a novel identification strategy to estimate the elasticity of the supply of endogenous amenities to the ratio of households without kids to households with kids. The identification challenge comes from the endogeneity of the number of households with no kids in a location. We overcome this challenge by exploiting the introduction of state-mandated insurance to cover infertility treatments, following Schmidt (2007). This state-level policy impacted the incentives to delay childbearing and thus the ratio of households with no kids to households with kids in each location, without a direct impact on the geography of the city.

To quantify the importance of fertility timing for urban revival, we perform a counterfactual exercise in which we keep the delay premium fixed to its level in 1990, when it was lower. As a consequence, a higher share of young high-skilled households have children and prefer to live in the suburbs. We find that changes in the delay premium account for 52% of the observed difference in income growth between downtown and the suburbs. Of this, the direct effect of the change in the fertility composition accounts for only 7.5 p.p., while the rest is generated by the endogenous response of housing prices and amenities. Lastly, we document a moderate feedback effect stemming from the fertility response to changes in housing prices and amenities.

Next, we use our model to analyze the consequences of urban revival for welfare inequality. We find that amenities related to the presence of high-skilled households were the most important contributors to the increase in welfare inequality between 1990 and 2010.[5] In addition, we find that the surge in the number of young households with no children led to a reaction of endogenous amenities that benefited the high-skilled more than the low-skilled.

[4]Estimates of the delay premium can be found in Miller (2011); Adda, Dustmann and Stevens (2017); and Gallen et al. (2023).

[5]This is consistent with previous results in the literature that points to a quantitatively important role of high-skill amenities. See, for instance, Diamond (2016); Almagro and Dominguez-Iino (2019); Couture et al. (2019); Su (2022)

Finally, we quantify the welfare consequences of the increase in the delay premium. Ignoring its effect on urban revival would lead to underestimating the change in high-skilled welfare by 2 p.p. and overestimating it by 1 p.p. for the low-skilled. This exercise highlights the importance of incorporating the spatial impact of changes in fertility to evaluate the welfare effects of long-run demographic trends.

Related literature. This paper contributes to three main strands of the literature. First, we contribute to the literature that analyzes the causes of urban revival. This literature has provided evidence for a wide range of drivers, such as changes in the characteristics of downtown neighborhoods (Glaeser, Kahn and Rappaport, 2008; Brueckner and Rosenthal, 2009; Ellen, Horn and Reed, 2019; Curci and Masera, 2018); increased valuation of downtown characteristics that did not change (Edlund, Machado and Sviatschi, 2015; Su, 2022); and changes in the demographic composition of the population (Baum-Snow and Hartley, 2019; Couture et al., 2019; Fogli and Guerrieri, 2019). In this paper, we focus on a novel demographic change: the delay in childbearing and the resulting increase in the number of young, high-skilled households with no children.

The most closely related paper is by Couture and Handbury (2020). They first estimate a model of residential choice by skill and age and show that urban revival is driven by an increase in the attraction of young college graduates to the city center. Next, they perform a decomposition exercise in which the propensity to live downtown remains fixed and show that changes in the family composition of young college graduates account for a sizeable share of the increased presence of this group downtown. Our work builds on their insight by adding fertility choices and endogenous amenities to a dynamic model of residential choice by age and skill. Therefore, in our setup, differences in the propensity to live downtown by family composition, age, and skill are endogenous. This allows us to quantify the general equilibrium effects of delayed childbearing on urban revival and evaluate the implications for welfare inequality. Moreover, by modeling fertility choices, we highlight a new amplification mechanism: Changes in amenities and housing prices have a feedback effect on fertility choices that reinforce urban revival. As central locations become more attractive for households with no kids, the incentives to delay childbearing increase.

Moreover, recent work by Guerrieri, Hartley and Hurst (2013); Behrens et al. (2018); Almagro and Dominguez-Iino (2019); Hoelzlein (2019) and Curci and Yusaf (2020) suggests that even initially small demographic changes in a location can induce a large transformation of the neighborhood because endogenous amenities amplify the effect. We contribute to this literature by quantifying the reaction of endogenous amenities to the share of childless households in a location.

Second, we contribute to the literature on the economic determinants and consequences of fertility (for a recent review, see Doepke et al. (2023)). The empirical work in this literature emphasizes the role of access to the pill, abortion, or infertility treatments in the delay in childbearing (see Bailey, Guldi and Hershbein (2014); Bailey and Hershbein (2018) for a review). This paper is most closely related to the macro literature that highlights the trade-off between career and fertility timing (Caucutt, Guner and Knowles, 2002; Erosa, Fuster and Restuccia, 2002; Attanasio, Low and Sánchez-Marcos, 2008; Adda, Dustmann and Stevens, 2017). We contribute to this strand of the literature by embedding a dynamic fertility choice model in a spatial equilibrium framework of residential location. By doing so, we can analyze how delayed fertility impacts welfare through its effect on urban structure. Moreover, we quantify a new driver of delayed fertility: the increased attractiveness of downtown areas for young, childless households.

Third, we contribute to a growing literature that studies the interaction of household choices and location choices. This literature has highlighted the role of spatial frictions in shaping gender gaps in labor markets (Petrongolo and Ronchi, 2020; Barbanchon, Rathelot and Roulet, 2021; Kwon, 2022; Liu and Su, 2022; Moreno-Maldonado, 2022; Farré, Jofre-Monseny and Torrecillas, 2023); the interaction between local marriage and labor markets (Fan and Zhou, 2021; Alonzo, 2022; Alonzo, Guner and Luccioletti, 2023); the role of housing markets on fertility (Coeurdacier et al., 2023); the importance of household composition for life-cycle patterns of sorting across cities (Albouy and Faberman, 2018); and the spatial determinants of school choice (Agostinelli, Luflade and Martellini, 2021; Pietrabissa, 2023). To the best of our knowledge, we are the first to estimate a structural spatial equilibrium model that includes the interaction of fertility decisions, including fertility-timing, and within-city location choices.

The rest of the paper is organized as follows. Section 2 presents evidence on delayed childbearing and urban revival. Section 3 introduces the structural model of fertility and within-city residential location choices. Section 4 quantifies the model, Section 5 presents results from our counterfactual exercise, and Section 6 from the welfare analysis. Section 7 concludes.

2 Empirical evidence

This section presents evidence to motivate the quantitative model. We start by documenting three stylized trends: urban revival, delayed childbearing, and the downtown overrepresentation of households with no children and young households. We then conduct a back-of-the-envelope decomposition of urban revival to quantify the role of changes in the fertility composition.

2.1 Data sources and definitions

The paper exploits information from three datasets. The first contains observations at the census tract level. The second contains observations at the individual level, but the smallest geographic unit is a Public Use Microdata Area, which is significantly larger than a census tract. We complement these data with a third dataset, the Vital Statistics Natality Birth Data, from the National Center for Health Statistics (NCHS). It contains information on every birth in the United States, including the age of the mother and whether it is their first birth.

The census-tract-level dataset combines decennial Census data and the American Community Survey (ACS) 2008-2012, from the National Historical Geographic Information System (NHGIS).[6] The analysis is conducted at constant 2010 census tract boundaries using the Longitudinal Tract Data Base (LTDB).[7] Census tracts are small geographical units that encompass between 2,500 and 8,000 people. The advantage of this dataset is the precision of the geographical location. The limitation is that we can only observe aggregate counts of population at census tract level. This limits the analysis to the subpopulations available in the NHGIS. For example,

[6]Data citation for National Historical Geographic Information System: Manson et al. (2021).
[7]Data citation for the Longitudinal Tract Data Base: Logan, Xu and Stults (2014)

it is not possible to simultaneously observe the number of individuals by age and the presence of children in the household.

The individual-level dataset is constructed from the ACS and contains information at the individual level.[8] This allows us to simultaneously analyze age, presence of children, and income quantiles. However, the smallest geographical area that is identifiable is a Public Use Microdata Area (PUMA) for 1990 to 2010; and county groups for 1980. PUMAs contain no fewer than 100,000 individuals each. We exclude from our sample individuals in group quarters or whose incomes are low enough to access public housing. We also drop individuals with top-coded incomes.

Cities. A city is defined as the Core-Based Statistical Area (CBSA) constructed by the Census Bureau. We restrict our sample to metropolitan areas with more than 1 million inhabitants. The sample size of the census-tract-level dataset includes 82,129 census-tract–year observations. The individual-level dataset includes 7.4 million individual-year observations.

City center, downtown, and suburbs. The center of a city is identified as the centroid of the census tracts that are included in the 1982 Census of Retail Trade, following Lee and Lin (2018). This definition is intended to capture the Central Business District of a city. In line with previous literature, downtown is defined as the smallest circle around the city center that includes 10% of the population of the city. Similarly, we define suburbs as the area of the city that contains the 50% of the population who live the farthest from the city center.

In the individual-level dataset, PUMAs are not always small enough to identify the downtown and suburbs of cities. A PUMA is classified as downtown if at least 50% of its population lives in a downtown census tract and as suburbs otherwise. A CBSA is included in the sample if at least 50% of downtown census tracts are within a PUMA classified as downtown.

2.2 Descriptive patterns

In this section, we document three stylized facts that suggest that delayed childbearing can be a quantitatively important driver of urban revival. First, urban revival

[8]Data citation for the American Community Survey: Ruggles et al. (2023).

was quantitatively large and started in 1980. Second, families tend to be overrepresented in the suburbs, while young individuals without children are overrepresented downtown. Third, there has been a pronounced increase in the age of first-time mothers over the last few decades.

Fact 1. Urban revival. The term urban revival refers to the transformation of a city characterized by faster income growth in downtown locations than in the suburbs. It is accompanied by an increase in housing prices downtown, which may lead to displacement of the original residents. Figure 1 plots the ratio of average income and average housing value in downtown relative to suburbs. The process of urban revival started around the 1980s and has continued until the present.[9]

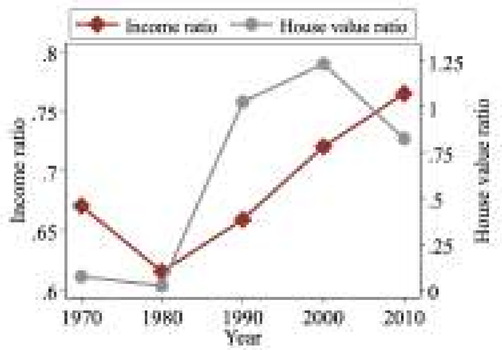


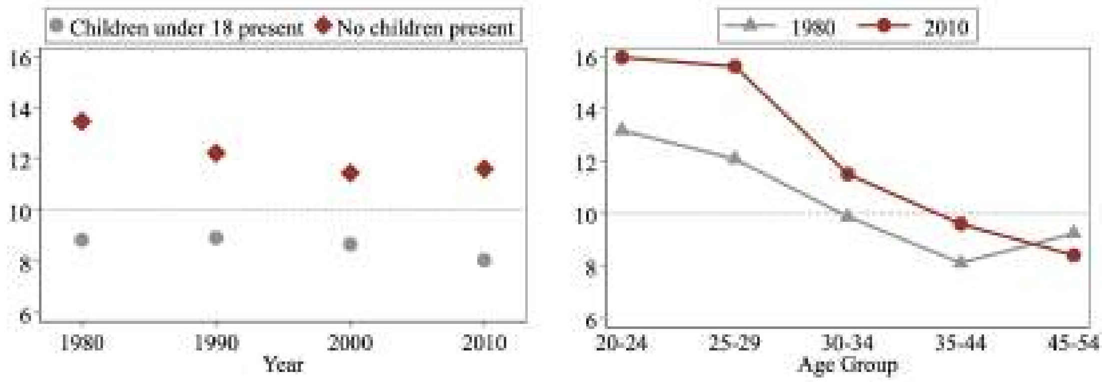
Figure 1: Urban revival

Notes: This figure plots two measures of urban revival. On the left axis, it plots the ratio of average income in downtown relative to suburban locations, and on the right axis, it plots the ratio of average housing values. It includes Core Based Statistical Areas above 1 million inhabitants. Source: NHGIS U.S. Census Data 1970 to 2010.

Fact 2. Families live in the suburbs and the young live downtown. Panel (a) in Figure 2 exhibits differences in location choices by households depending on the presence of own children under 18. Households with children are around 4 p.p. less likely to reside in downtown neighborhoods. This has been a persistent aspect of U.S. cities since at least 1980. Moreover, in Panel (b), we show that young individuals are more likely to live downtown than older individuals. Around 16% of households between 20 and 24 live downtown compared with around 8% of households between 45 and 54. This negative relationship between age and propensity to live downtown

[9]The figure shows a slowing down in the house value. However, in the quantification of the model, we estimate a hedonic price index and do not observe the slowdown. Thus, the pattern in house values is likely due to the relative quality of houses in the suburbs increasing faster than the relative hedonic price.

was already present in 1980 and had become steeper by 2010. The age category for which the propensity to locate downtown increased the most (by about 4 p.p.) is the group between 25 and 29, those most likely affected by the delay in childbearing.



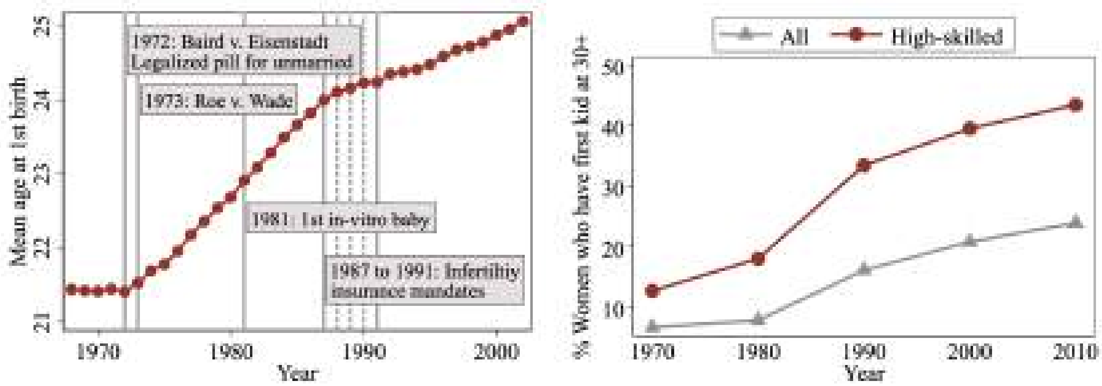
(a) Family type (b) Age

Figure 2: Percentage living downtown

Notes: This figure shows the percentage of households living downtown for different demographic groups in the US from 1980 to 2010. In panel 2a, we distinguish between households depending on whether own children under 18 years old are present. For the year 2000, this includes families in which children are present but not necessarily the children of the head of the household. Panel 2b displays the propensity to locate downtown for different age groups. Source: NHGIS U.S. Census Data 1980 to 2010.

Fact 3. Households increasingly delayed childbearing. Panel (a) in Figure 3 displays the average age of women at their first birth from 1970 to 2005, using Vital Statistics Natality Birth Data. It increased substantially from slightly above 21 years old in 1970 to 25 years old by 2003. The increase in age at first birth has been linked to supply-side factors, such as the introduction of the contraceptive pill, legalization of abortion, or advancements in assisted reproductive technologies Goldin and Katz (2002); Bailey (2006, 2010); Gershoni and Low (2021); Myers (2017); Schmidt (2007). Demand factors also played an important role in the timing of fertility. As pointed out by Goldin (2006), changes in fertility timing reflect the expansion of women's horizons in their decisions regarding human capital investments.

Panel (b) in Figure 3 uses census data from the ACS to show the differential effect for college-graduated women. This graph features the percentage of women who had their first child after age 30 among the cohort of women between 30 and 40 years old with children in that census year. We plot these series separately for the overall population and for women who are college-educated and above. In 1970, only about 10% of women had children after 30. This increased steadily, with a



(a) Mean age at 1st birth

(b) Percentage having kids after 30

Figure 3: Delay in Childbearing

Notes: Panel (a) plots the average age of first-time mothers in the US, together with some important events that may be related to this trend. The series ends in 2003, when there was a change in how age is collected. Panel (b) displays the fraction of women aged 30 years and older who had their first child after 30. High-skilled women include women with a college education or above. Source: ACS IPUMS data from 1970 to 2010 and NCHS's Vital Statistics Natality Birth Data 1968 to 2003

large increase between 1980 and 1990. By 2010, more than 20% of women overall and more than 40% of high-skilled women had children after the age of 30.

This rise in childbearing age implies a greater proportion of young individuals without children in the population who, as we have just shown, have the highest propensity to locate downtown. Hence, this demographic change could have fueled demand for downtown neighborhoods, particularly among college-graduated households, and thus contributed to urban revival. Next, we carry out a decomposition that exploits individual-level data to bring together facts 2 and 3.

2.3 Decomposition of urban revival

Urban revival is measured by the increase in the ratio of mean income downtown relative to the suburbs, which is defined as

$$\text{Inc. Ratio}_t = \frac{\sum_g \left(\text{Inc}_{g,dtown,t} \cdot \text{comp}_{g,dtown,t}\right)}{\sum_g \left(\text{Inc}_{g,sub,t} \cdot \text{comp}_{g,sub,t}\right)},$$

where g denotes a demographic group that is characterized by a combination of a 10-year age bin, whether there are kids present in the household, and, if so, whether they are older or younger than 10, and the income decile conditional on age and year. We decompose the changes in this measure of urban revival to isolate the

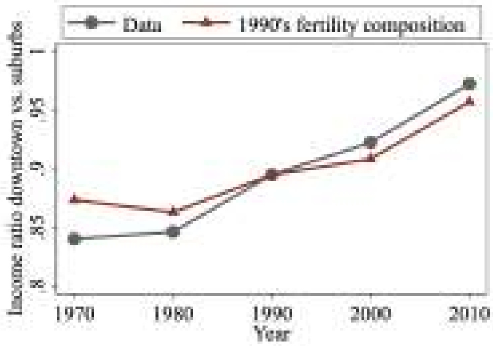


Figure 4: Decomposition of urban revival

Notes: This figure plots the decomposition of urban revival to isolate changes in the fertility composition. The vertical axis plots the ratio of average income downtown relative to the suburbs. The increase in the ratio observed in the data reflects urban revival. Age is captured by 10-year bins. The first bin includes everyone below 20, while the last bin includes everyone above 60 years old. Fertility is captured by the presence of kids below 10 years old, kids above 10 years old, or no kids. Data source: ACS from 1970 to 2010.

effect of changes in the fertility composition. We keep the percentage of households with and without children as in 1990, conditioning on age and income decile.

The role of the fertility composition. The demographic composition of downtown and suburbs can be written as follows:

$$\begin{aligned}\text{comp}_{g,dtwon,t} &= \frac{\text{Pop}_{g,dtown,t}}{\sum_l \text{Pop}_{g,l,t}} \Big/ \frac{\sum_j \text{Pop}_{j,dtown,t}}{\sum_{j,l} \text{Pop}_{j,l,t}} \cdot \frac{\sum_l \text{Pop}_{g,l,t}}{\sum_{j,l} \text{Pop}_{j,l,t}} \\ &= \text{RelProp}_{g,dtown,t} \cdot \text{PopComp}_{g,t},\end{aligned}$$

where g is the demographic group that is, age, kids' presence, and skill. The first term is the relative propensity to locate downtown of demographic group g relative to the population. The second term is the fraction of the total population that demographic group g represents. The change in the presence of group g downtown can arise either because this demographic group becomes relatively more likely to live downtown or because its presence in the population increases.

To isolate the effect of changes in the aggregate fertility composition, we write the population composition as the product of the composition by age; the composition by skill, given age; and the composition by fertility, conditional on age and skill:

$$\text{PopComp}_{a,k,s,t} = \text{comp}_{a,t} \cdot \text{comp}_{s|a,t} \cdot \text{comp}_{k|a,s,t}.$$

Figure 4 plots the counterfactual change in the income ratio if the presence of children, conditional on age and skill, had stayed as in 1990. It shows that there would have been a slower increase in the average income of downtown relative to the suburbs. The fertility composition alone can account for 20% of the increase in the average income between 1990 and 2010.

The decomposition suggests that changes in fertility were large enough to have played a direct role in urban revival. However, the conclusions are limited, since the propensity to live downtown is not independent of the fertility composition; for example, due to changes in housing prices or endogenous amenities. To capture interactions between the two, in the next section we estimate a model in which households choose where to live and whether and when to have children.

The stylized facts capture only correlations, and there are several reasons to be cautious in the causal interpretation. First, we cannot rule out reversed causality, that is, the possibility that, as downtown became more vibrant, young individuals chose to postpone childbearing in order to enjoy downtown amenities for longer. Second, there could be omitted common drivers of both urban revival and the delay in childbearing. To establish the direction of causality and to quantify the role of endogenous amenities, we estimate a structural model of fertility timing and within-city residential location choices in the next section.

3 A model of fertility and location choice

In this section, we propose a model of endogenous fertility timing and location choice. The goal of the model is to quantify how much of the observed urban revival can be explained by the delay in the age at first birth and the welfare consequences of this delay. The model is necessary in order to include general equilibrium effects of a counterfactual change in the incentives to delay childbearing (e.g., effects on the housing market and endogenous amenities) and to draw welfare implications.

3.1 Model setup

Geography and housing. The geography in this economy consists of a single city that features a set of locations indexed by $l = \{1, ..., N\}$. There are two types of locations: downtown (d) and suburbs (s). There is one downtown location, indexed $l = 1$, and $N - 1$ suburb locations, indexed $l = 2, ..., N$. All suburban locations are identical but agents have idiosyncratic preferences for each particular location. The suburban locations differ from the downtown location in two time-varying dimensions: amenities and housing supply.

There is no cost to move across locations and amenities are local, that is, they can only be consumed by residing in the location. Moreover, the city is a unique labor market, so income is independent of the within-city location of residence and there is no commuting.[10] There is free trade of the final commodity, which is used as the numeraire.

The housing stock is owned by absentee landlords who can add or subtract units from the market. The number of units supplied in a location by absentee landlords depends on the housing price in that location, p_l. The housing supply in location l is given by the following function:

$$H_l(p_l) = H_l^0 p_l^{\alpha_l}.$$

The parameters of the housing supply, H_l^0 and α_l, are allowed to differ between suburban and downtown locations, but they are the same in all of the suburban locations.[11]

Households. The economy is inhabited by a mass of households indexed by i. Households live for three periods. In the first period, they are young (y), then mature (m), and finally old (o). Let $a \in \{y, m, o\}$ index age. Households also differ in their skill level, z, which is constant throughout their lifetime. Households choose where to live and whether and when to have children. $L_t(a, z)$ denotes the exogenous mass of households of age a and skill z at period t in the city.

Households derive an idiosyncratic utility from residing in location l. Each period, they draw a vector of idiosyncratic preferences, $\vec{\varepsilon}_t^{\,i} = \left\{\varepsilon_{t,l}^i\right\}_{l=1}^N$ from a Fréchet distribution with shape parameter β_ε. Moreover, at the beginning of their lifetime, households draw an idiosyncratic preference for children, η^i, which is also distributed as a Fréchet with shape parameter β_η. This idiosyncratic amenity is enjoyed in each of the periods in which children are present in the household.

[10] Differences in commuting cost between downtown and suburbs will be captured in the downtown amenity.

[11] In the quantification, the housing supply elasticity, α_l, will be lower in the downtown location, as estimated by Baum-Snow and Han (2021).

Children. Children are present in the household for two periods. Let $k = 0$ if no kids are present, $k = 1$ if the household had kids that period, and $k = 2$ if they had kids the previous period. Conditional on wanting children, there is a probability that the household will be successful. We assume that the probability is equal to one when young, $\rho_y = 1$; zero when old $\rho_o = 0$; and a number between zero and one when mature, $0 < \rho_m < 1$.

Preferences. Households derive utility from consumption and amenities. They must buy one unit of housing in the location where they reside.[12] The unit of housing requirement does not depend on the presence of children.[13] Households consume the income that was not spent on housing. Thus, a household i at age a with kids aged k, skill z, and living in location l at time t derives the following indirect utility:

$$U_t^i\left(I_t, p_{t,l}; a, k, z, l\right) = \left[I_t\left(a, k, z\right) - p_{t,l}\right] \cdot \delta_t\left(a, k, z, l\right) \cdot \varepsilon_{t,l}^i \cdot \left[\eta^i\right]^{D_{k>0}},$$

where $D_{k>0}$ is a dummy taking value one in the period the household has kids, and $p_{t,l}$ is the price of housing at time t and location l. Agents apply discount factor ϕ to future periods and have perfect foresight. All agents derive utility from amenity $\delta_t\left(a, k, z, l\right)$ and from two idiosyncratic amenity shocks, one for location, $\varepsilon_{t,l}^i$, and one for the presence of children, η^i.

Amenities, $\delta_t\left(a, k, z, l\right)$, and income, $I_t\left(a, k, z\right)$, depend on the presence and age of kids, k, in addition to age, a, and skill, z. The flexibility on income can capture child penalty effects that vary by age and skill. To clarify identification, we impose the following structure on the amenity:

$$\delta_t\left(a, k, z, l\right) \equiv \underbrace{\lambda_t\left(a, k, z, l\right)}_{\text{Location amenity}} \cdot \underbrace{\left[\kappa_{y,t}\left(z\right)\right]^{D_{k>0\&\text{kids early}}}}_{\text{Taste for kids early}} \cdot \underbrace{\left[\kappa_{m,t}\left(z\right)\right]^{D_{k>0\&\text{kids delayed}}}}_{\text{Taste for kids delayed}}. \quad (1)$$

The amenity is the product of two components. First, residential-location amenities, $\lambda_t(a, k, z, l)$, which are any characteristic of a location l that affects households' utility in a way that may depend on their demographics. For example, this can

[12]The unit housing requirement is a simple way to introduce non-homothetic housing demand. For a discussion of how well it fits the data and alternative non-homothetic preferences, see Finlay and Williams (2020).

[13]Although a simplifying assumption, estimating the housing requirement separately for households with and without kids is unlikely to have a large quantitative impact. The reason is that the housing expenditure share conditional on household income, is only slightly higher when children are present. For details, see section A.1 in the Appendix.

capture any location-specific amenities related to kids, such as the availability of high-quality schools or proximity to parks. Second, a kid-related amenity, $\kappa_{y,t}(z)$ or $\kappa_{m,t}(z)$, that is independent of location and that depends on the skill and the age at childbearing. This captures the utility all households enjoy when children are present. We allow it to depend on whether the household has kids early or delays but do not impose any particular preference.

Supply of local amenities. The supply of amenities may respond to local demand. If the local population demands a certain type of amenities, such as restaurants or daycare centers, there will be entry of providers for those amenities and the neighborhood will change. We model this relationship in a reduced-form manner similar to Diamond (2016). Let the endogenous amenity supply in location l at time t for group $g = (a, k, z)$ of age a, kids k, and skill z be given by

$$\lambda_t(g,l) = \left(\frac{N_{l,t}^{High-Skill}}{N_{l,t}^{Low-Skill}}\right)^{\gamma_1(g)} \left(\frac{N_{l,t}^{No-Kids}}{N_{l,t}^{Kids}}\right)^{\gamma_2(g)} \chi_t(g,l), \tag{2}$$

where $\chi_t(g,l)$ is the exogenous portion of amenities in l at time t for demographic group g, and $\gamma_1(g)$, $\gamma_2(g)$ are the elasticities of amenities for demographic group g concerning the ratio of high-skill to low-skill households, and of households with no children relative to households with children in a location, respectively.

Timing. When households are born, they draw an idiosyncratic preference for children η^i. Each period, they observe whether they had kids in the previous period. If not, they decide whether to try to have kids this period. Once they have discovered their realized kid state, they draw a vector of idiosyncratic preferences for locations, $\vec{\varepsilon}_t^{\,i}$, and choose where to live. They then consume and produce.

The key timing assumption is that agents observe their location preference only after having decided whether to have children. Under this assumption, fertility choices are partly driven by downtown amenities that are common to everyone. For instance, if the amenity of living downtown increases for households with children, more households may want to have children. However, the timing assumption rules out selection into delayed fertility of households that idiosyncratically enjoy living downtown more. This assumption is reasonable to the extent that idiosyncratic preferences for location are more likely to change quickly and unexpectedly, while childbearing decisions are more permanent. In other words, at least some of the taste shocks related to location are realized only after making decisions on whether and when to have children.

3.2 Definition and characterization of equilibrium

Location choice. A household i of age a, kids aged k, skill z, and with an idiosyncratic location preference vector $\vec{\varepsilon}_t^{\,i}$ chooses the optimal location at each period in order to solve the following problem:

$$v_t\left(a,k,z,\vec{\varepsilon}_t^{\,i}\right)=\max_l\left\{x_t\left(a,k,z,l\right)\cdot\delta_t\left(a,k,z,l\right)\cdot\varepsilon_{t,l}^i\right\},$$

where $x_t\left(a,k,z,l\right)=I_t\left(a,k,z\right)-p_{t,l}$ is the observed component of the indirect utility from living in location l.

Given the assumption that $\varepsilon_{t,l}^i$ is distributed as a Fréchet with shape parameter β_ε, we can obtain the fraction of households that will choose to live downtown in a given period:

$$\pi_t^{loc}\left(d|a,k,z\right)=\frac{x_t\left(a,k,z,d\right)^{\beta_\varepsilon}\Delta_t\left(a,k,z\right)^{\beta_\varepsilon}}{x_t\left(a,k,z,d\right)^{\beta_\varepsilon}\Delta_t\left(a,k,z\right)^{\beta_\varepsilon}+\left(N-1\right)x_t\left(a,k,z,s\right)^{\beta_\varepsilon}},\tag{3}$$

where $\Delta_t\left(a,k,z\right)=\lambda_t\left(a,k,z,d\right)/\lambda_t\left(a,k,z,s\right)$ denotes the ratio of downtown to suburban amenity. Recall that $N-1$ is the number of identical suburban locations. We employ $d=1$ for the location index of the downtown location and $s\in\{2,...,N\}$ for the index of any of the suburban locations.

Fertility choice. Households choose among three possibilities: to have children while young, to postpone childbearing, or to not have children at all. Let $v_t^*\left(a,k,z\right)=Ev_t\left(a,k,z,\vec{\varepsilon}_t^{\,i}\right)$ denote the expected optimal utility of arriving in period t, at age a, with kids k, and skill z, knowing the household will then draw idiosyncratic preferences for location and choose the optimal location.

The lifetime utility of someone born at t from having kids when young, $v_{ky,t}^*$, kids when mature, $v_{km,t}^*$, or not at all, $v_{nk,t}^*$, for a household with idiosyncratic preference for children, η^i, is given by

$$\begin{aligned} v_{ky,t}^{*}\left(z;\eta^{i}\right) &= v_{t}^{*}\left(y,k=1,z\right)\eta^{i}+\phi v_{t+1}^{*}\left(m,k=2,z\right)\eta^{i}+\phi^{2}v_{t+2}^{*}\left(o,k=0,z\right) \\ v_{km,t}^{*}\left(z;\eta^{i}\right) &= v_{t}^{*}\left(y,k=0,z\right)+\phi v_{t+1}^{*}\left(m,k=1,z\right)\eta^{i}+\phi^{2}v_{t+2}^{*}\left(o,k=2,z\right)\eta^{i}, \\ v_{nk,t}^{*}\left(z\right) &= v_{t}^{*}\left(y,k=0,z\right)+\phi v_{t+1}^{*}\left(m,k=0,z\right)+\phi^{2}v_{t+2}^{*}\left(o,k=0,z\right). \end{aligned} \tag{4}$$

An agent with an idiosyncratic preference for children of η^i will then solve:

$$\max\left\{v_{ky,t}^{*}\left(z;\eta^{i}\right),\quad \rho_m\left(v_{km,t}^{*}\left(z;\eta^{i}\right)\right)+\left(1-\rho_m\right)v_{nk,t}^{*},\quad v_{nk,t}^{*}\right\},$$

where ρ_m is the probability of a successful pregnancy when mature.

Since agents have perfect foresight, the fertility decision is made once when households are young. The optimal decision can be characterized by two thresholds of η^i, according to which agents will choose one of the three paths: no kids, kids early, or delay.

Define a threshold for households born at t with skill z, $\bar{\eta}_{i,j}^{t}(z)$, such that at the threshold individuals born at t are indifferent between options i or j, and above the threshold, they prefer i to j, where i and j index each of the fertility choices, i.e., $i,j\in\{ky,km,nk\}$. In an equilibrium path in which a positive mass of households delay and preferences are transitive, it must be the case that

$$\bar{\eta}_{km,nk}^{t}\left(z\right)<\bar{\eta}_{ky,nk}^{t}\left(z\right)<\bar{\eta}_{ky,km}^{t}\left(z\right).$$

This implies that households with a high idiosyncratic preference for children will have children when young; those with a low idiosyncratic preference will choose not to have children; and those in the middle will delay and face the possibility of infertility. The shape of the Fréchet distribution for idiosyncratic preferences, β_η, determines how the fraction of agents making each choice reacts to changes in the thresholds. If the distribution is more dispersed, the same change in the thresholds will have smaller effects on the fraction of people choosing each fertility path.[14]

Now we can compute the fraction of households born at time t of each skill z

[14] Figure B.2 in the Appendix plots the distribution of idiosyncratic preferences together with the thresholds.

who choose each of the three options when young:

$$\begin{aligned} Prob_t\left(\text{kids young}; z\right) =& 1 - F_\eta\left(\bar{\eta}^t_{ky,km}\left(z\right)\right), \\ Prob_t\left(\text{delay}; z\right) =& F_\eta\left(\bar{\eta}^t_{ky,km}\left(z\right)\right) - F\left(\bar{\eta}^t_{km,nk}\left(z\right)\right), \\ Prob_t\left(\text{no kids}; z\right) =& F_\eta\left(\bar{\eta}^t_{km,nk}(z)\right), \end{aligned} \tag{5}$$

where $F_\eta(x)$ is the Fréchet distribution of the idiosyncratic preference for children and has shape parameter β_η. From here, we can obtain the fraction of households with realized fertility outcome k at time t, $\pi^{fert}_t\left(k|a,z\right)$, conditional on age, a, and skill, z.

Housing market. The housing price is such that the housing market will clear in each location l and period t. Namely,

$$H^0_{t,l} p^{\alpha_l}_{t,l} = \sum_{a,k,z} \pi^{loc}_t\left(l|a,k,z\right) \pi^{fert}_t\left(k|a,z\right) L_t\left(a,z\right). \tag{6}$$

Definition of a period equilibrium Given a sequence of fundamentals, that is, demographic composition, $\left\{L_t\left(a,z\right)\right\}^\infty_{t=t_0}$; exogenous amenities of locations and children, $\left\{\chi_t\left(a,k,z,l\right)\right\}^\infty_{t=t_0}$ and $\left\{\kappa_{y,t}(z), \kappa_{m,t}(z)\right\}^\infty_{t=t_0}$; income, $\left\{I_t\left(a,k,z\right)\right\}^\infty_{t=t_0}$; and housing supply shifters, $\left\{H^0_{t,l}\right\}^\infty_{t=t_0}$; an equilibrium for t_0 is a set of housing prices $\left\{p_{t_0,l}\right\}^N_{l=1}$, location choice probabilities $\left\{\pi^{loc}_{t_0}(l|a,k,z)\right\}^N_{l=1}$, and fertility outcome probabilities, $\left\{\pi^{fert}_{t_0}(k|a,z)\right\}^2_{k=0}$, such that:

1. Households correctly predict housing prices and endogenous amenities given perfect foresight of the fundamentals.
2. The supply of amenities is consistent with the population composition (Eq. 2)
3. Households location choices, $\left\{\pi^{loc}_{t_0}(l|a,k,z)\right\}^N_{l=1}$, are optimal (Eq. 3).
4. Fertility outcomes, $\left\{\pi^{fert}_{t_0}(k|a,z)\right\}^2_{k=0}$, are the result of optimal fertility choices given expectations (Eq. 5).
5. Housing markets clear in every location (Eq. 6),
6. Everyone chooses one location $\sum_l \pi^{loc}_{t_0}(l|a,k,z) = 1$ and has one fertility outcome $\sum_k \pi^{fert}_{t_0}(k|a,z) = 1$

Definition of a steady state equilibrium A period equilibrium is a steady state equilibrium if the set of future housing prices, location choice probabilities, and fertility outcome probabilities, are constant, that is,

$$\left\{ \{p_{t,l}\}_{l=1}^{N}, \left\{\pi_t^{loc}(l|a,k,z)\right\}_{l=1}^{N}, \left\{\pi_t^{fert}(k|a,z)\right\}_{k=0}^{2} \right\}_{t=t_0}^{\infty}$$
$$= \left\{ \{p_{ss,l}\}_{l=1}^{N}, \left\{\pi_{ss}^{loc}(l|a,k,z)\right\}_{l=1}^{N}, \left\{\pi_{ss}^{fert}(k|a,z)\right\}_{k=0}^{2} \right\}$$

For a period to be in steady state equilibrium, it must be the case that all the fundamentals are constant from that point on, including population. Importantly, we do not impose the population at the city level, $L_t(a,z)$ to be consistent with the fertility choices. In other words, the city is open to incoming and outgoing population and these population flows are taken as exogenous.

Existence and uniqueness. The existence and uniqueness of an equilibrium is not guaranteed for any set of parameters. Existence and uniqueness depend on how strongly the endogenous amenity supply reacts to the demographic composition, captured by $\gamma_1(g)$ and $\gamma_2(g)$. In the limit where these parameters tend to zero, equilibrium exists and is unique, since Equation 6 has a unique solution every period.

To estimate the model, we assume that all fundamentals stay constant starting in 2010, the last observed year. It is then enough to assume that we observe the economy in an equilibrium path to a steady state. However, when performing counterfactual exercises we cannot guarantee the existence and uniqueness of a new equilibrium. Instead of calculating a new equilibrium, we compute a counterfactual in which we allow an off-equilibrium one-shot reaction of endogenous amenities. This decomposition allows us to approximate the importance of endogenous amenities and avoid the complications of multiple equilibria.

4 Model quantification

4.1 Data and definitions

The quantification of the model employs census individual-level data for the years 1990, and 2000, and ACS multiyear 2008-2012 for 2010.[15] We employ PUMAs, which are the smallest units available each year. We select only couples in our data and

[15]Starting in 1990 allows us to include more cities in our sample.

treat each household as an individual agent in the model. Households are assigned to bins based on age, skill, fertility choices, and location.

Age. The age of a household is assigned based solely on the age of the female. Households between 20 and 30 are classified as young, between 30 and 40 as mature, and between 40 and 50 as old(er). This classification is meant to capture three fertility phases. Regarding fertility choices, we consider three fertility states: no kids, young kids if the household had them in the current age bin, or old kids if the household had them when in the previous age bin.

Location. Households' location is classified as downtown or suburban depending on the geographic unit where the couple lives, according to the following procedure. First, we establish the point location of a city center, as in Lee and Lin (2018).[16] Second, we employ the distance of each census tract to this point city center (provided by Lee and Lin (2018)) to classify as downtown all census tracts that are the closest to the center and include 10% of the population in the year 2000. Third, we classify a PUMA as downtown if at least 50% of the PUMA's population belongs to census tracts classified as downtown, and as suburban otherwise. Finally, we select only cities for which we can accurately identify the downtown in all of our sample years (1990-2010). Following Couture and Handbury (2020), we consider that we can identify the center in cities for which at least 50% of the population in the center lives in a PUMA (or county group) that is classified as downtown.[17] Table 1 reports the cities included in this sample.

Skill, income, and the child penalty. In our model, household income varies with the skill of the household as well as with the presence of children. Therefore, one of the challenges when quantifying the model is to correctly estimate the effect of children on income. If more productive households are more likely to have kids, we would quantify the wrong income effect of having kids. This is problematic because

[16] Lee and Lin (2018) use the procedure developed by Fee, Hartley et al. (2013) in which they identify the CBD of 268 MSAs using the 1982 Census of Retail Trade for the central city of the MSA. For the remaining 117 MSAs, the center is found by geocoding the MSA's central city, which is found using ArcGIS's 10.0 North American Geocoding Service.

[17] We use slightly more generous thresholds than Couture and Handbury (2020); they use 60% thresholds, while we employ 50%. We made this decision in order to obtain the largest possible sample of cities. Our goal was to have more power in the estimation.

Table 1: Sample of cities

Albany, NY	Indianapolis, IN
Allentown, PA	Los Angeles, CA
Atlanta, GA	Miami, FL
Boston, MA-NH-ME-CT	Milwaukee, WI
Charlotte, NC-SC	Minneapolis, MN-WI
Chicago, IL-IN-WI	New York, NY-NJ-CT-PA
Cleveland, OH	Philadelphia, PA-NJ-DE-MD
Dallas, TX	Pittsburgh, PA
Dayton, OH	Portland, OR-WA
Denver, CO	Sacramento, CA
Detroit, MI	San Francisco, CA
Grand Rapids, MI	Seattle, WA
Greensboro, NC	St.Louis, MO-IL
Hartford, CT	Tampa, FL
Houston, TX	Washington, DC-MD-VA-WV

Notes: This table includes the cities used to estimate the model. These are the cities for which we can accurately identify the downtown in all of the years used in estimation. We consider that we can identify the center in cities for which at least 50% of the population in the center lives in a PUMA that is classified as downtown.

when we perform the counterfactual we would like to predict how the income of individuals changes causally with fertility.

We proceed in four steps. First, we assign households a skill based on their income by dividing households into 10 income bins within age bins. Second, we estimate the child penalty as will be described in the next section. Third, we construct female income in the following way. For women with no children, we assign the median income of women with no children for each income and skill bin. For women with children, we assign the median income of women with no children, after subtracting the estimated child penalty. Fourth, we compute household income as the sum of the median male income by age and skill bin and our constructed female income by age, skill, and presence of children.

4.2 Quantification

Calibrated parameters There is a set of parameters that we quantify externally following the literature. To quantify the housing supply function, we first obtain the elasticity of housing supply in downtown and suburban locations. To do so, we use Baum-Snow and Han (2021)'s estimates of housing supply elasticities by distance to the CBD and adapt them to our definition of downtown and suburbs. Then, we obtain equilibrium housing prices from hedonic price regressions. Finally, we obtain the housing supply shifter, $H^0_{t,l}$, needed for the housing supply to equal the quantity demanded at the equilibrium price. In the counterfactual exercise, we keep the housing supply shifters as estimated and re-compute housing supply given the new counterfactual housing demand and the housing supply elasticity.

The remaining externally estimated parameters are the discount factor, the probability of pregnancy success for mature couples, the Fréchet shape for the idiosyncratic taste for location, and the elasticity of the supply of endogenous amenities to the ratio of high- to low-skilled for each skill level. Table 2 reports the values assigned to each of these parameters and their source.

Table 2: Externally estimated parameters

Parameter	**Definition**	**Value**	**Source or Target**
α_l	Elasticity of the housing supply	Downtown: 2.3 Suburbs: 4.8	Baum-Snow and Han (2021)
$H^0_{t,d}$	Housing supply shifter	–	Housing demand
$p_{d,t}/p_{s,t}$	Relative price downtown	1990: 0.95 2000: 1.13 2010: 1.31	Hedonic price regressions
$p_{s,t}$	Annual price in the suburbs	1990: \$12,341 2000: \$12,611 2010: \$13,077	24% housing exp. share
ϕ	Discount Factor	0.96	4% annual int. rate
ρ_m	Prob. have kids when mature	0.8	Rothman et al. (2013)
β_l	Fréchet parameter - taste for location	3	Couture et al. (2019)
$\gamma_1(z)$	Amenity supply elast. to skill ratio	High-skilled: 1.617 Low-skilled: 0.345	Su (2022)

The parameters left to estimate are the elasticity of the endogenous amenity supply to the ratio of no-kids to kids in a location, $\gamma_2(g)$; the exogenous amenity of downtown relative to the suburbs, $\Delta\chi(g)$; the shape of the idiosyncratic children amenity, β_η; and children-related amenities, $\kappa_y(z)$ and $\kappa_m(z)$.

Estimation of child penalties. We estimate child penalties following the most recent advances in the literature. In particular, Kleven (2023) develops a new approach to estimating child penalties based on cross-sectional data in which a pseudo-panel is constructed using matching techniques. He shows that event studies that use this method yield results very similar to those using panel data when the matching is done according to year, age, gender, education, marital status, race, and state of residence. We apply this methodology to estimate child penalties by fertility timing each year. Figure 5 summarizes the results from our estimation.

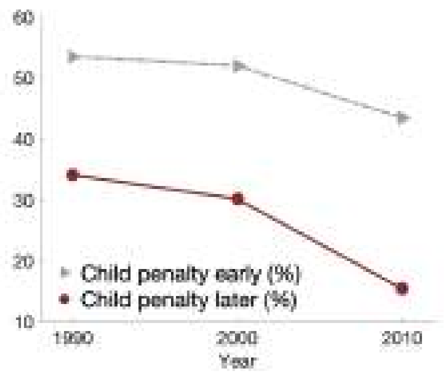


Figure 5: Child Penalties by Fertility Timing

Notes: This figure summarizes our estimation for child penalties by fertility timing for each census year. Child penalties are computed as the percentage drop in income for women having their first child compared with women with no children. Please, see Section B.1 in the Appendix for more details.

Consistent with the findings of Kleven (2023), our estimates reflect a reduction of the child penalty over time. Moreover, we find that women who have kids earlier experience a higher penalty than those delaying, as found by Miller (2011); Adda, Dustmann and Stevens (2017); and Gallen et al. (2023). Lastly, we document that even though child penalties have decreased since 1990, the decrease was larger for women having kids later, which means that the delay premium increased over our period of study and thus created incentives for delaying the arrival of children. A detailed explanation of how we apply the pseudo-panel methodology for our purposes is included in Section B.1 in the Appendix.

Estimation of the amenity supply elasticity. The equilibrium relationship between the log share of the population in the demographic group $g = (a, k, z)$ and the characteristics of a location l is given by

$$\ln(s_t(g, l)) = \beta_\varepsilon \ln\left(I_t(g) - p_{t,l}\right) + \beta_\varepsilon \gamma_1(g) \ln\left(\frac{N_{t,l}^{HighSkill}}{N_{t,l}^{LowSkill}}\right) \tag{7}$$

$$... + \beta_\varepsilon \gamma_2(g) \ln\left(\frac{N_{t,l}^{NoKids}}{N_{t,l}^{Kids}}\right) + \beta_\varepsilon \underbrace{\ln \xi_t(g, l)}_{\text{Unobserved}}, \tag{8}$$

where $\gamma_2(g)$ is the parameter to be estimated. We obtain estimates of β_ε and $\gamma_1(g)$ from Su (2022), who exploits exogenous variation in real income and in the ratio of high-skill residents. Instead, we focus on establishing the causal effect of the local composition of the share of households with no kids.

Su (2022) estimates $\gamma_1(g)$ for two groups: college-educated and non-college-educated. We map this division into the top one-third of skill and the bottom two-thirds of skill, since college-educated households represent one-third of the population. To account for the fact that households with and without children may differently value the demographic composition of the neighborhood or the resulting amenity arising from changes in neighborhood composition, we let $\gamma_2(g)$ depend only on whether households have children.

The identification challenge arises because the share of high-skill households and the share of households with no kids are likely correlated with the unobserved characteristics $\xi_t(g, l)$. For instance, locations that have exogenous characteristics that are attractive for households with no children will directly attract households with no kids and will feature a high ratio of households with no kids relative to households with kids as a result.

To identify the causal effect, we propose a shift-share instrument. The shift is the change in the city-level fertility composition and the share comes from the location choices of households with and without children in 1980, that is, before urban revival. Following Borusyak, Hull and Jaravel (2021), we note that exogeneity of the shift is sufficient for identification. To obtain an exogenous shift in city-level fertility, we exploit the enactment of infertility insurance mandates. Several U.S. states passed these policies in the late 1980s. The goal was to enhance access to assisted reproductive techniques. In practice, the mandates implied a substantial reduction in the price of infertility treatments that couples faced, and hence a decrease in infertility associated with delayed childbearing. Previous literature using differences-in-differences methodologies has established that the average age at first birth increased in states that passed these mandates compared with those that did not (Bitler and Schmidt, 2006). Therefore, we can exploit the exogenous increase in the number of young households without children induced by the policy to construct the shift. Recall that the share comes from the location of households with and without kids in 1980, before the infertility insurance mandates. The policy changed fertility decisions, and thus the share of households with no children, but did not directly affect the exogenous characteristics of locations. By looking at how different demographic groups changed their location decisions, we can estimate the reaction of the supply of endogenous amenities. Details on how we construct the shift-share instrument are included in Section B.2 in the Appendix.

Table 3 includes results from estimating Equation 8 by instrumenting the ratio of households with no kids relative to kids with the shift-share instrument. The first column presents the results from estimating the regression on households with no kids, and the second column for households with kids. As the share of households with no kids increases in a location, the supply of endogenous amenities increases for households with no kids, but decreases for those with kids. These estimates have to be divided by β_ε to obtain the structural elasticity of the local elasticity supply.

Table 3: Endogenous amenity supply

	No kids	Kids
	(1)	(2)
$\Delta \ln \left(\frac{N^{\text{No kids}}}{N^{\text{Kids}}} \right)$	0.484**	-0.309**
	(0.223)	(0.129)
Controls	Yes	Yes
Year FE	Yes	Yes
MSA FE	Yes	Yes
F-stat	237.382	277.260
Observations	6751	8926

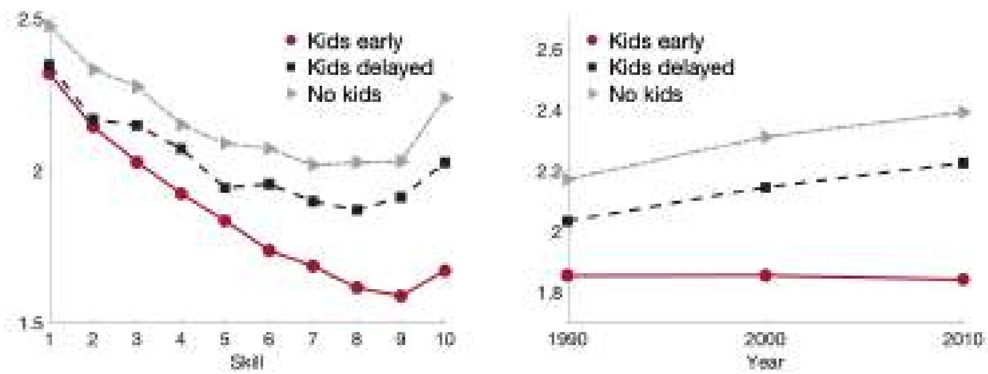
Notes: This table presents results from regressing the change in the share of households that choose to live in a location on the change in the log ratio of the number of households without kids relative to households with kids in that location. The results in Column (1) are from running this regression for households with no kids, and those in Column (2) for households with kids. The change in the log ratio of households without and with kids is instrumented with a shift-share IV where the shift is the change in the fertility composition at MSA level coming from the introduction of infertility insurance mandates, and shares come from the location decisions of households in 1980. Controls include the log mean income, population, and value of houses at the MSA level. Data source: Census and ACS IPUMS data from 1980 to 2010. **p<0.10, **p<0.05, ***p<0.01.*

Endogenous amenities can be thought of as a combination of all the characteristics that are valued by the household; these include physical amenities, such as daycare centers or recreation parks, as well as the composition in itself. For example, people may enjoy having neighbors with children even if no other characteristic of the neighborhood changes. Our estimation of the elasticity captures both effects.

Estimation of relative downtown amenities. For a given β_ε, the fraction of each group that lives in the city center allows us to estimate the difference in amenities between the center and the suburbs. Recall that the probability of choosing to live in the center is

$$\pi_t^{loc}\left(d|a,k,z\right) = \frac{\left(I_t(a,k,z) - p_{t,l}\right)^{\beta_\varepsilon} \cdot \Delta_t\left(a,k,z\right)^{\beta_\varepsilon}}{\left(I_t(a,k,z) - p_{t,d}\right)^{\beta_\varepsilon} \cdot \Delta_t\left(a,k,z\right)^{\beta_\varepsilon} + N_s\left(I_t(a,k,z) - p_{t,s}\right)^{\beta_\varepsilon}}.$$

Therefore, from the observed share of households in each demographic group g, we obtain $\Delta_t\left(a,k,z\right)$. From this estimation, we can quantify whether downtown is more attractive relative to the suburbs when households have children later in life. If so, the delay in childbearing will increase the relative attractiveness of downtown, and thus contribute to urban revival. To perform this comparison, we define the lifetime amenity of living downtown as the present value of both endogenous and exogenous amenities for a household living downtown throughout their life, relative to a household living in the suburbs. Panel (a) of Figure 6 shows the lifetime downtown amenities by fertility choice and skill in 1990, and Panel (b) exhibits the evolution of the lifetime downtown amenity by fertility choice since 1990 averaged by skill.



(a) 1990 (b) Evolution

Figure 6: Estimated lifetime downtown amenities.

Notes: This figure displays lifetime downtown amenities for each fertility choice and skill in 1990 (panel (a)) and their evolution averaging across skills (panel (b)). Lifetime amenities are the discounted sum of downtown amenities when young, mature, and old.

Panel (a) of Figure 6 shows that already in 1990, households without children or that had children at an older age enjoyed a higher level of lifetime downtown amenities than households that had children early. This should come as no surprise, since the kind of amenities that concentrate downtown such as bars or theaters, are usually visited more by childless individuals, and households that delay childbearing enjoy the same amenity as households without kids when they are young. Panel (b) of Figure 6 shows that the difference in lifetime amenities between having kids early versus delaying has increased.[18] This can be explained by the endogenous response of downtown amenities: As the proportion of young childless households increases downtown, amenities cater to this kind of household. Lastly, Figure B.4 in the Appendix shows all estimated downtown amenities for completeness.

Estimation of exogenous downtown amenities Using Equation 2 and given that $\Delta_t(a,k,z) = \frac{\lambda_t(a,k,z,d)}{\lambda_t(a,k,z,s)}$, we obtain the ratio of the exogenous characteristics of downtown relative to the suburbs. Namely,

$$\Delta\chi_t(a,k,z) = \Delta_t(a,k,z)\left(\frac{N_{t,\mathbf{d}}^{HighSkill}/N_{t,\mathbf{d}}^{LowSkill}}{N_{t.\mathbf{s}}^{HighSkill}/N_{t,\mathbf{s}}^{LowSkill}}\right)^{-\gamma_1(g)}\left(\frac{N_{t,\mathbf{d}}^{NoKids}/N_{t,\mathbf{d}}^{Kids}}{N_{t,\mathbf{s}}^{NoKids}/N_{t,\mathbf{s}}^{Kids}}\right)^{-\gamma_2(g)}.$$

Notice that it is not possible to identify the level of exogenous amenities for both downtown and the suburbs, $\chi_t(a,k,z,l)$, from the shares of households living downtown; it is only possible to identify the relative exogenous amenities. As a result, in the welfare analysis, we will not be able to speak to changes in welfare coming from the exogenous amenities. However, the relative exogenous amenities, $\Delta\chi_t(a,k,z)$ are sufficient to solve for the equilibrium.

Estimation of children amenities. We estimate the amenity associated with having children early and later in two steps. In the first step, we estimate the Fréchet parameter, β_η, that controls the distribution of idiosyncratic preferences. This parameter determines the elasticity of the fertility choices to changes in estimated incentives arising from income child penalties as well as the way downtown amenities change with the presence of children. We find a Fréchet parameter equal to 2.12, slightly lower than the Fréchet parameter of idiosyncratic preferences for locations. This implies a slightly larger elasticity of fertility choices than location choices. We provide more details of the estimation in Section B.3 of the Appendix.

In the second step, we employ the observed fraction of households with different fertility outcomes together with the probability of getting pregnant after 30, ρ_m, and the Fréchet parameter, β_η, to back out the amenity parameters $\kappa_{y,t}(z)$ and

[18]Changes in estimated lifetime downtown amenities by fertility choice and skill are displayed in Figure B.5. This figure shows that the increase in the lifetime downtown amenity was more pronounced for high-skilled households.

$\kappa_{m,t}(z)$. There is an additional complication: Since fertility decisions are dynamic, they depend on the price expectations of households.

We estimate children amenities in a loop in which we iterate over amenities and housing price expectations until the fertility outcomes match those in the data, the housing price path clears the market every period, and agents' expectations about the future are equal to the equilibrium housing prices. Details of the estimation can be found in Section B.4 of the Appendix.

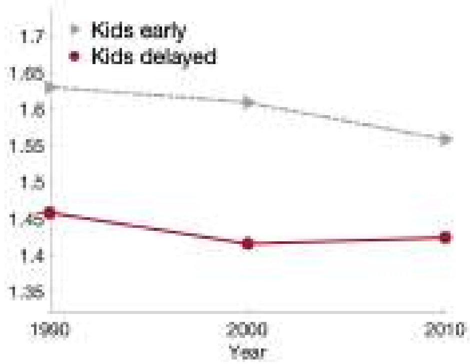


Figure 7: Evolution of average child amenity

Notes: This figure displays the evolution of the average child amenity for each fertility choice, averaging across skills.

The evolution of child amenities is displayed in Figure 7. Child amenities slightly decreased over time, especially for those having kids early. These amenities capture drivers of fertility choices beyond the delay premium and downtown amenities, such as changing norms regarding having children later in life.[19]

5 Delayed childbearing's impact on urban revival

In this section, we quantify the importance of changes in the timing of fertility for urban revival over our period of study. To do so, we keep the incentives to delay childbearing fixed and compute urban revival in the counterfactual equilibrium. Using the model, we can distinguish the direct impact of an increase in the fraction of young high-skilled individuals with no children from the amplification mechanisms that arise from the reaction of housing prices and amenities. Moreover, the model allows us to quantify an important aspect of the interaction between fertility

[19]Figure B.3 in the Appendix displays estimated child amenities from having kids early and delaying for each skill bin and year for completeness.

and location choices, that is, the feedback effect urban revival may have on fertility choices. As the share of young individuals without kids who remain in the center grows, downtown neighborhoods become more attractive to young childless individuals. This creates additional incentives to delay parenthood and reinforces the process of urban revival.

Incentives to delay childbearing In our model, there are three reasons why households increasingly delayed childbearing over our period of study. First, the delay premium increased. As we explained in Section 4, households face child penalties that vary with the timing of fertility and the census year. Our estimation reveals that the income penalties for having kids later decreased faster than those for having kids earlier, which implies that the delay premium increased over this period (as shown in Figure 5). Second, the lifetime amenities of living downtown for households postponing childbearing rose relative to those having children early, as shown in panel (b) of Figure 6. Finally, we estimate child amenities that depend on the timing of birth. We find that households' preferences for having children later in life increased relative to having them earlier, as can be seen in the estimation of child amenities displayed in Figure 7. This can capture, among other factors, changes in social norms or medical advancements that reduced the cost of delaying.

Quantitatively, we find that the most important reason for delay is the rise in the delay premium, consistent with studies that highlight the importance of career considerations for delaying childbearing.[20] Therefore, we perform a counterfactual in which we fix the delay premium to its 1990 level and study its implications for urban revival. In particular, we leave child penalties from having kids early as in the data and modify only the child penalties from having kids late in each period, such that the delay premium is constant and equal to its level in 1990.[21]

Increase in the delay premium In Figure 8, we display the lifetime delay premium for each skill bin in 1990 and 2010 *at the household level*. Even though our

[20]See, for instance, Caucutt, Guner and Knowles (2002); Erosa, Fuster and Restuccia (2002); Attanasio, Low and Sánchez-Marcos (2008); Adda, Dustmann and Stevens (2017).

[21]Section C.1 in the Appendix presents the results of an alternative counterfactual in which the percentage of households that delay childbearing is fixed to 1990, that is, if the timing of fertility not changed for any of the above-stated reasons. However, this counterfactual does not allow us to study how the feedback effect of urban revival on fertility choices reinforces the process.

estimation of child penalties does not vary with women's skill level, the figure shows that the delay premium is higher for high-skilled households.[22] This is because women's share of household income increases with skill.[23] In addition, this figure shows that the delay premium increased more for high-skilled households. This feature is driven by the higher income growth at the top of the distribution.[24]

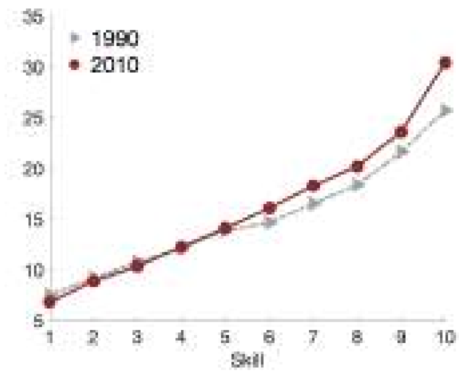


Figure 8: Delay Premium by Skill

Notes: This figure displays the lifetime delay premium for each skill bin in 1990 and 2010. Lifetime delay premia are calculated as the difference in the (present value) lifetime income between households delaying and households having kids when young, in thousands of 2000 USD.

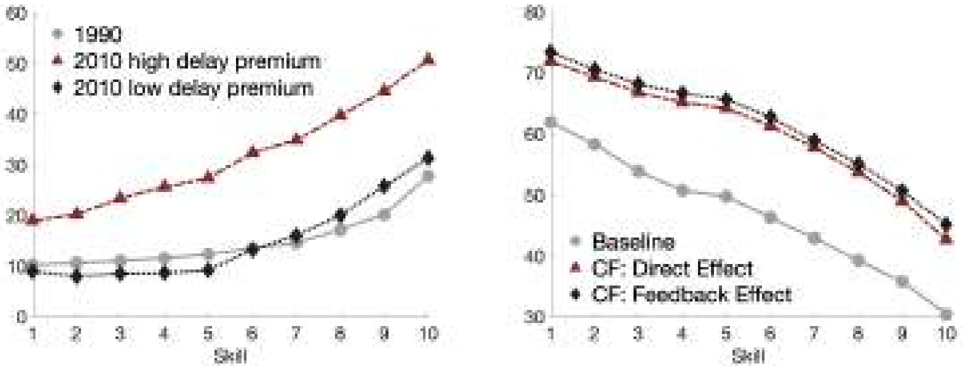
Counterfactual fertility choices. We start our analysis by looking at the response of fertility choices to the counterfactual delay premium, plotted in Figure 9. The left panel of this figure displays the percentage of households that delayed childbearing in 1990 and 2010. For 2010, we display this percentage in both the baseline equilibrium and the counterfactual equilibrium in which the delay premium does not increase over time. The percentage of households that delay is computed as the ratio of households that have young children when mature over all households having children for a given cohort. As expected, the percentage of households that postpone childbearing would have increased less than in the baseline. In the counterfactual, the percentage of high-skilled households that delay increases only from 21% in 1990 to 25% in 2010, while in the baseline it reaches 45% in 2010. For low-skilled households, we observe a small decline in the percentage of households

[22]Kleven (2023) finds only small differences in child penalties as a percentage of earnings between high- and low-skilled women.

[23]Figure C.7 in the Appendix displays the share of female income in household income in 1990, 2000, and 2010 by skill.

[24]Since child penalties are computed as a percentage of income, higher growth at the top of the distribution can explain why these penalties increase more for higher-income households in levels.

who postpone, from 12% in 1990 to 10% in 2010, compared with the increase to 26% in 2010 that took place in the baseline. For completeness, Figure C.9 in the Appendix exhibits changes in fertility choices by age and skill.



(a) % Households Delaying (b) % Young with kids in 2010

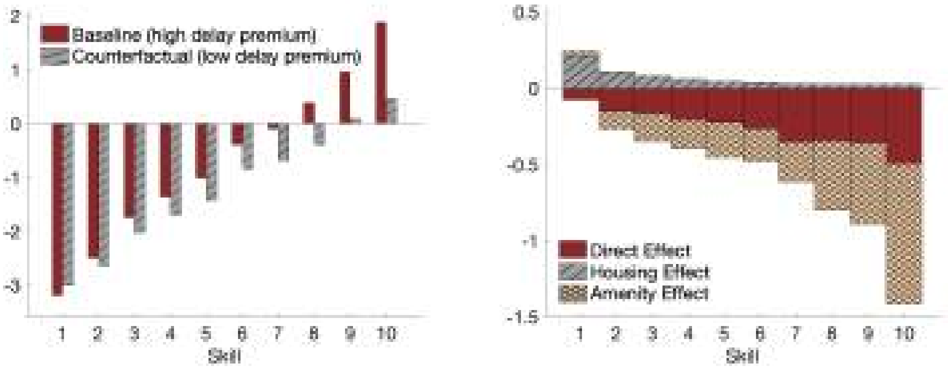
Figure 9: Changes in fertility choices

Notes: Panel (a) exhibits the percentage of households that delay childbearing in 1990 and 2010, in both the baseline (high delay premium) and the counterfactual (low delay premium). It is computed as the number of households that have kids when young over all households having kids for each cohort. Panel (b) shows the percentage of young households with kids in 2010 in the baseline; in the counterfactual when only fertility choices adjust (direct effect); and when fertility choices react to changes in housing prices and downtown amenities (feedback effect).

Panel (b) illustrates the magnitude of the feedback effect. We plot the percentage of young households that have children in 2010 in the baseline and the counterfactual before housing prices and amenities react (the direct effect) and after they react (the feedback effect). Once we allow amenities and housing prices to adjust, the percentage of young households with children increases, especially for high-skilled households.[25] The reason is that endogenous amenities give rise to an externality in fertility choices, by which being childless is more attractive when there are many other young households without children. Put another way, the incentives to delay childbearing or to not have kids at all are stronger when there are plenty of amenities to enjoy for childless individuals downtown than in an alternative scenario in which most young individuals have kids and move to the suburbs, and as a result the supply of downtown amenities valued by childless households is scarcer.

Counterfactual location choices. Next, we look at the effect of counterfactual changes in fertility choices on households' location decisions. The left panel of

[25]Figure C.8 in the Appendix shows that both young and mature households increase their fertility by around 1.5 p.p. (between 2% and 5%, depending on the skill) once housing prices and amenities react to the lower percentage of households postponing childbearing.



(a) 1990-2010 Changes (p.p.)

(b) Decomposition

Figure 10: The propensity to live downtown by skill

Notes: The left panel of this figure shows the 1990-2010 change in the propensity to live downtown by skill in the baseline and in the counterfactual. The right panel decomposes the difference into (i) *Direct Effect*, computed as the difference between the baseline change and the change when only the fertility composition adjusts; (ii) *Housing Effect*, computed as the difference between the counterfactual when housing prices react (but amenities do not) and the direct effect; (iii) *Amenity Effect*, computed as the difference between the counterfactual when amenities also react and the counterfactual when only housing prices react (but amenities do not).

Figure 10 displays the 1990-2010 changes in the propensity to live downtown for each skill bin in the baseline and the counterfactual. Focusing on baseline changes, we observe an influx of high-skilled households to the downtown location together with a displacement of the low-skilled, consistent with previous findings (Baum-Snow and Hartley 2019; Couture and Handbury 2020; Couture et al. 2019). Comparing the baseline to the counterfactual, we can see that, had the delay premium stayed as in 1990, all but very low-skilled households would have been less likely to live downtown, and especially higher-skilled households. The changes described here aggregate changes in the propensity to live downtown for each demographic group, included in Figure C.10 in the Appendix.

In the right panel of Figure 10, we decompose changes in the propensity to live downtown into a direct effect and the amplification effects through changes in housing prices and amenities. This decomposition is useful for understanding the mechanism behind the shifts in the spatial sorting of households. Solid bars show the difference in the change in the propensity to live downtown between the direct impact and the baseline. The higher proportion of young households with kids that results from a lower delay premium (the direct effect) translates into a lower increase in the propensity to live downtown for all skill levels compared with the baseline. Since the change in delay is larger among the high-skilled, the difference with the baseline is more pronounced for this group. Moreover, we showed in panel (a) of Figure 6 that differences in downtown amenities across households that have kids early versus late were more pronounced for high-skilled households. Therefore, changes in the propensity to locate downtown due to changes in fertility choices are more important for this group.

We now turn to the additional changes in spatial sorting that operate through changes in housing prices and amenities. First, we allow for housing prices to adjust. The reduction in the demand for the downtown location drives housing prices down, so the propensity to live downtown increases for all skills, but because of non-homothetic housing demand, this effect is larger for the low-skilled. Second, we add the response of amenities. The smaller increase in the proportion of high-skilled households downtown results in lower downtown amenities and a lower propensity to live downtown. High-skilled households are more affected by changes in downtown amenities, since they value amenities related to the presence of other high-skilled households more than the low-skilled. Lastly, because lower downtown amenities result in even lower housing prices, the effect of amenities is positive for the lowest-skilled households.

Impact on urban revival. Finally, to quantify the impact of changes in fertility choices on urban revival, we compare the difference in growth in average income, housing prices, and skill composition between downtown and suburban locations in the baseline and the counterfactual. To understand the importance of the different amplification mechanisms present in our model, we perform several counterfactual exercises that are summarized in Table 4.

The first column of Panel (a) in Table 4 displays the downtown/suburbs difference in the percentage growth of average income, housing price, and percentage of high-skilled, in the *baseline* economy, in which the delay premium increases over time. Columns (2) to (6) decompose the total effect of the counterfactual in which the delay premium does not increase over time. Each effect is computed as the difference in urban revival between the baseline economy and each counterfactual exercise. Column (2) displays the effect of holding the delay premium constant on urban revival when households adjust their fertility and location choices, but down-

Table 4: Impact on Urban Revival

Panel a: Decomposition of Urban Revival						
	Baseline	Direct	Direct & Housing	Direct & Amen.	Direct & Housing & Amen.	Direct & Housing & Amen. & Feedback
	(1)	(2)	(3)	(4)	(5)	(6)
Rel. income growth	10.46	0.78	1.27	1.64	5.29	5.42
Rel. housing price growth	14.16	0	1.2	0	4.4	4.44
Rel. high-skilled growth	9.09	0.72	0.98	1.22	3.1	3.2
Panel b: Percentage of Urban Revival Explained						
	Baseline	Direct	Direct & Housing	Direct & Amen.	Direct & Housing & Amen.	Direct & Housing & Amen. & Feedback
	(1)	(2)	(3)	(4)	(5)	(6)
Rel. income growth	0	7.52	12.16	15.74	50.54	51.84
Rel. housing price growth	0	0	8.45	0	31.09	31.39
Rel. high-skilled growth	0	7.87	10.78	13.45	34.14	35.16

Notes: The first column of Panel (a) displays the downtown/suburbs difference in the percentage growth of average income, housing price, and percentage of high-skilled in the baseline economy. Column (2) displays the difference in urban revival between the baseline and the counterfactual when only the fertility composition changes; Column (3) adds the effect of housing price adjustments only; Column (4) adds the effect of amenities only; Column (5) adds both housing prices and amenities to the initial change in the fertility composition; Column (6) adds the reaction of fertility choices to changes in housing prices and amenities. Panel (b) displays the percentage of the baseline explained by each counterfactual exercise.

town amenities and housing prices do not adjust (*the direct effect*). In columns (3) to (5), we evaluate the magnitude of the amplification effects through housing prices and amenities, *holding the fertility composition* as in column (2). Column (3) adds the effect of housing price adjustments to the direct effect, keeping downtown amenities as in the baseline; Column (4) adds the effect of downtown amenities adjustments, keeping housing prices as in the baseline; Column (5) adds both housing prices and downtown amenities; and Column (6) adds the reaction of fertility choices to changes in housing prices and amenities (*the feedback effect*). Finally, in Panel (b) we compute the percentage of the baseline urban revival that can be accounted for by each counterfactual exercise.

Between 1990 and 2010, average income and housing prices grew 10 p.p. and 14 p.p. faster downtown than in the suburbs, respectively. In addition, the share of high-skilled in the neighborhood population increased 9 p.p. faster downtown than in the suburbs. We obtain that the change in the fertility composition alone (the direct effect) explains around 7.5% of the observed difference in the average income growth rate. Once we allow for housing prices *or* amenities to adjust to the lower demand for downtown, the percentage explained doubles.[26] Interestingly, the effect on urban revival is much larger when both housing prices and amenities

[26] In Figure C.11 in Section C.2 in the Appendix, we show that in the counterfactual, downtown amenities grow less in the for individuals that choose not to have children or to postpone.

adjust, which suggests that these two amplification mechanisms complement each other. The combined reaction of housing prices and amenities amplifies the effect of the change in the fertility composition by around 40% of the observed difference in income growth between downtown and the suburbs. The magnitude of the amplification mechanism arising from endogenous changes in amenities is similar to the one obtained by Su (2022) and Couture et al. (2019). Lastly, we document a moderate feedback effect stemming from the fertility response to changes in housing prices and amenities. More high-skilled households have kids early as a consequence of a less attractive downtown (as shown in panel (b) in Figure 9). This further diminishes the proportion of high-skilled households downtown and reinforces urban revival.

6 Welfare analysis

In this section, we analyze the welfare implications of the link between the delay in fertility and urban revival. First, we revisit the impact of urban revival on welfare inequality. Second, we compute how the increase in the delay premium impacted welfare beyond its direct effect on income. In particular, we focus on how the delay premium changed welfare by contributing to urban revival.

6.1 Urban revival and welfare inequality

First, we analyze the welfare consequences of urban revival through the lens of our model. As in previous studies, the multinomial logit structure that results from extreme value idiosyncratic preferences implies that only relative amenities can be identified (see, for instance, Diamond (2016) or Su (2022)). Therefore, we cannot quantify how absolute levels of welfare changed nor can we make overall welfare comparisons across skill groups.[27] However, we can identify the differential contribution of some welfare components across skills. We focus on changes in

[27] Recall that the level of amenities in a given location is identified using the share of households in a given demographic and skill group that chooses each location. Amenities are only identified up to a constant. Thus, it is only possible to identify the amenity of downtown relative to the suburbs for each group. However, we cannot know whether increases in the relative amenity come from an improvement of amenities in this growing neighborhood or from a decline of amenities in the rest of the neighborhoods, nor whether amenities increased or decreased everywhere keeping the relative amenities constant.

welfare that arise from observable or estimated utility components: (i) changes in real income, (ii) changes in the proportion of high-skilled in a given neighborhood, and (iii) changes in the proportion of childless households in a given neighborhood.

To understand how urban revival affected welfare inequality, we compute, for each skill, the expected difference in the lifetime log utility of being born in 1990 relative to 2010 allowing agents to re-optimize; the expectation is taken over idiosyncratic preferences for location and fertility. Therefore, the expected welfare change takes into account that households' fertility and location decisions changed in response to differences in economic incentives across periods. For example, the lifetime real income received by a household born in 1990 compared with 2010 changed because downtown became more expensive, but also because households born in 2010 sorted differently into downtown locations.

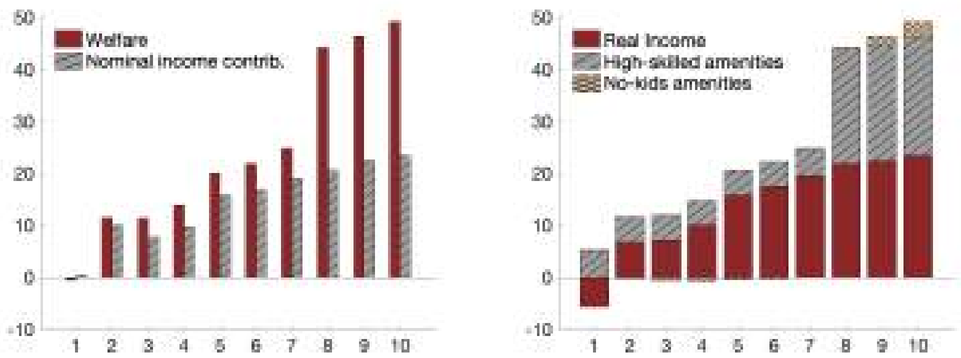
The expected welfare change by skill, $\Delta\mathcal{W}(z)$, can be written as

$$\Delta\mathcal{W}(z) = \Delta\mathcal{W}^{RI}(z) + \Delta\mathcal{W}^{HS}(z) + \Delta\mathcal{W}^{NK}(z) + \Delta\mathcal{W}^{U}(z),$$

where $\Delta\mathcal{W}^{RI}(z)$ is the expected difference in the lifetime log utility arising from differences in real income for a household of skill z born in 1990 compared with the same household if born in 2010. $\Delta\mathcal{W}^{HS}(z)$ and $\Delta\mathcal{W}^{NK}(z)$ are the expected differences in lifetime log utility from changes in endogenous amenities related to the ratio of high skill to low skill and the ratio of households with no kids to households with kids, respectively. Lastly, $\Delta\mathcal{W}^{U}(z)$ denotes other changes in expected lifetime log utility that are unobservable to us, including changes in exogenous amenities and expected utility from idiosyncratic preferences. In the remainder of the paper, we consider only the total measurable change in welfare, that is, excluding $\Delta\mathcal{W}^{U}(z)$. A detailed derivation of the decomposition can be found in Section D in the Appendix.

The contribution of nominal income to welfare between 1990 and 2010 underestimates the increase in welfare inequality. Panel (a) of Figure 11 plots percentage changes in the contribution of lifetime log nominal income to welfare and the log welfare of a household born in 1990 compared with one born in 2010.[28] By focusing

[28]The percentage welfare change from nominal income does not coincide with the percentage change in nominal income. This difference arises because we assume unit housing demand. As a result, the impact of changes in nominal income on welfare depends on disposable income, that is, nominal income minus the housing price. The same percentage increase in the nominal income



(a) Welfare vs. Nominal Income (b) Decomposition of Welfare

Figure 11: Welfare Changes 1990 vs. 2010

Notes: This figure displays percentage changes in welfare between a cohort born in 1990 and a cohort born in 2010. Panel (a) displays total (measurable) changes in welfare for each skill level and the contribution of nominal income to welfare. Panel (b) shows how each component contribute to the welfare change.

only on the impact of nominal income we would vastly underestimate the increase in welfare inequality, particularly at the top of the distribution.

As can be seen in Panel (b) of Figure 11, the increase in welfare inequality was mostly driven by the rise in the share of high-skilled households in the population, which shifted endogenous amenities in a way that benefited high-skilled households. First, high-skilled households have a higher valuation of high-skilled amenities (Su, 2022). Second, high-skilled households increased their presence downtown, where the growth of endogenous amenities was more pronounced, while low-skilled households left downtown over this period. This result is in line with the findings of Couture et al. (2019). We find that the increase in the share of households without children also contributed to increasing welfare inequality, but its effect is much more modest. These amenities enhanced welfare by 3% for the highest-skilled and by 0.3% for the lowest-skilled.

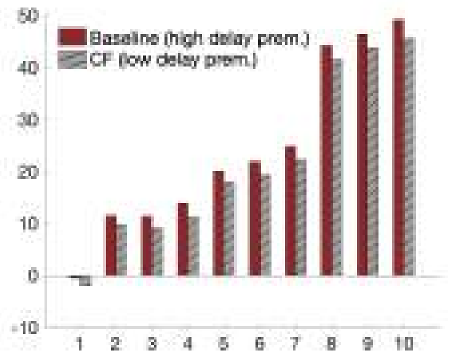
6.2 Welfare impact of the delay premium

Lastly, we compare changes in welfare in the baseline economy and in the counterfactual scenario in which the delay premium remains as in 1990. Recall that a lower premium from delaying childbearing leads to less delay and less urban revival.

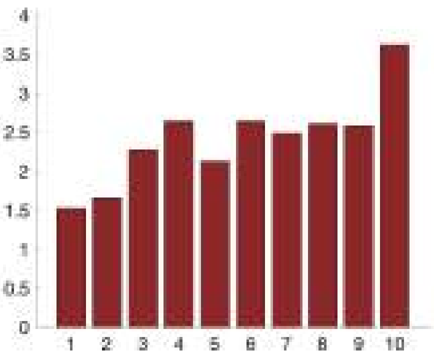
will have a larger impact on the welfare of households with lower disposable income. Please see Section D.1 in the Appendix for a more formal explanation.

Therefore, this comparison is useful in order to emphasize unexplored welfare consequences of the increase in the delay premium i.e., those that occur as a consequence of its impact on urban revival.

We compute the total expected 1990-2010 change in welfare in the baseline economy and the counterfactual with a lower delay premium, as displayed in Figure 12. We can see in panel (a) that welfare would have increased slightly less in the low delay premium equilibrium. Panel (b) of Figure 12 zooms into the difference between the baseline and the counterfactual. Welfare inequality increased as a result of the higher delay premium. Welfare increased about 2 p.p. more among the highest-skilled than for the lowest-skilled in the baseline compared with the counterfactual.



(a) Welfare Change
(2010 v. 1990)

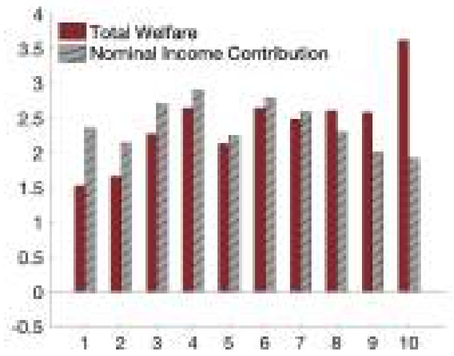


(b) Welfare Impact of Delay Premium
(Baseline v. CF Welfare Change)

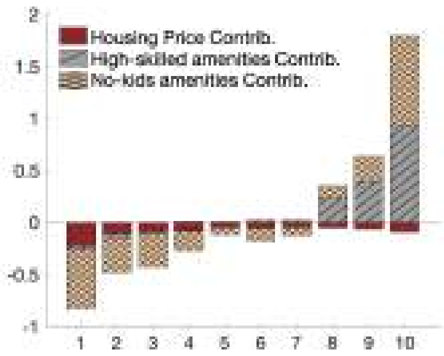
Figure 12: Welfare Evaluation

Notes: This figure summarizes differences in the expected 1990-2010 change in welfare in the baseline economy and the counterfactual. Panel (a) displays the total (measurable) expected 1990-2010 change in welfare in the baseline economy and the counterfactual. Panel (b) displays the difference between the two.

We are interested in understanding how neglecting the impact of the increase in the delay premium on urban revival would affect our welfare predictions. Consider a relatively naïve prediction of the welfare impact that relies only on the effect of the delay premium on nominal income, including changes in nominal income due to re-optimization of fertility and location decisions. Panel (a) of Figure 13 compares the difference between the baseline and the counterfactual in welfare changes and in the welfare changes that result from nominal income alone. It shows that considering only the welfare impact of nominal income would lead to overestimation of the growth in welfare for the lowest-skilled by 1 p.p. and underestimation for the highest-skilled by almost 2 p.p. This exercise highlights the importance of incorporating the spatial impact of changes in fertility to evaluate the welfare effects of long-run demographic trends.



(a) Welfare Impact &
Nominal Income's Contribution



(b) Urban Revival's Contribution
(Total - Nominal Income's Contrib.)

Figure 13: Decomposing the Welfare Impact of the Delay Premium

Notes: Panel (a) compares the total difference in 2010-1990 changes in welfare due to an increase in the delay premium and the contribution of changes in nominal income to this difference. Panel (b) displays the welfare contribution of changes in housing prices and endogenous amenities.

Panel (b) of Figure 13 summarizes how the increase in the delay premium affected welfare inequality through its impact on various dimensions of urban revival: housing prices and endogenous amenities. Since we are subtracting the welfare change in the counterfactual with less delay from the baseline welfare change, positive numbers in this graph illustrate welfare gains beyond the effect of nominal income, and negative numbers portray welfare losses from urban revival. The effect of the delay premium on urban revival led to an increase in welfare inequality. Rising housing prices were slightly more detrimental for low-skilled households, while the increase in endogenous amenities benefited the high-skilled considerably more. The graph reveals a novel welfare implication of the increase in delayed childbearing: downtown improvements for high-skilled households with no children.

7 Conclusions

Residential location choices in the United States are linked to the presence of children. Motivated by this insight, we study the interaction between demographic trends and urban structure. We do so by focusing on two salient trends in the U.S.: delayed childbearing and urban revival. To establish the causal link between the trends, we propose and estimate a structural spatial model of fertility timing and within-city location choice. We find that incentives to delay childbearing, and in particular the increase in the delay premium, contributed significantly to urban revival. Most of the effect is due to the endogenous reaction of amenities and housing prices.

Moreover, the model helps clarify that two conditions must hold for the delay in childbearing to lead to urban revival. First, young households with no children must value living downtown more than households with children. Second, the delay must be more pronounced among high-skilled households. We find both conditions hold in the United States. As long as they also hold in other countries, we expect the same link between fertility and urban revival.

Understanding the drivers of urban revival is important for two reasons. First, to understand who are the winners and losers. Second, to anticipate new waves of this phenomenon. Our findings suggest that new changes in the incentives to delay childbearing induced by changes in policies or social norms can create rising demand for downtown locations, with implications for welfare inequality. Predicting this impact can inform the design policies that safeguard households at risk of displacement.

Finally, understanding the impact of demographic trends on urban revival is key for the evaluation of policies that are directed at influencing demographic trends, such as subsidizing medical treatments to facilitate delaying childbearing.

References

Adda, Jérôme, Christian Dustmann, and Katrien Stevens. 2017. "The career costs of children." *Journal of Political Economy*, 125(2): 293–337.

Agostinelli, Francesco, Margaux Luflade, and Paolo Martellini. 2021. "On the Spatial Determinants of Educational Access." Human Capital and Economic Opportunity Working Group Working Papers 2021-042.

Albouy, David, and Jason Faberman. 2018. "Skills, migration, and urban amenities over the life cycle." Society for Economic Dynamics.

Almagro, Milena, and Tomas Dominguez-Iino. 2019. "Location Sorting and Endogenous Amenities: Evidence from Amsterdam." *Available at SSRN 4279562.*

Alonzo, Davide. 2022. "Marrying your job: Matching and mobility with geographic heterogeneity." *Available at SSRN 4328776.*

Alonzo, Davide, Nezih Guner, and Claudio Luccioletti. 2023. "Segregation and Sorting of US Households: Who Marries Whom and Where?" *Unpublished manuscript.*

Attanasio, Orazio, Hamish Low, and Virginia Sánchez-Marcos. 2008. "Explaining Changes in Female Labor Supply in a Life-Cycle Model." *American Economic Review*, 98(4): 1517–52.

Bailey, Martha J. 2006. "More power to the pill: The impact of contraceptive freedom on women's life cycle labor supply." *Quarterly Journal of Economics*, 121(1): 289–320.

Bailey, Martha J. 2010. "Momma's Got the Pill: How Anthony Comstock and Griswold v. Connecticut Shaped US Childbearing." *American Economic Review*, 100(1): 98–129.

Bailey, Martha J., and Brad J. Hershbein. 2018. "US Fertility Rates and Childbearing in American Economic History, 1800–2010." In *The Oxford Handbook of American Economic History Volume 1.* , ed. Louis P. Cain, Price V. Fishback and Paul W. Rhode. Oxford Handbooks. Accessed 9 Jan. 2024.

Bailey, Martha J., Melanie Guldi, and Brad J. Hershbein. 2014. "Is There a Case for a "Second Demographic Transition"? Three Distinctive Features of the Post-1960 Fertility Decline." In *Human Capital in History: The American Record. Book chapters authored by Upjohn Institute researchers*, , ed. Leah Boustan, Carola Frydman and Robert Margo, 273–312. W.E. Upjohn Institute for Employment Research.

Barbanchon, Thomas Le, Roland Rathelot, and Alexandra Roulet. 2021. "Gender Differences in Job Search: Trading off Commute against Wage." *Quarterly Journal of Economics*, 136(1): 381–426.

Baum-Snow, Nathaniel, and Daniel Hartley. 2019. "Accounting for central neighborhood change, 1980-2010." *Journal of Urban Economics*, 103228.

Baum-Snow, Nathaniel, and Lu Han. 2021. "The Microgeography of Housing Supply." Journal of Political Economy (Forth.).

Behrens, Kristian, Brahim Boualam, Julien Martin, and Florian Mayneris. 2018. "Gentrification and pioneer businesses." CEPR Discussion Papers 13296.

Bitler, Marianne, and Lucie Schmidt. 2006. "Health disparities and infertility: impacts of state-level insurance mandates." *Fertility and Sterility*, 85(4): 858–865.

Bitler, Marianne P, and Lucie Schmidt. 2012. "Utilization of infertility treatments: The effects of insurance mandates." *Demography*, 49(1): 125–149.

Borusyak, Kirill, Peter Hull, and Xavier Jaravel. 2021. "Quasi-Experimental Shift-Share Research Designs." *Review of Economic Studies*, 89(1): 181–213.

Brueckner, Jan K., and Stuart S. Rosenthal. 2009. "Gentrification and Neighborhood Housing Cycles: Will America's Future Downtowns Be Rich?" *The Review of Economics and Statistics*, 91(4): 725–743.

Buckles, Kasey. 2008. "Understanding the returns to delayed childbearing for working women." *American Economic Review*, 98(2): 403–07.

Caucutt, Elizabeth M, Nezih Guner, and John Knowles. 2002. “Why do women wait? Matching, wage inequality, and the incentives for fertility delay.” *Review of Economic Dynamics*, 5(4): 815–855.

Coeurdacier, Nicolas, Pierre-Philippe Combes, Laurent Gobillon, and Florian Oswald. 2023. “Fertility, Housing Costs, and City Growth.” Unpublished manuscript.

Couture, Victor, and Jessie Handbury. 2020. “Urban revival in America.” *Journal of Urban Economics*, 119: 103267.

Couture, Victor, and Jessie Handbury. 2023. “Neighborhood change, gentrification, and the urbanization of college graduates.” *Journal of Economic Perspectives*, 37(2): 29–52.

Couture, Victor, Cecile Gaubert, Jessie Handbury, and Erik Hurst. 2019. “Income growth and the distributional effects of urban spatial sorting.” National Bureau of Economic Research.

Curci, Federico, and Federico Masera. 2018. “Flight from urban blight: Lead poisoning, crime and suburbanization.” *The Review of Economics and Statistics (Forth.)*.

Curci, Federico, and Hasin Yusaf. 2020. “Gentrifying cities, amenities and income segregation: Evidence from San Francisco.” *Unpublished manuscript.*

Diamond, Rebecca. 2016. “The Determinants and Welfare Implications of US Workers’ Diverging Location Choices by Skill: 1980-2000.” *American Economic Review*, 106(3): 479–524.

Doepke, Matthias, Anne Hannusch, Fabian Kindermann, and Michèle Tertilt. 2023. “The economics of fertility: A new era.” In *Handbook of the Economics of the Family*. Vol. 1, 151–254. Elsevier.

Edlund, Lena, Cecilia Machado, and Maria Micaela Sviatschi. 2015. “Gentrification and the Rising Returns to Skill.” National Bureau of Economic Research.

Ellen, Ingrid Gould, Keren Mertens Horn, and Davin Reed. 2019. "Has falling crime invited gentrification?" *Journal of Housing Economics*, 46: 101636.

Erosa, Andrés, Luisa Fuster, and Diego Restuccia. 2002. "Fertility Decisions and Gender Differences in Labor Turnover, Employment, and Wages." *Review of Economic Dynamics*, 5(4): 856–891.

Fan, Jingting, and Ben Zhou. 2021. "The Dual Local Markets: Family, Jobs, and the Spatial Distribution of Skills." SSRN Working Paper 3922412.

Farré, Lídia, Jordi Jofre-Monseny, and Juan Torrecillas. 2023. "Commuting time and the gender gap in labor market participation." *Journal of Economic Geography*, 23(4): 847–870.

Fee, Kyle, Daniel Hartley, et al. 2013. "The relationship between city center density and urban growth or decline." *Revitalizing American Cities*, 45–64.

Finlay, John, and Trevor Williams. 2020. "Housing demand, inequality, and spatial sorting." SSRN.

Fogli, Alessandra, and Veronica Guerrieri. 2019. "The End of the American Dream? Inequality and Segregation in US Cities." National Bureau of Economic Research, Inc NBER Working Papers 26143.

Gallen, Yana, Juanna Schrøter Joensen, Eva Rye Johansen, and Gregory F Veramendi. 2023. "The Labor Market Returns to Delaying Pregnancy." *Available at SSRN 4554407.*

Gershoni, Naomi, and Corinne Low. 2021. "The power of time: The impact of free IVF on Women's human capital investments." *European Economic Review*, 133: 103645.

Glaeser, Edward L., Matthew E. Kahn, and Jordan Rappaport. 2008. "Why do the poor live in cities The role of public transportation." *Journal of Urban Economics*, 63(1): 1–24.

Goldin, Claudia. 2006. "The Quiet Revolution That Transformed Women's Employment, Education, and Family." *American Economic Review*, 96(2): 1–21.

Goldin, Claudia, and Lawrence F Katz. 2002. "The power of the pill: Oral contraceptives and women's career and marriage decisions." *Journal of Political Economy*, 110(4): 730–770.

Guerrieri, Veronica, Daniel Hartley, and Erik Hurst. 2013. "Endogenous gentrification and housing price dynamics." *Journal of Public Economics*, 100(C): 45–60.

Hamilton, Barton H, and Brian McManus. 2012. "The effects of insurance mandates on choices and outcomes in infertility treatment markets." *Health Economics*, 21(8): 994–1016.

Hoelzlein, Mattias. 2019. "Two-Sided Sorting and Spatial Inequality in Cities." *Unpublished manuscript.*

Kleven, Henrik. 2023. "The Geography of Child Penalties and Gender Norms: Evidence from the United States." National Bureau of Economic Research Working Paper 30176.

Kleven, Henrik, Camille Landais, and Jakob Egholt Søgaard. 2019. "Children and Gender Inequality: Evidence from Denmark." *American Economic Journal: Applied Economics*, 11(4): 181–209.

Kwon, Eunjee. 2022. "Why Do Improvements in Transportation Infrastructure Reduce the Gender Gap in South Korea Distribution of Skills." SSRN Working Paper 4081288.

Lee, Sanghoon, and Jeffrey Lin. 2018. "Natural amenities, neighbourhood dynamics, and persistence in the spatial distribution of income." *Review of Economic Studies*, 85(1): 663–694.

Liu, Sitian, and Yichen Su. 2022. "The geography of jobs and the gender wage gap." *Review of Economics and Statistics*, 1–27.

Logan, John R., Zengwang Xu, and Brian J. Stults. 2014. "Interpolating US Decennial Census Tract Data from as Early as 1970 to 2010: A Longitudinal Tract Database." *Professional Geographer*, 66(3): 412–420.

Manson, Steven, Jonathan Schroeder, David Van Riper, Tracy Kugler, and Steven Ruggles. 2021. "IPUMS National Historical Geographic Information System: Version 16.0 [dataset]. Minneapolis, MN: IPUMS."

Miller, Amalia R. 2011. "The effects of motherhood timing on career path." *Journal of Population Economics*, 24(3): 1071–1100.

Moreno-Maldonado, Ana. 2022. "Mums and the City: Household labour supply and location choice." *Available at SSRN 3466171.*

Myers, Caitlin Knowles. 2017. "The Power of Abortion Policy: Reexamining the Effects of Young Women's Access to Reproductive Control." *Journal of Political Economy*, 125(6): 2178–2224.

Petrongolo, Barbara, and Maddalena Ronchi. 2020. "Gender gaps and the structure of local labor markets." *Labour Economics*, 64(C).

Pietrabissa, Giorgio. 2023. "School Access and City Structure." Unpublished manuscript.

Rothman, Kenneth J., Lauren A. Wise, Henrik T. Sørensen, Anders H. Riis, Ellen M. Mikkelsen, and Elizabeth E. Hatch. 2013. "Volitional determinants and age-related decline in fecundability: a general population prospective cohort study in Denmark." *Fertility and Sterility*, 99(7): 1958–64.

Ruggles, Steven, Sarah Flood, Matthew Sobek, Daniel Backman, Annie Chen, Grace Cooper, Stephanie Richards, Renae Rogers, and Megan Schouweiler. 2023. "IPUMS USA: Version 14.0." *Dataset.*

Schmidt, Lucie. 2007. "Effects of infertility insurance mandates on fertility." *Journal of Health Economics*, 26(3): 431–446.

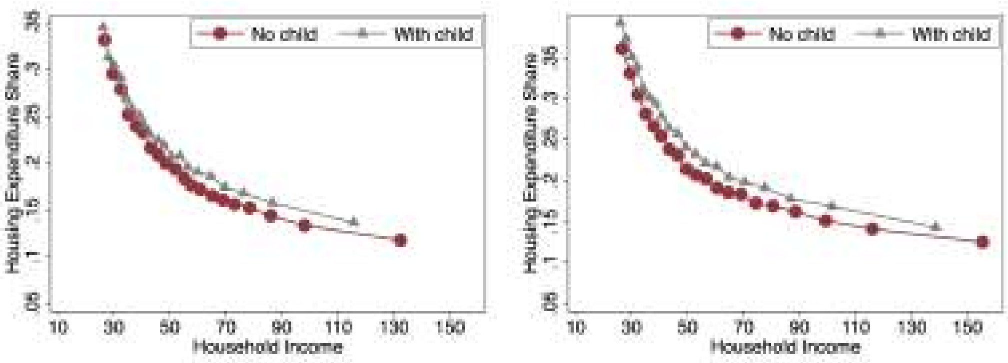
Su, Yichen. 2022. "The rising value of time and the origin of urban gentrification." *American Economic Journal: Economic Policy*, 14(1): 402–439.

Appendix

A Model appendix

A.1 Housing requirement by child's presence

In this section, we explore the quantitative importance of allowing the presence of children to impact the housing requirement. First, we document how the share of income spent on housing differs by the presence of children, controlling for income. Figure A.1 shows that housing expenditures represent a slightly higher fraction of income for households with children, especially at the top of the income distribution, in both 1990 and 2010. This may be surprising, since children certainly impose the need to have at least one additional separate room. However, the data suggest that households, especially those with lower income, adjust other margins with the presence of children. They may change location or give up other desirable housing characteristics in order to increase the number of rooms while keeping the share of income spent on housing roughly constant.



(a) 1990

(b) 2010

Figure A.1: Housing expenditure share

Notes: This figure plots for each decade the relationship between household income and the share of household income that is spent on annual gross rent. Each plot includes this relationship separately for households with children present and those without children present in the household. Source: American Community Survey 1990 and 2010.

Second, we estimate the housing requirement by computing the real housing expenditure, that is, dividing housing expenditure by the hedonic price, which we allow to differ by city and suburbs versus downtown. We normalize the housing requirement by assuming that a household with no children consumes one unit of housing. The housing requirement of households with children is the ratio of real housing expenditures with versus without children. We compute the housing requirement by decade to allow for possible changes in the socially acceptable amount of space dedicated to children. Table A.1 shows that the housing requirement of families with children would be only slightly above one. The implied housing requirement for households with children is only about 6% higher than for families with no children in 2010 and it was 2% higher in 1990. This small difference is unlikely to change the main results from the model, so we prefer to keep the model simple and assume a common housing requirement for all households. If the suburbs offer larger houses that families with children enjoy more, this will be captured in the relative amenity of downtown for families with children.

Table A.1: Housing requirement by child's presence

	Real Exp. without Kids (a)	Real Exp. with Kids (b)	Housing Requirement with Kids (c)
1990	10.44	10.72	1.02
2000	10.48	10.27	.98
2010	11.96	12.69	1.06

Notes: Columns (a) and (b) present the real annual expenditure on housing measured in thousands of 2000 dollars by dividing housing expenditure by the hedonic price index, which varies by city and within-city area (suburbs or downtown). Columns (c) includes the implied housing requirement for households with kids, relative to households with no children for which the housing requirement is one. Source: ACS 1990-2010.

B Model quantification appendix

B.1 Estimation of child penalties by fertility timing

Estimating child penalties is challenging because individuals with different (unobservable) work preferences or labor market prospects may sort into different fertility choices, which biases OLS estimations. Therefore, to estimate the causal effect of having children on earnings, researchers usually rely on high-quality panel data that allows them to control for individual fixed effects. However, this kind of data is not widely available and even when it is, the sample size is usually small, which limits the study of potential heterogeneity. In a recent paper, Kleven (2023) develops a new approach to estimating child penalties based on cross-sectional data, in which a pseudo-panel is constructed using matching techniques. We apply this methodology to estimate child penalties by period and fertility timing.

First, we construct a pseudo-panel using Census and ACS data from 1960 to 2019. Kleven, Landais and Søgaard (2019) estimate child penalties using event studies around the birth of the first child (which is indexed as event time $t = 0$). The main problem when using cross-sectional data is that we observe individuals only once, so we see both individuals with children and individuals without children, but it is not possible to know whether and when childless individuals will have children (i.e., negative event times are unobserved). However, we can match individuals with children to individuals without with the same demographic characteristics observed in earlier periods, which we will use as observations with negative event times in the event studies. Kleven (2023) validates this approach using data from the Panel Study of Income Dynamics and the National Longitudinal Survey of Youth. He shows that event studies that use the pseudo-panel approach yield results very similar to those that use panel data, when the matching is done according to year, age, gender, education, marital status, race, and state.

Second, we follow the event study approach to estimate child penalties on our pseudo-panel data. We employ the same specification as Kleven, Landais and Søgaard (2019) and run it separately for men (m), for women who had kids early (before turning 30, w^e), and for women who delayed (w^d):[29]

$$\text{Earnings}_{it}^{g} = \alpha^g \mathbb{D}_{it}^{Event} + \beta^g \mathbb{D}_{it}^{Age} + \gamma^g \mathbb{D}_{it}^{Year} + \nu_{it}$$

where i is the index on the individual and $g \in \{w^e, w^d, m\}$ indexes the different groups. The first term on the right-hand side includes dummies for each event time t, from 5 years before the birth of the child to 10 years after, omitting the year before childbirth. Event times coefficients, α^g, measure the impact of the birth of the first child on earnings in event year t, relative to the base year. The specification includes age and year dummies to control for life-cycle trends and time trends. Given that we want to capture changes in child penalties and the delay premium over our period of

[29]To be consistent with the model, we only consider women who had kids from age 20 to age 40.

study, we run separate regressions for each of the years we use in our model (1990, 2000, and 2010). As argued by Kleven, Landais and Søgaard (2019), identification of the short-term impact of children relies on a smoothness assumption, while long-term impacts require further assumptions. For this reason, we focus on the short-term impact (one year after childbirth) to infer changes in child penalties over time.

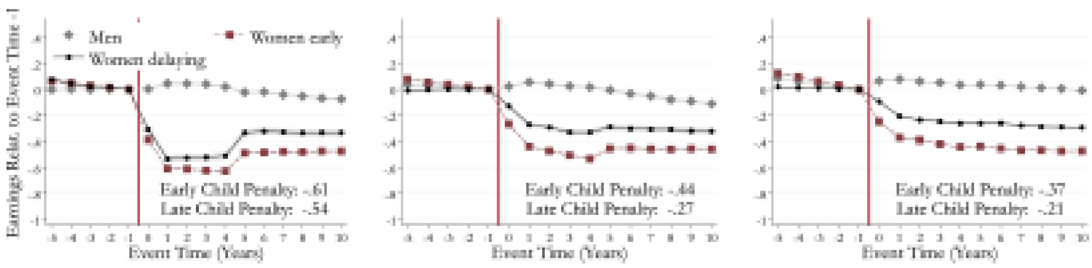
Third, we define the percentage impact for each group, P_t^g, and child penalties as in Kleven, Landais and Søgaard (2019):

$$P_t^g \equiv \frac{\hat{\alpha}_t^g}{\mathbb{E}[\tilde{Y}_{it}^g|t]} \tag{9}$$

$$\text{Child Penalty}_t \equiv P_t^m - P_t^{w^a} \tag{10}$$

where $\tilde{Y}_{it}^g|t$ is the counterfactual outcome absent children.

Figure B.1 displays our estimates for the percentage impact of children on earnings at each event time for each given group and period. Consistent with the findings in Kleven (2023), our estimates reflect a reduction of the child penalty over time. Moreover, women who have kids early experience a higher penalty than those delaying.[30] Lastly, we document that even though all child penalties have been decreasing, the decrease has been larger for women having kids later, which means that the delay premium increased over our period of study, and thus created incentives for delaying the arrival of children.



(a) 1990 (b) 2000 (c) 2010

Figure B.1: Percentage impact of children on earnings

Notes: This figure plots the estimated percentage impact of children on earnings at each event time for all groups in a given year: men, women who have kids early (before age 30) and women who delay. Each panel also displays the short-term (t=1) child penalty defined as in equation 10.

[30] This finding aligns with studies documenting the existence of a delay premium, such as Miller (2011); Adda, Dustmann and Stevens (2017); and Gallen et al. (2023).

B.2 Estimation of the elasticity of the amenity supply

To identify the causal effect of an increase in the proportion of households without children in a neighborhood on the supply of amenities, we propose a shift-share instrument that leverages exogenous changes in fertility choices that arise from the enactment of infertility insurance mandates. In this section, we describe how we construct our shift-share instrument.

Location demand The (log) share of households of age a, kids k, and skill z who at time t choose to live in MSA m and location j is given by

$$\ln\left(s_{j,m,a,k,z,t}\right) = \underbrace{\tilde{\phi}_{m,a,k,z,t}}_{\text{Fixed effect}} + \beta_{\varepsilon} \ln x(j,m,a,k,z,t) + \beta \ln \delta(j,m,a,k,z,t),$$

where $x_t\left(j,m,a,k,z\right) = I_t\left(m,a,k,z\right) - p_t\left(j,m\right)$ is the real income and $\delta(j,m,a,k,z,t)$ is the amenity from living in location l, and is common to all individuals of the same demographic group.

Endogenous amenity supply In our model, the supply of amenities reacts to the demographic composition of a location. Let the supply of amenities in location j within MSA m at time t, for demographic group g, where g indicates age, skill, and presence of kids be given by

$$\ln \delta(j,g,t) = \eta_1^g \ln\left(\frac{N_{j,m,t}^{High-Skill}}{N_{j,m,t}^{Low-Skill}}\right) + \eta_2^g \ln\left(\frac{N_{j,m,t}^{No-Kids}}{N_{j,m,t}^{Kids}}\right) ...$$
$$... + \tilde{\theta}^g \underbrace{X_{j,m,t}}_{\text{Observ. charc.}} + \underbrace{\phi_{j,m}^g}_{\text{Tract FE}} + \underbrace{\phi_{m,t}^g}_{\text{MSA-time FE}} + \underbrace{\xi_{j,m,t}^g}_{\text{Unobserv. charc.}}$$

There are two structural elasticities in the amenity supply, the first, η_1^g, captures how amenities react to the skill composition and the second, η_2^g, how they react to the composition of households with children. For now, we allow the elasticities to depend on the detailed demographic group g of age, skill, and kids. In the baseline estimation, we will assume that η_1^g depends only on skill and η_2^g depends only on the presence of children. This captures the idea that if the ratio of households with kids increases in an area, there may be an increase in children's parks, schools, or children's museums and/or shows, etc.. These can be positive or negative amenities, depending on whether a household has children. In particular, we expect amenities to increase for other households with kids and decrease for those without kids.

Substituting the supply of amenities into the demand for location, we can write the equilibrium share of a demographic group in a location as an iso-elastic function of skill and no-kids ratios, governed by the reduced-form elasticities γ_1^g and γ_2^g. These reduced-form parameters are a combination of demand and supply elasticities. Namely,

$$\ln\left(s_{j,m,g,t}\right) = \underbrace{\tilde{\phi}_{m,g,t}}_{\text{FE}} + \underbrace{\tilde{\phi}_{j,m,g}}_{\text{FE}} + \beta \ln x(j,m,g,t) + \gamma_1^g \ln\left(\frac{N_{j,m,t}^{High-Skill}}{N_{j,m,t}^{Low-Skill}}\right)$$
$$... + \gamma_2^g \ln\left(\frac{N_{j,m,t}^{No-Kids}}{N_{j,m,t}^{Kids}}\right) + \theta^g \underbrace{X_{j,m,t}}_{\text{Observ. charc.}} + \underbrace{\xi_{j,m,g,t}}_{\text{Unobserv. charc.}}$$

We estimate the equation in first differences so that time-invariant observed and unobserved differences between locations are eliminated and we are left with changes in unobserved characteristics.

$$\Delta \ln\left(s_{j,m,g,t}\right) = \Delta \underbrace{\tilde{\phi}_{m,g,t}}_{\text{FE}} + \beta \Delta \ln x(j,m,g,t) + \gamma_1^g \left(\Delta \ln N_{j,m,t}^{High-Skill} - \Delta \ln N_{j,m,t}^{Low-Skill}\right) ...$$
$$... + \gamma_2^g \left(\Delta \ln N_{j,m,t}^{No-Kids} - \Delta \ln N_{j,m,t}^{Kids}\right) + \theta^g \underbrace{\Delta X_{j,m,t}}_{\text{Observ. charc.}} + \underbrace{\Delta \xi_{j,m,g,t}}_{\text{Unobserv. charc.}}$$

The parameter of interest is γ_2^g, the reaction of the share of demographic group g to changes in the ratio of households with no children. To identify this parameter, we need an instrument that shifts the ratio of households with no kids but is uncorrelated with the change in $\Delta\xi_{j,m,g,t}$. We leverage variation arising from the introduction of state infertility mandates to construct this instrument.

B.2.1 Exogenous source of variation: Infertility insurance mandates

To identify the elasticity of local amenities to the ratio of households without children, γ_2^g, we exploit variation in the incentives to delay childbearing coming from the introduction of state-level mandates to cover infertility diagnosis and treatment.

Background. Women often find it desirable to postpone motherhood, given the well-documented positive effects of delaying childbearing on women's lifetime earnings (Buckles 2008; Caucutt, Guner and Knowles 2002; Miller 2011). However, fertility decays sharply with age. In this context, assisted reproductive technology[31] (ART) decreases the risk of delaying by increasing the probability of pregnancy at later ages. However, ART treatments and especially in-vitro fertilization (IVF), are expensive and insurers rarely cover their cost unless required by law.[32]

Starting in the 1980s, several US states enacted mandates that require private insurers to cover infertility diagnosis and treatment. The mandates decreased the cost of infertility treatments borne by patients. Utilization rates of infertility treatments increased significantly after the introduction of such mandates. Bitler and Schmidt (2012) estimate a 4.1 p.p. increase in the probability of using ART treatments for high-skilled women older than 30, using a difference-in-differences strategy.

The effect of the mandates on the delay of parenthood went above and beyond the increase in utilization rates. The reason is that ART treatments serve as an option value that creates incentives for women to delay pregnancy. However, the vast majority of women who delay by a few years are able to become pregnant without the need for ART. On top of that, the mandates may have increased awareness about the availability of IVF and consequently changed women's misconceptions about its effectiveness. Lastly, increased IVF usage may have reduced the stigma associated with marrying and having children at an older age for the whole population of women.

Therefore, the mandates provide us with a variation in the fertility composition of a city that is not driven by urban revival. In particular, it is important for our identification that the mandates were not driven by amenities that changed differently across locations.

Assignment to the treatment. There is substantial heterogeneity in the strength of the mandates across states. First, while most states require that insurers *cover* ARTs treatments in every available insurance policy, mandates in California and Texas only require insurers to *offer* infertility treatments. In addition, not all man-

[31] ART is defined by the Centers for Disease Control and Prevention (CDC) as "all fertility treatments in which both eggs and sperm are handled". In-vitro fertilization (IVF) is a process of fertilization in which an egg is combined with sperm in a laboratory. IVF is one of the most common ART techniques.

[32] One cycle of IVF entails an out-of-pocket cost of \$10,000 to \$15,000 to the patient and it is common to attempt multiple cycles of treatment (Hamilton and McManus, 2012).

dates include IVF, which is the most expensive and effective infertility treatment. Table B.1 lists all states that have enacted mandates that affect the insurance of ART procedures over the five decades covering our census samples (1970-2010) and summarizes their main features.

Table B.1: States with mandated infertility insurance

State	**Date enacted**	**Mandate to cover**	**Mandate to offer**	**IVF coverage**
Arkansas	1987	X		X
California	1989		X	
Connecticut	1989	X		X
Hawaii	1987	X		X
Illinois	1991	X		X
Louisiana	2001	X		
Maryland	1985	X		X
Massachusetts	1987	X		X
Montana	1987	X		
New Jersey	2001	X		X
New York	1990	X		
Ohio	1991	X		X
Rhode Island	1989	X		X
Texas	1987		X	X
West Virginia	1977	X		

Notes: This table summarizes the main features of acts that mandate infertility insurance in all states that ever passed a mandate of this type. Source: Resolve [2004], National Conference of State Legislatures [2004].

Therefore, we choose to restrict our attention to strongly treated states that mandated health insurers to cover infertility treatments (as opposed to only offering them) and that included IVF. In addition, we do not exploit variation in the timing when the mandates were enacted by pooling all states that passed reforms between 1980 and 1990.[33] Moreover, some US metropolitan areas belong to several states, such as Boston. In those cases, we consider that a city is treated if at least 10% of its population belongs to a state in our treated group. The rationale for this

[33]In our analysis, we use census data because it allows us to identify neighborhoods' location. However, since these data are only available every 10 years, we include 1970 and 1980 in the pre-treatment period, and consider 1990, 2000, and 2010 as part of the post-treatment period.

choice is that it is likely that residents in parts of the metropolitan area that belong to other states were also affected by the policy, since MSAs have a high degree of economic and social integration. In the baseline specification, weakly treated cities are dropped from the final sample. Lastly, given that urban revival is a large-city phenomenon, we focus on cities whose population in 2010 was above 1.5 million. After applying these criteria, we are left with the following list of treated and non-treated cities:

Table B.2: List of cities by treatment

Control	**Treated**
Allentown-Bethlehem-Easton, PA	Boston-Worcester-Lawrence, MA-NH-ME-CT
Atlanta, GA	Chicago-Gary-Kenosha, IL-IN-WI
Denver-Boulder-Greeley, CO	Cleveland-Akron, OH
Detroit-Ann Arbor-Flint, MI	New York, Northern New Jersey, Long Island, NY-NJ-CT-PA
Greensboro–Winston Salem–High Point, NC	St. Louis, MO-IL
Indianapolis, IN	Washington-Baltimore, DC-MD-VA-WV
Miami-Fort Lauderdale, FL	
Milwaukee-Racine, WI	
Minneapolis-St. Paul, MN-WI	
Norfolk-Virginia Beach-Newport News, VA-	
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	
Pittsburgh, PA	
Portland-Salem, OR-WA	
Seattle-Tacoma-Bremerton, WA	
Tampa-St. Petersburg-Clearwater, FL	

B.2.2 Data source and description

We employ data from the ACS and the population census from 1970 to 2010. We employ constant PUMAs in order to obtain a panel of sub-city constant areas in every year. Constant PUMAs are a bit larger than year PUMAs since they are the smallest constantly identifiable unit. There are no constant PUMAs for 1970. However, it is enough to employ city-level data for 1970 in order to test for parallel trends and for the event study. Importantly, we do not need to classify PUMAs into downtown and suburbs. We run the main analysis at constant puma level.

B.2.3 Shift-share instrumental variable

The identification strategy employs a shift-share instrument. We exploit the introduction of state insurance mandates in an event study to obtain shocks to the demographic composition of treated MSAs. This constitutes the shift and exogenous part of our instrument. We then combine the shift with the initial shares of each demographic group by local area within a city in 1980, before the mandates were introduced.

Event study specification. The shift part of the instrument comes from an event study specification that compares cities in treated and non-treated states by year. We denote the causal effect of the policy on fertility outcome k for an age a in treated cities at time t, $\beta^{a,k}_{DD,t}$. The dependent variable is the fertility composition in city m for a given age. We estimate $\beta^{a,k}_{DD,t}$ from the following regression, separately for each age and fertility outcome:

$$Prob\,(k|a)_{m,t} = \delta_t + \delta_m + \beta^{a,k}_{DD,t} Ever\ Treated_m \times \delta_t + X_{m,t} + \epsilon_{m,a,k,t}, \tag{11}$$

where $Prob\,(k|a)_{m,t}$ denotes the share of the population in m with age a that have kids k at time period t; δ_t, and δ_m are time and city fixed effects; $Ever\ Treated_m$ is a dummy variable taking value 1 if the city m is ever treated; $X_{m,t}$ is a set of controls for city m at time t such that conditional on these controls the treatment is as good as random; and $\epsilon_{m,a,k,t}$ is the error term, which must be uncorrelated with the treatment for identification. In the baseline specification controls include log population, log mean income, and log mean house value at city and year level.

Balancedness. For the event study specification to identify a causal effect from the treatment on the fertility composition, it is necessary that treated and non-treated cities not only were on parallel trends before the treatment, but that they would have stayed on parallel trends had it not been for the treatment. It is never possible to prove what would have happened absent the treatment, but it helps to understand why the treatment happened in some cities and not others in order to render the argument convincing. Introducing infertility insurance mandates was a policy choice, and as such it was not random but driven in part by the characteristics of the places where it was introduced. Schmidt (2007) finds that Democratic states were more likely to introduce this policy, since they had more favorable views on government intervention in healthcare insurance markets. The key assumption for us is that the introduction of this policy affected the urban structure only through its impact on the fertility composition. The policy preceded urban revival and thus was not a response to it. However, we need to rule out the possibility that cities that introduced the policy were different in some characteristics that caused them to be more likely to experience urban revival. In this section, we test for balancedness between treatment and control for different sets of variables that could render cities more likely to both implement the policy and experience urban revival later on.

Table B.3: Balancedness of treatment in 1970

	Difference	Treated	Control
	(1)	(2)	(3)
Log population	1.264***	15.46***	14.20***
	(0.319)	(0.270)	(0.170)
Log mean income	0.102*	2.886***	2.784***
	(0.0580)	(0.0490)	(0.0310)
Ln mean income, emp. males	0.106*	3.363***	3.257***
	(0.0598)	(0.0506)	(0.0320)
Ln mean income, emp. females	0.123*	2.110***	1.987***
	(0.0672)	(0.0568)	(0.0359)
% males with college +	1.203	10.05***	8.849***
	(0.907)	(0.767)	(0.485)
% females with college +	0.327	6.035***	5.708***
	(0.563)	(0.476)	(0.301)
% males not in the labor force	-0.636	17.40***	18.04***
	(1.698)	(1.435)	(0.908)
% females not in the labor force	-0.787	43.00***	43.79***
	(1.788)	(1.511)	(0.956)
% pop in female-intensive occ	0.633	69.99***	69.35***
	(0.920)	(0.778)	(0.492)
Mean age of houses	0.370**	4.500***	4.130***
	(0.185)	(0.156)	(0.0987)
Mean monthly rent	0.0618***	0.211***	0.149***
	(0.0201)	(0.0169)	(0.0107)

	Difference	Treated	Control
	(1)	(2)	(3)
Log mean house value	0.202***	9.961***	9.759***
	(0.0725)	(0.0613)	(0.0388)
Hedonic price downtown	-0.0267	0.976***	1.002***
	(0.0206)	(0.0174)	(0.0110)
Mean age	-0.116	31.91***	32.03***
	(1.173)	(0.991)	(0.627)
Ratio retired-to-working age	-0.0102	0.168***	0.178***
	(0.0316)	(0.0267)	(0.0169)
% non-white population	3.849	15.54***	11.69***
	(3.571)	(3.018)	(1.909)
% foreign born population	2.810	8.114***	5.304***
	(2.334)	(1.973)	(1.248)
% 20-30 yo with young kids	-3.068	43.59***	46.66***
	(1.974)	(1.668)	(1.055)
% 30-40 yo with young kids	0.550*	5.493***	4.942***
	(0.307)	(0.259)	(0.164)
% 30-40 yo with old kids	-3.994***	70.18***	74.18***
	(1.286)	(1.087)	(0.687)
% 40-50 yo with old kids	0.394	18.08***	17.69***
	(0.441)	(0.372)	(0.235)

Notes: This table presents the results from regressing, at the MSA level, aggregate characteristics of the city on an indicator for treatment. Column (1) includes the difference between treated and control cities, column (2) the average in treated, and column (3) in non-treated cities. Source: 1970 Census IPUMS. **p<0.1, **p<0.05, ***p<0.01.*

Table B.3 presents results of the balancedness test for variables related to overall economic activity (income and population); variables on women's emancipation (income of men and women, percentage of men and women who had college education, percentage of men and women in the labor force, and percentage of the population in female-intensive occupations); variables on the housing market (age of buildings, rent, and housing value); variables on aging of the population (mean age, ratio of retired to working population); variables on other aspects of the demographic composition (percentage of non-white and percentage of foreign); and variables on fertility (percentage of young, mature, and old with young or old kids).

Table B.4: Conditional balancedness of treatment in 1970

	Difference	Treated	Control	Ln Inc	Ln Pop	Ln House Val
	(1)	(2)	(3)	(4)	(5)	(6)
Ln mean income, emp. males	0.0217	1.741***	1.719***	0.00942	0.971***	-0.133***
	(0.0189)	(0.439)	(0.428)	(0.00968)	(0.0533)	(0.0401)
Ln mean income, emp. females	-0.0193	-3.584***	-3.565***	-0.0147	0.930***	0.325**
	(0.0598)	(1.389)	(1.353)	(0.0306)	(0.168)	(0.127)
% males with college +	0.0773	-26.78	-26.86	0.147	5.511	1.874
	(1.357)	(31.55)	(30.73)	(0.695)	(3.826)	(2.877)
% females with college +	-0.338	-24.54	-24.21	-0.0823	2.817	2.381
	(0.839)	(19.50)	(19.00)	(0.430)	(2.366)	(1.779)
% males not in the labor force	0.509	104.8***	104.3***	1.900***	-27.08***	-3.884
	(1.333)	(30.99)	(30.19)	(0.682)	(3.758)	(2.827)
% females not in the labor force	0.197	154.3***	154.1***	2.531***	-26.02***	-7.566**
	(1.541)	(35.81)	(34.89)	(0.789)	(4.343)	(3.266)
% pop in female-intensive occ	-0.472	43.34	43.82	0.879	-6.235*	3.116
	(1.308)	(30.40)	(29.62)	(0.669)	(3.687)	(2.773)
Mean age of houses	0.282	7.402	7.121	0.133	0.473	-0.635
	(0.275)	(6.398)	(6.233)	(0.141)	(0.776)	(0.584)
Mean monthly rent	-0.0126	-2.471***	-2.458***	0.0175**	0.0774*	0.220***
	(0.0150)	(0.348)	(0.339)	(0.00766)	(0.0422)	(0.0317)
Hedonic price downtown	-0.0131	0.548	0.561	-0.0275*	0.0878	0.0601
	(0.0300)	(0.696)	(0.678)	(0.0153)	(0.0845)	(0.0635)
Mean age	0.703	87.41***	86.70***	1.115*	-17.32***	-2.285
	(1.123)	(26.10)	(25.42)	(0.575)	(3.165)	(2.380)
Ratio retired-to-working age	0.0209	1.987***	1.966***	0.0273*	-0.466***	-0.0900
	(0.0295)	(0.686)	(0.668)	(0.0151)	(0.0832)	(0.0625)
% non-white population	-1.028	-148.1	-147.1	1.684	-1.007	14.11
	(5.503)	(127.9)	(124.6)	(2.816)	(15.51)	(11.66)
% foreign born population	-2.682	-88.16	-85.48	4.506***	-17.07**	7.616
	(2.812)	(65.35)	(63.67)	(1.439)	(7.926)	(5.961)
% 20-30 yo with young kids	2.073	199.6***	197.6***	-2.040	-0.859	-12.25**
	(2.649)	(61.56)	(59.97)	(1.356)	(7.466)	(5.615)
% 30-40 yo with young kids	-0.283	-12.74	-12.46	0.535***	-1.033	1.298
	(0.393)	(9.144)	(8.909)	(0.201)	(1.109)	(0.834)
% 30-40 yo with old kids	1.052	206.7***	205.6***	-2.600***	4.412	-10.94***
	(0.994)	(23.09)	(22.49)	(0.508)	(2.800)	(2.106)
% 40-50 yo with old kids	-0.148	3.310	3.458	0.173	1.795	0.694
	(0.680)	(15.80)	(15.40)	(0.348)	(1.917)	(1.442)

Notes: This table presents the results from regressing, at MSA level, aggregate characteristics of the city on an indicator for treatment controlling for log income, population, and housing value. Column (1) includes the difference between treated and control cities, column (2) the average in treated, and column (3) in non-treated cities. Columns (4) to (6) include the coefficients on the controls. Source: 1970 Census IPUMS. **p*<0.1, ***p*<0.05, ****p*<0.01.*

Treated cities tend to be larger, offer higher income for both employed men and women, have older houses that are more expensive, and households tend to have children a bit later. These are the only dimensions in which treated cities are statistically significantly different from control cities. They are a concern because they could indeed be in themselves predictors of urban revival even in the absence of treatment. For instance, Brueckner and Rosenthal (2009) show that the age of the housing stock contributes to urban revival. For this reason, we will include log population, log mean income, and log mean house value as controls in both the event study and the estimation of the elasticity for endogenous amenities. Table B.4 includes the balancedness of the treatment after controlling for these variables. Once we include the controls, there is no significant difference between treatment and control groups in any other variable.

Parallel trends and causal effect of the policy. Next, we test whether there are parallel trends in the variable of interest, that is, the fertility composition conditional on age. The previous section showed that in 1970, after controlling for log population, there were no significant differences in the fertility composition between treatment and control. However, it is still possible that the treated cities were on different trends. Since we only have data starting in 1970, the test for parallel trends will consist of testing that $\beta^{a,k}_{DD,1970}$ is not significantly different from zero. This coefficient captures the difference between treatment and control in changes in the fertility composition from 1970 to the event year, 1980.

Table B.5 presents results from the event study, in which we regress the fertility composition conditional on age on the interaction of treatment with year dummies, controlling for log population, log mean income, and log mean house value. The coefficient of the interaction of treatment with the 1970 dummy captures the pre-trend in the fertility composition. The pre-trend is not significant for the fertility composition of any of the age groups. Since there are no pre-trends, the coefficients on the following years can be interpreted as the causal effect of the introduction of fertility insurance mandates.

The introduction of fertility insurance mandates had a causal effect on the fertility decisions of the population. Table B.5 presents how the policy impacted the probability of having children at different ages.[34] Households in which the female was between 20 and 30 years old became 8 p.p. less likely to have children in 1990. This effect grew over time: By 2010, these young households were 21 p.p. less likely to have children. This effect is relative to a baseline of 47% of young households having children in 1970 in the control group, as shown in Table B.3.

Table B.5: Event study on fertility composition

	Age 20-30 w. young kids	Age 30-40 w. young kids	Age 30-40 w. old kids	Age 40-50 w. old kids
	(1)	(2)	(3)	(4)
Ever treated=1 × 1970	0.145	0.0926	0.0785	0.0648
	(0.145)	(0.0643)	(0.113)	(0.0509)
Ever treated=1 × 1990	-0.0821*	0.0844***	-0.214***	0.00250
	(0.0491)	(0.0212)	(0.0383)	(0.0172)
Ever treated=1 × 2000	-0.102	0.131***	-0.287***	0.0597***
	(0.0626)	(0.0270)	(0.0488)	(0.0220)
Ever treated=1 × 2010	-0.206**	0.193***	-0.388***	0.129***
	(0.0821)	(0.0355)	(0.0640)	(0.0288)
Log population	-0.00700	-0.00519	-0.0193	0.00726
	(0.0209)	(0.00902)	(0.0163)	(0.00732)
Log mean income	-0.00181	0.0104**	-0.00424	0.00701*
	(0.0118)	(0.00522)	(0.00917)	(0.00413)
Log mean house value	-0.0385	0.0267	-0.0956**	0.0476**
	(0.0550)	(0.0238)	(0.0429)	(0.0193)
Observations	1050	1019	1050	1050
City FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Notes: This table presents results from an event study in which an ever-treated dummy is interacted with decade fixed effects. The regression is run at MSA level. The dependent variable in column (1) is the fraction of households between 20 and 30 years old who have kids younger than 10 years in that city. In Columns (2) and (3), the dependent variable is the fraction of households between 30 and 40 years old with young and old kids, respectively. Finally, in Column (4), the dependent variable is the fraction of households between 40 and 50 years old with children older than 10 years. Census and ACS IPUMS data from 1970 to 2010. **p<0.1, **p<0.05, ***p<0.01.*

The decrease in fertility between 20 and 30 years old was accompanied by an

[34]Notice that for households with ages between 30 and 40, the age of children allows us to infer whether they had children when young or delayed instead.

increase in fertility for households between 30 and 40 years old, consistent with households affected by the policy increasingly delaying childbearing. In particular, households with ages between 30 and 40 became 8 p.p. more likely to have kids younger than 10 years in 1990 and the effect grew to 19 p.p by 2010. Moreover, these households became much less likely to have kids older than 10. The effect was 21 p.p in 1990 and grew to 39 p.p by 2010. This effect was out of a baseline of 74% of households between 30 and 40 years old having kids older than 10 years in 1970 in the control.

Finally, the last column of Table B.5 shows a corresponding increase in the percentage of households between 40 and 50 years old that have children older than 10 years As expected, this effect is delayed in time. There is no effect in 1990, followed by a 6 p.p. increase in 2000 and a 13 p.p. increase in 2010.

The introduction of fertility mandates had a causal impact on the delay of childbearing, which led to large changes in the fertility composition of the population at city level. Next, we employ this shift in the city-level fertility composition and interact it with the initial shares of households living downtown by age and fertility to obtain a shift-share instrument at local within-city level.

Shares in 1980. To predict the number of households in a demographic group g, in treated MSA m in each location j, $N_{j,m,g,t}$ for each year after the enactment of the policy, we combine the 1980's share of that demographic group g in location j of city m, $s_{j,m,g,80}$, with the 1980's number of people in this demographic group in a city m, $N_{m,g,80}$, plus the exogenous shock to the city-level composition, $\hat{\beta}^{g}_{DD,t}$ from the event study. Namely,

$$\hat{N}_{j,m,g,t} = \begin{cases} s_{j,m,g,80} \cdot N_{m,g,80}, & \text{if } t = 1980 \\ s_{j,m,g,80} \cdot \left(N_{m,g,80} + \underbrace{\hat{\beta}^{a,k}_{DD,t} N_{m,a,z,80}}_{\text{Effect of policy}} \right), & \text{if } t > 1980 \end{cases}$$

where g denotes a demographic group that corresponds to an age, skill, and presence of kids bin. Notice that for non-treated cities $\beta^{a,k}_{DD} = 0$, so we predict the population will stay as in 1980. Thus, we can only exploit treated cities in the identification of amenities. Importantly, the shift does not exploit any variation from changes in the age or skill distribution in the city, since these were not exogenous. It only incorporates changes in the population that arise from the exogenous change in the composition of fertility driven by the introduction of the policy.

Finally, we construct the predicted log changes in the ratio of households with no kids relative to those with kids driven only by the change in the fertility composition that resulted from the introduction of the infertility mandates. We will use this predicted change as the instrument for the actual change in the ratio of households with no kids. The predicted change is constructed by summing over young and mature households of all skills as follows:

$$\Delta \ln \left(\frac{\hat{N}^{\text{No kids}}_{j,m,t}}{\hat{N}^{\text{Kids}}_{j,m,t}} \right) = \ln \left(\frac{\sum_{a=\{y,m\}} \sum_z \hat{N}_{j,m,a,k=0,z,t}}{\sum_{a=\{y,m\}} \sum_z \hat{N}_{j,m,a,k>0,z,t}} \right) - \ln \left(\frac{\sum_{a=\{y,m\}} \sum_z \hat{N}_{j,m,a,k=0,z,t-1}}{\sum_{a=\{y,m\}} \sum_z \hat{N}_{j,m,a,k>0,z,t-1}} \right) \quad (12)$$

Relevance of the shift-share instrument. In the first stage, $\Delta \ln \left(\frac{N^{\text{No kids}}_{j,m,t}}{N^{\text{Kids}}_{j,m,t}} \right)$ is regressed on the instrument, $\Delta \ln \left(\frac{\hat{N}^{\text{No kids}}_{j,m,t}}{\hat{N}^{\text{Kids}}_{j,m,t}} \right)$, to show that the instrument has predictive power and the relevance assumption holds. Table B.6 presents the results. The instrument has predictive power in all 3 years of the study.

Table B.6: First stage

	1990	2000	2010
	(1)	(2)	(3)
$\Delta \ln \left(\frac{\hat{N}^{\text{No kids}}}{\hat{N}^{\text{Kids}}} \right)$	0.490***	1.396***	0.454***
	(0.0292)	(0.0492)	(0.0365)
Observations	5247	5248	5182
IV stage	First-stage	First-stage	First-stage

Notes: This table displays the first-stage regression. *$p<0.1$, **$p<0.05$, ***$p<0.01$.

B.2.4 Estimation results

Recall that the estimating equation of interest, the combination of location demand and supply of amenities, regresses the log change in the share of demographic group g (age, skill, kid's presence and kid's age) in location j of city m at time t on city and time fixed effects:

$$\begin{aligned}\Delta \ln \left(s_{j,m,g,t}\right) = \; & \underbrace{\Delta\tilde{\phi}_{m,K(g)}}_{\text{City \& kids FE}} + \underbrace{\Delta\tilde{\delta}_{t,K(g)}}_{\text{Year \& kids FE}} + \beta^{S(g)} \Delta \ln x(j,m,g,t) + ... \\ & \gamma_1^{S(g)} \Delta \ln \left(\frac{\hat{N}_{j,m,t}^{High-Skill}}{\hat{N}_{j,m,t}^{Low-Skill}} \right) + \gamma_2^{K(g)} \Delta \ln \left(\frac{\hat{N}_{j,m,t}^{NoKids}}{\Delta \ln \hat{N}_{j,m,t}^{Kids}} \right) ... \\ & + \theta^{K(g)} \underbrace{\Delta X_{m,t}}_{\text{Observ. charc.}} + \underbrace{\Delta \xi_{j,m,g,t}}_{\text{Unobserv. charc.}} ,\end{aligned} \tag{13}$$

where $K(g)$ denotes whether demographic group g belongs to one of two kids bins, either no kids or kids both young and old; $S(g)$ indicates whether the skill of demographic groups falls into the high-skill bin, which includes the top 30% of skill, or the low-skill bin, which includes the bottom 70% of skill;[35] $x\left(j,m,g,t\right)$ is a model-based composite of income and rent; and $X_{m,t}$ includes observable city characteristics that vary over time and includes log population, log mean income, and log mean house value. The parameter of interest is $\gamma_2^{K(g)}$, which will allow us to identify the supply elasticity of amenities to the no-kids ratio.

Identification of β^S. The coefficient β^S captures the location demand of demographic group g driven by the real income the location offers. However, the income and housing rent in a location is itself a function of how many people from a demographic group live there. For instance, there could be agglomeration economies whereby more people choosing a location can lead to higher income and more expensive housing as long as the housing elasticity is not infinite. Therefore, we would need to instrument income and housing rent. Since we lack this instrument, we instead borrow the causal estimates of Su (2022), who estimates the same parameter in the same context, MSAs in the United States from 1990 to 2010. He allows different skill groups, college and non-college-educated individuals, to value housing rent differently. We also leverage this heterogeneity and allow β^S to vary for a comparable heterogeneity, that is, the top 30% and bottom 70% of the income distribution conditional on age.

[35] Skill is classified into this group to resemble as closely as possible the classification into college and non-college. Roughly 30% of the U.S. population is college-educated.

Identification of γ_1^S. The coefficient γ_1^S captures how the share of a demographic group that chooses a location reacts to the ratio of high-skill to low-skill residents. The identification challenge comes from the fact that the high-skill ratio is the result of how desirable a location is for these two groups and, at the same time, it can drive desirability through the reaction of amenities. Once again, we borrow Su (2022)'s estimates for this parameter. He instruments the ratio of high- to low-skill residents by exploiting changes in the time value by different occupations.

Identification of γ_2^K. Finally, we arrive at the coefficient γ_2^K, which captures how the share of a given demographic group that chooses to live in a particular location reacts to the ratio of residents with no kids in that location. We allow this reaction to depend on whether this group has kids or not. This captures the fact that the presence of households with no children will affect the supply of amenities, such as bars, which may be a positive or negative amenity for households depending on whether they have children. The identification challenge comes from the fact that the ratio of households with no children will depend directly on the supply of children-related amenities. Therefore, we need an exogenous change in the ratio that is not driven by changes in amenities in order to quantify the reaction in the propensity to choose that location as a result of the change in the local composition. We exploit the introduction of infertility insurance mandates in order to construct an instrument for the ratio of households with no children. The identifying assumption is that the instrument is uncorrelated with the unobserved shocks to neighborhood characteristics. That is,

$$E\left[\Delta\xi_{j,m,g,t}\Delta\ln\left(\frac{\hat{N}^{\text{No kids}}_{j,m,t}}{\hat{N}^{\text{Kids}}_{j,m,t}}\right)\right]=0,$$

where $\Delta\xi_{j,m,g,t}$ is the error term in Equation 13 and the instrument is defined in Equation 12. We next show that this identification assumption can be rewritten to show that exogeneity of the shift is enough for identification, following a similar logic to that of Borusyak, Hull and Jaravel (2021).

First, we write the above condition as equivalent to two separate conditions: one for the number of households with no kids and one for the number with kids.

$$E\left[\sum_j \xi_{j,m,g,t}\Delta\hat{N}_{j,m,g,t}\right] = 0;$$
$$E\left[\sum_j \xi_{j,m,g,t}\cdot s_{j,m,g,80}\cdot \hat{\beta}^{a,k}_{DD,t}N_{m,a,z,80}\right] = 0;$$
$$E\left[\hat{\beta}^{a,k}_{DD,t}N_{m,a,z,80}\cdot \sum_j \xi_{j,m,g,t}\cdot s_{j,m,g,80}\right] = 0;$$
$$E\left[\hat{\beta}^{a,k}_{DD,t}N_{m,a,z,80}\cdot \bar{\xi}_{m,g,t}\right] = 0.$$

The identification assumption requires that the shift in population arising from the policy, $\hat{\beta}^{a,k}_{DD,t}N_{m,a,z,80}$, is uncorrelated with a weighted average of unobservable shocks at the demographic group g and within-city location j in m, weighted by 1980's shares of demographic group g that chooses to live in each location j. Identification will hold if the introduction of the policy is not triggered by a factor that caused future unobservable shocks. The balancedness and parallel trends tests in the event study showed that the treatment was not correlated with a wide range of potential factors that could have triggered urban revival independent of the policy.

Moreover, a threat to identification would arise if the introduction of the insurance mandates was in response to a particular spatial distribution of the population in 1980 within the city, for example, if state governments decided to introduce the policy in response to households with children living disproportionately downtown. Although there are not any obvious reasons why this would be the case, we test for the possibility that treated cities had a statistically different distribution of the population in 1980. Table B.7 includes the balancedness test in 1980 between control and treated cities on variables that relate to the differential spatial sorting of the groups of interest. We find that households are a bit more likely to live downtown in treated cities in 1980, though the difference is not significant. Younger households and households with no children are more likely to live downtown in both treated and control cities to a similar and statistically equal extent. Therefore, the exogeneity of treatment with respect to the initial shares holds and the shift-share instrumental variable identification assumption holds.

Table B.7: Balancedness on the spatial distribution of the population

	Difference	Treated	Control
	(1)	(2)	(3)
% Households downtown	2.343	8.630***	6.287***
	(2.161)	(1.826)	(1.155)
Diff. in % downtown: Households aged 20 to 30 - 40 to 50	0.119	0.731***	0.613***
	(0.281)	(0.237)	(0.150)
Diff. in % downtown: Houseolds with no kids - with kids	0.0340	0.653	0.619*
	(0.643)	(0.543)	(0.344)

Notes: This table summarizes differences in the spatial distribution of the population between treated and non-treated cities in 1980. **p<0.1, **p<0.05, ***p<0.01.*

Finally, Table B.8 presents estimation results of the elasticity of amenities with respect to the ratio of households with no kids relative with households with either young or old kids. As expected, we find a positive elasticity for households with no kids and a negative elasticity for households with kids. The estimated elasticities are a combination of the response of the amenity supply to the higher concentration of households with no kids in the neighborhood and the possibility that households with no children may simply value being around other households with no children. We can compare the magnitude with the elasticity of endogenous amenities to the ratio of high-skill households. The elasticity of no-kids amenities for households with no kids is about one-fifth of the elasticity of high-skilled households to the ratio of high-skilled to low-skilled households.

B.3 Estimation of the distribution of idiosyncratic preferences for children

This section outlines how we estimate the Fréchet parameter for the distribution of idiosyncratic preferences for children, β_η. Recall that the decision of whether to have children young, mature, or not at all is pinned down by two thresholds on the distribution of idiosyncratic preferences: One that makes households indifferent between having kids young or delaying, $\bar{\eta}^t_{ky,km}(z)$, and another that makes households indifferent between delaying childbearing or not having kids at all, $\bar{\eta}^t_{km,nk}(z)$.

Table B.8: Elasticity of the endogenous amenity

	No kids	Kids
	(1)	(2)
$\Delta \ln \left(\frac{N^{\text{No kids}}}{N^{\text{Kids}}} \right)$	0.484**	-0.309**
	(0.223)	(0.129)
Controls	Yes	Yes
Year FE	Yes	Yes
MSA FE	Yes	Yes
F-stat	237.382	277.260
Observations	6751	8926

Notes: This table presents results from regressing the change in the share of households that choose to live in a location on the change in the log ratio of the number of households without kids relative to households with kids in that location. Results in Column (1) are from running this regression for households with no kids, and those in Column (2) for households with kids. The change in the log ratio of households without and with kids is instrumented with a shift-share IV in which the shift is the change in the fertility composition at MSA level coming from the introduction of infertility insurance mandates, and shares come from the location decisions of households in 1980. Controls include the log mean income, population, and value of houses at MSA level. Data source: Census and American Community Survey IPUMS data from 1980 to 2010. **p<0.10, **p<0.05, ***p<0.01.*

Figure B.2 illustrates the relationship between the thresholds in the idiosyncratic preference distribution and the mass of households that make each fertility choice.

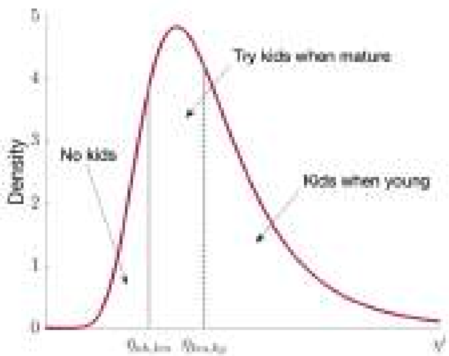


Figure B.2: Fertility decision at period t

The shape of the Fréchet distribution, β_η parameter determines how changes in the thresholds affect choices and consequently fertility outcomes. Thus we can estimate this parameter by exploiting the relationship between the thresholds, which are given by optimality conditions from the model, and observed fertility outcomes.

To derive the estimating equation, we first notice that the relationship between the fraction of mature households mature households with young or old children by skill in the data and the distribution of preferences, which is given by

$$\pi_t^{fert}(2,1,z) = \rho_m \left(F_\eta \left(\bar{\eta}_{ky,km}^{t-1}(z) \right) - F_\eta \left(\bar{\eta}_{km,nk}^{t-1}(z) \right) \right),$$
$$\pi_t^{fert}(2,2,z) = 1 - F_\eta \left(\bar{\eta}_{ky,km}^{t-1}(z) \right).$$

Combining the two equations above, we obtain:

$$F_\eta \left(\bar{\eta}_{km,nk}^{t-1}(z) \right) = 1 - \pi_t^{fert}(2,2,z) - \frac{1}{\rho_m} \pi_t^{fert}(2,1,z).$$

Notice that $F_\eta \left(\bar{\eta}_{km,nk}^{t-1}(z) \right)$ coincides with the fraction of mature households that *choose* not to have children, which we denote as $\pi_t^*(2,0,z)$. This fraction is not equal to the fraction of mature households that do not have children, $\pi^{fert_t(2,0,z)}$, because some mature households may not have been successful after delaying. Therefore, we obtain that:

$$\pi_t^*(2,0,z)_\eta \left(\bar{\eta}_{km,nk}^{t-1}(z) \right) = 1 - \pi_t^{fert}(2,2,z) - \frac{1}{\rho_m} \pi_t^{fert}(2,1,z).$$

Using the definition of the Fréchet distribution:

$$-\ln\left(-\ln\left(\pi_t^*(2,0,z)\right)\right) = \beta_\eta \ln \bar{\eta}_{km,nk}^{t-1}(z).$$

Next, we substitute in the expression for the threshold $\bar{\eta}_{km,nk}^{t-1}(z)$. The threshold on idiosyncratic preferences is such that households born at $t-1$ are indifferent between delaying childbearing or not having children. Namely,

$$\rho_m v_{km,t-1}^* \left(z; \bar{\eta}_{km,nk}^{t-1} \right) + (1-\rho_m) v_{nk,t-1}^*(z) = v_{nk,t-1}^*(z),$$

where $v_{km,t}^*(z;\eta^i)$ and $v_{nk,t}^*(z)$ are the present discounted utilities of having children mature or not having children respectively, for households born in period t with skill z and who draw an idiosyncratic preference for children equal to η^i. They are formally defined in Equation 4 as the present discount value of period utilities.

Re-arranging the indifference condition we obtain:

$$\bar{\eta}_{km,nk}^{t-1} = \frac{\tilde{\Phi}_{m,0,z}^{t} + \phi\tilde{\Phi}_{o,0,z}^{t+1}}{\tilde{\Phi}_{m,1,z}^{t} + \phi\tilde{\Phi}_{o,2,z}^{t+1}} \cdot \frac{1}{\kappa_{m,t}(z)},$$

where $\tilde{\Phi}_{a,k,z}^{t} \equiv \left(x_t\left(a,k,z,d\right)^{\beta_\eta}\Delta_t(a,k,z)^{\beta_\eta} + (N-1)\,x_t\left(a,k,z,s\right)^{\beta_\eta}\right)^{1/\beta_\eta}\Gamma\left(1-\frac{1}{\beta_\eta}\right)$, d and s denote downtown and suburb locations, $x_t\left(a,k,z,l\right) \equiv I\left(a,k,z\right) - p_{t,l}$, and Γ is the Gamma function: $\Gamma\left(n\right) = \left(n-1\right)!$, and where $\kappa_{m,t}(z)$ is the utility that households of skill z who have children as mature at time t obtain for every period the child is present in the household.

We can finally write the estimating equation as follows:

$$-\ln\left(-\ln\left(\pi_t^*\left(2,0,z\right)\right)\right) = \beta_\eta \underbrace{\ln\frac{\tilde{\Phi}_{m,0,z}^{t} + \phi\tilde{\Phi}_{o,0,z}^{t+1}}{\tilde{\Phi}_{m,1,z}^{t} + \phi\tilde{\Phi}_{o,2,z}^{t+1}}}_{\equiv\Delta_{nk,km}\ln\tilde{\Phi}^t(z)} - \beta_\eta \ln\kappa_{m,t}(z), \tag{14}$$

where an observation is a skill and time and $\beta_\eta \ln\kappa_{m,t}(z)$ is the unobserved error term and we have denoted with $\Delta_{nk,km}\ln\tilde{\Phi}^t(z)$ the log difference in incentives between having no kids or delaying kids at time t for skill z. Intuitively, β_η captures how the fraction of households that choose not to have children react to the ratio of incentives that arise from disposable income and downtown amenities.

The identification of β_η requires that the common amenity associated with having children mature $\ln\kappa_{m,t}(z)$ is uncorrelated with the incentives from income, housing prices, and amenities. In other words, whatever drives households to enjoy the presence of children does not affect the child penalties or the downtown amenities. This assumption would be violated if, for example, the fact the households of skill z in time t derive a high utility from children leads to the delay premium for these households to be particularly high. Then we would not be able to distinguish if households delay because of the income premium or the preference. We include skill fixed effects in the regression to attenuate this concern.

Table B.9 presents the results from estimating Equation 14.

Table B.9: Fréchet shape of children's idiosyncratic preferences

	(1)
$\Delta_{nk,km} \ln \tilde{\Phi}^t(z)$	2.121***
	(0.490)
Skill FE	Yes
Observations	40

Notes: This tables presents the results from regressing a monotonic transformation of the fraction of households that choose not to have children, on a measure the incentives not to have children that arise from disposable income and downtown amenities. The resulting coefficient is the estimate of the Fréchet shape parameter of the idiosyncratic preference for children. Source: Census and American Community Survey IPUMS data 1980-2010. **p<0.1, **p<0.05, ***p<0.01,*

In the next section, we outline how, once we have β_η, we obtain $\kappa_{km}(z)$ and $\kappa_{ky}(z)$ that perfectly replicate the observed fertility outcomes in the population.

B.4 Estimation of children amenities

In this section, we describe in detail the procedure for estimating the amenities that households enjoy from the presence of children. The first step is to back out the thresholds $\bar{\eta}^t_{ky,km}(z)$ and $\bar{\eta}^t_{km,nk}(z)$ from the observed fertility outcomes. By observing at time t the fraction of young households with no kids, and mature households with young kids and with old kids we can back out the following three thresholds:

$$\begin{aligned}
\bar{\eta}^t_{ky,km}(z) &= \left(-\ln \pi^{fert}_t(1,0,z)\right)^{-\frac{1}{\beta_\eta}} \\
\bar{\eta}^{t-1}_{ky,km}(z) &= \left(-\ln\left(1-\pi^{fert}_t(2,2,z)\right)\right)^{-\frac{1}{\beta_\eta}} \\
\bar{\eta}^{t-1}_{km,nk}(z) &= \left(-\ln\left(1-\frac{\pi^{fert}_t(2,1,z)}{\rho_m}-\pi^{fert}_t(2,2,z)\right)\right)^{-\frac{1}{\beta_\eta}},
\end{aligned}$$

where $\pi^{fert}_t(a,k,z)$ is the fraction of households of age a and skill z who at time t have kids k, ρ_m is the probability of having children when trying as a mature household, and β_η is the shape of the Fréchet distribution of idiosyncratic preferences for children. The previous section outlined how this parameter was estimated in a preceding step.

Once we have the thresholds, we exploit the indifference conditions to obtain an expression for the children amenities as a function of the thresholds. The threshold on idiosyncratic preferences is such that households born at t are indifferent between delaying childbearing or not having children. Namely,

$$\begin{aligned} v^*_{ky,t}\left(z;\bar{\eta}^t_{ky,km}\right) &= \rho_m\left(v^*_{km,t}\left(z;\bar{\eta}^t_{ky,km}\right)\right) + (1-\rho_m)\, v^*_{nk,t}(z) \\ v^*_{nk,t}(z) &= \rho_m\left(v^*_{km,t}\left(z;\bar{\eta}^t_{km,nk}\right)\right) + (1-\rho_m)\, v^*_{nk,t}(z), \end{aligned}$$

where $v^*_{f,t}\left(z;\eta^i\right)$ is the present discounted utility of fertility path f for households born in period t with skill z and who draw an idiosyncratic preference for children equal to η^i. They are formally defined in Equation 4 as the present discount value of period utilities. Re-arranging the indifference conditions, we obtain:

$$\begin{aligned} \kappa_{m,t+1}(z) &= \frac{\tilde{\Phi}^{t+1}_{m,0,z} + \phi\tilde{\Phi}^{t+1}_{o,0,z}}{\tilde{\Phi}^{t}_{m,1,z} + \phi\tilde{\Phi}^{t+2}_{o,2,z}} \cdot \frac{1}{\bar{\eta}^t_{km,nk}(z)} \\ \kappa_{y,t}(z) &= \rho_m\left(\frac{\tilde{\Phi}^{t}_{y,0,z} + \phi\tilde{\Phi}^{t+1}_{m,1,z}\kappa_{m,t+1}(z)\,\bar{\eta}^t_{ky,km}(z) + \phi^2\tilde{\Phi}^{t+2}_{o,2,z}\kappa_{m,t+1}(z)\,\bar{\eta}^t_{ky,km}(z)}{\tilde{\Phi}^{t}_{y,1,z}\bar{\eta}^t_{ky,km}(z) + \phi\tilde{\Phi}^{t+1}_{m,2,z}\bar{\eta}^t_{ky,km}(z)}\right) \\ &\quad + \frac{(1-\rho_m)\left(\tilde{\Phi}^{t}_{y,0,z} + \phi\tilde{\Phi}^{t+1}_{m,0,z} + \phi^2\tilde{\Phi}^{t+2}_{o,0,z}\right) - \phi^2\tilde{\Phi}^{t+2}_{o,0,z}}{\tilde{\Phi}^{t}_{y,1,z}\bar{\eta}^t_{ky,km}(z) + \phi\tilde{\Phi}^{t+1}_{m,2,z}\bar{\eta}^t_{ky,km}(z)} \end{aligned}$$

where $\tilde{\Phi}^t_{a,k,z} \equiv \left(x_t(a,k,z,d)^{\beta_\eta}\,\Delta_t(a,k,z)^{\beta_\eta} + (N-1)\,x_t(a,k,z,s)^{\beta_\eta}\right)^{1/\beta_\eta}\Gamma\left(1-\frac{1}{\beta_\eta}\right)$, d and s denote downtown and suburb locations, $x_t(a,k,z,l) \equiv I(a,k,z) - p_{t,l}$, and Γ is the Gamma function: $\Gamma(n) = (n-1)!$ and where $\kappa_{a,t}(z)$ is the utility that households skill z who have children at age a and time t obtain for every period the child is present in the household.

Notice that the equilibrium relationship between amenities and the idiosyncratic preference thresholds depends on the expectations that households have on housing prices. Therefore we need to solve for the path of housing prices at the same time as for children amenities. We do so by iterating over the path of housing prices.

- Step 0: We first guess that households expect housing prices to stay constant from 2010 on.
- Step 1: Given this expectation, we solve for children amenities until 2010 and

assume they stay constant from then on.[36]

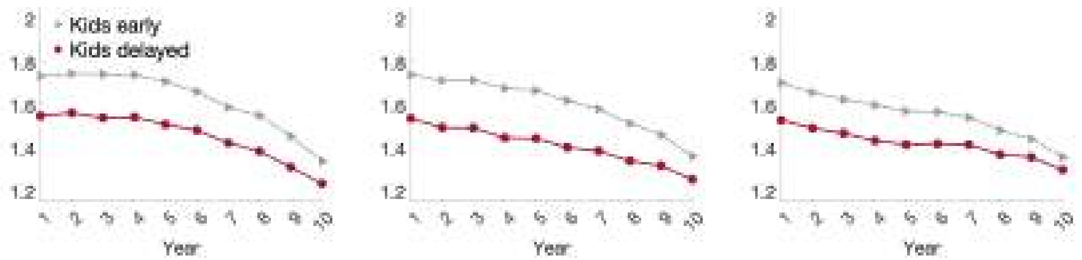
- Step 2: Then we solve for the model going forward and obtain the equilibrium housing prices that clear housing markets.
- Step 3: Use this equilibrium path of housing prices for the next guess of households' expectations and start again in Step 1.

We keep iterating until the expected path of housing prices converges to the equilibrium path.

B.5 Additional estimation results

B.5.1 Child amenities by fertility timing

Figure B.3 displays estimated child amenities from having kids when young and mature for each skill bin and year.



(a) 1990 (b) 2000 (c) 2010

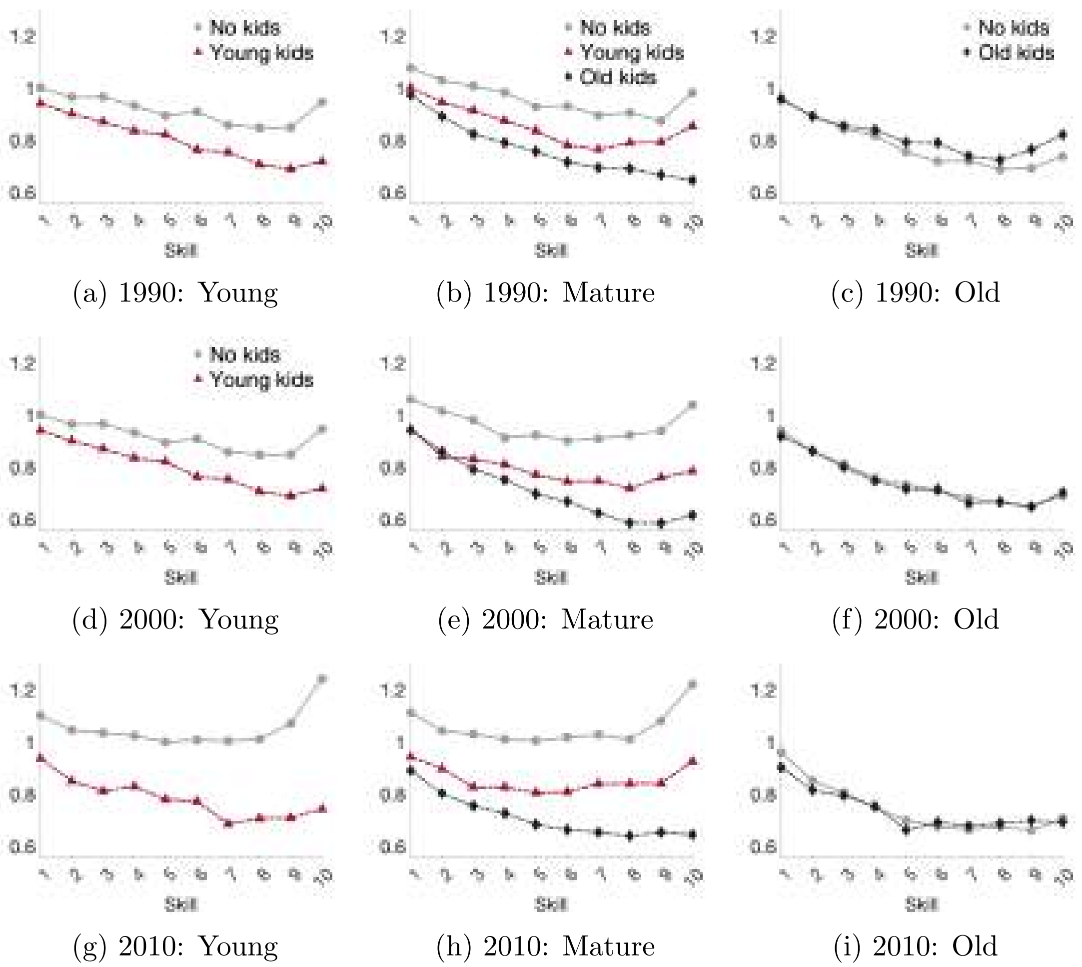
Figure B.3: Child amenities

Notes: This figure displays the results of our estimation of child amenities for households that had children when they were young and when they were mature for each skill bin and year.

B.5.2 Estimated downtown amenities

Figure B.4 displays all the results of our estimation for downtown amenities. Therefore, it includes the downtown amenity for each age, kids' age, skill, and year.

[36]Recall that we assume all the exogenous parameters remain constant starting in 2010, that is, income, exogenous downtown amenities, the population composition of age and skill, and the housing supply level parameter.



(a) 1990: Young (b) 1990: Mature (c) 1990: Old

(d) 2000: Young (e) 2000: Mature (f) 2000: Old

(g) 2010: Young (h) 2010: Mature (i) 2010: Old

Figure B.4: Downtown amenities estimation

Notes: This figure exhibits all the estimated downtown amenities. Thus, it shows the estimated downtown amenity for each age, kids' age, skill, and year.

B.5.3 Changes in downtown amenities by age, skill, and kids' age.

Figure B.5 displays the 1990-2010 change in the lifetime downtown amenity for each skill bin by fertility choice. It can be seen that the downtown amenity grew more for households that chose not to have kids or that delayed childbearing as compared to households that had their kids when they were young. The difference is larger for high-skilled households.

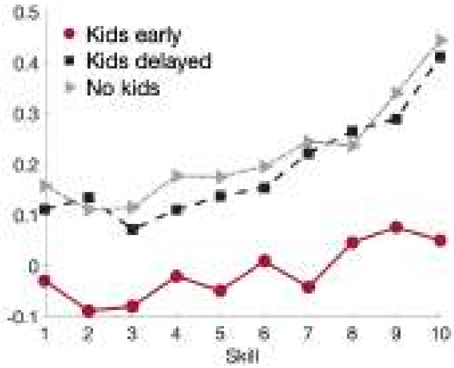


Figure B.5: 1990-2010 Change in lifetime downtown amenities.

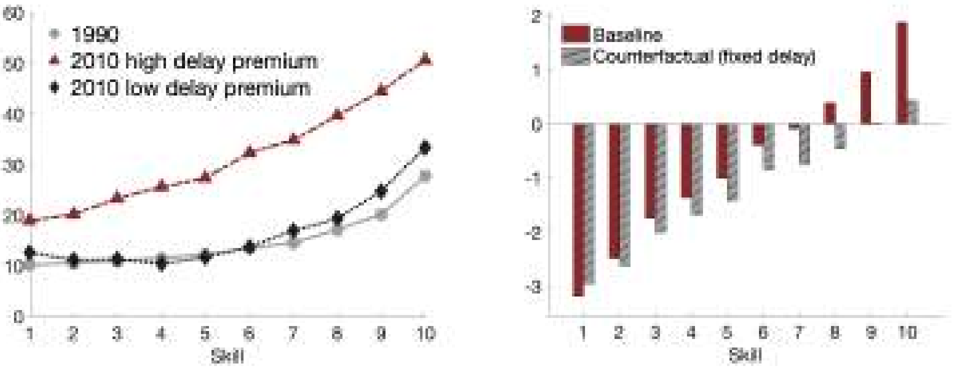
Notes: This figure displays the 1990-2010 change in lifetime downtown amenities for each fertility choice. Lifetime amenities are the discounted sum of downtown amenities when young, mature, and old.

C Counterfactual Exercises Appendix

C.1 Alternative Counterfactual: No Increase in Delayed Childbearing

In this section, we summarize the results of a counterfactual in which the percentage of households that delay is fixed to 1990. To perform this counterfactual, we shut down the choice of fertility and keep the fraction of households that delay exogenously fixed to 1990 levels, while we allow for everything else to evolve as in the data. Thus, this counterfactual allows us to compute how much less urban revival there would have been had choices about the timing of fertility not changed.

Figure C.6 summarizes the changes in fertility and location choices between the baseline equilibrium and a counterfactual equilibrium in which the percentage of people who delay is kept to its level in 1990. Panel (a) displays the percentage of households that delay childbearing in 1990 and 2010. This percentage is computed as the number of households that have young children when mature over all households having children for each cohort. Panel (b) displays the 1990-2010 changes in the percentage of households that live downtown by skill, in both the baseline and the counterfactual. Given that according to our model, almost all the delay in childbearing is explained by changes in the delay premium, the results are very similar to the counterfactual in which the delay premium is kept as in 1990.



(a) % Households Delaying (b) Δ % Living Downtown

Figure C.6: Changes in fertility and location choices

Notes: This figure summarizes changes in fertility and location choices between the baseline equilibrium and a counterfactual equilibrium in which the percentage of households that delay is kept to its level in 1990. Panel C.6a displays the percentage of households that delayed childbearing in 1990 and 2010. This percentage is computed as the number of households that have young children when mature over all households having children for each cohort. Panel C.6b displays the 1990-2010 changes in the percentage of households that live downtown by skill, in both the baseline and the counterfactual.

Table C.10 is the analogue of Table 4 in the main text. Thus, this table displays several measures of urban revival in the baseline and each counterfactual exercise we use to decompose the effects of a change in delay. The first column of Panel (a) in Table 4 displays the downtown/suburbs difference in the percentage growth of average income, housing price, and percentage of high-skilled, in the *baseline* economy, in which the percentage of households that delay increases over time. Columns (2) to (6) decompose the total effect of the counterfactual in which this percentage remains as in 1990. Each effect is computed as the difference in urban revival between the baseline economy and each counterfactual exercise. Column (2) displays the effect of fixing delay to 1990 and letting households optimize their location choices, assuming that downtown amenities and housing prices do not adjust (*the direct effect*). In columns (3) to (5), we evaluate the magnitude of the amplification effects through housing prices and amenities, *holding the fertility composition* as in column (2). Column (3) adds the effect of housing price adjustments to the direct effect, keeping amenities as in the baseline; Column (4) adds the effect of amenity adjustments, keeping housing prices as in the baseline; and, Column (5) adds both housing prices and amenities. Finally, in Panel (b) we compute the percentage of the baseline urban revival that can be accounted for each counterfactual exercise.

Notice that, since fertility is shut down the feedback effect is not present in this counterfactual.

Table C.10: Measures of Urban Revival

Panel a: Decomposition of Urban Revival					
	Baseline (1)	Direct (2)	Direct & Housing (3)	Direct & Amen. (4)	Direct & Housing & Amen. (5)
Rel. income growth	10.46	0.54	1.26	3.59	5.62
Rel. housing price growth	14.16	0	1.66	0	4.69
Rel. high-skilled growth	9.09	0.72	1.11	2.45	3.5
Panel b: Percentage of Urban Revival Explained					
	Baseline (1)	Direct (2)	Direct & Housing (3)	Direct & Amen. (4)	Direct & Housing & Amen. (5)
Rel. income growth	0	5.13	12.04	34.37	53.72
Rel. housing price growth	0	0	11.7	0	33.13
Rel. high-skilled growth	0	7.93	12.21	26.94	38.48

Notes: The first column of Panel (a) displays the downtown/suburbs difference in the percentage growth of average income, housing price, and percentage of high-skilled in the baseline economy. Column (2) displays the difference in urban revival between the baseline and the counterfactual when only the fertility composition changes; Column (3) adds the effect of housing price adjustments only; Column (4) adds the effect of amenities only; and Column (5) adds both housing prices and amenities to the initial change in the fertility composition. Panel (b) displays the percentage of the baseline explained by each counterfactual exercise.

C.2 Additional Results on Baseline Counterfactual No Increase in the Delay Premium

Percentage of female income in household income. Figure C.7 displays the percentage of female income in household income for 1990, 2000, and 2010. This percentage increases with household skill.

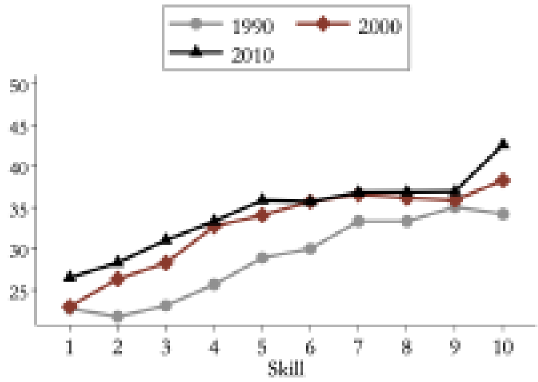


Figure C.7: % Female income in household income

Feedback effect. Figure C.8 illustrates the magnitude of the feedback effect. It displays the difference in the percentage of households having kids in 2010 in the counterfactual before fertility reacts to changes in housing prices and amenities (the direct effect) and afterward (the feedback effect). It can be seen that as a consequence of the decline in amenities for childless households, both young and mature households increase their fertility rates by 1-2.5 p.p..

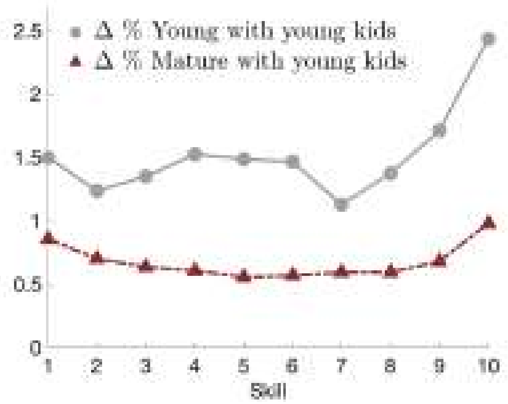
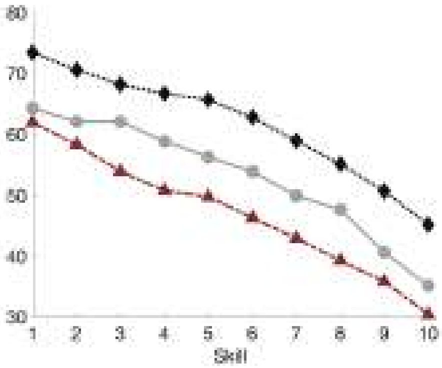


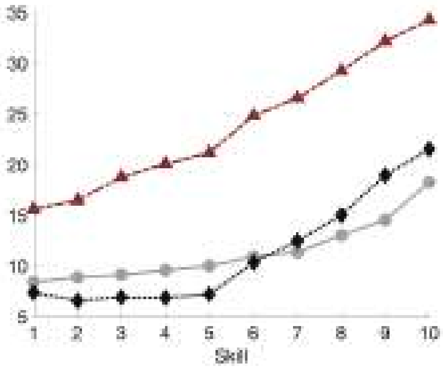
Figure C.8: Feedback effect

Notes: This figure shows the difference in the percentage of young and mature households with children in 2010 in the counterfactual before fertility choices react to housing prices and amenities (direct) and afterward (feedback).

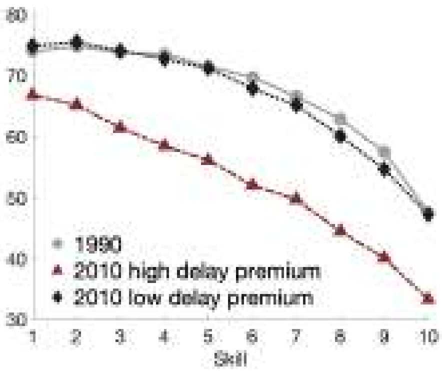
Changes in fertility choices by age and skill. Figure C.9 summarizes changes in fertility choices between 1990 and 2010 in both the baseline economy and the counterfactual with no increase in the delay premium. It displays the percentage of young, mature, and old households that have young or old children by skill. We can see that in the baseline economy, the percentage of households that have children early (those that have young kids when they are young and old kids when mature) decreased over time, while the percentage of households that delay childbearing (those that have young kids when they are mature and old kids when old) increased. In the counterfactual, there is lower increase in the percentage of households that delay: There are more young with young kids and fewer mature with young kids.



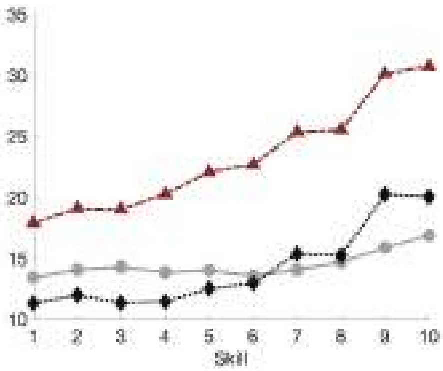
(a) Young with young kids



(b) Mature with young kids



(c) Mature with old kids

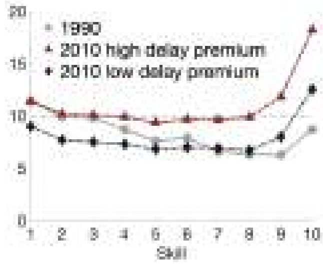


(d) Old with old kids

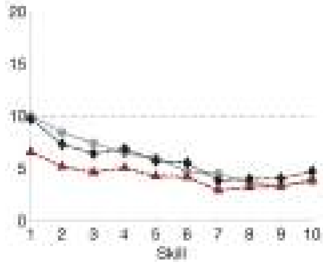
Figure C.9: Changes in fertility choices by age and skill

Notes: This figure displays the percentage of households that have children by age of the household, kids' age, and skill in 1990 and 2010. For 2010 we display the percentage in the baseline economy in which the delay premium increases over time and in the counterfactual economy in which it does not. Panel C.9a shows the percentage of young households that have children. Panels C.9b and C.9c show the percentage of mature households that have young/old kids out of all mature households. Finally, panel C.9d displays the percentage of old households that have old kids.

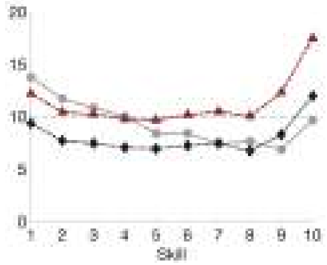
Changes in the propensity to live downtown by age, kid's age, and skill Figures C.10 summarizes changes in the propensity to live downtown by age, kid's age, and skill of the household between 1990 and 2010, in both the baseline economy and the counterfactual economy in which the delay premium does not increase over time. In the baseline economy, households with no children increase their presence downtown, especially the high-skilled, while households with children decrease it, especially the low-skilled. In contrast, in the counterfactual economy fewer households with no children locate downtown and fewer low-skilled households leave downtown.



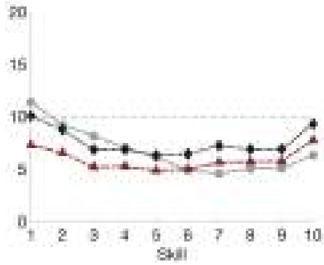
(a) Young: No kids



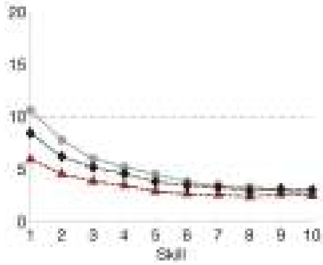
(b) Young: Young kids



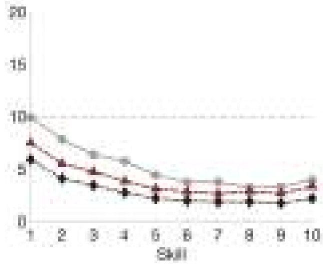
(c) Mature: No kids



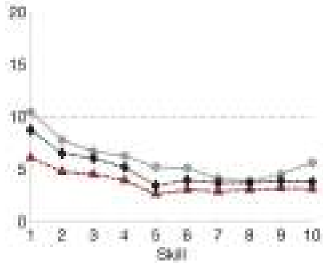
(d) Mature: Young kids



(e) Mature: Old kids



(f) Old: No kids

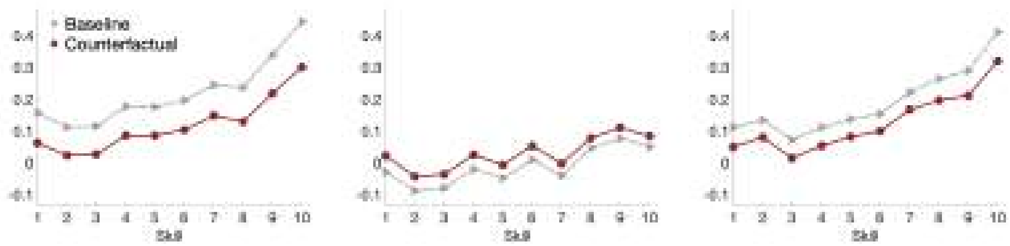


(g) Old: Old kids

Figure C.10: Propensity to locate downtown of households by age

Notes: This figure exhibits the propensity to live downtown for young, mature, and old households with young kids, old kids, or no kids, for each skill in 1990 and 2010 (baseline and counterfactual). Dashed gray lines indicate that downtown is set to contain 10% of the population in 1990.

Counterfactual changes in downtown amenities Figure C.11 shows the changes in downtown amenities arising from changes in the composition of downtown neighborhoods by fertility choice and skill. Since in this counterfactual fewer people without kids live downtown, childless amenities decrease in this location, which has a stronger effect on households that do not have kids or that choose to postpone.



(a) No kids (b) Kids when young (c) Kids when mature

Figure C.11: Endogenous amenities response

Notes: This figure displays the 1990-2010 change in the lifetime downtown amenity by skill and fertility choice, in the baseline economy and in the counterfactual economy in which we keep delay as they were in 1990.

D Welfare Appendix

D.1 Welfare Decomposition

The goal of the welfare section is to investigate welfare changes both in time and across counterfactuals. The changes that took place in the economy affected welfare differently not only for households of different skills, but also for households with different fertility preferences. Households with a high preference for having children may have gained more from the decrease in the child penalty, but they also benefited less from the improvement in downtown amenities. We are interested in understanding the average effect by skill, averaging over fertility preferences.

Let's start by defining $W_t(z,\eta^i)$ as the expected lifetime welfare of a household born at time t with skill z and idiosyncratic preferences for children equal to η^i, where the expectation is taken over idiosyncratic preferences for locations in each period.

$$W_t(z,\eta^i) = \max\left\{W_{ky,t}(z,\eta^i), \rho W_{km,t}(z,\eta^i) + (1-\rho)\, W_{nk,t}(z,\eta^i), W_{nk,t}(z,\eta^i)\right\},$$

where $W_{ky,t}$, $W_{km,t}$, and $W_{nk,t}$ denote the expected lifetime log welfare from having kids when young, when mature, or not to have kids, respectively. They are each defined as follows:

$$W_{ky,t}(z,\eta^i) = \sum_{a=1}^{3} \phi^{a-1} \sum_{l} \pi_{t+a-1}\left(l|a, k_{ky}(a), z\right) \ln V_{t+a-1}(a, k_{ky}(a), z, l) + \ln \eta^i + \phi \ln \eta^i;$$

$$W_{km,t}(z,\eta^i) = \sum_{a=1}^{3} \phi^{a-1} \sum_{l} \pi_{t+a-1}\left(l|a, k_{km}(a), z\right) \ln V_{t+a-1}(a, k_{km}(a), z, l) + \phi \ln \eta^i + \phi^2 \ln \eta^i;$$

$$W_{nk,t}(z,\eta^i) = \sum_{a=1}^{3} \phi^{a-1} \sum_{l} \pi_{t+a-1}\left(l|a, k_{nk}(a), z\right) \ln V_{t+a-1}(a, k_{nk}(a), z, l),$$

where $k_f(a)$ is the kid's age and presence at household age a under fertility path f, where $f = \{ky, km, nk\}$ if having kids young, mature, or not having kids, respectively. To be more explicit, $k_{ky}(1) = 1$, $k_{ky}(2) = 2$, $k_{ky}(3) = 0$, $k_{km}(1) = 0$, $k_{km}(2) = 1$, $k_{km}(3) = 2$, and $k_{nk}(.) = 0$; and $\pi_t\left(l|a, k, z\right)$ denotes the optimal location choice conditional on age, kids and skill. Finally, $\ln V_t(a, k, z, l)$ is the mean log welfare of households of skill z who at age a and time t choose optimally to live in location l. It is given by:

$$\begin{aligned}
\ln V_t\left(a, k, z, l\right) = & \ln\left[I_t\left(a, k, z\right) - p_{t,l}\right] \\
& + \gamma_{1(z)} \ln\left(\frac{N_{l,t}^{High-Skill}}{N_{l,t}^{Low-Skill}}\right) + \gamma_{2(k)} \ln\left(\frac{N_{l,t}^{No-Kids}}{N_{l,t}^{Kids}}\right) + \ln \chi_t\left(a, k, z, l\right) \\
& + \ln\left[\kappa_{y,t}\left(z\right)\right]^{D_{k>0\&\text{kids early}}} + \ln\left[\kappa_{m,t}\left(z\right)\right]^{D_{k>0\&\text{kids delayed}}} \\
& + \mathbb{E}_t \ln(\varepsilon_{t,l}|l, a, k, z),
\end{aligned}$$

where $\mathbb{E}_t \ln(\varepsilon_{t,l}|l, a, k, z)$ denotes the expected log idiosyncratic utility of living in l conditional on l being the optimal choice for a household of age a, kids k, and skill z at time t.

Next, we define the difference in welfare from being born at time t_1 rather than at time t_0 with the same skill, z, and idiosyncratic fertility preference, η^i, as:

$$\Delta_{t_0}^{t_1} W(z, \eta^i) = W_{t_1}(z, \eta^i) - W_{t_0}(z, \eta^i).$$

Finally, $\Delta_{t_0}^{t_1} \mathcal{W}(z) \equiv \mathbb{E}\Delta_{t_0}^{t_1} W(z)$ denotes the expected difference in welfare between being born at period t_0 or t_1 with skill z before drawing idiosyncratic preferences. Namely,

$$\Delta_{t_0}^{t_1}\mathcal{W}(z) = \mathbb{E}_\eta \left[W_{t_1}(z,\eta) - W_{t_0}(z,\eta)\right].$$

Next, let's define $\pi\left(f_{t_0}, f_{t_1}|z\right)$ as the fraction of households that at time t_0 experience fertility f_{t_0} and at time t_1 choose fertility f_{t_1} in equilibrium, conditional on their skill z. This probability has an analytical solution that only requires the estimated thresholds of fertility preferences in each period, $\bar{\eta}^t_{nk,km}(z)$ and $\bar{\eta}^t_{km,ky}(z)$, combined with the probability of having kids as mature, ρ^m. Therefore, we can then write the expected difference in welfare between being born at period t_0 or t_1 with skill z as:

$$\begin{aligned}
\Delta_{t_0}^{t_1}\mathcal{W}(z) = &\sum_{f_{t_0}}\sum_{f_{t_1}} \pi\left(f_{t_0}, f_{t_1}|z\right) \times \Bigg(\sum_{a=1}^{3}\phi^{a-1} \\
&\sum_{l_{t_1}=1}^{N} \pi_{t_1+a-1}\left(l_{t_1}|a, k_{f_{t_1}}(a), z\right) \ln V_{t_1+a-1}(a, k_{f_{t_1}}(a), z, l_{t_1}) - \\
&\sum_{l_{t_0}=1}^{N} \pi_{t_0+a-1}\left(l_{t_0}|a, k_{f_{t_0}}(a), z\right) \ln V_{t_0+a-1}(a, k_{f_{t_0}}(a), z, l_{t_0}) \\
&+ D_{k(f_{t_1})>0}\mathbb{E}_{t_1+a-1}[\ln(\eta|a, k_{f_{t_1}}(a), z)] - D_{k(f_{t_0},a)>0}\mathbb{E}_{t_0+a-1}[\ln(\eta|a, k_{f_{t_0}}(a), z)]\Bigg)
\end{aligned}$$

Multiplying by the sum of location probabilities conditional on age, kids' age and skill over all locations under the other scenario for each time t and rearranging, we get to the following linear expression:[37]

$$\begin{aligned}
\Delta_{t_0}^{t_1}\mathcal{W}(z) = &\sum_{f_{t_0}}\sum_{f_{t_1}} \pi\left(f_{t_0}, f_{t_1}|z\right) \times \Bigg(\sum_{a=1}^{3}\phi^{a-1} \\
&\sum_{l_{t_1}=1}^{N}\sum_{l_{t_0}=1}^{N} \pi_{t_1+a-1}\left(l_{t_1}|a, k_{f_{t_1}}(a), z\right) \pi_{t_0+a-1}\left(l_{t_0}|a, k_{f_{t_0}}(a), z\right) \times ... \\
&\Big(\ln V_{t_1+a-1}(a, k_{f_{t_1}}(a), z, l_{t_1}) - \ln V_{t_0+a-1}(a, k_{f_{t_0}}(a), z, l_{t_0})\Big) \\
&+ D_{k(f_{t_1})>0}E_{t_1+a-1}[\ln(\eta|a, k_{f_{t_1}}(a), z)] - D_{k(f_{t_0})>0}E_{t_0+a-1}[\ln(\eta|a, k_{f_{t_0}}(a), z)]\Bigg).
\end{aligned}$$

[37]We can multiply by the sum of location probabilities on age, kids' age and skill over all locations because it is equal to 1, that is, $\sum_l^N \pi(l|a,k,z) = 1$.

Finally, since this expression is linear, we can decompose $\Delta_{t_0}^{t_1}\mathcal{W}(z)$ into the drivers of welfare change, which allows us to focus on those elements that are observable to us. Namely,

$$\Delta_{t_0}^{t_1}\mathcal{W}(z) = \Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z) + \Delta_{t_0}^{t_1}\mathcal{W}^{HS}(z) + \Delta_{t_0}^{t_1}\mathcal{W}^{NK}(z) + \Delta_{t_0}^{t_1}\mathcal{W}^{U}(z),$$

where each of the elements is defined as follows. To save on notation, let

$$\pi_{t_0,t_1}(f_{t_0}, l_{t_0}, f_{t_1}, l_{t_1}|a,z) = \pi_{t_0,t_1}\left(f_{t_0}, f_{t_1}|z\right)\pi_{t_0}\left(l_{t_0}|a, k_{f_{t_0}}(a), z\right)\pi_{t_1}\left(l_{t_1}|a, k_{f_{t_1}}(a), z\right).$$

First, the contribution of real income is given by

$$\Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0}, l_{t_0}, f_{t_1}, l_{t_1}|a,z\right)\times\Bigg($$
$$\ln\left[I_{t_1+a-1}\left(a, k_{f_{t_1}}(a), z\right) - p_{t_1+a-1,l_{t_1}}\right] - \ln\left[I_{t_0+a-1}\left(a, k_{f_{t_0}}(a), z\right) - p_{t_0+a-1,l_{t_0}}\right]\Bigg). \tag{15}$$

Second, the contribution of endogenous amenities responding to the high-skill ratio is given by

$$\Delta_{t_0}^{t_1}\mathcal{W}^{HS}(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0}, l_{t_0}, f_{t_1}, l_{t_1}|a,z\right)\times\Bigg[$$
$$\gamma_1(z)\ln\left(\frac{N_{l_{t_1},t_1+a-1}^{High-Skill}}{N_{l_{t_1},t_1+a-1}^{Low-Skill}}\right) - \gamma_1(z)\ln\left(\frac{N_{l_{t_0},t_0+a-1}^{High-Skill}}{N_{l_{t_0},t_0+a-1}^{Low-Skill}}\right)\Bigg].$$

Third, the contribution of endogenous amenities responding to the no-kids ratio is given by:

$$\Delta_{t_0}^{t_1}\mathcal{W}^{NK}(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0}, l_{t_0}, f_{t_1}, l_{t_1}|a,z\right)\times\Bigg[$$
$$\gamma_2(k(f_{t_1})\ln\left(\frac{N_{l_{t_1},t_1+a-1}^{No-Kids}}{N_{l_{t_1},t_1+a-1}^{Kids}}\right) - \gamma_2(k(f_{t_0})\ln\left(\frac{N_{l_{t_0},t_0+a-1}^{No-Kids}}{N_{l_{t_0},t_0+a-1}^{Kids}}\right)\Bigg].$$

Fourth, the contribution of unobserved components is given by the sum of unobserved components, idiosyncratic preferences, and not-identified components of welfare:

$$\Delta_{t_0}^{t_1}\mathcal{W}^U(z) = \Delta_{t_0}^{t_1}\mathcal{W}^\chi(z) + \Delta_{t_0}^{t_1}\mathcal{W}^\kappa(z) + \Delta_{t_0}^{t_1}\mathcal{W}^\varepsilon(z) + \Delta_{t_0}^{t_1}\mathcal{W}^\eta(z)$$

which are given by

$$\Delta_{t_0}^{t_1}\mathcal{W}^\chi(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}|a,z\right)\times\Bigg(\ln\chi_{t_1+a-1}\left(a,k_{f_{t_1}}(a),z,l_{t_1}\right)-\ln\chi_{t_0+a-1}\left(a,k_{f_{t_0}}(a),z,l_{t_0}\right)\Bigg),$$

$$\Delta_{t_0}^{t_1}\mathcal{W}^\kappa(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}|a,z\right)\times\Bigg(D_{k_{f_{t_1}}(a)>0\&\text{kids early}}\ln\left[\kappa_{y,t_{1+a-1}}(z)\right]+D_{k_{f_{t_1}}(a)>0\&\text{kids delayed}}\ln\left[\kappa_{m,t_{1+a-1}}(z)\right]\ldots$$
$$-D_{k_{f_{t_0}}(a)>0\&\text{kids early}}\ln\left[\kappa_{y,t_{0+a-1}}(z)\right]-D_{k_{f_{t_0}}(a)>0\&\text{kids delayed}}\ln\left[\kappa_{m,t_{0+a-1}}(z)\right]\Bigg),$$

$$\mathbb{E}\Delta_{t_0}^{t_1}\mathcal{W}^\varepsilon(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}|a,z\right)\times\Bigg(\mathbb{E}_{t_1+a-1}\left[\ln(\varepsilon_{l_{t_1}}|l_{t_1},a,k_{f_{t_1}}(a),z)\right]-\mathbb{E}_{t_0+a-1}\left[\ln(\varepsilon_{l_{t_0}}|l_{t_0},a,k_{f_{t_0}}(a),z)\right]\Bigg)$$

$$\Delta_{t_0}^{t_1}\mathcal{W}^\eta(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{a,z}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}\right)\times\Bigg(D_{k(f_{t_1})>0}\mathbb{E}_{t_1+a-1}[\ln(\eta|a,k_{f_{t_1}}(a),z)]-D_{k(f_{t_0})>0}\mathbb{E}_{t_0+a-1}[\ln(\eta|a,k_{f_{t_0}}(a),z)]\Bigg).$$

The contribution of nominal income and housing prices. To decompose the effect of real income into the effect of nominal income and the effect of housing prices on welfare, we first add and subtract $\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_1+a-1}(l_{t_0})\right]$ for each $a,f_{t_0},f_{t_1},l_{t_0},l_{t_1}$ to the expression for $\Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z)$ (equation 15). This allows us to isolate the effect of re-optimizing the fertility and location choices. The re-optimization affects both the nominal income and housing prices.

$$\Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}|a,z\right)\times\Bigg(\underbrace{\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_1}}(a),z\right)-p_{t_1+a-1,l_{t_1}}\right]-\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]}_{\text{Re-optimization}}$$
$$+\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]-\ln\left[I_{t_0+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_0+a-1,l_{t_0}}\right]\Bigg).$$

Next, we add and subtract $\ln\left[I_{t_1+a-1}\left(a,k(f_{t_1}),z\right)-p_{t_1+a-1,l_{t_0}}\right]$ for each $a,f_{t_0},f_{t_1},l_{t_0},l_{t_1}$ to separate the effect of re-optimization for income and housing prices. Finally, we add and subtract $\ln\left[I_{t_1+a-1}\left(a,k(f_{t_0}),z\right)-p_{t_0+a-1,l_{t_0}}\right]$ for each $a,f_{t_0},f_{t_1},l_{t_0},l_{t_1}$ to separate the effect of changing nominal income and housing prices conditional on choices. This allows us to arrive at the following expression:

$$\Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z) = \sum_{a=1}^{3}\phi^{a-1}\sum_{f_{t_0},f_{t_1},l_{t_0},l_{t_1}}\pi_{t_0+a-1,t_1+a-1}\left(f_{t_0},l_{t_0},f_{t_1},l_{t_1}|a,z\right)\times\Bigg($$
$$\underbrace{\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_1}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]-\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]}_{\text{Re-optimization for income}}$$
$$\underbrace{\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_1}}(a),z\right)-p_{t_1+a-1,l_{t_1}}\right]-\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_1}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]}_{\text{Re-optimization for housing prices}}$$
$$\underbrace{\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_0+a-1}(l_{t_0})\right]-\ln\left[I_{t_0+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_0+a-1,l_{t_0}}\right]}_{\text{Effect of nominal income conditional on choices}}$$
$$\underbrace{\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_1+a-1,l_{t_0}}\right]-\ln\left[I_{t_1+a-1}\left(a,k_{f_{t_0}}(a),z\right)-p_{t_0+a-1,l_{t_0}}\right]}_{\text{Effect of housing prices conditional on choices}}\Bigg).$$

Re-arranging terms, we can decompose the welfare impact of real income into the nominal income and the housing prices effects. Namely,

$$\Delta_{t_0}^{t_1}\mathcal{W}^{RI}(z) = \Delta_{t_0}^{t_1}\mathcal{W}^{I}(z) + \Delta_{t_0}^{t_1}\mathcal{W}^{HP}(z),$$

where $\Delta_{t_0}^{t_1}\mathcal{W}^{I}(z)$ and $\Delta_{t_0}^{t_1}\mathcal{W}^{HP}(z)$ include both the re-optimization and the effect conditional on choices for each element.