A Dynamic Model of Authoritarian Social Control

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Abstract

Authoritarian regimes often use targeted social control - unequal application of the law to limit expressive freedom and enforce social conformity. At the same time, their methods appear less draconian than in the past. In this model, an authority structures punishments and rewards to compel adherence to its preferred norm. The authority's commitment is time-limited and depends on imperfectly informative signals of a citizen's behavior. Given two citizens with the same observed behavior, the authority imposes harsher punishments on the poorer and/or ex ante dissident individual. Lighter punishments are imposed on the wealthier citizen to prevent "overcompliance." Wealth inequality increases over time. Some citizens become prosperous "lackeys" while others become destitute from confiscation. In stable regimes with high state capacity, the authority reduces punishments and/or increases rewards to allow citizens to accumulate wealth, leading to social conformity and balanced growth in the long run. In unstable regimes with low capacity, the citizenry splits into groups of wealthy lackeys and destitute proles.

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1 Introduction

All societies use some method of social control to maintain order. This study examines social control in authoritarian regimes. These regimes typically rationalize their control by claiming to represent a socially cohesive society (Arendt (1951); Feldman (2003); Acemoglu and Robinson (2006); Applebaum (2012); Svolik (2012); Von Soest and Grauvogel (2017); Guriev and Treisman (2019); Egorov and Sonin (2020); Guriev and Treisman (2022)). In some cases, cohesion is claimed along ethnic lines. In other cases, it is ideological. In either case, the claims are intended to make electoral consent and civil liberties unnecessary.

For this reason, *expressive* behavior has always been a problem for authoritarians. The free exercise of artistic expression, cultural identification, religion, or ideology troubles them precisely because it undermines the appearance of social cohesion (Arendt (1951); Feldman (2003)). Autocrats repress "intellectual, spiritual, and artistic initiative" because individual expression is more "dangerous [to the autocrat] than mere political opposition."[1]

Repression of expressive freedom has a long history. The Spanish Inquisition sought to eliminate heresy; Cromwell suppressed Catholicism in 17th century England; Nazi Germany burned "degenerate" books. In the Soviet Union, "Socialist realism, 'the art of social command,' was the official, and therefore the only acceptable, style. The artist became a state functionary and an illustrator of an imposed ideology, his work judged primarily by accuracy of political conformity rather than by aesthetic criteria. It was the duty of the State Committee for the Affairs of Art constantly to scrutinize artistic production and to reprimand and correct any deviation."[2] More recent examples include Iran's "morality police," Orban's anti-LBGTQ campaign, Putin's crackdown on anti-war dissidents, and the Taliban's Ministry of Vice and Virtue.

This paper posits a dynamic agency model of authoritarian social control. It examines its central features and addresses questions: How effective are authoritarians in achieving social conformity and at what cost? What are the long run social and economic implications?

In the model, a ruling *authority* ("it") faces a continuum of citizens. A citizen ("she") is characterized by her current wealth and a preference type. The citizen and the authority each have distinct single peaked preferences over expressive behavior. Initially, the model focuses on social conformity as the authority's only motive. Other motives like rent extraction are added later on as extensions of the baseline model.

At the beginning of each period t, the authority announces a targeted *compliance rule* to impel behavior toward its preferred norm. The rule can be tailored to each par-

[1] Arendt (1951), p. 37.

[2] Mclanathan (1960), p.75.

ticular citizen-type and prescribes punishments or rewards on the basis of an imperfectly informative signal of the citizen's expressed behavior. The punishment for "bad signals" includes possible confiscation of some or all of the citizen's wealth. However, the punishment can go well beyond confiscation. It can include, for instance, imprisonment or torture. The citizen can also be co-opted with rewards for "good" signals - those that indicate behavior closer to the authority's preferred norm.

After the authority's announced rule, each citizen chooses her action. The signal is then realized, prescribed punishments or rewards are doled out, and the citizen's remaining wealth, if any, grows at a fixed rate and is carried over to the next period. Citizens' choices reflect the tradeoff between expressive freedom and wealth retention (or jail avoidance).

There is no independent judiciary to limit the authority's power, and so the authority cannot commit to a long term rule. Hence, it continually re-optimizes, choosing a new rule each period. Without coordination or commitment, the Markov Perfect equilibria characterize dynamic interactions between the authority and its citizens.

The structure of these equilibria is subtle. Various combinations of parameters can produce divergent outcomes. Yet, they all share certain qualitative features broadly consistent with recent trends (described in the Literature Review in Section 4) in authoritarian control. Four key results are outlined here:

1. Despite the availability of harsh punishments, more limited penalties - wealth confiscation in particular - emerge endogenously.

A number of recent studies show that modern autocracies achieve social control with less draconian methods than those in the past.[3] The result here is not due to any direct cost of repression. Instead, it arises from the moral hazard of social conformity enforcement. The authority must construct punishments and rewards to move a citizen toward, but not surpass, its ideal norm. If the punishment for poor signals and/or rewards for good ones are excessive, then the citizen responds by "overcomplying," i.e., she flanks the authority by moving too far to the right or left. A citizen who overcomplies adopts even more extreme behavior than the authority desires. To prevent overcompliance, the authority reduces the severity of punishment. I show that over time, the authority comes to rely mainly on limited wealth confiscation.

The authority also faces an indirect, dynamic cost of repression. Confiscation reduces the citizen's future wealth. This is bad for authority in future periods because it reduces the authority's future leverage. In response, the authority reduces punishment even further. This dynamic cost is larger for some citizens than others:

[3] A partial list includes Gandhi and Przeworski (2007); Scheppele (2018); Arias et al. (2018); Guriev and Treisman (2019); Xu (2020); Egorov and Sonin (2020); Guriev and Treisman (2022); Xu et al. (2022). See Section 4.

2. Ceteris paribus, a wealthier citizen is more willing to conform than a poorer one. The authority's rule targets the poor and/or ex ante dissident for harsher treatment. It chooses regressive confiscation rates.

Specifically, if two different citizens produce the exact same observed signal, the wealthier one faces a lower rate of confiscation. In addition, the individuals who are more aligned, *ex ante*, with the authority also face lower rates of confiscation. Hence, authoritarian control entails unequal treatment across both wealth and ideological dimensions.

Ideologically aligned citizens are more conforming for obvious reasons. The overcompliance problem leads to lower penalties for these individuals. The effect of wealth on conformity is more complicated. A wealthy citizen has more to lose from confiscation in the current period. But by being more compliant now she may lose even more wealth if the authority is more punitive in the future. Realizing this, a patient authority is willing to sacrifice some current compliance for improved future compliance. Here, the model introduces a novel *constraint relaxation effect*. This effect captures the additional *dynamic* incentive to lower confiscation taxes on the wealthy. Because wealthier citizens are expected to be more compliant, the current authority allows a citizen to retain and accumulate wealth. In turn, greater future wealth increases the confiscatory leverage of the future authority.

As a result of regressive confiscation, inequality will generally increase over time. Notably, the time series and cross sectional observations in this society can look very different. The model's time series tracks individuals of a given ideological type. A given citizen becomes more (less) compliant as her wealth grows (shrinks) over time. Along the transition path, two citizens who start with the same wealth will generally diverge. One who is less ideologically aligned ex ante, or one who unluckily produces negative signals faces higher confiscation rates, fewer rewards, and possibly imprisonment. In the cross section, the relation between wealth and compliance depends on the correlation between wealth and ideology. If, for instance, wealthier citizens are less aligned with the authority, then greater compliance could be observed among the poor in the cross section.

3. For each citizen, there is a wealth threshold above which the citizen is fully compliant. If the authority is patient, it allows the citizen to reach this threshold and ultimately converge to a balanced growth path.

This threshold creates an absorbing region in wealth space in which both full compliance and monotonic wealth accumulation occur. A citizen whose wealth enters this region becomes a "lackey" who fully conforms to the prescribed norm.

By allowing its citizen to reach this threshold, the authority creates a "Singapore effect." Specifically, by reducing punishments and increasing rewards, it allows the citizens to accumulate wealth, making them more compliant, which then increases the likelihood of rewards and decreases the likelihood of punishment, leading to higher wealth, and so on. In the long run, this type of regime produces a wealthy, compliant, and not too repressive country - like Singapore. However, different parameters can produce very different outcomes.

4. Authorities that are more impatient or have low state capacity are more confiscatory and produce greater wealth inequality and lower growth. In extreme cases, this leads to a poverty trap. Highly rapacious authorities produce lower growth rates, mainly by reducing rewards for compliance.

Interpreting the authority's discount factor as a survival probability, a more patient regime is a more stable one. A patient/stable authority takes the long view and allows citizens to build up wealth. By contrast, highly impatient authorities are uninterested in long run benefits of wealth accumulation. Consequently, punishments are larger. Citizens spend longer periods in destitution.

State capacity refers to the authority's ability to administer rewards or calibrate punishment. If an authority lacks capacity to reward good signals then some portion of the citizenry gets stuck in a poverty trap of destitution and noncompliance. This leads to a highly polarized and unequal society. Over time, citizens divide into groups wealthy lackeys and destitute proles.

Even in countries with higher state capacity, the Singapore effect is a long run outcome. Depending on parameters, the speed of adjustment may be slow relative to the regime's or citizens' survival rates. Viewed from the outside, two regimes with similar initial conditions can diverge quickly. At a point in time, an authoritarian regime could resemble Singapore or it could resemble, for instance, Venezuela.

Finally, the model is extended to allow for rent-extraction motives. A *rapacious* authority uses control to enrich itself with confiscated wealth. Intuitively, one expects higher confiscation taxes and lower rewards from a rapacious authority, all leading toward lower growth. Rewards are indeed reduced. However, the effects of rapaciousness on confiscation are ambiguous. A higher confiscation tax is partially offset by increased compliance which, in turn, lowers the confiscatory rents to the authority.

The paper's organization follows a familiar format. Section 2 describes the pure conformity model. It characterizes the equilibrium and main results as outlined above. Section 3 analyzes comparative statics and extensions. It examines the roles of discounting, state capacity, and exogenous growth. It also extends the model to analyze rapacious regimes. Section 4 reviews the literature. It describes patterns documented in the empirical literature and reviews related models of autocratic control.[4] Section 5 concludes with a discussion of assumptions and omissions that could guide future work. Section 6 contains the proofs.

[4]The reader should consult Gehlbach et al. (2016), Egorov and Sonin (2020), and Paine and Tyson (2024) for broader surveys of the vast literature on authoritarian motives and methods.

2 A Baseline Model of Pure Social Conformity

This section describes a dynamic model of authoritarian social control. The model is based on three premises. First, an authoritarian ruler (the "authority") seeks universal adherence to an ideological ideal and, consequently, attempts to impel citizens toward that ideal. In the baseline model, conformity to this ideal is the authority's lone motive. Second, the authority is not constrained to apply laws equally across citizens. Third, the authority can only commit to rules in the short run, but not in the longer run. It must therefore re-optimize each period.

A point of terminology: the terms *increasing* and *decreasing*, when applied to functions, is taken to mean weakly increasing everywhere on the domain and strictly increasing on an open set. The term *strictly increasing* is taken to mean strictly increasing everywhere on the domain.

2.1 An Authoritarian Ruler and the Citizenry

Consider an ongoing society with time indexed by $t = 0, 1, 2, \ldots$. At each date t an authoritarian regime, the "authority" (it), seeks to influence the behavior of each citizen ("she"). Citizens are differentiated by type, indexed by $i \in [0, 1]$. A citizen i enters the period with wealth w_{it}. In the absence of any interference from the authority, the citizen's flow payoff is

$$u_{it} \;=\; -\frac{1}{2}(a_{it} - a_i^c)^2 \;+\; w_{it} \tag{1}$$

where $a_{it} \in \mathbb{R}$ is her act in date t. The act a_{it} represents an expressive act such as a religious practice, claim of cultural identity, work of art, or ideological stance. A citizen's *preference type* is her ideal act a_i^c. Her full type is her preference type and her initial wealth w_{i0}. The payoff (1) assumes that a citizen's ideal act is personal; she has no opinion about behavior of other citizens. The appearance of wealth w_{it} directly in the citizen's payoff is a reduced form for the consumption of a flow of services from asset w_{it}.[5] In the absence of any interference from the authority, wealth grows according to $w_{i\,t+1} = w_{it}(1+\gamma)$ with $\gamma \geq 0$ being the natural growth rate in the economy.

The flow payoff of the authority in date t is

$$v(a_t) \;=\; -\int \frac{1}{2}(a_{it} - \bar{a})^2 di \tag{2}$$

where $a_t = (a_{it})_{i\in[0,1]}$ is the behavior profile of all citizens' acts, and $\bar{a}$ is the authority's ideal act. The authority seeks university conformity with act $\bar{a}$ from all citizens.

[5] One unit of wealth generates one unit of consumption. Assume, for instance, the production of services is linear in the stock. Specifically, $c_{it} = w_{it}\ell_{it}$, where c_{it} is i's consumption, w_{it} is i's asset stock, and $\ell_{it} \in [0, 1]$ the citizen's labor. If labor is supplied inelastically, then the wealth consumption is as in (1).

For now, I focus on the specific agency relationship between the authority and a given citizen i. Long run payoffs of both the authority and citizen i are discounted sums of flow payoffs. Let δ_A and δ_c denote the discount factors of the authority and the citizen, respectively. The differences can arise from differing time horizons, or different views about regime stability. Discount factor δ_A reflects the regime's view of its own survival probability.

Without loss of generality, assume $a_i^c < \bar{a}$ so the citizen's ideal behavior lies to the left of the regime's. A citizen who is induced to choose $a_{it} = \bar{a}$ will be said to be *fully compliant.* A citizen who chooses $a_{it} \in (a_i^c, \bar{a})$ is partially compliant.

The authority's monitoring technology, however, is imperfect. The citizen's act a_{it} at t generates an imperfectly informative signal x_{it} drawn from a conditional distribution $F(\cdot|a_{it})$. Assume $E[x_{it}|a_{it}] = a_{it}$ and F admits a smooth, unimodal, symmetric density $f(\cdot|a)$ satisfying the monotone likelihood ratio property.[6] Assume also $F(x|a) = F(x - \epsilon|a - \epsilon)$ for all ϵ, x and a. That is, the class of distributions obtained by varying a are all mean translations of one another. Under these assumptions $F_a(x|x) \equiv \frac{\partial F(x|a)}{\partial a}\Big|_{a=x}$ is independent of x and is an increasing function of signal precision.

From here out, the subscript t is dropped unless it is necessary to distinguish variables across time. At the beginning of a period the authority chooses a *compliance rule* $\mathcal{T}_i(w_i, x)$ which applies a punishment or reward to a citizen of type i whose wealth is w_i, and given signal realization x.

Citizen i receives a punishment if $\mathcal{T}_i(w_i, x) > 0$. She receives a reward if $\mathcal{T}_i(w_i, x) < 0$. Expressing $\mathcal{T}_i(w_i, x)$ in units of wealth is a straightforward way to examine confiscation. The citizen's wealth net of punishment/reward is $w_i - \mathcal{T}_i(w_i, x)$. We allow for the possibility that $\mathcal{T}_i(w_i, x_i) > w_i$, i.e., the rule imposes punishment like imprisonment that exceeds full confiscation. A *feasible* compliance rule satisfies

$$-R \;\le\; \mathcal{T}_i(w_i, x) \;\le\; w_i + P. \tag{3}$$

The value $w_i + P$ is the maximal punishment, while R is the maximal reward a citizen can receive. A citizen receiving the maximal punishment has her wealth entirely confiscated by the authority and, in addition, receives punishment P which can be interpreted as imprisonment or even torture. A citizen receiving the maximal reward keeps her current wealth and receives an additional wealth transfer R. This can include property like a nice apartment or an ownership stake in oil wealth.

The bounds in (3) place limits on the authority's coercive powers. These limits could come from external pressure, budget constraints, or costs of repression. However, the only assumptions on P and R is they are finite. Even with potentially draconian punishment, one of the main takeaways is that the constraints will often not bind. The authority will sometimes opt to limit punishments and rewards.

[6] Formally, $\frac{df(x|a)/da}{f(x|a)}$ is increasing in x

Once the authority chooses $\mathcal{T}_i$, it is committed to the scheme for the duration of the period, but cannot commit beyond the period. Given $\mathcal{T}_i$, the citizen then chooses a_i. This, in turn, generates signal x according to density $f(x|a_i)$ which then determines the punishment/reward $\mathcal{T}_i(w_i, x)$.

Finally, the citizen cannot enter the next period with negative wealth,[7] and so the law of motion for wealth is

$$w_{i\,t+1} = G(w_{it}, x) \equiv \max\{0, (1+\gamma)(w_{it} - \mathcal{T}_i(w_{it}, x))\} \tag{4}$$

According to (4), next period's wealth is the growth-augmented, post-confiscation wealth. To bound dynamic payoffs for the citizen, assume $\delta_c(1+\gamma) < 1$.

So far the regime derives no payoff directly from confiscated wealth $\mathcal{T}_i(w_i, x)$. The confiscated assets can be either set aside or refunded to the citizens lump sum at the end of the period. This "pure conformity" motive will be relaxed later on when rent-extraction motives are examined.

2.2 Equilibrium

This Section analyzes the Markov Perfect equilibria of the local game between a citizen and the authority. A Markov Perfect equilibria is a Subgame Perfect equilibrium in Markov strategies.[8]

Formally, a strategy profile consists of a pair $(\mathcal{T}_i, \alpha_i)$ where $\mathcal{T}_i$ is the authority's compliance rule, and $\alpha_i(w_{it}, \mathcal{T}_i) = a_{it}$ is the citizen's response given her wealth w_{it} at t. A Markov Perfect equilibrium strategy pair $(\mathcal{T}_i^*, \alpha_i^*)$ satisfies, for each date t and wealth w_{it},

[7] Without some bound on penury, the citizen could accumulate and suffer unlimited punishment. It would also mean that consumption would be negative if the citizen were to start with negative wealth. Even a destitute citizen, however, can be punished with penalty P.

[8] The restriction to Markov Perfect equilibria seems natural in a setting with an atomized citizenry and no external enforcement or commitment device. By contrast, firms operate in an environment where contracts are externally enforceable. Consequently, agency models of the firm typically assume the principal is able to fully commit to a history contingent contract at $t = 0$. In an authoritarian government, there is no independent judiciary to hold the regime to its commitment. So, history contingency arises only if citizens and the authority all jointly coordinate on a common understanding of the past. In the pure authoritarian regime, individual citizens are too small to exert strategic influence, and large coalitions are suppressed. Finally, with few additional assumptions, the setting here conforms to Bhaskar et al. (2012) who show that all Subgame Perfect equilibria are necessarily Markov Perfect in sequenced games if the players' social memory is bounded. Section 5 discusses extensions to history-contingent equilibria.

- The authority's compliance rule $\mathcal{T}_i^*$ solves the Bellman equation

$$V_i(w_{it}) \;=\; \max_{\mathcal{T}_i}\left\{\; -\frac{1}{2}(\alpha_i^*(w_{it},\mathcal{T}_i)-\bar{a})^2 \;+\; \delta_A\int V_i(G(w_{it},x))f(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\;\right\} \tag{5}$$

  over all feasible $\mathcal{T}_i$.

- For any feasible compliance rule $\mathcal{T}_i$, the citizen's strategy $\alpha_i^*(w_{it},\mathcal{T}_i)$ solves the Bellman equation

$$U_i(w_{it},\mathcal{T}_i) \;=\; \max_{a_{it}}\left\{-\frac{1}{2}(a_{it}-a_i^c)^2+\int\left[w_{it}-\mathcal{T}_i(w_{it},x)+\delta_c\,U_i^*(G(w_{it},x))\right]f(x|a_{it})dx\right\} \tag{6}$$

  with $U^*(G(w_{it},x)) \equiv U_i(G(w_{it},x),\mathcal{T}_i^*)$, and

- $w_{i\,t+1} = G(w_{it},x) = \max\{0,(1+\gamma)(w_{it}-\mathcal{T}_i(w_{it},x)\}$ (as given by (4)).

Notice that the continuation payoff U_i^* is evaluated at the equilibrium $(\mathcal{T}_i^*,\alpha_i^*)$. Additionally, $-\frac{P}{1-\delta_c}$ is a lower bound for U^*. The citizen can do no worse than to choose $a_{it}=a_i^c$, have her wealth be fully confiscated, and get hit with imprisonment P every period. The authority's payoff depends on wealth only through the citizen and from the boundary constraint on $\mathcal{T}_i$.

From here on, any Markov Perfect equilibrium will be referred to simply as an "equilibrium." The rest of this section characterizes the set of equilibria, first analyzing the Euler equations associated with the Bellman equations above, then characterizing path properties of equilibria.

2.3 Incentives to Comply

The citizen's first order condition can be expressed as

$$a_{it} \;=\; a_i^c-\int\mathcal{T}_i(w_{it},x))f_a(x|a_{it})dx+\delta_c\int U_i^*(G(w_{it},x))f_a(x|a_{it})dx \tag{7}$$

Because the set of acts is not constrained, the first order condition holds with equality as longs as the solution is finite. The act a_{it} appears on both sides of (7), and so a solution is a fixed point of this map.

Lemma 1 *For any wealth w_i and feasible compliance rule $\mathcal{T}_i$, every solution to the citizen's Bellman equation is a fixed point of the map in (7), and there is a unique fixed point of (7) that solves the citizen's Bellman equation.*

Depending on the chosen $\mathcal{T}_i$, the citizen's objective function may not be globally concave, and so multiple fixed points can exist. However, there is only one that solves the Bellman equation. The authority will anticipate the citizen's strategy α_i^* when choosing compliance rule $\mathcal{T}_i$.

Equation (7) has an intuitive interpretation. The last two terms on the right-hand side of (7) can be viewed as shifts away from the citizen's ideal a_i^c due to the punishments and rewards encoded in the compliance rule. These shifts result from the change in the distribution of signals. The term, $-\int \mathcal{T}_{it}(w_{it}, x) f_a(x|\alpha_i(w_i, \mathcal{T}_i))dx$ represents an immediate *wealth enhancement effect*. It constitutes the marginal change in one's wealth due to an incremental increase in compliance. The term $\int U_i^*(w_{i\,t+1}) f_a(x|a_{it})dx$ is the *long run effect*, representing the incremental change in future payoffs due to an increase in one's compliance.

Thus we have

$$\underbrace{a_{it} - a_i^c}_{\textit{(marginal) cost of compliance}} = \underbrace{\int -\mathcal{T}_{it}(w_{it}, x) f_a(x|a_{it})dx}_{\textit{wealth enhancement}} + \underbrace{\delta_c \int U_i^*(G(w_{it}, x)) f_a(x|a_{it})dx}_{\textit{long run effect}} . \tag{8}$$

Notice that if $\mathcal{T}_i(w_i, x) = 0$ for all x, there are no consequences for non-compliance, and so the solution is $a_{it} = a_i^c$.

The decomposition is a useful one since the wealth enhancement effect dominates if δ_c is low, while the long run effect is obviously more prominent when δ_c is large. How does compliance respond to changes in punishments and/or rewards? Are patient citizens more or less compliant? These are partially answered in the following result.

Proposition 1 *Fix an equilibrium $(\mathcal{T}_i^*, \alpha_i^*)$ and wealth w_i for citizen i.*

(i) The wealth enhancement and long run effects of compliance in (8) are both positive.

(ii) Consider any incremental change $\Delta\mathcal{T}_i$ in $\mathcal{T}_i^*$ satisfying $\Delta\mathcal{T}_i(x) \geq 0$ if $x < \alpha_i^*(w_i, \mathcal{T}_i^*)$ and $\Delta\mathcal{T}_i(x) \leq 0$ if $x \geq \alpha_i^*(w_i, \mathcal{T}_i^*)$ with strict inequality either for all $x < \alpha_i^*(w_i, \mathcal{T}_i^*)$ or all $x > \alpha_i^*(w_i, \mathcal{T}_i^*)$ or both. Then

$$\alpha_i^*(w_i, \mathcal{T}_i^* + \Delta\mathcal{T}_i) > \alpha_i^*(w_i, \mathcal{T}_i^*).$$

(iii) $\alpha_i^*(w_i, \mathcal{T}_i^*) \in (a_i^c, \bar{a}]$ and is increasing in preference type a_i^c.

Part (i) establishes the reinforcing effects of current and future incentives. Together, they move the citizen's behavior away from her own ideal behavior and toward that of the authority. An implication of Part (i) is that a more patient (higher δ_c) citizen is more compliant than an impatient one. Intuitively, greater compliance today reduces punishment or increases rewards that, in turn, increase future wealth. The logic requires an equilibrium continuation in which authority's rule in the future is not expected to be more punitive.

Part (ii) shows how the citizen responds to changes in different regions of the signal space. The citizen is more responsive to punishments for low signals and to rewards for high signals. Her best response determines the cutoff point between these regions. She is more compliant in equilibrium if punishment is higher on $x < \alpha_i^*(w_i, \mathcal{T}_i^*)$ and/or if rewards (negative punishment) are higher on $x \geq \alpha_i^*(w_i, \mathcal{T}_i^*)$. Intuitively one might conclude that the authority should then maximize punishments when $x < \alpha_i^*(w_i, \mathcal{T}_i^*)$ and maximize rewards when $x \geq \alpha_i^*(w_i, \mathcal{T}_i^*)$. In fact, this will not be the case, as shown in the next section.

By Part (iii) a citizen never "overshoots" the authority's ideal behavior $\bar{a}$ in equilibrium. Nor does she "undershoot" her own ideal behavior a_i^c. The no-undershooting argument is clear: authority can always guarantee at least a_i^c by making $\mathcal{T}_i^*$ independent of x.

The argument against $a_{it} > \bar{a}$ is more nuanced. To illustrate, suppose the authority is restricted to use a simple two-step compliance rule. It fixes a cutoff signal x^*. Then the authority fixes a constant punishment $\tau(w_i)$ on all signals $x < x^*$, and a constant reward $\rho(w_i)$ on all signals $x \geq x^*$. Equilibrium compliance rules will not generally be two-step rules. Nevertheless, the two-step rule provides useful intuition. Suppose $w_i \geq \tau_i(w_i)$. Under the two-step rule, the first order condition (7) reduces to

$$\begin{aligned} a_i \;=\; & a_i^c \;+\; [\; (\tau(w_i) + \rho(w_i)) \\ & +\;\; \delta_c[U_i^*((1+\gamma)(w_i + \rho(w_i))) - U^*((1+\gamma)(w_i - \tau(w_i)))]] \,|F_a(x^* | \, a_i)| \end{aligned} \tag{9}$$

Now suppose the citizen chooses $\alpha_i^*(w_i, \mathcal{T}_i) > \bar{a}$ (i.e., she overcomplies). The obvious solution for the authority is to reduce punishment $\tau(w_i)$. Lessening punishment $\tau(w_i)$ decreases the right-hand side of (9) which can be shown to reduce the solution $\alpha_i^*(w_i, \mathcal{T}_i)$. Scaling back punishment has in fact a dual advantage: it moves the citizen's choice back toward $\bar{a}$ and, at the same time, increases the citizen's expected future wealth - a possible benefit for the authority as shown later on.

The reduction of punishment may not be enough, however, to prevent overcompliance. Once $\tau(w_i)$ hits zero in (9), the reward $\rho(w_i)$ may have to be reduced too. Fortunately for the authority, if it ever gets to the point where its only remaining instrument is the reward $\rho(w_i)$, the reduction of $\rho(w_i)$ pulls the citizen's behavior back to $\bar{a}$ while the citizen's wealth accumulates with certainty. But by definition, this means the citizen reaches a point where she fully complies and wealth accumulation occurs regardless of the signal. The latter means that the citizen fully complies in all continuations.

This example also hints at why confiscation tax *rate* will be regressive. Here the rate is $\tau(w_i)/w_i$. This ratio is decreasing in w_i since overcompliance is a bigger problem with the wealthy. Moreover, even if the authority is not overly concerned about overcompliance, it may still want lessen the punishment in order allow the citizen to build up of wealth. The next section fleshes out the logic behind the authority's incentives.

2.4 Incentives to Control

Consider the authority's incentives. For any instantaneous change $d\mathcal{T}_i$, the authority's first order condition is expressed as

$$\underbrace{\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a}-\alpha_i^*(w_{it},\mathcal{T}_i)) \;+\; \delta_A \int V(G(w_{it},x)) f_a(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\right] d\mathcal{T}_i}_{\textit{compliance effect}}$$
$$+ \;\; \delta_A \underbrace{\int \frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} f(x|\alpha_i^*(w_{it},\mathcal{T}_i)) d\mathcal{T}_i(x)}_{\textit{constraint relaxation effect}} \;=\; (\geq)(\leq)\; 0 \tag{10}$$

where either $\frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} = -(1+\gamma)$ or $= 0$ depending on whether the nonnegativity of wealth in (4) binds. The first order condition (10) holds with equality if $\mathcal{T}(w_i,x)$ is an interior solution on the domain of $d\mathcal{T}_i$. The inequality "$\geq$" applies if the solution hits its upper bound $\mathcal{T}(w_i,x) = w_i + P$ while "$\leq$" holds if the solution hits its lower boundary $\mathcal{T}(w_i,x) = -R$.

As with the citizen's solution, the first order condition can broken into two distinct effects. The *compliance effect* refers to the direct change in the authority's payoff due to a change in the citizen's compliance, both current and future. The term depends on the change in α_i^* on both flow and continuation payoffs.

The *constraint relaxation effect* is the indirect change in the authority's payoff due to a change in the constraints on a future authority's attempted confiscation. The constraint relaxation effect captures the additional incentive of the authority to alter the constraints on its future self's confiscatory punishment. The citizen's wealth enters directly into the feasibility constraint (3), and so the future constraint is the maximal allowable confiscation $w_{i\,t+1} + P$ next period.

Proposition 2 *Let $(\mathcal{T}_i^*, \alpha_i^*)$ be an equilibrium. Then for each w_i, the constraint relaxation effect is always (weakly) negative. The compliance effect is positive when $d\mathcal{T}_i(x)$ applies to signals $x < \alpha_i^*(w_i,\mathcal{T}_i^*)$ and is negative when $d\mathcal{T}_i(x)$ applies to signals $x \geq \alpha_i^*(w_i,\mathcal{T}_i^*)$.*

Considering only the constraint relaxation effect, the current authority should reduce punishment or increase its rewards. This is in contrast with a common result in the dynamic contracting literature (e.g., Ray (2002)) that the principal should backload payment to avoid disincentivizing future effort. Here, for very different reasons, it is the opposite. The constraint relaxation effect induces the authority/principal to "front-load" wealth for the citizen/agent. The main reason is that the bound on punishment in $t+1$ depends on wealth $w_{i\,t+1}$ entering the next period. The larger the wealth the laxer the authority's constraint, whereas in the contracting literature the two are typically independent.

The constraint relaxation effect can be interpreted as a "commitment problem" in reverse. Rather than tie the hands of its future self, the authority has an incentive to free them by "front-loading" the citizen's wealth. The logic is reminiscent of Harstad (2020) who formulates a model of an authority with time inconsistent preferences over green investments and shows that it sometimes front-loads the investment for later generations.

According to Proposition 2, when the realized signal is above a switching point $\alpha_i^*(w_i, \mathcal{T}_i^*)$ the compliance effect is in sync with the constraint relaxation effect. In that case, if the two effects are nonzero, the first order condition defines a corner solution at R, the maximal reward. This means that the authority pays out the maximal reward after observing above-average signals.

By contrast, when the realized signal is below $\alpha_i^*(w_i, \mathcal{T}_i^*)$, the two effects are at odds. On the one hand, the authority would prefer to increase compliance via increased punishment. On the other hand, this results in decreased wealth for the citizen which, in turn, reduces future punishment capacity. To resolve this tension, the punishment may may be less than maximal, or even negative, for signals that are closer to the switching point $\alpha_i^*(w_i, \mathcal{T}_i^*)$.

A third possibility arises if full compliance is obtained in the current period and in all future periods. In that case, both effects are zero, and the solution is necessarily interior. The punishment (and possibly rewards) must be reduced to prevent overcompliance. Since wealthier citizens are more likely to overshoot, the confiscation tax will be regressive; wealthier citizens give up a lower portion of their wealth than poorer ones. The next section formalizes this intuition.

2.5 Equilibrium Path Compliance and Control

This section characterizes equilibria and their path properties. To state the main result, some additional notation is needed. Given an equilibrium $(\mathcal{T}_i^*, \alpha_i^*)$, let $\mathbb{P}_i^*$ denote the equilibrium probability on signal paths $x^\infty = (x_1, x_2, \ldots, x_t, \ldots)$. Let $\{\omega_{i\,t+s}^*(x^\infty)\}_{s=0}^\infty$ be the equilibrium wealth path of citizen i starting from date t when the realized signal path is x^∞.[9]

Proposition 3 *There exists a wealth threshold w_i^* such that for any equilibrium $(\mathcal{T}_i^*, \alpha_i^*)$, the interval $[w_i^*, \infty)$ is an absorbing region of full compliance. Specifically, for any t and wealth state w_{it}, there are signal cutoffs $\underline{x}(w_{it})$, $x^*(w_{it})$, and $\bar{x}(w_{it})$ with $-\infty \leq \underline{x}(w_{it}) \leq x^*(w_{it}) \leq \bar{x}(w_{it}) \leq \alpha_i^*(w_{it}, \mathcal{T}_i^*)$ such that*

(i) If $w_{it} < w_i^*$, then

$$\mathbb{P}_i^* \left(x^\infty : \ \exists s \geq t, \ \alpha_i^*(\omega_{i\,s}^*(x^\infty), \mathcal{T}_i^*) < \bar{a} \ \middle| \ w_{it} < w_i^* \right) \ > \ 0$$

(the citizen is not fully compliant at some point). The rule $\mathcal{T}_i^*$ then satisfies

$$\mathcal{T}_i^*(w_{it}, x) \ = \ \begin{cases} w_{it} + P & if \quad x < \underline{x}(w_{it}) \\ \in (0, w_{it} + P] & if \quad x \in [\underline{x}(w_{it}), x^*(w_{it})) \\ \in (-R, 0] & if \quad x \in [x^*(w_{it}), \bar{x}(w_{it})) \\ -R & if \quad x \geq \bar{x}(w_{it}) \end{cases} \tag{11}$$

where $\underline{x}(w_{it}) > -\infty$ and $\bar{x}(w_{it}) < \alpha_i^*(w_{it}, \mathcal{T}_i^*)$.

(ii) If $w_{it} \geq w_i^*$ then

$$\mathbb{P}_i^* \left(x^\infty : \ \ \forall \ s \geq t, \ \omega_{i\,s}^*(x^\infty) \geq w_i^* \ and \ \alpha_i^*(\omega_{i\,s}^*(x^\infty), \mathcal{T}_i^*) = \bar{a} \ \middle| \ w_{it} \geq w_i^* \right) \ = \ 1$$

(wealth remains above w_i^* and the citizen is forever, fully compliant). Rule $\mathcal{T}_i^*$ then satisfies $\mathcal{T}_i^*(w_{it}, x) < w_{it}$ (no jail).

Proposition 3 identifies a threshold wealth level w_i^* such that, above w_i^*, the citizen will always choose $a_i = \bar{a}$ and wealth will never fall below w_i^*. That is, the interval $[w_i^*, \infty)$ *is an absorbing region of full compliance.* Below the absorbing region, the citizen is partially compliant and the authority sets cutoffs in the signal space for extreme punishments, extreme rewards, and regions of intermediate punishments and intermediate rewards.

Figure 1 illustrates an equilibrium compliance rule when Part (i) applies. The citizen's wealth w_{it} is below w_i^*. Then there is some signal path in which she not fully compliant, either at t or at some future date. Suppressing the notational dependence on w_{it}, the Figure displays a cutoff x^* that divides the signal space into a punishment region

[9] The formal constructions of $\mathbb{P}_i^*$ and ω_i^* are in the Appendix.

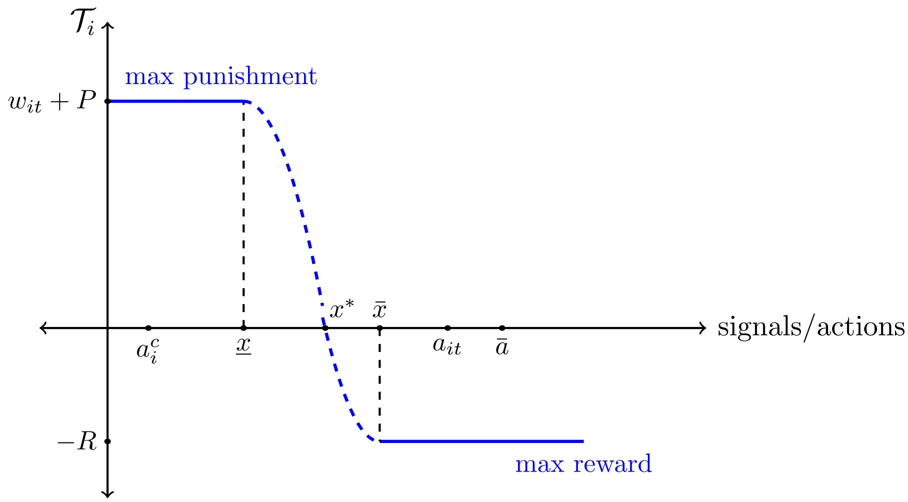

Figure 1: Equilibrium compliance rule when citizen's wealth is below w_i^*.

$(-\infty, x^*)$ and a reward region $[x^*, \infty)$. Within the reward region, Figure 1 shows the citizen receiving the maximal reward R on the upper interval $[\bar{x}, \infty)$. This maximal reward region strictly contains $[\alpha_i^*(w_{it}, \mathcal{T}_i^*), \infty)$. This means that the citizen receives a maximal reward R even for some signals below $\alpha_i^*(w_{it}, \mathcal{T}_i^*)$. In other words, the citizen receives a reward, in fact the maximal one, on signals that actually disincentivize compliance. To understand why, recall that a patient authority is not interested in simply maximizing immediate compliance. The fact that the cutoff $\bar{x}$ lies to the left of $\alpha_i^*(w_{it}, \mathcal{T}_i^*)$ indicates a tradeoff between present and future compliance. Under the constraint relaxation effect, a larger expected reward increases a citizen's future wealth which can then be used as leverage to increase compliance in the future. The authority increases the likelihood of rewards, even as it sacrifices current compliance, to obtain higher future compliance.

The Proposition implies that when $w_{it} < w_i^*$,

$$F(\bar{x}|\, \alpha^*(w_{it}, \mathcal{T}^*)) < 1 - F(\bar{x}|\, \alpha^*(w_{it}, \mathcal{T}^*)). \tag{12}$$

That is, the citizen is more likely to receive the maximal reward than either a lower reward or a punishment.

Within the punishment region, Figure 1 shows the citizen receiving the maximal punishment $w_{it} + P$ on $(-\infty, \underline{x}]$. In this region, the citizen has her wealth entirely confiscated and in addition receives jail time P. Finally, intermediate punishments are implemented in region $(\underline{x}, x^*]$.[10]

The second part of the Proposition characterizes equilibria when wealth w_{it} exceeds w_i^*. In this case, wealth never falls below w_i^*, and the citizen is fully compliant from

[10]The dashed line in the Figure indicates an intermediate punishment which is continuous and decreasing on its domain.

period t onward. The authority therefore receives its maximal payoff $V(w_{it}) = 0$. The idea is the following. Above w_i^*, any compliance rule is weakly dominated by a canonical rule $\tilde{\mathcal{T}}_i^*$ satisfying

$$\tilde{\mathcal{T}}_i^*(w_{it}, x) \;=\; \begin{cases} \frac{\gamma}{1+\gamma} w_i^* & if \quad x < x_i^* \\ -\rho_i^* & if \quad x \geq x_i^* \end{cases} \tag{13}$$

where $\rho_i^* \leq R$, $x_i^* \equiv x^*(w_i^*)$, $\underline{x}(w_{it}) = -\infty$, and $\bar{x}(w_{it}) = \infty$.

Figure 2 displays $\tilde{\mathcal{T}}_i^*$ when $w_i \geq w_i^*$. The proof in the Appendix verifies that for an appropriate choice of ρ_i^* and x_i^*, the constructed $\tilde{\mathcal{T}}_i^*$ is an equilibrium compliance rule which yields payoff $V(w_i) = 0$ for all $w_i \geq w_i^*$. Under $\tilde{\mathcal{T}}_i^*$, two things happen. First, the rule provides just the right incentives for the citizen to be fully, but not overly, compliant. Second, the citizen's wealth never falls, even after the worst possible signals are realized. In fact, w_i^* is the lowest wealth level in which both of these properties hold. When w_i^* is reached, punishments are moderated to allow the citizen's wealth to grow regardless of signal. To verify this, observe that if $w_i \geq w_i^*$, then

$$(1+\gamma)(w_i - \tilde{\mathcal{T}}_i^*(w_i, x)) \geq (1+\gamma)(w_i - \frac{\gamma}{1+\gamma} w_i^*) = w_i + \gamma(w_i - w_i^*) \;\geq\; w_i, \quad \forall\, x$$

This obviously holds with equality when $w_i = w_i^*$. So, once w_i^* reached or exceeded, all future wealth levels will remain above w_i^* permanently and the citizen is fully compliant from that point onward. The payoff to the authority in that case is $V(w_i) = 0$, its global maximum. Since the authority can switch to $\tilde{\mathcal{T}}_i^*$ in any period, the one-shot deviation principle implies that any alternative equilibrium rule must also guarantee $V(w_i) = 0$.

The salient fact in this wealth region is $\tilde{\mathcal{T}}_i^*(w_i, x) < w_i$, or in words, *punishment is limited to partial confiscation of wealth.* Jailing does not occur.

When $w_i \geq w_i^*$, the prescribed punishment $\frac{\gamma}{1+\gamma} w_i^*$ is a moderate lump sum confiscation tax that allows for continued wealth accumulation, albeit at some rate below γ. As before, $x_i^*(w_i^*)$ separates the reward and punishment regions.[11] The fixed reward ρ_i^* is the largest reward that combines with punishment $\frac{\gamma}{1+\gamma} w_i^*$ to prevent over-shooting.

Two other points must be emphasized. First, since w_i^* is the minimal wealth that guarantees growth, the punishment after w_i^* is reached is minimal among all compliance rules that are payoff equivalent to the authority. The rule $\tilde{\mathcal{T}}_i^*$ is therefore the one most favorable to the citizen. The next section discusses growth implications of $\tilde{\mathcal{T}}_i^*$.

Second, the confiscation *rate* $\frac{\gamma}{1+\gamma} w_i^* / w_i$, is declining. Poor people have a larger portion of their wealth confiscated than rich people. This follows from the overcompliance problem. The reward rate also declines as wealth accumulates.

[11] In addition $\underline{x}(w_{it}) = -\infty$ by construction since punishments are set at interior level Γw_i^*.

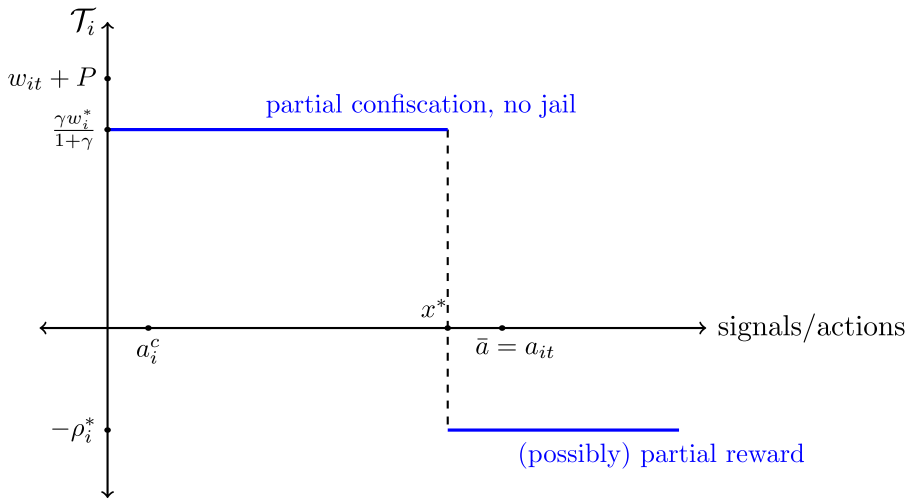

Figure 2: Equilibrium compliance rule when citizen's wealth is above w_i^*.

2.6 The Long Run

If the citizen's wealth starts out at or above w_i^* the authority can do no worse than to employ the compliance rule $\tilde{\mathcal{T}}_i^*$ defined in (13). Because all future wealth levels remain above w_i^*, any equilibrium that reaches w_i^* must have full compliance from that point onward, regardless of the signal. Since this is the best possible outcome for the authority, it will choose a rule payoff-equivalent to $\tilde{\mathcal{T}}_i^*$.

The next result states that as long as the authority has capacity to reward the citizen, it can ensure that she reaches the absorbing region of full compliance. Formally:

Proposition 4 *Suppose $R > 0$. Let $(\mathcal{T}_i^*, \alpha_i^*)$ be an equilibrium. For any initial wealth w_{i0},*

$$\mathbb{P}_i^* \left(x^\infty : \lim_{t\to\infty} \omega_{it}(x^\infty) > w_i^* \;\middle|\; w_{i0} \right) \;=\; 1, \tag{14}$$

and so

$$\mathbb{P}_i^* \left(x^\infty : \; \exists\, t < \infty \;\; \forall\; s \geq t, \quad \alpha^*(\omega_{i\,t+s}^*(x^\infty), \mathcal{T}_i^*) = \bar{a} \;\right) \;=\; 1 \tag{15}$$

Equation (14) asserts that the equilibrium wealth path converges to the interior of the absorbing region almost surely. Proposition 3 can then be applied to establish convergence to full compliance in Equation (15). This property is verified for the canonical compliance rule $\tilde{\mathcal{T}}_i^*$ in (13). Under $\tilde{\mathcal{T}}_i^*$, if $w_{it} \geq w_i^*$, the citizen's expected growth rate at t is

$$\gamma(w_{it}) \;\equiv\; \frac{E[w_{i\,t+1}|\, w_{it}] \;-\; w_{it}}{w_{it}} \;=\; \gamma - \frac{F(x^*(w_{it})|\bar{a})\gamma w_i^* \;-\; (1 - F(x^*(w_{it})|\bar{a}))(1+\gamma)\rho_i^*}{w_{it}} \tag{16}$$

which converges to γ as $t \to \infty$. The citizen's wealth path therefore converges to the balanced growth path. Growth rate $\gamma(w_{it})$ converges to γ from below (above) if the expected net punishment is positive (negative), i.e., if

$$F(x^*(w_{it})|\bar{a})\frac{\gamma}{1+\gamma}w_i^* \; - \; (1 - F(x^*(w_{it})|\bar{a}))\rho_i^* \; > \; (<) \; 0. \tag{17}$$

The sign of (17) has implications for inequality. If the expected net punishment is positive (i.e., if ">" holds), then the growth rate $\gamma(w_{it})$ is increasing in wealth. Since wealth is always increasing after w_i^*, it follows that wealth inequality among all fully compliant individuals is increasing over time. If, however, the inequality is reversed then wealth inequality is decreasing over time.

State capacity plays a large role. Since $\rho^* \leq R$, a sufficient condition for increasing inequality is that R, the capacity to co-opt a citizen, is small. Regimes with low state capacity therefore exhibit increasing wealth inequality over time.

3 Comparative Statics and Extensions

This Section explores the effects of changes in parameters of the model. Later on it introduces an extension whereby the authority is also motivated to extract rents from its compliance rule.

3.1 Stopping Times

Given an equilibrium, define the random stopping time to reach i's absorbing region of full compliance as

$$T_i(x^\infty) = \inf\{t : \; \omega_{it}(x^\infty) \geq w_i^*\}. \tag{18}$$

Equation (14) asserts that $T_i(x^\infty)$ is finite for $\mathbb{P}_i^*$-almost every path x^∞.

Proposition 5 *In any equilibrium, $T_i(x^\infty)$ is decreasing pointwise in discount factors δ_c and δ_A, in the citizen's preference type a_i^c, in initial wealth w_{i0}, in the growth rate γ, and in the maximal reward R.*

The effect of growth rate γ is straightforward. As for other parameters, an individual whose ideal a_i^c is closer to $\bar{a}$ is both more compliant and receives lower confiscation rates. She therefore accumulates wealth more quickly than a dissident. Citizens with high initial wealth w_{i0} reach the threshold more quickly simply because they start out closer. The stopping time is also decreasing in both discount factors δ_c and δ_A. Generally, a more patient citizen is more compliant and therefore accumulates wealth quickly. A more patient authority benefits from reaching the threshold w_i^* more quickly and therefore alters punishments and rewards to make it so. Section 3.3 discusses the comparative statics of discounting in more detail.

3.2 Low State Capacity: A Poverty Trap

Besley and Persson (2010) refer to a regime's *state capacity* as its ability to raise revenue or enforce property rights. This Section analyzes state capacity in the form of administrative capability to reward citizens for good signals. In the extreme, a lack of state capacity means $R = 0$.

If $R = 0$ then long run convergence to the fully absorbing region (Eq. (14)) no longer holds. For one thing, state capacity is needed to lift citizens out of destitution. If $R = 0$, then $w_i = 0$ is an absorbing state of the equilibrium dynamics.

Even in the case where $w_{it} > 0$ for some t, low enough signal realizations result in full confiscation, the result of which is the citizen ends up permanently destitute. The probability of this happening, according to Proposition 3, is at least $F(\underline{x}(w_{it})|\,\alpha_i^*(w_i, \mathcal{T}_i^*))$. This is the probability that in the current period, the citizen draws a signal in the region $(-\infty, \underline{x}(w_{it})]$ of full confiscation. By Proposition 3, this probability is positive if $w_{it} < w_i^*$. In terms of stopping time,

$$\mathbb{P}_i^* \left(x^\infty :\ T_i(x^\infty) = \infty \;\middle|\; w_{it} < w_i^*,\ R = 0 \right) \ \geq\ F(\underline{x}(w_{it})|\,\alpha_i^*(w_i, \mathcal{T}_i^*)) > 0. \tag{19}$$

Because full confiscation occurs below w_i^* with positive probability, it implies a positive probability of ending up permanently destitute - a *poverty trap*.

How compliant is a destitute citizen? Consider her incentives when $w_i = 0$. The long run effect in the citizen's Euler equation (8) zeroes out because destitution is an absorbing state. The best case scenario for the authority is a maximal punishment P levied against signals below the citizen's anticipated act a_i. The maximal likelihood of receiving this punishment is $Z = |F_a(a_i|a_i)|$. Thus the Euler equation of a destitute citizen reduces to

$$a_i \ = \ a_i^c \ + \ \mathcal{T}_i^*(0, x)|F_a(x^*|\,a_i)| \leq a_i^c + PZ \tag{20}$$

Notice, if jail is not feasible (i.e., if $P = 0$), then a destitute citizen chooses her ideal a_i^c. An authority without state capacity must be able to jail the destitute to impel even partial compliance.[12]

[12] One might argue that state capacity is also related to the ability to impose punishments. Low state capacity could impede monitoring which, in the case of (20), refers to Z. The lower the signal precision, the lower is Z. If an authority that can neither co-opt nor monitor well, the level of compliance expected from this citizen is low.

More generally, a low but positive capacity $R > 0$ allows the citizen to escape destitution. However, the stopping time $T_i(x^\infty)$ is decreasing pointwise in R by Proposition 5. Hence, average stopping time $E[T_i(x^\infty)]$ is high when R is low.

One way of understanding the result is to compare average stopping time to the citizen's average lifespan $L_c = \sum_{t=1}^{\infty} t\delta_c^{t-1}(1-\delta_c)$, where δ_c, represents her survival rate. A simple corollary of Proposition 5 is:

Corollary 1 *For w_{i0} and R sufficiently small,*

$$E[T_i(x^\infty)] > L_c.$$

In that case, the citizen in her lifetime is unlikely to reach the absorbing region of full compliance and guaranteed wealth accumulation. The citizen then spends her life either cycling between low and intermediate levels of wealth, or hitting a poverty trap. Note that $E[T_i(x^\infty)] = \infty$ if $w_{i0} = R = 0$.

3.3 Impatient Authoritarians

Viewing δ_A as the probability of survival each period, an impatient authoritarian is a short-lived one.[13] In this sense, δ_A is a proxy for regime stability.

Consider an authority with $\delta_A = 0$. In this case the authority's Bellman equation reduces to a static optimization problem which, from its perspective, is solved as the first mover in a Stackelberg game. The authority's first order condition is

$$\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}(\bar{a} - \alpha_i^*(w_{it}, \mathcal{T}_i)) = 0 (> 0)(< 0) \tag{21}$$

which yields an interior solution if $\bar{a} = \alpha_i^*(w_{it}, \mathcal{T}_i)$.

Unlike in the dynamic model where future payoffs moderate current punishments, the authority has no incentive to hold back. It maximizes current compliance up to $\bar{a}$.

Proposition 6 *Suppose $\delta_A = 0$. Let $(\mathcal{T}_i^*, \alpha_i^*)$ be an equilibrium and suppose w_{it} satisfies $\bar{a} > \alpha_i^*(w_{it}, \mathcal{T}_i^*)$ (the citizen is not fully compliant). Then $\mathcal{T}_i^*(w_{it}, x) = w_{it} + P$ if $x < \alpha_i^*(w_{it}, \mathcal{T}_i)$ and $\mathcal{T}_i^*(w_{it}, x) = -R$ if $x \geq \alpha_i^*(w_{it}, \mathcal{T}_i)$.*

For an impatient authority, the anticipated act $a_{it}^* \equiv \alpha_i^*(w_{it}, \mathcal{T}_i)$ becomes the switching point between punishment and reward. For signals in the punishment region $(-\infty, a_{it}^*)$,

[13] The connection between present bias in effective discount factor and political replacement is examined in, for instance, Persson and Svensson (1989), Alesina and Tabellini (1990), Battaglini and Coate (2008), Lagunoff (2009), and Harstad (2023).

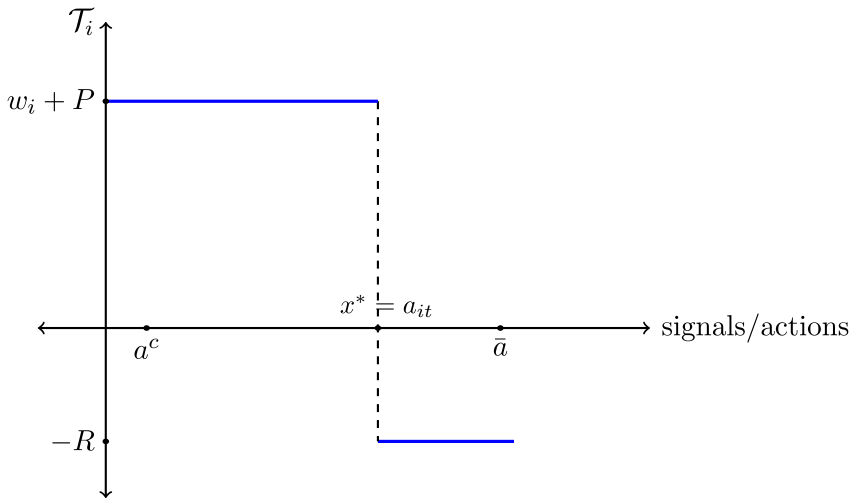

Figure 3: Equilibrium compliance rule for an impatient authoritarian when citizen's wealth is below $\hat{w}_i$.

the first order condition hits a corner solution at $w_i + P$. For signals in the reward region $[a^*_{it}, \infty)$, the first order condition hits the other corner solution, $-R$. The proof applies Proposition 1. Because the authority cares only about immediate social conformity, it imposes extreme punishments and rewards for all citizens who have not reached the full compliance threshold. This results in full confiscation whenever signals are low.

Given a wealth level w_{it} in which the citizen is not fully compliant, her Euler equation is

$$\alpha^*_i(w_i, \mathcal{T}^*) \;=\; a^c_i \;+\; [(w_i + P + R) \;+\; \delta_c(U^*_i((1+\gamma)(w_i + R)) - U^*_i(0)\;)\;]\,Z \tag{22}$$

where $Z \equiv F_a(a_i|a_i)$.

Since the right-hand side of equation in (22) is increasing in wealth, there is a unique wealth threshold above which overcompliance occurs unless punishment and/or rewards are reduced. Let $\hat{w}_i$ denote this threshold. Then $\hat{w}_i$ satisfies

$$\alpha^*(\hat{w}_i, \mathcal{T}^*) \;=\; \bar{a} \;=\; a^c_i \;+\; [(\hat{w}_i + P + R) \;+\; \delta_c(U^*_i((1+\gamma)(\hat{w}_i + R)) - U^*_i(0)\;)\;]\,Z \tag{23}$$

Wealth $\hat{w}_i$ is the lowest wealth level that achieves, but not exceeds, $\bar{a}$. Clearly, $\hat{w}_i$ is decreasing in a^c_i and in Z. Ideological allies of the authority are fully compliant at lower $\hat{w}_i$. Superior monitoring (higher Z) produces full compliance at lower $\hat{w}_i$. The compliance rule when $w_i < \hat{w}_i$ is displayed in Figure 3.

3.3.1 A Draconian Equilibrium

By definition, when $w_i > \hat{w}_i$ then rewards and/or punishments must be reduced to prevent overcompliance. There are multiple combinations of rewards and punishments to accomplish this. The impatient authority is indifferent among these since they all achieve full compliance above $\hat{w}_i$. I describe a "draconian" equilibrium in which punishments are maximal and rewards are minimal. In this draconian equilibrium the authority first reduces the reward to prevent overcompliance. Only after rewards are eliminated does it address punishment. It maintains maximal punishment $w_i + P$ but reduces its reward to $\rho_i = \hat{w} + R - w_i$. Whenever the citizen's wealth increases, the reward is reduced so as to maintain $\alpha^*(w_i, \mathcal{T}^*) = \bar{a}$. This continues up until another wealth threshold $w_i^\circ = R + \hat{w}_i$ at which $\rho = 0$, i.e., no rewards are given. Beyond this point, the punishment remains constant at $\tau_i = w_i^\circ + P$. Though no rewards are given at this point, a series of good signals allow wealth to grow at rate γ whenever $x > \alpha_i^*(w_i, \mathcal{T}^*)$.

Formally, the draconian equilibrium is given by: $\mathcal{T}_i^*(w_i, x) = \tau_i(w_i)$ if $x < \alpha_i^*(w_i, \mathcal{T}^*)$ and $\mathcal{T}_i^*(w_i, x) = -\rho_i(w_i)$ if $x \geq \alpha_i^*(w_i, \mathcal{T}^*)$, and $\tau_i(w_i)$ and $\rho(w_i)$ are given by

$$\rho_i(w_i) = \begin{cases} R & if \quad w_i < \hat{w}_i \\ \hat{w} + R - w_i & if \quad \hat{w}_i \leq w_i < w_i^\circ \equiv \hat{w}_i + R \\ 0 & if \quad w_i \geq w_i^\circ \end{cases}$$

and

$$\tau_i(w_i) = \begin{cases} w_i + P & if \quad w_i < w_i^\circ \\ w_i^\circ + P & if \quad w_i \geq w_i^\circ \end{cases}$$

Again, all equilibria are identical when $w_i < \hat{w}_i$. The compliance rule for $w_i \geq \hat{w}_i$ is depicted in Figures 4 and 5. Above $\hat{w}_i$, the authority reduces rewards linearly while punishment remains maximal and fully confiscatory - the "draconian" part. Wealth w_i° is the level at which rewards hits zero, and confiscation remains constant above w_i°.

As before, the confiscation *rate* $\tau_i(w_i)/w_i$ is declining in wealth, and there is an absorbing full compliance region defined by a threshold w_i^* satisfying $(1+\gamma)(w_i^* - \hat{w} - P) = w_i^*$ or $w_i^* = \frac{1+\gamma}{\gamma}(w_i^\circ + P)$.[14]

More generally, for arbitrary δ_A the reward region is smaller the more impatient the authority. When $\delta_A = 0$ the region is minimized at $x^* = \alpha_i^*(w_i, \mathcal{T}_i^*)$. Recall the lower is

[14] This threshold is different than in the patient player case because the equilibrium here is draconian, and there are other more citizen-friendly equilibria. The equilibrium constructed in Proposition 3 is more citizen-friendly which makes sense since continuation values of the authority and citizen are both monotone in citizen wealth.

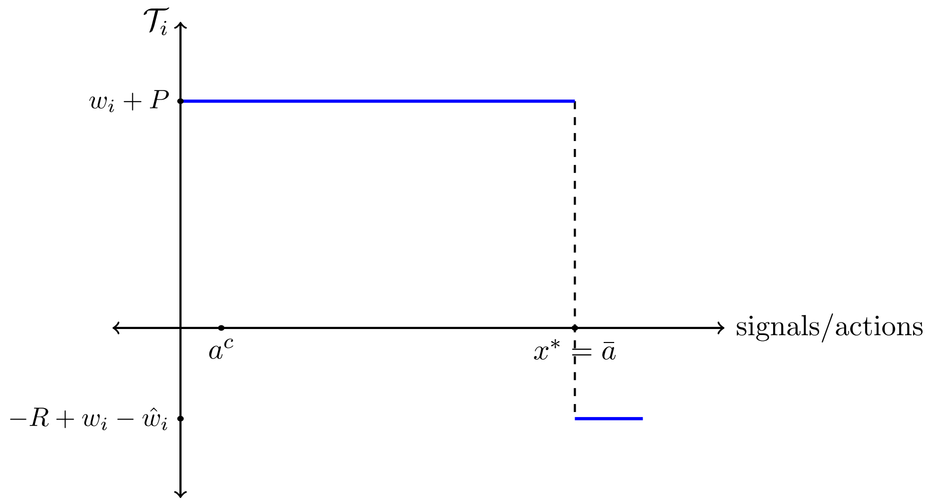

Figure 4: Draconian equilibrium compliance rule for an impatient authoritarian when citizen's wealth is between $\hat{w}_i$ and w_i°.

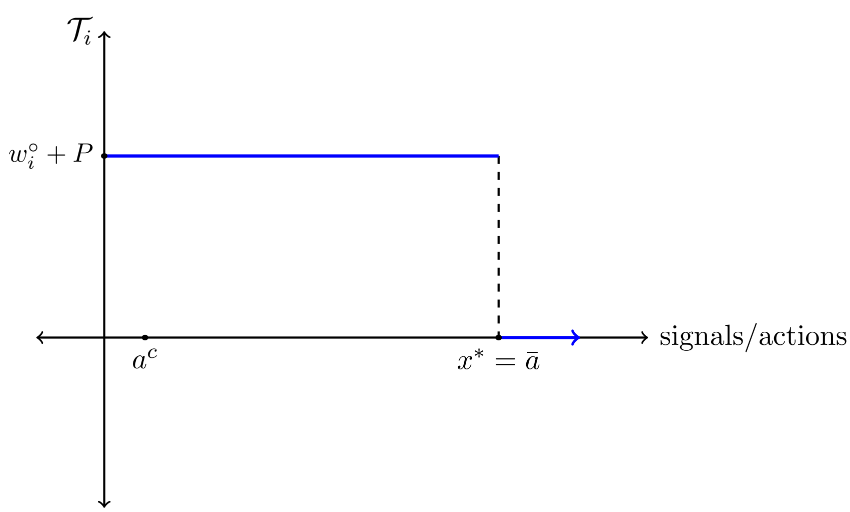

Figure 5: Draconian equilibrium compliance rule for an impatient authoritarian when citizen's wealth is above w_i°.

δ_A, the longer the stopping time to absorbing full compliance.

3.3.2 Draconian Equilibrium when Everyone is Impatient: Another Poverty Trap

The model can be solved in closed form when $\delta_c = \delta_A = 0$. The citizen's first order condition reduces to

$$a_{it} = a_i^c - \int \mathcal{T}_i(w_{it}, x)) f_a(x|a_{it}) dx \tag{24}$$

In the draconian equilibrium, (24) reduces to

$$\alpha_i^*(w_i, \mathcal{T}^*) = a_i^c + (\tau_i(w_i) + \rho(w_i)) Z \tag{25}$$

The draconian equilibrium rule $\mathcal{T}_i^*$ has the same structure as before, but now the full compliance wealth threshold has a closed form solution: $\hat{w} = \frac{\bar{a}-a_i^c}{Z} - (P+R)$. This wealth level is the threshold at which $\alpha_i^*(w_i, \mathcal{T}^*) = \bar{a}$ in (25). The wealth level w_i° at which rewards disappear is given by $w_i^\circ = \hat{w}_i + R = \frac{\bar{a}-a_i^c}{Z} - P$. The compliance rule is characterized by its punishment and reward schedules, respectively, τ_i and ρ_i. The closed form solutions for citizen's behavior and the schedules are illustrated in Figure 6.

Significantly, there are parameter configurations in which some individuals do not reach the absorbing region of full compliance. For instance, suppose $\frac{\bar{a}-a_i^c}{Z} < P$. Then wealth distribution for those with $0 < w_{i0} < P$ bifurcates. When $w_{i0} < P$ the citizen's wealth is fully confiscated at low signals, then hits zero, and has insufficient reward at high signals to escape destitution. This results in another *poverty trap*. In such case, the citizen has nothing to gain by complying. She therefore chooses $a_i = a_i^c$. Hence, this combination of impatience with low state capacity is particularly polarizing. Some individuals receive strings of high signals and escape the trap. Others, hit with a single low signal, fall into and remain in the poverty trap.

3.4 Rapacious Authoritarians

This Section extends the pure control model to one where the authority directly values the confiscated assets. This rent extraction motive alters the view of punishment versus rewards. In the pure control model, the authority favored rewards over punishment because of the greater long run value in citizen wealth accumulation. Rent-extraction can reverse this ordering.

To simplify the analysis, suppose there is no punishment beyond confiscation, i.e., $P = 0$. Consider the case of a rapacious authority that places weight weight λ on rent

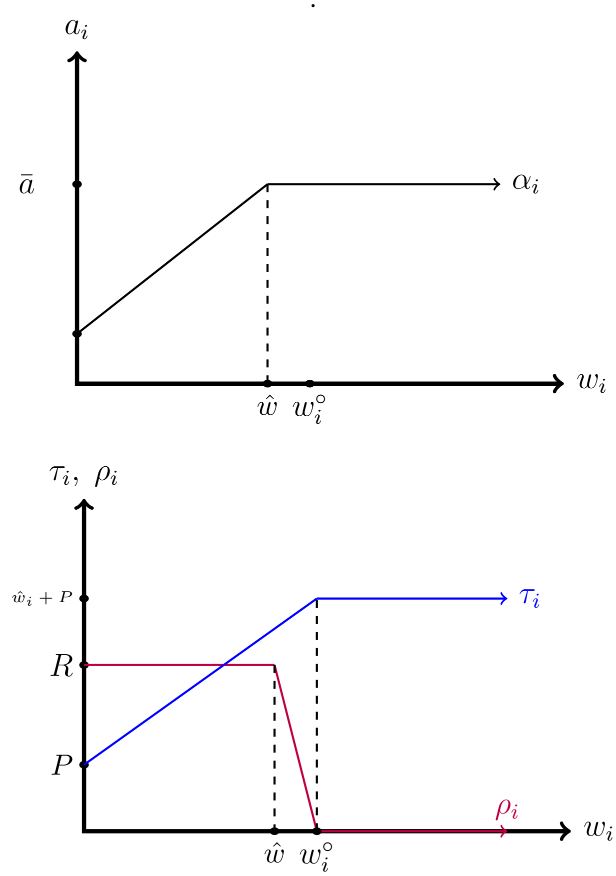

Figure 6: Draconian Eqm for Impatient Players: (a) Top - Citizen Behavior, and (b) Bottom - Authority's Punishment and Reward schedules.

extraction. Specifically, it puts weight λ on each dollar gained in confiscation or lost from reward payouts. The parameter λ determines the relative weight the authority's puts on rent extraction relative to social conformity. The authority's compliance rule $\mathcal{T}_i^*$ solves the Bellman equation

$$\begin{aligned} V_i(w_{it}) \;=\; \max_{\mathcal{T}_i} \Big\{ \; -\frac{1}{2}(\alpha_i^*(w_{it},\mathcal{T}_i)-\bar{a})^2 \;+\; \lambda \int \mathcal{T}_i(w_{it},x))f(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx \\ +\; \delta_A \int V_i(G(w_{it},x))f(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx \;\Big\} \end{aligned} \tag{26}$$

over all feasible $\mathcal{T}_i$.

The pure conformity model is obviously a special case where $\lambda = 0$.[15] The Bellman equation omits the past assets held by the authority since they play no role in the authority's optimization problem.

3.4.1 Ambiguous Effects of Rapaciousness

The first order condition is for a change $d\mathcal{T}_i$ on some subset D in the signal space is

$$\begin{aligned} &\underbrace{\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a}-\alpha_i^*(w_{it},\mathcal{T}_i)) \;+\; \delta_A \int V(G(w_{it},x))f_a(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\right]}_{\textit{compliance effect}} \\ &+\; \delta_A \underbrace{\int_D \frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} f(x|\alpha_i^*(w_{it},\mathcal{T}_i))d\mathcal{T}_i(x)}_{\textit{constraint relaxation effect}} \\ &+\; \underbrace{\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\lambda\left[\int \mathcal{T}_i(w_{it},x))f_a(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx + \int_D f(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\right]}_{\textbf{\textit{rent extraction effect}}} \;=\; (\geq)(\leq)\; 0 \end{aligned} \tag{27}$$

Equation (27) differs from the first order condition (10) in the benchmark model. Here, there is an additional term: the *rent extraction effect*. This rent extraction effect comprises the marginal revenue generated by a change $\mathcal{T}_i$ in the compliance rule. The rent extraction effect includes both the direct change in rents due to a change in confiscation, and the indirect effect due to a change in the citizen's behavior in response to a change

[15] I thank Bard Harstad for suggesting this formulation. Setting $P = 0$ avoids the notational complexity of restricting λ to the confiscatory portion of punishment.

in confiscation. Inside the brackets "[·]" in the rent extraction effect, the direct effect is positive and the indirect effect is negative. An increase in confiscation (or reduction in rewards) directly increases rents to the authority. However, the higher confiscation also increases citizen compliance which then reduces rents. It is unclear which part, direct or indirect, dominates.

Rapaciousness exposes and interesting asymmetry between rewards and punishments. A rapacious authority extracts more rents from both decreased rewards and increased punishments. Ceteris paribus the authority decreases reward to the citizen when compared to the pure conformity ($\lambda = 0$) model. By decreasing rewards, the authority increases its rents and, at the same time, reduces the citizen's compliance which increases the likelihood of confiscation - which further increases rents to the authority.

Yet, the effects of rapaciousness on punishment are ambiguous. There are multiple margins to consider. Increasing punishment increases rents directly, but reduces rents indirectly because it increases citizen compliance.

3.4.2 An Impatient, Rapacious Authoritarian: No Trap but No Growth

To better assess the rent extraction effect, we shut down the inter-temporal tradeoffs. Suppose $\lambda > 0$ and $\delta_A = 0$. The authority is both impatient and rapacious. The authority's Euler equation (27) collapses to

$$\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a} - \alpha_i^*(w_{it}, \mathcal{T}_i)) + \lambda \int \mathcal{T}_i(w_{it}, x)) f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx\right] + \lambda \int_D f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx$$
$$= (\geq)(\leq)\ 0 \tag{28}$$

When $\lambda = 0$ then the rent-extraction incentive vanishes and once again the impatient authoritarian seeks only to maximize behavior up to $\bar{a}$. When $\lambda > 0$, then the authority may prefer a lower level of compliance with a reduced reward to the citizen in order to increase extracted wealth.

Consider a "guess and verify" solution similar to the non-rapacious impatient authority. Namely, suppose the cutoff signal x^* satisfies $x^* = \alpha_i^*(w_{it}, \mathcal{T}_i)$. Set $\mathcal{T}_i(w, x) = \tau_i(w_i)$ if $x < x^*$ and $\mathcal{T}_i(w, x) = -\rho_i(w_i)$ if $x \geq x^*$. The first order condition (28) collapses to

$$\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a} - \alpha_i^*(w_{it}, \mathcal{T}_i)) + \lambda(\tau_i(w_i) + \rho_i(w_i))F_a(x^*|\alpha_i^*(w_{it}, \mathcal{T}_i))\right] + \lambda F(x^*|\alpha_i^*(w_{it}, \mathcal{T}_i)) = (\geq)(\leq)\ 0 \tag{29}$$

The middle term, $\lambda(\tau_i(w_i) + \rho_i(w_i))F_a(x^*|\alpha_i^*(w_{it}, \mathcal{T}_i))$, is negative. Suppose the citizen is not fully compliant and consider a change $d\mathcal{T}_i$ in the reward region $x > x^* = \alpha_i^*(w_{it}, \mathcal{T}_i)$. From Lemma 1, $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} < 0$. A non-rapacious authority would seek to increase the reward up to its maximum R. With a rapacious authority, if λ is large enough the authority would reduce the reward. The rent extraction effect is positive, and so a highly rapacious ruler is willing to sacrifice some compliance in exchange for more rents. For λ sufficiently large, the authority chooses $\rho = 0$, no reward.

Now consider a change $d\mathcal{T}_i$ in the punishment region $x < x^*$. From Lemma 1, $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} > 0$. A non-rapacious ruler maximizes punishment. Intuitively, one might think a rapacious ruler would do so as well. After all, confiscation serves both to increase compliance *and* to extract rents.

However, the logic is more subtle. The sign of the rent extraction effect is itself ambiguous in this case. The direct effect from increased punishment is of course positive. Higher confiscation directly produces more rents to the authority. However, there is an indirect effect: by increasing punishment, the authority increases compliance which reduces the likelihood of a low signal and, in turn, reduces expected rents. This indirect effect may well outweigh the direct effect. To see why, take $\tau_i(w_i) = w_i$, so that wealth is fully confiscated. A higher wealth level increases the indirect effect on rents relative to other influences. It increases the marginal loss in rents from confiscation. To offset the cost, the authority actually increases the threshold x^* above $\alpha_i^*(w_{it}, \mathcal{T}_i)$, in contrast with results from the prior section. This means that there are good signals that receive punishment in order increase rent extraction

It follows that $x^* \to \infty$ as $w_i \to \infty$. If the citizen is fortunate enough to accumulate wealth, her wealth is increasingly likely to be fully confiscated leading to repeated bouts of destitution.

The combination of authoritarian rapaciousness coupled with impatience/instability, differs from a more stable and conformity-motivated regime. There need not be long run convergence to balanced growth, and the level compliance is lower across all citizens.

4 Patterns of Authoritarian Rule: A Review

The model contributes to a large and expanding political economy literature on authoritarian regimes. Gehlbach et al. (2016), Egorov and Sonin (2020), and Paine and Tyson (2024) all provide comprehensive surveys. This paper focuses specifically on social control as a mechanism to obtain conformity. Authoritarians direct their efforts to repress expressive freedom, not just economic behavior. In doing so, authoritarians apply their methods unequally - they target certain types of individuals more harshly than others. This Section reviews existing studies as they relate to this premise and to the main results.

One obvious motive for social conformity is survival. The survival motive is a prominent theme in both theoretical and empirical work, including Gandhi and Przeworski (2006); Padro i Miquel (2007); Myerson (2008); Escriba-Folch (2020); Guriev and Sonin

(2009); Svolik (2012); Albertus and Menaldo (2012); Xu and Jin (2017); Tyson (2018); Garfias (2018); Shadmehr (2019). An accumulating body of evidence shows that repression of expressive freedom is an effective method of survival. Using cross country measures of political terror, physical integrity, civil rights, and modes of exit, Escriba-Folch (2020) show that restrictions on civil liberties rather than political terror are more effective in maintaining survival of autocrats of all types. Using data on autocratic expropriation in Latin America, Albertus and Menaldo (2012) show that expropriation increases the odds of survival. Xu and Jin (2017) find evidence that targeted repression diminishes social trust and political participation, reducing the potential threat of an organized opposition.

The present model provides a complementary perspective by distinguishing the pure social conformity motive from its instrumental uses. There are practical reasons for doing so. Survival may not be a primary concern of an entrenched autocrat like Xi Jinping in China. In such cases, repression may be used for other purposes like rent-extraction (Padro i Miquel (2007)).[16] In addition, some actors exercise extreme forms of control without apparent concern for survival (e.g., ISIS in Iraq and Syria).

In the model, authoritarians endogenously choose to limit punishment. The harshest measures are not used on certain types of citizen, and these measures become less harsh for all citizens over time. This is consistent with observed trends toward less severe repression. Alesina and Rodrik (1994) and Arias et al. (2018) document the deterrent effects of globalization of investment and information on autocratic repression. It also contrasts with the experience of "older" autocracies. For instance, Gregory et al. (2011) access data on Soviet archives in the Stalin era to document more severe punishment of innocents occurring when external information on repression activities is poor.

Escriba-Folch (2020) evaluates data on autocratic exit and repression to show restrictions on civil liberties are more effective in securing power than the gulag. Guriev and Treisman (2019); Egorov and Sonin (2020); Guriev and Papaioannou (2021); Guriev and Treisman (2022) emphasize (in both models and data) the linkages between communication/propaganda and repression, the former facilitating less draconian forms of the latter. Gehlbach and Keefer (2012) model limited social control as a commitment device used by the autocrat to ties its hands against excessive rent-extraction.

Gandhi and Przeworski (2007) provide evidence that autocrats use superficially democratic processes to increase their odds of survival. Scheppele (2018) elaborates further, using the term *autocratic legalism* to refer to the autocrats' successful mimicry of the legal architecture of democracies while undermining the rule of law. Under autocratic legalism, "the new autocrats eliminate their opponents by pressuring them differently: they drive their opponents out of the country rather than jail them, and they punish

[16] See also Alesina and Tabellini (1990); Benabou (2000); Li (2009); Gehlbach and Keefer (2012); Albertus (2015, 2021); Cao and Lagunoff (2022) who study rapaciousness as a motivating force in the authoritarian drive for control.

those who defy them through economic measures...Opponents are fired from their jobs, denied social benefits for technical reasons, and evicted from their buildings because of small and technical violations..."[17]

In a study of China's Social Credit System (CSCS), a compliance rule based on "social scoring," Xu et al. (2022) observe: "Repression under the SCS takes even milder, lower-profile forms. Instead of putting dissidents into jails, the government can lower their social scores to ban them from traveling, buying property, or taking out a loan."[18] In another study of CSCS, Lin and Milhaupt (2023) summarize the scoring as a "regime of rewards and punishments (in the form of 'redlists' and 'blacklists')." In an extensive study of autocratic land distribution in Latin America, Albertus (2015) quantifies the extent of confiscations and rewards used by autocrats to maintain control.

Notably, there is no consensus about whether recent trends toward lighter control methods are due to higher costs of repression. The present study points instead to the salience of wealth dynamics and the overcompliance problem. Empirical results by Shih (2008) suggest that recent authoritarians are in fact sensitive to potential overcompliance. He examines control mechanisms in China and documents a compliance system that produces "nauseating displays of loyalty" as a way for ordinary citizens to signal conformity without substantively overreacting.[19]

It is an interesting question as to why these factors are more salient now than in the past. On a theoretical level, the choice of "under-punishment" here is comparable to laxity in tax compliance (Reinganum and Wilde (1988)) or to efficiency wages (Shapiro and Stiglitz (1984)) where agents are "over-rewarded" to deter future shirking. Both theories require a modern economy with modern methods of property enforcement. Similarly, the society modeled here must be wealthy enough and have sufficient state capacity so that confiscation and rewards can replace the gulag. In a poorer society, such as one where $w_i < P$ in Section 3.3.2, or one with $R = 0$, the compliance rule will still rely on imprisonment.

The main results produce compliance rules highly tailored to wealth and ideology. Lin and Milhaupt (2023) collect data on China's Social Credit System in the Zhejiang province. They find that firms with high *a priori* political connections received higher scores on average. Xu (2020) links higher levels of targeting to counties in China that spend more on surveillance, in line with the result that improved monitoring increases differential treatment between citizen-types.

In our model, wealthier citizens are more compliant and experience lower rates of confiscation. Using data from the World Values Survey Ceka and Magalhaes (2020) show that the higher one's socioeconomic position the stronger the support for the institutional

[17] Scheppele (2018), p. 575-6.

[18] Xu et al. (2022), p. 3.

[19] Other examples of, for instance, sycophancy or bellicose demands for harsh penalties on others are described in Applebaum (2012) on post-war Eastern Europe, or Beevor (2022) on early Soviet Russia.

status quo. They show this is true in both democracies and autocracies, suggesting that compliance by wealthier individuals is a function of social standing along with the threat of losing it.[20] Egorov et al. (2009) also show that compliance correlates with wealth. They show that more wealth (in the form of oil wealth) in autocracies is associated with a more compliant media.

The relation between wealth and compliance in our model produces a time tradeoff: an authority that lets up on punishment sacrifices present compliance in order to secure greater compliance in the future. However, the more unstable/impatient the regime the less willing it is to allow for wealth accumulation. The logic is similar to over-extraction of a limited resource by an unstable authority in Harstad (2023). Our results suggest that stable autocracies are expected to be wealthier, appear more socially cohesive, and impose less repression on the whole - the Singapore effect. This is borne out by Escriba-Folch (2020) who show that increased increased probability of exit, an obvious indicator of unstable autocracy, is associated with increases in repression levels. Using data on Latin American autocrats, Albertus and Menaldo (2012) show larger confiscation occurs mostly in less mature regimes.

The role of wealth is reminiscent of a trade off between bribery and lobbying in Harstad and Svensson (2011). In their model, the ruler tries to extract a higher bribe from firms that are more capitalized. As response, highly capitalized firms switch to lobbying. In the present model the principal - the authority - switches to a lower punishment regime when citizens are wealthier to avoid citizen over-responses. Since bribes (in their model) and high confiscation (in ours) reduce possibilities for wealth accumulation, poverty traps can occur in both models.

Our theory has implications for growth and inequality. Regarding the latter, large recent increases in inequality in Russian and China have been documented by Novokmet et al. (2018) and Piketty et al. (2019), respectively. The present paper focuses on unequal treatment in the application of law as one driver of wealth inequality. Unequal treatment means that wealth paths among the citizenry diverge, even among those with the same initial wealth. Dissident individuals, or those who were simply unlucky, do not catch up. In the long run, wealth converges to a balanced growth path that merely locks in existing inequality. A number of studies show that authoritarian regimes without constraints on individual targeting will exhibit lower growth and greater confiscation than those that have constraints in place. Huber et al. (2021) examine effects of an infamous case of targeting - the expulsion of Jews from business in Nazi Germany. They find that the expulsions led to a 1.8% reduction in GDP. Funke et al. (2020) conduct a comprehensive study of populist regimes and find that after a period of 15 years, "GDP per capita is more than 10% lower compared to a plausible non-populist counterfactual," and that

[20]Evidence also suggests that regressive confiscation, though less draconian, sometimes occurs in democracies as well (ACLU (2016)). Moreover, in both democracies and autocracies sufficient state capacity is required to exercise control. Grigoryan and Polborn (2018) show that without that capacity, confiscation by non-state actors may be non-monotone in wealth.

"both variants of populism [left-wing and right-wing] are equally bad for the economy." While explicit methods of targeted control in these regimes are not identified, their study includes a number of indirect measures of control (e.g., extent of media freedom and judicial constraints). Because the regime in our model treats poor and rich individuals differently, our results suggest that authoritarian regimes produces larger inequality than those that treat all citizens equally under the law.[21] In extreme cases of autocratic instability, low state capacity, or rapaciousness, the wealth distribution bifurcates. Some citizens transition to the absorbing region of full compliance and wealth accumulation while others become locked in a poverty trap.

While this study focusses more on the authority's choice of repression, the concessionary motives of the citizenry also play a role. Each citizen must effectively choose between comfort and freedom. A similarly stark choice is modeled in different settings by Gratton and Lee (2024) and Shadmehr (2019). Gratton and Lee (2024) posit a choice by the citizen-voter between a liberal and an illiberal democracy. The voter values the liberty offered by a liberal government but also values the security offered by an illiberal one. The illiberal government is perceived to be unencumbered and more adaptable to negative shocks. As with lackeys in the present model, their citizen-voter sometimes chooses security over freedom.[22] Shadmehr (2019) explains how a portion of the citizenry might even impose repression on themselves and others. He examines a global game model of capital owners and workers. The former can choose to invest at home or abroad. The latter can work or foment revolution. Because capital flight and revolts are complementary, capitalists may choose to limit freedom on capital movement as a way to avoid regime change.

The citizens here are less powerful than in these models. The role of principal and agent are reversed here with the authoritarian government taking the role of principal. Tyson (2018) places the regime in the position of the principal, but adds a layer to the agency problem by examining incentives of those who carry out repression. The presence of an intermediary constrains the authority who cannot then target everyone. However, the authority's choice of target is endogenous, and so non-targeted individuals may end up complying as well.

The role of authority-as-principal is presented in novel framework by Persico (2023). He models political competition with non-anonymous voting. Each citizen's vote is observed and is therefore punishable ex post if she votes the wrong way. Authoritarian rule is maintained since voters will tend to support the favored candidate rather than their own favorite. His model complements ours since it proposes a particular surveillance

[21] In the model, each citizen's wealth accumulation converges to balanced growth wealth accumulation. Since inequality remains constant along the balanced growth path, it locks in the prior inequality attained along the transition path.

[22] Their mechanism is quite different. Gratton and Lee (2024) consider a dynamic agency model of Bayesian persuasion in which signal garbling in the persuasion policy can bias the voter in favor of keeping an illiberal government.

tool for controlling expressive choices: the election mechanism itself.

Finally, our results for authoritarian regimes usefully contrast with the literature on social control in democratic polities. Lagunoff (2001) shows that expansive civil liberties emerge over time in societies where the rule of law prevents targeting. Mukand and Rodrik (2020) argue that societies with strong identity cleavages and large levels of inequality are less likely to evolve laws that prevent targeted repression. The present model shows that the forces of identity cleavages and targeting work symbiotically.

5 Conclusions and Future Directions

This paper posits a model in which authoritarians repress expressive freedom to achieve social conformity. The model characterizes equilibria in which compliance rules chosen by the authority can incorporate potentially severe punishments for "bad" signals and rewards for "good" ones. Despite this latitude, the authority limits punishment, albeit in a way that applies laws unequally. Over time it relies mainly on partial asset confiscation and rewards. The results are driven in part by the overcompliance problem and by the dynamic tradeoffs between present and future control.

Wealthier citizens are more compliant, ceteris paribus. Consequently, the wealthy face lower confiscation rates. Over time, each citizen's compliance positively varies with her wealth trajectory. Above a wealth threshold, a citizen becomes a fully compliant "lackey" whose wealth converges to the balance growth path. A relatively patient/stable authority reduces punishment and increase rewards in order to allow the citizen to reach this threshold.

Finally, authorities that are more impatient, have lower state capacity, or grow more slowly are more repressive. High confiscation in these cases can lead to poverty traps. In these regimes, wealth will be more unequal and behavior more polarized. Rapaciousness adds an extra dimension. A rapacious ruler reduces the rewards, and this may increase the likelihood of falling into the poverty trap. However, the overall effect of rapaciousness on repression is unclear.

Even in cases most favorable to growth, different citizen-types reach their respective thresholds at different rates.[23] Social control therefore leads to higher wealth inequality. When citizens reach the absorbing region of full compliance, convergence to the balanced growth path merely locks in existing inequality.

For tractability the model makes some parametric assumptions and omits some po-

[23]Scholars of authoritarian regimes have long recognized the heterogeneity of citizens in response to repression but differ on the reasons. Osborne et al. (2023) characterize a process of psychological activation process whereby threats to safety and security move ordinary citizens toward extreme predispositions.

tentially important considerations. This Section concludes with a discussion of some of the assumptions and omissions, and discusses a path forward for future work.

Shocks. The model excludes economic shocks. Resilience against shocks is sometimes cited as a determining factor in regime stability. Some studies postulate a liberty-security trade off in which the autocrat is more nimble in handling shocks (e.g., Shadmehr (2019)). Others describe rigidities in authoritarian regimes that render them less flexible (e.g., Funke et al. (2020)). Under our formulation the effects of uncertainty would be ambiguous. Negative shocks reduce growth and wealth accumulation. If shocks are persistent then there is no absorbing region of full compliance. Presumably, an authority in this position is less willing to sacrifice current confiscation in favor of future confiscation. In that case, shocks have an effect similar to a less patient regime.

Allowing for consumption smoothing motives. If flow payoffs of citizens were strictly concave in wealth, on might argue that wealthy citizens would be harder to control, *ceteris paribus*, since the marginal payoff loss of an extra dollar is small. For common payoff specifications, this will not be the case. It is true that concavity reduces the regime's control, but not enough to alter the results. Take the simple case analyzed in Section 3.3.2 where $\delta_A = \delta_c = 0$. Both citizen and authority are impatient. Next consider a standard reformulation of (1):

$$u_i \; = \; -\frac{1}{2}(a_i - a_i^c)^2 \; + \; \frac{1}{\nu}\left(w_i - \int \mathcal{T}_i(x) f(x|a_i)dx\right)^{\nu} \tag{30}$$

with $0 < \nu \leq 1$. When $\nu = 1$ we recover the original model. When $\nu < 1$, the citizen's payoff on wealth is strictly concave, and so confiscation penalties have a diminishing marginal effect on wealthier individuals.

Since the authority's first order conditions are the same, the citizen's equilibrium response in (25) generalizes to

$$\alpha_i^*(w_i, \mathcal{T}^*) \; = \; a_i^c + (\tau_i(w_i) + \rho(w_i))Z(w_i + \frac{1}{2}(\rho_i(w_i) - \tau_i(w_i))^{\nu-1} \tag{31}$$

In (31) citizen's compliance is still increasing in wealth. Because the over-compliance problem remains, the confiscation rate will still be regressive, albeit less so than before. The qualitative results go through with minor changes and much more notation.

History contingency and resistance. The restriction to Markov Perfect equilibria assumes individual citizens are atomistic. Individual citizens are too small, by definition, to resist the authority's scheme.[24]

Yet, Desai et al. (2007); Jia et al. (2021); Cao and Lagunoff (2022), among others, all show that coordinated history-contingent strategies can loosen the authoritarian's con-

[24] Footnote 8 also references Bhaskar et al. (2012) who provide a formal foundation for Markov Perfect equilibria which arguably applies here.

trol.[25] To justify history-contingency, these models typically depart from pure autocracy. Specifically, in order to have a strategic effect vis-a-vis the authority, individuals need to be part of a large strategic group. Regimes where this happens are sometimes referred to as *anocracies*. Anocracies are authoritarian-minded regimes that permit civil society groups such as unions, trade associations, and civic organizations. Cao and Lagunoff (2022) show that these groups can act as strategic partners, bargaining with the authority to enforce commitments to the rule of law. Persico (2023) makes a similar point in focussing on bottom-up approaches to democratization. Large players can coordinate on history to enforce less punitive outcomes and ensure greater freedom.

It may be easier for the rich to form such groups; they have more resources to organize. This may explain the persistence of wealthy elites in many regimes. Regardless of who forms the group, any group powerful enough to negotiate with the authority may also be powerful enough to replace it. Hence, anocracies are unstable. Future work could incorporate group formation to account for these possibilities.

Private investment incentives. Arendt (1951) long ago observed that totalitarian states stifle individual initiative and creativity. Growth impediments show up in the model via confiscation. The model does not consider citizens' productive incentives under the threat of confiscation. These incentives could be added to the model explicitly in an endogenous growth model with consumption and investment.

Endogenous state capacity. State capacity is an exogenous parameter in the model. A natural extension posits state capacity as an investment choice. After investing, the authority then allocates rewards across citizens depending on signal realizations. The allocation would depend on which citizens were easiest to co-opt. Because the budget allocation depends relative, not absolute wealth positions of the citizens, the dynamics are non-trivial. The authority must discern relative differences in speeds of adjustment toward full conformity and then compare these differences with relative losses of conformity in the present. This approach is beyond the scope of the present paper, but an important topic for future work.

Regime Change. The model examines regime change obliquely, via the discount factor. The more unstable regime is more repressive. Adding an explicit survival motive adds a wrinkle. On the one hand, if enforced conformity increases survival as much of the empirical literature suggests, then repression in the model may be understated. On the other hand, enforced conformity reduces growth which strains the authority's credibility. It is also worth considering what would happen to an authoritarian who is replaced. Depending on the type of autocracy, the authority may be less repressive if it survives the loss of power. One might suppose a probability $1-\delta_A$ each period that the authority becomes an ordinary citizen.

[25] This is not a given since Persico (2023) shows how history-contingency can be used to punish voters who vote the wrong way.

Democracies and electoral competition. Are things different in democracies? There are a number of reasons to believe so. First, constitutions in liberal democracies place limits on "cruel and unusual" punishments. Just as important, democracies avoid targeting as a matter of principle. Control will be coarser, resulting in more varied behavior. Finally, democracies' control emerges from elections where voters cannot isolate themselves from the consequences of their votes. Any scheme will apply to the decisive voter as well.[26] For that reason, punishment thresholds will be laxer. Each of these factors separately will reduce inequality and allow individuals to accumulate wealth in democracies.

Notably, some authoritarians face electoral competition even when they manipulate the electoral process. Jair Bolsonaro's electoral loss in Brazil shows that autocratic control of elections is imperfect. Future work could explore whether and to what extent competition negates authoritarian control. As Persico (2023) shows, the answer depends on information flows among other things. The Bernhardt et al. (2022) model of political competition between demagogues and pro-democracy parties seems a natural starting point. Future work could also integrate *de jure* choice of control with "organic" emergence of control via evolutionary dynamics (Cerqueti et al. (2013)) or network effects (Genicot (2022)).

6 Appendix

Proof of Lemma 1. Equation (7) is the Euler equation derived from the citizen's Bellman equation (6). Any solution to (6) is a critical point (7).

We first show a fixed point solution to (7) exists. Observe $f_a(x|a_{it}) < 0$ if $x < a_{it}$, $f_a(x|a_{it}) > 0$ if $x > a_{it}$, and $f_a(a_{it}|a_{it}) = 0$ is the unique crossing point. Moreover, $\int f_a(x|a_{it})dx = 0$. Observe also that since $\mathcal{T}_i(w_i, \cdot)$ is uniformly bounded in x, then $|\int \mathcal{T}_i(w_i, x))f_a(x|a_i)dx|$ is uniformly bounded in a_i. Denote this bound by $B(w_i)$.

Also, U^* is bounded in the interval $[-\frac{(a_i^c-\bar{a})^2}{2(1-\delta)} - \frac{P}{1-\delta}, \frac{w_{it}+R}{1-\delta_c(1+\gamma)}]$. The lower bound is the case where the citizen is compelled to comply but nevertheless receives the maximal penalty each period. The upper bound is the balanced growth path in which the citizen chooses her ideal $a_{it} = a_i^c$ and is rewarded with $R + w_{is}$ each period $s \geq t$. (Neither of these bounds are tight).

These bounds imply $\int U_i^*(G(w_{it}, x))f_a(x|a_i)dx$ is uniformly bounded in a_i as well. Denote the absolute value of this bound by $D(w_i)$. Consequently, for any w_i, there is

[26] This was a key point in Lagunoff (2001).

are two behavior choices a_i' and a_i'' such that

$$\begin{aligned} a_i' &> a_i^c + B(w_i) + \delta_c D(w_i) \\ &\geq a_i^c - \int \mathcal{T}_i(x, w_{it}) f_a(x|a_i') dx + \delta_c \int U_i^*(G(w_{it}, x)) f_a(x|a_i') dx. \end{aligned} \tag{32}$$

and

$$\begin{aligned} a_i'' &< a_i^c - B(w_i) - \delta_c D(w_i) \\ &\leq a_i^c - \int \mathcal{T}_{it}(x, w_{it}) f_a(x|a_i'') dx + \delta_c \int U_i^*(G(w_{it}, x)) f_a(x|a_i'') dx. \end{aligned} \tag{33}$$

By the Intermediate Value Theorem there exists a fixed point $a_i = \alpha_i(w_i, \mathcal{T}_i) \in [a_i^c, a_i']$ of the map (7). That is,

$$\alpha_i(w_i, \mathcal{T}_i) = a_i^c - \int \mathcal{T}_{it}(x, w_{it}) f_a(x|\alpha_i(w_i, \mathcal{T}_i)) dx + \delta_c \int U_i^*(G(w_{it}, x)) f_a(x|\alpha_i(w_i, \mathcal{T}_i)) dx \tag{34}$$

Let $\Psi_i(w_i, \mathcal{T}_i, a_i)$ denote the right-hand side map in (7), i.e., Ψ_i is the "fixed point map." The above argument shows that the map starts above a_i^c and crosses from above. Hence at least one fixed point of $\Psi_i(w_i, \mathcal{T}_i, a_i)$ is a local maxima and, consequently, there is a solution to the citizen's Bellman equation. Since $f_a(x|a) \to 0$ as $a \to \infty$, then $\Psi_i(w_i, \mathcal{T}_i, a_i) \to a_i^c$ as $a_i \to \infty$. It follows that there is a maximal fixed point which is a local optimum for the citizen. Generically, there are finitely many maxima, only one of which is the global maxima. ∎

The following Remark is useful for the next few Propositions.

Remark 1 *Holding fixed $\mathcal{T}_i(\hat{w}_i, \cdot)$ evaluated at an arbitrary $\hat{w}_i$, the fixed point map $\Psi_i(w_i, \mathcal{T}_i(\hat{w}_i, \cdot), a_i)$ is constant in w_i. Wealth only enters the citizen's response through the compliance rule $\mathcal{T}_i$. Thus the fixed point $\alpha_i(w_i, \mathcal{T}_i(\hat{w}_i, \cdot))$ of $\Psi_i(w_i, \mathcal{T}_i(\hat{w}_i, \cdot), a_i)$ is constant in w_i as well. As a result, one can write $\alpha_i^*(w_i, \mathcal{T}) = \alpha_i^*(\mathcal{T}(w_i, \cdot))$.*

Proof of Part (i) in Proposition 1. We first show positive compliance shifts. To show that static compliance shift is positive, recall from the assumptions on F that f_a is strictly increasing, $f_a(x|a_{it}) < 0$ if $x < a_{it}$, $f_a(x|a_{it}) > 0$ if $x > a_{it}$, and $f_a(a_{it}|a_{it}) = 0$ is the unique crossing point. Moreover, $\int f_a(x|a_{it}) dx = 0$. By monotonicity of $\mathcal{T}_{it}$ in x the term

$$\int -\mathcal{T}_{it}(x, w_{it}) f_a(x|a_{it}) dx > 0.$$

Regarding the dynamic compliance shift, This follows from the proof in part (ii) below establishing $\frac{\partial U_i^*}{\partial w_{it}} \geq 0$ By the properties of f_a, we obtain $\int U_i^*(G(w_{it}, x)) f_a(x|a_{it}) dx > 0$.

Proof of Part (ii) in Proposition 1. To prove the result, the following claim must be established: **Claim.** $\frac{\partial U_i^*}{\partial w_{i\,t}} \geq 0$**.**

Using the Envelope Theorem:

$$\frac{\partial U_i^*}{\partial w_{i\,t}} \;=\; \int (1 - \int \frac{\partial \mathcal{T}^*}{\partial w_{it}}) f(x|\alpha_i^*(w_i, \mathcal{T}_i^*))dx + \delta_c \int \frac{\partial U_i^*(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G}{\partial w_{it}} f(x|\alpha_i^*(w_i, \mathcal{T}_i^*))dx. \tag{35}$$

where either $\frac{\partial G}{\partial w_{it}} = 0$, or $\frac{\partial G}{\partial w_{it}} = (1+\gamma)(1-\frac{\partial \mathcal{T}^*}{\partial w_{it}})$, depending on whether the nonnegativity of wealth in (4) binds.

If the wealth constraint binds, then $\frac{\partial U_i^*}{\partial w_{i\,t}} = \int (1 - \frac{\partial \mathcal{T}^*}{\partial w_{it}}) f(x|\alpha_i^*(w_i, \mathcal{T}_i^*))dx$. If the wealth constraint does not bind, iterate Equation (35) forward. After forward iteration, each period $s \geq t$ has a term of the form:

$$\delta^{s-t}(1+\gamma)^{s-t} \prod_{\ell=1}^{s} \left(1 - \int\int \cdots \int \frac{\partial \mathcal{T}^*}{\partial w_{i\ell}}) f(x_\ell|\alpha_i^*(w_{i\ell}, \mathcal{T}_i^*))dx_\ell dx_{\ell-1} \cdots dx_1\right), \quad \ell = t, t{+}1, \ldots, s.$$

We now show $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} \leq 1$ for all x. Any solution to the authority's Bellman equation (26) must satisfy the constraint (3). Consequently, the solution $\mathcal{T}_i(w_i, x)$ is either a corner solution at $-R$ implying $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} = 0$, or it is a corner solution at $w_i + P$ implying $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} = 1$, or $\mathcal{T}_i(w_i, x)$ is an interior solution: $-R < \mathcal{T}_i(w_i, x) < w_i + P$.

We now show that if $\mathcal{T}_i(w_i, x)$ is an interior solution at w_i, then $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} \leq 0$. The authority's first order condition is

$$\begin{aligned} &\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a} - \alpha_i^*(w_{it}, \mathcal{T}_i)) \;+\; \delta_A \int V(G(w_{it}, x)) f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx\right] \\ &\quad +\; \delta_A \int \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} f(x|\alpha_i^*(w_{it}, \mathcal{T}_i)) d\mathcal{T}_i(x) \;=\; 0 \end{aligned} \tag{36}$$

Because $\frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} = -(1+\gamma)$ if the zero wealth boundary is not reached, the last term is strictly negative. Since the term inside $[\cdot]$ in the first term is nonnegative, the hypothesized interior solution implies $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} > 0$.

Recall, $\alpha^*(w_i, \mathcal{T}) = \alpha^*(\mathcal{T}(w_i, \cdot))$. Suppose by contradiction, $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} > 1$ for $x < \alpha^*(\mathcal{T}(w_i, \cdot))$. This implies a decrease in $w_i - \mathcal{T}(w_i, x)$ and a decrease in $\bar{a} - \alpha^*(\mathcal{T}(w_i, \cdot))$.

The reduction of future wealth implies $\int V(G(w_{it}, x)) f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx$ (weakly) decreases; the decrease in $G(w_{it}, x)$ tightens constraints on future compliance rules $\mathcal{T}_{i\,t+1}, \mathcal{T}_{i\,t+2}, \cdots$. For the same reason, the last term in (36) decreases due to the reduction in $G(w_{it}, x)$. To maintain an interior solution, $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} < 0$, a contradiction. Consequently, $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} \leq 1$ whenever $\mathcal{T}(w_i, x)$ is an interior solution.

This confirms that $\frac{\partial \mathcal{T}^*(w_{it},x)}{\partial w_{it}} \leq 1$ for all x, interior or otherwise. Since $\frac{\partial U_i^*}{\partial w_{i\,t}}$ is the sum of these terms, for both binding and non-binding wealth constraints, it follows that $\frac{\partial U_i^*}{\partial w_{i\,t}} \geq 0$.

We now proceed with the remainder of the proof of Part (ii). Consider a choice $a_i = \alpha_i^*(w_i, \mathcal{T}_i)$ anticipated by the authority. Consider the change $\Delta\mathcal{T}_i$ described in the hypothesis. Namely, $\Delta\mathcal{T}_i(x) > 0$ when $x < a_i$ and/or $\Delta\mathcal{T}_i(x) < 0$ when $x > a_i$.

Consider the value $\Psi_i(w_i, \mathcal{T}_i + \Delta\mathcal{T}_i, a_i)$ where, recall, Ψ is the fixed point map in (7). To simplify notation let $\Delta\mathcal{R}_i(x) \equiv -\Delta\mathcal{T}_i(x)$ for $x > a_i$ so that $\Delta\mathcal{R}_i(x)$ is the incremental reward on the high signal set. Using the definition of G in (4),

$$
\begin{aligned}
&\Psi_i(w_i, \mathcal{T}_i + \Delta\mathcal{T}_i, a_i) - a_i^c \; = \\
&\int (-\mathcal{T}_i(x, w_{it}) - \Delta\mathcal{T}_i(x)) f_a(x|a_i)dx + \delta_c \int U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it}) - \Delta\mathcal{T}_i(x)\}) f_a(x|a_i)dx \; = \\
&\int_{-\infty}^{a_i} (-\mathcal{T}_i(x, w_{it}) - \Delta\mathcal{T}_i(x)) f_a(x|a_i)dx + \delta_c \int_{-\infty}^{a_i} U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it}) - \Delta\mathcal{T}_i(x)\}) f_a(x|a_i)dx \\
&+ \int_{a_i}^{\infty} (-\mathcal{T}_i(x, w_{it}) + \Delta\mathcal{R}_i(x)) f_a(x|a_i)dx + \delta_c \int_{a_i}^{\infty} U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it}) + \Delta\mathcal{R}_i(x)\}) f_a(x|a_i)dx \\
&\qquad > \\
&\int_{-\infty}^{a_i} (-\mathcal{T}_i(x, w_{it})) f_a(x|a_i)dx + \delta_c \int_{-\infty}^{a_i} U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it})\}) f_a(x|a_i)dx \\
&+ \int_{a_i}^{\infty} (-\mathcal{T}_i(x, w_{it})) f_a(x|a_i)dx + \delta_c \int_{a_i}^{\infty} U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it})\}) f_a(x|a_i)dx \\
&= \int (-\mathcal{T}_i(x, w_{it})) f_a(x|a_i)dx + \delta_c \int U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it})\}) f_a(x|a_i)dx \\
&= \Psi_i(w_i, \mathcal{T}_i, a_i) - a_i^c
\end{aligned}
\tag{37}
$$

To explain the inequality in (37), observe that the terms $-\mathcal{T}_i(x, w_{it}) - \Delta\mathcal{T}_i(x)$ and $U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it}) - \Delta\mathcal{T}_i(x)\})$ have decreased on $(-\infty, a_i)$ while $-\mathcal{T}_i(x, w_{it}) + \Delta\mathcal{R}_i(x)$ and $U_i^*((1+\gamma)\max\{0, w_{it} - \mathcal{T}_i(x_{it}, w_{it}) + \Delta\mathcal{R}_i(x)\})$ have increased on $[a_i, \infty)$. Moreover, both $\Delta\mathcal{T}_i(x) f_a(x|a_i) < 0$ when $x \in (-\infty, a_i)$ and $\Delta\mathcal{R}_i(x) f_a(x|a_i) < 0$ when $x \in [a_i, \infty)$. Then Ψ_i increased following the change $\Delta\mathcal{T}_i$ which means that its fixed point $\alpha_i(w_i, \mathcal{T}_i + \Delta\mathcal{T}_i)$ is larger than $\alpha_i(w_i, \mathcal{T}_i)$.

Proof of Part (iii) in Proposition 1. Finally, I utilize the result from Part (ii) to show $\alpha_i^*(w_i, \mathcal{T}_i^*) \in [a_i^c, \bar{a}]$. Suppose that under a given w_i and $\mathcal{T}_i$, $\alpha_i(w_i, \mathcal{T}_i) < a_i^c$. In the absence of punishment or reward, the citizen will choose her ideal behavior a_i^c. Hence, as $||\mathcal{T}_i|| \to 0$, the right-hand side of (7) converges to a^c. Moreover, a decrease in $\mathcal{T}_i(w_i, x)$ on a positive measure set $C \subseteq \mathbb{R}$ (either a decrease in punishment or increase in reward) moves the citizen'sact closer to a_i^c. This is preferable for both the citizen and the authority since it moves behavior closer to both individuals' ideal points and increases the post-confiscation wealth of the citizen. Next, suppose that under a given w_i and $\mathcal{T}_i$, $\alpha_i(w_i, \mathcal{T}_i) > \bar{a}$. Again, the ruler can decrease punishment $\mathcal{T}_i(w_i, x)$ for low signals x, moving behavior in the direction of a_i^c and, in this case, back toward $\bar{a}$. Again, this is preferable for both agents, leaving the citizen with higher post-confiscation wealth.

Now suppose that even after elimination of punishment for low signals, it is still the case that $\alpha_i(w_i, \mathcal{T}_i) > \bar{a}$. From the previous argument on the consequences of reducing $||\mathcal{T}_i|| \to 0$, a reduction of reward sufficiently allows the authority to fully eliminate overcompliance in the present. It also reduces wealth in the future. However, does not reduce the authority's control because in the worst case, $w_{i\,t+1} = (1+\gamma)w_{it}$. So even without reward the citizen's wealth always grows (recall there is no punishment for low signals). If full compliance is obtainable at w_{it}, then it is obtainable at $w_{i\,t+1}$ by lowering the reward if necessary. One concludes $a_i^c < \alpha_i(w_i, \mathcal{T}_i) \leq \bar{a}$. ∎

Construction of Equilibrium Distribution of Signal path. Formally, let $x^t = (x_0, x_1, \ldots, x_t) \in \mathbb{R}^{t+1}$. Let $x^\infty = (x_0, x_1, \ldots,) \in \mathbb{R}^\infty$. Endow the space $\mathbb{R}^\infty$ with the product topology. Let $\mathcal{X}$ denote the Borel σ-algebra generated from this topology. For any $\mathcal{X}$-measurable set X, define

$$\mathbb{P}_i^* \left(x^\infty \in X\right) \;=\; \int_{x^\infty=(x_1,x_2,\ldots)\in X} \times_{t=0}^{\infty} f(x_t \,|\, \alpha_i^*(G^t(w_{i\,0}, x^t), \mathcal{T}_i^*) dx^t$$

where G^t is built up recursively from $\mathcal{T}_i^*$ starting from default signal x^0, and $w_{i\,1} = G(w_{i\,0}, x_0)$, $w_{i\,2} = G(G(w_{i\,0}, x_0), x_1), \ldots$, and so

$$G^t(w_{i\,0}, x^t) = \overbrace{G(G(\cdots G(G(w_{i\,0}, x_0), x_1,) \cdots), x_t)}^{t}$$

Proof of Proposition 3. Before proceeding with the proofs of Parts (i) and (ii), I establish some general properties of equilibria based on the authority's first order equation.

Changes in α_i^* due to an incremental change in $\mathcal{T}_i$ can be expressed as $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}$. To simplify notation, set $a_i^* = \alpha_i^*(w_i, \mathcal{T}_i^*)$. By Proposition 1, $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} > 0$ when $d\mathcal{T}_i = d\mathcal{T}_i(D^-)$ where $D^- \equiv (-\infty, a_i^*)$. In addition, $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} < 0$ when $d\mathcal{T}_i = d\mathcal{T}_i(D^+)$ where $D^+ \equiv [a_i^*, \infty)$. In words, the citizen's behavior shifts rightward when the compliance rule increases penalties on low signals and/or increases rewards on high signals.

Now consider the authority's first order condition. For any instantaneous change $d\mathcal{T}_i$, the authority's first order condition is expressed as

$$\begin{aligned} &\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i}\left[(\bar{a}-\alpha_i^*(w_{it},\mathcal{T}_i)) \;+\; \delta_A \int V(G(w_{it},x)) f_a(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\right] d\mathcal{T}_i \\ &\quad + \;\delta_A \int \frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} f(x|\alpha_i^*(w_{it},\mathcal{T}_i)) d\mathcal{T}_i(x) \;=\; (\geq)(\leq)\; 0 \end{aligned} \tag{38}$$

where either $\frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} = -(1+\gamma)$ or $= 0$ depending on whether the nonnegativity of wealth in (4) binds. The first order condition (38) holds with equality if $\mathcal{T}(w_i, x)$ is interior a.e. x, and with inequality otherwise.

In the first order condition (38),

$$V(w_{i\,t+1}) = 0 \iff \frac{\partial V}{\partial w_{i\,t+1}} = 0. \tag{39}$$

That is, the authority reaches its maximal continuation value of zero when the citizen is fully compliant in all future states on the equilibrium path. In that case, any increase in wealth will have no effect on the authority's continuation value.

Observe that the last term in (38),

$$\delta_A \int \frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} f(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx \leq 0$$

and is strictly negative if $\frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} > 0$ and $\frac{\partial G(w_{it},x)}{\partial \mathcal{T}_i(x)} = -(1+\gamma)$ on a nonnull set of x.

As for the term inside the brackets $[\cdot]$ in first term in (38), both parts inside $[\cdot]$ are nonnegative. Thus

$$\left[(\bar{a}-\alpha_i^*(w_{it},\mathcal{T}_i)) \;+\; \delta_A \int V(G(w_{it},x)) f_a(x|\alpha_i^*(w_{it},\mathcal{T}_i))dx\right] \;\geq\; 0$$

with strict inequality if either $\bar{a} > \alpha_i^*(w_{it},\mathcal{T}_i)$ (current non-compliance) and/or if $V(G(w_{it},x)) \neq 0$ for a nonnull set of x (future non-compliance).

Part (i). Suppose $V(G(w_{it},x)) < 0$ for a nonnull set of x. By (39), $\frac{\partial V(G(w_{it},x))}{\partial w_{i\,t+1}} > 0$.

Consider sets D^+, D^- such that $D^+ \equiv [a_i^*, \infty)$ and $D^- \equiv (-\infty, a_i^*)$ as per Proposition 1. Consider first an increment $d\mathcal{T}_i$ on D^+: $d\mathcal{T}_i(x) > 0$ if $x \in D^+$ and $d\mathcal{T}_i(x) = 0$ otherwise. Then I show the value of (38) is negative. To see why, observe from Proposition 1 that $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} < 0$ for the change $d\mathcal{T}_i$. Then the first term (38), is strictly negative while the second is so as well. Thus, $\mathcal{T}_i$ is hitting the lower corner solution $-R$ on D^+. But since D^+ can be a subset of interval $[\alpha_i^*(w_{it},\mathcal{T}_i), \infty)$, it follows that $\mathcal{T}_i(w_{it},x) = -R$ for all $x \geq \alpha_i^*(w_{it},\mathcal{T}_i)$. Moreover, since the corner solution has strict inequality at $x = \alpha_i^*(w_{it},\mathcal{T}_i)$, it follows that there is $\epsilon > 0$ such that $\mathcal{T}_i(w_{it},x) = -R$ for all $x = \alpha_i^*(w_{it},\mathcal{T}_i) - \epsilon$.

Next, consider an increment $d\mathcal{T}_i$ on D^-: $d\mathcal{T}_i(x) > 0$ if $x \in D^-$ and $d\mathcal{T}_i(x) = 0$ otherwise. Then $\frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} > 0$ in which case (38) can hold with equality, or with inequality in either direction. Since we are in the case where $[\cdot] > 0$ hold in the first term, there exists $\underline{x}(w_{it})$ such that (38) is positive if $D^- \subset (-\infty, \underline{x}(w_{it})]$. To see why, drop the δ_A term and write the last two terms in (38) as

$$\left[\int_{x \notin D^-} V(G(w_{it}, x)) f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx + \int_{x \in D^-} V(G(w_{it}, x)) f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))dx\right] \frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} d\mathcal{T}_i +$$
$$\left[\int_{x \notin D^-} \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} d\mathcal{T}_i(x) f(x|\alpha_i^*(w_{it}, \mathcal{T}_i)) + \right.$$
$$\left.\int_{x \in D^-} \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} d\mathcal{T}_i(x) f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))\right]$$

Notice this holds since $d\mathcal{T}_i(x) = 0$ on $x \notin D^-$.

Dividing through by $f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))$ yields

$$\left[\int_{x \notin D^-} V(G(w_{it}, x)) \frac{f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}{f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}dx + \int_{x \in D^-} V(G(w_{it}, x)) \frac{f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}{f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}dx\right] \frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} d\mathcal{T}_i +$$
$$\left[\int_{x \notin D^-} \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} d\mathcal{T}_i(x) + \int_{x \in D^-} \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} d\mathcal{T}_i(x)\right]$$

which reduces to

$$\begin{gathered}\left[\int_{x} V(G(w_{it}, x)) \frac{f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}{f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}dx\right] \frac{\partial \alpha_i^*}{\partial \mathcal{T}_i} d\mathcal{T}_i + \\ \left[\int_{x \in D^-} \frac{\partial V(G(w_{it}, x))}{\partial w_{i\,t+1}} \frac{\partial G(w_{it}, x)}{\partial \mathcal{T}_i(x)} d\mathcal{T}_i(x)\right]\end{gathered} \tag{40}$$

Notice that $f(x|\alpha_i^*(w_{it}, \mathcal{T}_i)) \to 0$ as $x \to -\infty$ and so the last term in (40) vanishes. Applying the Monotone Likelihood Ratio property, $\frac{f_a(x|\alpha_i^*(w_{it}, \mathcal{T}_i))}{f(x|\alpha_i^*(w_{it}, \mathcal{T}_i))} < 0$ for $x < \alpha^*(w_{it}, \mathcal{T}_i)$ and the likelihood ratio decreases away from 0 as $x \to -\infty$. Thus the first term in (40) remains positive and bounded away from zero on some interval $(-\infty, \underline{x}(w_{it})]$. It follows that the two terms in (38) are jointly positive on $(-\infty, \underline{x}(w_{it})]$, and the first term remains so as well. Therefore, $\mathcal{T}_i(w_{it}, x) = P + w_{it}$ for all $x < \underline{x}(w_{it})$.

Thus far we have shown $\mathcal{T}_i(w_i, x) = P + w_i$ if $x \leq \underline{x}(w_i)$, and $\mathcal{T}_i(w_i, x) = -R$ if $x \geq \alpha^*(w_i, \mathcal{T}_i) - \epsilon = \bar{x}(w_i)$. This establishes the Proposition on a set of wealth levels in which full compliance does not occur somewhere in the continuation path.

To complete the proof of Part (i) and establish Part (ii) we now establish that there is a minimum wealth threshold in which full compliance *does occur* in the continuation, and in this region, the compliance rule is strictly interior.

Part (ii). To establish Part (ii), construct a specific equilibrium $(\tilde{\mathcal{T}}_i^*, \tilde{\alpha}_i^*)$ and threshold w^* and show that the authority's long run payoff V^* satisfies $V^*(w_i) = 0$ for all

$w_i \geq w_i^*$. Let $\Gamma \equiv \frac{\gamma}{1+\gamma}$. Fix w_i. Consider a compliance rule $\mathcal{T}_i$, parameterized by threshold signal $\hat{x}$ and a reward ρ of the form

$$\mathcal{T}_i(w_i, x; \hat{x}, \rho) = \begin{cases} \Gamma w_i & if \quad x < \hat{x} \\ -\rho & if \quad x \geq \hat{x} \end{cases} \tag{41}$$

and $\rho \leq R$. In this formulation, $\mathcal{T}_i$ is parameterized by fixed reward ρ and a threshold signal $\hat{x}$ that separates punishment from reward. The constant punishment is set at $\Gamma w_i = \frac{\gamma}{1+\gamma} w_i$ where γ is the natural growth rate. This compliance rule has the property that

$$G(w_i, x) = (1+\gamma)(w_i - \mathcal{T}_i(w_i, x)) \geq (1+\gamma)(w_i - \Gamma w_i) = (1+\gamma)(w_i - \frac{\gamma}{1+\gamma} w_i) = w_i \tag{42}$$

for all x. Since the confiscation tax Γw_i is the worst case event, Equation (42) shows that regardless of the signal, future wealth beyond level $\frac{\gamma}{1+\gamma} w_i$ never decreases: $w_{i\,t+1} \geq w_{it}$ under this rule.

By Lemma 1, any solution to the citizen's Bellman equation must satisfy the fixed point problem defined by this $\mathcal{T}_i$. This fixed point map is given by

$$a_i \;=\; a_i^c + [(\Gamma w_i + \rho) + \delta_c[U_i^*((1+\gamma)(w_i+\rho)) - U_i^*((1+\gamma)w_i(1-\Gamma))]]\, F_a(\hat{x}|\, a_i) \tag{43}$$

Equation (43) is simply the fixed point map evaluated at the constructed rule $\mathcal{T}_i$. By Lemma 1, there is at least one such fixed point of (43) that solves the citizen's Bellman equation. Denote the solution map by α_i.

Notice that $F_a(\hat{x}|\, a_i)$ is maximal at $a_i = \hat{x}$ and declines monotonically as a_i moves away from threshold $\hat{x}$. In addition, the last term, $U_i^*((1+\gamma)(w_i+\rho)) - U_i^*((1+\gamma)(w_i - \Gamma)) \geq 0$.

As before, denote the fixed point (right-hand side) map in (43) by

$$\Psi(w_i, \rho, \hat{x}, a_i) \equiv a_i^c + [(\Gamma w_i + \rho) + \delta_c[U_i^*((1+\gamma)(w_i+\rho)) - U_i^*((1+\gamma)w_i(1-\Gamma))]]\, F_a(\hat{x}|\, a_i)$$

where ρ and $\hat{x}$ replace $\mathcal{T}_i$ in the notation since these parameters characterize $\mathcal{T}_i$.

Observe that $\Psi(0, 0, \hat{x}, a_i) = a_i^c$ for all a_i, that is, by setting $\rho = w_i = 0$, the fixed point map gives the citizen's ideal behavior. Observe also that there is w_i' and ρ' sufficiently large such that $\Psi(w_i', \rho', \hat{x}, a_i) > \bar{a}$ for all a_i. By the Intermediate Value Theorem, there exists a triple $(w_i', \rho', \hat{x}')$ satisfying

$$\Psi(w_i', \rho', \hat{x}', \bar{a}) = \bar{a}$$

or, equivalently,

$$\bar{a} \;=\; a_i^c + [(\Gamma w_i' + \rho') + \delta_c[U_i^*((1+\gamma)(w_i'+\rho')) - U_i^*((1+\gamma)w_i'(1-\Gamma))]]\, F_a(\hat{x}'|\, \bar{a}) \tag{44}$$

Observe that if we set $\rho' = 0$, then (44) will still hold if w_i' is large enough. Hence we can ensure $\rho' \leq R$. Moreover, since $F_a(x|\,\bar{a})$ crosses the axis at $x = \bar{a}$, we can restrict attention to $\hat{x}' \leq \bar{a}$.

$$w_i^* = \min\{w_i : \ \exists \rho \leq R, \hat{x} \leq \bar{a}, \ \ (w_i, \rho, \hat{x}) \text{ solves the citizen's Bellman equation}\} \quad (45)$$

Let ρ^* be the associated reward and $\hat{x}^*$ the associated punishment/reward threshold. Again by Lemma 1, the triple $(w_i^*, \rho^*, \hat{x}^*)$ satisfies (44) and so w_i^* is the minimum threshold to do so. By (45), $\alpha_i(w_i^*, \mathcal{T}_i) = \bar{a}$.

To summarize so far: Lemma 1 shows that the set of fixed points of $\Psi(w_i, \rho, \hat{x}, a_i) = a_i$ contains a solution to the Bellman equation, and every solution to the Bellman equation is a fixed point of Ψ. The rule in (41) has parameters and wealth that can be scaled to obtain any $a_i \in [a_i^c, \bar{a}]$, including $a_i = \bar{a}$.

Thus there is a triple $(w_i^*, \rho^*, \hat{x}^*)$ that (1) solves the Bellman equation, and (2) yields $\alpha_i(w_i^*, \mathcal{T}_i) = \bar{a}$, and w_i^* is chosen to be minimal with respect to these two properties.

We now construct $\tilde{\mathcal{T}}_i^*$ from $\mathcal{T}_i$ by setting

$$\mathcal{T}_i^*(w_i, x) = \mathcal{T}_i(w_i^*, x; \hat{x}^*, \rho^*) \quad \forall\ w_i \geq w_i^*. \quad (46)$$

Then let $\tilde{\alpha}_i^*$ be the solution map α_i evaluated at any $\mathcal{T}_i$ with continuation payoffs evaluated at $\tilde{\mathcal{T}}_i^*$.

Suppose under $\tilde{\mathcal{T}}_i^*$ that full compliance, $\tilde{\alpha}_i^*(w_i, \mathcal{T}_i^*) = \bar{a}$ holds for all wealth levels $w_i \geq w_i^*$. Since $\tilde{\mathcal{T}}_i^*$ is constructed so that (42) holds, once w_i^* is reached, subsequent wealth never falls below w_i^* regardless of the signal x. If that holds then $w_{it} \geq w_i^*$ would imply $\alpha_i^*(\omega_{i\,t+s}(x^\infty), \mathcal{T}^*) = \bar{a}, \forall s \geq 0$. This in turn gives the authority its optimal payoff $V_i^*(w_{it}) = 0$.

To verify this and complete Part (ii) we need only that the citizen chooses full compliance, $\alpha_i^*(w_i, \tilde{\mathcal{T}}_i) = \bar{a}$ for all wealth levels $w_i \geq w_i^*$.

Under this rule, observe that if $w_i \geq w^*$, then the payoff difference in the last term (43) satisfies

$$\begin{aligned} & U_i^*((1+\gamma)(w_i + \rho^*)) - U_i^*((1+\gamma)(w_i - w_i^*\Gamma)) \\ = \ & U_i^*((1+\gamma)w_i + (1+\gamma)\rho^*)) - U_i^*((1+\gamma)w_i - (1+\gamma)w_i^*\Gamma)) \\ = \ & K(1+\gamma)(\rho^* + w_i^*\Gamma) \end{aligned} \quad (47)$$

with K a constant. The last step asserts that the payoff difference is a linear function of $\rho^* + w_i^*\Gamma$ which, in particular, is independent of w_i.[27] This follows if full compliance

[27]Under full compliance everywhere in the continuation, under the rule $\tilde{\mathcal{T}}_i^*$, if $w_i \geq w_i^*$, a payoff

holds along the equilibrium path for all signal paths x^∞. In that case, continuation payoffs are linear in current wealth w_i will all increments wealth additively separable. Hence, w_i vanishes in (47).

Thus we have

$$\alpha_i^*(w_i, \tilde{\mathcal{T}}_i^*) = \bar{a} \;\; = \;\; a_i^c + [(\Gamma w_i^* + \rho^*) + \delta_c[K(1+\gamma)(\rho^* + \Gamma w_i^*)]]\,|F_a(\hat{x}^*|\bar{a})| \tag{49}$$

for all $w_i \geq w_i^*$. We conclude that $\tilde{\mathcal{T}}_i^*$ is an optimal compliance rule given α_i^*, starting starting from all wealth levels in $[w_i^*, \infty)$. The constructed rule does not vary in wealth when $w_i > w_i^*$. We have shown that

$$\tilde{\mathbb{P}}_i^* \left(x^\infty : \; \omega_{i\,s}^*(x^\infty) \geq w_i^*, \; \alpha_i^*(\omega_{i\,s}^*(x^\infty), \mathcal{T}_i^*) = \bar{a}, \; \forall\; s \geq t \;\middle|\; w_{it} \geq w_i^* \right) \; = \; 1$$

where $\tilde{\mathbb{P}}_i^*$ is the equilibrium probability on signal paths corresponding to $\tilde{\mathcal{T}}_i^*$.

By Lemma 1, the citizen's response in any period is unique, and by Remark 1 any optimal response α_i depends on w_i only through the compliance rule. This means that α_i^* is stationary. Now suppose there is an equilibrium compliance rule $\mathcal{T}_i^*$ such that $V(w_{it}) \neq 0$ for some $w_{it} \geq w_i^*$. At some present or future date the citizen must not be fully compliant with positive probability. Without loss of generality, consider w_{it} itself to satisfy $\alpha_i^*(w_{it}, \mathcal{T}_i^*) < \bar{a}$. Then, replacing $\mathcal{T}_i^*$ with $\tilde{\mathcal{T}}_i^*$ as a one shot deviation produces $\alpha_i^*(w_{it}, \tilde{\mathcal{T}}_i^*) = \bar{a}$ and also leads to states $w_{i\,t+s} \geq w_{it}$ regardless of signal realization. This leads to an immediate improvement in the authority's flow payoff, and produces future continuation states that allow for full compliance under some selection of a compliance rule. Successive forward iteration produces full compliance given the citizen's stationary response α^*. The deviation is therefore profitable. One concludes $V^*(w_{it}) = 0$ which implies

$$\mathbb{P}_i^* \left(x^\infty : \; \omega_{i\,s}^*(x^\infty) \geq w_i^*, \; \alpha_i^*(\omega_{i\,s}^*(x^\infty), \mathcal{T}_i^*) = \bar{a}, \; \forall\; s \geq t \;\middle|\; w_{it} \geq w_i^* \right) \; = \; 1,$$

completing the proof of Part (ii).

Turning to Part (i), since w_i^* satisfies (45) then $w_i < w_i^*$ implies either $\alpha_i^*(w_i, \mathcal{T}_i^*) < \bar{a}$ (the citizen is not fully compliant at w_i or the solution to (44) requires $\rho > R$. If the citizen is not fully compliant, then (11) holds, and reward must be maximal for larger

$U_i^*(w_i)$ is a discounted binary expansion

$$\begin{aligned} U_i^*(w_i) \;&=\; w_i + \frac{\delta}{2}\left[(1+\gamma)(w_i+\rho) + (1+\gamma)w_i\right] \\ &+ \frac{\delta^2}{4}\left[(1+\gamma)^2 w_i + (1+\gamma)\rho + (1+\gamma)^2(w_i+\rho) + (1+\gamma)^2 w_i + \rho + (1+\gamma)^2 w_i\right] \\ &+ \frac{\delta}{8}\left[\cdots\right] + \cdots \end{aligned} \tag{48}$$

Consequently, in any difference $U_i^*(w_i+\rho) - U_i(w_i)$ the w_i terms cancels.

than average signals. If $\rho > R$ is required to solve (44) then any solution achieving $\alpha_i^*(w_i, \mathcal{T}_i^*) < \bar{a}$ requires $\mathcal{T}^*(w_i, x) > \frac{\gamma}{1+\gamma} w_i$ for an open set of signals x. In that case if one of these signals is realized, future wealth falls below w_i. In words, full compliance can possibly be reached at w_i, but is not guaranteed to obtain in the future. Once again, (11) holds and

$$\mathbb{P}_i^* \left(x^\infty : \; \alpha_i^*(\omega_{i\,s}^*(x^\infty), \mathcal{T}_i^*) = \bar{a}, \; \forall \; s \geq t \;\middle|\; w_{it} < w_i^* \right) \; < \; 1.$$

This concludes the proof. ∎

Proof of Proposition 4. Clearly the result holds trivially if $w_{i0} \geq w_i^*$. So suppose that $w_{i0} < w_i^*$, indeed, suppose $w_{i0} = 0$. This is the lowest state and is a reflective barrier since the citizen can go higher but not lower than zero. Given the equilibrium, observe from prior results that $\mathcal{T}_i^*(w, x) < 0$ for all $x > \alpha^*(w, \mathcal{T}_i^*)$. The citizen is rewarded for high signals. Recall the threshold $\bar{x}(w)$ for which the reward is maximal: $\mathcal{T}_i^*(w, x) = -R < 0$ for $x > \bar{x}_i(w)$. The maximal reward threshold $\bar{x}_i(w)$ from Proposition 3 is bounded over all $w \in [0, w_i^*]$. Letting $\bar{x}^* = \sup\{\bar{x}_i(w) : \; w \in [0, w_i^*]\}$, it therefore follows $\bar{x}^* < \infty$.

Starting from 0 there is a finite time T and signal sequence x^T with $x_t \geq \bar{x}_i^*$ of high enough rewards such that $\omega_{i\,T}(x^T) \geq w_i^*$. Since $\alpha^*(w, \mathcal{T}_i^*)$ is bounded between a_i^c and $\bar{a}$, there is $\varepsilon > 0$ such that $(1 - F(\bar{x}^* | \alpha^*(w, \mathcal{T}_i^*)) > \varepsilon$ (i.e. the probability of maximal reward signals is bounded below by ε). Thus the probability of reaching w_i^* from zero is no lower than ε^T.

Consider the infinite signal path x^∞. The probability of $x_{t+s} \geq \bar{x}^*, \; s = 0, \ldots, T-1$ for infinitely many t is one. Applying Proposition 3, the interval $[w_i^*, \infty)$ is an absorbing wealth region, we conclude

$$\mathbb{P}_i^* \left(x^\infty : \; \lim_{t \to \infty} \omega_{it}(x^\infty) > w_i^* \;\middle|\; w_{i0} = 0 \right) \; = \; 1$$

∎

Proof of Proposition 5. As shown earlier, all equilibria are payoff equivalent to the equilibrium in Proposition 3 once w_i^* has been reached. Equation (49) characterizes w_i^*. From the equation, w_i^* is decreasing in the citizen's discount factor δ_c, in growth rate γ, and in the location of the citizen's ideal behavior a_i^c.

Next observe the first term in the authority's Euler equation (38) is strictly negative and this term is strictly negative and increasing in absolute value as $\delta_A \to 1$. The threshold w_i^* at which corner solution for maximal reward is reached is therefore decreasing in δ_A.

Finally, applying Proposition 3, since $\bar{x}_i(w_i) < \alpha_i^*(w_i, \mathcal{T}_i^*)$, the maximal reward R is always paid out at least half the time along the transition path when $w_i < w_i^*$. In the worst case, given $w_{i0} = 0$, let

$$s(R) = \min\{t : \; R(1+\gamma)^t \geq w_i^*\}.$$

The integer $s(R)$ is the number of steps needed to reach w_i^* from zero. Clearly, $s(R)$ is decreasing in R. A signal path x^∞ contains a string $s(R)$ of positive signal realizations almost surely. Since the first hitting time of string $s(R)$ in x^∞ is decreasing in R, stopping time $T(x^\infty)$ is pointwise decreasing in R.

Applying the comparative statics to the stopping time defined in (18), it follows that $T_i(x^\infty)$ is decreasing $\delta_c, a_i^c, \gamma, \delta_A$, R, and by construction initial wealth w_{i0}. ∎

Proof of Proposition 6. I show the compliance maximizing switching point for an impatient authority is precisely $x^* = \alpha_i^*(w_{it}, \mathcal{T}_i)$, the anticipated act of the citizen. The anticipated action is also the expected value of the signal conditional on the citizen choosing $\alpha_i^*(w_{it}, \mathcal{T}_i)$. To verify this for any state w_{it} in which the citizen is not fully compliant, her Euler equation is

$$a_i \;=\; a_i^c \;+\; [(w_i + P + R) \;+\; \delta_c(U_i^*((1+\gamma)(w_i+R)) - U_i^*(0) \;) \;] \times |F_a(x^*|a_i)| \qquad (50)$$

The term $U_i^*((1+\gamma)(w_i+R)) - U_i^*(0)$ is the difference in continuation values between receiving a maximal reward and incurring a maximal punishment. Totally differentiating a_i with respect to x^* yields

$$\frac{\partial a_i}{\partial x^*} \;=\; [(w_i + P + R) \;+\; \delta_c(U_i^*((1+\gamma)(w_i+R)) - U_i^*(0) \;) \;] \times |F_{aa}(x^*|a_i)\frac{\partial a_i}{\partial x^*} + f_a(x^*|a_i)|.$$

Solving for $\frac{\partial a_i}{\partial x^*}$ yields

$$\frac{\partial a_i}{\partial x^*} \;=\; \frac{[(w_i + P + R) \;+\; \delta_c(U_i^*((1+\gamma)(w_i+R)) - U_i^*(0) \;) \;] \times |f_a(x^*|a_i)|}{1 + [(w_i + P + R) \;+\; \delta_c(U_i^*((1+\gamma)(w_i+R)) - U_i^*(0) \;) \;] \times |F_{aa}(x^*|a_i)|}$$

Taking $x^* = \alpha_i^*(w_i, \mathcal{T}_i^*)$, one obtains $\frac{\partial \alpha_i^*}{\partial x^*}\Big|_{x^* = \alpha_i^*(w_i, \mathcal{T}_i^*)} = 0$. This establishes $x^* = \alpha_i^*(w_i, \mathcal{T}_i^*)$ as a compliance-maximizing solution for the authority. Thus until full compliance is achievable all punishments and rewards will be maximal with the switching point between punishment and reward is the anticipated behavior. ∎

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