The Great Accretion and the Great Depression*

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Abstract

The Second Industrial Revolution sparked a wave of new products and industrial processes, fueling an optimistic Roaring Twenties. But did excitement about technological progress contribute to an over accumulation of investment, despite a slowdown in new product development and satiated demand during the 1920s? And, was this over investment worsened by continuous process innovation? Could these factors have played a role in triggering the Great Depression? To explore these questions, a macroeconomic model that incorporates both process and product innovation is proposed. Proof-of-concept simulations are performed to assess whether these factors can help explain the Great Depression. The answer is yes.

Keywords: Great Depression, Over Accumulation, Process Innovation, Product Innovation, Rational Exuberance, Roaring Twenties, Satiation, Second Industrial Revolution, Technological Progress

JEL Nos: E13, E22, E24, E32, N12, 040

* *First Draft*: The latest version of the paper is here.

1 Opening

The Roaring Twenties was a period of rapid economic growth and unprecedented prosperity in the United States. Driven by the Second Industrial Revolution, this era saw transformative developments such as electrification, the rise of the automobile and airplane industries, and the emergence of petrochemicals. It was a time of optimism marked by sweeping cultural shifts—including the Art Deco movement, the Jazz Age, and the first sexual revolution—as well as major economic expansion.

This period ended abruptly end 1929. Figure 1 shows the precipitous drop in employment, GDP, and manufacturing investment that occurred. Investment dropped the most, followed by output, while employment declined the least. By 1933, output had fallen to 71% of its 1929 level, employment stood at 82%, and investment had plummeted to just 37%. Notably, while GDP and investment were rising throughout the 1920s, employment was constant, and labor share of income falling.

The causes of the Great Depression remain debated. Commonly cited explanations include poor monetary policy, widespread banking failures, stock market crashes, and technological back sliding. An alternative hypothesis is entertained here: Could the Great Depression, *at least in part*, be the result of exuberant beliefs during the 1920s–specifically, a massive overinvestment in plant and equipment based on expectations of an ever-increasing consumer demand?

To explore this question, a model of the 1920s—referred to here as the Great Accretion—is developed. The analysis draws on some ideas presented in Szostak (1995) and Yorukoglu (Undated). They argue that the 1920s was a period of comparatively slow product innovation. In contrast, rapid process innovation made it possible for businesses to produce the existing range of products while employing a much less labor. The combination of satiated demand, due to slow product innovation, and rapid process innovation set the stage for Great Depression.

This concept is formalized using a framework introduced by Yorukoglu (2000), which incorporates both product and process. The central insight is that different economic periods vary in their dominant form of innovation: at times, the introduction of new goods (product innovation) dominates, while at other times, reductions in production costs (process innovation) are more prominent. Consumption adjusts accordingly, operating along either the extensive *or* intensive margins depending on which type of innovation prevails. When product innovation outpaces process innovation—resulting in a wide variety of goods relative to income—consumers move along the extensive margin, as acquiring new goods provides greater utility than consuming more of existing ones. In this situation, aggregate consumption is high. Additionally, prices are competitively determined and equal marginal cost. Conversely, when process innovation dominates and income grows faster than product variety, consumption

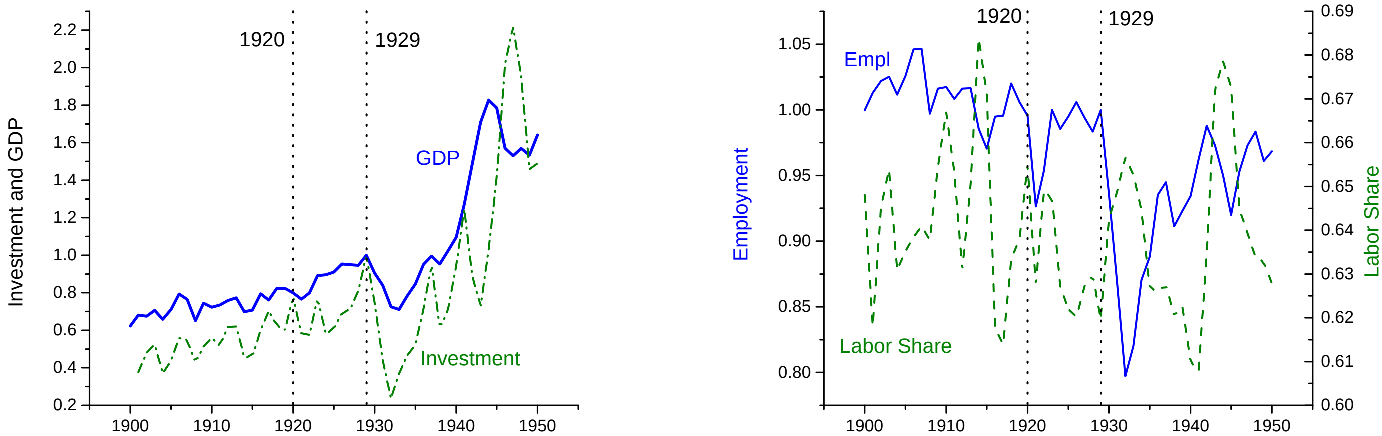

Figure 1: The left panel shows chain-indexed real investment for manufacturing and real GDP per capita, while employment per capita and labor share of income are displayed in the right one. The numbers for employment, GDP, and investment are normalized so that 1929=1.0. Source: *Historical Statistics* (2006), Series Aa7, Ba471, Ca11, Dd721, and Charpe et al. (2020), Figure 1a.

shifts to the intensive margin. Aggregate consumption is lower, relative to when consumption proceeds along the extensive margin. Households purchase all available goods, and labor effort may decline. Here, prices exceed marginal cost. The equilibrium market structure is *endogenous* and has a significant influence on the behavior of consumption, investment, GDP, and labor.

Suppose that during the 1920s, despite emerging signs of a slowdown in product innovation, firms expected a return to a future marked by strong product innovation and invested accordingly. As the decade came to a close, however, it became clear that this expectation would not be realized. Process innovation continued, reducing the demand for labor in producing existing goods. This divergence between optimistic expectations and actual developments may have triggered a sharp economic correction—ultimately contributing to the onset of the Great Depression.

The narrative unfolds in three stages. First, a perfect-foresight simulation is computed that replicates the key quantitative features of the 1920s Great Accretion. Here, despite a slowdown in product innovation during the 1920s, individuals anticipate a strong future rebound, which drives a boom in GDP and investment, while process innovation keeps labor supply constant (recall Figure). Second, an unexpected halt in future product innovation in 1929 leads to drops in GDP, investment, and labor, closely matching patterns from the late 1920s and early 1930s. Now, there is both the Great Accretion and the onset of the Great

Depression. Third, the risk of a stall is incorporated into a rational exuberance framework, where optimism grows until a stall occurs, again triggering a sharp economic crash resembling the onset of the Great Depression. These proof-of-concept simulations are intended to be illustrative and not conclusive. The hypothesis here is presented as a *potential trigger* for the Great Depression, alongside other contributing factors discussed after the main analysis.

2 Stylized Facts

Product Innovation

The 1920s was a consumer goods revolution when many new products were introduced. Since businesses trademark their products, the trend in trademarks reflects the arrival of new products. Figure 2, left panel, shows the evolution of trademarks per capita. As can be seen, there was a burst of trademark activity during the 1920s that began to taper off at decade's end. Businesses also use design patents to protect from others emulating the appearance of their product. Therefore, the trend in design patent applications also provides information on the arrival of new goods. There was a surge of design patent applications in the early part of the 20th century reaching a peak in the 1921. One of the key innovations of the Second Industrial Revolution was the automobile, which became a significant new product in the marketplace. Auto registrations per capita grew continuously until 1929 and then flattened out until 1946. This is shown in Figure 3, left panel. Since registrations are a stock concept, the change in registrations is shown in the right panel. This peaks in 1925, suggesting that a saturation point had been reached. Likewise, factory sales grew until 1929, slumped (left panel), and didn't regain their momentum before 1946.

The automobile brought about suburbanization. As can also be seen from Figure 4, the number of housing units started and the value of new construction rose in the first part of the 20th century cresting in 1925. It then slumped and didn't recover until 1950. Again, perhaps the housing market had reached a saturation point in the 1920s.

The electric age started with the opening of Niagara Falls in 1903. Electrification occurred in the United States. It is evident from Figure 5, left panel, that by 1929 electricity accounted for 78 percent of the horsepower used in factories and remained there until the mid 1950s. Likewise, 68 percent of houses were already electrified in 1929. The right panel suggests that the flow of electrification for both factories and homes had slowed down before 1929. Spending on construction by private utilities for electric light and power peaked in 1926 and didn't surpass this level until 1947–see the left panel. Expenditure on gas crested in 1927 and did not fully recover before 1947. Total spending by private utilities–electric, gas, petroleum, railroads, and telephone and telegraph–reached a high point in 1927, a value only exceeded after 1947. Again, a case can be made that these markets began to be saturated in the late

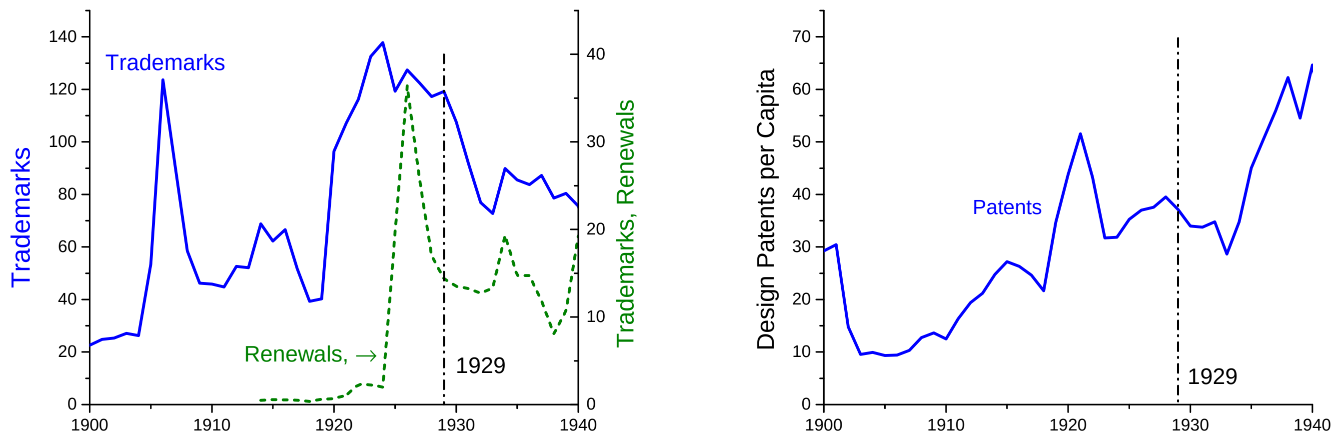

Figure 2: Trademarks and design patent applications, 1900-1940. Source: *Historical Statistics* (2006), Series Aa7, Cg28, Cg108, and Cg109

1920s.

Process Innovation

Unlike product innovation, process innovation persisted undaunted throughout the 1920s and 1930s. In 1913 Henry Ford introduced the moving assembly line at his High Park plant in Detroit. This symbolizes the process innovation that occurred in the Second Industrial Revolution. Figure 6, left panel, shows the rapid decline in the number of manhours that it took to make a car. The assembly line spread to other industries such as the manufacturing of consumer durables. According to Fabricant (1942) (pp 323 and 324), it took 4 times as much labor to produce a lamp in 1920 versus 1929, 2.4 times as much to produce tires, tubing, and other rubber goods in 1919 compared with 1929, while the amount of required labor to produce washing and ironing machines dropped by 30 percent just between 1927 and 1929. Additionally, by 1930 55 percent of households had a washing machine (which compares with only 75 percent by 1990, sixty years later), rendering the effects of process innovation even more severe. Process innovation also occurred in the production of petrochemicals and the manufacturing of steel–see Figure 6, right panel. In the petrochemical industry batch production techniques were replaced by continuous flow techniques. So, process innovation became widespread in manufacturing. Process innovation combined with saturated consumer goods markets spelled trouble for employment. To relay Freeman and Soete (1997) (p.139),

> (t)his triumph of mass production and flow production was, however, achieved only after a very painful worldwide structural adjustment in the 1920s and 1930s.

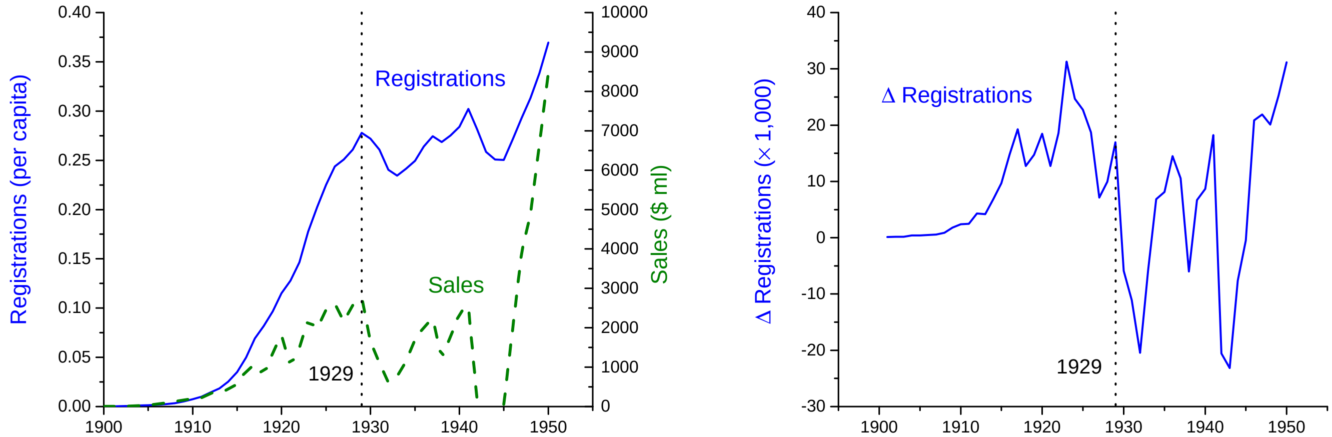

Figure 3: Automobiles. Left panel. Automobile registrations and factory sales, 1900-1950. Source: *Historical Statistics* (2006), Series Aa7, Df340, and Df344. Right panel. The change in per capita registrations, derived from Series Df340 and multiplied by 1,000.

> Mass production capacity for automobiles and other goods had outstripped the absorptive capacity of the (then) very limited market for the new goods.

Around the mid-1920s, the number of active business enterprises declined–see Figure 7. This decline may have been driven by the rationalization of business practices resulting from process innovation as well as slowdown in the introduction of new products and processes.

Evidence of overoptimism during the 1920s can be seen in several key trends. First, both GDP and investment steadily increased from 1924 to 1929; recall Figure 1. This growth suggests that investors did not anticipate an impending crash—people typically don't invest heavily when they expect a downturn. Second, economists generally agree that the stock market reflects collective expectations about the future. The dramatic rise in stock prices from 1921 to 1929 (see Figure 8) indicates that public sentiment during this period was highly optimistic. The climb in the price/earnings ratio, in particular, suggests that investors were expecting future earnings to be high relative to current earnings. Perhaps this was due to buoyant expectations about the profitability of oncoming new products. Third, business confidence was also reflected in the surge of initial public offerings (IPOs) and mergers and acquisitions (M&As) leading up to 1929. Notably, the market highs of 1929 would not be reached again until 1950.

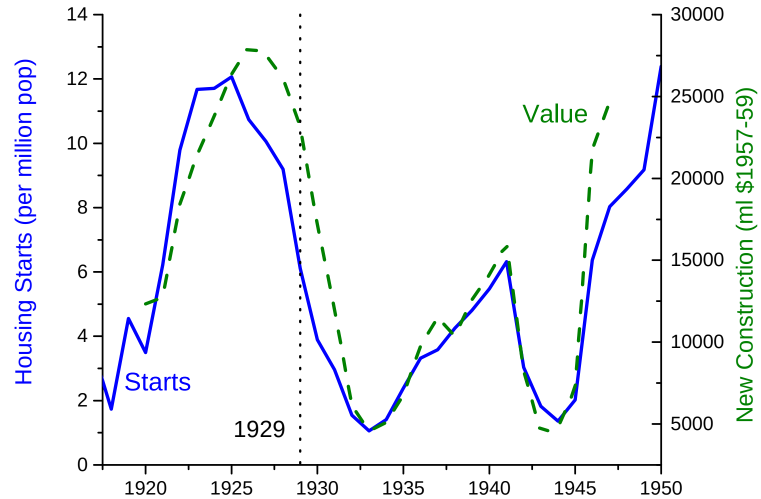

Figure 4: The number of housing units started per million population, 1900-1950, and the value of new construction in million of $1957-59, 1920-1947. Source: *Historical Statistics* (2006), Series Aa141, Dc23, and Dc510.

3 Setting

At the heart of the model is a representative individual who consumes a variety of final products. Consumption has both an extensive and intensive margin. Sometimes there is an abundance of varieties and the individual will choose not to consume all of them due to a lower bound on the consumption of a product. Other times the person would like to consume an extra variety but cannot because of a lack of availability. What varieties are available or not depends on the state of technology in the economy. Each variety of final consumption goods is produced by a monopolistic competitor. These firms produce final output using an intermediate goods. The profits they make are distributed back to the representative individual.

The person supplies capital and labor to the economy on a competitive factor market, which are hired by firms to produce intermediate inputs. The capital income, labor income, and profits earned by the individual are used to buy final goods and to invest in capital. Capital is also produced using intermediate goods.

There are two sources of exogenous technological progress. First, there is process innovation that allows the cost of final goods to decrease over time. Second, there is product innovation whereby new varieties of final goods are introduced. The state of the economy hinges on whether process innovation, to date, has outpaced product innovation, to date, or vice versa.

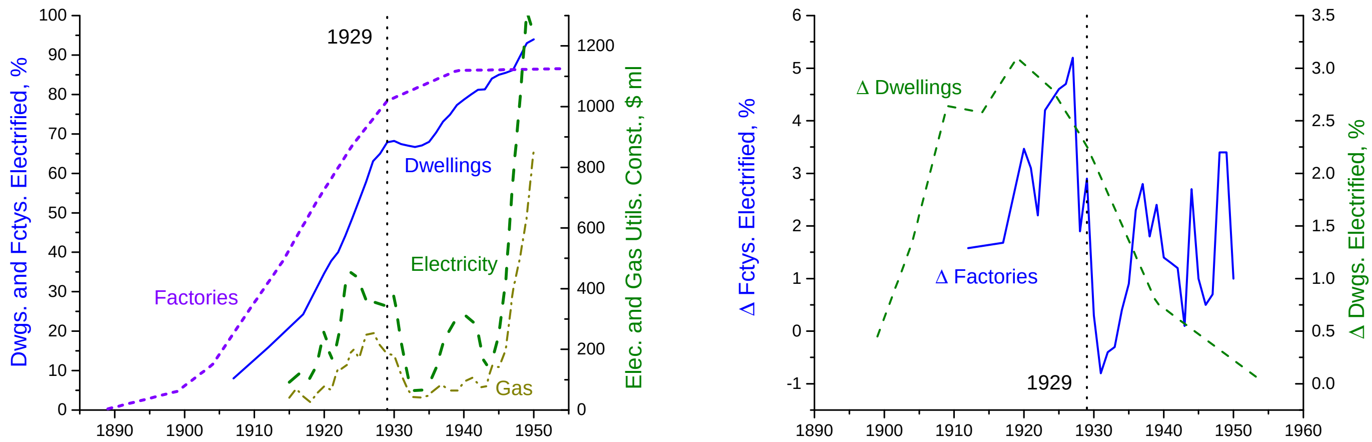

Figure 5: Left panel. The percentage of horsepower for mechanical drive in factories supplied by electricity, 1889-1954; the percentage of dwelling that were electrified, 1907-1950; and the value of new construction by private utilities for electric and gas in millions of dollars, 1907-1950. Right panel. The change in mechanical drive and dwellings powered by electricity, as computed from the two associated diffusion curves in the left panel. Sources: David (1989) and *Historical Statistics* (1975, 2006), Series S71, Dc323, and Dc324.

The analysis that follows focuses on a symmetric equilibrium, which can endogenously fall into one of three distinct zones:

1. *Extensive Margin Zone (Zone 1)*: When product innovation outpaces process innovation, the economy generates more product varieties than a consumer chooses to purchase. As a result, consumption expands only along the extensive margin—that is, by increasing the number of varieties consumed. In this zone, with excess varieties, perfect competition prevails. Things here work as if in the standard neoclassical growth model.

2. *Intensive Margin Zone (Zone 3)*: When process innovation dominates, individuals consume all available varieties, and any change in consumption occurs along the intensive margin—by increasing the quantity consumed of each variety. Here the monopolistic competitors charge a fixed markup and the economy behaves like the standard model with monopolistic competition. Since the price of consumption is higher in this zone relative to the extensive margin zone, real wages are lower. Additionally, aggregate consumption tends to be lower in this zone, because consuming along the intensive margin lowers the marginal of consumption as opposed to consuming along the extensive one. These factors reduce the incentive to work relative to the extensive margin zone.

3. *Shackled Margins Zone (Zone 2)*: In this case, consumers are constrained on both mar-

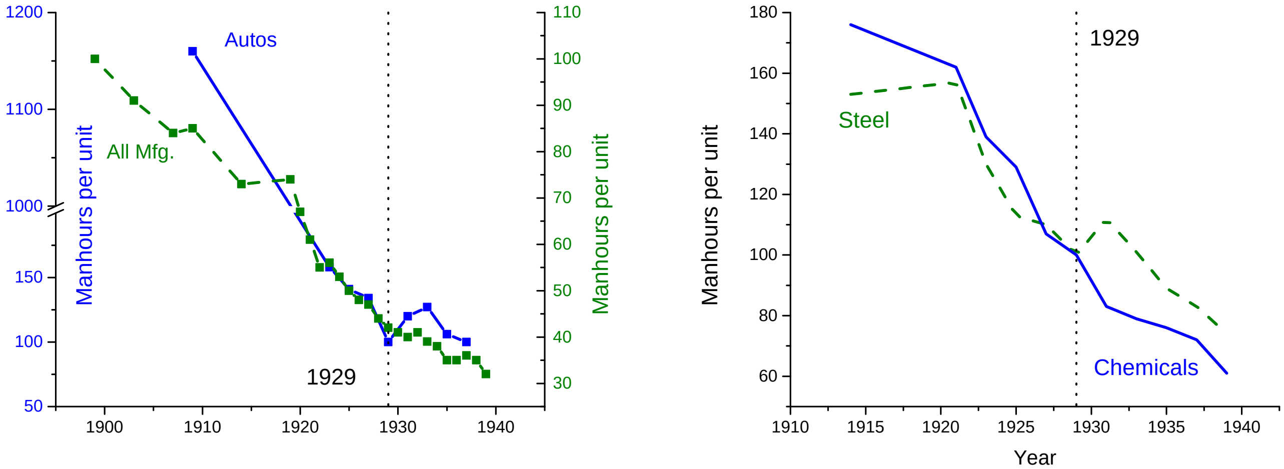

Figure 6: Left panel. Process innovation in autos and all manufacturing. Source: Fabricant (1942) (pp. 325 and 331). Right panel. Process innovation in chemical products and blast Furnace and steel-mill production, 1914-1939. Fabricant (1942) (pp. 304 and 316).

gins. They neither endogenously expand the range of varieties consumed nor the quantity per variety. Instead, only investment and labor supply adjust in response to economic conditions. Now each monopolistic competitors sets their price so that consumers are indifferent between purchasing an extra variety or not. Here profits can be high and consequently labor share of income low. The markup varies with the state of the economy. The high price of goods lowers the real wage and consequently the incentive to work, relative to the extensive margin zone. Here continual process innovation leads to fall in labor supply given shackled consumption. It also stimulates investment due to the fact that the marginal product of capital is increasing. This allows employment to decrease. This zone plays an important role in replicating the facts characterizing the Great Accretion.

So, the economy behaves differently depending on the zone it is in.

3.1 Individuals

In each period t an infinitely-lived individual consumes N_t varieties of final goods out of a possible $\mathfrak{N}_t$ varieties. Increases in $\mathfrak{N}_t$ over time will reflect product innovation. The period-t consumption of variety j is denoted by c_{jt}, where there is a lower bound on consumption represented by $\mathfrak{c}$. Yorukoglu (2000) presents suggestive evidence of the presence of lower bounds on consumption. The person supplies labor in the amount l_t. The individual's lifetime

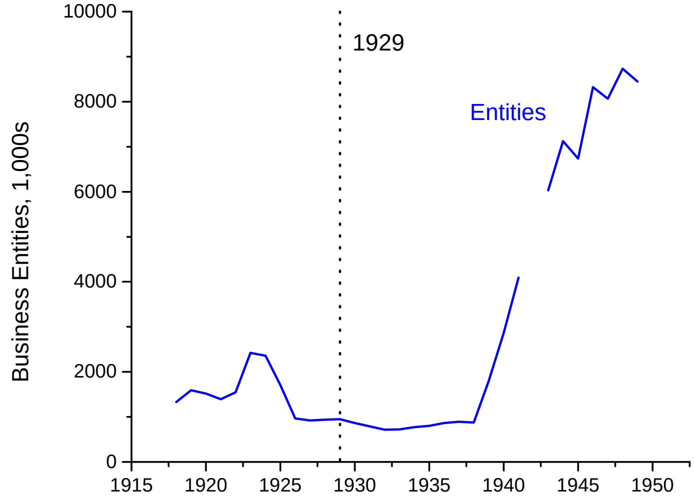

Figure 7: Active corporations, partnerships, and proprietorships, 1919-1949. Source: *Historical Statistics* (2006), SeriesCh1.

utility function is given by

$$\sum_{t=0}^{\infty} \beta^t \left[\alpha \ln \left(\int_0^{N_t} c_{jt}^{\theta} dj \right)^{1/\theta} - (1-\alpha) \frac{l_t^{1+\chi}}{1+\chi} \right], \text{ with } 0 < \alpha, \theta < 1 \text{ and } \chi > 0, \tag{1}$$

where $0 < \beta < 1$ is the discount factor.

The person has three sources of income in period t. The first is labor income, $w_t l_t$, where w_t is the wage rate. The second is capital income, $r_t k_t$, where r_t is the rental rate on capital and k_t is the capital stock that the person owns. Capital depreciates at the rate δ. The third is the profits, π_t, accruing from the portfolio of firms that the individual owns. The individual can use their income to purchase variety j at the unit price p_{jt} or to acquire capital for next period, k_{t+1}, at a unit price of one. The person's period-t budget constraint reads

$$\int_0^{N_t} p_{jt} c_{jt} dj + k_{t+1} - (1-\delta) k_t = w_t l_t + r_t k_t + \pi_t. \tag{2}$$

The person's problem is to choose $c_{jt} \in \{0, [\mathfrak{c}, \infty)\}, k_{t+1}, l_t$, and $N_t \leq \mathfrak{N}_t$ to maximize (1) subject to (2). Let $\beta^t \lambda_t$ be the Lagrange multiplier attached to the budget constraint (2). The

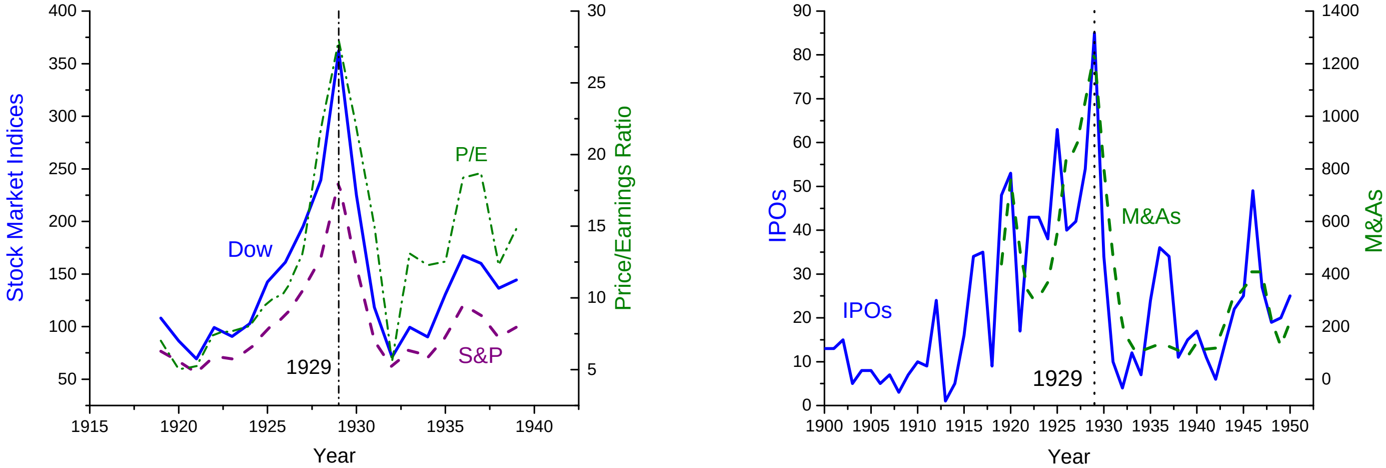

Figure 8: Irrational exuberance. Left panel, The September Dow and S&P, 1919-1939. Source: *Historical Statistics* (2006), Table Cb52-54. Also plotted is the cyclically adjusted June P/E ratio, taken from Shiller (2006). Right Panel, IPOs, 1900-1950, and M&As, 1919-1950. Sources: Jovanovic and Rousseau (2001) and *Historical Statistics* (1975, 2006), Series S71, Dc323 and Dc324.

first-order conditions for c_{jt} and N_t are given by (3) and (4):

$$\alpha \frac{c_{jt}^{\theta-1}}{\int_0^{N_t} c_{jt}^{\theta} dj} \leq \lambda_t p_{jt} \text{ (with equality if } c_{jt} > \mathfrak{c}) \tag{3}$$

and

$$\alpha \frac{1}{\theta} \frac{1}{\int_0^{N_t} c_{jt}^{\theta} dj} c_{N_t,t}^{\theta} \geq \lambda_t p_{N_t,t} c_{N_t,t} \text{ (with equality if } N_t < \mathfrak{N}_t). \tag{4}$$

These two first-order conditions govern consumption along the intensive and extensive margins. The lefthand side of (3) is the marginal benefit from consuming an extra unit of variety j in period t, while the righthand side is the marginal cost. When the lefthand side is less than the righthand side, marginal benefit is less than marginal cost, and consumption will be at its lower bound. Similarly, when the lefthand side of (4) exceeds the righthand side, the marginal benefit of consuming an extra variety is larger than its marginal cost and the number of varieties consumed will be at the upper bound.

Some interesting features obtain from these two first-order conditions, starting with the lemma below.

Lemma 1. *(Three Consumption Zones) In a symmetric equilibrium where $p_{jt} = p_t$ for all j, either $c_{jt} = \mathfrak{c}$ and/or $N_t = \mathfrak{N}_t$.*

Proof. Both (3) and (4) cannot hold with equality. To the contrary suppose that they do. Divide (3) into (4) to obtain $1/\theta = 1$, which is the desired contradiction. □

Remark. (Only One Interior Solution) The lemma implies that there does not exist an interior solution with $c_{jt} > \mathfrak{c}$ and $N_t < \mathfrak{N}_t$. Hence, consumption cannot operate simultaneously along both the intensive and extensive margins.

The upshot is that in a symmetric equilibrium ($p_{jt} = p_t$ for all j) there are three possible zones for consumption. These zones play an important role in the subsequent analysis and are enumerated now.

Zone 1 (Extensive Margin)

Here $c_{jt} = \mathfrak{c}$ and $N_t < \mathfrak{N}_t$; that is, the first-order condition (3) is slack while (4) holds with equality. When possible, an individual would always prefer to move along the extensive margin as opposed to the intensive one. To understand why, imagine giving the person an extra $p_t\mathfrak{c}$ in income to spend on consumption. Would they prefer to boost their spending on each of the varieties they are currently consuming by $p_t\mathfrak{c}/N_t$ or purchase an extra variety for the same cost? The answer is that they would prefer to consume an extra variety, as Figure 9 illustrates. This transpires because consumption along the intensive margin suffers from diminishing marginal utility because the utility function for a variety is strictly concave. Consumption along the extensive margin adds, in a linear fashion, the level of utility from the extra variety.

Zone 2 (Shackled Margins)

Here $c_{jt} = \mathfrak{c}$ and $N_t = \mathfrak{N}_t$. Here the first-order condition (3) is slack. As is discussed in Section 3.2, monopolistic competitors will set their prices so that (4) holds with equality at the corner $N_t = \mathfrak{N}_t$. In this zone aggregate consumption spending is simply $p_t\mathfrak{N}_t\mathfrak{c}$.

Zone 3 (Intensive Margin)

Here $c_{jt} \geq \mathfrak{c}$ and $N_t = \mathfrak{N}_t$; that is, the first-order condition (3) holds with equality and (4) is slack. Using (3) it can be seen that the demand for variety j is given by

$$c_{jt} = D(p_{jt}) \equiv \left[\frac{\alpha}{\lambda_t p_{jt} \int_0^{\mathfrak{N}_t} c_{it}^{\theta} di}\right]^{1/(1-\theta)}. \tag{5}$$

To complete the individual's problem, the first-order condition for capital accumulation is

$$\lambda_t = \beta\lambda_{t+1}[r_{t+1} + (1-\delta)] = \lambda_{t+j}\beta^j \prod_{i=1}^{j}[r_{t+i} + (1-\delta)], \tag{6}$$

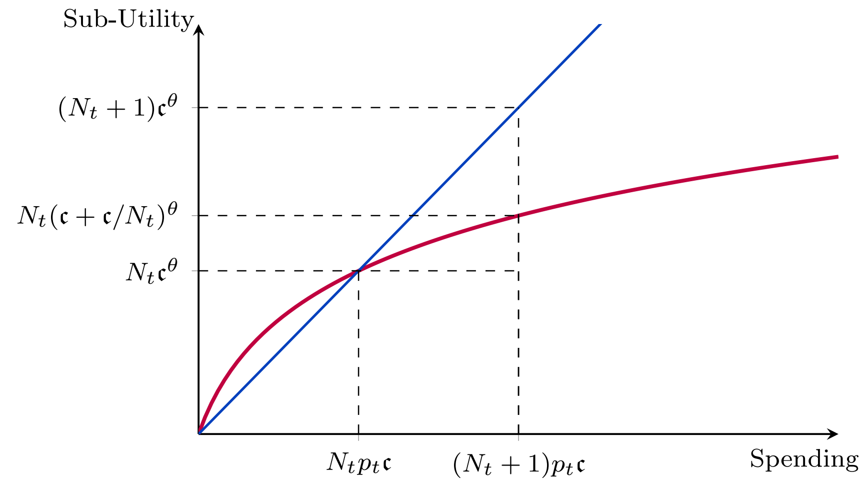

Figure 9: Extensive vs intensive margin consumption. In a symmetric equilibrium the utility from consumption can be written as $(1/\theta)\ln(N_t c^\theta)$. Consider the sub-utility function, $N_t c^\theta$, inside of the ln function. By consuming an extra variety sub-utility moves up from $N_t\mathfrak{c}^\theta$ to $(N_t + 1)\mathfrak{c}^\theta$, while by using the same amount of spending to purchase a little more of all N_t varieties currently consumed sub-utility is boosted from $N_t\mathfrak{c}^\theta$ to $N_t(\mathfrak{c} + 1/N_t)^\theta$. The diagram shows that the former movement is bigger, due to the fact that c^θ is concave in c.

where λ_t is the period-t marginal utility of income. As usual, this equation sets the marginal cost of investing a unit of capital, λ_t, equal to its marginal benefit, $\beta\lambda_{t+1}[r_{t+1}+(1-\delta)]$. Observe that the Euler equation can be cascaded forward to period $t+j$, where $\beta^j \prod_{i=1}^{j}[r_{t+i} + (1-\delta)]$ is the discounted value of return on investment from period t to $t+j$. Last, the first-order condition for labor reads

$$\lambda_t w_t = (1-\alpha)l_t^\chi. \tag{7}$$

In standard fashion, this equation sets the marginal return from working, $\lambda_t w_t$, equal to the marginal disutility of effort, $(1-\alpha)l_t^\chi$.

3.2 Monopolistic Competitors

Each variety of the final good is produced by a monopolistic competitor. Specifically, monopolistic competitor j produces variety j in quantity o_j using intermediate goods in the quantity m_j according to the linear production function

$$o_{jt} = m_{jt}.$$

The price of intermediate good j is normalized to be one. The price, p_{jt}, at which the monopolistic competitor sells their variety depends on which of the three zones the economy is in.

Zone 1 (Extensive Margin)

In this zone the number of varieties consumed by an individual is less than the total number available; $c_{jt} = \mathfrak{c}$ and $N_t < \mathfrak{N}_t$. In symmetric equilibrium the monopolistic competitor has no market power. Thus, the price for a variety must equal its marginal cost, so that

$$p_{jt} = 1, \text{ for all } j. \tag{8}$$

This transpires because from a consumer's viewpoint all varieties are completely substitutable. If a producer tries to charge a higher price, consumers will just switch to another variety.

In a symmetric equilibrium the first-order condition for consumption along the extensive margin (4) simplifies to

$$\alpha \frac{1}{\theta} \frac{1}{N_t \mathfrak{c}} = \lambda_t, \tag{9}$$

where use has been made of (8). This allows the efficiency condition for labor (7) to be rewritten as

$$\frac{\alpha}{\theta} \frac{1}{N_t \mathfrak{c}} \times w_t = (1 - \alpha) l_t^{\chi}. \tag{10}$$

Here w_t is the real wage, because $p_t = 1$, and $(\alpha/\theta)/(N_t\mathfrak{c})$ is the marginal utility of consumption along the extensive margin. Familiar income and substitution effects will be at play here. Shifts in the number of varieties consumed, N_t, operate as an income effect, while changes in the real wage, w_t, work as a substitution effect.

Zone 2 (Shackled Margins)

Here an individual would like to consume more varieties but can't because they are in limited supply; i.e., $c_{jt} = \mathfrak{c}$ and $N_t = \mathfrak{N}_t$. The monopolistic competitor charges the highest price possible. This is the price at which the consumer is indifferent between dropping the variety, and consuming more of the remaining varieties in their consumption bundle, or retaining the last variety. This can be done because, at the competitive price, $p = 1$, the loss in utility from dropping a variety exceeds the gain in utility from using the money to consume more of the other varieties–recall Figure 9. The monopolist seizes all of this surplus. From (4) it is easy to deduce that the pricing condition in a symmetric equilibrium is

$$p_{jt} = p_t = \frac{\alpha}{\theta} \frac{1}{\mathfrak{N}_t \mathfrak{c}} \frac{1}{\lambda_t}, \text{ for all } j. \tag{11}$$

This pricing condition is instrumental for generating the fall in labor share of income observed in the 1920s–recall Figure 1, right panel.

The term $(\alpha/\theta)/(\mathfrak{N}_t\mathfrak{c})$ is the marginal utility of consumption along the extensive margin, while λ_t is the marginal utility of income. So, the righthand side is the amount of income that a person is willing to give up in order to get an extra variety (or the marginal rate of substitution between income and a variety). This price can be higher than in the standard monopolistic competition model. This happens when the marginal utility from spending an extra dollar along the extensive consumption margin, at the price p_t, exceeds that from spending the dollar along intensive one, at the standard monopolistic competitive price, as is explained in Section 3.5.

The efficiency condition for labor (7) now appears as

$$\frac{\alpha}{\theta}\frac{1}{\mathfrak{N}_t\mathfrak{c}} \times \frac{w_t}{p_t} = (1-\alpha)l_t^{\chi}, \tag{12}$$

where use has been made of the pricing condition (11) to solve out for the marginal utility of income, λ_t, in (7). Now, w_t/p_t is the real wage while the marginal utility of consumption is $(\alpha/\theta)/(\mathfrak{N}_t\mathfrak{c})$. By comparing (10) with (11) it appears that labor will be lower in Zone 2 relative to Zone because $p_t > 1$, at least when other things are equal (i.e., assume that Zone-1 N_t and w_t equal Zone-2 $\mathfrak{N}_t$ and w_t). In Zone 2 the real wage is lower.

Zone 3 (Intensive Margin)

Once again an individual would like to consume more varieties but can't because they are in limited supply; however, they have the wherewithal to consume more than the lower bound for each of the available varieties; i.e., $c_{jt} > \mathfrak{c}$ and $N_t = \mathfrak{N}_t$. Again, the monopolistic competitor has some market power. Monopolistic competitor j will choose the price p_{jt} to maximize their profits, π_{jt}, as given by

$$\pi_{jt} = \max_{p_{jt}}\{p_{jt}D(p_{jt}) - D(p_{jt})\},$$

where the demand for the variety, $D(p_{jt})$, is given by (5). The associated first-order condition is

$$D(p_{jt}) + pD'(p_{jt}) = D'(p_{jt}),$$

so that marginal revenue equals marginal cost (which is one). This implies that

$$p_{jt} = \frac{1}{pD(p_{jt})/D'(p_{jt}) + 1} = \frac{\varepsilon}{\varepsilon - 1},$$

where ε is the price elasticity of demand. From (5) the price elasticity of demand is $-1/(1-\theta)$ delivering the familiar condition

$$p_{jt} = \frac{1}{\theta}. \tag{13}$$

Table 1: Synopsis

Zone 1 (Extensive Margin)	**Zone 2 (Shackled Margins)**	**Zone 3 (Intensive Margin)**
Consumption		
$c = \mathfrak{c}$	$c = \mathfrak{c}$	$c > \mathfrak{c}$
$N < \mathfrak{N}$	$N = \mathfrak{N}$	$N = \mathfrak{N}$
Prices and Profits		
$p = 1$	$p > 1$	$p = 1/\theta > 1$
$\pi = 0$	$\pi = (p-1)\mathfrak{N}\mathfrak{c} > 0$	$\pi = (1/\theta - 1)\mathfrak{N}c > 0$
Labor Supply		
$l = \Big(\{\alpha/[(1-\alpha)\theta N\mathfrak{c}]\}$	$l = \Big(\{\alpha/[(1-\alpha)\theta\mathfrak{N}\mathfrak{c}]\}$	$l = \Big(\{\alpha/[(1-\alpha)\mathfrak{N}c]\}$
$\times w/1\Big)^{1/\chi}$	$\times w/p\Big)^{1/\chi}$	$\times w/(1/\theta)\Big)^{1/\chi}$

Note on Labor Supply: The real wages in each zone, $w/1$, w/p, and $w/(1/\theta)$, are different, as reflected in the last line of the table. Other things equal, it is highest in Zone 1. Additionally, the marginal utilities of consumption differ across the extensive and intensive zones–the terms in braces on the penultimate line. They tend to be higher in the extensive margin zones because $\theta > 1$ and $\mathfrak{c} < c$.

In Zone 3 the first-order condition for consumption along the intensive margin (3) simplifies in a symmetric equilibrium to

$$\alpha\theta\frac{1}{\mathfrak{N}_t c_t} = \lambda_t, \tag{14}$$

where use has been made of the pricing condition (13). Therefore, the labor supply condition reads

$$\alpha\frac{1}{\mathfrak{N}_t c_t} \times \frac{w_t}{1/\theta} = (1-\alpha)l_t^{\chi}. \tag{15}$$

The real wage is $w_t/(1/\theta)$. The marginal utility of consumption, $\alpha/(\mathfrak{N}_t c_t)$, is lower along the intensive margin as opposed to the extensive one, other things equal, because consuming more of an existing variety contributes less to utility than consuming an additional variety–recall Figure 9. A juxtaposition of (10) and (15) suggests that labor supply will be lower in Zone 3 relative to Zone 1, ceteris paribus, because $\theta < 1/\theta$. Things look ambiguous when comparing Zone 2 to Zone 3. On the one hand, in Zone 2 the marginal utility of consumption will be higher than in Zone 3, but on the other hand the real wage in this Zone is lower (assuming that Zone-2 $\mathfrak{c}$, $\mathfrak{N}_t$, and w_t equal Zone-3 c, $\mathfrak{N}_t$, and w_t). Labor supply will be bigger in Zone 3 if $p > 1/\theta^2$, or when prices are very high, and will be smaller otherwise. A synopsis of the three zones is presented in Table 1.

3.3 Intermediate Goods Production

Intermediate goods are used to produce final consumption and investment goods. In period t intermediate goods, m_t, are produced competitively in line with a standard Cobb-Douglas production function using capital, k_{mt}, and labor, l_{mt}, according to

$$m_t = k_{mt}^{\gamma}(z_t l_{mt})^{1-\gamma},$$

where z_t is a labor-augmenting technology factor. Increases in z_t reflect process innovation. The output of intermediate goods, m_t, can be converted in a one-to-one fashion into intermediate goods for any variety, m_{jt}, with the same being true for investment goods, i_t. The price of intermediate goods is normalized to be one. Capital and labor are chosen to maximize profits:

$$\pi_m = \max_{k_{mt}, l_{mt}} \{k_{mt}^{\gamma}(z_t l_{mt})^{1-\gamma} - r_t k_{mt} - w_t l_{mt}\}. \tag{16}$$

Since the intermediate goods sector is competitive, profits will be zero so that $\pi_m = 0$.

The cost function for a unit of intermediate goods is $z_t^{-(1-\gamma)}\gamma^{-\gamma}(1-\gamma)^{1-\gamma} r_t^{\gamma} w_t^{1-\gamma}$. Recall that the price of intermediate goods is normalized to be one. Thus, equation (17) obtains. As z_t rises the price of the intermediate goods going into final goods production falls, relative to the cost of inputs as reflected by the input price index $r_t^{\gamma} w_t^{1-\gamma}$. Therefore, process innovation results in the cost of producing final goods decreasing.

$$z_t^{-(1-\gamma)}\gamma^{-\gamma}(1-\gamma)^{1-\gamma} r_t^{\gamma} w_t^{1-\gamma} = 1. \tag{17}$$

3.4 Symmetric Equilibrium

Attention is on a symmetric equilibrium. In equilibrium the demand for labor must equal its supply implying that

$$l_{mt} = l_t. \tag{18}$$

Likewise, the capital market must clear

$$k_{mt} = k_t. \tag{19}$$

Finally, a market clearing condition must hold for intermediate goods. In a symmetric equilibrium $c_{jt} = c_t$ for all j so the demand for intermediate goods from the final consumption goods sector is $N_t c_t$. The demand for intermediate goods from the capital goods sector, or gross investment, is simply $i_t = k_{t+1} - (1-\delta)k_t$. Thus, intermediate goods market clearing condition is

$$N_t c_t + i_t = k_t^{\gamma}(z_t l_t)^{1-\gamma} \equiv o_t. \tag{20}$$

It now time to define the equilibrium that is being cast.

Definition. (Symmetric Equilibrium) Given exogenous sequences for process and product innovation, $\{z_t\}_{t=1}^{\infty}$ and $\{\mathfrak{N}_t\}_{t=1}^{\infty}$, an equilibrium consists of a solution for: (i) consumption along the intensive and extensive margins, c_t and N_t, capital investment, k_{t+1}, labor supply, l_t, and the Lagrange multiplier, λ_t; (ii) the capital and labor hired by intermediate goods firms, k_{mt} and l_{mt}; and (iii) the prices for renting capital, r_t, hiring labor, w_t, and purchasing final consumption goods, p_t, plus profits, π_t, such that:

1. Consumption, c_t and N_t, capital investment, k_{t+1}, and labor supply, l_t, maximize the individual's lifetime utility (1) subject to their budget constraint (2), taking as given final consumption goods prices, p_t, the rental rate on capital, r_t, profits, π_t, and wages, w_t.

2. Intermediate goods firms hire capital, k_{mt}, and labor, l_{mt}, to maximize profits in accordance with (16), taking as given the rental rate on capital, r_t, and wages, w_t.

3. The rental rate on capital, r_t, clears the capital market as given by (19).

4. The wage rate for labor, w_t, clears the labor market as given by (18).

5. The Lagrange multiplier, λ_t, is determined so that the market for intermediate goods always clears so that (20) holds.[1]

6. The price for final consumption goods, p_t, maximizes the monopolistic competitor's profits whereby in Zone 1 p_t is given by (8), p_t in Zone 2 is determined in accordance with (11), and in Zone 3 p_t is governed by (13). In Zone 1 profits are zero so that $\pi_t = 0$, in Zone 2 profits are given by $\pi_t = (p_t - 1)\mathfrak{N}_t\mathfrak{c}$, while in Zone 3 they read $\pi_t = (1/\theta - 1)\mathfrak{N}_t c_t$.

3.5 Balanced Growth

It's possible to have balanced growth paths in each of the three zones depending on the profiles for technological progress. Let $g_z \equiv z_{t+1}/z_t$ and $g_{\mathfrak{N}} \equiv \mathfrak{N}_{t+1}/\mathfrak{N}_t$ denote constant gross rates of process and product innovations.

Lemma 2. *(Balanced growth) Balanced growth paths may exist in each of the three zones as specified below.*

Zone 1 (Extensive Margin). If $g_{\mathfrak{N}} \geq g_z \geq 1$*, then a balanced growth path may exist where* $c_t = \mathfrak{c}, l_t, p_t$*, and* r_t *are constant, and* $k_t, N_t < \mathfrak{N}_t$*,and* w_t *grow at rate* g_z*.*

Zone 2 (Shackled Margins). If $g_{\mathfrak{N}} = g_z \geq 1$*, then a balanced growth path may exist where* $c_t = \mathfrak{c}, l_t, p_t$*, and* r_t *are constant, and* $k_t, N_t = \mathfrak{N}_t$*,and* w_t *grow at rate* g_z*.*

[1] The requirement (20) that intermediate goods market must clear, in conjunction with the condition for profits, implies that the individual's budget constraint (2) must hold.

Zone 3 (Intensive Margin). If $g_z \geq g_{\mathfrak{N}} \geq 1$, then a balanced growth path may exist where l_t, p_t, and r_t are constant, $N_t = \mathfrak{N}_t$ grows at rate $g_{\mathfrak{N}}$, $c_t > \mathfrak{c}$ grows at rate $g_z/g_{\mathfrak{N}}$, and k_t and w_t grow at rate g_z.

Proof. See Appendix A. □

Corollary. *(Invariant Ratios) Along a balanced growth path, each of the ratios $k_t/(z_t l_t)$, $N_t c_t/o_t$, and i_t/o_t are the same across zones and constant over time.*

Proof. Again, see Appendix A. □

Which zone a balanced growth path lies in depends upon the relative strengths of process versus product innovation, as the next lemma establishes.

Lemma 3. *(Characterization of Balanced Growth Zones) There exists three thresholds, $\underline{z/\mathfrak{N}}$, $\widetilde{z/\mathfrak{N}}$, and $\overline{z/\mathfrak{N}}$, such that:*

1. *If $z/\mathfrak{N} < \underline{z/\mathfrak{N}}$, then the economy is in Zone 1.*
2. *If $\underline{z/\mathfrak{N}} < z/\mathfrak{N} < \widetilde{z/\mathfrak{N}}$, then the economy is in Zone 2.*
3. *If $\widetilde{\mathfrak{N}/z} < z/\mathfrak{N} < \overline{z/\mathfrak{N}}$, then the economy can be in either Zone 2 or Zone 3.*
4. *If $z/\mathfrak{N} > \overline{z/\mathfrak{N}}$, then the economy is in Zone 3.*

Proof. Once again, see Appendix A. □

Discussion

Figure 10 illustrates the key concepts underlying Lemmas 2 and 3. When values of $z/\mathfrak{N}$ is below $\underline{z/\mathfrak{N}}$, the economy follows a Zone-1 balanced growth path. In this zone, the product innovation factor, z, is relatively low compared to process innovation one, $\mathfrak{N}$, leading to a relative abundance of varieties. In this case, $c = \mathfrak{c}$, $N < \mathfrak{N}$, and $p = 1$. Hours worked, l, are elevated due to the high marginal utility of consumption.

As $z/\mathfrak{N}$ increases and surpasses $\underline{z/\mathfrak{N}}$, the relative abundance of varieties decreases, and the economy shifts to a Zone-2 balanced growth path. In this Zone as $z/\mathfrak{N}$ continues to rise, $c = \mathfrak{c}$, $N = \mathfrak{N}$, and prices, p, are rising. Hours worked, l, are falling because less labor is required in production due to process innovation. Observe that for high values of $z/\mathfrak{N} \in [\underline{z/\mathfrak{N}}, \widetilde{z/\mathfrak{N}}]$, the monopolistic competitor charges a price $p > 1/\theta$, where $1/\theta$ is the price that would be set in the standard model of monopolistic competition. This transpires because reducing the price will not increase a consumer's demand for the variety, because it is stuck at the lower bound $c = \mathfrak{c}$. Here the extra utils gained per extra dollar spent moving along the extensive margin, $\alpha/(\theta p \mathfrak{N}\mathfrak{c})$, exceed that from proceeding along intensive margin, $\alpha\theta/(\mathfrak{N}\mathfrak{c})$ at the price $p = 1/\theta$.

If $z/\mathfrak{N}$ increases further, it will eventually meet $\widetilde{z/\mathfrak{N}}$. At this point the number of varieties becomes more limited relative to the economy's productivity. After this threshold two possibilities arise. The economy can either remain on a Zone-2 balanced growth path or shifts to a Zone-3 one. In the Zone-3 case, $c > \mathfrak{c}$, $N = \mathfrak{N}$, prices drop down to $p = 1/\theta$, and hours worked lie below their Zone-1 level.

In the Zone-2 case hours worked will continue to fall as $z/\mathfrak{N}$ increases and prices will keep moving up. Eventually, $z/\mathfrak{N}$ will cross the $\overline{z/\mathfrak{N}}$ threshold, marking a transition to a Zone-3 balanced growth path, where the number of products is restricted. The transition from Zone 2 to Zone 3 will be accompanied by a drop down in prices and jumps up in consumption and labor. The jump in variables between Zones 2 and 3 occurs because the marginal utility of consumption drops discontinuously from $(\alpha/\theta)/(\mathfrak{N}\mathfrak{c})$ to $\alpha/(\mathfrak{N}c)$ as consumption switches from the extensive to the intensive margin. Note that at the threshold $\overline{z/\mathfrak{N}}$ the extra utils gained per extra dollar spent moving along the extensive margin, $\alpha/(\theta p\mathfrak{N}\mathfrak{c})$, at the price $1/\theta^2$ exactly equals that from proceeding along intensive margin, $\alpha/(p\mathfrak{N}\mathfrak{c})$ at the price $p = 1/\theta$.

The model has many of the standard features of the neoclassical growth model. In particular, from Lemma 2, output, o, in all three zones grows at the rate of process innovation, g_z, and therefore is not affected by the rate of product innovation, $g_{\mathfrak{N}}$. Additionally, the Corollary establishes that the capital-to-effective labor ratio, $k/(zl)$, the consumption-to-output ratio, Nc/o, and the investment-to-output ratio, i/o, are constant over time. More surprisingly, each of the ratios are the same across zones. Importantly, the levels of capital, k, labor, l, consumption, Nc, investment, i, and output, o, differ across zones for a given z and depend on the state of process innovation relative to process innovation, or $z/\mathfrak{N}$–Lemma 3 and Figure 10.

Remark. (Perpetual Zones) If Lemma 3 holds, then Lemma 2 must be true. If an economy is on a Zone-1 balanced growth path as specified by Lemma 3, it can only remain there when $z/\mathfrak{N}$ is shrinking over time. This implies that $g_{\mathfrak{N}} \geq g_z$, as stated in Lemma 2. Similarly, according to Lemma 3, when the economy is on a Zone-2 balanced growth, $z/\mathfrak{N}$ is constant. For this to be the case, $g_{\mathfrak{N}} = g_z$, which is the condition in Lemma 2. Last, to remain on Zone-3 balanced growth path, $z/\mathfrak{N}$ must be rising, implying $g_z \geq g_{\mathfrak{N}}$.

4 Proof-of-Concept Simulations

Imagine a scenario where individuals are aware in the 1920s that product innovation has slowed down relative to process innovation but expect that the former will pick up entering into the 1930s. The uptick in product innovation never materializes, however, leading to an over accumulation of inputs. This causes a crash when entering into the 1930s. Is this framework capable of replicating both the Great Accretion of the 1920s, where GDP and investment grew

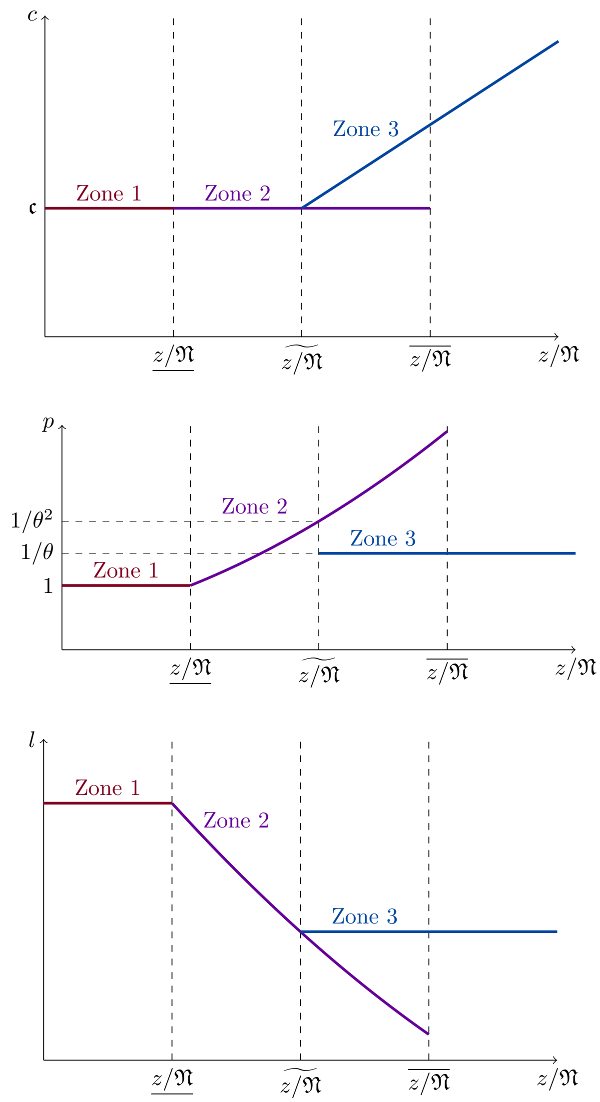

Figure 10: Characterization of the Balanced Growth Path Zones. Each of the 3 possible zones–extensive, shackled, and intensive–are color coded differently. The zone a balanced growth path resides in depends on the relative strengths of process, z, versus product, $\mathfrak{N}$, innovation. Over the region $[\widetilde{z/\mathfrak{N}}, \overline{z/\mathfrak{N}}]$ there are two possible equilibria. Properties of the graph are established in Appendix A during the course of the proof for Lemma 3.

Table 2: Parameter Values for Tastes and Technology

Parameter	Value	Basis
Tastes		
α	0.5	Consumption Share, 75%
θ	0.9	Markup, 11%
χ	1.33	Chetty et al. (2011)
β	0.96	Standard
Technology		
γ	1/3	Standard
δ	0.08	Standard
Period length	1 yr	Standard

at a reasonable pace with a constant labor supply, as well as the onset of the Great Depression in the early 1930s, or a sizable crash in GDP, investment, and labor supply?

To answer this question, some proof-of-concept simulations are undertaken. The analysis is meant to be illustrative, not definitive. The simulations are done at the annual frequency. Table 2 gives the parameter values used for tastes and technology. The values chosen for the depreciation rate, discount factor, and capital share of income in the intermediate goods sector are standard. The Frisch elasticity of labor supply is picked to be 1.3, which is in the range of numbers reported in Chetty et al. (2011). The exponent on a variety is chosen to generate a markup of around 11 percent, while the weight on consumption gives a consumption share of GDP of around 75 percent. The values used for process and product innovation differ by experiment and are shown in Figures 11 and 13.

4.1 The Rosy 1920s

The economy enters the 1920s from the early stages of the second industrial revolution where product innovation outpaces process innovation, or when $g_{\mathfrak{N},t} \equiv \mathfrak{N}_{t+1}/\mathfrak{N}_t \geq g_{z,t} \equiv z_{t+1}/z_t$. Specifically, assume that the economy enters the 1920s from a Zone-1 balanced growth path. Product innovation slows down relative to process innovation so that $g_{\mathfrak{N},t} < g_{z,t}$. People believe that things will revert back to the pre-1920s balanced growth path at the end of the 1920s. More precisely, they believe that in 1929 the economy will start a transition back to a Zone-1 balanced growth.[2] Beliefs concerning the time paths for $\mathfrak{N}_t$ and z_t are shown in Figure 11. Process innovation is constant at 2.5% a year. Over the period 1921 to 1929 product innovation grows at 1.0% annually. Over the next 5 years, $\mathfrak{N}_t$ is allowed to catch up with z_t, converging subsequently to $g_{\mathfrak{N},t} = g_{z,t} = g = 1.025$. For this to happen, product innovation

[2]Pushing back the date when people believe product innovation will revert back to the pre-1920s balanced growth path weakens the overaccumulation mechanism for the 1920s, other things equal. This transpires because people discount dates further off in the future more heavlity. But, simular qualititave results would still occur for post-1929 dates.

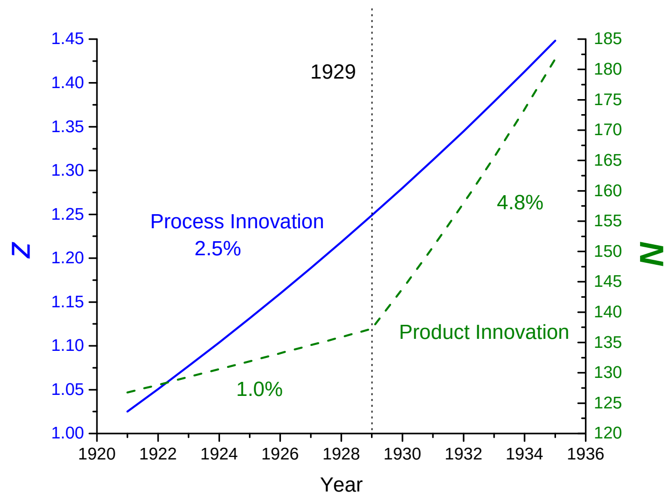

Figure 11: Process and Product Innovation, Expected Paths in the Rosy 1920s. The assumed annual growth rates for process and product innovation are indicated on the diagram.

must grow at 4.8%. This assumed combination of process and product innovation generates an annual growth rate in real per-capita GDP of 3.4% for the period 1921 to 1929, the same as in the U.S. data.

The resulting time paths for gross investment and GDP are shown in Figure 12, left panel. The economy starts off and ends in Zone 1, where there is an abundance of varieties. Most of the time, though, the economy is in Zone 2, where the number of available varieties is relatively scarce due to the slow down in product innovation. Real GDP and investment per capita rise continually throughout the 1920s at annual rates of 3.4 and 4.5%, close to the 3.4 and 5.2% observed in the U.S. data.[3] Investment begins to taper off prior to the mid 1930s as product innovation converges back to the rate of process innovation.

Employment declines slightly over the 1920s in the model while it is constant the U.S. data, despite the fact that GDP is growing–see Figure 12, right panel[4]. In Zone 2 real wages are increasing due process innovation. But, the rate of increase in real wages is less than the rise in the number of varieties due to rising prices. This leads to a fall in labor supply as per equation (12); i.e., the marginal utility of consumption is falling faster than the rise in real wages. If in Zone 2 process innovation was proceeding at the same constant rate as product innovation, the economy would be on a balanced growth path with constant prices–recall Lemma 2. The fact that employment can decrease while GDP and investment are increasing is due to the

[3]The data numbers for the 1920s are computed from *Historical Statistics* (2006): (i) For real GDP per capita, Table Ca11; (ii) For per-capita real investment, Table Dd721 was combined with Table Aa141.

[4]The share of the labor force employed in the data was computed by combining Table Ba471 with Table Ba470 (Civilian Labor Force - Total)–*Historical Statistics* (2006).

relentless process innovation. Less and less labor is needed to produce a unit of final output. Interestingly, labor share of income falls in the model as it does in the data–see Figure 1, right panel. In the model this transpires due to the increase in monopoly rents that are earned in Zone 2.

People use the rise in their incomes both to work less and save more. The Euler equation (6) for Zone 2 can be rewritten as

$$\frac{\mathfrak{N}_{t+1}}{\mathfrak{N}_t}\frac{p_{t+1}}{p_t} = \beta[\gamma k_{t+1}^{\gamma-1}(z_{t+1}l_{t+1})^{1-\gamma} + (1-\delta)]. \tag{21}$$

Here use has been made of the facts that $\lambda_t = (\alpha/\theta)/(p_t\mathfrak{N}_t\mathfrak{c})$, using the pricing condition (11), and that $r_t = \gamma k_t^{\gamma-1}(z_t l_t)^{1-\gamma}$, since the rental rate must equal the marginal product of capital. On the one hand, in Zone 2 the number of new varieties and prices are rising and this operates to dissuade capital accumulation. When next period's consumption is expensive relative to current consumption, the motivation to save is lower. Additionally, an increase in the number of new varieties over time implies that the marginal utility of consumption is falling over time, which also reduces the incentive to invest. On the other hand, process innovation implies that the marginal product of capital is continuously rising and this encourages capital accumulation. This latter effect dominates spurring along investment. The higher stock of capital also makes it feasible to cut back on labor. To see this, suppose—counterfactually—that the number of new varieties remains constant and prices stay the same, so the lefthand side of (21) does not change. Process innovation would then imply that some combination of increasing capital and falling labor is required to keep the Euler equation holding. The Euler equation can also be cascaded forward to a future period in Zone 2, say period $t+j$, to get

$$\frac{\mathfrak{N}_{t+j}}{\mathfrak{N}_t}\frac{p_{t+j}}{p_t} = \beta^j \prod_{i=1}^{j}[\gamma k_{t+i}^{\gamma-1}(z_{t+i}l_{t+i})^{1-\gamma} + (1-\delta)].$$

Using the same logic, the continual process innovation allows some combination of increasing capital and falling labor between periods t and $t+j$ in Zone 2.

4.2 The Crash

Suppose that in 1929 people realize that their rosy beliefs about product innovation are not going to transpire. In particular, they now recognize that product innovation has stalled; i.e. that is $g_{\mathfrak{N},t} \equiv \mathfrak{N}_{t+1}/\mathfrak{N}_t = 1$, for $t \geq 1929$. Thus, the economy now starts a transition toward a Zone-3 balanced growth path where $g_{\mathfrak{N}} < g_z$. The actual time paths for process and product innovation are displayed in Figure 13. Individuals made their plans for the 1930s based on the time paths for process and product innovation shown in Figure 11 and not Figure 13. What

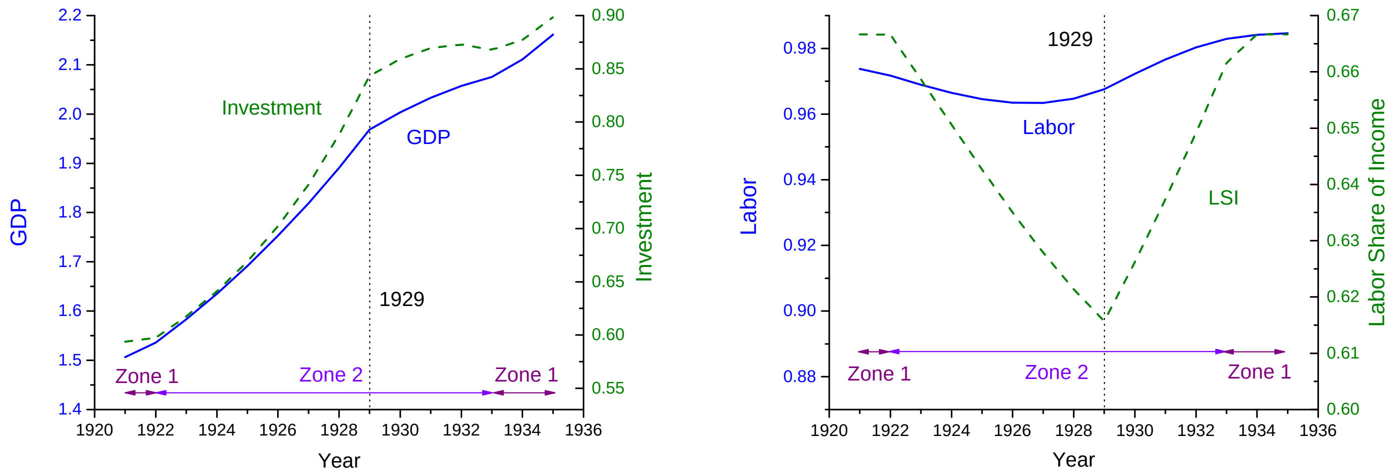

Figure 12: The Rosy 1920s. The left panel shows GDP and Gross Investment, while Labor and Labor Share of Income are displayed in the right one. As discussed in the main text, the model matches reasonably well the U.S. data for 1920s.

are the implications of this for the economy?

The behavior of the economy over the 1920s is the same as before. Not surprisingly, the economy enters a recession as people's rosy expectations are not fulfilled–see Figure 14, left panel. Process innovation is now far outpacing product innovation resulting in the economy entering Zone 3, where it ultimately converges to a Zone-3 balanced growth path. The transition to Zone 3 occurs because individuals must switch any increases in consumption, occurring from the continual process innovation, away from the extensive margin to the intensive one. The movement into Zone 3 results in a drastic fall in employment–see Figure 14, right panel. This causes GDP to drop by 1.9%, short of the 9.6% observed in the U.S. data. Investment does better, dropping by 13.5%, relative to 26.9% in the data. Labor share of income rises because profits (relative to GDP) are lower in Zone 3 relative to Zone 2, due to a drop in the markup for specialized varieties. Between 1929 and 1930 employment drops by 7.8%, close to the 6.3% in the data. Zone-3 employment is lower in 1935 than Zone-1 employment was in 1921. This, despite the fact that real wages are higher in 1935 than in 1920 because of the process innovation that had transpired. The return to working in 1935 is low because the marginal utility of consumption in 1935 is less than in 1921, both due to an increase in extensive margin (Zone 2) and intensive margin (Zone 3) consumption over the course of time, as can be understood by comparing equation (10) with (13).

As can be seen from Figure 14, right panel, investment drops upon entering Zone 3. The

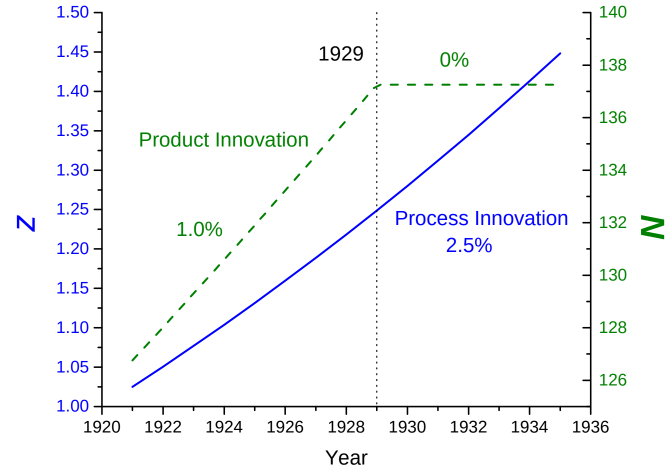

Figure 13: Process and Product Innovation, Realized Paths when the Bubble Bursts. The assumed annual growth rates for process and product innovation are indicated on the diagram.

Euler equation for capital accumulation switches from (21) to

$$\frac{c_{t+1}}{c_t} = \beta[\gamma k_{t+1}^{\gamma-1}(z_{t+1}l_{t+1})^{1-\gamma} + (1-\delta)]. \tag{22}$$

The drastic cut in employment reduces the marginal product of capital causing a fall in investment, working to offset the benefit from process innovation. Additionally, a comparison of (21) with (22) suggests that investment will drop when $c_{t+1}/c_t > (\mathfrak{N}_{t+1}/\mathfrak{N}_t)(p_{t+1}/p_t)$, which occurs due to the shift into intensive margin consumption.

4.3 A Deeper Crash

Now, individuals were expecting the time path for product innovation given by Figure 11 but instead the one displayed in Figure 13 materialized. The drop in GDP in previous simulation was a bit anemic. Can it be made deeper? The time path for new varieties from 1929 on was slashed from $\{\widehat{\mathfrak{N}}_t\}_{t=1929}$ to $\{\mathfrak{N}_t\}_{t=1929}$, where the $\wedge$ over a variable denotes the expected path. It's unrealistic to believe that the inputs developed for the $\{\widehat{\mathfrak{N}}_t - \mathfrak{N}_t\}_{t=1929}$ failed varieties can be seamlessly reallocated to the remaining $\{\mathfrak{N}_t\}_{t=1929}$ ones. The reorganization of production processes can be costly, a fact emphasized in David (1989) and formalized in Greenwood and Yorukoglu (1997). To capture this assume that there are two forms of adjustment costs; viz, those internal to a variety producing firm and those external to it. These are discussed in turn.

1. *Internal Adjustment Costs.* With internal adjustment costs, the final goods producer

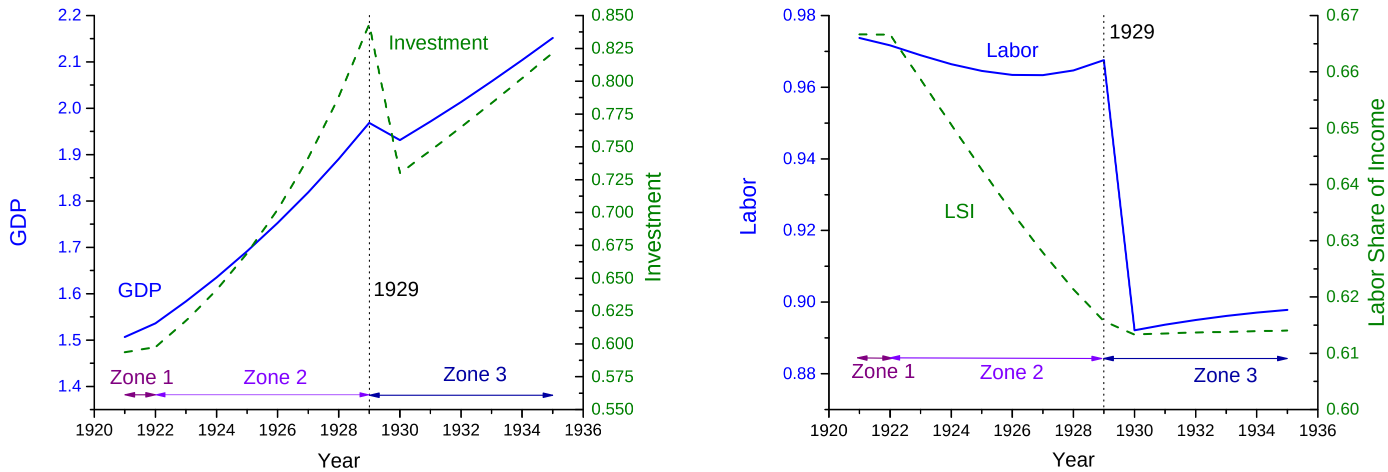

Figure 14: The Crash. The left panel shows GDP and Gross Investment, while Labor and Labor Share of Income are displayed in the right one. For a comparison of the model with the U.S. data see the main text.

takes into account how changes in its own output affects these costs. The form of adjustment costs assumed here is shorthand for the things needed to effect a short-run expansion in firm-level output such as: having to pay overtime or hire and train new workers; the disruptive costs of installing and setting up new equipment; and the organizational costs of reorganizing factory layouts. Let $\hat{o}_{1929}$ be the expected demand for a variety in 1929 and assume that the adjustment cost for a firm is given by

$$A(o_{1929}) = \frac{\phi}{\gamma}\left(\frac{\widehat{\mathfrak{N}}_{1929}}{\mathfrak{N}_{1929}}\hat{o}_{1929} - o_{1929}\right)^{-\gamma}\hat{o}_{1929},$$

which is increasing and convex in o_{1929}. Now, $\widehat{\mathfrak{N}}_{1929}/\mathfrak{N}_{1929}$ is the ratio of expected varieties to those that actually materialized, so think about $(\widehat{\mathfrak{N}}_{1929}/\mathfrak{N}_{1929})\hat{o}_{1929}$ as being the level of output for a remaining variety if production could be costlessly reallocated from a cut variety to the remaining one. There cannot be a full reallocation: if $o_{1929} = (\widehat{\mathfrak{N}}_{1929}/\mathfrak{N}_{1929})\hat{o}_{1929}$, then the marginal adjustment cost is infinitely large. The constant ϕ and the exponent γ are important for regulating the marginal cost of adjustment. Since intermediate goods can be converted into final goods in a one-to-one fashion, this adjustment cost function can be thought of as being imposed on a final goods producer's inputs of intermediate goods. The effective cost of an input is now $1 + A'(o_{1929})$, where recall that the market price of an intermediate good is one.

Table 3: Adjustment Cost Parameter Values

Parameter	Value
Internal	
ϕ	1.9×10^{-6}
γ	2×10^{-3}
External	
ω	0.9
$g_z - 1$	0.25
ρ	0.8
τ	5

2. *External Adjustment Costs.* Changes in the final goods producer's own output do not affect external adjustment costs. For example, upon a crash: the search process in labor markets with frictions may become less efficient due to mass layoffs; likewise, similar disruptions may occur in the resale markets for used plant and equipment; and business-to-business networks and supply chains may have to be reorganized due to firm exits. The form of the external adjustment costs function adopted here is shorthand for these type of things. Suppose that aggregate productivity in 1929 is given by

$$z_{1929} = \frac{\hat{z}_{1929}}{(\hat{\mathbf{o}}_{1929}/\mathbf{o}_{1929})^{\omega}},$$

where $\hat{\mathbf{o}}_{1929}$ and $\mathbf{o}_{1929}$ are the expected and realized levels of aggregate intermediate goods production. Upon the start of the economic contraction where $\mathbf{o}_{1929} < \hat{\mathbf{o}}_{1929}$, aggregate productivity, z_{1929}, falls. Overtime as the economy recovers productivity slowly reverts back to its old trend. In particular, let the subsequent evolution of z_{t+1}, for $t \geq 1930$, be characterized by

$$z_{t+1} = g_z^{t-1930}\hat{z}_{1929} - \frac{1}{1+\exp[-\rho(\tau - t)]}(g_z^{t-1930}\hat{z}_{1929} - z_t).$$

Here g_z is the gross rate of growth for z_t shown in Figure 11. The first term on right is just the trend rate of growth. The second term specifies that when z_t is below trend so will be z_{t+1}. As $t \to \infty$, $z_{t+1} \to \hat{z}_{t+1}$, so process innovation converges back to the pre-crash trend. The speed of convergence is governed by a logistic function. Hence, convergence will be slow at first, then accelerate up to the point where $t = \tau$, and subsequently slow down.

The parameter values for the internal and external adjustment costs are shown in Table 3. The values are chosen to mimic the observed behavior of GDP, discussed below. The resulting series for process innovation, z_t, is displayed in Figure 15.

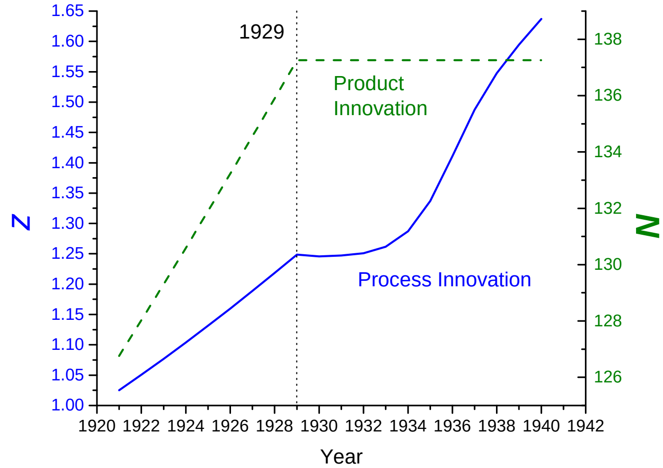

Figure 15: Process and Product Innovation, Realized Paths with Adjustment Costs

The crash in GDP and investment is now much deeper–see Figure 16, left panel. There is a fall in GDP of 8.9% between 1929 and 1930, which is close to the observed value 9.6%. Additionally, the slump is prolonged: GDP doesn't catch up with the pre-crisis level before 1935, similar to the 1936 recorded in the data. Investment now falls by 24.1% compared with the actual 26.9%. Labor falls by 6.8% (vs 6.3%). The model matches the ∪-shape in labor share of income that is observed in the data over the course of the 1920s and early 30s–Figure 16, right panel. Again, the fall in labor share of income during the Great Accretion is due to the monopoly profits being earned in Zone 2. These profits are wiped out upon the impact of the crash. Last, by lowering adjustments costs the size of the crash due to process innovation is reduced–see Appendix B for an example. This leaves room for some other factors that may have contributed to the crash that are discussed in Section 5.

4.4 Rational Exuberance

Can the above story be embedded in setting where individuals believe all along that there is the possibility of a crash occurring? To this end, suppose people believe that it possible for product innovation to continue along the upward sloping path $\{\mathfrak{N}_t\}_{t=1}^{T}$, but at any date t there is the chance that product innovation will forever stop. For simplicity, assume that at the terminal date T, if a stall hasn't occurred, then the economy moves deterministically toward a Zone-1 balanced growth path where $g_{\mathfrak{N}} > g_z$. If up to time t product innovation hasn't stopped, the person enters period t with the belief that a move up to $\mathfrak{N}_{t+1}$ in period $t+1$ will occur the transition probability $\tau_{t,t+1}$. This implies that a stall may happen with

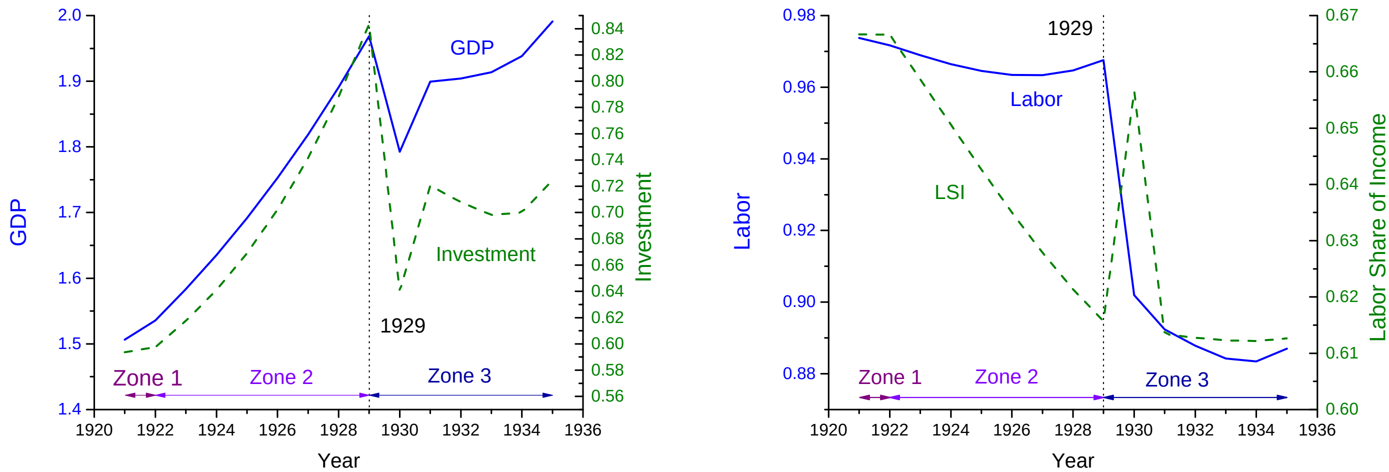

Figure 16: A Deeper Crash. The left panel shows GDP and Gross Investment, while Labor and Labor Share of Income are displayed in the right one. The model matches reasonably well the U.S. data for 1920s and early 30s, see the main text.

the complementary probability $1 - \tau_{t,t+1}$. The situation is illustrated in Figure 17.

The time-t belief that the economy will move up from t to $t + j$ is given by

$$\pi^t_{t,t+j} = \prod_{i=1}^{j} \tau_{t+i-1,t+i}.$$

Now,

$$\pi^t_{t,T} = \prod_{i=1}^{T-t} \tau_{t+i-1,t+i} < 1,$$

so people are expecting a stall. The actual stall date is $1929 < T$, which is unknown. The time-t belief that the economy will not move up (or crash) at period $t + j - 1$ is

$$\prod_{i=1}^{j-1} \tau_{t+i-1,t+i}(1 - \tau_{t+j-1,t+j}) = \pi^t_{t,t+j-1}(1 - \tau_{t+j-1,t+j}).$$

Lemma 4. *(Increasing Optimism) Suppose that a stall does not occur at time t. Then, people believe that the likelihood of a future move up to $\mathfrak{N}_{t+j}$ is higher and that the odds of stall occurring at some time before T are lower.*

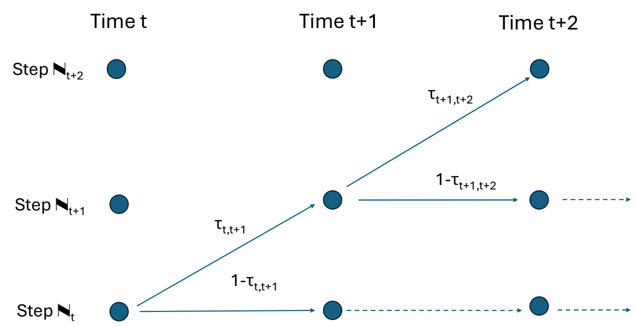

Figure 17: Product Innovation Ladder. Along the diagonal are movements up the product innovation ladder. But, at any node a permanent stall can occur, as shown by the horizontal lines. People believe in period t that the transition probability of moving up in period $t+1$ from $\mathfrak{N}_t$ to $\mathfrak{N}_{t+1}$ is $\tau_{t,t+1}$. The odds of a stall in period t are $1-\tau_{t,t+1}$. Note that the step size between $\mathfrak{N}_{t+1}$ to $\mathfrak{N}_{t+2}$ is bigger than the one between $\mathfrak{N}_t$ to $\mathfrak{N}_{t+1}$.

Proof. Suppose a stall does not occur at time t. Then,

$$\pi_{t+1,t+j}^{t+1} = \prod_{i=2}^{j} \tau_{t+i-1,t+i} > \prod_{i=1}^{j} \tau_{t+i-1,t+i} = \pi_{t,t+j}^{t},$$

so the time-$(t+1)$ belief of moving up the ladder to $\mathfrak{N}_{t+j}$ is higher than the time-t one. The time-t odds of a stall occurring *some time* before T are given by

$$1-\tau_{t,t+1}+\tau_{t,t+1}(1-\tau_{t+1,t+2})+\tau_{t,t+1}\tau_{t+1,t+2}(1-\tau_{t+2,t+3})+\cdots = 1-\prod_{i=1}^{T-t}\tau_{t+i-1,t+i},$$

where $\prod_{i=1}^{T-t}\tau_{t+i-1,t+i}$ is the probability of climbing up to the end of the ladder at time T. The probability of a crash occurring some time is decreasing in t. □

Remark. (Bayesian Learning) The evolution of beliefs is consistent with Bayesian updating. If a stall does not occur, then people have no new information that could refute their priors.

This formulation does not guarantee that a significant crash can occur upon a stall. When people foresee the possibility of a crash, they may invest less. The stochastic period-t Euler equation for capital accumulation in the situation where product innovation hasn't stopped now appears as

$$\lambda_t^{\uparrow} = \beta\tau_{t,t+1}\lambda_{t+1}^{\uparrow}[k_{t+1}^{\gamma-1}(z_{t+1}l_{t+1}^{\uparrow})^{1-\gamma} + (1-\delta)] \\ + \beta(1-\tau_{t,t+1})\lambda_{t+1}^{\rightarrow}[k_{t+1}^{\gamma-1}(z_{t+1}l_{t+1}^{\rightarrow})^{1-\gamma} + (1-\delta)]. \quad (23)$$

The first line on the righthand side represents the event when a stall hasn't happened, which occurs with probability $\tau_{t,t+1}$. The vertical arrow superscript, $\uparrow$, signifies the state-contingent value of a variable when this event occurs. The second line does the same thing for when a stall does transpires, with the horizontal arrow superscript, $\rightarrow$, denoting a variable's state-contingent value. If $l_{t+1}^{\rightarrow} < l_{t+1}^{\uparrow}$, then there will be less incentive to invest in capital in period-t, as the marginal product of capital will be lower when a crash occurs. The same is true if $\lambda_{t+1}^{\rightarrow} < \lambda_{t+1}^{\uparrow}$. This can occur if the economy switches to Zone 3 upon a crash, which results in consumption switching to the intensive margin, lowering the marginal utility of income. Solving the stochastic Euler equation (23) is a bit tricky. The unique algorithm developed to do so is detailed in Appendix C.

When making their investment decisions people believe that a stall can occur at any point along the product innovation ladder, as reflected in the Euler equation (23). The left panel of Figure 18 shows how beliefs about a potential stall happening before period T decline over time.[5] This has the flavor of Zeira (1999)'s rational stock market bubbles where optimism rises over time, but the statistical mechanics here are different. In a similar vein, Boz and Mendoza (2014) model the 2008 U.S. credit crisis using a model with Bayesian learning. Some potential stall paths for product innovation are shown in Figure 18, right panel. The solid green line shows the evolution of the economy when a stall does not occur or when the economy keeps on moving up the product innovation ladder. Figure 19 shows the impact on employment of these potential stalls.

The crash in GDP and investment when the stall occurs in 1929 are shown in the left panel of Figure 20. Again, the simulation matches the 3.4% annual growth rate in real per-capita GDP for the 1920s. Additionally, between 1921 and 1929 investment per capita grows at 4.4% a year, reasonably close to the observed value of 5.2%. Labor over this period is roughly constant, in accord with data. When the crash occurs, GDP falls by 9.6%, identical to the actual value recorded in the data. Moreover, in the data investment falls by 26.9% between 1929 and 1930, and by 23.8% in the simulation. The drop in investment is a little more muted now because people are aware that a crash can happen and invest less in anticipation. The

[5] Note that the probabilities fall in a convex pattern. This reflects the assumption that agents place a higher likelihood on a stall occurring at later transition stages than at earlier ones (i.e., $\tau_{t,t+1} < \tau_{t',t'+1}$ for all $t < t'$). The assumption helps limit incentives to overaccumulate and produces an investment growth path from 1921 to 1929 that matches the data. Intuitively, since stalls are viewed as more likely in the distant future, optimism remains contained during early moves up the product ladder. Importantly, the main results do not hinge on this assumption, as alternative belief profiles yield similar dynamics.

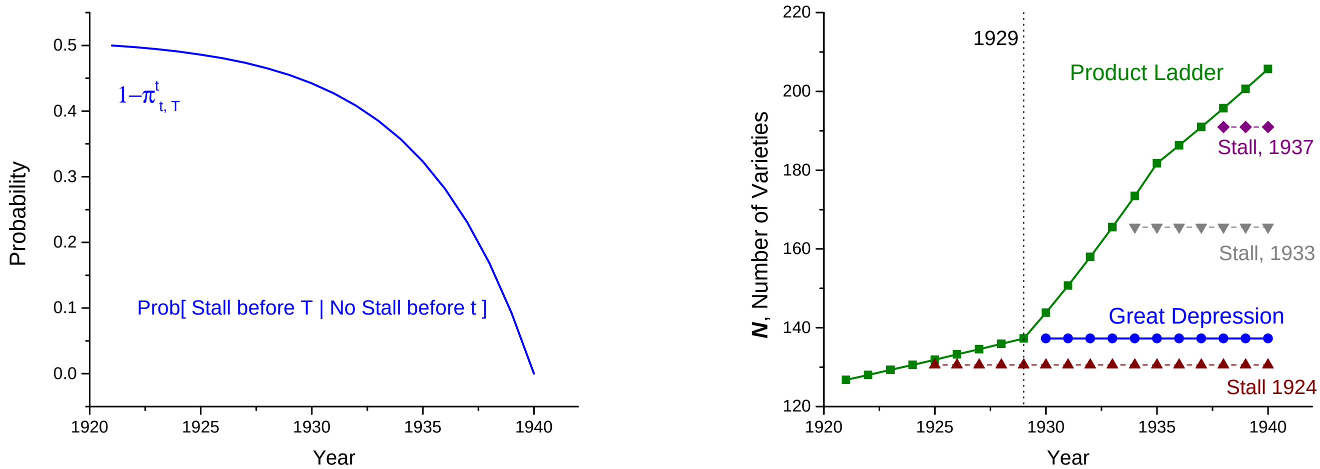

Figure 18: Rational Exuberance. The left panel shows the time-t odds of a stall happening before period T. Some potential stall paths along the product innovation ladder are displayed in the right panel, where again adjustment costs are assumed.

right panel displays the 6.3% decline in labor (identical to the 6.3% in the data) and the rise in labor share of income. These results are similar to the earlier ones with adjustment costs.

5 Other Contributing Factors: Amplification and Propagation

Consider the hypothesis presented here as a potential trigger for the Great Depression: that the widespread optimism of the 1920s—fueled by a wave of rapid product and process innovations—led to excessive investment throughout the economy. When the overly optimistic expectations failed to materialize, the resulting disappointment triggered an economic downturn. Several amplification and propagation mechanisms then deepened the crisis:

1. *The 1929 Stock Market Crash.* The speculative boom in the stock market during the 1920s is consistent with the hypothesis outlined above. When the crash occurred, it caused a sharp decline in both business and consumer confidence. Fortunes were wiped out, reducing household wealth and leading to a collapse in durable goods purchases and business investment.

2. *Bank Failures and Credit Crunches.* Beginning in the early 1930s, thousands of banks failed. Gorton and Ordoñez (2023) highlight a significant credit boom during the 1920s, particularly in the mortgage sector—again supporting the over investment hypothesis. These bank failures drastically curtailed the availability of credit and reduced household

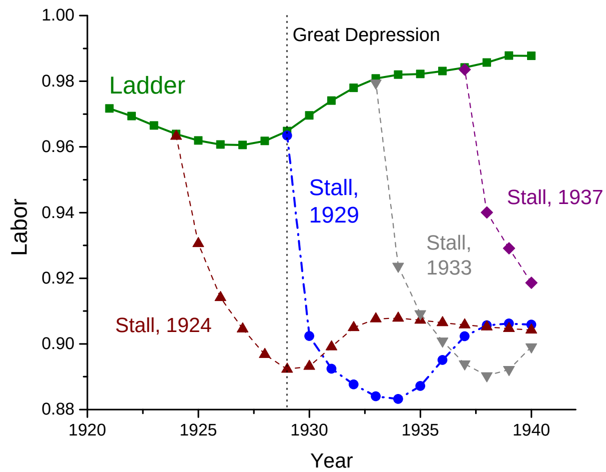

Figure 19: employment implications of potential stall paths along the product innovation ladder, where again adjustment costs are assumed.

savings. Bernanke (1983) argues that this financial disruption transformed a severe recession into a prolonged depression.

3. *Federal Reserve Policy.* In 1928 and 1929, the Federal Reserve raised interest rates to curb stock market speculation. Friedman and Schwartz (2008) emphasize this as a key factor contributing to the crash. After the crash and subsequent bank failures, the Fed failed to provide adequate liquidity, leading to a sharp contraction in the money supply—another factor identified by Friedman and Schwartz as worsening the downturn.

4. *Smoot-Hawley Tariff Act (1930).* The enactment of the Act triggered a global trade war, further exacerbating the downturn. For a deeper discussion, see Crucini and Kahn (1996).

5. *Dust Bowl (mid-1930s).* A severe drought struck the prairies. Years of unsustainable farming practices had left the land vulnerable, compounding the environmental and economic damage.

6. T*he New Deal (1933-1938).* The New Deal policies concentrated power in large businesses and unions. According to Cole and Ohanian (2004), these cartelization policies suppressed competition and hindered recovery, delaying the return to pre-Depression economic activity.

Last, related to the hypothesis advanced here, Ohanian and Ozturk (2024) argue that the diffusion of scientific management during the 1920s allowed intangible organizational capital

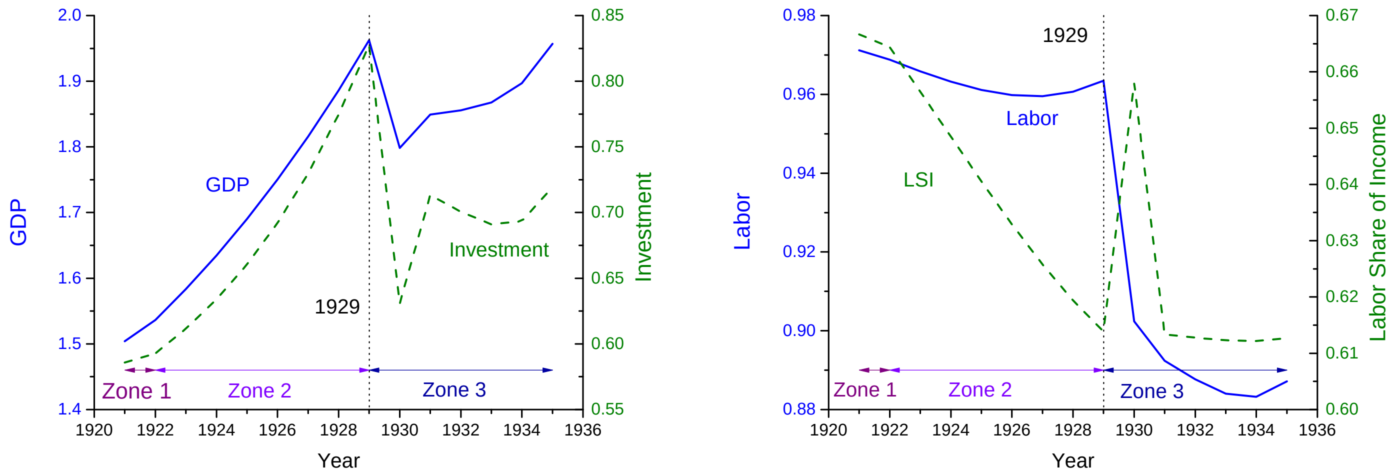

Figure 20: Rational Exuberance. The left panel shows GDP and Gross Investment, while Labor and Labor Share of Income are displayed in the right one. Again, adjustment costs are assumed. The model is roughly in accord with the U.S. data for the 1920s and early 30s.

to be substituted for labor and tangible capital–this could be viewed as a form of process innovation. They argue that this can explain some of the anomalies of the 1920s, such as stagnating employment in face of rising GDP. Organization capital is not considered here and no such direct substitution between tangible capital and labor is allowed in the current work. Ohanian and Ozturk (2024) do not address the Great Depression.

6 Closing

The Second Industrial Revolution, which unfolded around the turn of the 20th century, ushered in a wave of transformative new products—such as automobiles, motion pictures, and washing machines—alongside groundbreaking industrial processes like the assembly line, the Bessemer steel-making method, and continuous flow techniques in petrochemical production. This surge in technological progress helped fuel the widespread optimism of the Roaring Twenties, which met its end with the onset of the Great Depression.

To understand the dynamics of the Roaring Twenties and the Great Depression, a model is developed that incorporates both product and process innovation. During times when product innovation outpaces process innovation, leading to an abundance of new products, consumption tends to expand on the extensive margin. When new products flood the market, prices are competitively determined and align with the marginal cost of production. As incomes rise, people prefer to explore a wider variety of products rather than simply buying more of the same goods.

By contrast, when process innovation outstrips product innovation, and the introduction of new products slows down, consumption moves along the intensive margin. In this scenario, people buy more of the products they already consume, and they may even reduce the amount of labor they supply. Prices in this case tend to be higher than the marginal cost of production. The way people consume—whether focusing on a wider range of products or consuming more of what they already have—plays a significant role in shaping the broader economy.

The hypothesis proposed here is that when the flow of new product development began to slow in the 1920s, widespread optimism about the future still fueled excessive investment. During this period process innovation outpaced product innovation, investment and GDP rose, while employment declined. This speculative boom, combined with process innovation—where less labor was required to produce the same output—ultimately deepened the impact of the Great Depression. Proof-of-concept simulations supports this hypothesis, lending credibility to the proposed relationship.

References

Bernanke, Ben S. "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression". *American Economic Review*, v. 73, no. 3: 257–276, 1983.

Boz, Emine and Enrique G. Mendoza. "Financial innovation, the discovery of risk, and the U.S. credit crisis". *Journal of Monetary Economics*, v. 62, no. March: 1–22, 2014.

Carter, Susan B., Scott Sigmund Gartner, Michael R. Haines, Alan L. Olmstead, Richard Sutch, and Gavin Wright, editors. *Historical Statistics of the United States: Millennial Edition*. Cambridge University Press, New York, 2006.

Charpe, Matthieu, Slim Bridji, and Peter Mcadam. "Labor Share and Growth in the Long Run". *Macroeconomic Dynamics*, v. 24, no. 3: 1720–1757, 2020.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins". *American Economic Review*, v. 101, no. 3: 471–475, 2011.

Cole, Harold L. and Lee E. Ohanian. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis". *Journal of Political Economy*, v. 112, no. 4: 779–816, 2004.

Crucini, Mario J. and James Kahn. "Tariffs and Aggregate Economic Activity: Lessons from the Great Depression". *Journal of Monetary Economics*, v. 38, no. 3: 427–467, 1996.

David, Paul A. "Computer and Dynamo: The Modern Productivity Paradox in a Not-Too-Distant Mirror". 1989. Center for Economic Policy Research, no. 172, Stanford University.

Fabricant, Solomon. *Employment in Manufacturing, 1899–1939: An Analysis of Its Relation to the Volume of Production*. NBER, 1942. New York, NY.

Freeman, Chris and Luc Soete. *The Economics of Industrial Innovation*. The MIT Press, 1997. Cambridge, MA.

Friedman, Milton and Anna J. Schwartz. *A Monetary History of the United States, 1867–1960*. Princeton University Press, 2008. Princeton, NJ.

Gorton, Gary B. and Guillermo L. Ordoñez. *Macroeconomics and Financial Crises: Bound Together by Information Dynamics*. Princeton University Press, 2023. Princeton, NJ.

Greenwood, Jeremy and Mehmet Yorukoglu. "1974". *Carnegie-Rochester Conference Series on Public Policy*, v. 46, no. June: 49–95, 1997.

Jovanovic, Boyan and Peter L. Rousseau. "Why Wait? A Century of Life before IPO". *American Economic Review*, v. 91, no. 2: 336–341, 2001.

Ohanian, Lee E. and Fatih Ozturk. "The Great Depression May Have Started Sooner than We Thought". *Draft, UCLA*, 2024.

Shiller, Robert. *Irrational Exuberance*. Princeton University Press, 2006. Princeton, NJ.

Szostak, Richard. *Technological Innovation and the Great Depression*. Westview Press, 1995. Boulder, CO.

United States Census Bureau. *Historical Statistics of the United States, Colonial Times to 1970*. 1975.

Yorukoglu, Mehmet. "Product vs. Process Innovations and Economic Fluctuations". *Carnegie–Rochester Conference Series on Public Policy*, v. 52: 137–163, 2000.

Yorukoglu, Mehmet. "From Model T to the Great Depression: Automobilization and Suburbanization of the U.S.". Undated. Slides, University of Chicago.

Zeira, Joseph. "Informational Overshooting, Booms, and Crashes". *Journal of Monetary Economics*, v. 43, no. 1: 237–257, 1999.

A Balanced Growth

A.1 Proof of Lemma 2

It's possible to have balanced growth paths in Zones 1, 2, and 3, depending on the configurations for exogenous technological progress. Denote the constant gross rates of process and product innovation by $g_z \equiv z_{t+1}/z_t$ and $g_{\mathfrak{N}} \equiv \mathfrak{N}_{t+1}/\mathfrak{N}_t$. Let process and product innovation be in balance so that $g_z = g_{\mathfrak{N}} > 1$. The analysis proceeds using a guess-and-verify technique. Conjecture that along any balanced growth path that labor supply, l_t, and the rental rate on capital, r_t, are constant. Also, suppose that k_{t+1} and $1/\lambda_t$ grow at rate g_z. For l_t to be constant it is clear from the efficiency condition for labor supply (7) that the wage rate, w_t, must grow at the same rate as z_t or g_z. Additionally, the Euler equation (6) for capital accumulation implies that along a balanced growth $r_t = g_z/\beta - (1-\delta)$.

Zone 1 (Extensive Margin)

Suppose that $g_{\mathfrak{N}} \geq g_z \geq 1$, but start with the case where $g_{\mathfrak{N}} = g_z$. Conjecture that N_t grows at *gross* rate g_z. In this Zone $c_t = \mathfrak{c}$, $p_t = 1$ and $\pi_t = 0$, so it is clear that the budget constraint (2) holds along a balanced growth path. The first-order condition (4) for the number of varieties consumed is satisfied because N_t grows at rate g_z while λ_t declines at the same rate. Turn now to the situation where $g_{\mathfrak{N}} > g_z$. Here, along a balanced growth path $N_t/\mathfrak{N}_t \leq 1$, with $N_t/\mathfrak{N}_t = 0$ asymptotically. It's easy to verify that above analysis still holds. Intuitively speaking, the logic for $N_t < \mathfrak{N}_t$ is now even stronger, with condition (4) for N_t still holding.

Zone 2 (Shackled Margins)

Assume that $g_{\mathfrak{N}} = g_z \geq 1$. In this Zone $c_t = \mathfrak{c}$, $N_t = \mathfrak{N}_t$. Equation (11) implies that prices, p_t, will be constant, because $\lambda_t \mathfrak{N}_t$ is constant. It is clear that the budget constraint (2) holds along a balanced growth path. The first-order conditions for intensive and extensive margin consumption, (3) and (4), remain slack because the lefthand and righthand sides both grow at the rates $1/g_z$.

Zone 3 (Intensive Margin)

Let $1 \leq g_{\mathfrak{N}} \leq g_z$. In this Zone $N_t = \mathfrak{N}_t$ and $p_t = 1/\theta$. Conjecture that consumption grows at the rate $g_c \equiv c_{t+1}/c_t = g_z/g_{\mathfrak{N}}$. The budget constraint (2) will hold along a balanced growth path, given this conjecture. The first-order condition for consumption (3) along the intensive margin is fulfilled because $1/(N_t c_t)$ declines at rate g_z, the same as λ_t. Observe that when $g_{\mathfrak{N}} = g_z$ consumption will be constant along a balanced growth path, otherwise it grows less that the rate of process innovation, g_z, because $g_{\mathfrak{N}} \geq 1$.

Corollary. *[Capital-to-effective-labor ratio] Along a balanced growth, $k_t/(z_t l_t)$, is the same and constant in all three zones implying that k_t grows at rate g_z.*

Proof of Corollary

Start with the capital-to-effective-labor ratio, $k_t/(z_t l_t)$. The Euler equation (6) governing capital accumulation is the same in all three zones. Along it balanced growth path it reads $\lambda_t/\lambda_{t+1} = \beta[\gamma k_{t+1}^{\gamma-1}(z_{t+1}l_{t+1})^{1-\gamma} + (1-\delta)]$. In all three zones $\lambda_t/\lambda_{t+1} = g_z$, implying that $k_t/(z_t l_t) \equiv \kappa$ must be the same and time invariant. This can be deduced from the equations for λ_t in each of three zones–(9), (11), and (14)–while applying the above properties for c_t, N_t, and p_t. Now, along a balanced growth path investment and output can be written as $i_t = [\gamma_z - (1-\delta)]k_t$ and $o_t = k_t(z_t l_t/k_t)^{1-\gamma}$. Thus, i_t/o_t is the invariant across zones. It is now immediate from the national income identity that c_t/o_t must be the same across zones.

A.2 Proof of Lemma 3

Before proceeding to the proof, some equations that will be utilized are presented.

Preliminaries

First note that the wage rate can be written as $w_t = (1-\gamma)k_t^{\gamma}(z_t l_{mt})^{-\gamma} = z_t(1-\gamma)\kappa^{\gamma}$,where $\kappa \equiv k_t/(z_t l_t)$ is the stationary capital-to-effective-labor ratio. From the national income identity along a balanced growth path $N_t c_t = (\kappa^{\gamma} z_t - \hat{\delta}\kappa z_t)l_t$, where $\hat{\delta} \equiv g_z - (1-\delta)$.

Zone-2 price equation. From equation (12),

$$p_t = \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{1}{\mathfrak{N}_t \mathfrak{c}} \times \frac{w_t}{l_t^{\chi}}.$$

Using the above formulae for the wage rate and the national income identity, this can be rewritten as

$$p_t = \frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{(1-\gamma)\kappa^{\gamma}\left(\kappa^{\gamma} - \hat{\delta}\kappa\right)^{\chi}}{\mathfrak{c}^{1+\chi}}\left(\frac{z_t}{\mathfrak{N}_t}\right)^{1+\chi}. \tag{24}$$

Observe prices rise with $z_t/\mathfrak{N}_t$, as Figure 10 illustrates.

Zone-3 consumption equation. Plugging in the formula for the wage rate and the national income identity into the first-order condition for labor supply (15) results in

$$l_t = \left[\frac{\alpha}{1-\alpha}\theta\frac{(1-\gamma)\kappa^{\gamma}}{\kappa^{\gamma} - \hat{\delta}\kappa}\right]^{1/(1+\chi)}. \tag{25}$$

Note that labor supply is constant in Zone 3.[6] Next, using the national income identity again yields

$$c_t = (\kappa^\gamma - \hat{\delta}\kappa)\frac{z_t}{\mathfrak{N}_t} l_t = (\kappa^\gamma - \hat{\delta}\kappa)\left[\frac{\alpha}{1-\alpha}\theta\frac{(1-\gamma)\kappa^\gamma}{\kappa^\gamma - \hat{\delta}\kappa}\right]^{1/(1+\chi)} \frac{z_t}{\mathfrak{N}_t}. \tag{26}$$

Consumption is an increasing function of $z_t/\mathfrak{N}_t$, as displayed in Figure 10.

Proof. Attention is now directed to determining the three zones, $\underline{z/\mathfrak{N}}$, $\widetilde{z/\mathfrak{N}}$, and $\overline{z/\mathfrak{N}}$.

Determination of $\underline{z/\mathfrak{N}}$. To have a smooth transition in prices, p, from Zone 1 to 2 as shown in Figure 10, it must be the case that $p = 1$ at the start of Zone 2.[7] Hence, using formula (24) for Zone-2 pricing

$$1 = \frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{(1-\gamma)\kappa^\gamma\left(\kappa^\gamma - \hat{\delta}\kappa\right)^\chi}{\mathfrak{c}^{1+\chi}}(\underline{z/\mathfrak{N}})^{1+\chi}.$$

Determination of $\widetilde{z/\mathfrak{N}}$. This defines the first transition point between Zone 2 and Zone 3. Here there is a discrete drop in price when there is a switch to the Zone 3 equilibrium. Can this Zone 3 equilibrium be sustained? Suppose that *all* other firms are charging a price of $1/\theta$ and are selling $c \geq \mathfrak{c}$. Is it profitable to deviate and charge a price of $p > 1/\theta$ and sell $c = \mathfrak{c}$? The answer is no, because in Zone 3 $p = 1/\theta$ is the profit maximizing price for an individual producer. Move next to the determination of $\widetilde{z/\mathfrak{N}}$, which defines one of the transition points between Zone 2 and Zone 3. Now it must be the case that in Zone 3

$$c_t = \frac{\kappa^\gamma - \hat{\delta}\kappa}{\mathfrak{N}_t}\left(\frac{\alpha}{1-\alpha}\theta\frac{(1-\gamma)\kappa^\gamma}{\kappa^\gamma - \hat{\delta}\kappa}\right)^{\frac{1}{1+\chi}} z_t \geq \mathfrak{c},$$

where formula (26) for Zone-3 consumption has been used. This implies that[8] [9]

$$\widetilde{z/\mathfrak{N}} = 1/\left\{\frac{(\kappa^\gamma - \hat{\delta}\kappa)}{\mathfrak{c}}\left[\frac{\alpha}{1-\alpha}\theta\frac{(1-\gamma)\kappa^\gamma}{\kappa^\gamma - \hat{\delta}\kappa}\right]^{\frac{1}{1+\chi}}\right\}. \tag{27}$$

[6] Following the same procedure for labor supply in Zone 1 results in

$$l_t = [\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{(1-\gamma)\kappa^\gamma}{\kappa^\gamma - \hat{\delta}\kappa}]^{1/(1+\chi)}.$$

So, labor supply is constant in Zone 1 and is greater that in Zone 3. Thus, another property of Figure 10 is established.

[7] There will also be a smooth transition in labor supply. The national income identity implies that $l_t = (N_t/z_t)\mathfrak{c}/(\kappa^\gamma - \hat{\delta}\kappa)$ in Zone 1 while in Zone 2 it reads $l_t = (\mathfrak{N}_t/z_t)\mathfrak{c}/(\kappa^\gamma - \hat{\delta}\kappa)$. As the threshold $\underline{z/\mathfrak{N}}$ is approached N_t converges to $\mathfrak{N}_t$ implying that labor supply in the two Zones is the same at this point–see Figure 10.

[8] Inserting (27) into (24) leads to $p = 1/\theta^2$. Therefore the Zone-2 price is greater than the Zone-3 one where $p = 1/\theta$. So, at threshold $\widetilde{z/\mathfrak{N}}$ the Zone-2 price is as shown in Figure 10.

[9] Recall that Zone-2 labor supply is given by $l_t = (\mathfrak{N}_t/z_t)\mathfrak{c}/(\kappa^\gamma - \hat{\delta}\kappa)$. At the threshold $z_t/\mathfrak{N}_t = \widetilde{z/\mathfrak{N}}$. Substituting this into the Zone-2 labor supply condition gives the Zone-3 value for labor supply or equation (25). Thus, the labor supplies in Zones 2 and 3 connect at the point $z_t/\mathfrak{N}_t = \widetilde{z/\mathfrak{N}}$. Again, see Figure 10.

This can be rewritten as

$$\widetilde{z/\mathfrak{N}} = \frac{1}{\theta^{2/(1+\chi)}}\underline{z/\mathfrak{N}}. \tag{28}$$

Determination of $\overline{z/\mathfrak{N}}$. This is the second transition point, $\overline{z/\mathfrak{N}}$, between Zone 2 and Zone 3. (Note when in Zone 3 to begin with there will not be a transition.) If the economy is in Zone 2, there will be a discrete drop in prices upon the switch to $1/\theta$. Is it optimal for Zone-2 firms to drop their prices? The answer should be yes. Consider an individual producer who chooses to deviate and charge a price of $1/\theta$ instead of the Zone-2 price p. This producer faces the demand function given by equation (5):

$$c = D(p) = \left[\frac{\alpha}{\lambda p \int_0^{\mathfrak{N}} c_i^{\theta} di}\right]^{\frac{1}{1-\theta}} = \left[\frac{\alpha\theta}{\lambda \mathfrak{N}\mathfrak{c}^{\theta}}\right]^{\frac{1}{1-\theta}},$$

where every other producer sells the quantity $\mathfrak{c}$. From equation (11),

$$\lambda = \frac{\alpha}{\theta}\frac{1}{p\mathfrak{N}\mathfrak{c}},$$

where p is the equilibrium Zone-2 price. Plugging this into the demand function yields

$$c = \left(\theta^2 p\right)^{\frac{1}{1-\theta}}\mathfrak{c}.$$

The individual producer will deviate if and only if

$$(p-1)\,\mathfrak{c} \leq \left(\frac{1}{\theta}-1\right)c$$

or

$$(p-1) \leq \left(\frac{1}{\theta}-1\right)\left(\theta^2 p\right)^{\frac{1}{1-\theta}},$$

which rewrites as

$$\left(\frac{1}{\theta}-1\right)\theta^{\frac{2}{1-\theta}}p^{\frac{1}{1-\theta}} - p + 1 \geq 0.$$

Finally using the price equation (24) for Zone 2, delivers the threshold equation

$$\left(\frac{1}{\theta}-1\right)\theta^{\frac{2}{1-\theta}}\left(\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{(1-\gamma)\,\kappa^{\gamma}\left(\kappa^{\gamma}-\hat{\delta}\kappa\right)^{\chi}}{\mathfrak{c}^{1+\chi}}\left(\overline{z/\mathfrak{N}}\right)^{1+\chi}\right)^{\frac{1}{1-\theta}}$$
$$-\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{(1-\gamma)\,\kappa^{\gamma}\left(\kappa^{\gamma}-\hat{\delta}\kappa\right)^{\chi}}{\mathfrak{c}^{1+\chi}}\left(\overline{z/\mathfrak{N}}\right)^{1+\chi} + 1 = 0.$$

Now from (27),

$$(\frac{1}{\widetilde{z/\mathfrak{N}}})^{1+\chi} = \frac{\alpha}{1-\alpha}\theta\frac{(1-\gamma)\kappa^{\gamma}(\kappa^{\gamma}-\hat{\delta}\kappa)^{\chi}}{\mathfrak{c}^{1+\chi}}.$$

This implies that

$$\left(\frac{1}{\theta}-1\right)\theta^{\frac{2}{1-\theta}}(\frac{1}{\theta^2})^{\frac{1}{1-\theta}}\left(\frac{\overline{z/\mathfrak{N}}}{\widetilde{z/\mathfrak{N}}}\right)^{\frac{1+\chi}{1-\theta}} = \frac{1}{\theta^2}\left(\frac{\overline{z/\mathfrak{N}}}{\widetilde{z/\mathfrak{N}}}\right)+1. \tag{29}$$

Establishing that $\underline{z/\mathfrak{N}} < \widetilde{z/\mathfrak{N}} < \overline{z/\mathfrak{N}}$. The fact that $\underline{z/\mathfrak{N}} < \widetilde{z/\mathfrak{N}}$ is immediate from (28). Is $\overline{z/\mathfrak{N}} > \widetilde{z/\mathfrak{N}}$? To answer this, evaluate (29) at $\overline{z/\mathfrak{N}}/\widetilde{z/\mathfrak{N}} = 1$. The left hand side of (29) collapses to $(1-\theta)/\theta$, while the right hand side reduces to $1/\theta^2+1$. Therefore, the left hand side is smaller than the right hand one at $\overline{z/\mathfrak{N}}/\widetilde{z/\mathfrak{N}} = 1$, implying that $\overline{z/\mathfrak{N}}/\widetilde{z/\mathfrak{N}}$ must be increased to obtain equality. Thus, it has been established that $\underline{z/\mathfrak{N}} < \widetilde{z/\mathfrak{N}} < \overline{z/\mathfrak{N}}$. Now, by following the above chain of logic, the left hand side of (29) will exceed the right hand one for any $z/\mathfrak{N} > \overline{z/\mathfrak{N}}$, so that a Zone-2 firm will want to deviate and reduce its price to $p = 1/\theta$. □

B Smaller Adjustment Costs Generate a Smaller Crash

In the example shown in Figure 21, the adjustment cost parameter ϕ has been reduced to generate a crash half as large as shown in Figure 16. Even without adjustments costs the simulation furnishes a crash in GDP and investment as displayed in Figure 14.

C Solution Algorithm

Even though it is stochastic, the model with rational exuberance can be solved using multiple shooting. Before proceeding, recall that a vertical arrow superscript, ↑, signifies the state-contingent value of a variable when a move up the ladder is possible–again refer to Figure 17. The horizontal arrow superscript, →, denotes a variable's state-contingent value when a stall has happened. Represent the stochastic Euler equation (23) by

$$MC_t^{\uparrow}(k_t^{\uparrow}, k_{t+1}^{\uparrow}) = \beta\tau_{t,t+1}MB_{t+1}^{\uparrow}(k_{t+1}^{\uparrow}, k_{t+2}^{\uparrow}) + \beta(1-\tau_{t,t+1})MB_{t+1}^{\rightarrow}(k_{t+1}^{\uparrow}, k_{t+2}^{\rightarrow}). \tag{30}$$

In the above equation, $MC_t^{\uparrow}(k_t^{\uparrow}, k_{t+1}^{\uparrow})$ characterizes the marginal cost of investing $k_{t+1}^{\uparrow}$ units of capital in period t. The term $MB_{t+1}^{\uparrow}(k_{t+1}^{\uparrow}, k_{t+2}^{\uparrow})$ represents the marginal benefit in period $t+1$ of investing $k_{t+1}^{\uparrow}$ units of capital should $\mathfrak{N}_t$ move up to $\mathfrak{N}_{t+1}$, which happens with probability $\tau_{t,t+1}$. In this state the individual will be choosing $k_{t+2}^{\uparrow}$ units of capital to take over to period $t+2$. Likewise, term $MB_{t+1}^{\rightarrow}(k_{t+1}^{\uparrow}, k_{t+2}^{\rightarrow})$ indicates the marginal benefit in period $t+1$ of

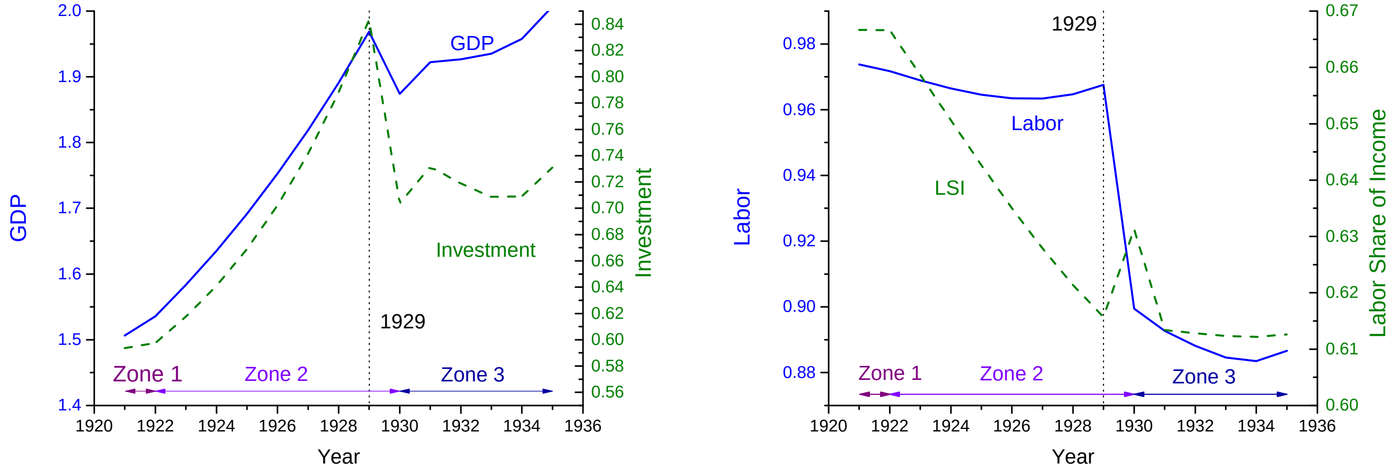

Figure 21: The Deeper Crash with Smaller Adjustment Costs. The left panel shows GDP and Gross Investment, while Labor and Labor Share of Income are displayed in the right one. The model matches reasonably well the U.S. data for 1920s, but now generates about half of the observed crash for the early 1930s.

investing $k^{\uparrow}_{t+1}$ units of capital should $\mathfrak{N}_t$ stall. In this state the individual will be choosing $k^{\rightarrow}_{t+2}$ units of capital to take over to period $t+2$.

Now the time path for capital accumulation is deterministic after a stall occurs. Denote the solution for $k^{\rightarrow}_{t+2}$ by

$$k^{\rightarrow}_{t+2} = K^{\rightarrow}_{t+1}(k^{\uparrow}_{t+1});$$

more on this in Section C.2. This allows the Euler equation (30) to be rewritten as

$$MC^{\uparrow}_t(k^{\uparrow}_t, k^{\uparrow}_{t+1}) = \beta\tau_{t,t+1}MB^{\uparrow}_{t+1}(k^{\uparrow}_{t+1}, k^{\uparrow}_{t+2}) + \beta(1-\tau_{t,t+1})MB^{\rightarrow}_{t+1}\big(k^{\uparrow}_{t+1}, K^{\rightarrow}_{t+1}(k^{\uparrow}_{t+1})\big). \quad (31)$$

Equation (31) can then be thought of as a second-order difference equation in $k^{\uparrow}_t, k^{\uparrow}_{t+1}$, and $k^{\uparrow}_{t+2}$. If one knew $k^{\uparrow}_t$ and $k^{\uparrow}_{t+1}$, this equation could be solved for $k^{\uparrow}_{t+2}$. This is where multiple shooting comes into play.

C.1 Multiple Shooting

To implement multiple shooting two boundary conditions are needed. The first boundary condition is the capital stock at the start of time, k_1. The second boundary condition is the terminal capital stock when the top of the product innovation ladder has been reached. For illustration purposes, suppose one move up to $\mathfrak{N}_T$. Then, the economy will enter a steady state situation where $k_t = k^*_{T+1}$ for all $t \geq T+1$. The idea of the multiple shooting algorithm is to pick $k^{\uparrow}_{t+2}$ so that $|k^{\uparrow}_{T+1} - k^{*}_{T+1}| < \varepsilon$. To do this a function needs to be written that returns a value for $k^{\uparrow}_{T+1}$, given a guess for $k^{\uparrow}_{t+2}$; write this as

$$k^{\uparrow}_{T+1} = \mathbf{K}_{T+1}(k^{\uparrow}_{t+2}).$$

So, a solution for $k^{\uparrow}_{t+2}$ in the equation below is being sought, which can be found using a non-linear equation solver.

$$\mathbf{K}_{T+1}(k^{\uparrow}_{t+2}) - k^{*}_{T+1} = 0.$$

Embedded in the function $\mathbf{K}_{T+1}(k^{\uparrow}_{t+2})$ is a loop running from $t = 1, \cdots, T$. The loop proceeds as follows:

1. Start off in period 3. Given the starting value, k_1, and guess for $k^{\uparrow}_2$, a solution for $k^{\uparrow}_3$ can be found from

$$MC^{\rightarrow}_1(k_1, k^{\uparrow}_2) = \beta\tau_{1,2} MB^{\uparrow}_2(k^{\uparrow}_2, k^{\uparrow}_3) + \beta(1-\tau_{1,2}) MB^{\rightarrow}_2\big(k^{\uparrow}_2, K^{\rightarrow}_2(k^{\uparrow}_2)\big).$$

2. Next, the given the guess, $k^{\uparrow}_2$, and the value for $k^{\uparrow}_3$ that was just obtained, a solution for $k^{\uparrow}_4$ can be found from

$$MC^{\rightarrow}_2(k^{\uparrow}_2, k^{\uparrow}_3) = \beta\tau_{2,3} MB^{\uparrow}_3(k^{\uparrow}_3, k^{\uparrow}_4) + \beta(1-\tau_{2,3}) MB^{\rightarrow}_3\big(k^{\uparrow}_3, K^{\rightarrow}_3(k^{\uparrow}_3)\big).$$

3. Keep iterating down the path until period $T+1$ is reached. Solve for $k^{\uparrow}_{T+1}$ using

$$MC^{\rightarrow}_{T-1}(k^{\uparrow}_{T-1}, k^{\uparrow}_T) = \beta\tau_{T-1,T} MB^{\uparrow}_T(k^{\uparrow}_T, k^{\uparrow}_{T+1}) + \beta(1-\tau_{T-1,T}) MB^{\rightarrow}_T\big(k^{\uparrow}_T, K^{\rightarrow}_T(k^{\uparrow}_T)\big),$$

   while utilizing the previous solutions for $k^{\uparrow}_{T-1}$ and $k^{\uparrow}_T$.

C.2 The Function $K^{\rightarrow}_{t+1}(k^{\uparrow}_{t+1})$

To implement the algorithm, a function for the stall paths needs to be written. Once a stall has happened, however, the model is deterministic. The subsequent time path for the capital stock is now governed by the simpler Euler equation

$$MC^{\rightarrow}_{t+1}(k^{\rightarrow}_{t+1}, k^{\rightarrow}_{t+2}) = \beta MB^{\rightarrow}_{t+2}\big(k^{\rightarrow}_{t+2}, k^{\rightarrow}_{t+3}\big).$$

This can also be solved using multiple shooting. This function will be utilized for every potential stall path. There are T of these.