Unemployment Cycles*

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June 2016

Abstract

The labor market by itself can create cyclical outcomes, even in the absence of exogenous shocks. We propose a theory that shows that the search behavior of the employed has profound aggregate implications for the unemployed. There is a *strategic complementarity* between active on-the-job search and vacancy posting by firms: active search changes the number of searchers and the duration of a job, and in the presence of sorting, it improves the quality of the pool of searchers. More vacancy posting in turn makes costly on-the-job search more attractive, a self-fulfilling belief. The absence of on-the-job search discourages vacancy posting, rendering costly on-the-job search unattractive. This model of multiple equilibria can account for large fluctuations in vacancies, unemployment, and job-to-job transitions; it provides a rationale for the Jobless Recovery through a novel channel of the employed searchers crowding out the unemployed; and it gives rise to a shift in the Beveridge Curve (the unemployment-vacancy locus).

Keywords. On-the-job search. Strategic Complementarity. Unemployment Cycles. Sorting. Mismatch. Job-to-job flows. Jobless Recovery.

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*We are grateful to Jess Benhabib, Katarina Borovickova, Hector Chade, Mike Golosov, Boyan Jovanovic, Guido Menzio, Fabien Postel-Vinay, Edouard Schaal and Randy Wright for helpful discussion and comments. We also benefited from the feedback of many seminar audiences. Eeckhout gratefully acknowledges support from the ERC, Grant 339186, and from RecerCaixa. Throughout, we benefitted from the excellent research assistance of Julia Faltermeier and Adrià Morrón-Salmerón.

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1 Introduction

Business cycles have a wide variety of origins, ranging from financial crises, over oil price shocks, to productivity spurts and slowdowns. Often, of all economic agents, workers are those affected most dramatically, mainly through unemployment. For long, researchers – most notably Diamond (1982) – have asked whether frictional markets can generate cyclical outcomes, even in the absence of any exogenous shocks or changes in fundamentals. But so far there has been no compelling mechanism where the *labor market by itself* can generate cycles and that fits the facts. In this paper, we propose a simple theory that generates self-fulfilling unemployment fluctuations and that can account for the key labor market facts: fluctuations in unemployment, vacancies and job-to-job flows. We will argue that our model provides a simple rationale for the Jobless Recovery, the fact that it takes a long time for unemployment to drop even after vacancies and productivity have recovered, and for the outward shift of the Beveridge Curve.

The main driving force behind unemployment cycles is the *search behavior of those with a job*. Singling out the employed to explain unemployment may seem counterintuitive. But with a share of over ninety percent of the labor force, any minor change in the behavior of the employed, who vie for job openings side by side with the unemployed in the same labor market, has profound aggregate implications for unemployment. Even if they search much less intensively than the unemployed, because of their shear size, *on average* still about half of the job searchers are workers who were employed already. Most importantly, we document that there is also a huge *variation* in the composition of the searcher pool, ranging from 42% of employed workers in the recession to 62% in the boom, mostly due to the change in the search behavior of the employed over the business cycle. And it is precisely this variation that has drastic implications for the unemployed during the recession and recovery.

We contribute to the theoretical literature by spelling out a model that features a *strategic complementarity* between on-the-job search and vacancy creation, giving rise to multiple equilibria. In their search decision, workers trade off the matching probability against the cost of searching. In turn, in their vacancy posting decision, firms take into consideration both the expected quality (or productivity) as well as duration of the job. When workers actively search while on-the-job, this tends to push up the firm's value of a job: In the presence of sorting, searchers tend to move to jobs with higher match quality, and with active search the relative number of on-the-job searchers compared to unemployed is higher. This gives rise to a *composition externality* in the pool of searchers. At the same time, under active search, the expected duration of a job is shorter, pushing the value of a job down. It is precisely the combination of the composition externality and the different duration of a job that is at the root of the multiplicity. With active on-the-job search, the higher expected match quality dominates the shorter job duration, which creates incentives for vacancy creation. More vacancies in turn create incentives for workers to actively search on-the-job since it is easier to find one. Active job search has become self-fulfilling. This high churning outcome corresponds to an economic boom with active on-the-job search, high employment-to-employment (EE) transitions, little mismatch, abundant job creation, low unemployment and high aggregate output. But there is also another equilibrium, the recession, where workers do not actively search on-the-job, where the pool of searchers has relatively few on-the-job searchers and the expected productivity of a job is low. For firms, the shorter duration of jobs here dominates the impact of the composition externality. As a result, firms have little incentives to post vacancies, which generates a low matching probability for workers that does not compensate the cost of search. Again, this low search intensity is self-fulfilling. It leads to low job turnover and high mismatch, low aggregate output and high unemployment. In the recession, workers experience grim job prospects and hang on to their existing jobs, even if mismatched. Firms take solace in the long duration of jobs, even if they are of low productivity.[1]

This economic mechanism is based on three components: 1. on-the-job search; 2. sorting (wage ladder) with mismatch; 3. endogenous vacancy creation. Each of these ingredients is crucial for the self-fulfilling equilibria. Without endogenous on-the-job search, there is no choice on the worker side and the firms always face jobs of equal expected duration as well as the same composition of job searchers. Without sorting, say if all jobs are identical and there is no mismatch, there is no efficiency gain from moving from the first job and as a result, firms would not prefer hiring on-the-job searchers - there is a unique equilibrium. It is not only necessary that there is sorting, there must be a sufficiently large output gain in order to obtain the multiplicity. This is due to the fact that firms face shorter job durations with active on-the-job search, and only a high enough output gain will compensate that loss. Finally, for the strategic complementarity between firm and worker behavior to arise, firms must respond to the on-the-job search behavior of workers, so vacancy creation needs to be endogenous.

The strategic complementarity and the resulting endogenous fluctuations can account for three features of the labor market and how it evolves between boom and recession. First, even in the absence of any exogenous shocks (for instance to productivity and to financial markets) there are big *fluctuations* in unemployment, vacancies and job-to-job transitions generated by the labor market itself. As self-fulfilling beliefs change and workers switch from active to passive search (while firms go from a high to a low rate of vacancy posting) this results in sizable fluctuations of unemployment and employment-to-employment (EE) flows. In a boom, workers switch jobs frequently whereas in a recession they do so much less, as can be seen from Figure 1.A. EE flows in the US exhibit large fluctuations, up to 13% above trend during expansion and up to 25% below trend during the last recession. This is consistent with the share of on-the-job searchers in overall searchers fluctuating between 42% and 62%, as reported above. In our model, when beliefs switch, there is precisely such a sudden drop in job-to-job transitions. Because employed workers constitute the large majority of the labor force, even a minor change in their behavior has an enormous impact on the labor market, in particular on the unemployed.

The second labor market feature the model can account for is the *Jobless Recovery*. Even after

[1] Notice that the strategic complementarity that drives the multiplicity stems from the non-contractibility of on-the-job search: firms cannot sign contracts with workers that are contingent on their search behavior.

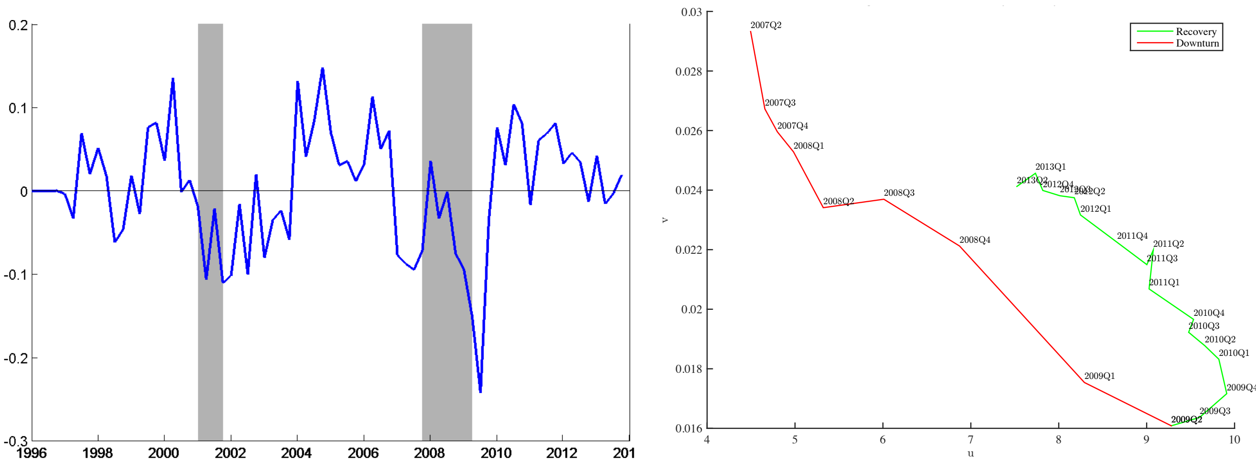

Figure 1: A. Employment-to-Employment Rate (de-trended using HP-filter, see Section 5 for details). B. Outward Shift of the Beveridge Curve in the Great Recession.

productivity has picked up following a recession, unemployment remains sluggishly high. In late 2014 for example, five years after the end of the great recession, unemployment was still at 6%, more than one percentage point above the rate observed in the preceding boom in 2007. Why does it take so long for unemployment to recover, even after financial markets have turned healthy, productivity has recovered and the economy is growing? Ever since the term was first used in the aftermath of the great depression in 1935, economists have speculated about its possible origins and mechanisms. Popular and largely unfounded explanations aside such as permanent technological change (the Luddite fallacy[2]) or irreversible structural change in the labor market, academics struggle to find a satisfactory explanation for it. Here we argue that a jobless recovery is inherent in the cyclical behavior of unemployment due to fluctuations in on-the-job search. We identify a new channel where the employed are *crowding out* the unemployed. At the end of a recession, as beliefs switch to an active on-the-job search regime and firms add many vacancies, the composition of the pool of searchers changes. Almost instantaneously, the on-the-job searchers crowd out the unemployed searchers. Overall, job flows pick up, but with random matching they go disproportionately to the on-the-job searchers who are abundant after the recession, and not to the unemployed. All the renewed activity thus initially translates in better jobs for the employed and higher labor productivity, but in no improvement in the prospects for the unemployed. After some time this crowding out fades as the fraction of those with a job who have found a good match has grown, and the number of on-the-job searchers decreases again relative to the unemployed searchers.

Third, the endogenous job market fluctuations naturally give rise to a *shift in the Beveridge Curve* (the locus of vacancies and unemployment). While researchers have pondered over observed shifts of the Beveridge Curve ever since it was introduced, recently there has been a renewed interest because

[2]During the second industrial revolution, the Luddites, textile workers in England, feared that automated looms would lead to mass unemployment. It is considered a fallacy because the laid off textile workers were eventually hired in new, higher productivity jobs in different industries.

of a sizable outward shift following the Great Recession (see Figure 1.B).[3] An outward shift is often interpreted as a decrease in matching efficiency: to maintain a given level of unemployment, a larger number of vacancies needs to be posted. A deterioration of labor market efficiency during the recovery is puzzling. One would expect to the contrary that part of the recovery is due to improved matching. We argue that the shift of the Beveridge Curve is not due to matching efficiency but rather due to a shift in the *effective* market tightness. While the Beveridge Curve depicts the ordinary market tightness, i.e., ratio of vacancies to unemployed, the *effective* market tightness is the ratio of vacancies to *all searchers*, including in the denominator the on-the-job searchers as well as the unemployed. Consider the comparison between an equilibrium with active and passive on-the-job search. When employed workers start to actively search on-the-job, effective tightness drops. This affects the value of creating a vacancy in multiple ways: negatively, because the job duration on average is shorter due to active on-the-job search; positively, because there are more searchers so it is easy to fill a vacancy; and positively also, because the pool of searchers has improved. Active on-the-job search equilibrium requires that the net effect for the firm is positive, but for any given level of vacancies, with more searchers, the *effective* labor market tightness measured as vacancies over *all* searchers has gone down. Responsible for the shifting Beveridge Curve is therefore a large difference across equilibria in the *endogenous argument* of the matching function – i.e., the effective market tightness – and not the exogenous matching technology. This crowding out results in lower job finding probabilities for the unemployed and contributes to the Jobless Recovery during the transition to a new steady state.

Much of our theoretical analysis focuses on steady state equilibria where we are agnostic about what drives changes in equilibrium beliefs but we also address the issue of equilibrium selection and explore the dynamics of this model. We adopt the history-dependent criterion to select equilibrium first proposed by Cooper (1994). This selection criterion relies on changes in the economy's fundamentals, which we specify here as Markov productivity shocks in the presence of forward-looking agents: agents' beliefs only switch if the productivity shock is large enough to destroy the previous equilibrium and pushes the economy towards the other (now unique) equilibrium. After a recession, a reasonably large positive productivity shock can push the economy into the boom equilibrium. In this interpretation of the model, the *original cause* of a recession or a boom is a productivity shock that moves the economy outside the range of existing equilibria. This leads the economy in to a new equilibrium regime that is very different from the former one, generating large changes in labor market outcomes.

The contribution of this paper is predominantly theoretical in that we identify a new mechanism behind labor market fluctuations, which is driven by the search behavior of the employed. However, to illustrate the economic relevance of our mechanism, we also conduct an empirical exercise and include an application of the model to the Great Recession. We first provide direct evidence in favor of pro-

[3]The outward shift is substantial. For example, with 1.9% of vacancy creation, the unemployment rate at the end of 2008 was 7.5% while with the same vacancy creation, the unemployment rate at the beginning of 2010 was 9.5%. Most people (including the NBER) would argue that 2010 was a solid recovery, yet unemployment was higher.

cyclical search intensity of employed workers. Second, we analyze the quantitative implications of the model's mechanism. To this end, we calibrate the model to the US economy and show that its multiple equilibria can account for a significant part of the observed fluctuations in unemployment, vacancies and EE flows over the business cycle. Third, we show that along the transition path from the recession into a boom (i.e. during the recovery), the economy exhibits a Jobless Recovery, with unemployment *increasing* after the peak of the crisis, due to a crowding out of unemployed by employed searchers.

Related Literature. We are intellectually indebted to several earlier contributions and ideas that have shaped our thinking on this topic. The most celebrated model of self-fulfilling employment fluctuations is without a doubt Diamond (1982).[4] He proposes a very interesting search mechanism with search in production as well as in exchange in the goods market. Trade in the goods market is faster if the population of traders is larger. This in turn, leads to faster production and lower unemployment due to search in production. In addition to this high production, low unemployment equilibrium, there is also an equilibrium with a small population in the goods market and long unemployment in the production market. The multiplicity is due to the thick market externality: the more people search, the higher the probability of trading. While our source of multiplicity is similar since it also stems from endogenous behavior affecting the matching function, we do not rely on increasing returns to matching, a counterfactual feature of the matching technology (Pissarides and Petrongolo (2001)).

The source of multiplicity in our model is also related to Burdett and Coles (1997). In a simple two-type model with two-sided heterogeneity and non-transferable utility, they show that even with constant returns in the matching technology, there can be multiple equilibria. The driving force is thus not Diamond (1982)'s market size or network externality, but rather a *selection externality.* If high types believe other high types are not selective and accept matches with low types, the equilibrium distribution of searchers will have a low fraction of high types provided the economy-wide share of high types is low enough. This in turn makes the expected value of search low, inducing high types to accept low types. Instead, if the high types believe other high types will be selective and accept matches only with other high types (rejecting all low types), then the steady state fraction of high types is high (provided the economy-wide fraction of high types is below a threshold) and the continuation value of search is high. The best response therefore is to accept high types only. These beliefs are therefore self-fulfilling, driven by the externality from the composition in the stationary distribution of searchers. While the composition effects of active on-the-job search in our model is somewhat similar to the selection externality first proposed in Burdett and Coles (1997), the model has quite different predictions. In Burdett and Coles (1997) it is difficult to map the two equilibria into a boom or a recession. In the selective equilibrium, the mismatch is low and output is high (as in a boom), but unemployment is *high* as well, whereas in the non-selective equilibrium, mismatch is high and output is low (as in the recession), but unemployment is *low.*

[4]Diamond and Fudenberg (1989) further elaborate on the model and analyze the non-stationary rational expectations equilibrium properties.

In the spirit of Diamond (1982)'s goods market externality, Kaplan and Menzio (2014) ask the archetypical Keynesian question whether externalities in the goods market can affect production in the labor market: if people do not demand goods, there will be less production, leading to unemployment and hence lower demand.[5] It is well known that with competitive goods markets, prices instantaneously adjust and there is no feedback or propagation from of demand on unemployment. Kaplan and Menzio (2014) model goods demand by means of a frictional goods market where the unemployed search more intensely for low goods prices. As unemployment in the labor market is higher, markups in the goods market are lower which in turn leads to lower production, lower vacancies, and more unemployment. They show that this shopping externality can thus generate multiple self-fulfilling steady state equilibria, which they, as we do here, interpret as business cycles. In Kaplan and Menzio (2014) of course, the business cycle is due to a demand externality, and not and externality in the labor market itself. Moreover, their mechanism does not speak to the jobless recovery.[6] In an interesting approach in a related framework, Schaal and Taschereau-Dumouchel (2014) have a model with multiple equilibria but focus on equilibrium selection using global games. While this guarantees a unique equilibrium, it maintains the amplification through multiple steady states. Finally, a recent paper by Golosov and Menzio (2015) also obtains fluctuations driven exclusively by the labor market. It features a mechanism of providing incentives in the presence of moral hazard and it is most efficient to do so through firing during recessions. Interestingly, to obtain the results, the model has decreasing, not increasing returns to matching.

Two interesting mechanisms in very different settings are related to ours. In the marriage market, Burdett, Imai, and Wright (2004) unearth a source of multiplicity from the strategic interaction *within a match* that stems from the fact that both partners in the match search (in addition to the economy wide, aggregate feedback of beliefs that drives the multiplicity in our model). If my partner searches "on the marriage", my match will dissolve with higher probability than if she does not search, and therefore my match value is lower. This in turn increases the incentives to search myself, a self-fulfilling belief. In the housing market, Moen, Nenov, and Sniekers (2015) derive multiple equilibria where homeowners who switch houses decide to sell their current house before or after they buy the new house. While this has small implications for the value of housing for a given individual (the transition of a few months is very small compared to the average duration of ownership, about eight years in the US), it has big aggregate implications which are reflected in housing prices. Using micro level data from housing in Denmark, they find huge changes in the fraction of "on-the-home" buyers versus those who go through a period of being without a home (which is comparable to unemployment in the labor market).

In his seminal paper, Shimer (2005) argues that in the standard Mortensen-Pissarides model of unemployment, productivity fluctuations cannot account for the fluctuations in unemployment and

[5] In an early contribution, Howitt and McAfee (1992) address a similar question.

[6] Mortensen (1999) and Sniekers (2013) also consider a demand externality but explain cycles through a limit cycle around one steady state.

vacancies observed in the data.[7] Hall (2005) (wage stickiness) and Hagedorn and Manovskii (2008) (the high value of unemployment) have offered explanations to counter Shimer's finding, and can indeed create labor market volatility from small productivity shocks. We do not see our main contribution in adding to this debate. We do however need to take a stance on the parameter values we use in our calibration. In the light of this debate and of additional recent work,[8] we will use an intermediate value of the replacement rate (75%). In any event, our mechanism can generate multiple steady states for *any* value of unemployment benefits and we show in our counterfactual exercises that on-the-job search behavior is more important in generating labor market fluctuations than productivity movements, even when unemployment benefits are set relatively high.

The paper is organized as follows. In Section 2, we set out the environment. In Section 3, we show that multiple steady state equilibria arise. In Section 4, we analyze the dynamic equilibria and their stability. Section 5 contains an empirical validation exercise of pro-cyclical search behavior and a quantitative exercise where we assess the model's implications for labor market fluctuations and jobless recovery. Finally, Section 6 offers some concluding remarks.

2 Environment

The model economy is a simplified version of Postel-Vinay and Robin (2002). It features on-the-job search with two productivity types and a highly stylized form of job ladder. It is aimed to capture the main forces of search models with on-the-job search and sorting (à la Shimer and Smith (2000)): a worker who already has a job will only move to a new job if the new job is more productive. Therefore, the types of jobs out of unemployment are on average less productive than those that are accepted when moving from an existing job.

The most natural way of modeling this would be exactly as in Postel-Vinay and Robin (2002) with random arrival of job types of high and low productivity.[9] We analyze that model in Appendix B.1 and can show that the multiplicity survives, but it is much less tractable and notationally more cumbersome. It turns out that we can capture all the aspects of interest in this economy in a very simple way through a two-step job ladder: we assume that all jobs out of unemployment have low match productivity, and all jobs out of an existing job have high match productivity. This is a drastic assumption, but it turns out that the logic of our argument is similar to that in the general model, as we show in the Appendix

[7] In a theory of rest and search unemployment, a variation of the Mortensen-Pissarides search model, Jovanovic (1987) shows that productivity fluctuations also generate pro-cyclical search behavior (in addition to pro-cyclical productivity and countercyclical unemployment) as here, but without the amplification from equilibrium multiplicity that we highlight.

[8] Chodorow-Reich and Karabarbounis (2013) *directly* estimate from micro data the value of being unemployed. They find a value of unemployment relative to productivity of 0.75 that is higher than Shimer (2005)'s (0.4), and lower than Hagedorn and Manovskii (2008) (0.95). In addition, they find the contribution of unemployment benefits to the value of unemployment is very small (16%) and that the estimated value of unemployment is pro-cyclical.

[9] See also Moscarini and Postel-Vinay (2012) for a similar job ladder model without explicit sorting. The full blown model of sorting with on-the-job search and a continuum of types is analyzed computationally in Robin and Lise (2013) and Lamadon, Lise, Meghir, and Robin (2013).

and explain in the *Remarks on Assumptions* at the end of Section 3.1.[10] We have therefore opted to use the simpler, stylized setup for analytical tractability and exposition in which all the relevant action reduces to two value functions per worker/job.

Agents and Technology. Time is continuous, $t \in \mathbb{R}_+$. There is a measure one of risk neutral workers in the economy. A worker is unemployed and searching for a job, or employed, in which case he can choose to search *actively* or *passively* on-the-job. We assume that on-the-job search only takes place in low productivity jobs (see Appendix B.2 for a model that relaxes this assumption but preserves the key mechanism). The flow utility from being unemployed is b and the flow utility of employment is equal to the wage, w. The search cost when unemployed or under passive on-the-job search is normalized to zero and the search cost for *active* search when employed is k, so search costs increase in search intensity. Workers maximize the value of employment: They search actively if the gain from active search exceeds the cost. Otherwise, they search passively at no cost (more below).

There is a large measure of potential jobs (firms). Firms can open a job paying a flow cost c. If they stay inactive their payoff is zero. Firms are risk neutral and maximize the discounted sum of profits. Denote the measure of job openings by v. All jobs are ex ante identical, but ex post heterogeneous in their job productivity y. We assume the technology is given by $f(y) = py$, where p is aggregate and $y \in \{\underline{y}, \overline{y}\}$ is match-specific productivity.[11] When a job is filled by an unemployed worker, the productivity is $\underline{y}$ and when it is filled by a formerly employed worker the productivity is $\overline{y}$, with $\underline{y} \leq \overline{y}$.[12] This captures in a stylized way the economy's job ladder: Workers tend to be better matched to the new job after they switch, which is reflected in the data by substantial wage gains after EE transitions. We model this job ladder as improvements in the match-specific component of a worker-firm pair.

Denote the measure of the unemployed by u; the measure of the employed in a low productivity job $\underline{y}$ by γ; the measure of the employed in high productivity jobs $\overline{y}$ by ξ. Since the measure of workers is equal to one, feasibility requires that $u + \gamma + \xi = 1$.

Market Frictions, Search and Wage Setting. Meetings between jobs and workers are stochastic, and are modeled by means of a standard matching function $m(v, s)$, where m is increasing, concave and

[10]To demonstrate the robustness of our multiplicity result we consider three more general models in Appendix B: 1. One where the job quality is match-specific and each match (both with employed and unemployed) is of high productivity $\overline{y}$ with probability π, and where workers search until they extract the full rents. This is exactly the specification in Postel-Vinay and Robin (2002) with two productivity types; and 2. One with the setup as in our benchmark model with deterministic productivity upgrade, except that workers can search until they extract the full rents; 3. One where firms choose to open either a high or a low productivity job where the productivity is permanent (instead of match-specific). We will discuss both at the end of Section 3 and also in the Appendix why multiple equilibria emerge in these models.

[11]Below we assume that not only match-specific productivity y but also the cost of on-the-job search k and unemployment benefits b are proportional to p. This is consistent with the findings of Chodorow-Reich and Karabarbounis (2013) that the value of unemployment is pro-cyclical.

[12]If the surplus of a low type match is positive, it is optimal for the firm to accept this match even if that surplus is lower than the surplus of a high type match. In our calibration below, the low type match surplus is positive. If the low type match surplus were negative instead, our formulation implicitly assumes that firms commit to hire any worker type, whether she is hired out of unemployment or from an on-the-job move.

has constant returns to scale, and where v denotes the measure of vacancies and s the measure of job searchers. Therefore the matching probability for a worker is $m(\theta)$, where $\theta = \frac{v}{s}$, and that of a firm is $q(\theta) = \frac{m(\theta)}{\theta}$. Job separation is exogenous and occurs at rate δ.[13]

The employed always engage in passive search at no cost (some job opportunities arrive without search effort) which leads to a match with probability $\lambda_0 m(\theta)$, where λ_0 is the search intensity of passive on-the-job searchers relative to the search intensity during unemployment, which is normalized to one. They can also engage in active search and search at intensity $\lambda_0 + \lambda_1$ by incurring the search cost k.[14] In return they get a higher chance of a match, $(\lambda_0 + \lambda_1)m(\theta)$. Therefore the effective measure of workers searching for a job, s, is given by $u + \lambda_0\gamma$ if no employed worker actively searches on-the-job and by $u + (\lambda_0 + \lambda_1)\gamma$ if all employed worker actively search on-the-job. λ_0 and λ_1 denote the efficiency of the on-the-job matching technology. The resulting market tightness is then a function of the total measure of searchers expressed in "efficiency units", denoted by the *effective* number of searchers s, equal to $u + \lambda_0\gamma$ or $u + (\lambda_0 + \lambda_1)\gamma$. For example, when all workers actively search on the job, the market tightness is given by $\theta = \frac{v}{u+(\lambda_0+\lambda_1)\gamma}$ and when none search actively on the job it is given by $\theta = \frac{v}{u+\lambda_0\gamma}$. Notice that we distinguish the effective market tightness $\theta = \frac{v}{s}$ that takes into account *all* effective job searchers from the conventional market tightness in the Mortensen-Pissarides search model, here denoted by $\Theta = \frac{v}{u}$, which takes into account only unemployed job searchers.

Wages are determined in a sequential auction framework as in Postel-Vinay and Robin (2002) (see also Dey and Flinn (2005)). Employment contracts stipulate a fixed wage over time that the employer commits to and that can be renegotiated only if both parties agree. The contract cannot condition on the search intensity, which is private information. Firms can fire workers and workers are free to quit at will. As a result, when workers receive outside offers, wages may be renegotiated. Current and outside employers Bertrand-compete for the worker. The worker goes to the match that generates higher total value and receives a wage such that his continuation value equals the match value with the least productive of the two competing firms. If no outside offer arrives, wages remain unchanged. If the worker is hired out of unemployment, wages are such that she receives the option value of unemployment.

Individual Decision Problems and Bellman Equations. We denote the value of a vacant job by V, of a filled job by J, of an unemployed worker by U, and of an employed worker by E. Unless there is cause for confusion, we drop the time subscript t to keep the notation parsimonious. When we denote the value of a job for an employed worker, we use the notation $\underline{E}$ ($\overline{E}$) to indicate that a worker is employed in a low (high) productivity job. Likewise, $\underline{J}$ ($\overline{J}$) denotes the value of a low (high) productivity job that is filled with a worker coming out of unemployment (a low productivity job).

[13]Even though endogenous separations in the presence of stochastic match surplus is an important determinant of labor market dynamics (see amongst others Mortensen and Pissarides (1994) and Bils, Chang, and Kim (2011)), we aim to make the point in this paper under exogenous separations.

[14]An alternative way of modeling this would be through continuous search intensity on-the-job, where workers *choose* an interior non-zero search intensity under a convex cost. This could also give rise to multiple equilibria with a high and a low intensity on-the-job search outcome that are determined endogenously. Unfortunately we cannot solve that case analytically. Observe that our cost is a step function and hence convex.

Denote by $\omega \in [0,1]$ the decision by the individual worker whether to actively search on-the-job and which equals one when the worker searches actively and is zero otherwise. And denote by $\mathbf{\Omega} \in [0,1]$ the behavior of all other workers in a symmetric strategy equilibrium. Throughout we assume that individual search behavior ω is private information, so a firm cannot make the wage contingent on search effort. For the remainder, we also use the notation $\lambda(\omega) = \lambda_0 + \omega\lambda_1$ for the individual search intensity and $\lambda(\mathbf{\Omega}) = \lambda_0 + \mathbf{\Omega}\lambda_1$ for the aggregate search intensity.

Workers. We can write the values of workers as follows.

$$\begin{aligned}
rU &= pb + m(\theta(\mathbf{\Omega}))(\underline{E} - U) + \dot{U} && (1)\\
r\underline{E} &= \underline{w}(\mathbf{\Omega}) - \omega pk + \lambda(\omega)m(\theta(\mathbf{\Omega}))(\overline{E} - \underline{E}) - \delta(\underline{E} - U) + \dot{\underline{E}} && (2)\\
r\overline{E} &= \overline{w}(\mathbf{\Omega}) - \delta(\overline{E} - U) + \dot{\overline{E}} && (3)
\end{aligned}$$

where $\dot{U}$ is the time derivative of U (likewise for all other values) and where $\theta(\mathbf{\Omega}) = \frac{v}{s(\mathbf{\Omega})} = \frac{v}{u+\lambda(\mathbf{\Omega})\gamma}$.

It is key to observe here that individual search decisions ω affect only the value of the employed in low productivity jobs, $\underline{E}$, through the cost of job search k and the probability of finding a job λ_1. Aggregate outcomes from the population behavior at large enter into values through two channels: 1. market tightness $\theta(\mathbf{\Omega})$ and 2. wages $w(\mathbf{\Omega})$. The market tightness induces a search externality that affects the job finding probabilities of all job searchers: the value of the employed in a low productivity job as well as the value of the unemployed. But also wages change depending on the belief whether workers search on-the-job or not. Whenever it is clear from the context whether $\mathbf{\Omega}$ is zero or one, we will drop the dependence of the market tightness and wages on $\mathbf{\Omega}$ in the notation.

Firms. The value of a vacancy to a firm deciding to open one is given by:

$$rV = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{s(\mathbf{\Omega})}\underline{J} + \frac{\lambda(\mathbf{\Omega})\gamma}{s(\mathbf{\Omega})}\overline{J} - V\right] + \dot{V}. \quad (4)$$

Because we assume free entry and a large measure of potentially entering firms, the value of a vacancy V is driven to zero. So, in equilibrium, equation (4) is evaluated at $V = 0$. Observe that the measure of vacancies adjusts instantaneously. Whenever there is a positive value for vacancies, vacancies are created frictionlessly to set the profits back to zero.

The value of a filled job, low and high productivity, is given by:

$$\begin{aligned}
r\underline{J} &= p\underline{y} - \underline{w}(\mathbf{\Omega}) - [\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})) + \delta](\underline{J} - V) + \dot{\underline{J}} && (5)\\
r\overline{J} &= p\overline{y} - \overline{w}(\mathbf{\Omega}) - \delta(\overline{J} - V) + \dot{\overline{J}}. && (6)
\end{aligned}$$

The flow value of a high type job is output net of wages. Once it is filled, the job lasts until there is exogenous separation with probability δ. The low type job similarly generates a flow value of $p\underline{y} - \underline{w}$

and separates exogenously at rate δ, but in addition faces separation from on-the-job search.

Labor Market Dynamics. At any point in time, the laws of motion for unemployment and employment satisfy:

$$\begin{aligned} 1 &= u + \gamma + \xi && (7) \\ \dot{\gamma} &= u m(\theta(\mathbf{\Omega})) - \gamma[\delta + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega})] && (8) \\ \dot{\xi} &= \gamma \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega})) - \xi\delta && (9) \end{aligned}$$

Equation (7) is simply feasibility. The measure of workers consisting of unemployed u, on-the-job searchers γ, and high productivity workers who obtained the job through on-the-job search ξ and is equal to the measure of the entire worker population. In (8), the change in the stock of on-the-job searchers equals the difference between flow into low productivity jobs out of unemployment and the flow out of low productivity job, which consists of separation δ and the outflow due finding a better job. Finally, the change in the stock of high productivity workers equals the difference between in- and outflow from high productivity jobs.

Definition of equilibrium. We can now define equilibrium.

Definition 1 *A Perfect Foresight Equilibrium is a tuple* $\{U, \underline{E}, \overline{E}, V, \underline{J}, \overline{J}, \theta, u, \gamma, \xi, \underline{w}, \overline{w}, \omega, \mathbf{\Omega}\}$ *such that*

1. $U, \underline{E}, \overline{E}, V, \underline{J}, \overline{J}$ *satisfy the Bellman equations above;*
2. *Given* $\mathbf{\Omega}$*,* $\omega = \mathbf{\Omega}$ *maximizes* $\underline{E}$*;*
3. *There is free entry:* $V = 0$*;*
4. *Wages:* $\underline{w}$ *is such that* $\underline{E} = U$ *and* $\overline{w}$ *is such that* $\underline{J} = V$*;*
5. u, γ, ξ *satisfy the laws of motion;*
6. $\lim_{t\to\infty} \theta_t = 0$ *is finite for initial conditions* $u_{-1}, \gamma_{-1}, \xi_{-1}$*.*

3 Steady State Equilibrium

We first focus on steady state equilibrium. We solve the system of equilibrium equations, taking into account that values and stocks are constant over time. First, wages are determined following the sequential auction wage setting (Postel-Vinay and Robin (2002)) and are pinned down by the worker's outside option. A firm that hires an unemployed worker will offer her a wage $\underline{w}(\mathbf{\Omega})$ that makes her indifferent between accepting the job and remaining unemployed, i.e. $\underline{E} = U$. Likewise, the firm offers a wage $\overline{w}(\mathbf{\Omega})$ that makes the worker with an on-the-job offer indifferent between joining the new, high productivity firm $\overline{y}$ and staying at the existing low productivity firm $\underline{y}$, extracting the whole surplus from the previous firm, i.e. $\underline{J} = V$. Some algebra (see Appendix A) reveals that

$$\underline{w}(\mathbf{\Omega}) = p\left[b\left(\frac{r+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))+\delta}{r+\delta}\right) - \frac{\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}{r+\delta}\underline{y} + \mathbf{\Omega}k\right] \tag{10}$$

$$\overline{w}(\mathbf{\Omega}) = p\underline{y}. \tag{11}$$

Observe that we have used the population-wide average of active searchers $\mathbf{\Omega}$ and not the individual level ω since the wage reflects the firm's belief about the workers' search behavior but cannot condition on the actual (unobserved) search behavior of the particular worker that is hired. Having solved for wages, we next pin down θ from free entry where firms take wages and search behavior as given. Finally, for given θ, we determine u, γ, ξ from the flow equations in steady state, setting time derivatives to zero.

3.1 Multiple Steady States

We now turn to the multiplicity of steady state equilibria. We construct two candidate equilibria in which either no worker searches actively on-the-job, $\mathbf{\Omega} = 0$, or all search actively on-the-job $\mathbf{\Omega} = 1$. To ascertain whether these candidate equilibria are indeed equilibria, we have to check whether there is no deviation by any individual worker or firm.

One shot deviations. For a steady-state to exist, it is sufficient to check that one-shot deviations by a firm or a worker are not profitable. To exclude the firms' one-shot deviations is straightforward since firms only have a participation decision to make in the presence of free entry and hence zero profits. In our current setup we have restricted the contract space to constant wages until the arrival of an outside offer as is customary in much of the literature. In Appendix C we show that even if firms can offer a wage contract with backloading, they nonetheless do not want to deviate from constant wages under natural parameter restrictions and there continue to be multiple equilibria.

On the worker side, it is sufficient to check one-shot deviations from the worker's strategy in the low-productivity job. The value of unemployment is pinned down by the exogenous value of unemployment benefits b. As a result, the value of unemployment is independent of the equilibrium search intensity $\mathbf{\Omega}$. Likewise, the worker value in high productivity jobs is independent of endogenous behavior since those workers do not search and only exogenous separation leads to unemployment. This implies that U and $\overline{E}$ are independent of the search decision ω, and we can directly check the deviations of those employed in a low productivity job $\underline{E}$.

To evaluate deviations by an individual worker, we introduce the following notation. We define $\underline{E}(\omega|\mathbf{\Omega})$ as the worker's value in the low productivity job, while taking action ω *for an instant* dt and then reverting to the equilibrium action $\mathbf{\Omega}$. This captures the notion of a one shot deviation, or equivalently Bellman optimality.

We now check two possible deviations and derive conditions under which those deviations are not individually rational: 1. when all are actively searching on-the-job, there is no deviation by a single agent to stop active search if:

$$\underline{E}(1|\mathbf{1}) > \underline{E}(0|\mathbf{1}) \quad \Longleftrightarrow \quad m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right) < \theta(\mathbf{1}).$$

2. when no worker is actively searching on-the-job, there is no deviation of a single agent to search if:

$$\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0}) \quad \Longleftrightarrow \quad \theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right).$$

These two no-deviation conditions give rise to the following result.

Lemma 1 *There exist multiple steady state equilibria if and only if*

$$\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right) < \theta(\mathbf{1}).$$

All proofs of this section are in Appendix A. Under the condition that the value of market tightness is not too extreme, there exists two pure strategy equilibria, one where all search actively and one where no one actively searches on-the-job. Whenever the two pure strategy equilibria exist, there is also a mixed strategy equilibrium where every agent actively searches on-the-job with probability $\omega = \mathbf{\Omega} \in (0,1)$ in every interval of time dt, i.e. workers randomize between the choice of search effort (see Appendix A for the formal statement). This is illustrated in Figure 2, where we plot the mutual best-responses of workers' search effort and firms' vacancy posting (reflected by tightness). The workers' best response to tightness is an increasing step function and the firms' best response of tightness to workers search effort is an increasing function as well, indicating the strategic complementarity. Points A and C mark the pure strategy steady state equilibria while B indicates the steady state equilibrium in workers' mixed strategies. For the remainder of the paper, we will focus attention on the two pure strategy steady states.

Of course, tightness $\theta(\mathbf{\Omega})$ is an endogenous object. Unfortunately, we cannot in general compute conditions under which Lemma 1 is satisfied. We do find a necessary and sufficient condition in terms of the primitives of the model under a particular matching function, the telegraph matching function:

$$m(\theta) = \phi\frac{\alpha\theta}{\alpha\theta+1}, \tag{12}$$

where ϕ is the overall matching efficiency of the matching technology and α is a parameter that determines curvature.[15] In much of what follows, we will use the telegraph for three reasons. First, it has all

[15]This is also the matching function used in the money and search literature, where ϕ and α are set to one. There it is interpreted as a matching process where buyers (money holders) and sellers are one population, and hence under uniform random matching, the likelihood of meeting a buyer is proportional to the number of buyers in the total population of buyers and sellers: $m(\theta) = \frac{\theta}{1+\theta} = \frac{b}{b+s}$ where $\theta = \frac{b}{s}$ is the ratio of buyers to sellers.

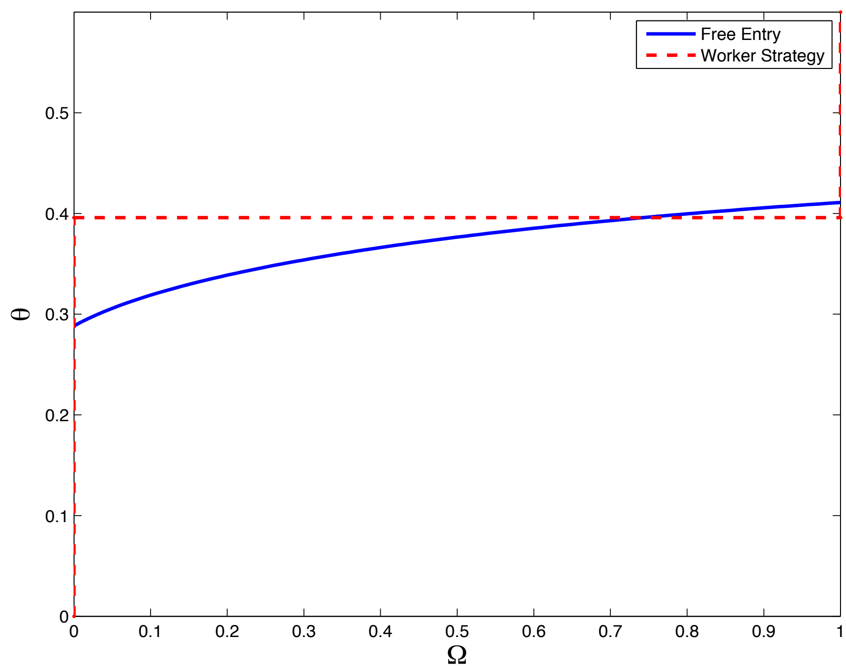

Figure 2: Set of Equilibria: A. No active on-the-job search $\mathbf{\Omega} = 0$; C. Active on-the-job search $\mathbf{\Omega} = 1$; B. Mixed Strategy Equilibrium $\mathbf{\Omega} \in (0, 1)$. (Parameters taken from the calibrated economy below.)

the desirable features of a matching function, being a special case of the CES matching function. Second, with the level parameter ϕ and the shape parameter α, we can approximate precisely the matching functions used in the literature (such as Shimer (2005), Hagedorn and Manovskii (2008)). Finally, it is tractable and allows us to prove many results analytically.

Under these assumptions, we can then establish the following result:

Proposition 1 *Let* $m(\theta) = \phi \frac{\alpha\theta}{\alpha\theta+1}$. *Then there are multiple steady state equilibria if and only if* $p \in [p_l, p_h]$. *The set* $[p_l, p_h]$ *is non-empty for an open set of parameters.*

See Appendix A for the exact expressions for $[p_l, p_h]$. In other words, this result rewrites the necessary and sufficient condition for multiple equilibria in Lemma 1 as a condition on exogenous parameters. This in turn is here interpreted as an interval of values for aggregate productivity p: values for productivity in that interval are a necessary and sufficient condition for multiplicity. Moreover, that interval is non-empty for a robust set of parameters. The intuition is straightforward: If aggregate productivity is too high, $p > p_h$, then all workers want to search actively to take advantage of high match-specific productivity jobs whose productivity is now augmented by high aggregate productivity. The passive search equilibrium breaks down and the active search equilibrium is unique. The opposite occurs if productivity is too low, $p < p_l$. In Figure 3.A we illustrate the multiplicity region, by plotting equilibrium outcome θ for different values of p. Observe that θ is always increasing in productivity, both with and without active on-the-job search.

It is important to note that these p-bounds are functions of all other parameters of this economy. We have performed detailed comparative statics, though we do not report them here. In general, a change in a parameter value shifts both bounds in *the same* direction.

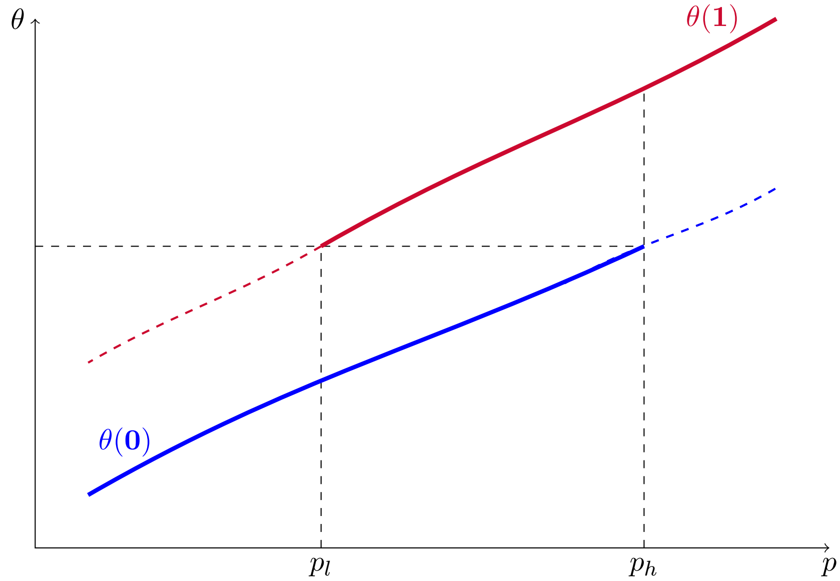

Figure 3: Effective market tightness $\theta = \frac{v}{s}$: equilibrium with multiplicity range as a function of aggregate productivity p.

This condition for multiplicity can of course also be expressed in terms of any of the exogenous variables other than p. The next result states that the existence of multiple equilibria is closely related to the gains from sorting, i.e. the difference $\overline{y} - \underline{y}$. For low gains from sorting, there is a unique equilibrium with no on-the-job search. On-the-job search has two costs: 1. the direct search cost k incurred by the worker; and 2. the indirect search cost incurred by the firm due to shorter expected duration of a job. As a result, everything else equal, the opportunity cost of opening a job to the firm is higher. This indirect cost then explains why there cannot be active on-the-job search in equilibrium when the productivity gains from on-the-job search (measured by $\overline{y} - \underline{y}$) are arbitrarily small. If there is hardly any output gain when filling a job with an employed worker but a discrete increase in the opportunity cost due to shorter job duration, this discourages vacancy posting, and in turn disincentivizes workers' search effort. As a result, it is a dominant strategy not to search.

At the other extreme, when the output gain $\overline{y} - \underline{y}$ is arbitrarily large for given search cost k and vacancy cost c, the gains from on-the-job search swamp costs, irrespective of the behavior of other workers. It is then a dominant strategy to always search. For tractability, we focus in this result on the case $\delta \to 0$, which, by continuity of the free entry condition in δ, implies that the result holds for small $\delta > 0$.

Proposition 2 *(Sorting Gains Needed for Active On-The-Job Search). Let* $\delta \to 0$*, and* $m(\theta) = \phi\frac{\alpha\theta}{\alpha\theta+1}$.

1. *When the gains from sorting are small* ($\overline{y} - \underline{y} < \epsilon$) *then there is a unique steady state with non-active on-the-job search;*

2. *When the gains from sorting are arbitrarily high, there is a unique steady state with active on-the job search.*

3. *For intermediate gains from sorting* $\overline{y} \in [\overline{y}_l, \overline{y}_h]$ *(for given* $\underline{y}$*) there are multiple equilibria.*

Our results in this section illustrate the mechanism that gives rise to the strategic complementarity and hence multiplicity. Firms trade off the expected quality or productivity of a job (which changes due to the composition externality) against job duration. And workers trade off the matching probability against the cost of searching. With active on-the-job search, there is more sorting and the value to the firm of a job is higher, which creates incentives for vacancy posting. More vacancies in turn create incentives for workers to actively search on-the-job since it is easier to find a job. Likewise, there is also an equilibrium where workers do not actively search on-the-job, where the pool of searchers has relatively few on-the-job searchers and is of relatively low match quality. For firms, the shorter duration of jobs then dominates the impact of the composition externality, and as a result, they post few vacancies. This in turn leads workers not to search actively. Our results from this section show that too large or too small sorting or productivity gains from on-the-job search resolve the trade-offs for firms and workers in an unambiguous way, leading to a unique steady state equilibrium.

Remarks on the Assumptions. Before analyzing the equilibrium properties, we discuss some of our main assumptions, their motivation and whether they are crucial for the multiplicity of steady state equilibria.

First, we assume in our baseline model that there is a two-step job ladder, where on-the-job searchers receive a *deterministic* match-specific productivity upgrade. One of the implications is that (under certain parameter restrictions), firms prefer hiring on-the-job searchers compared to unemployed workers, fueling the strategic complementarity between vacancy posting and search effort that leads to multiplicity. Besides analytical tractability, this assumption is based on evidence that unemployed workers accept lower-quality jobs offers (Faberman, Mueller, Sahin, and Topa (2016)) and there is substantial wage growth as workers climb up the job ladder (see for example Faberman (2015), Haltiwanger, Hyatt, and McEntarfer (2015), and Gertler, Huckfeldt, and Trigari (2016)), which could have various causes: 1. OJS improves the pool of job applicants (i.e. there is selection); 2. Employed workers are better in directing their search compared to unemployed workers, leading to a mismatch-reducing effect of OJS; 3. Employed workers accumulate more human capital through learning-by doing on the job compared to unemployed workers. All three channels can micro-found the reduced-form job ladder that we assume in our model. It is, however, beyond the scope of our paper to analyze these channels in more detail.

Even though this shows that there is supporting evidence that our simplified job ladder is not a poor approximation, we also show in the Appendix that the multiplicity is robust to introducing *stochastic* productivity upgrades, where both unemployed and employed workers receive them with the same probability, similar to Postel-Vinay and Robin (2002) where heterogenous job offers randomly arrive to both employed and unemployed workers from the job sampling distribution (see Appendix B, Section 1).[16] There we also relax the assumption of a single round of on-the-job search (Appendix B, Section 2).

What transpires from these exercises is that similar strategic complementarities between on-the-job search and vacancy posting generate multiple steady state equilibria in more general environments. This shows that the multiplicity is *not* due to the specific job ladder we assume but roots more deeply in the interplay between composition externality and job duration. Even though overall job duration is lower under active search, the heterogeneity of productivity (i.e. firms with a high match specific shock can extract surplus if they meet an employed worker who is currently in a firm with low match-specific shock) and the fact that a match is of longer duration when formed with an employed compared to an unemployed searcher, make on-the-job searchers for firms attractive. This again triggers a strategic complementarity between search intensity and vacancy posting as in our baseline model. We thus establish that our simple model setup inherits all the key features of the general setup with the benefit of a substantial gain in tractability.

Second, we assume in our baseline model that productivity is match-specific. Appendix B, Section 3 shows that multiplicity of equilibria does not hinge on this assumption either. There we introduce *ex-ante* productivity differences of firms that are permanent, i.e. firms can either open a low or a high productive vacancy. Employed and unemployed workers meet both types of vacancies with the same probabilities.

Third, we further assume that unemployed workers (as opposed to employed workers) do not choose their search intensity endogenously, for two reasons: 1. The empirical studies on how search intensity of the unemployed varies over the cycle are inconclusive. There is evidence on both (slightly) counter-cyclical search intensity (Mukoyama, Patterson, and Sahin (2014)) and pro-cyclical search intensity (Schwartz (2014)). Note that our results would go through if we interpreted the on-the-job search intensity *relative* to the unemployed search intensity. 2. In our model, even if we introduced endogenous search intensity of the unemployed, they would always (independent of the business cycle) choose a *unique* level of search intensity. This is due to the sequential auction wage setting with constant value of unemployment where the gains for the unemployed from search are constant. Note also that we do not include the flows of those Not in the Labor Force. While they are sizable, the cyclical properties of the search intensity of those Not in the Labor Force is the opposite of that of the employed: it is counter-cyclical and less pronounced. In Appendix A we report the measure for search intensity. As a

[16] In this Postel-Vinay and Robin (2002) model with 2 types, there are many more value functions and most importantly, there are $2^4 = 16$ candidate equilibria, depending on various choices of search intensity at different parts of the job ladder. In principle, one needs to check no-deviation conditions for each of these candidate equilibria. We pick two specific equilibria out of the 16 candidate ones and show that they can co-exist for certain parameter ranges.

result, our mechanism is not convoluted by the fact that those Not in the Labor Force flood the labor market during the boom (in addition to the employed), quite to the contrary.

Fourth, we assume that both employed and unemployed workers randomly search for jobs in a single labor market. It is true that for our mechanism to work, there cannot be completely segregated labor markets for these two groups of searchers since in that case the discussed composition externality would be shut down. But we could allow for partly directed search.

Fifth, one may think that the multiplicity hinges on the wage setting protocol that we assume and, in particular, on the assumption that wages remain unchanged if no outside offer arrives. It is well known that in the presence of endogenous on-the-job search, commitment to a fixed wage can be improved upon with a time varying contract (see for example Lentz (2014)). However, nothing about the mechanism that generates multiplicity here is particular to our contractual setting. We show in Appendix C that even if firms can deviate from a fixed wage to a simple contract with backloading, there is multiplicity.

Last, we assume that separations are constant in the model. This is clearly not borne out in the data, see Fujita and Ramey (2009). However, the focus of our attention is on the interaction between search intensity of the employed and vacancy creation by firms in a model of multiplicity, which is why we make the simplifying assumption of exogenous and constant separations.

3.2 Steady State Equilibrium Properties

We depict the equilibrium in the canonical Beveridge Curve diagram, i.e., the unemployment-vacancy space. Since this is the convention, we continue to do so in the (u, v)-space with corresponding market tightness $\Theta = \frac{v}{u}$. However, since matching probabilities are a function of the effective market tightness $\theta = \frac{v}{u+\lambda(\mathbf{\Omega})\gamma} = \frac{v}{s}$, we then also report the *effective* Beveridge curve, in the (s, v) space.

From the flow equations (7)–(9), evaluated in steady state, we obtain the stock of the unemployed and workers employed in low-productivity jobs:

$$u = \frac{\delta}{\delta + m(\theta(\mathbf{\Omega}))} \tag{13}$$

$$\gamma = \frac{\delta m(\theta(\mathbf{\Omega}))}{[\delta + m(\theta(\mathbf{\Omega}))][\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))]}. \tag{14}$$

Equation (13) is the Beveridge curve, which through $\theta = \frac{v}{u+\lambda\gamma}$ now not only depends on u and v but also on γ. Even though the equilibrium system is expressed in terms of u, γ, θ, as is common practice in this literature, we *plot* the system in terms of u, γ, v, where v is an immediate transformation of θ: $v = \theta(u + \lambda\gamma)$. This allows us to interpret the well-known Beveridge curve (BC). The BC is a steady state relationship between vacancies and unemployment, in this setting, for a given stock of on-the-job searchers γ, derived from the flow equation (13) in and out of unemployment. In our setting there is also a second flow equation (14) in and out of low productivity jobs, which we label the γ-curve (γC) and which also depicts a relationship between vacancies, unemployment and γ. In Figure 4, we plot the equilibrium. For the sake of clarity, we plot the intersection between the BC and the γC, which is now a two-dimensional manifold instead of a three-dimensional one. For the assumed matching function, this intersection BC $\cap$ γC is given by

$$v = -\frac{\delta s(2\delta(-1+s) + \phi(\lambda(-2+s) + s - \sqrt{\lambda^2(-2+s)^2 + s^2 - 2\lambda s^2}))}{-2\alpha\delta(\delta + 2\lambda\phi) + 2\alpha(\delta+\phi)(\delta+\lambda\phi)s} \tag{15}$$

(which is derived from (13) and (14) by summing $\lambda\gamma + u$ and solving for v as a function of s) and is denoted by BC^s, i.e. the Beveridge curve in (u, s) space. In turn, the conventional Beveridge curve can be explicitly expressed as:

$$v = \frac{\delta u(1-u)[2\lambda(\mathbf{\Omega})(1-u) + u]}{\alpha[u(\delta+\phi) - \delta][\lambda(\mathbf{\Omega})(1-u) + u]} \tag{16}$$

Note that the effective BC^s has the same properties as the BC in the standard Pissarides framework (it is downward sloping and convex).

Finally, we plot the Free Entry condition (from equation (28) in Appendix A) which gives the equilibrium measure of posted vacancies given any level of unemployment u and on-the-job searchers γ. The intersection between the Free Entry manifold and BC^s marks the steady state. In the Figure 4, we plot both the steady state equilibrium for active and for passive on-the-job search (and for clarity we omit the equilibrium in mixed strategies).

For our purpose, of interest are the properties across the multiple equilibria, which are summarized in the following Proposition.

Proposition 3 *Let there be multiple steady state equilibria. Then:*

1. *unemployment is lower with active on-the-job search:* $u(\mathbf{1}) < u(\mathbf{0})$*;*

2. *EE flows are higher with active on-the-job search:* $EE(\mathbf{1}) > EE(\mathbf{0})$*;*

and under the telegraph matching function:

3. $BC(\mathbf{1})$ *is shifted outward relative to* $BC(\mathbf{0})$

4. $BC^s(\mathbf{1})$ *is shifted outward relative to* $BC^s(\mathbf{0})$ *(given* $\lambda(\mathbf{1}) \leq 1$*).*

5. *vacancies are higher with active on-the-job search:* $v(\mathbf{1}) > v(\mathbf{0})$*;*

6. *market tightness is higher with active on-the-job search:* $\Theta(\mathbf{1}) > \Theta(\mathbf{0})$*;*

7. *the share of on-the-job searchers in overall searchers is higher with active on-the-job search:* $\frac{\lambda(\mathbf{1})\gamma(\mathbf{1})}{s(\mathbf{1})} > \frac{\lambda(\mathbf{0})\gamma(\mathbf{0})}{s(\mathbf{0})}$.

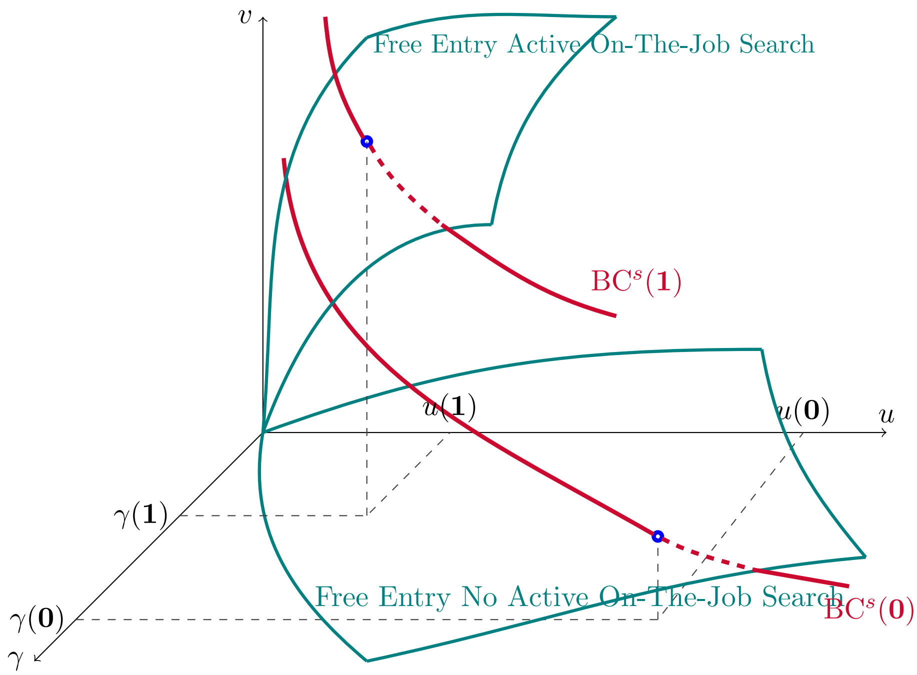

Figure 4: The Equilibrium in 3D with Free Entry manifold and the intersection of the Beveridge Curve and the γ-Curve, for both pure strategy equilibria.

This proposition describes in greater detail what exactly is the difference between the two steady state equilibria whenever they coexist. Many of these features can be observed in Figure 5. It plots the conventional Beveridge Curve that relates vacancies v to unemployment u with the standard market tightness $\Theta = \frac{v}{u}$ for both equilibria. (A graph of the effective Beveridge curves looks qualitatively identical). In line with Lemma 1, if conventional market tightness under active on-the-job search is high enough (intersecting with the bold part of the red Beveridge curve), then this equilibrium exists. In turn, if market tightness under non-active on-the-job search is low enough (intersecting with the bold part of the blue Beveridge curve), then the equilibrium with low search intensity exists.

First, the BC shifts out under active on-the-job search, both in the (u, v) and (u, s) space (3. and 4. of Proposition 3), and hence corresponds to lower match efficiency. There are many more jobs, increasing the matching probability, but the on-the-job searchers crowd out the unemployed. Hence the match efficiency per unemployed worker is lower. Vacancies are higher (5.): there are more job searchers that generate a high productivity match, hence firms have more incentives to open vacancies. Despite the lower match efficiency, unemployment is lower under active on-the-job search (1.). This follows immediately from the flow equation for unemployment and the fact that under multiplicity $\theta(\mathbf{1}) > \theta(\mathbf{0})$ (from Lemma 1): the matching probability increases while job separation is unchanged. Not surprisingly, the EE flow is higher under active on-the-job search (2.). Across the two equilibria, the stock of overall searchers, s, can in fact be smaller under high search intensity. But the quality of

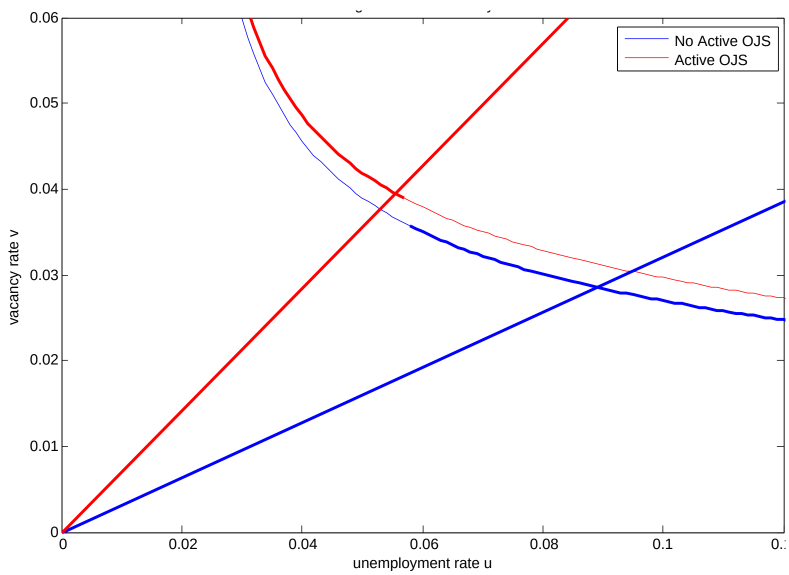

Figure 5: Multiplicity and The Beveridge Curve: The conventional BC, in (u, v) with $\Theta = \frac{v}{u}$ (parameters taken from the calibrated economy below).

the pool of searchers is higher since there are relatively more effective on-the job-searchers, which is the key factor behind higher vacancy posting by firms (7.). Finally, market tightness $\Theta = \frac{v}{u}$ is higher under active on-the-job search (6.), which follows from vacancies being higher and unemployment is lower.

4 Dynamic Equilibrium

The multiplicity of stationary equilibria generates multiplicity of equilibria in the dynamic economy, i.e. there are multiple equilibrium paths. As in the case of stationary equilibria, agents optimally choose actions given their beliefs about other agents. In addition, in the dynamic equilibrium the transversality condition on $\underline{J}$ (or equivalently on θ) must be satisfied. Below in our analysis of the dynamic properties of equilibrium, we will clarify when this is the case. We now turn to the equilibrium dynamics of this economy, focussing on local stability. We can solve this three dimensional dynamic system only by relying on linear approximations, which precludes us from saying anything about global stability.[17]

To start, we rewrite the dynamic system in terms of the two state variables u, γ and the choice variable θ. From the value functions and the flow equations, we can reduce the dynamic economy to

[17]Moreover, in the numerical analysis below, we did not encounter any calibration that yielded a bifurcation point, which would have allowed us, despite linearization, to use the local bifurcation as a building block to construct the global bifurcation diagram.

the following system (see Appendix A for the derivations in this Section):

$$\dot{u} = \quad \delta(1-u) - um(\theta(\mathbf{\Omega})) \tag{17}$$

$$\dot{\gamma} = \quad um(\theta(\mathbf{\Omega})) - (\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\gamma \tag{18}$$

$$\begin{aligned}\dot{\theta}(\mathbf{\Omega}) = \quad & \frac{m(\theta(\mathbf{\Omega}))u}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \times \Bigg[\frac{\lambda}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\left(-\dot{u}\frac{\gamma}{u} + \dot{\gamma}\right) - (p\underline{y} - \underline{w}(\mathbf{\Omega})) \\ & + \left(\frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J}\right)(r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\Bigg]\end{aligned} \tag{19}$$

We assess the local stability of this system by considering the linearized system around the steady state

$$\begin{bmatrix} \dot{u} \\ \dot{\gamma} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{u}}{\partial u}^* & \frac{\partial \dot{u}}{\partial \gamma}^* & \frac{\partial \dot{u}}{\partial \theta}^* \\ \frac{\partial \dot{\gamma}}{\partial u}^* & \frac{\partial \dot{\gamma}}{\partial \gamma}^* & \frac{\partial \dot{\gamma}}{\partial \theta}^* \\ \frac{\partial \dot{\theta}}{\partial u}^* & \frac{\partial \dot{\theta}}{\partial \gamma}^* & \frac{\partial \dot{\theta}}{\partial \theta}^* \end{bmatrix} \begin{bmatrix} u_t - u_i^* \\ \gamma_t - \gamma_i^* \\ \theta_t - \theta_i^* \end{bmatrix} \tag{20}$$

where all partial derivatives are functions of $\mathbf{\Omega}$ and are evaluated at the steady state under consideration (indicated by *) with $\dot{u} = 0, \dot{\gamma} = 0, \dot{\theta} = 0$. The eigenvalues of the Jacobian matrix of (20) give insights into the local stability of system (17)–(19). Since an analytical solution is infeasible, we approach the problem numerically. We compute the eigenvalues of this system below for our calibration and verify the existence of multiple equilibria as well as the stability properties ex-post. Under our calibration (and without imposing any restriction on the parameter space) both steady states are characterized by two negative real eigenvalues and one positive eigenvalue.

Following Theorem 2.1. in Kuznetsov (1998), this implies the dynamics are characterized by a stable manifold of dimension two, and an unstable manifold of dimension one. As a result, each of the steady states is a saddle that forms a node. From any point on the stable two-dimensional manifold, the system converges to the corresponding steady state. Outside the stable manifold, the system diverges. Since the number of negative eigenvalues is equal to the number of predetermined state variables u, γ, this solution is unique (see Acemoglu (2008), Theorem 7.18). As a result, for any u, γ in the neighborhood of the steady state, the choice variable θ will adjust in order to bring the system on the stable manifold, from which it will converge to the steady state.

The topology of the dynamic system is illustrated in Figure 6, where we plot the dynamic properties of the two equilibria via a three-dimensional phase diagram in u, γ, v (instead of v we could have chosen to display θ). Both equilibria essentially exhibit the same dynamic properties, which is saddle-path stability. In each of them, the stable manifold, denoted by $v^S(u, \gamma; \mathbf{\Omega})$, is given by the two eigenvectors that correspond to the negative eigenvalues of the system's Jacobian. In turn, the unstable manifold, denoted by $v^U(u, \gamma; \mathbf{\Omega})$, is given by the eigenvector corresponding to the positive eigenvalue. For any given initial conditions of the predetermined state variables γ_0 and u_0 in the neighborhood of a steady state and given a belief about aggregate search intensity $\mathbf{\Omega}$, v_0 (and thus θ_0) will adjust so as to position

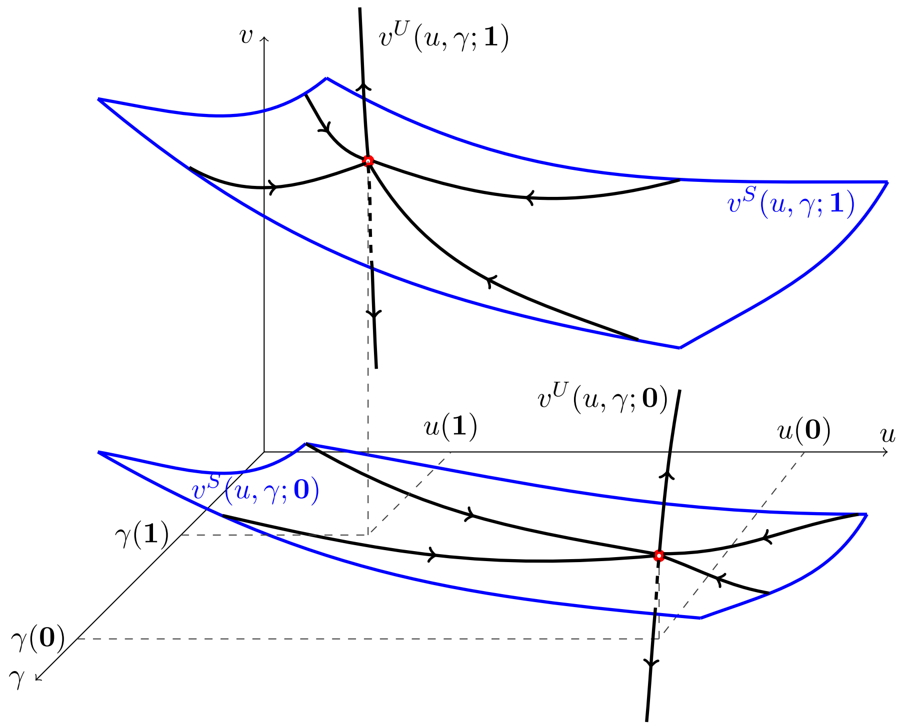

Figure 6: Dynamic System: Phase Diagram

the system along the saddle path, given by the stable manifold. Along the stable manifold the system is in equilibrium, characterized by mutual best responses of all agents for given beliefs, the flow equations and the transversality condition. Notice that outside the stable manifold, the transversality condition is violated because, for given beliefs, v (and thus θ) diverges to $\pm\infty$, meaning that the economy is not in equilibrium.

Note that so far we have not addressed the issue of equilibrium selection. We will do so in Section 5.4.1 in the context of our quantitative exercise.

5 Validation and Quantitative Exercise

In this section, we link the model to the data. We first provide direct evidence on pro-cyclical search behavior of the employed and the mismatch-reducing effect of booms that lend support to our key mechanism. We then evaluate our model quantitatively: The model is calibrated to the US economy. We show that the model admits multiple equilibria and large labor market fluctuations across them (considerably larger than those generated by realistic TFP shocks). We also explore the dynamic behavior of our model and show that the transition from recession to boom exhibits a Jobless Recovery, with unemployment *increasing* once the economy starts recovering *after* the peak of the crisis.

We will provide evidence for our model from the US economy. For the calibration exercise, our framework requires data on vacancies, unemployment, as well as labor market transitions (UE, EE, EU). Our main data source for labor flows and unemployment rates is the Current Population Survey (CPS), where we aggregate the monthly series up to quarterly frequency. For vacancies, we use the JOLTS data from the Bureau of Labor Statistics, supplemented by data from the Conference Board on online help-wanted ads.[18] Using this supplement for vacancies, our data spans the period 1996-2013.

5.1 Direct Evidence for the Theoretical Mechanism

The key premise underlying our model is that workers search more actively on-the-job during the boom. This biases the composition of searchers towards on-the-job searchers, it induces firms to post more vacancies and it leads to higher EE-flows compared to the recession. In this section, we provide evidence for this mechanism. We also refer to the abundant evidence on countercyclical mismatch between workers and firms, which is an important consequence of the model's mechanism.

Direct Evidence from EE Flows. We compute EE flows from the CPS panel by counting all the individuals who have answered 'no' to the question whether they still work for the same employer as last month. The quarterly series is then computed by summing these responses over three months (and applying a seasonal adjustment). We first decompose EE flows, given by $EE = \lambda\gamma m(\theta)$, into its determinants: the effective stock of on-the-job searchers $\lambda\gamma$, and matching probability $m(\theta)$.[19] To separate the cyclical component from the trend (which our paper has little to say about), we plot the de-trended variables. We measure the matching probability in the data from $m(\theta) = UE/u$, and $\lambda\gamma$ from the definition of EE flows, taking into account that $UE = m(\theta)u$:

$$\lambda\gamma = \frac{uEE}{UE}. \tag{21}$$

As our model predicts, in the data EE flows are highly pro-cyclical (Figure 7.A). At first glance, this seems to stem from a sharp drop in v and sharp increase of u during the recession, rendering matching probabilities pro-cyclical (Figure 7.B). In turn, the number of effective on-the-job searchers $\lambda\gamma$ is counter-cyclical (Figure 7.B): The *number* of effective on-the-job searchers actually increases in the recession. The reason is that towards the recession, many workers are stuck in low productivity jobs at the bottom of the job ladder due to low matching rates. With fewer job-to-job transitions, the fraction of those in low productivity jobs, γ, is increasing. In the boom, all search actively, hence they leave the pool at a higher rate, implying that the stock in steady state is smaller. Below, we provide direct evidence that the counter-cyclicality of $\lambda\gamma$ is driven by the stock of on-the-job searchers γ and *not* by their search intensity λ, which we will document is *pro-cyclical*. In fact, the pro-cyclical EE flows can be attributed to both, pro-cyclical matching probabilities and pro-cyclical search intensity.

[18] We thank Regis Barnichon for making this data available to us.

[19] We use the raw series of EE transitions (HP de-trended, with multiplier equal to 1600) from the CPS. Mukoyama (2014) shows that time-aggregation has no significant impact on the cyclical properties of EE transitions.

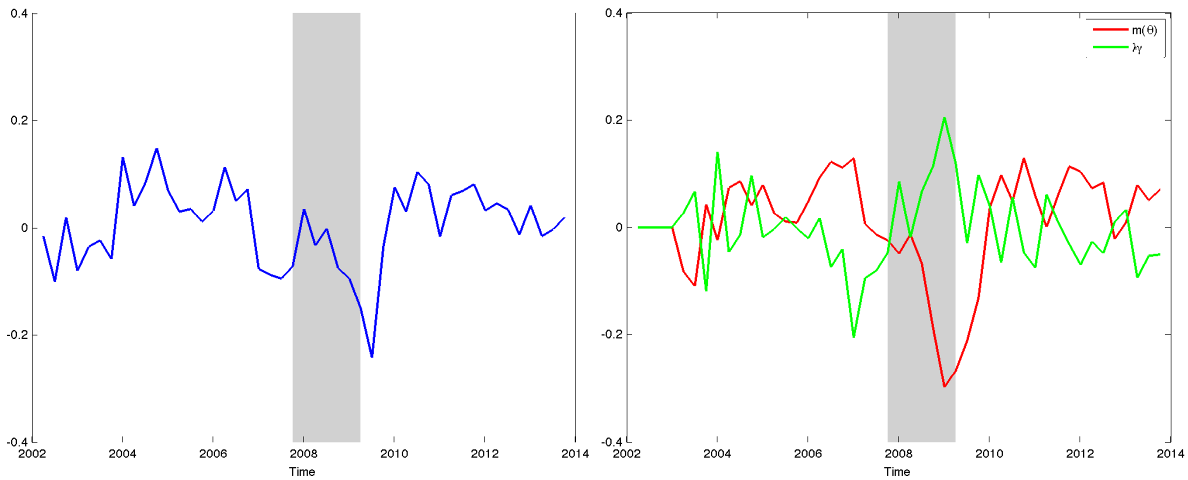

Figure 7: Decomposition of EE Flows (de-trended using HP filter). A. EE Flows. B. Matching Probability and Number of Effective On the Job Searchers.

Direct Evidence on Search Intensity and the Composition of Jobs and Searchers. While up to now in this section, we were not able to disentangle the effective number of on-the-job searchers into the number of searchers γ and search intensity λ, this is what we aim to achieve here. We do so by exploiting the panel structure from the CPS Micro-data in order to assess whether employed individuals transited to their current jobs from unemployment, which we count into the stock of on-the-job searchers γ, or from another job, indicating an EE move (which we then count into the stock ξ). The CPS interviews individuals for 4 consecutive months, then gives them a break for 8 consecutive months, and finally interviews them again for 4 consecutive months. We aggregate the monthly data up to the quarterly level. Figure 8 shows the results of this exercise.

In line with the calibrated model below, the stock of on-the-job searchers is countercyclical (with a slight lag; Figure 8.A, blue line), indicating that during downturns many workers are stuck at the bottom of the job ladder. But few workers are in high productivity jobs at the top, reflected by a low stock ξ (Figure 8.A, green line). This shows that the recession negatively affects the composition of jobs, with a considerable bias towards low-productivity jobs. This resembles the findings by Moscarini and Postel-Vinay (2012), who argue that the job ladder has failed during the Great Recession.

Most importantly, this direct measure of the stock of on-the-job searchers γ allows us to back out our measure of search intensity λ in the data using (21). Figure 9 plots the de-trended search intensity, which is *pro*-cyclical, high in booms and low in recessions. Entering the recession, search intensity dropped abruptly, followed by a sudden relapse during recovery, providing strong support for our mechanism.

This decomposition shows that our mechanism that relies on cyclical fluctuations in search intensity and firms responding endogenously to this behavior, has bite. Higher search intensity of workers is

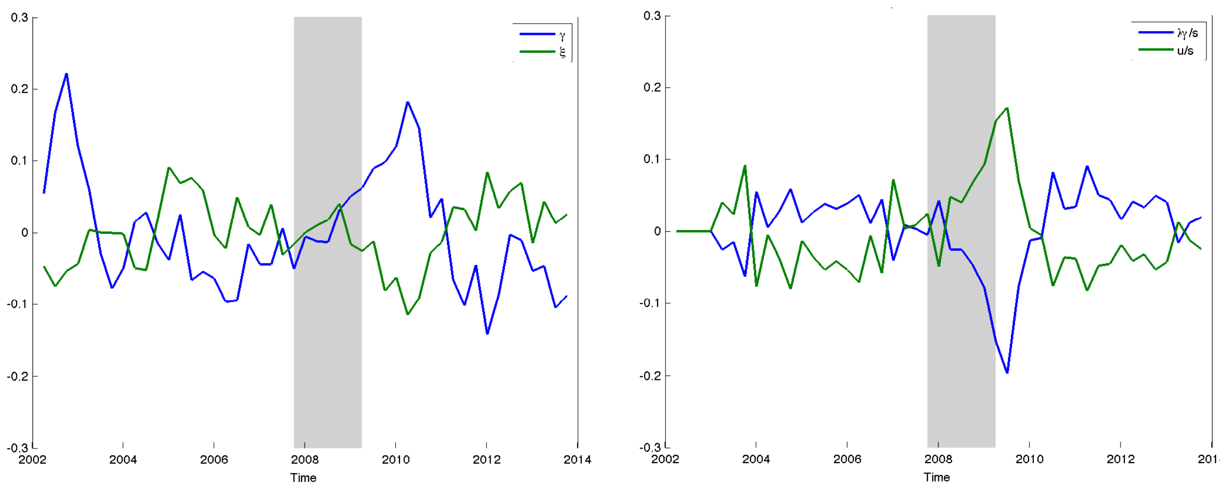

Figure 8: Composition of Jobs and Searchers. A. Stock of workers in low and high productivity jobs, γ and ξ; B. Shares of on-the-job searchers and unemployed searchers. Both de-trended (using HP-filter).

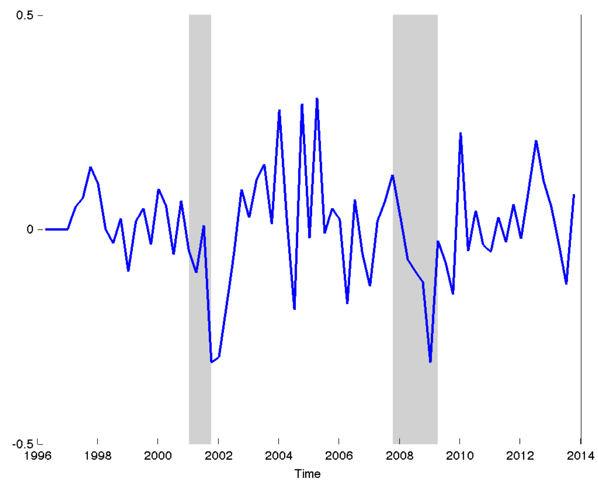

Figure 9: Search Intensity λ (log-deviations from trend; de-trending by HP filter).

rewarded by higher matching rates during booms compared to recessions.[20] Firms are encouraged to post more vacancies because the *composition* of the search pool is tilted towards actively searching employed workers that generate higher match quality compared to the unemployed. Figure 8.B captures this by showing how the composition of searchers varies over the business cycle. In a boom, the share of on-the-job searchers is high while the share of unemployed searchers is low – in line with our model. The opposite holds true during recession.

Pro-cyclical fluctuations in EE flows are thus not merely driven by pro-cyclical fluctuations in market tightness and thus matching probabilities. They are also prompted by pro-cyclical search intensity of

[20]This is in line with the literature that documents that search intensity correlates with real outcomes, predicting how likely it is to find a job. Krueger and Mueller (2011) show that the amount of time devoted to job search helps predict early exits from Unemployment Insurance. Carillo-Tudela, Hobijn, Perkowski, and Visschers (2015) show that active search increases the likelihood of finding a job by a factor of six, both from UE and EE.

the employed. Active search intensity has both a direct positive impact on EE-transitions and an indirect positive effect since it encourages vacancy posting.

A natural alternative source of information about search intensity draws from the American Time Use Survey (ATUS). In line with the findings on search intensity of the unemployed (Mukoyama, Patterson, and Sahin (2014)), we find that the cyclical pattern of search intensity of the employed is inconclusive. In Appendix A (Figure 14), we report the time spent searching by employed workers, first unconditionally, and then conditional on reporting non-zero search activity. In line with the predictions of our model and the CPS evidence, we find that the *unconditional* search intensity is counter-cyclical. This is due to the fact that in the recession, there are more employed workers that are mismatched at the bottom of the job ladder and searching for a better job. In our model this is reflected in the fact that during recessions the number of searchers, γ, is higher. However, when *conditioning* on non-negative search activity, search intensity is slightly pro-cyclical. One issue to bear in mind is that the ATUS generally records very low durations for search intensity, possibly because it does not include attendance of professional events, social networking, etc.

Pro-cyclical search intensity of on-the-job searchers is also in line with suggestive evidence by Carillo-Tudela, Hobijn, Perkowski, and Visschers (2015). They use U.S. data on the search behavior of employed workers from the Contingent Worker Supplement (CWS) to the CPS, conducted in February of 1995, 1997, 1999, 2001, and 2005. The advantage of our method of backing out the cyclicality of search behavior is that we can get insights for all years, most importantly the period before, during and after the Great Recession.

Evidence on Cyclical Properties of Mismatch and Wages. Finally, our model also predicts that mismatch – here measured by the fraction of $\underline{y}$ jobs – is increasing in the recession (see Bowlus (1995), Lazear (2014), and Gertler, Huckfeldt, and Trigari (2016) for evidence), and which was referred to as the *sullying effect* of recessions where workers get stuck in poor matches at the bottom of the job ladder (Barlevy (2002)). This is also supported by the fact, as our model predicts, that wage growth during the boom is higher (Faberman (2015)).

5.2 Calibration of Steady State Equilibria

We calibrate our model to quarterly data. We set the parameters $(r, b, \delta, p, \underline{y})$ outside the model and report the values in Table 1. Our model features a constant separation rate δ across boom and recession, which we set equal to average observed quarterly separations over time. Moreover, from the theory we can generate multiple steady states even in the absence of productivity changes. Initially therefore, we normalize aggregate productivity p to one. Below, when exploring the model's dynamics, we will relax the constant productivity assumption. We set b such that the average replacement rate is 75%, which is an intermediate value considering the calibrations in the existing literature.[21]

[21] For instance, our value is higher than Shimer (2005)'s (0.4), and lower than Hagedorn and Manovskii (2008) (0.95).

Table 1: Exogenously Set Parameters

Variable	Value		Notes
r	0.0113	discount factor	standard
$\underline{y}$	1	productivity first job	normalization
b	0.919	unemployment value	75% average replacement rate
δ	0.05	job separation rate	average quarterly separation rate across peak and trough
p	1	productivity	normalization

We determine the remaining parameters endogenously, including those that relate to the search technology, active and passive on-the-job search intensity (λ_0, λ_1), the parameters of the matching function (α, ϕ) and the cost of vacancies c and of on-the-job search k, as well as productivity $\overline{y}$. To pin down these parameters, we use our model of multiple equilibria to target business cycle moments from the Great Recession, i.e. moments from the peak (*boom* or *equilibrium with active OJS*) before the recession and the trough (*recession* or *equilibrium with passive OJS*). We focus on those two data points from the last business cycle where the difference between EE transition rates was most pronounced (since EE fluctuations are at the heart of our model's mechanism). We denote the third quarter of 2006 as *boom* $(\omega = 1)$ and the third quarter of 2009 as *recession* $(\omega = 0)$.

Central to our calibration strategy is to target EE transition rates in both boom and recession, since we would like to explain business cycle fluctuations through differences in on-the-job search. Moreover, we target matching probabilities in boom and recession as well as wage dispersion in the boom. Last we aim to choose parameters such that the model matches the empirical vacancy and unemployment levels in the *boom*. Notice that we do *not* target unemployment and vacancy levels in the recession. Instead, we want to assess how well the calibrated model of multiple equilibria can quantitatively match the observed fluctuations in vacancies and unemployment over the business cycle in the absence of any TFP movements (i.e. for constant p).

We use General Method of Moments to estimate our model, with equal number of moments and parameters. Targeted moments (data and model) and estimated parameter values are in Tables 2 and 3.

Table 2: Targeted Moments

	Data	**Model**
$EE(\mathbf{1})$	0.066	0.035
$EE(\mathbf{0})$	0.036	0.022
$u(\mathbf{1})$	0.047	0.055
$v(\mathbf{1})$	0.029	0.039
$m(\theta(\mathbf{1}))$	0.852	0.853
$m(\theta(\mathbf{0}))$	0.511	0.513
$\frac{\overline{w}(\mathbf{1})}{\underline{w}(\mathbf{1})}$	1.230	1.230

Table 3: Estimated Parameters

	Estimate	**Parameter Description**
λ_0	0.092	passive OJS intensity
λ_1	0.073	active OJS intensity
α	0.863	curvature matching function
ϕ	3.258	overall matching efficiency
c	9.404	vacancy posting cost
$\overline{y}$	1.577	high productivity
k	0.080	search cost

Overall, the model performs fairly well. We match the EE flows in boom and recession reasonably well, most importantly we match the difference between the two. We match observed matching probabilities, wage dispersion as well as unemployment rate nearly exactly. Only regarding vacancies, the model slightly over-predicts the level. The main reason for the gap between model and data are constant separation rates and constant aggregate productivity across boom and recession.

The parameter estimates suggest that search intensity of on-the-job searchers, $\lambda_0 + \lambda_1 = 0.165$, is considerably lower than search intensity of unemployed workers that was normalized to 1 (in line with evidence by Faberman, Mueller, Sahin, and Topa (2016)). Moreover, on-the-job searchers are about twice as actively searching in boom compared to recession. The curvature of the matching technology is estimated to be nearly linear, and matching efficiency ϕ is estimated to be about 3. Notice that the matching efficiency is estimated to be higher than what is suggested by most estimates in the literature, which stems from using a different tightness measure v/s (which is smaller than the conventional v/u since it takes all searchers into account). The costs of on-the-job search are estimated to be a relatively small fraction of the first job's output (about 8%). Finally, the estimated costs of posting a vacancy c, which reflects the overall resources that a firm spends on hiring, are comparably high, corresponding to a hiring cost of more than a year's output of a high productivity match. These estimated costs are large but in line with a growing literature that argues hiring costs are substantial and, depending on the worker type, can take up more than an annual wage.[22]

It is important to note that our calibrated economy admits multiple steady state equilibria, where aggregate productivity $p = 1 \in [p_l, p_h] = [0.994, 1.026]$, where p_l and p_h are the multiplicity bounds coming out of the calibration. Thus, the necessary and sufficient condition for multiplicity (Proposition 1) is satisfied. Importantly, we did *not* restrict the estimation to parameter estimates that are in line with multiplicity. Moreover, under this calibration, we check the eigenvalues of the model's dynamic system (20) numerically and find that both steady states are saddle-path stable. Given any starting values (u_0, γ_0), in the neighborhood of a steady state, tightness θ adjusts in order to bring the economy on the saddle-path. The economy converges to the active-search equilibrium if beliefs are optimistic and to the passive-search equilibrium if they are pessimistic. We discuss these dynamics in more detail below but first analyze differences in labor market outcomes across steady states.

5.3 Labor Market Fluctuations across Steady States

Using this calibration, we are interested in the unemployment and vacancy fluctuations that the model predicts only based on multiplicity and without any movements of aggregate productivity. Table 4 suggests that our model of multiple equilibria performs well in matching the *non-targeted* moments of unemployment and vacancies in the recession. Comparing vacancy and unemployment levels across boom and recession (Table 2 and 4), the model generates sizable fluctuations: Our model explains

[22] For evidence on substantial hiring costs, see, for instance, Blatter, Muehlemann, and Schenker (2012) as well as Dube, Freeman, and Reich (2010) and the references therein.

about 57% of the observed increase in unemployment and about 60% of the drop in vacancies during the Great Recession. Moreover, it predicts about 70% of the change in the composition of the pool of searchers: While in the data the proportion of on-the-job searchers in overall searchers declined by 33% during the recession, in the model it declined by 23%. It is important to note that these fluctuations are obtained through multiple equilibria alone and without alluding to any decline in aggregate productivity p, which is held fixed in this exercise. This suggests that differences in the intensity at which workers search on-the-job in boom versus recession can have a profound impact on labor market fluctuations.

Table 4: Non-Targeted Moments

	Data	**Model**
$u(\mathbf{0})$	0.096	0.089
$v(\mathbf{0})$	0.016	0.029
$\frac{\lambda(\mathbf{0})\gamma}{s(\mathbf{0})}$	0.423	0.327
$\frac{\lambda(\mathbf{1})\gamma}{s(\mathbf{1})}$	0.625	0.425

We now contrast these results on labor market fluctuations with those that can be obtained by aggregate productivity movements. Shimer (2005) argues that in the standard Mortensen-Pissarides model of unemployment, productivity fluctuations cannot account for the fluctuations in unemployment and vacancies observed in the data. Several studies have offered explanations to counter Shimer's finding, and can indeed create labor market volatility from small productivity shocks, for instance, sufficiently high value of unemployment b (Hagedorn and Manovskii (2008)). To shed light on the role of productivity changes and to isolate it from our channel of multiplicity, we perform the following counterfactual. We pretend the boom equilibrium with active on-the-job search is the unique equilibrium and feed a TFP drop of 5% that was observed during the Great Recession into the model (see, e.g., Schaal and Taschereau-Dumouchel (2014)). Can the observed decline in productivity generate fluctuations of similar magnitudes as our model of multiple equilibria?

To assess the explanatory power of our mechanism (Model 1) against a model where fluctuations are driven by productivity changes alone, we focus on the boom equilibrium and feed in the observed productivity drop from the Great Recession (Model 2). See Table 5, where Δ indicates a change (and $\Delta \times 100$ a percentage change).

Compared to our mechanism that relies on equilibrium multiplicity, a model with unique equilibrium and productivity shocks generates relatively small fluctuations (Model 2, Table 5). This holds across variables, but especially the increase in unemployment and the decline in vacancies are negligible, namely +26% and -12%, respectively, compared to fluctuations in the data of +106% and -47%. This exercise suggests that changes in search behavior on the job are more important in generating labor market fluctuations than productivity movements.

Finally, we are interested in whether the two steady state equilibria can be Pareto-ranked. While our theory does not yield any unambiguous result regarding welfare, we can provide an answer using

Table 5: Labor Market Fluctuations and Counterfactual with Varying Productivity

	Data	**Model 1**	**Model 2**
ΔEE	-0.46	-0.37	-0.08
$\Delta m(\theta)$	-0.40	-0.40	-0.22
Δv	-0.47	-0.28	-0.12
Δu	1.06	0.60	0.26
$\Delta\theta$	-0.61	-0.47	-0.28
$\Delta\Theta$	-0.74	-0.55	-0.30
$\Delta\lambda\gamma/s$	-0.32	-0.23	-0.04

the calibrated economy. We compare the output net of search costs in boom and recession and find that this measure is larger in the boom than in the recession: $Y(\mathbf{1}) = 0.9561, Y(\mathbf{0}) = 0.861$. In other words, a difference of $\left(\frac{Y(\mathbf{1})}{Y(\mathbf{0})} - 1\right) * 100 = 11\%$. For parameter values of aggregate productivity around $p = 1$ (more precisely $\pm 10\%$), we find that net output is always higher in the boom equilibrium where search intensity is high. In this calibrated version of the model, we can therefore clearly rank the two equilibria in terms of welfare.

5.4 Dynamics

This section addresses the issue of equilibrium selection and investigates the transition from one steady state to another, in particular from the recession to the boom. We then show the economic implications of the transition dynamics. In particular, we show that our model can reproduce the observed phenomenon of jobless recovery based on the crowding out of unemployed searchers by employed searchers.

5.4.1 Equilibrium Selection

Up to this point, our exercises focused on the comparison of two steady state equilibria without specifying how these different equilibria are selected. The underlying assumption was that an equilibrium changes in response to a switch in agents' belief that we did not model explicitly. Moreover, to highlight the model's mechanism and the role of equilibrium multiplicity, we have so far not considered any changes in the fundamentals across equilibria.

In contrast, we now propose one possible equilibrium selection explanation that connects multiplicity of equilibria with *changes in the economy's fundamentals*. Following Cooper (1994), we choose the following equilibrium selection criterion to resolve the model's multiplicity: We assume that the economy stays in the equilibrium played in the previous period *if* it continues to exist. This selection criterion generates history-dependent beliefs where individuals use the past to guide their current actions and they believe that others behave this way as well. Periods of pessimistic (optimistic) equilibrium behavior yield pessimistic (optimistic) beliefs in the future. As long as the number of equilibria does not vary, nor will the choice of search effort.

The discussed selection criterion ensures that there are no equilibrium switches unless aggregate productivity p takes a value *outside* of $[p_l, p_h]$, (i.e. the economy moves from $p \in [p_l, p_h]$ to $p' \notin [p_l, p_h]$) for then there is a unique equilibrium and no selection issue arises. The equilibrium *must* change if the previous equilibrium ceases to exist. In turn, if productivity moves from $p \notin [p_l, p_h]$ to $p' \in [p_l, p_h]$, then the equilibrium will be selected that is in line with the previously played unique equilibrium. It is worth mentioning that for our calibration, the p-bounds for existence of multiple equilibria are very narrow, $[p_l, p_h] = [0.994, 1.026]$. It therefore does not take unrealistically big productivity shocks to make the economy switch equilibria.

For simplicity and illustration, we assume that there are two productivity states, $p_0 < p_l$ and $p_1 > p_h$. We model the transition between them through a 2-state Markov chain,

$$\begin{bmatrix} \pi(\mathbf{0}) & 1-\pi(\mathbf{0}) \\ 1-\pi(\mathbf{1}) & \pi(\mathbf{1}) \end{bmatrix}$$

where $\pi(\mathbf{0})$ is the probability to stay in the low productivity state, and $1-\pi(\mathbf{0})$ is the probability to switch from the low to the high productivity state. Similarly, $\pi(\mathbf{1})$ and $1-\pi(\mathbf{1})$ give the probability to stay in the high productivity state or to switch from high to low productivity, respectively. In case the productivity state changes, labor market tightness jumps to a value on the new stable manifold from where the economy begins to converge to the new steady state. Given the state with low productivity p_0, the economy converges to the steady state with only passive on-the-job search (which is the unique equilibrium under p_0), conditional on remaining in the low productivity state. Similarly, given the state with high productivity p_1, the economy converges to the steady state with active on-the-job search (which is the unique equilibrium under p_1), conditional on remaining in the high productivity state. This is illustrated in Figure 10. Observe that immediately after the productivity shock, tightness θ is not immediately at its steady state level $\theta(\mathbf{0})$ (or $\theta(\mathbf{1})$) but under- or over-shoots first.

We follow the approach of Kaplan and Menzio (2014) in analyzing the transition dynamics in response to a (productivity) shock. Because our three-dimensional dynamic system is very complex, fully solving the transition dynamics of this economy is difficult, if not impossible. We therefore use their approximation approach and add (subtract) a constant value to the choice variable, in our case θ, upon realization of a positive (negative) shock, that moves the economy to the stable manifold of the new steady state. This approximation captures all the changes embedded in the value functions. Because in our system there is only one choice variable that instantaneously adjusts, and the other variables, u and γ, are state variables, this method gives an approximation for the changes to all value functions U, V, E, J.[23] Formally, this implies that upon the realization of a shock, there is a constant term Δ_θ added to the dynamic equation in the choice variable.

Adjusting the differential equations (17)-(19), the approximate dynamic system with productivity

[23] This is identical to Kaplan and Menzio (2014) who introduce a jump in the value J, the only choice variable in their dynamic system, and not in the values for the unemployed workers and the vacant firms.

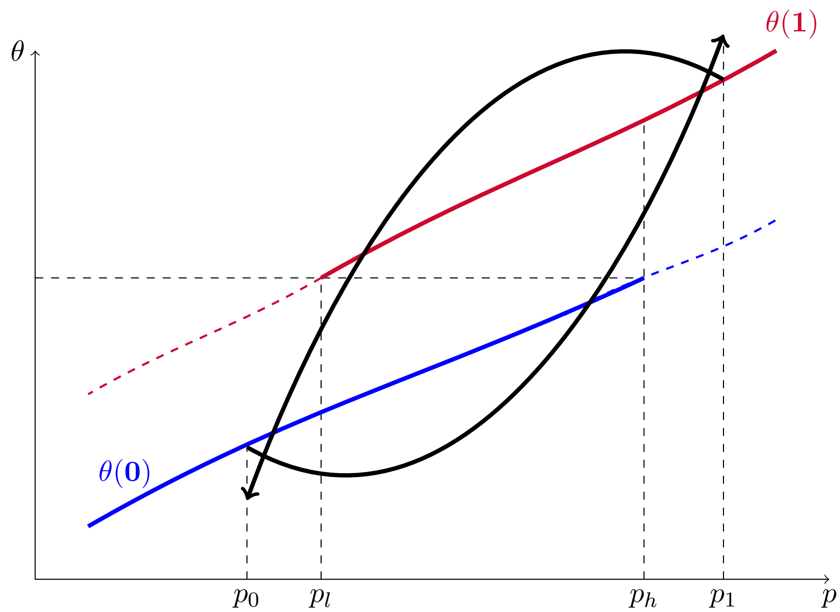

Figure 10: Productivity Induced Dynamics.

shocks can be written as (48)-(50) in Appendix A, where forward-looking agents take into account that a productivity shock (that moves the economy to a new equilibrium) occurs with probability $1-\pi(\mathbf{\Omega})$, thus shifting the choice variable instantaneously by $\Delta_\theta(\mathbf{\Omega})$. With probability $\pi(\mathbf{\Omega})$ there is no change and the shift in value is zero. For the dynamic system to converge, the choice variable must jump exactly onto the stable manifold. We therefore pick the value of $\Delta_\theta(\mathbf{\Omega}) = \theta(\neg\mathbf{\Omega}|u,\gamma) - \theta(\mathbf{\Omega}|u,\gamma)$ to be such that for any u and γ, tightness θ in the new equilibrium jumps on the stable manifold. For example, if the economy is currently in the recession and a positive shock hits, the choice variable $\theta(\mathbf{0}|u,\gamma)$ will jump to $\theta(\mathbf{1}|u,\gamma)$, which is on the stable manifold of the active search equilibrium. This implies that if the economy stayed in the high productivity state forever, it would converge to the active search steady state.

Even with this approximation to the value functions using the jump in θ given by Δ_θ, this three-dimensional dynamic system is extremely difficult to solve numerically. We therefore further approximate the solution by focusing on the linear approximation, pinned down by the eigenvectors $\mathbf{v}(\lambda_i)$ and eigenvalues λ_i of the Jacobian of the linearized system (see Appendix A):

$$\begin{bmatrix} u_t - u_i^* \\ \gamma_t - \gamma_i^* \\ \theta_t - \theta_i^* \end{bmatrix} = [\mathbf{v}(\lambda_1) \quad \mathbf{v}(\lambda_2) \quad \mathbf{v}(\lambda_3)] \begin{bmatrix} C_1 \exp(\lambda_1 t) \\ C_2 \exp(\lambda_2 t) \\ C_3 \exp(\lambda_3 t) \end{bmatrix}. \tag{22}$$

This linear approximation is unlikely to capture the transition dynamic system quantitatively. Moreover, it is well known that the Pissarides model with free entry, as in ours, features saddle path dynamics that lead to drastic overshooting of vacancies in response to a positive productivity shock which is not borne out in the data. It is therefore clear that the transition dynamics in this linearized model with free entry will have some unrealistic features. We nonetheless think it is instructive to consider the dynamic pattern of what is driving the transition. In particular, in our model it is the composition of searchers (unemployed versus on-the-job searchers) that drives the transition dynamics of vacancies and unemployment in response to productivity shocks.

5.4.2 Jobless Recovery

Within this dynamic environment and the discussed equilibrium selection criterion, we now illustrate the dynamical properties of the model applied to the phenomenon of the *jobless recovery*. Our objective, given the simplicity of the model and some unrealistic features of its dynamics, is not to provide a full quantitative analysis of the jobless recovery that gets all magnitudes right, but to show that our model provides a new mechanism that qualitatively accounts for the several features in the data.[24]

Consider an economy in the recession that has been there for a long period since the negative productivity shock p_0. That implies that the state variables u, γ and the choice variable θ are stationary, i.e., $\dot{u}, \dot{\gamma}, \dot{\theta}$ equal zero. We therefore can compute the values from setting $\mathbf{\Omega} = 0, \dot{u} = \dot{\gamma} = \dot{\theta} = 0$ in (48), (49), and (50). We treat this as the initial equilibrium denoted by u_0, γ_0, θ_0.

Now consider a positive productivity shock from p_0 to $p_1 > p_h$, pushing the economy in the region of a unique equilibrium with active on-the-job search.[25] Our objective is to track the economy's recovery regarding u, γ, θ and its transition to the new equilibrium of active on-the-job search. Using u_0, γ_0, θ_0 as initial values, we compute the transitions setting $\mathbf{\Omega} = 1$ using the linearized system (22).

Figure 11 depicts the transition dynamics of θ and u in model and data. Like in the Pissarides model, our model features saddle path stability and hence a jump in θ. As a result, the magnitudes of the model's transitions do not match the data. In addition, and unlike the standard Pissarides model, there is also a jump in unemployment: while job finding rates increase, new jobs are all taken by the on-the-job searchers. This is the jobless (or rather job destructive) recovery. And while there is no evidence of the overshooting of θ in the data, we do see evidence of an increase in unemployment following the recovery. This increase of unemployment in the model is larger than that in the data, which is again a consequence of the saddle path dynamics.

The root of jobless recovery lies in the abrupt change of the *composition of searchers* after the crisis, illustrated by Figure 12. This effect becomes apparent once we derive the share of unemployed searchers out of all searchers: $\frac{u}{s}$ where $s = u + \lambda\gamma$. While we cannot observe $\lambda\gamma$ directly in the data, we can derive it from the flows as we mentioned above: $EE = m(\theta)\lambda\gamma$ and $UE = m(\theta)u$ so that $\lambda\gamma = \frac{EE}{UE}u$. Again,

[24] In the Appendix D. we also perform a simple exercise that illustrates how on impact when the recovery starts, unemployment increases even if the economy is recovering. There, as in the fully dynamic exercise, we illustrate that this is due to the composition externality where employed workers who search actively crowd out unemployed workers.

[25] We choose the smallest possible productivity increase that pushes the economy into the unique equilibrium with active on-the-job search, which is $p_1 = 1.0262$ (recall that $p_h = 1.0261$). This corresponds to a 2-3% TFP increase during the recovery, which is in line with observed TFP changes (see, for instance, Schaal and Taschereau-Dumouchel (2014)).

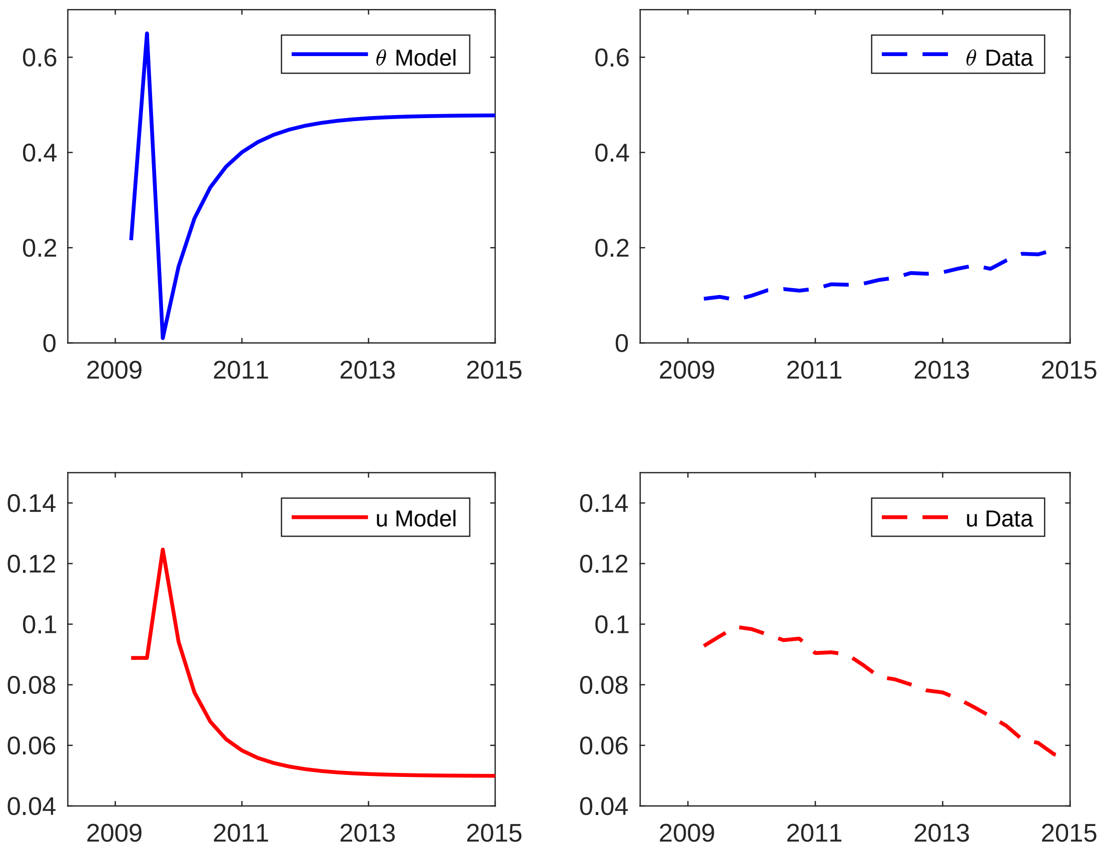

Figure 11: Transition Dynamics: Market Tightness and Unemployment.

the change in the composition of unemployed searchers $\frac{u}{s}$ in the data does not parallel the magnitude of the jumps in the model immediately after the productivity shock, but it does qualitatively match the inverted U-shape pattern. Coming out of the recession, the fraction of unemployed searchers is low initially. At impact, due to the increase in search intensity, the on-the-job searchers flood the pool of searchers (lower panel, Figure 12) and find jobs faster, crowding out the unemployed and thus increasing the proportion of unemployed searchers shortly after (upper panel, Figure 12). Once the pool of on-the-job searchers is heavily biased towards unemployed workers, the opposite is true and the unemployed are being hired relatively more. This in turn decreases the fraction of unemployed searchers, gradually reducing unemployment after a phase of jobless recovery.

This exercise indicates that the immediate impact of the recovery out of the recession looks even bleaker than the recession itself. Due to crowding out by on-the-job searchers, the unemployed initially match at a slower rate and the unemployment rate goes up. Rather than a jobless recovery, this indicates a more accurate term might be a *job-destructive recovery*.

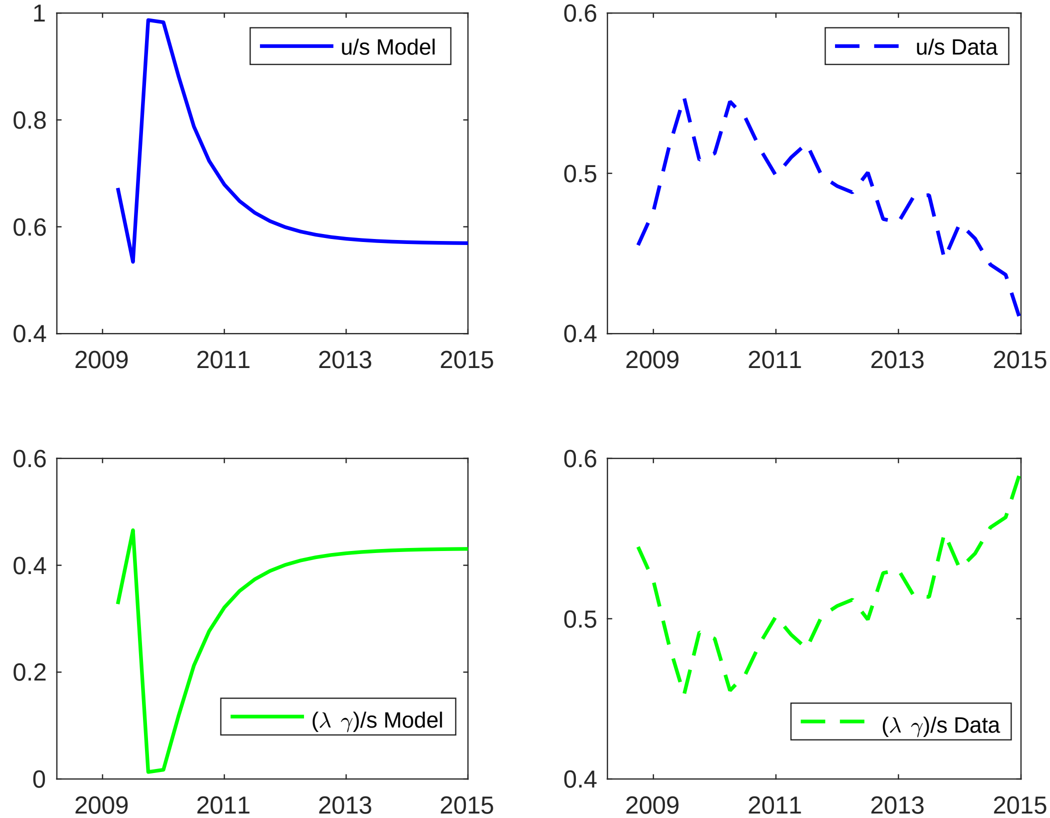

Figure 12: Transition Dynamics: the share of unemployed out of all job searchers ($\frac{u}{s}$), model (A) and data (B); the share of employed out of all job searchers ($\frac{\lambda\gamma}{s} = 1 - \frac{u}{s}$), model (C) and data (D).

It is important to note that a comparable model with on-the-job search but *unique* equilibrium (for all p), where business cycle fluctuations are solely driven by variations in productivity p, would *not* feature a jobless recovery: Tightness and hence matching probabilities go up during the recovery but crucially – and in contrast to our model of multiple equilibria – the composition of the pool of searchers remains unchanged. This prevents the crowding out of unemployed searchers. As a result, unemployment would *not* grow during the recovery.

6 Conclusion

We have argued that the labor market behavior of the employed can have profound implications for the unemployed. Moreover, even in the absence of exogenous shocks, search behavior of employed workers by itself can create multiple equilibria and hence cyclical outcomes due to a strategic complementarity in active on-the-job search and vacancy creation. Active on-the-job search by the employed makes it more attractive for firms to post vacancies, which in turn makes on-the-job search more attractive.

Self-fulfilling beliefs can thus give rise to either a high activity on-the-job search equilibrium which we interpret as a boom as well as a low activity equilibrium interpreted as a recession.

These beliefs give rise to large fluctuations in vacancies, unemployment and job-to-job transitions, even without any change in the productivity or other primitives. Moreover, in the transition from a low on-the-job search equilibrium to an equilibrium with active on-the-job search, this model naturally generates a jobless recovery. As the employed start to search, they crowd out the unemployed, possibly making the recovery for the unemployed even worse than the recession. In fact, unemployment initially rises. We believe that this crowding out channel is new in the literature. The model also gives rise to a shift in the Beveridge Curve (the unemployment-vacancy locus). This shift is not driven by an exogenous change in the matching efficiency or the simple transition dynamics around a unique steady state. Instead, it is driven by the transition from one steady state to another where the composition of searchers changes and where as a result an endogenous argument of the matching function adjusts.

We calibrate the model to the US economy and confirm the importance of our mechanism for labor market fluctuations and the phenomenon of jobless recovery.

Appendix A: Additional Data and Omitted Proofs

UE, EE, UE+EE Rates

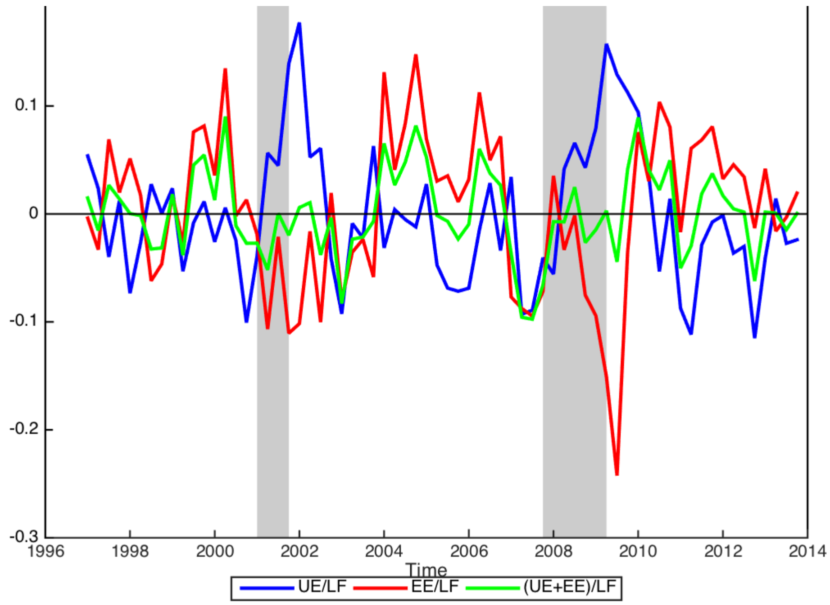

Figure 13: Labor Market Flows (detrended).

Search Intensity ATUS

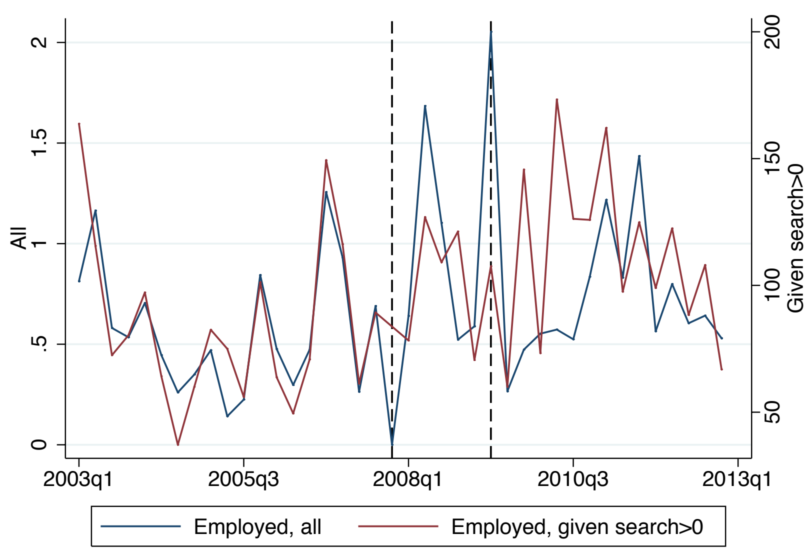

Figure 14: Search Intensity as measured by the American Time Use Survey (ATUS).

Search Intensity of those Not in the Labor Force

Similar to how we calculate the search intensity of the employed, we calculate the search intensity of those Not in the Labor Force. Denote their stock by N and their flow into employment by NE. Then $NE = \lambda_N m(\theta) N$ where λ_N is the search intensity and $m(\theta) = UE/U$ is the matching probability derived from the Unemployment to Employment flow. Therefore we calculate search intensity as $\lambda_N = \frac{NE \cdot U}{UE \cdot N}$.

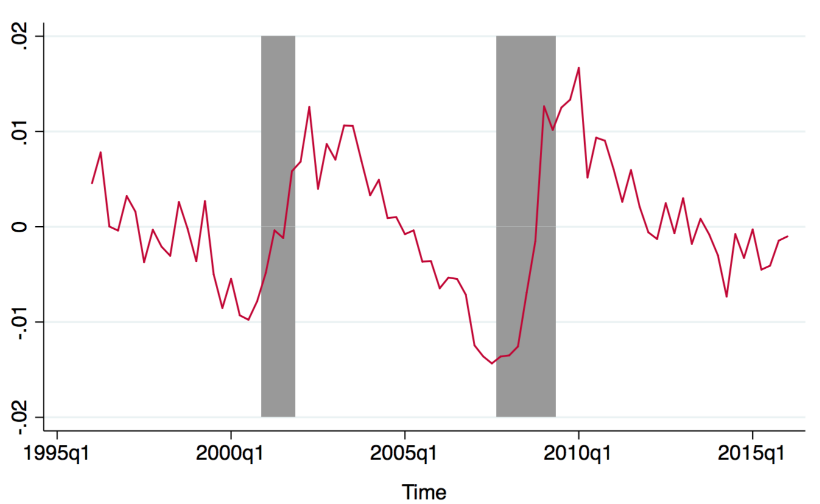

Figure 15: Search Intensity of those Not in the Labor Force (log-deviations from trend; de-trending by HP filter).

Equilibrium Value Functions

Firms believe workers take an individual action ω consistent with the equilibrium belief $\mathbf{\Omega}$, i.e., $\omega = \mathbf{\Omega}$. Wage setting requires that $\underline{E} = U$, which implies that $\dot{\underline{E}} = \dot{U}$. Using this and solving for U in the Bellman equation (1), implies:

$$U = \frac{pb}{r} + \frac{\dot{U}}{r} = \frac{pb}{r}, \tag{23}$$

where the second equality follows from the fact that the first term $\frac{pb}{r}$ is a constant, which immediately implies that $\dot{U} = 0$, and as a result, $\dot{\underline{E}} = 0$. Using this expression for U to solve for $\underline{E}$ in (2) we get:

$$\underline{E} = \frac{\underline{w}(\mathbf{\Omega}) - \mathbf{\Omega} pk + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega}))\overline{E}}{r + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega}))}. \tag{24}$$

Solving for $\overline{E}$ in (3) implies:

$$\overline{E} = \frac{\overline{w}(\mathbf{\Omega}) + \delta \frac{pb}{r} + \dot{\overline{E}}}{r + \delta}.$$

The equilibrium wage for the high productivity job $\overline{w}$ is pinned down by the sequential auction framework, setting $\underline{J} = V = 0$, which applies to any new formed high type match. Since $\dot{V} = 0$, this implies that the wage set in the high productivity job is time invariant and independent of the equilibrium $\mathbf{\Omega}$. Solving for the wage from $\underline{J} = V = 0$ implies:

$$\overline{w} = p\underline{y}.$$

This further implies that

$$\overline{E} = \frac{p\underline{y} + \delta \frac{pb}{r}}{r+\delta},$$

where $\dot{\overline{E}} = 0$ since all other terms are constants.

Similarly, the equilibrium wage for the low productivity job $\underline{w}(\mathbf{\Omega})$ is pinned down by the sequential auction framework setting $\underline{E} = U$. Using (23) and (24) to solve for $\underline{w}(\mathbf{\Omega})$ implies:

$$\underline{w}(\mathbf{\Omega}) = pb\left(\frac{r + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})) + \delta}{r+\delta}\right) - \frac{\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}{r+\delta}p\underline{y} + \mathbf{\Omega}pk.$$

From free entry, $V = 0$, and therefore (4), (5) and (6) can be written as (where we make use of $\overline{J} = 0$):

$$\begin{aligned}
0 &= -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\underline{J} + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\overline{J}\right] \\
\overline{J} &= \frac{p\overline{y} - \overline{w}}{r+\delta} \\
\underline{J} &= \frac{p\underline{y} - \underline{w}(\mathbf{\Omega}) + \dot{\underline{J}}}{r + \delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}.
\end{aligned}$$

Now using the equilibrium wages and substituting all explicit solutions for the values, we obtain the following complete set of equilibrium Bellman equations:

$$U = \frac{pb}{r} \tag{25}$$

$$\underline{E} = \frac{pb}{r} \tag{26}$$

$$\overline{E} = \frac{p\underline{y} + \delta\frac{pb}{r}}{r+\delta} \tag{27}$$

$$V = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\left(\frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega} - \dot{\underline{J}}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}\right) + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\frac{p(\overline{y}-\underline{y})}{r+\delta}\right] = 0 \tag{28}$$

$$\underline{J} = \frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega} - \dot{\underline{J}}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} \tag{29}$$

$$\overline{J} = \frac{p(\overline{y}-\underline{y})}{r+\delta}. \tag{30}$$

Proof of Lemma 1

Proof. 1. No deviation when no one searches: $\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0})$.

In this case, when no one actively searches on-the-job ($\mathbf{\Omega} = \mathbf{0}$), a worker in a low productivity job deviating during an interval dt chooses $\omega = 1$ and gets a payoff

$$\underline{E}(1|\mathbf{0}) = \frac{1}{1+rdt}\left[dt(\underline{w}(\mathbf{0}) - pk) + (1-\delta dt)dt\lambda(1)m(\theta(\mathbf{0}))\overline{E} + (1-\delta dt)(1-dt\lambda(1)m(\theta(\mathbf{0})))\underline{E}(0|\mathbf{0}) + \delta dtU\right]$$

where $\overline{E} = \overline{E}(0|\mathbf{0})$ since that value is the same independent of the argument. There is no deviation provided $\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0})$ or:

$$\underline{E}(0|\mathbf{0})(1+rdt) > dt(\underline{w}(\mathbf{0}) - pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\overline{E} + [1 - \delta dt - dt\lambda(1)m(\theta(\mathbf{0})) + dt^2\delta\lambda(1)m(\theta(\mathbf{0}))]\underline{E}(0|\mathbf{0}) + \delta dtU.$$

After subtracting $\underline{E}(0|\mathbf{0})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$r\underline{E}(0|\mathbf{0}) > \underline{w}(\mathbf{0}) - pk + \lambda(1)m(\theta(\mathbf{0}))\overline{E} + (-\delta - \lambda(1)m(\theta(\mathbf{0})))\underline{E}(0|\mathbf{0}) + \delta U.$$

Substituting the equilibrium values for $\underline{E}(0|\mathbf{0}), \overline{E}, U$ and $\underline{w}(\mathbf{0})$ we get:

$$(\underline{y} - b)[\lambda(1) - \lambda(0)]m(\theta(\mathbf{0})) - k(r+\delta) < 0. \tag{31}$$

2. No deviation when all search: $\underline{E}(1|\mathbf{1}) > \underline{E}(0|\mathbf{1})$.

In this case, when all actively search on-the-job ($\mathbf{\Omega} = \mathbf{1}$), a worker in a low productivity job deviating for an interval dt chooses $\omega = 0$ and gets a payoff

$$\underline{E}(0|\mathbf{1}) = \frac{1}{1+rdt}\left[dt\underline{w}(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\overline{E} + (1-\delta dt)(1-dt\lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta dtU\right].$$

There is no deviation provided $\underline{E}(1|\mathbf{1}) > \underline{E}(0|\mathbf{1})$:

$$\underline{E}(1|\mathbf{1})(1+rdt) > dt\underline{w}(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\overline{E} + (1 - \delta dt - dt\lambda(0)m(\theta(\mathbf{1})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta dtU.$$

After subtracting $\underline{E}(1|\mathbf{1})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$r\underline{E}(1|\mathbf{1}) > \underline{w}(\mathbf{1}) + \lambda(0)m(\theta(\mathbf{1}))\overline{E} + (-\delta - \lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta U.$$

Substituting the equilibrium values for $\underline{E}(1|\mathbf{1}), \overline{E}, U$ and $\underline{w}(\mathbf{1})$ we get:

$$(\underline{y} - b)[\lambda(1) - \lambda(0)]m(\theta(\mathbf{1})) - k(r+\delta) > 0. \tag{32}$$

Combining (31) and (32) gives the condition in the Lemma. ■

Steady State in Mixed Stategies

Denote by $\underline{E}(\omega|\mathbf{\Omega})$ the value of playing ω for one instant dt while every one else pursues strategy $\mathbf{\Omega}$. This payoff is the same as the one-shot deviation payoff in Lemma 1.

For $\mathbf{\Omega} \in [0,1]$, mixing requires that $\underline{E}(0|\mathbf{\Omega}) = \underline{E}(1|\mathbf{\Omega})$, where these value functions refer to a dt-period play (after that instant the agents play $\omega = \mathbf{\Omega}$ again). If this condition is satisfied, then any mixed strategy ω (including $\mathbf{\Omega}$) is optimal from a worker's point of view. To see this, denote $\underline{E}(0|\mathbf{\Omega}) = \underline{E}(1|\mathbf{\Omega}) \equiv \underline{E}$. Then, any ω leaves the worker indifferent $\omega\underline{E} + (1-\omega)\underline{E} = \underline{E}$, i.e. there is an equilibrium in mixed strategies.

We now provide details.

$$\begin{aligned}
\underline{E}(1|\mathbf{\Omega}) &= \frac{1}{1+rdt}\left[dt(\underline{w}(\mathbf{\Omega}) - pk) + (1-\delta dt)dt\lambda(1)m(\theta(\mathbf{\Omega}))\overline{E} + (1-\delta dt)(1-dt\lambda(1)m(\theta(\mathbf{\Omega})))\underline{E}(\omega|\mathbf{\Omega}) + \delta dtU\right] \\
\underline{E}(0|\mathbf{\Omega}) &= \frac{1}{1+rdt}\left[dt\underline{w}(\mathbf{\Omega}) + (1-\delta dt)dt\lambda(0)m(\theta(\mathbf{\Omega}))\overline{E} + (1-\delta dt)(1-dt\lambda(0)m(\theta(\mathbf{\Omega})))\underline{E}(\omega|\mathbf{\Omega}) + \delta dtU\right]
\end{aligned}$$

Set these values equal and simplify (divide by dt and let $dt \to 0$) to obtain:

$$\lambda(0)m(\theta(\mathbf{\Omega}))\overline{E} - \lambda(0)m(\theta(\mathbf{\Omega}))\underline{E}(\omega|\mathbf{\Omega}) = -pk + \lambda(1)m(\theta(\mathbf{\Omega}))\overline{E} - \lambda(1)m(\theta(\mathbf{\Omega}))\underline{E}(\omega|\mathbf{\Omega})$$

Note that (as any equilibrium value of employment in the low-productivity job), $\underline{E}(\omega|\mathbf{\Omega}) = U = \frac{bp}{r}$. Using this, we obtain a necessary and sufficient condition for the mixing steady state to exist,

$$\theta(\mathbf{\Omega}) = m^{-1}\left(\frac{k(\delta+r)}{\lambda_1(\underline{y}-b)}\right)$$

where the RHS is the same constant as in Lemma 1. In sum, there co-exist three steady states iff

$$\theta(\mathbf{0}) < \theta(\mathbf{\Omega}) = m^{-1}\left(\frac{k(\delta+r)}{\lambda_1(\underline{y}-b)}\right) < \theta(\mathbf{1}).$$

The mixing probability $\mathbf{\Omega}$ can be found by plugging $\theta(\mathbf{\Omega}) = m^{-1}\left(\frac{k(\delta+r)}{\lambda_1(\underline{y}-b)}\right)$ into the FE condition of the firm and solving for $\mathbf{\Omega}$. We obtain the following result.

Proposition A1 *(Existence of Mixed Strategy Steady State).*
If there exist both active and passive search steady states, then there also exists a steady state in mixed strategies.

Proof. We showed in the Proof of Lemma 1 that the active on-the-job search steady state exists if

$$E(1|\mathbf{1}) > E(0|\mathbf{1}). \tag{33}$$

In turn, the passive on-the-job search steady state exists if

$$E(0|\mathbf{0}) > E(1|\mathbf{0}). \tag{34}$$

We provided conditions in terms of exogenous parameters such that both (33) and (34) hold. So, for Ω close to one,

$$E(1|\mathbf{\Omega}) > E(0|\mathbf{\Omega}) \tag{35}$$

but not

$$E(0|\mathbf{\Omega}) > E(1|\mathbf{\Omega}). \tag{36}$$

In turn, for $\mathbf{\Omega}$ close to zero, (36) holds but not (35). Since $E(\omega|\mathbf{\Omega}) - E(\omega|\mathbf{\Omega})$ is continuous in $\mathbf{\Omega}$, there exist a $\mathbf{\Omega} \in (0, 1)$, such that $E(0|\mathbf{\Omega}) = E(1|\mathbf{\Omega})$. ■

Proof of Proposition 1

Proof. At the multiplicity bounds, the market tightness in the equilibrium with active and passive on-the-job search satisfies according to Lemma 1:

$$\theta_l = \frac{-\delta k - kr}{\alpha(b\lambda_1\phi + \delta k + kr - \lambda_1\phi\underline{y})} = \theta_h$$

where θ_l is the lowest tightness that sustains the equilibrium with active on-the-job search and θ_h is the highest tightness that sustains the equilibrium with passive on-the-job search.

To obtain the p-bounds, we evaluate the free entry condition of the active on-the-job search equilibrium at θ_l to obtain p_l, given by:

$$\begin{aligned} p_l \quad = \quad & -\left[c\lambda_1(b-\underline{y})(\delta+r)(\lambda_1(\underline{y}-b)+k(\lambda_0+\lambda_1))(\delta(\lambda_1(\underline{y}-b)+2k(\lambda_0+\lambda_1))+2kr(\lambda_0+\lambda_1))\right] \times \\ & \big[\alpha(\lambda_1\phi(b-\underline{y})+k(\delta+r))\left(b^3\delta\lambda_1^2 - b^2\lambda_1(\delta k(2\lambda_0+\lambda_1)+3\delta\lambda_1\underline{y}+kr(\lambda_0+\lambda_1))+\right. \\ & b\left(k(\delta+r)(\lambda_0+\lambda_1)(k\lambda_0+\lambda_1\overline{y})+k\lambda_1\underline{y}(\delta(3\lambda_0+\lambda_1)+r(\lambda_0+\lambda_1))+3\delta\lambda_1^2\underline{y}^2\right) \\ & \left.-k^2\overline{y}(\delta+r)(\lambda_0+\lambda_1)^2+k\lambda_1\underline{y}(\delta+r)(k-\overline{y})(\lambda_0+\lambda_1)-\delta k\lambda_0\lambda_1\underline{y}^2-\delta\lambda_1^2\underline{y}^3\right)\big]^{-1} \end{aligned}$$

and likewise for p_h (evaluating the free entry condition of the passive on-the-job search equilibrium at θ_h):

$$p^h \quad = \quad -\frac{c\lambda_1(b-\underline{y})(\delta+r)(b\delta\lambda_1-2\delta k\lambda_0-\delta\lambda_1\underline{y}-2k\lambda_0 r)}{\alpha(b\lambda_1\phi+\delta k+kr-\lambda_1\phi\underline{y})\left(b^2\delta\lambda_1-b\delta k\lambda_0-2b\delta\lambda_1\underline{y}-bk\lambda_0 r+\delta k\lambda_0\overline{y}+\delta\lambda_1\underline{y}^2+k\lambda_0 r\overline{y}\right)}$$

We aim to show that $p_h > p_l$ for some range of parameters. To simplify the expressions, let $\underline{y} = 1$ (a normalization), and $\delta = 0$, which yields:

$$p^h - p^l = \frac{2(b-1)^2ck\lambda_1^2 r}{\alpha(\overline{y}-b)\left(b^2\lambda_1 - b(k\lambda_0+\lambda_1\overline{y}+\lambda_1)+k\overline{y}(\lambda_0+\lambda_1)-k\lambda_1+\lambda_1\overline{y}\right)\left(-(1-b)\lambda_1\phi+kr\right)}$$

which needs to be ensured to be positive. The numerator is positive. The denominator is positive if:

$$kr \quad > \quad (1-b)\lambda_1\phi \tag{37}$$

$$\overline{y} \quad > \quad \frac{b(\lambda_1(1-b)+k(\lambda_0+\lambda_1))}{\lambda_1(1-b)+k(\lambda_0+\lambda_1)} \tag{38}$$

where the RHS of (38) is smaller than one. Therefore, a set of sufficient conditions for the existence of multiple steady states, i.e. for $p^h - p^l > 0$, is: (i) $k > ((1-b)\lambda_1\phi)/r$, (ii) $\overline{y} > 1 = \underline{y} > b$. (iii) By continuity of p_h and p_l in δ, this holds for $\delta > 0$ but small. ■

Proof of Proposition 2

Proof. For tractability, we focus on the case with $\delta \to 0$, which, by continutity, implies that the result holds for small δ.

1. We aim to show that there will be a deviation. By Lemma 1, a sufficient condition for a unique equilibrium is $\theta(\mathbf{0}) > \theta(\mathbf{1})$. To show this obtains, observe that $\underline{J}(\mathbf{1}) = \underline{J}(\mathbf{0}) - K$ where $K = \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}$ (by equation (29)).

Free entry condition (28), when $\delta \to 0$ and $\overline{y} \to \underline{y}$, in both equilibria, can be written as:

$$\begin{aligned} \frac{2c}{q(\theta(\mathbf{1}))} &= \underline{J}(\mathbf{0}) - \frac{pk}{r+\delta+\lambda(\mathbf{1})m(\theta(\mathbf{1}))} \\ \frac{2c}{q(\theta(\mathbf{0}))} &= \underline{J}(\mathbf{0}) \end{aligned}$$

Since the LHS is increasing in θ and $\frac{pk}{r+\delta+\lambda(\mathbf{1})m(\theta(\mathbf{1}))} > 0$, it immediately follows that $\theta(\mathbf{1}) < \theta(\mathbf{0})$. By continuity, this holds for $\overline{y} - \underline{y} < \varepsilon$.

2. Fix $\underline{y} < \infty$ and let $\overline{y} \to \infty$. We aim to show that when there is no active on-the-job search by all workers, there is always a profitable deviation by an individual. From Lemma 1 this is equivalent to violating $\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right)$. From equation (28) we obtain:

$$\begin{aligned} \frac{2c}{q(\theta(\mathbf{\Omega}))} &= \underline{J}(\mathbf{0}) - \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} + \lim_{\overline{y}\to\infty} \frac{p(\overline{y}-\underline{y})}{r+\delta} \\ &= \underline{J}(\mathbf{0}) - \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} + \infty \end{aligned}$$

which under no active on-the-job search is

$$\frac{2c}{q(\theta(\mathbf{0}))} = \underline{J}(\mathbf{0}) + \infty.$$

This can only be satisfied if $\theta(\mathbf{0}) \to \infty$, thus violating no-deviation condition $\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right) < \infty$. By continuity, this holds for finite but large $\overline{y}$.

3. We can compute the bounds for $\overline{y}$ explicitly from a system of two equations (per bound), namely free entry and the no-deviation condition.

$$\begin{aligned} \overline{y}_l &= b + k + \frac{2cr}{\alpha\phi p} - \frac{2ckr^2}{\alpha b\lambda_1\phi^2 p + \alpha k\phi pr - \alpha\lambda_1\phi^2 p\underline{y}} - \frac{k^2(\lambda_0+\lambda_1)}{k(\lambda_0+\lambda_1)+\lambda_1(-b+\underline{y})} \\ \overline{y}_h &= \frac{\alpha bp(kr+\lambda_1\phi(b-\underline{y})) + 2c\lambda_1 r(b-\underline{y})}{\alpha p(kr+\lambda_1\phi(b-\underline{y}))} \end{aligned}$$

The set $[\overline{y}_l, \overline{y}_h]$ can be shown to be non-empty for an open set of parameters.

We have shown here that our results hold for $\delta \to 0$. By continuity of the free entry condition in δ, they also hold for $\delta > 0$ and small. ■

Proof of Proposition 3

Proof. Each of the items in the proposition hinges on the fact that $\theta(\mathbf{1}) > \theta(\mathbf{0})$, which follows from Lemma 1.

1. From equation (13), $u(\mathbf{1}) < u(\mathbf{0})$ immediately follows from the fact that $\theta(\mathbf{1}) > \theta(\mathbf{0})$ and $m(\theta)$ is increasing.
2. EE flows are defined as:

$$\begin{aligned} EE(\mathbf{\Omega}) &= \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))\gamma(\mathbf{\Omega}) \\ &= \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))\frac{\delta m(\theta(\mathbf{\Omega}))}{(\delta+m(\theta(\mathbf{\Omega})))(\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))}, \end{aligned}$$

using (14). Then $EE(\mathbf{1}) > EE((\mathbf{0})$ provided

$$\begin{aligned} \frac{\delta\lambda(\mathbf{1})m(\theta(\mathbf{1}))^2}{(\delta+m(\theta(\mathbf{1})))(\delta+\lambda(\mathbf{1})m(\theta(\mathbf{1})))} - \frac{\delta\lambda(\mathbf{0})m(\theta(\mathbf{0}))^2}{(\delta+m(\theta(\mathbf{0})))(\delta+\lambda(\mathbf{0})m(\theta(\mathbf{0})))} &> 0 \\ \delta^2\left(\lambda(\mathbf{1})m(\theta(\mathbf{1}))^2 - \lambda(\mathbf{0})m(\theta(\mathbf{0}))^2\right) + \lambda(\mathbf{0})\lambda(\mathbf{1})m(\theta(\mathbf{0}))m(\theta(\mathbf{1}))\left[m(\theta(\mathbf{1})) - m(\theta(\mathbf{0}))\right] & \\ +m(\theta(\mathbf{0}))m(\theta(\mathbf{1}))\delta\left[\lambda(\mathbf{1})m(\theta(\mathbf{1})) - \lambda(\mathbf{0})m(\theta(\mathbf{0}))\right] &> 0, \end{aligned}$$

which is holds since $\lambda(\mathbf{1}) > \lambda(\mathbf{0})$ and under multiplicity $m(\theta(\mathbf{1})) > m(\theta(\mathbf{0}))$.

3. Since we need an explicit expression for the matching function to solve explicitly for v, we derive this for the telegraph matching function. For a given unemployment rate u, from (16) we find that $v(\mathbf{1}) > v(\mathbf{0})$ provided

$$\frac{2\lambda(\mathbf{1})(1-u)+u}{\lambda(\mathbf{1})(1-u)+u} > \frac{2\lambda(\mathbf{0})(1-u)+u}{\lambda(\mathbf{0})(1-u)+u}$$

which is satisfied since $\lambda(\mathbf{1}) > \lambda(\mathbf{0})$ and $u \in [0,1]$.

4. It suffices to show that the derivative of (17) w.r.t $\lambda(\Omega)$ is non-negative, i.e. for given s, v is (weakly) increasing in $\lambda(\mathbf{\Omega})$ (recall that $\lambda(\mathbf{1}) > \lambda(\mathbf{0})$). We obtain:

$$\begin{aligned} \frac{\partial v}{\partial\lambda(\mathbf{\Omega})} = \delta s &\frac{\left(2\alpha\phi(\delta(s-2)+\phi s)\left(2\delta(s-1)+\phi\left(-\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}+\lambda(\Omega)(s-2)+s\right)\right)\right)}{(2\alpha\delta(\delta+2\lambda(\Omega)\phi)-2\alpha s(\delta+\phi)(\delta+\lambda(\Omega)\phi))^2} \\ -\delta s &\frac{\phi\left(\frac{s^2-\lambda(\Omega)(s-2)^2}{\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}}+s-2\right)(2\alpha s(\delta+\phi)(\delta+\lambda(\Omega)\phi)-2\alpha\delta(\delta+2\lambda(\Omega)\phi))}{(2\alpha\delta(\delta+2\lambda(\Omega)\phi)-2\alpha s(\delta+\phi)(\delta+\lambda(\Omega)\phi))^2} \end{aligned}$$

Since the denominator is positive, focus on the numerator.

$$\begin{aligned} &\left(\frac{s^2-\lambda(\Omega)(s-2)^2}{\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}}+s-2\right)(2\alpha s(\delta+\phi)(\delta+\lambda(\Omega)\phi)-2\alpha\delta(\delta+2\lambda(\Omega)\phi)) \\ &+2\alpha(\delta(s-2)+\phi s)\left(2\delta(s-1)+\phi\left(-\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}+\lambda(\Omega)(s-2)+s\right)\right) \end{aligned}$$

Simplifying (and taking into account that $\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}$ in the denominator is positive) implies that the following expression needs to be signed.

$$\begin{aligned} &\delta^2(s-1)\left(s\left(\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}-s\right)-2\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}+\lambda(\Omega)(s-2)^2\right) \\ &+2\delta\phi(s-1)s\left(\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}+\lambda(\Omega)(s-2)-s\right) \\ &+\phi^2 s^2\left(\sqrt{\lambda(\Omega)^2(s-2)^2-2\lambda(\Omega)s^2+s^2}+(\lambda(\Omega)-1)s\right) \end{aligned}$$

By assumption $s \leq 1$ (due to $\lambda(\mathbf{1}) \leq 1$), so all three lines are non-negative: Line 1 is non-negative since both factors are non-positive due to $s \leq 1$. Line 2 is non-negative since the second factor is

$$\begin{aligned}
\left(\sqrt{\lambda(\Omega)^2(s-2)^2 - 2\lambda(\Omega)s^2 + s^2} + \lambda(\Omega)(s-2) - s\right) &\leq 0 \\
\lambda(\Omega)^2(s-2)^2 - 2\lambda(\Omega)s^2 + s^2 &\leq \lambda(\Omega)^2(s-2)^2 - 2\lambda(\Omega)s(s-2) + s^2 \\
-4\lambda(\Omega)s \leq 0.
\end{aligned}$$

Line 3 is non-negative since $\left(\sqrt{\lambda(\Omega)^2(s-2)^2 - 2\lambda(\Omega)s^2 + s^2} + (\lambda(\Omega) - 1)s\right)$ is decreasing in s and is positive for $s = 0$ and equals zero for $s = 1$.

5. We know from 1. that $u(\mathbf{1}) < u(\mathbf{0})$, and from 3. that for a given u, v is higher under active on-the-job search (outward shift of Beveridge Curve). Because the BC is downward sloping, it follows that also for $u(\mathbf{1}) < u(\mathbf{0})$ it must be the case that $v(\mathbf{1}) > v(\mathbf{0})$.

6. This follows from Lemma 1 (i.e. necessary condition for multiplicity $\theta(\mathbf{1}) > \theta(\mathbf{0})$) and

$$\theta(\mathbf{\Omega})\frac{u(\mathbf{\Omega}) + \lambda(\omega)\gamma(\mathbf{\Omega})}{u} = \Theta(\mathbf{\Omega})$$

where $\frac{u+\lambda\gamma}{u} = 2 - \frac{\delta}{\delta+\lambda m(\theta)}$ is larger for $\mathbf{\Omega} = 1$ since $\lambda(1)m(\theta(\mathbf{1}) > \lambda(0)m(\theta(\mathbf{0}))$.

7. $\lambda(\mathbf{1})\gamma(\mathbf{1})/s(\mathbf{1}) > \lambda(\mathbf{0})\gamma(\mathbf{0})/s(\mathbf{0})$ follows from $\lambda(\mathbf{\Omega})\gamma(\mathbf{\Omega})/s(\mathbf{\Omega}) = 1 - u(\mathbf{\Omega})/s(\mathbf{\Omega}) = 1 - (\delta^2 + \delta\lambda m(\theta(\mathbf{\Omega})))/(\delta^2 + 2\delta\lambda m(\theta(\mathbf{\Omega})))$ and

$$\frac{\partial(u(\mathbf{\Omega})/s(\mathbf{\Omega}))}{\partial\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} = -\frac{\delta^3}{(\delta^2 + 2\delta\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))^2} < 0$$

as well as $\lambda(\mathbf{1})m(\theta(\mathbf{1})) > \lambda(\mathbf{0})m(\theta(\mathbf{0}))$. ■

Dynamics

Local Stability

To analyze the dynamic properties, we take the following three dynamic equations into account. They hold both for boom and recession,

$$\dot{u} = \delta(1-u) - um(\theta(\mathbf{\Omega})) \tag{39}$$

$$\dot{\gamma} = um(\theta(\mathbf{\Omega})) - (\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\gamma \tag{40}$$

$$\dot{\underline{J}} = -(p\underline{y} - \underline{w}(\mathbf{\Omega})) + \underline{J}(r + \delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))) \tag{41}$$

where (39) describes unemployment dynamics, (40) gives the dynamics for employed workers after a UE transition and (41) describes how the value of a filled job evolves over time.

It will be more convenient to work with $\dot{\theta}$, so we first transform the equation for $\dot{J}$ into and equation in $\dot{\theta}$.

Notice that from the free entry condition we can find an expression for $\underline{J}$:

$$\underline{J} \quad = \quad \frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J} \tag{42}$$

Take the time derivative of $\underline{J}$ (taking into account $q(\theta(\mathbf{\Omega})) = \frac{m(\theta(\mathbf{\Omega}))}{\theta(\mathbf{\Omega}))}$) to obtain:

$$\begin{aligned}
\dot{\underline{J}} &= \dot{\theta}(\mathbf{\Omega})\frac{c}{m(\theta(\mathbf{\Omega}))^2}(m(\theta(\mathbf{\Omega})) - \theta(\mathbf{\Omega})m'(\theta(\mathbf{\Omega})))\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} + \dot{u}\frac{\lambda(\mathbf{\Omega})\gamma}{u^2}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) - \dot{\gamma}\frac{\lambda(\mathbf{\Omega})}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) \\
&= \dot{\theta}(\mathbf{\Omega})\frac{c}{m(\theta(\mathbf{\Omega}))}(1-\eta(\theta(\mathbf{\Omega})))\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} + \dot{u}\frac{\lambda(\mathbf{\Omega})\gamma}{u^2}\left(-\frac{\theta c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) - \dot{\gamma}\frac{\lambda(\mathbf{\Omega})}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) \qquad (43)
\end{aligned}$$

where we define the elasticity of the matching function as $\eta(\theta) = \frac{\theta m'(\theta)}{m(\theta)}$.

Plug the expressions for $\dot{\underline{J}}$, (43), and for $\underline{J}$ from free entry (42), into (41),

$$\begin{aligned}
&\dot{\theta}(\mathbf{\Omega})\frac{c}{m(\theta(\mathbf{\Omega}))}(1-\eta(\theta(\mathbf{\Omega})))\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} + \dot{u}\frac{\lambda(\mathbf{\Omega})\gamma}{u^2}\left(-\frac{\theta c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) - \dot{\gamma}\frac{\lambda(\mathbf{\Omega})}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right) \\
= \;& -(p\underline{y} - \underline{w}(\mathbf{\Omega})) + \left(\frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J}\right)(r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))
\end{aligned}$$

and solve for $\dot{\theta}$, to obtain:

$$\begin{aligned}
\dot{\theta}(\mathbf{\Omega}) = \frac{m(\theta(\mathbf{\Omega}))u}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \quad \times \quad & \left[\frac{\lambda}{u}\left(-\frac{\theta c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\left(-\dot{u}\frac{\gamma}{u} + \dot{\gamma}\right) - (p\underline{y} - \underline{w}(\mathbf{\Omega}))\right. \\
& \left. + \left(\frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J}\right)(r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\right]
\end{aligned}$$

So our dynamic system is given by:

$$\dot{u} = \quad \delta(1-u) - um(\theta(\mathbf{\Omega})) \tag{44}$$

$$\dot{\gamma} = \quad um(\theta(\mathbf{\Omega})) - (\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\gamma \tag{45}$$

$$\begin{aligned}
\dot{\theta}(\mathbf{\Omega}) = \quad & \frac{m(\theta(\mathbf{\Omega}))u}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \quad \times [\frac{\lambda}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\left(-\dot{u}\frac{\gamma}{u} + \dot{\gamma}\right) - (p\underline{y} - \underline{w}(\mathbf{\Omega})) \\
& + \left(\frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J}\right)(r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))]
\end{aligned} \tag{46}$$

where $\underline{w}(\mathbf{0})$and $\underline{w}(\mathbf{1})$ are given by (10).

To analyze the stability of system (44)-(46), we further have to specify the Jacobian matrix,

$$J^*(\mathbf{\Omega}) = \begin{bmatrix} \frac{\partial \dot{u}}{\partial u}^* & \frac{\partial \dot{u}}{\partial \gamma}^* & \frac{\partial \dot{u}}{\partial \theta}^* \\ \frac{\partial \dot{\gamma}}{\partial u}^* & \frac{\partial \dot{\gamma}}{\partial \gamma}^* & \frac{\partial \dot{\gamma}}{\partial \theta}^* \\ \frac{\partial \dot{\theta}}{\partial u}^* & \frac{\partial \dot{\theta}}{\partial \gamma}^* & \frac{\partial \dot{\theta}}{\partial \theta}^* \end{bmatrix}$$

where all partial derivatives are functions of $\mathbf{\Omega}$ and are *evaluated at the steady state* under consideration (indicated by *): $\dot{u} = 0, \dot{\gamma} = 0, \dot{\theta} = 0, \dot{\underline{J}} = 0$. In particular, we obtain

$$\begin{aligned}
\frac{\partial \dot{u}}{\partial u} &= -(\delta + m(\theta(\mathbf{\Omega}))) \\
\frac{\partial \dot{u}}{\partial \gamma} &= 0 \\
\frac{\partial \dot{u}}{\partial \theta} &= -um'(\theta(\mathbf{\Omega})) \\
\frac{\partial \dot{\gamma}}{\partial u} &= m(\theta(\mathbf{\Omega})) \\
\frac{\partial \dot{\gamma}}{\partial \gamma} &= -(\delta + \lambda m(\theta(\mathbf{\Omega}))) \\
\frac{\partial \dot{\gamma}}{\partial \theta} &= m'(\theta(\mathbf{\Omega}))(u - \lambda(\mathbf{\Omega})\gamma) \\
\frac{\partial \dot{\theta}}{\partial u} &= \frac{m(\theta(\mathbf{\Omega}))\lambda(\mathbf{\Omega})\gamma}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \times \left[\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\left(\frac{\gamma}{u}\left[r + 2\delta + (\lambda(\mathbf{\Omega})+1)m(\theta(\mathbf{\Omega}))\right] + m(\theta(\mathbf{\Omega}))\right)\right] > 0 \\
\frac{\partial \dot{\theta}}{\partial \gamma} &= \frac{m(\theta(\mathbf{\Omega}))\lambda(\mathbf{\Omega})}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \times \left[-\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)(r + 2\delta + 2\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\right] < 0 \\
\frac{\partial \dot{\theta}}{\partial \theta} &= \frac{m(\theta(\mathbf{\Omega}))u}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \times \left[\lambda(\mathbf{\Omega})m'(\theta(\mathbf{\Omega}))\left(-\frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} + \frac{\gamma(1-\lambda(\mathbf{\Omega}))+u}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\right)\right] \\
&\quad + r + \delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))
\end{aligned}$$

The linearized system (around the steady state) of differential equations is then given by

$$\begin{bmatrix} \dot{u} \\ \dot{\gamma} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{u}}{\partial u}^* & \frac{\partial \dot{u}}{\partial \gamma}^* & \frac{\partial \dot{u}}{\partial \theta}^* \\ \frac{\partial \dot{\gamma}}{\partial u}^* & \frac{\partial \dot{\gamma}}{\partial \gamma}^* & \frac{\partial \dot{\gamma}}{\partial \theta}^* \\ \frac{\partial \dot{\theta}}{\partial u}^* & \frac{\partial \dot{\theta}}{\partial \gamma}^* & \frac{\partial \dot{\theta}}{\partial \theta}^* \end{bmatrix} \begin{bmatrix} u_t - u_i^* \\ \gamma_t - \gamma_i^* \\ \theta_t - \theta_i^* \end{bmatrix} \tag{47}$$

Transition Dynamics

We follow Stemp and Herbert (2006) to use an approximation for the stable manifold that we cannot solve explicitly.

Using the linearized system (47) we can write the system as:

$$\begin{bmatrix} u_t - u_i^* \\ \gamma_t - \gamma_i^* \\ \theta_t - \theta_i^* \end{bmatrix} = \left[\mathbf{v}(\lambda_1) \quad \mathbf{v}(\lambda_2) \quad \mathbf{v}(\lambda_3)\right] \begin{bmatrix} C_1 \exp(\lambda_1 t) \\ C_2 \exp(\lambda_2 t) \\ C_3 \exp(\lambda_3 t) \end{bmatrix}$$

where λ_i is the i-th eigenvalue and $\mathbf{v}(\lambda_i)$ is the corresponding eigenvector. In our case all three eigenvalues are real, with one eigenvalue positive (the unstable eigenvalue) and two eigenvalues negative (the stable eigenvalues). Without loss, we assume that λ_3 is positive. Then the stable solution is given by setting $C_3 = 0$, and we obtain:

$$\begin{bmatrix} u_t - u_i^* \\ \gamma_t - \gamma_i^* \\ \theta_t - \theta_i^* \end{bmatrix} = [\mathbf{v}(\lambda_1) \;\; \mathbf{v}(\lambda_2)] \begin{bmatrix} C_1 \exp(\lambda_1 t) \\ C_2 \exp(\lambda_2 t) \end{bmatrix}$$

$$= \begin{bmatrix} v_1(\lambda_1) & v_1(\lambda_2) \\ v_2(\lambda_1) & v_2(\lambda_2) \\ v_3(\lambda_1) & v_3(\lambda_2) \end{bmatrix} \begin{bmatrix} C_1 \exp(\lambda_1 t) \\ C_2 \exp(\lambda_2 t) \end{bmatrix}$$

Productivity Induced Dynamics

$$\dot{u} = \delta(1-u) - um(\theta(\mathbf{\Omega})) \tag{48}$$

$$\dot{\gamma} = um(\theta(\mathbf{\Omega})) - (\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\gamma \tag{49}$$

$$\begin{aligned}\dot{\theta}(\mathbf{\Omega}) = {} & \frac{m(\theta(\mathbf{\Omega}))u}{c(1-\eta(\theta(\mathbf{\Omega})))(u+\lambda(\mathbf{\Omega})\gamma)} \times \left[\frac{\lambda}{u}\left(-\frac{\theta(\mathbf{\Omega})c}{m(\theta(\mathbf{\Omega}))} + \overline{J}\right)\left(-\dot{u}\frac{\gamma}{u} + \dot{\gamma}\right) - (p\underline{y} - \underline{w}(\mathbf{\Omega}))\right. \\ & \left. + \left(\frac{c}{q(\theta(\mathbf{\Omega}))}\frac{u+\lambda(\mathbf{\Omega})\gamma}{u} - \frac{\lambda(\mathbf{\Omega})\gamma}{u}\overline{J}\right)(r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))\right] + (1-\pi(\mathbf{\Omega}))\Delta_\theta(\mathbf{\Omega}). \end{aligned} \tag{50}$$

Appendix B: Alternative Model Specifications

We will analyze three generalizations of the model to show that the multiplicity of steady states is not specific to our simplified baseline model.

1. We generalize our baseline model in two dimensions: first, we allow for *stochastic* match-specific productivity upgrades (i.e. when employed or unemployed workers search, they all have the same probability of obtaining an $\overline{y}$ match (with probability π) or $\underline{y}$ match (with probability $1-\pi$). Second, the number of rounds of OJS is no longer restricted to one. Workers can search as many rounds they like.

2. We keep the deterministic match-specific productivity upgrade from OJS (as in baseline model) but allow for an unrestricted number of rounds of OJS.

3. We introduce ex-ante heterogeneity of firms (there is free entry into low productivity vacancies that produce $\underline{y}$ and free entry into high productivity vacancies that produce $\overline{y}$ once matched). Both unemployed and employed workers meet these different vacancies with the same probabilities. We keep the assumption of a single round of OJS from the baseline model.

1. Stochastic Match-Specific Types with Unrestricted Number of Search Periods

Suppose that any realized match has a probability π to be of productivity $\overline{y}$ and a probability $1-\pi$ to be of productivity $\underline{y}$. Once matched, the worker decides whether to continue to search. Now the exact history matters for the continuation. There are 6 possible states that workers can be in, depending on their history:

1. u: unemployment

2. γ_L: employed out of u in a $\underline{y}$ job

3. γ_H: employed out of u in a $\overline{y}$ job

4. γ_{LL}: employed in an $\underline{y}$ job after being employed in $\underline{y}$ in a previous period

5. γ_{LH}: employed in a $\overline{y}$ job after being employed in an $\underline{y}$ job or after having received at least once an offer from a $\underline{y}$ job.

6. γ_{HH}: employed in a $\overline{y}$ job after being employed in $\overline{y}$ in a previous period.

We denote the corresponding values of (un)employment and wages by:

1. U: unemployed

2. E_L: Employed at $\underline{y}$ out of U, i.e. coming from u into γ_L. Get wage w_L.

3. E_H: Employed at $\overline{y}$ out of U, i.e. coming from u into γ_H. Get wage w_H.

4. E_{LL}: Employed at $\underline{y}$ after a match with at least one other $\underline{y}$ and no $\overline{y}$, i.e. coming from γ_L into γ_{LL}. Get wage w_{LL}.

5. E_{LH}: Employed at $\overline{y}$ after having matched with at least one $\underline{y}$ and not matched with another $\overline{y}$, i.e. coming from γ_L or γ_{LL}, or after not having been matched with any $\underline{y}$ before but having received an offer from $\underline{y}$ while matched with one $\overline{y}$ (i.e. coming from γ_H). Get wage w_{LH}.

6. E_{HH}: Employed at $\overline{y}$ after matching with at least one other $\overline{y}$, i.e. coming from γ_H or γ_{LH}. Get wage w_{HH}.

Apart from the stochastic (as opposed to deterministic) upgrading of productivity as workers search, there is a second change compared to the baseline model: We now leave the number of rounds of on-the-job search unrestricted, i.e. workers will stop searching only once they extract the entire rents from a match.

As in the baseline model, ties are broken in favor of the incumbent: in case of a tie, assume the worker does not move or equivalently, that there is an ε moving cost. For the values of a filled job, we adopt a similar notation below.

We now make a distinction between the action of the different active searching workers, depending on which state they are in. Denote by $\mathbf{\Omega}_L$ the search intensity of all workers in state L (and the individual search intensity is ω_L). Likewise for $\mathbf{\Omega}_H, \mathbf{\Omega}_{LL}$ and $\mathbf{\Omega}_{LH}$ (note that $\mathbf{\Omega}_{HH} = 0$ since at this stage the worker's wage equals the entire output). The overall search intensity that enters the market tightness θ is a vector $\mathbf{\Omega} = (\mathbf{\Omega}_L, \mathbf{\Omega}_H, \mathbf{\Omega}_{LL}, \mathbf{\Omega}_{LH})$. Then we will determine the number of searchers as $s = u + (\vec{\lambda}_0 + \lambda_1 \mathbf{\Omega})\gamma^T$ where $\vec{\lambda}_0$ is the vector of identical constants λ_0 and γ is the vector $[\gamma_L, \gamma_H, \gamma_{LL}, \gamma_{LH}]$. Then $s = u + (\lambda_0 + \lambda_1 \mathbf{\Omega_L})\gamma_L + (\lambda_0 + \lambda_1 \mathbf{\Omega_H})\gamma_H + (\lambda_0 + \lambda_1 \mathbf{\Omega_{LL}})\gamma_{LL} + (\lambda_0 + \lambda_1 \mathbf{\Omega_{LH}})\gamma_{LH}$.

The laws of motion satisfy:

$$\begin{aligned}
1 &= u + \gamma_L + \gamma_H + \gamma_{LL} + \gamma_{LH} + \gamma_{HH} \\
\dot{\gamma_L} &= um(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_L[\delta + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))] \\
\dot{\gamma_H} &= um(\theta(\mathbf{\Omega}))\pi - \gamma_H[\delta + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))] \\
\dot{\gamma_{LL}} &= \gamma_L \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_{LL}[\delta + \lambda(\mathbf{\Omega}_{LL})m(\theta(\mathbf{\Omega}))\pi] \\
\dot{\gamma_{LH}} &= \gamma_L \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))\pi + \gamma_{LL}\lambda(\mathbf{\Omega}_{LL})m(\theta(\mathbf{\Omega}))\pi + \gamma_H \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_{LH}[\delta + \lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega}))\pi] \\
\dot{\gamma_{HH}} &= \gamma_H \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))\pi + \gamma_{LH}\lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega}))\pi - \gamma_{HH}\delta
\end{aligned}$$

Equilibrium. As before, we focus on steady state equilibria.

We can write the value functions of the worker in steady state as:

$$\begin{aligned}
rU &= pb + m(\theta(\mathbf{\Omega}))((1-\pi)E_L + \pi E_H - U) \\
rE_L &= w_L(\mathbf{\Omega}) - \omega_L pk + \lambda(\omega_L)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LL} + \pi E_{LH} - E_L) - \delta(E_L - U) \\
rE_H &= w_H(\mathbf{\Omega}) - \omega_H pk + \lambda(\omega_H)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LH} + \pi E_{HH} - E_H) - \delta(E_H - U) \\
rE_{LL} &= w_{LL}(\mathbf{\Omega}) - \omega_{LL} pk + \lambda(\omega_{LL})m(\theta(\mathbf{\Omega}))(\pi(E_{LH} - E_{LL})) - \delta(E_{LL} - U) \\
rE_{LH} &= w_{LH}(\mathbf{\Omega}) - \omega_{LH} pk + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))(\pi(E_{HH} - E_{LH})) - \delta(E_{LH} - U) \\
rE_{HH} &= w_{HH}(\mathbf{\Omega}) - \delta(E_{HH} - U)
\end{aligned}$$

The steady state values of a filled job, high or low productivity (and depending on the workers previous position), are given by:

$$\begin{aligned}
rJ_L &= p\underline{y} - w_L(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))[(1-\pi)(J_{LL} - J_L) + \pi(J_{LH} - J_L)] - \delta(J_L - V) \\
rJ_H &= p\overline{y} - w_H(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))[(1-\pi)(J_{LH} - J_H) + \pi(J_{HH} - J_H)] - \delta(J_H - V) \\
rJ_{LL} &= p\underline{y} - w_{LL}(\mathbf{\Omega}) + \lambda(\Omega_{LL})m(\theta(\mathbf{\Omega}))\pi(V - J_{LL}) - \delta(J_{LL} - V) \\
rJ_{LH} &= p\overline{y} - w_{LH}(\mathbf{\Omega}) + \lambda(\Omega_{LH})m(\theta(\mathbf{\Omega}))(\pi(J_{HH} - J_{LH})) - \delta(J_{LH} - V) \\
rJ_{HH} &= p\overline{y} - w_{HH}(\mathbf{\Omega}) - \delta(J_{HH} - V)
\end{aligned}$$

The value of a vacancy to the firm is

$$rV = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{s(\mathbf{\Omega})}(1-\pi)J_L + \frac{u}{s(\mathbf{\Omega})}\pi J_H + \frac{\lambda(\mathbf{\Omega}_L)\gamma_L\pi + \lambda(\mathbf{\Omega}_{LL})\gamma_{LL}\pi}{s(\mathbf{\Omega})}J_{LH} - V\right]$$

where s denotes the number of searchers:

$$s(\mathbf{\Omega}) = u + \lambda(\mathbf{\Omega}_L)\gamma_L + \lambda(\mathbf{\Omega}_H)\gamma_H + \lambda(\mathbf{\Omega}_{LL})\gamma_{LL} + \lambda(\mathbf{\Omega}_{LH})\gamma_{LH}$$

The value of a vacancy V reflects that workers stay with the incumbent firm in case the worker draws the same match-specific productivity.

Then the equilibrium tightness can be written as:

$$\theta(\mathbf{\Omega}) = \frac{v}{s(\mathbf{\Omega})}.$$

We now derive the steady state equilibrium values (where $\omega = \Omega$):

$$\begin{aligned}
U &= \frac{pb}{r} \\
E_L &= \frac{w_L(\mathbf{\Omega}) - \omega_L pk + \lambda(\omega_L)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LL} + \pi E_{LH}) + \delta U}{r + \lambda(\omega_L)m(\theta(\mathbf{\Omega})) + \delta} \\
E_H &= \frac{w_H(\mathbf{\Omega}) - \omega_H pk + \lambda(\omega_H)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LH} + \pi E_{HH}) + \delta U}{r + \lambda(\omega_H)m(\theta(\mathbf{\Omega})) + \delta} \\
E_{LL} &= \frac{w_{LL}(\mathbf{\Omega}) - \Omega_{LL} pk + \lambda(\omega_{LL})m(\theta(\mathbf{\Omega}))\pi E_{LH} + \delta U}{r + \lambda(\omega_{LL})m(\theta(\mathbf{\Omega}))\pi + \delta} \\
E_{LH} &= \frac{w_{LH}(\mathbf{\Omega}) - \omega_{LH} pk + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))E_{HH} + \delta U}{r + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))\pi + \delta} \\
E_{HH} &= \frac{w_{HH}(\mathbf{\Omega}) + \delta U}{r + \delta}
\end{aligned}$$

The equilibrium wage is set to the maximum amount that the “losing” firm (which is the firm who loses the worker to another firm or that unsuccessfully tries to poach the worker) would be able to pay for the worker. In turn, if the worker is hired out of unemployment than the wage is set such that he is indifferent between remaining unemployed and taking the job. That implies:

$$\begin{aligned}
w_L(\mathbf{\Omega}) :& \quad E_L = U \\
w_H(\mathbf{\Omega}) :& \quad E_H = U \\
w_{LL}(\mathbf{\Omega}) :& \quad J_{LL} = V \quad \rightarrow \quad w_{LL} = p\underline{y} \\
w_{LH}(\mathbf{\Omega}) :& \quad J_{LH} = V \quad \rightarrow \quad w_{LH} = p\underline{y} \\
w_{HH}(\mathbf{\Omega}) :& \quad J_{HH} = V \quad \rightarrow \quad w_{HH} = p\overline{y}
\end{aligned}$$

This implies that the values for the firm (imposing free entry of firms, $V = 0$) are given by:

$$0 = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{s}(1-\pi)J_L + \frac{u}{s}\pi J_H + \frac{\lambda(\mathbf{\Omega}_L)\gamma_L\pi + \lambda(\mathbf{\Omega}_{LL})\gamma_{LL}\pi}{s}J_{LH}\right] \tag{51}$$

$$\begin{aligned}
J_L &= \frac{p\underline{y} - w_L(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))((1-\pi)J_{LL} + \pi J_{LH})}{r + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega})) + \delta} \\
J_H &= \frac{p\overline{y} - w_H(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))((1-\pi)J_{LH} + \pi J_{HH})}{r + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega})) + \delta} \\
J_{LL} &= 0 \\
J_{LH} &= \frac{p\overline{y} - w_{LH}(\mathbf{\Omega})}{r + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))\pi + \delta} \\
J_{HH} &= 0
\end{aligned}$$

Multiple Steady State Equilibria.

Now there are four states at which workers make search decisions. Hence, there are $2^4 = 16$ potential equilibria. Multiplicity obtains if any two out of 16 equilibria coexist for some range of parameters. We can verify numerically that the the following two equilibria can coexist: one where workers search in *all* four states $\gamma_L, \gamma_H, \gamma_{LH}, \gamma_{HH}$ and one where workers search in all states but γ_L. In short we will indicate the equilibrium strategy of in the first equilibrium by 1111 and the strategy in the second equilibrium by 0111. (Of course, other equilibria may coexist as well. Establishing multiplicity of all other equilibria is beyond the purpose of this exercise which is to show that the mechanism that leads to multiplicity does not hinge on the particular job ladder that we have in the baseline model.)

We need to verify the four sets of double no-deviation conditions, where, as in the baseline model, it suffices to consider *one-shot deviations*. This implies that we need to check the conditions:

1. In state γ_L: $E_L(0|\mathbf{0111}) > E_L(1|\mathbf{0111})$ and $E_L(1|\mathbf{1111}) > E_L(0|\mathbf{1111})$
2. In state γ_H: $E_H(1|\mathbf{0111}) > E_H(0|\mathbf{0111})$ and $E_H(1|\mathbf{1111}) > E_H(0|\mathbf{1111})$
3. In state γ_{LL}: $E_{LL}(1|\mathbf{0111}) > E_{LL}(0|\mathbf{0111})$ and $E_{LL}(1|\mathbf{1111}) > E_{LL}(0|\mathbf{1111})$
4. In state γ_{LH}: $E_{LH}(1|\mathbf{0111}) > E_{LH}(0|\mathbf{0111})$ and $E_{LH}(1|\mathbf{1111}) > E_{LH}(0|\mathbf{1111})$

In more detail:

1. Check Deviations from Search Decisions in γ_L:

1.1 $E_L(0|\mathbf{0111}) > E_L(1|\mathbf{0111})$ (no deviation from 'no search') if

$$E_L(0|\mathbf{0111})(1+rdt) > dt(w_L(\mathbf{0111}) - pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0111}))[(1-\pi)E_{LL}(1|\mathbf{0111}) + \pi E_{LH}(1|\mathbf{0111})]$$
$$+[1-\delta dt - dt\lambda(1)m(\theta(\mathbf{0111})) + dt^2\delta\lambda(1)m(\theta(\mathbf{0111}))]E_L(0|\mathbf{0111}) + \delta dtU.$$

After subtracting $E_L(0|\mathbf{0111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_L(0|\mathbf{0111}) > w_L(\mathbf{0111}) - pk + \lambda(1)m(\theta(\mathbf{0111}))[(1-\pi)E_LL(1|\mathbf{0111}) + \pi E_{LH}(1|\mathbf{0111})]$$
$$+(-\delta - \lambda(1)m(\theta(\mathbf{0111})))E_L(1|\mathbf{0111}) + \delta U.$$

1.2 $E_L(1|\mathbf{1111}) > E_L(0|\mathbf{1111})$ (no deviation from 'search') if

$$E_L(1|\mathbf{1111})(1+rdt) > dtw_L(\mathbf{1111}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1111}))[(1-\pi)E_{LL}(1|\mathbf{1111}) + \pi E_{LH}(1|\mathbf{1111})]$$
$$+(1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1111})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1111})))E_L(1|\mathbf{1111}) + \delta dtU.$$

After subtracting $E_L(1|\mathbf{1111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_L(1|\mathbf{1111}) > w_L(\mathbf{1111}) + \lambda(0)m(\theta(\mathbf{1111}))[(1-\pi)E_{LL}(1|\mathbf{1111}) + \pi E_{LH}(1|\mathbf{1111})] \\ +(-\delta - \lambda(0)m(\theta(\mathbf{1111})))E_L(1|\mathbf{1111}) + \delta U.$$

2. Check Deviations from Search Decisions in γ_H:

2.1 $E_H(1|\mathbf{0111}) > E_H(0|\mathbf{0111})$ (no deviation from 'search') if

$$E_H(1|\mathbf{0111})(1+rdt) > dtw_H(\mathbf{0111}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{0111}))[(1-\pi)E_LH(1|\mathbf{0111}) + \pi E_{HH}] \\ +[1-\delta dt - dt\lambda(0)m(\theta(\mathbf{0111})) + dt^2\delta\lambda(0)m(\theta(\mathbf{0111}))]E_H(1|\mathbf{0111}) + \delta dtU.$$

After subtracting $E_H(1|\mathbf{0111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_H(1|\mathbf{0111}) > w_H(\mathbf{0111}) + \lambda(0)m(\theta(\mathbf{0111}))[(1-\pi)E_{LH}(1|\mathbf{0111}) + \pi E_{HH}(1|\mathbf{0111})] \\ +(-\delta - \lambda(0)m(\theta(\mathbf{0111})))E_H(1|\mathbf{0111}) + \delta U.$$

2.2 $E_H(1|\mathbf{1111}) > E_H(0|\mathbf{1111})$ (no deviation from 'search') if

$$E_H(1|\mathbf{1111})(1+rdt) > dtw_H(\mathbf{1111}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1111}))[(1-\pi)E_{LH}(1|\mathbf{1111}) + \pi E_{HH}(1|\mathbf{1111})] \\ +(1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1111})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1111})))E_H(1|\mathbf{1111}) + \delta dtU.$$

After subtracting $E_H(1|\mathbf{1111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_H(1|\mathbf{1111}) > w_H(\mathbf{1111}) + \lambda(0)m(\theta(\mathbf{1111}))[(1-\pi)E_{LH}(1|\mathbf{1111}) + \pi E_{HH}(1|\mathbf{1111})] \\ +(-\delta - \lambda(0)m(\theta(\mathbf{1111})))E_H(1|\mathbf{1111}) + \delta U.$$

3. Check Deviations from Search Decisions in γ_{LL}:

3.1 $E_{LL}(1|\mathbf{0111}) > E_{LL}(0|\mathbf{0111})$ (no deviation from 'search') if

$$E_{LL}(1|\mathbf{0111})(1+rdt) > dtew_{LL} + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{0111}))\pi E_{LH}(1|\mathbf{0111}) \\ +[1-\delta dt - dt\lambda(0)m(\theta(\mathbf{0111}))\pi + dt^2\delta\lambda(0)m(\theta(\mathbf{0111}))\pi]E_{LL}(1|\mathbf{0111}) + \delta dtU.$$

After subtracting $E_{LL}(1|\mathbf{0111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LL}(1|\mathbf{0111}) > w_{LL} + \lambda(0)m(\theta(\mathbf{0111}))\pi E_{LH}(1|\mathbf{0111}) + (-\delta - \lambda(0)m(\theta(\mathbf{0111}))\pi)E_{LL}(1|\mathbf{0111}) + \delta U.$$

3.2 $E_{LL}(1|\mathbf{1111}) > E_{LL}(0|\mathbf{1111})$ (no deviation from 'search') if

$$\begin{aligned} E_{LL}(1|\mathbf{1111})(1+rdt) &> dtw_{LL} + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1111}))\pi E_{LH}(1|\mathbf{1111}) \\ &+ (1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1111}))\pi + dt^2\delta\lambda(0)m(\theta(\mathbf{1111}))\pi)E_{LL}(1|\mathbf{1111}) + \delta dtU. \end{aligned}$$

After subtracting $E_{LL}(1|\mathbf{1111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LL}(1|\mathbf{1111}) > w_{LL} + \lambda(0)m(\theta(\mathbf{1111}))\pi E_{LH}(1|\mathbf{1111}) + (-\delta - \lambda(0)m(\theta(\mathbf{1111}))\pi)E_{LL}(1|\mathbf{1111}) + \delta U.$$

4. Check Deviations from Search Decisions in γ_{LH}:

4.1 $E_{LH}(1|\mathbf{0111}) > E_{LH}(0|\mathbf{0111})$ (no deviation from 'search') if

$$\begin{aligned} E_{LH}(1|\mathbf{0111})(1+rdt) &> dtw_{LH} + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{0111}))\pi E_{HH} \\ &+ [1-\delta dt - dt\lambda(0)m(\theta(\mathbf{0111}))\pi + dt^2\delta\lambda(0)m(\theta(\mathbf{0111}))\pi]E_{LH}(1|\mathbf{0111}) + \delta dtU. \end{aligned}$$

After subtracting $E_{LH}(1|\mathbf{0111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LH}(1|\mathbf{0111}) > w_{LH} + \lambda(0)m(\theta(\mathbf{0111}))\pi E_{HH} + (-\delta - \lambda(0)m(\theta(\mathbf{0111}))\pi)E_{LH}(1|\mathbf{0111}) + \delta U.$$

4.2 $E_{LH}(1|\mathbf{1111}) > E_{LH}(0|\mathbf{1111})$ (no deviation from 'search') if

$$\begin{aligned} E_{LH}(1|\mathbf{1111})(1+rdt) &> dtw_{LH} + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1111}))\pi E_{HH} \\ &+ (1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1111}))\pi + dt^2\delta\lambda(0)m(\theta(\mathbf{1111}))\pi)E_{LH}(1|\mathbf{1111}) + \delta dtU. \end{aligned}$$

After subtracting $E_{LH}(1|\mathbf{1111})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LH}(1|\mathbf{1111}) > w_{LH} + \lambda(0)m(\theta(\mathbf{1111}))\pi E_{HH} + (-\delta - \lambda(0)m(\theta(\mathbf{1111}))\pi)E_{LH}(1|\mathbf{1111}) + \delta U.$$

Analogously to the baseline model, when evaluating these 8 conditions at equalities we obtain the corresponding labor market tightness θ for these 8 bounds. We then evaluate the free entry condition (one by one) at these 8 different values of θ to obtain multiplicity bounds on primitive p. This gives us 8 potential p-bounds: one $\overline{p}$ (i.e. it must be that $p < \overline{p}$, otherwise workers would be incentivized to *always* search, that is also in γ_L) and 7 candidates for $\underline{p}$ (i.e. $p > \underline{p}$, otherwise workers would not want to search in $\gamma_L, \gamma_H, \gamma_{LL}, \gamma_{LH}$ in the boom and in $\gamma_H, \gamma_{LL}, \gamma_{LH}$ in the recession). Hence, for multiple steady state equilibria to exist it must be that $\overline{p} > max(\underline{p})$. It is not possible to report an analytical solution for the p-bounds but we show numerically that there exists parameter ranges for which $\overline{p} > max(\underline{p})$.

An example of parameter intervals, for which there exist multiple steady state equilibria (i.e. co-existence of the two equilibria **1111** and **0111**) is: $\overline{y} \in [1.85 - \Delta, 1.85 + \Delta]$, $\underline{y} \in [1 - \Delta, 1 + \Delta]1$, $b \in [0.79 - \Delta, 0.79 + \Delta]$, $\lambda_0 \in [.04 - \Delta, .04 + \Delta]$, $\lambda_1 \in [0.109 - \Delta, 0.109 + \Delta]$, $k \in [0.05 - \Delta, 0.05 + \Delta]$, $r \in [0.013 - \Delta, 0.013 + \Delta]$, $c \in [3.15 - \Delta, 3.15 + \Delta]$, $\alpha \in [0.03 - \Delta, 0.03 + \Delta]$, $\phi \in [3.16 - \Delta, 3.16 + \Delta]$, $\delta \in [0.05 - \Delta, 0.05 + \Delta]$, $\pi \in [0.46 - \Delta, 0.46 + \Delta]$ for $\Delta > 0$ small.

The mechanism that generates multiplicity in this general setup is similar to the mechanism in our baseline model (main text). It is driven by a composition externality in the pool of searchers in productive and non-productive jobs. Of course, there are more states, which makes the analysis different.

First of all, firms do not want to meet workers already in a $\overline{y}$ job, i.e., in states γ_H and γ_{LH}, because workers in those states will not move but stay in the incumbent firm and extract all the surplus from the match. Also, existing firms do not like their matched workers to receive offers because that instantaneously depletes their surplus. Firms therefore only generate a positive surplus hiring workers out of unemployment (creating both $\underline{y}$ and $\overline{y}$ jobs) or out of existing $\underline{y}$ jobs, i.e., out of states u, γ_L and γ_{LL}. Now what generates the strategic complementarity between the firm's profits from opening a vacancy and the employed worker incentive to search on the job can be read of from the firm's free entry condition (51). What needs to be satisfied there is that the value of hiring out of an existing job πJ_{LH} is larger than hiring out of unemployment with surplus $(1-\pi)J_L + \pi J_H$. This will be the case whenever (i) the duration of a J_H job is low enough (because workers receive outside offers) and (ii) J_L is not too high, i.e., when the productivity of an $\underline{y}$ job is relatively close to the outside option $b + k$. Given that (i) and (ii) are satisfied, similar to the baseline model, when the composition of jobs changes in the boom due to endogenous search intensity, i.e. $\frac{\lambda_L \gamma_L}{s} + \frac{\lambda_{LL}\gamma_{LL}}{s}$ grows relative to $\frac{u}{s}$, then the value of opening jobs goes up. Instead, when the composition of jobs is reversed due to low search intensity in γ_L jobs, then $\frac{\lambda_L \gamma_L}{s} + \frac{\lambda_{LL}\gamma_{LL}}{s}$ is relatively small compared to $\frac{u}{s}$ and the value of opening vacancies falls. This is at the root of the strategic complementarity and generates self-fulfilling prophecies (even though productivity upgrades on the job ladder are *not* deterministic here, unlike in the baseline model).

2. Deterministic Match-Specific Types with Unrestricted Number of Search Periods

In this section, we extend the baseline model in the following sense. We let workers search on the job until they extract the entire output, i.e. there are *two* states of OJS instead of one.

There are 4 possible states that workers can be in, depending on their history:

1. u: unemployment

2. γ_L: employed out of u in a $\underline{y}$ job

3. γ_{LH}: employed in a $\overline{y}$ job after being employed in an $\underline{y}$ job

4. γ_{HH}: employed in a $\overline{y}$ job after having received an outside offer from another $\overline{y}$ job.

We denote the corresponding values of (un)employment and wages by:

1. U: unemployed

2. E_L: Employed at $\underline{y}$ out of U, i.e. coming from u into γ_L. Get wage w_L.

3. E_{LH}: Employed at $\overline{y}$ after having matched with at least one $\underline{y}$ and not matched with another $\overline{y}$, i.e. coming from γ_L. Get wage w_{LH}.

4. E_{HH}: Employed at $\overline{y}$ after received at least one outside offer from another $\overline{y}$, i.e. coming from γ_H. Get wage w_{HH}.

As in the baseline model, ties are broken in favor of the incumbent: in case of a tie, assume the worker does not move or equivalently, that there is an ε moving cost. For the values of a filled job, we adopt a similar notation below.

We now make a distinction between the action of the different active searching workers, depending on which state they are in. Denote by $\mathbf{\Omega}_L$ the search intensity of all workers in state L (and the individual search intensity is ω_L). Likewise for $\mathbf{\Omega}_{LH}$ (note that $\mathbf{\Omega}_{HH} = 0$ since at this stage the worker's wage equals the entire output). The overall search intensity that enters the market tightness θ is a vector $\mathbf{\Omega} = (\mathbf{\Omega}_L, \mathbf{\Omega}_{LH})$. Then we will determine the number of searchers as $s = u + (\vec{\lambda}_0 + \lambda_1 \mathbf{\Omega})\gamma^T$ where $\vec{\lambda}_0$ is the vector of identical constants λ_0 and γ is the vector $[\gamma_L, \gamma_{LH}]$. Then $s = u + (\lambda_0 + \lambda_1 \mathbf{\Omega_L})\gamma_L + (\lambda_0 + \lambda_1 \mathbf{\Omega_{LH}})\gamma_{LH}$.

The laws of motion satisfy:

$$\begin{aligned}
1 &= u + \gamma_L + \gamma_{LH} + \gamma_{HH} \\
\dot{\gamma_L} &= um(\theta(\mathbf{\Omega})) - \gamma_L[\delta + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega})] \\
\dot{\gamma_{LH}} &= \gamma_L \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega})) - \gamma_{LH}[\delta + \lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega}))] \\
\dot{\gamma_{HH}} &= \gamma_{LH}\lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega})) - \gamma_{HH}\delta
\end{aligned}$$

Equilibrium. As before, we focus on steady state equilibria.

We can write the value functions of the worker in steady state as:

$$
\begin{aligned}
rU &= pb + m(\theta(\mathbf{\Omega}))(E_L - U) \\
rE_L &= w_L(\mathbf{\Omega}) - \omega_L pk + \lambda(\omega_L)m(\theta(\mathbf{\Omega}))(E_{LH} - E_L) - \delta(E_L - U) \\
rE_{LH} &= w_{LH}(\mathbf{\Omega}) - \omega_{LH} pk + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))(E_{HH} - E_{LH}) - \delta(E_{LH} - U) \\
rE_{HH} &= w_{HH}(\mathbf{\Omega}) - \delta(E_{HH} - U)
\end{aligned}
$$

The steady state values of a filled job, high or low productivity (and depending on the workers previous position), are given by:

$$
\begin{aligned}
rJ_L &= p\underline{y} - w_L(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))(V - J_L) - \delta(J_L - V) \\
rJ_{LH} &= p\overline{y} - w_{LH}(\mathbf{\Omega}) + \lambda(\Omega_{LH})m(\theta(\mathbf{\Omega}))(J_{HH} - J_{LH}) - \delta(J_{LH} - V) \\
rJ_{HH} &= p\overline{y} - w_{HH}(\mathbf{\Omega}) - \delta(J_{HH} - V)
\end{aligned}
$$

The value of a vacancy to the firm is

$$
rV = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{s(\mathbf{\Omega})}J_L + \frac{\lambda(\mathbf{\Omega}_L)\gamma_L}{s(\mathbf{\Omega})}J_{LH} - V\right]
$$

where s denotes the number of searchers:

$$
s(\mathbf{\Omega}) = u + \lambda(\mathbf{\Omega}_L)\gamma_L + \lambda(\mathbf{\Omega}_{LH})\gamma_{LH}
$$

The value of a vacancy V reflects that workers stay with the incumbent firm in case the worker draws the same match-specific productivity. The equilibrium tightness can be written as:

$$
\theta(\mathbf{\Omega}) = \frac{v}{s(\mathbf{\Omega})}.
$$

We now derive the steady state equilibrium values (where $\omega = \Omega$):

$$
\begin{aligned}
U &= \frac{pb}{r} \\
E_L &= \frac{w_L(\mathbf{\Omega}) - \omega_L pk + \lambda(\omega_L)m(\theta(\mathbf{\Omega}))E_{LH} + \delta U}{r + \lambda(\omega_L)m(\theta(\mathbf{\Omega})) + \delta} \\
E_{LH} &= \frac{w_{LH}(\mathbf{\Omega}) - \omega_{LH} pk + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega}))E_{HH} + \delta U}{r + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega})) + \delta} \\
E_{HH} &= \frac{w_{HH}(\mathbf{\Omega}) + \delta U}{r + \delta}
\end{aligned}
$$

The equilibrium wage is set to the maximum amount that the "losing" firm (which is the firm who loses the worker to another firm or that unsuccessfully tries to poach the worker) would be able to pay for the worker. In turn, if the worker is hired out of unemployment than the wage is set such that he is indifferent between remaining unemployed and taking the job. That implies:

$$\begin{aligned}
w_L(\mathbf{\Omega}):& \quad E_L = U \quad \rightarrow \\
w_L(\mathbf{\Omega}) &= \frac{p\left(b(\delta+r)(\delta k\omega_L + k\omega_L(2(\lambda_0+\lambda_1)m(\Omega)+r) - (\lambda_0+\lambda_1)m(\Omega)((\lambda_0+\lambda_1)m(\Omega)\overline{y}+\underline{y})) + r^2(\delta+(\lambda_0+\lambda_1)m(\Omega)+r)^2\right)}{b(\delta+r)(\delta+(\lambda_0+\lambda_1)m(\Omega)+r)} \\
w_{LH}(\mathbf{\Omega}):& \quad J_{LH} = V \quad \rightarrow \quad w_{LH} = p\underline{y} \\
w_{HH}(\mathbf{\Omega}):& \quad J_{HH} = V \quad \rightarrow \quad w_{HH} = p\overline{y}
\end{aligned}$$

This implies that the values for the firm (imposing free entry of firms, $V = 0$) are given by:

$$\begin{aligned}
0 &= -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{s(\mathbf{\Omega})}J_L + \frac{\lambda(\mathbf{\Omega}_L)\gamma_L}{s(\mathbf{\Omega})}J_{LH}\right] \\
J_L &= \frac{p\underline{y} - w_L(\mathbf{\Omega})}{r + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega})) + \delta} \\
J_{LH} &= \frac{p(\overline{y} - \underline{y})(\mathbf{\Omega})}{r + \lambda(\omega_{LH})m(\theta(\mathbf{\Omega})) + \delta} \\
J_{HH} &= 0
\end{aligned}$$

Multiple Steady State Equilibria.

Now there are two states at which workers make search decisions. Hence, there are $2^2 = 4$ potential equilibria. Multiplicity obtains if any two out of 4 equilibria coexist for some range of parameters. We can verify numerically that the following two equilibria, that we are particularly interested, in can coexist: one where workers search in *all* two states γ_L, γ_{LH} and one where workers search not at all (i.e. in neither state). In short we will indicate the equilibrium strategy of in the first equilibrium by 11 and the strategy in the second equilibrium by 00.

We need to verify the two sets of double no-deviation conditions, where, as in the baseline model, it suffices to consider *one-shot deviations*. This implies that we need to check the conditions:

1. In state γ_L: $E_L(0|\mathbf{00}) > E_L(1|\mathbf{00})$ and $E_L(1|\mathbf{11}) > E_L(0|\mathbf{11})$
2. In state γ_{LH}: $E_{LH}(0|\mathbf{00}) > E_{LH}(1|\mathbf{00})$ and $E_{LH}(1|\mathbf{11}) > E_{LH}(0|\mathbf{11})$

In more detail:

1. Check Deviations from Search Decisions in γ_L:

1.1 $E_L(0|\mathbf{00}) > E_L(1|\mathbf{00})$ (no deviation from 'no search') if

$$\begin{aligned}
E_L(0|\mathbf{00})(1+rdt) > \; & dt(w_L(\mathbf{00}) - pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{00}))E_{LH}(0|\mathbf{00}) \\
& + [1 - \delta dt - dt\lambda(1)m(\theta(\mathbf{00})) + dt^2\delta\lambda(1)m(\theta(\mathbf{00}))]E_L(0|\mathbf{00}) + \delta dtU.
\end{aligned}$$

After subtracting $E_L(0|\mathbf{00})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$\begin{aligned} rE_L(0|\mathbf{00}) > w_L(\mathbf{00}) - pk + \lambda(1)m(\theta(\mathbf{00}))E_{LH}(0|\mathbf{00}) \\ +(-\delta - \lambda(1)m(\theta(\mathbf{00})))E_L(0|\mathbf{00}) + \delta U. \end{aligned}$$

1.2 $E_L(1|\mathbf{11}) > E_L(0|\mathbf{11})$ (no deviation from 'search') if

$$\begin{aligned} E_L(1|\mathbf{11})(1 + rdt) > dtw_L(\mathbf{11}) + dt\lambda(0)(1 - \delta dt)m(\theta(\mathbf{11}))E_{LH}(1|\mathbf{11}) \\ +(1 - \delta dt - dt\lambda(0)m(\theta(\mathbf{11})) + dt^2\delta\lambda(0)m(\theta(\mathbf{11})))E_L(1|\mathbf{11}) + \delta dtU. \end{aligned}$$

After subtracting $E_L(1|\mathbf{11})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$\begin{aligned} rE_L(1|\mathbf{11}) > w_L(\mathbf{11}) + \lambda(0)m(\theta(\mathbf{11}))E_{LH}(1|\mathbf{11}) \\ +(-\delta - \lambda(0)m(\theta(\mathbf{11})))E_L(1|\mathbf{11}) + \delta U. \end{aligned}$$

2. Check Deviations from Search Decisions in γ_{LH}:

2.1 $E_{LH}(0|\mathbf{00}) > E_{LH}(1|\mathbf{00})$ (no deviation from 'no search') if

$$\begin{aligned} E_{LH}(0|\mathbf{00})(1 + rdt) > dt(w_{LH} - pk) + dt\lambda(1)(1 - \delta dt)m(\theta(\mathbf{00}))E_{HH} \\ +[1 - \delta dt - dt\lambda(1)m(\theta(\mathbf{00})) + dt^2\delta\lambda(1)m(\theta(\mathbf{00}))E_{LH}(0|\mathbf{00}) + \delta dtU. \end{aligned}$$

After subtracting $E_{LH}(0|\mathbf{00})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LH}(0|\mathbf{00}) > w_{LH} - pk + \lambda(1)m(\theta(\mathbf{00}))E_{HH} + (-\delta - \lambda(1)m(\theta(\mathbf{00}))\pi)E_{LH}(0|\mathbf{00}) + \delta U.$$

2.2 $E_{LH}(1|\mathbf{11}) > E_{LH}(0|\mathbf{11})$ (no deviation from 'search') if

$$\begin{aligned} E_{LH}(1|\mathbf{11})(1 + rdt) > dtw_{LH} + dt\lambda(0)(1 - \delta dt)m(\theta(\mathbf{11}))E_{HH} \\ +(1 - \delta dt - dt\lambda(0)m(\theta(\mathbf{11})) + dt^2\delta\lambda(0)m(\theta(\mathbf{11})))E_{LH}(1|\mathbf{11}) + \delta dtU. \end{aligned}$$

After subtracting $E_{LH}(1|\mathbf{11})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_{LH}(1|\mathbf{11}) > w_{LH} + \lambda(0)m(\theta(\mathbf{11}))E_{HH} + (-\delta - \lambda(0)m(\theta(\mathbf{11})))E_{LH}(1|\mathbf{11}) + \delta U.$$

Analogously to the baseline model, when evaluating these 4 conditions with equalities we obtain the corresponding labor market tightness θ for these 4 bounds. We then evaluate the free entry condition (one by one) at these 4 values of θ to obtain multiplicity bounds on primitive p. This gives us 4 potential p-bounds: two for $\overline{p}$ (i.e. it must be that $p < \overline{p}$, otherwise workers would be incentivized to *always* search, that is also in γ_L, γ_{LH} in the 'no-search' equilibrium) and 2 candidates for $\underline{p}$ (i.e. $p > \underline{p}$, otherwise workers would *never* want to search in γ_L, γ_{LH} in the 'search' equilibrium). Hence, for multiple steady state equilibria to exist it must be that $min(\overline{p}) > max(\underline{p})$. The analytical solution for the p-bounds are very tedious. We show numerically that there exists parameter ranges for which $min(\overline{p}) > max(\underline{p})$. An example of parameter intervals, for which there exist multiple steady state equilibria is: $\overline{y} \in [4, 4.2]$, $\underline{y} \in [1-\Delta, 1+\Delta]1$, $b \in [0.438-\Delta, 0.438+\Delta]$, $\lambda_0 \in [0.053-\Delta, 0.053+\Delta]$, $\lambda_1 \in [0.217-\Delta, 0.217+\Delta]$, $k \in [0.4-\Delta, 0.4+\Delta]$, $r \in [0.02-\Delta, 0.02+\Delta]$, $c \in [2.79-\Delta, 2.79+\Delta]$, $\alpha \in [8.56-\Delta, 8.56+\Delta]$, $\phi \in [8.78-\Delta, 8.78+\Delta]$, $\delta \in [0.53-\Delta, 0.53+\Delta]$, for $\Delta > 0$ small.

3. Ex-Ante Firm Heterogeneity and Restricted Number of Search Rounds

Now we study a setting with two types of jobs $\overline{y}$ and $\underline{y}$. A firm can open vacancies of either type, which is permanent. Let the number of vacancies be $v \in \{\overline{v}, \underline{v}\}$. Following Robin and Lise (2013), we model the employment production technology by assuming the the cost of vacancies is increasing in the aggregate number of vacancies of each type: $c(v)$ with $c(0) \geq 0, c' > 0, c'(0) = 0, c'' = 0$.

Suppose that a worker meets a high productivity firm $\overline{y}$ with probability π and a low productivity firm $\underline{y}$ with probability $1-\pi$. Once matched, the worker decides whether to continue to search. Now the exact history matters for the continuation. Similar to Extension 1 of the baseline model, there are 6 possible states, depending on the worker history:

1. u: unemployment
2. γ_L: employed out of u in a $\underline{y}$ job
3. γ_H: employed out of u in a $\overline{y}$ job
4. γ_{LL}: employed in an $\underline{y}$ job after being employed in $\underline{y}$ in a previous period
5. γ_{LH}: employed in a $\overline{y}$ job after being employed in an $\underline{y}$ job or after having received at least once an offer from a $\underline{y}$ job.
6. γ_{HH}: employed in a $\overline{y}$ job after being employed in $\overline{y}$ in a previous period.

The corresponding values of workers are given by:

1. U: unemployed
2. E_L: Employed at $\underline{y}$ out of U (get wage w_L)
3. E_H: Employed at $\overline{y}$ out of U (get wage w_H)
4. E_{LL}: Employed at $\underline{y}$ after a match with another $\underline{y}$ (wage w_{LL})
5. E_{LH}: Employed at $\overline{y}$ after first having been employed by a $\underline{y}$ or employed at $\overline{y}$ after getting an outside offer from a $\underline{y}$ (get wage w_{LH});

6. E_{HH}: Employed at $\overline{y}$ after matching with another $\overline{y}$ (get wage w_{HH})

As in the baseline version of the model, we assume that search costs are prohibitively high (or the gains are too low) when the wage offer has been matched once (i.e. after one round of on-the-job search), so that no more search occurs to increase the wage further after one round of on-the-job search. Ties are broken in favor of the incumbent: in case of a tie, assume the worker does not move or equivalently, that there is an ε moving cost. For the values of a filled job as well as for the labor market stocks, we adopt a similar notation below.

The laws of motion satisfy:

$$\begin{aligned}
1 &= u + \gamma_L + \gamma_H + \gamma_{LL} + \gamma_{LH} + \gamma_{HH} \\
\dot{\gamma_L} &= um(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_L[\delta + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))] \\
\dot{\gamma_H} &= um(\theta(\mathbf{\Omega}))\pi - \gamma_H[\delta + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))] \\
\dot{\gamma_{LL}} &= \gamma_L\lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_{LL}[\delta + \lambda(\mathbf{\Omega}_{LL})m(\theta(\mathbf{\Omega}))\pi] \\
\dot{\gamma_{LH}} &= \gamma_L\lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))\pi + \gamma_{LL}\lambda(\mathbf{\Omega}_{LL})m(\theta(\mathbf{\Omega}))\pi + \gamma_H\lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))(1-\pi) - \gamma_{LH}[\delta + \lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega}))\pi] \\
\dot{\gamma_{HH}} &= \gamma_H\lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))\pi + \gamma_{LH}\lambda(\mathbf{\Omega}_{LH})m(\theta(\mathbf{\Omega}))\pi - \gamma_{HH}\delta
\end{aligned}$$

Equilibrium.

Define as π the equilibrium fraction of high type vacancies (determined below by free entry conditions): $\pi = \frac{\overline{v}}{\overline{v}+\underline{v}}$. We use a similar notation for values and stocks as in Appendix B, Section 1. The value of a filled job, high or low productivity, in steady state is given by:

$$\begin{aligned}
rJ_L &= p\underline{y} - w_L(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_L)m(\theta(\mathbf{\Omega}))[(1-\pi)(J_{LL} - J_L) + \pi(J_{LH} - J_L)] - \delta(J_L - V) \\
rJ_H &= p\overline{y} - w_H(\mathbf{\Omega}) + \lambda(\mathbf{\Omega}_H)m(\theta(\mathbf{\Omega}))[(1-\pi)(J_{LH} - J_H) + \pi(J_{HH} - J_H)] - \delta(J_H - V) \\
rJ_{LL} &= p\underline{y} - w_{LL}(\mathbf{\Omega}) - \delta(J_{LL} - V) \\
rJ_{LH} &= p\overline{y} - w_{LH}(\mathbf{\Omega}) - \delta(J_{LH} - V) \\
rJ_{HH} &= p\overline{y} - w_{HH}(\mathbf{\Omega}) - \delta(J_{HH} - V)
\end{aligned}$$

We can write the value functions of the worker in steady state as:

$$\begin{aligned}
rU &= pb + m(\theta(\mathbf{\Omega}))((1-\pi)E_L + \pi E_H - U) \\
rE_L &= w_L(\mathbf{\Omega}) - \omega_L pk + \lambda(\omega_L)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LL} + \pi E_{LH} - E_L) - \delta(E_L - U) \\
rE_H &= w_H(\mathbf{\Omega}) - \omega_H pk + \lambda(\omega_H)m(\theta(\mathbf{\Omega}))((1-\pi)E_{LH} + \pi E_{HH} - E_H) - \delta(E_H - U) \\
rE_{LL} &= w_{LL}(\mathbf{\Omega}) - \delta(E_{LL} - U) \\
rE_{LH} &= w_{LH}(\mathbf{\Omega}) - \delta(E_{LH} - U) \\
rE_{HH} &= w_{HH}(\mathbf{\Omega}) - \delta(E_{HH} - U)
\end{aligned}$$

The steady state laws of motions for the labor market stocks are identical to the previous extension with math-specific types (Section 1, Appendix B), with the only difference that $\pi = \frac{\overline{v}}{\overline{v}+\underline{v}}$ is endogenous. Then the equilibrium tightness can be written as:

$$\theta(\mathbf{\Omega}) = \frac{v}{s(\mathbf{\Omega})} = \frac{v}{u + \lambda(\mathbf{\Omega})\left[\gamma_L + \gamma_H\right]}.$$

The equilibrium wage is set to the maximum amount that the "losing" firm (which is the firm who loses the worker to another firm or that unsuccessfully tries to poach the worker) would be able to pay for the worker. In turn, if the worker is hired out of unemployment than the wage is set such that he is indifferent between remaining unemployed and taking the job. That implies:

$$\begin{aligned}
w_L(\mathbf{\Omega}): &\quad E_L = U \quad \rightarrow \quad w_L(\mathbf{\Omega}) = pb\left(\frac{r+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))+\delta}{r+\delta}\right) - \frac{\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}{r+\delta}p\underline{y} + \mathbf{\Omega}pk \\
w_H(\mathbf{\Omega}): &\quad E_H = U \quad \rightarrow \quad w_H(\mathbf{\Omega}) = pb\left(\frac{r+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))+\delta}{r+\delta}\right) - \frac{\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}{r+\delta}p[(1-\pi)\underline{y} + \pi\overline{y}] + \mathbf{\Omega}pk \\
w_{LL}(\mathbf{\Omega}): &\quad J_{LL} = V \quad \rightarrow \quad w_{LL} = p\underline{y} \\
w_{LH}(\mathbf{\Omega}): &\quad J_{LH} = V \quad \rightarrow \quad w_{LH} = p\underline{y} \\
w_{HH}(\mathbf{\Omega}): &\quad J_{HH} = V \quad \rightarrow \quad w_{HH} = p\overline{y}
\end{aligned}$$

The objective of the vacancy posting firm (as in Robin and Lise (2013), we assume that these are handled by competing intermediaries; in contrast to their setup, our intermediaries operate in a CRS environment and have zero profits, meaning that one firm can post many vacancies) is to maximize the value of vacancies by choosing the number of either low or high type vacancies. In particular, the value of a low and high-type vacancies, $\underline{v}$ and $\overline{v}$, are given by

$$\begin{aligned}
\underline{v} &= -c(\underline{v}) + \underline{v}q(\theta(\mathbf{\Omega}))\frac{u}{s}J_L \\
\overline{v} &= -c(\overline{v}) + \overline{v}q(\theta(\mathbf{\Omega}))\left[\frac{u}{s}J_H + \frac{\lambda(\mathbf{\Omega_L})\gamma_L}{s}J_{LH}\right]
\end{aligned}$$

In equilibrium, with free entry, $\underline{v} = \overline{v} = 0$, and so the marginal cost of a vacancy is equal to the value of a job of each type:

$$\begin{aligned}
c'(\underline{v}) &= q(\theta(\mathbf{\Omega}))\frac{u}{s}J_L \\
c'(\overline{v}) &= q(\theta(\mathbf{\Omega}))\left[\frac{u}{s}J_H + \frac{\lambda(\mathbf{\Omega_L})\gamma_L}{s}J_{LH}\right] \equiv q(\theta(\mathbf{\Omega}))J_{\overline{v}}
\end{aligned}$$

In addition, with CRS, the levels will be such that profits from opening any vacancy are zero or equivalently, the equilibrium value of opening either vacancy is zero, $q(\theta(\mathbf{\Omega}))vJ_v - c(v) = 0$, where write J_v is the value of a low or high vacancy.

Dependence of cost on y. Let $c(v) = c_0 v y$. Then the first order conditions of both types of firms are given by:

$$
\begin{aligned}
q(\theta(\mathbf{\Omega}))\frac{u}{s}J_L &= c_0\underline{y} \\
q(\theta(\mathbf{\Omega}))\left[\frac{u}{s}J_H + \frac{\lambda(\mathbf{\Omega})\underline{\gamma_L}}{s}\overline{J}_{LH}\right] &= c_0\overline{y}
\end{aligned}
$$

Recall that we assume CRS, which is why both types of firms make on average zero profits (i.e. $\underline{v} = \overline{v} = 0$), so that firms are indifferent between posting low and high type vacancies. From the FOCs, $\frac{u}{s}J_L\overline{y} = \left[\frac{u}{s}J_H + \frac{\lambda(\mathbf{\Omega})\gamma_L}{s}J_{LH}\right]\underline{y}$. We can then solve this equation for $m(\theta)$ (and, assuming the telegraph matching function, also for θ) as a function of π.

$$
\begin{aligned}
m(\theta(\mathbf{\Omega})) \quad = \quad & -\frac{1}{2\lambda^2(\overline{y}-\underline{y})(b+(\pi-1)(\overline{y}-\underline{y}))} \\
& \times\left[\sqrt{\lambda^2(\overline{y}-\underline{y})^2\left((b(2\delta+r)+\delta k\Omega-(\pi-1)\underline{y}(\delta+r)+\delta\pi\overline{y}-\delta\overline{y}+k\Omega r)^2-4\delta(\delta+r)(b+k\Omega)(b+(\pi-1)(\overline{y}-\underline{y}))\right)}\right. \\
& \left.+\lambda(\overline{y}-\underline{y})(b(2\delta+r)+\delta k\Omega-(\pi-1)\underline{y}(\delta+r)+\delta\pi\overline{y}-\delta\overline{y}+k\Omega r)\right] \qquad (52)
\end{aligned}
$$

where θ can then immediately be calculated from inverting the matching function $m(\theta) = \frac{\phi\alpha\theta}{\alpha\theta+1}$. This condition pins down θ as a function of π.

Multiple Equilibria. We focus on multiplicity of two equilibria, one where workers always search actively, i.e. in *both* states γ_L and γ_H, with $\omega_L = \omega_H = 1$. And one where workers never search, i.e. $\omega_L = \omega_H = 0$. We thus need to verify *two* no-deviation conditions for those workers in two states γ_L and γ_H. This implies that we need to check the conditions:

1. No deviation when no one searches ever: $E_L(0|\mathbf{0}) > E_L(1|\mathbf{0})$ and $E_H(0|\mathbf{0}) > E_H(1|\mathbf{0})$
2. No deviation when all search always: $E_L(1|\mathbf{1}) > E_L(0|\mathbf{1})$ and $E_H(1|\mathbf{1}) > E_H(0|\mathbf{1})$

The next proof, adapted from the proof of the baseline model, shows that the condition for multiplicity is very similar to (but stronger than) the condition from the baseline model, i.e.,

$$
\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1\left[\pi\overline{y}+(1-\pi)\underline{y}-b\right]}\right) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right) < \theta(\mathbf{1}).
$$

Proof. 1.1. No deviation in γ_L jobs when no one searches: $E_L(0|\mathbf{0}) > E_L(1|\mathbf{0})$.

In this case, when no one actively searches on-the-job ($\mathbf{\Omega} = \mathbf{0}$), a worker in a low productivity job deviating during an interval dt chooses $\omega = 1$ and gets a payoff

$$
E_L(1|\mathbf{0}) = \frac{1}{1+rdt}\left[dt(w_L(\mathbf{0})-pk)+(1-\delta dt)dt\lambda(1)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LL}+\pi E_{LH}\right]+(1-\delta dt)(1-dt\lambda(1)m(\theta(\mathbf{0})))E_L(0|\mathbf{0})+\delta\right.
$$

where $E_{LL} = E_{LL}(0|\mathbf{0})$ and $E_{LH} = E_{LH}(0|\mathbf{0})$. There is no deviation provided $E_L(0|\mathbf{0}) > E_L(1|\mathbf{0})$ or:

$$\begin{aligned} E_L(0|\mathbf{0})(1+rdt) \quad > \quad & dt(w_L(\mathbf{0}) - pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LL} + \pi E_{LH}\right] \\ & +(1 - \delta dt - dt\lambda(1)m(\theta(\mathbf{0})) + dt^2\delta\lambda(1)m(\theta(\mathbf{0})))E_L(0|\mathbf{0}) + \delta dtU. \end{aligned}$$

After subtracting $E_L(0|\mathbf{0})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_L(0|\mathbf{0}) > w_L(\mathbf{0}) - pk + \lambda(1)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LL} + \pi E_{LH}\right] + (-\delta - \lambda(1)m(\theta(\mathbf{0})))E_L(0|\mathbf{0}) + \delta U.$$

Substituting the equilibrium values for $E_L(0|\mathbf{0}), E_{LL}, E_{LH}, U$ and $w_L(\mathbf{0})$ we get:

$$(\underline{y} - b)[\lambda(\mathbf{1}) - \lambda(\mathbf{0})]m(\theta(\mathbf{0})) - k(r+\delta) < 0.$$

So there is no deviation provided that:

$$\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y} - b))}\right)$$

1.2. No deviation in γ_H jobs when no one searches: $E_H(0|\mathbf{0}) > E_H(1|\mathbf{0})$.

$$\begin{aligned} E_H(1|\mathbf{0}) = \quad & \frac{1}{1+rdt}\left[dt(w_H(\mathbf{0}) - pk) + (1-\delta dt)dt\lambda(1)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right]\right. \\ & \left. +(1-\delta dt)(1 - dt\lambda(1)m(\theta(\mathbf{0})))E_H(0|\mathbf{0}) + \delta dtU\right] \end{aligned}$$

where $E_{LL} = E_{LL}(0|\mathbf{0})$ and $E_{LH} = E_{LH}(0|\mathbf{0})$. There is no deviation provided $E_H(0|\mathbf{0}) > E_H(1|\mathbf{0})$ or:

$$\begin{aligned} E_H(0|\mathbf{0})(1+rdt) \quad > \quad & dt(w_H(\mathbf{0}) - pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right] \\ & +(1 - \delta dt - dt\lambda(1)m(\theta(\mathbf{0})) + dt^2\delta\lambda(1)m(\theta(\mathbf{0})))E_H(0|\mathbf{0}) + \delta dtU. \end{aligned}$$

After subtracting $E_H(0|\mathbf{0})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_H(0|\mathbf{0}) > w_H(\mathbf{0}) - pk + \lambda(1)m(\theta(\mathbf{0}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right] + (-\delta - \lambda(1)m(\theta(\mathbf{0})))E_H(0|\mathbf{0}) + \delta U.$$

Substituting the equilibrium values for $E_H(0|\mathbf{0}), E_{LH}, E_{HH}, U$ and $w_H(\mathbf{0})$ we get:

$$(\underline{y} - b)[\lambda(1) - \lambda(\mathbf{0})]m(\theta(\mathbf{0})) - k(r+\delta) + (\lambda(1) - \lambda(\mathbf{0}))m(\theta(\mathbf{0}))\left[\pi(\overline{y} - \underline{y})\right] < 0$$

This condition is stronger than the one under 1.1. (that one is implied by this condition) since $(\lambda(1) - \lambda(\mathbf{0}))m(\theta(\mathbf{0}))\left[\pi(\overline{y} - \underline{y})\right] > 0$. Therefore, the requirement for multiplicity is (using the fact that

$\lambda(1) - \lambda(\mathbf{0}) = \lambda_1$:

$$(\underline{y} - b)\lambda_1 m(\theta(\mathbf{0})) - k(r+\delta) + \lambda_1 m(\theta(\mathbf{0})) \left[\pi(\overline{y} - \underline{y})\right] < 0$$

or

$$\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1\left[\pi\overline{y} + (1-\pi)\underline{y} - b\right]}\right).$$

2.1. No deviation in γ_L job when all search: $E_L(1|\mathbf{1}) > E_L(0|\mathbf{1})$.

In this case, when all actively search on-the-job ($\mathbf{\Omega} = \mathbf{1}$), a worker in a low productivity job deviating for an interval dt chooses $\omega = 0$ and gets a payoff

$$E_L(0|\mathbf{1}) = \frac{1}{1+rdt}\left[dtw_L(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LL} + \pi E_{HH}\right] + (1-\delta dt)(1 - dt\lambda(0)m(\theta(\mathbf{1})))E_L(1|\mathbf{1}) + \delta dtU\right].$$

There is no deviation provided $E_L(1|\mathbf{1}) > E_L(0|\mathbf{1})$:

$$\begin{aligned} E_L(1|\mathbf{1})(1+rdt) \quad > \quad & dtw_L(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LL} + \pi E_{HH}\right] \\ & + (1 - \delta dt - dt\lambda(0)m(\theta(\mathbf{1})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1})))E_L(1|\mathbf{1}) + \delta dtU. \end{aligned}$$

After subtracting $E_L(1|\mathbf{1})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_L(1|\mathbf{1}) > w_L(\mathbf{1}) + \lambda(0)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LL} + \pi E_{HH}\right] + (-\delta - \lambda(0)m(\theta(\mathbf{1})))E_L(1|\mathbf{1}) + \delta U.$$

Substituting the equilibrium values for $E_L(1|\mathbf{1}), E_{LL}, E_{HH}, U$ and $w_L(\mathbf{1})$ we get:

$$(\underline{y} - b)[\lambda(1) - \lambda(0)]m(\theta(\mathbf{1})) - k(r+\delta) > 0.$$

So there is no deviation provided that:

$$\theta(\mathbf{1}) > m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y} - b))}\right)$$

2.2. No deviation in γ_H job when all search: $E_H(1|\mathbf{1}) > E_H(0|\mathbf{1})$.

In this case, when all actively search on-the-job ($\mathbf{\Omega} = \mathbf{1}$), a worker in a high productivity job deviating for an interval dt chooses $\omega = 0$ and gets a payoff

$$E_H(0|\mathbf{1}) = \frac{1}{1+rdt}\left[dtw_H(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right] + (1-\delta dt)(1 - dt\lambda(0)m(\theta(\mathbf{1})))E_H(1|\mathbf{1}) + \delta dtU\right].$$

There is no deviation provided $E_H(1|\mathbf{1}) > E_H(0|\mathbf{1})$:

$$\begin{aligned} E_H(1|\mathbf{1})(1+rdt) \quad > \quad & dtw_H(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right] \\ & +(1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1})))E_H(1|\mathbf{1}) + \delta dtU. \end{aligned}$$

After subtracting $E_H(1|\mathbf{1})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$rE_H(1|\mathbf{1}) > w_H(\mathbf{1}) + \lambda(0)m(\theta(\mathbf{1}))\left[(1-\pi)E_{LH} + \pi E_{HH}\right] + (-\delta - \lambda(0)m(\theta(\mathbf{1})))E_H(1|\mathbf{1}) + \delta U.$$

Substituting the equilibrium values for $E_H(1|\mathbf{1}), E_{LH}, E_{HH}, U$ and $w_H(\mathbf{1})$ we get:

$$(\underline{y}-b)[\lambda(1)-\lambda(0)]m(\theta(\mathbf{1})) - m(\theta(\mathbf{1}))[\lambda(1)-\lambda(0)]\pi(\overline{y}-\underline{y}) - k(r+\delta) > 0.$$

And therefore, there is no deviation in this particular point of the tree if:

$$\theta(\mathbf{1}) > m^{-1}\left(\frac{k(r+\delta)}{\lambda_1\left[\pi\overline{y} + (1-\pi)\underline{y} - b\right]}\right).$$

Since $\lambda_1(\pi\overline{y} + (1-\pi)\underline{y} - b) > \lambda_1(\underline{y} - b)$ this condition is less strict than the condition under 2.1. As a result, the conditions for no deviation when all search is:

$$\theta(\mathbf{1}) > m^{-1}\left(\frac{k(r+\delta)}{\lambda_1\left[\underline{y} - b\right]}\right).$$

The necessary and sufficient conditions for the existence of multiple steady state equilibria is therefore:

$$\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1\left[\pi\overline{y} + (1-\pi)\underline{y} - b\right]}\right) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y} - b))}\right) < \theta(\mathbf{1}) \tag{53}$$

■

Notice that this condition is *more stringent* than the one from our baseline model. This is intuitive since workers who obtain the high-productivity match right after unemployment have strong incentives to keep searching in order to obtain another $\overline{y}$ match and extract all rents from matching. Only a *sufficiently* low market tightness prevents them from wanting to do so always.

Notice that these bounds still depend on the endogenous variable π. To obtain the bounds in terms of π that only depend on parameters, we evaluate 53 at equality, using the expression for $\theta(\mathbf{\Omega})$ above (i.e. obtain θ from (52)) and solve for π. We obtain the following two expressions

$$
\begin{aligned}
\pi_H(\mathbf{0}) &= \frac{1}{2\lambda_1(\delta+r)(\overline{y}-\underline{y})(\delta(\overline{y}-\underline{y})(b\lambda_1+k\lambda_0)-k\lambda_0 r\underline{y})} \times \\
&\quad \big\{ \big[k^2\lambda_0^2(\delta+r)^2 \left(\delta^2(\overline{y}-\underline{y})^2(\lambda_1(\overline{y}-b)+k\lambda_0)^2 + 2\delta r(\overline{y}-\underline{y})(\lambda_1(\overline{y}-b)+k\lambda_0)(\lambda_1(b\overline{y}+2b\underline{y}-\overline{y}\underline{y})+k\lambda_0(\overline{y} - \right. \\
&\quad \left. + r^2\left(2k\lambda_0\lambda_1(\overline{y}-\underline{y})(b\overline{y}+2b\underline{y}-\overline{y}\underline{y})+\lambda_1^2(b\overline{y}-2b\underline{y}+\overline{y}\underline{y})^2+k^2\lambda_0^2(\overline{y}-\underline{y})^2\right)\right)\big]^{\frac{1}{2}} \\
&\quad +\delta^2(-(\overline{y}-\underline{y}))\left(k\lambda_0\lambda_1(b-\overline{y}+2\underline{y})+2b\lambda_1^2(\underline{y}-b)+k^2\lambda_0^2\right)+\delta r\left(k\lambda_0\lambda_1\left(b(\underline{y}-2\overline{y})+(\overline{y}-2\underline{y})^2\right)\right. \\
&\quad \left. +2b\lambda_1^2(b-\underline{y})(\overline{y}-\underline{y})+2k^2\lambda_0^2(\underline{y}-\overline{y})\right)-k\lambda_0\lambda_1 r^2(b\overline{y}+\underline{y}(\overline{y}-2\underline{y}))+k^2\lambda_0^2 r^2(\underline{y}-\overline{y})\big\} \\
\pi_L(\mathbf{1}) &= \frac{1}{k(\lambda_0+\lambda_1)(\delta(\overline{y}-\underline{y})(\lambda_1(\underline{y}-b)+k(\lambda_0+\lambda_1))+r(\lambda_1\underline{y}(b-\underline{y})+k(\lambda_0+\lambda_1)(\overline{y}-\underline{y})))} \times \\
&\quad \big\{b^3(-\delta)\lambda_1^2+b^2\lambda_1(\delta k(2\lambda_0+\lambda_1)+2\delta\lambda_1\underline{y}+kr(\lambda_0+\lambda_1)) \\
&\quad +b\left(-k(\lambda_0+\lambda_1)(k\lambda_0(\delta+r)+\delta\lambda_1\overline{y})+\delta k\lambda_1\underline{y}(\lambda_1-\lambda_0)-\delta\lambda_1^2\underline{y}^2\right) \\
&\quad +k\left(\delta(k(\lambda_0+\lambda_1)+\lambda_1\underline{y})(\overline{y}(\lambda_0+\lambda_1)-\underline{y}(\lambda_0+2\lambda_1))+r(\lambda_0+\lambda_1)\left(k\overline{y}(\lambda_0+\lambda_1)-k\underline{y}(\lambda_0+2\lambda_1)-\lambda_1\underline{y}^2\right)\right)\big\}
\end{aligned}
$$

where $\pi_H(\mathbf{0})$ gives the highest value of π that is consistent with an equilibrium where no one searches actively and $\pi_L(\mathbf{1})$ is the lowest π that is consistent with an equilibrium where everyone searches actively.

Finally, to obtain the bounds in terms of primitive p, we use the zero profit conditions (either $\overline{V}=0$ or $\underline{V}=0$) for both the equilibrium of active and non-active search and solve for p_l and p_h respectively

$$
\begin{aligned}
p_l &= \frac{c\underline{y}}{\frac{m(\theta(\mathbf{1}))u(\underline{y}-b)}{\theta(\mathbf{1})(\delta+r)(\gamma(\lambda_0+\lambda_1)+u)} - \frac{km(\theta(\mathbf{1}))u}{\theta(\mathbf{1})(\gamma(\lambda_0+\lambda_1)+u)(\delta+m(\theta(\mathbf{1}))(\lambda_0+\lambda_1)+r)} + \frac{m(\theta(\mathbf{1}))^2(1-\underline{\pi}(\mathbf{1}))u(\lambda_0+\lambda_1)(\overline{y}-\underline{y})}{\theta(\mathbf{1})(\delta+r)(\gamma(\lambda_0+\lambda_1)+u)(\delta+m(\theta(\mathbf{1}))(\lambda_0+\lambda_1)+r)}} \\
p_h &= -\frac{c\theta(\mathbf{0})\underline{y}(\delta+r)(\gamma\lambda_0+u)(\delta+\lambda_0 m(\theta(\mathbf{0}))+r)}{m(\theta(\mathbf{0}))u(b\delta+b\lambda_0 m(\theta(\mathbf{0}))+br-\delta\underline{y}+\lambda_0 m(\theta(\mathbf{0}))\overline{\pi}(\mathbf{0})\overline{y}-\lambda_0 m(\theta(\mathbf{0}))\overline{\pi}(\mathbf{0})\underline{y}-\lambda_0 m(\theta(\mathbf{0}))\overline{y}-r\underline{y})}
\end{aligned}
$$

which we evaluate at $\underline{\pi}(\mathbf{\Omega}), \theta(\mathbf{\Omega})$ as well as $u, \underline{\gamma}$ from above to obtain expressions that solely depend on parameters. The expressions are involved but one can show via simulations that there exists a parameter range for which $p_h > p_l$ and $\underline{\pi}(\mathbf{1}) > \overline{\pi}(\mathbf{0})$. We have the following result (exact expressions available upon request).

Proposition 4 *Let $m(\theta) = \phi\frac{\alpha\theta}{\alpha\theta+1}$. Then there are multiple steady states if and only if $p \in [p_l, p_h]$. The set $[p_l, p_h]$ is non-empty for an open set of parameters.*

Also in this set-up with *ex-ante firm heterogeneity*, the strategic complementarity and thus multiplicity survives. When both equilibria coexist, the active on-the-job search equilibrium is characterized by more search effort, larger market tightness and a larger share of high-productivity vacancies $\overline{v}$ (with $\overline{y}$). What gives rise to this strategic complementarity between on-the-job search and (high-type) vacancies? Here workers are incentivized to search actively, not only if tightness is high enough (as before) but also if the fraction of high productivity vacancies is high enough. High type vacancies encourage on-the-job search because it offers workers the chance to end up in a job where they extract the entire surplus (after having met a high productivity firm not only after unemployment but, crucially, also after on-the-job search). In turn, firms are encouraged to not only post more vacancies but especially more high type vacancies in the presence of active on-the-job search because on-the-job search biases the pool of searcher towards the employed. This bias implies that high type vacancies *match faster* ($\underline{v}$ cannot attract on-the-job searchers) and the *match duration* with employed searchers is longer than with unemployed ones (due to the restriction to a finite number of search rounds).

Appendix C. Firm Deviation to Back-Loaded Wage Contract

One concern of our analysis is that the assumption of fixed wages drives the multiplicity result. While the fixed wage assumption is common in this literature, it is well-known that it is not necessarily the optimal contract. A firm may find it optimal to offer *time-varying wages* to discourage workers' on-the-job search in the equilibrium with active on-the-job search. Here we make a modest attempt to address this issue. We extend the contract space to a two-part wage with back loading, and ask whether firms would want to deviate and offer a wage different from the constant wage. In particular, we allow a firm to deviate from the current contract with constant wages and to post a relatively low wage for T periods (where T is optimally chosen by the firm) which incentivizes on-the-job search, followed by a relatively high wage from $T+1$ onward that discourages active search. We find that for the relevant parameter values, a firm is worse off when deviating and posting the time-varying wages compared to the equilibrium contract with stable wages. In this case, the value of a filled low productivity job under the deviating contract approaches the equilibrium value of a filled low productivity job in the limit for $T \to \infty$ and is strictly below for finite T, implying that the firm would *not* want to deviate from fixed wages and pay a higher wages to discourage search. Only if discounting is (unnaturally) high, such that workers do not value much the benefits of search, and if at the same time search costs are high, then it is profitable for the firm to deviate from the equilibrium wage contract because discouraging search is cheap. Of course, this result does not prove that there exists no profitable firm deviation through some more complicated contract. But it does demonstrate that allowing for a natural class of wage contracts does not induce profitable deviations by the firms that would destroy the equilibrium with on-the-job search (under reasonable parameter restrictions).

Here, we sketch the analysis if we allow firms to commit to the above mentioned wage contract, given that other firms offer fixed wages. We do not aim to provide a full analytical characterization at this stage but rather to give the conceptual framework and the intuition for the results. For convenience, we do the analysis in discrete time. There is only one stage in which this deviating contract may be profitable to the firm, and that is when employing a worker in a low productivity job (this is the only stage at which the worker searches). We therefore focus on a deviation by a single firm regarding the contract in the low productivity job in the equilibrium with active on-the-job search.

Denote by $\underline{E}_T$ the value of a low productivity job to a worker, in which he will receive the low wage $\underline{w}_1$ for T periods and the high wage $\underline{w}_2$ from $T+1$ onward. This implies that E_T is the value of a job from the perspective of an unemployed worker. Denote by $\underline{E}_0$ the value of a job to a worker from period $T+1$ onward. We adopt the same notation for the firm, i.e. $\underline{J}_T$ is the value of a filled low productivity job to a firm when paying the worker a low wage for T periods; $\underline{J}_0$ is the value of that job when starting to pay the worker a higher wage from $T+1$ on.

In steady state, these values are given by

$$
\begin{aligned}
\underline{J}_T &= \frac{1-(\beta(1-\delta)(1-\lambda(1)m))^T}{\beta(1-\delta)(1-\lambda(1)m)}(p\underline{y}-\underline{w}_1)+(\beta(1-\delta)(1-\lambda(1)m))^T\underline{J}_0 \\
\underline{J}_0 &= \frac{p\underline{y}-\underline{w}_2}{\beta(1-\delta)(1-\lambda(0)m)} \\
\underline{E}_T &= \frac{1-(\beta(1-\delta)(1-\lambda(1)m))^T}{\beta(1-\delta)(1-\lambda(1)m)}(\underline{w}_1-pk)+(\beta(1-\delta)(1-\lambda(1)m))^T\underline{E}_0+\beta\lambda(1)m\frac{1-(\beta(1-\delta)(1-\lambda(1)m))^T}{\beta(1-\delta)(1-\lambda(1)m)}\overline{E} \\
&\quad +\delta\beta\frac{1-(\beta(1-\delta)(1-\lambda(1)m))^T}{\beta(1-\delta)(1-\lambda(1)m)}U
\end{aligned}
$$

$$
\begin{aligned}
\underline{E}_0 &= \frac{w_2+\beta\lambda(0)m\overline{E}+\beta\delta U}{1-\beta(1-\delta)(1-\lambda(0)m)} \\
\overline{E} &= \frac{\overline{w}+\beta\delta U}{1-\beta(1-\delta)} \\
U &= \frac{b}{1-\beta}
\end{aligned}
$$

which take into account that $V=0$ due to free entry.

The firm's objective is to choose a triple $T, \underline{w}_1, \underline{w}_2$ to maximize the value of a low productivity job $\underline{J}_T$ subject to three constraints:

$$
\begin{aligned}
&\max_{T,\underline{w}_1,\underline{w}_2} \underline{J}_T \\
s.t.\quad &\underline{E}_T \geq U \\
&\underline{E}_0(0) \geq E_0(1) \\
&\underline{E}_{T-1}(1) \geq E_{T-1}(0)
\end{aligned}
$$

The first constraint states that the wages must be such that the worker is at least as well off taking the job as in unemployment; the second states that after T periods, wages must be such that the worker weakly prefers not to search; the third constraint ensures that the worker does not want to deviate from the strategy *search until period T (and not thereafter)*: If the worker prefers to search in period $T-1$, he also prefers to search in all periods $t<T-1$ since in period $T-1$ it is most tempting to not search due to the soon-to-be-expected wage increase. The first two constraints will hold with equality, otherwise the firm would forgo profits. We recover $\underline{w}_2$ from constraint 2 and, given $\underline{w}_2$, we recover $\underline{w}_1$ from constraint 1. Last, we verify that constraint 3 holds for any parameter constellation; it is slack.

We then evaluate the objective function $\underline{J}_T$ at the wages and check its properties: Our simulations (available upon request) reveal that it is either monotonic increasing or decreasing, i.e. T^* is at a corner. For most parameter ranges, $\underline{J}_T$ is increasing, always weakly below the value of the on-the-job search equilibrium $\underline{J}$ with $\lim_{T\to\infty}\underline{J}_T=\underline{J}$. This implies that the deviation is *not* profitable; firms do not seek to discourage search by backloading wages. For some parameter constellations (in particular, for unnaturally low β and high k), $\underline{J}_T$ can be decreasing with its maximum at $T^* = 0$. In this case, in which on-the-job search is costly and workers do not value much future benefits, it is worth it for the firm to discourage search, and the firm would do so immediately after hiring.

Appendix D. Jobless Recovery – A Simple Exercise

Suppose we are at the bottom of the recession, and we investigate the impact of an unexpected change in workers' beliefs: all workers in a low productivity job start searching actively for a job and firms instantaneously adjust by posting vacancies so that profits are driven to zero.

The immediate implication is an increase in the number of active searchers, implying $s(\mathbf{0}) = u(\mathbf{0}) + \lambda_0\gamma(\mathbf{0})$ rises to $s^R = u(\mathbf{0}) + (\lambda_0 + \lambda_1)\gamma(\mathbf{0})$, where the superscript R stands for Recovery. This also leads to *crowding out*. Conditional on forming a match, the probability that it is with an unemployed worker is now lower since more are searching on-the-job. Denote by κ the fraction of hires with the worker coming from unemployment. Then:

$$\kappa(\mathbf{0}) = \frac{u(\mathbf{0})}{u(\mathbf{0}) + \lambda_0\gamma(\mathbf{0})} \quad \text{and} \quad \kappa^R = \frac{u(\mathbf{0})}{u(\mathbf{0}) + (\lambda_0 + \lambda_1)\gamma(\mathbf{0})}$$

The stocks $u(\mathbf{0})$ and $\gamma(\mathbf{0})$ have not changed yet, but there is already an immediate response in the search activity. As a result it follows that $\kappa^R < \kappa(\mathbf{0})$. According to the estimates of our calibration, this implies that conditional on the formation of a match, the probability that it is with an unemployed worker goes from $\kappa(\mathbf{0}) = 0.67$ to $\kappa^R = 0.53$. As soon as the recovery starts, the likelihood, conditional on a match formation, that an unemployed workers is selected over a worker with a job significantly drops. This is what we refer to as *crowding out* during the recovery.

Certainly, what matters is not just the conditional likelihood of being drawn. It is also important how fast the matching is. Under the belief that employed workers actively search for a job, the matching rate for firms goes up, and in response, new vacancies are created until profits are driven down to zero again. As a result, the market tightness changes as does the matching probability $m(\theta)$. Therefore, the unconditional matching probability for an unemployed worker is $\kappa m(\theta)$, which drops from $\kappa(\mathbf{0})m(\theta(\mathbf{0})) = 0.34$ in the recession to $\kappa^R m(\theta^R) = 0.30$ in the recovery. The implication of the lower matching rates is that the unemployment rate initially edges up marginally: $\dot{u} > 0$ since the separation rate δ is unchanged. In turn, the matching rate of the employed workers who search significantly increases, going from $(1 - \kappa(\mathbf{0}))m(\theta(\mathbf{0})) = 0.17$ in the recession to $(1 - \kappa^R)m(\theta^R) = 0.26$ in the recovery.

Within the framework of our model, we can also highlight the role of the effective market tightness for jobless recovery. Since we observe vacancies and unemployment, we can readily construct the conventional market tightness $\Theta = \frac{v}{u}$. We want to compare Θ to the effective market tightness $\theta = \frac{v}{s}$, which we obtain from the data as follows

$$\theta = \frac{v}{u + \lambda\gamma} = \frac{v}{\frac{UE}{m(\theta)} + \frac{EE}{m(\theta)}} = \Theta\frac{UE}{UE + EE},$$

since $EE = m(\theta)\lambda\gamma$ and $UE = m(\theta)u$. Figure 16.A plots both θ and Θ. It is apparent that there is not only less fluctuation in θ than there is in Θ but in particular, after the crisis, the recovery

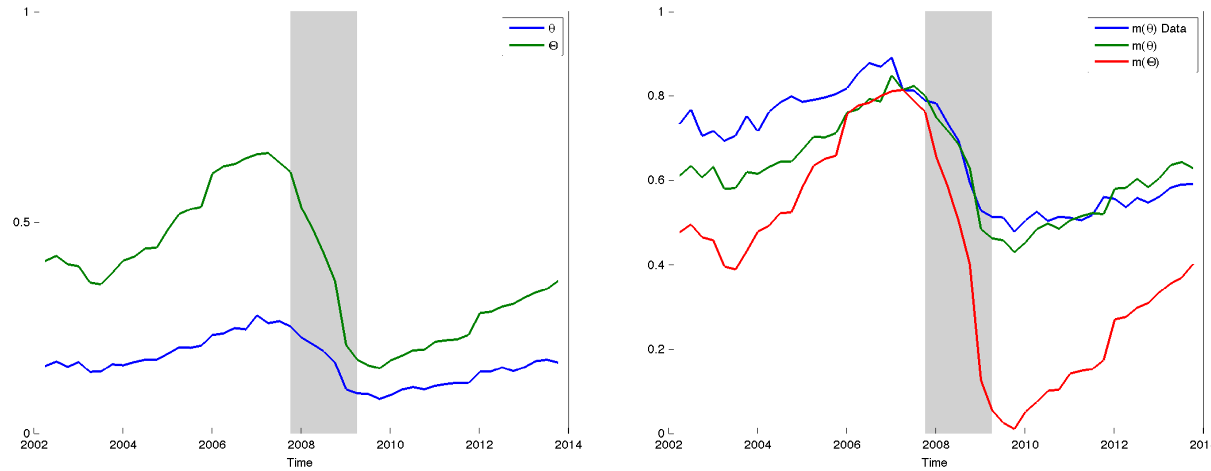

Figure 16: A. Market tightness $\Theta = \frac{v}{u}$ and effective market tightness $\theta = \Theta\frac{UE}{UE+EE}$ in the data; B. Matching probabilities in data and with telegraph matching (using Θ and θ from the data and calibrated parameters for the matching function). Matching probabilities from model are normalized to match the data in the pre-crisis quarter.

Table 6: Jobless Recovery and Counterfactual with Varying Productivity

	Model 1	**Model 2**
$\Delta\kappa$	-0.21	0.00
$\Delta m(\theta)$	0.08	0.34
$\Delta\kappa m(\theta)$	-0.14	0.34
$\Delta(1-\kappa)m(\theta)$	0.54	0.34

of θ is much flatter than that indicated by Θ. This is due to the fact that the effective market tightness θ reflects the change in the number of on-the-job searchers. The implications for fluctuations in matching probabilities follow immediately (Figure 16.B): While the matching probability according to $m(\Theta)$ (using the telegraph matching function) indicates fast recovery, matching probability $m(\theta)$ recovers more slowly, closely resembling the slow recovery of matching probability in the data and fueling jobless recovery.

Again, we assess the explanatory power of our mechanism in jobless recovery against a model where changes in matching probabilities are driven by productivity fluctuations (Model 2). The results are displayed in Table 6. Contrary to our model, a model with single steady state and productivity shocks (Model 2) does not feature jobless recovery.

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