Redistributive Bargaining under the Shadow of Protests*

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Abstract

We consider an alternating-offers redistributive bargaining model where an affected third party can protest against proposals under review. Protests are costly and only stochastically successful. When successful, they secure the status quo. Stationary equilibria feature either inefficient protests or excessive accommodation to the third party. In both cases, the bargainers do not extract the full surplus. *Strategic delay* is necessary and sufficient to curb this issue: By delaying a harmful agreement with positive probability only after acquiescence, the bargainers create an endogenous punishment device that allows them to extract more surplus from the third party without triggering protests. The bargainers' misaligned interests are key for this result: If they internalized each other's payoff, strategic delay would not be credible.

Keywords: Strategic Delay; Redistributive Bargaining; Protests

JEL codes: C78; D61; D72; D74

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"If you are not at the table, you are on the menu."[1]

1 Introduction

Bilateral agreements often benefit the bargaining parties at the expense of third parties who are not included in the negotiations. For example, a trade agreement or military alliance between two countries may worsen a neighboring country's economic and political power, a merger of two companies may lead to worker layoffs, and a coalition contract between two political parties may weaken the position of an opposing interest group. Even though they are not formally involved in the bargaining process, these affected groups can still interfere with the negotiations through non-institutional channels. Countries can wage preventive war, workers can strike, and interest groups can spend resources to break down coalition talks. In this paper, we study how the possibility of such *protests* may affect the bargaining outcomes and how the bargainers can limit the impact of protests.

There are two ways in which protests can benefit an affected third party. First, a successful protest may cement the status quo, completely avoiding any harm due to the agreement. Second, the threat of a protest alone may convince the bargainers to refrain from making proposals that are particularly harmful to the third party.

Our main finding is that the bargainers can limit the influence of protests on negotiations by using *strategic delay*, i.e., by delaying an agreement that is harmful to the third party with positive probability in case no protests occur, while reaching an immediate consensus otherwise. This contingent behavior creates an endogenous punishment device for protests and allows the bargainers to extract more surplus from the third party without triggering any protest. If protesting is costly enough, strategic delay is incentive-compatible. However, this is the case only if the bargainers compete, i.e., only if they are self-interested. If they internalized each other's payoffs, they would not be able to commit to any agreement delay.

Formally, we consider a model with three agents. Two of them – Ann and Bob – bargain over how to redistribute a finite set of (continuously divisible) resources between themselves and a third party. Every period, they interchange the roles of proposer and responder as in Rubinstein (1982). To emphasize the role of redistribution, our baseline analysis focuses on settings where feasible proposals are zero-sum.[2] The remaining agent – Charlie – represents the third party and is not included in the negotiations. Despite the lack of formal representation, Charlie can interfere by protesting against any proposal that is currently under review. Protesting is costly and only stochastically successful. If a protest succeeds, negotiations break down immediately and the status quo persists. All agents are impatient expected utility maximizers. Our solution concept is subgame perfect equilibrium (SPE).

As a leading example, consider two firms, Ann Inc. and Bob Corp., facing a one-time opportunity to merge. Suppose the only effect of the merger is a possibility for restructuring which would make some workers dispensable. Charlie represents these workers, e.g., through

[1]We thank Luigi Zingales for suggesting this compelling quote.

[2]In Section 6.1, we show how the main insights of our analysis extend beyond this purely redistributive setting.

their union. Ann and Bob benefit from the merger if they successfully lay off redundant workers. To prevent this, the workers can strike, launch formal complaints, lobby politicians to intervene, or protest on the streets. In our model, all of these actions are captured by Charlie's decision to protest, which is costly but nullifies the merger opportunity if successful. Our analysis studies how these potential protests may influence firms' negotiations in equilibrium and what strategies the firms can employ to limit their adverse effects.

If Charlie did not have the possibility to protest, the outcome of our bargaining game would resemble that of Rubinstein (1982): The bargainers immediately agree on a split of Charlie's resources. The main difference is that we model bargaining as redistributive.[3] In particular, the surplus the bargainers consume is not created exogenously but extracted from the third party during the game. Our first result characterizes when this *Rubinstein outcome* prevails as the unique SPE outcome of the model even though Charlie is allowed to protest. We show that this is the case whenever Charlie has access to a protesting technology with low cost-effectiveness, i.e., whenever the cost of protesting is too high relative to its success probability. In such cases, the standard predictions of Rubinstein bargaining apply: The SPE payoffs are pinned down uniquely for all agents, and the unique equilibrium outcome, which can be sustained in stationary strategies, features no delay. The remainder of the paper shows that these predictions are reversed if Charlie has access to a protesting technology whose cost-effectiveness is high enough to impact the bargaining outcome in equilibrium.

To show this, we first focus on symmetric Markov Perfect Equilibria, or stationary equilibria for short. In stationary equilibria, agents' strategies can depend only on the payoff-relevant state. In our model, this corresponds to the current-period proposal. Two equilibrium regions exist. If protesting costs are low, Charlie protests against any significant worsening of his position. Therefore, *Conflict* prevails: The proposer offers to extract everything from Charlie, hoping for an unsuccessful protest. The stationary equilibrium outcome for intermediate protesting costs is *Accommodation*. In this case, the proposer offers to extract less from Charlie to avoid his protest.

Counter-intuitive comparative statics emerge. In particular, Charlie can benefit from access to a *less* cost-effective protesting technology. An increase in the cost of protesting, or a decrease in its success probability, makes Charlie less inclined to protest. Therefore, the proposer can extract more resources from him without triggering a protest, possibly encouraging Accommodation.[4] Since Charlie receives his min-max payoff under Conflict, it follows that Charlie may benefit from a worsening in his protesting technology when this leads to a change in the equilibrium regime from Conflict to Accommodation. In the context of our leading example, this indicates that workers might sometimes find laws and regulations that increase their cost to mobilize advantageous. Such measures reduce the concessions the firms have to make to avoid strikes. As a result, the equilibrium outcome may change from one where protests occur

[3]The redistributive nature of our model implies that agreement delay is not socially inefficient. While the bargainers dislike delay like as in Rubinstein (1982), Charlie enjoys delay since it postpones the implementation of a harmful agreement.

[4]While an increase in the cost of protesting always favors Accommodation, this is not necessarily the case for a decrease in the protest success probability. This is because the proposer's payoff from inducing Conflict also increases as protests become less likely to succeed. See Section 4 for a detailed discussion.

since the proposed merger does not include any concessions, to one where the merger proceeds without such interference because the firms commit to dampening the adverse effect on ex-post redundant workers.

Under stationary strategies, the bargainers cannot extract the full surplus in equilibrium. We say that the bargainers extract the full surplus if their joint payoff equals the negative of Charlie's min-max payoff.[5] Under Conflict, socially wasteful protests occur on path, implying that the outcome is not Pareto efficient. Under Accommodation, Charlie receives more than his min-max payoff. Thus, when restricting to stationary strategies, protests create enough *de facto* bargaining power for third parties to make full surplus extraction impossible.

This conclusion, however, no longer holds if the bargainers can use what we call *strategic delay*, i.e., if the responder can condition his acceptance probability not only on the proposal under review but also on Charlie's current-period protesting decision. Our main results show that such strategic delay is both necessary and sufficient for full surplus extraction in equilibrium.

If the responder's behavior does not depend on Charlie's current-period action, the bargainers can only punish protests through continuation play from next period onwards. However, since Charlie discounts future punishments, avoiding protests requires awarding him a payoff strictly larger than his min-max payoff. This proves our first main result: Any full surplus extraction SPE must feature strategic delay.

Our second main result shows that an SPE where the bargainers extract the full surplus through strategic delay exists as long as the protesting cost is not too low. In this equilibrium, the proposer offers to extract all resources from Charlie, while leaving the responder exactly indifferent between accepting and rejecting her offer. Given the negative impact of such an offer on his payoff, Charlie strictly benefits from implementation delay. This allows the responder to create an endogenous punishment device for protests: He delays the agreement with positive probability if Charlie acquiesces, but accepts the proposer's offer immediately after an unsuccessful protest. Intuitively, delay can serve as a way to accommodate to Charlie, just like a more favorable proposal. But if delay is contingent on Charlie's protesting decision, it can further be used as a punishment device, allowing the bargainers to extract more surplus. In fact, by choosing a high enough probability of delay, Charlie optimally refrains from protesting, even though the proposal under review prescribes extracting all of his resources.

Thus, strategic delay serves as a "carrot and stick" mechanism to encourage acquiescence. In the context of our leading example, this is because deferring the merger benefits the ex-post redundant workers, as they remain employed longer. Since this reward for acquiescence becomes more pronounced the more *impatient* agents are, the threat of punishing strikes by speeding up the merger becomes a more powerful deterrent the *less* forward-looking workers are.[6] This can be seen most clearly when the agents are perfectly myopic. In this case, the full surplus extraction equilibrium can be supported regardless of the parametric details of the protesting

[5] Since proposals are zero-sum, the negative of Charlie's min-max payoff serves as an upper bound on the bargainers' joint payoff in any equilibrium.

[6] This is in stark contrast to Folk theorems in the repeated games literature, where punishment typically obtains through future-period continuation play.

technology.

Key for sustaining strategic delay in equilibrium is that, unlike in standard models of complete information bargaining, the proposer does not necessarily benefit from avoiding agreement delay in our framework. To see why, note that if the proposer were to offer slightly more to the responder to break his indifference, this would remove the punishment for protests: Charlie would understand that the acceptance probability no longer depends on his action, making it optimal for him to protest. As long as the cost of protesting is not too small, the benefit of extracting the full surplus more than compensates the proposer for her loss of agenda-setting power due to agreement delay.

Full surplus extraction can be achieved in (symmetric) time-invariant strategies, i.e., strategies where agents' behavior can only depend on current-period events. This is the minimal relaxation of the stationarity assumption that allows the bargainers to use strategic delay, since agents' behavior remains independent of calendar time, and the proposer and Charlie's strategy space are left unchanged. It is in this sense that strategic delay is also sufficient to obtain full surplus extraction in equilibrium.

To sustain strategic delay, the bargainers' interests must be misaligned, i.e., they need to compete for how much of Charlie's resources each of them receives. Instead, if the bargainers were to share the objective of maximizing the amount of resources they *jointly* extract from Charlie, they would still like to commit to such delay. But since they prefer to extract his resources as soon as possible, they cannot. Intuitively, the responder only delays an agreement in equilibrium if his discounted continuation value is larger than the current proposed extraction of Charlie's resources. With aligned preferences, no such continuation value exists, implying that colluding bargainers can do no better than to receive their optimal no-delay equilibrium payoff.

Conversely, misaligned interests allow the bargainers to commit to strategic delay in equilibrium. To see why, observe that strategic delay produces two effects. It mechanically postpones when the bargainers extract Charlie's resources, hurting both the proposer and the responder. However, it also increases the responder's agenda-setting power, since he becomes the next-period proposer. This second effect, which is mute under collusion, is precisely what makes strategic delay credible under competition. Therefore, in redistributive bargaining settings where protests can interfere with the negotiations, the bargainers can extract more surplus when they compete, rather than collude.

In the Discussion section, we extend our analysis beyond purely redistributive settings, i.e., we consider settings where an agreement can create or destroy economic resources. When this is the case, delaying an agreement is no longer welfare-neutral. Nevertheless, strategic delay can still be used by the bargainers to extract more surplus as long as the redistributive nature of the model is prevalent, i.e., as long as the welfare gains generated by an agreement are not too large.

Related literature. Our model builds upon the alternating-offers bargaining game of Rubinstein (1982). However, compared to this canonical framework, our approach features two key distinctions: First, as we focus on redistributive bargaining settings, the "pie" the bargainers can split is not exogenous, but equals the resources extracted from a third party. Second, since the third party can only interfere with the ongoing negotiations by protesting against proposals under review, the agents do not have symmetric bargaining power. In particular, the third party has only access to an imperfect veto technology. Our analysis shows that, under these assumptions, any equilibrium where the bargainers extract the full surplus must feature strategic delay. Thus, unlike standard Rubinstein (1982), stationary strategies not only restrict equilibrium outcomes but also preclude the bargainers from maximizing their joint equilibrium payoff.

The existence of equilibria featuring agreement delay connects our work to the large literature investigating delay in bargaining. The traditional explanation for this phenomenon is the bargainers' incomplete information about preferences (e.g., Rubinstein, 1985; Admati and Perry, 1987; Cho, 1990; Cramton, 1992).[7] Alternative foundations are, among others, uncertainty about higher order beliefs (Feinberg and Skrzypacz, 2005), deadline effects (Fershtman and Seidmann, 1993; Ma and Manove, 1993), multi-issue bargaining (e.g., Acharya and Ortner, 2013), reputational concerns (e.g., Abreu and Gul, 2000; Abreu, Pearce, and Stacchetti, 2015), the presence of non-standard time preferences (Schweighofer-Kodritsch, 2018), and second-order optimism (Friedenberg, 2019). Our contribution identifies a new rationale for delay in bargaining: In redistributive settings where negotiations take place under the shadow of protests, (strategic) delay can be used as an endogenous punishment device, allowing the bargainers to extract more surplus from the third party without triggering a protest.

Since one can interpret the third party as a veto player, our work is also related to the political economy literature on veto bargaining following Romer and Rosenthal (1978).[8] Compared to this literature, our approach differs in at least two aspects. First, since a protest can fail to break down negotiations, the third party only holds an imperfect veto technology in our framework. Second, the principal making offers to the veto player is not a unitary agent in our setup. Rather, it consists of two competing bargainers as in Rubinstein (1982).

Our work further relates to the legislative bargaining literature (Baron and Ferejohn, 1989). In a recent article within this tradition, Miettinen and Vanberg (2020) study the *decision costs* (i.e., the risk of gridlocks) associated with the unanimity rule and the majority rule. Since an agreement between the bargainers in our setting can be interpreted as the formation of a simple majority, we complement their work by emphasizing how the *external costs* (i.e., the risk the agreement does not consist of a Pareto improvement) vary across these different decision rules.

Our paper also belongs to the vast political economy literature on protests and revolutions. Following the global games approach of Morris and Shin (1998), most of this literature focuses on understanding how protesters can overcome their collective-action problem in equilibrium

[7] This traditional explanation, however, can be subject to the Coase conjecture, which implies that agreement delay fades away as offers become more frequent (see, e.g., Gul and Sonnenschein, 1988). As Theorem 2 shows, this is not the case in our redistributive framework when the bargainers use strategic delay.

[8] See Cameron (2000) for a book, Cameron and McCarty (2004) for a survey, and Ali, Kartik, and Kleiner (2023) for a recent publication on the topic.

(see, e.g., Bueno De Mesquita, 2010; Edmond, 2013).[9] In our model, we abstract away from free-riding considerations. Nonetheless, we show that the influence of protests can still be significantly undermined if the bargainers coordinate on equilibria featuring strategic delay. Within the protest literature, our approach is similar to that of Passarelli and Tabellini (2017), where a policymaker ponders the costs associated with protests when designing a new policy. One main difference is that the policymaker is not a benevolent social planner in our framework, but consists of two competing rent-seeking bargainers.

Since an application of our model concerns negotiations over military alliances and the possibility that neighboring countries start preventive wars in response to them, our work also contributes to the large political science literature on conflicts and wars (Fearon, 1995; Powell, 2004). In particular, we are connected to a recent article by Benson and Smith (2023). Like us, these authors find that agreement delay can serve as an instrument to accommodate to the third party and avoid conflict. However, delay in alliance formation is exogenous in their model. Conversely, (strategic) delay arises endogenously in our framework as the best strategy the bargainers have to undermine the influence of protests on negotiations in equilibrium.

Finally, our finding that competition helps the bargainers to commit to an optimal course of action resembles the "competing to commit" result of Cusumano, Fabbri, and Pieroth (2024) obtained in an oligopolistic setting with rationally inattentive consumers. Conceptually, the difference between the two papers lies in the source of the commitment problem: Consumers' costly information processing in their setting, and bargainers' impatience in the present setup.

Outline. The remainder of the paper is organized as follows. Section 2 describes the model. In Section 3, we analyze a benchmark in which protests are forbidden or prohibitively expensive. In Section 4, we restrict attention to stationary equilibria and show that full surplus extraction is impossible. Section 5 explains how and when the bargainers can limit the influence of protests on negotiations by using strategic delay. Finally, Section 6 considers several model extensions: We allow for welfare effects of agreements, change the timing and impact of protests, and alter the observability of the proposer's offer. All omitted proofs are in the Appendix.

2 Model

Two agents – Ann and Bob – bargain over how to redistribute a finite amount of (continuously divisible) resources between themselves and a third party: In every period, they take turns in proposing how agents' resource allocation should change relative to the status quo. The third party – Charlie – is not part of the negotiations but can interfere by protesting against any proposal that is currently under review. Protests are costly and only stochastically successful. If successful, protesting terminates negotiations immediately, cementing the status quo.

Formally, let $I := \{A, B, C\}$ denote the set of agents. Time is discrete and infinite. Each period $t \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$ is divided into three stages: The *proposing stage*, the *protesting stage*,

[9] Another important strand of the protest literature focuses on the role that public protests, like petitions or referenda, might play in aggregating dispersed information within society (Lohmann, 1993; Battaglini, 2017). Our framework highlights a different kind of "protests." Namely, those employed instrumentally to preserve the status quo.

and the *decision stage*. When t is even, Ann is the *proposer*, and Bob is the *responder*, while when t is odd, the roles between these two agents are reversed. The game proceeds as follows:

Proposing stage. At the start of every period t, we enter the proposing stage. In this stage, the current-period proposer makes an offer (equivalently, a proposal) $x^t \in X$ to the current-period responder. The set X, representing all feasible redistributions of resources across the agents, is given by:[10]

$$X := \Big\{ x \in \mathbb{R}^I : \sum_{i \in I} x_i = 0, \text{ and } x_C \geq -1 \Big\}.$$

Let $x_0 := (0, 0, 0)$ denote the *status quo allocation*. An offer consists of a proposed redistribution of resources among the agents: For every $x \in X$, each component x_i represents agent i's allocative well-being under proposal x relative to the status quo. In particular, if $x_i > 0$, agent i gains resources compared to x_0, while if $x_i < 0$, agent i loses resources. The constraint $x_C \geq -1$ indicates the finiteness of the resources available for redistribution.[11] Note that $x_0 \in X$: Not redistributing is always a viable option.

Protesting stage. After the proposal x^t is made, we enter the protesting stage. Here, Charlie observes x^t and decides whether to *protest* or not. If Charlie *abstains* from protesting, the play immediately reaches the *decision stage*, where it is the current-period responder's turn to move. Otherwise, Charlie pays a cost of $f > 0$,[12] and two outcomes can arise: With probability $0 \leq p < 1$, Charlie's protest is successful, which implies that the game ends immediately with the status quo allocation x_0 being implemented;[13] With complementary probability, Charlie's protest is unsuccessful, and once again the play reaches the decision stage. Charlie's protesting decision is observable.

Decision stage. In the decision stage, the current-period responder can either approve or reject x^t. In case of rejection, x^t expires and the play moves to the next period, where the same sequence of events unfolds except for the interchange of the roles of proposer and responder between Ann and Bob. If the proposal is accepted, x^t is implemented.

Payoffs. The game continues until either a proposal is agreed upon or one of Charlie's protests successfully terminates negotiations. Thus, a *consequence* of this game can be described by a profile $(\tau, x, \mathcal{T}_C)$ where $\tau \leq \infty$ is the (possibly infinite) period when the game ends, $x \in X$ indicates the implemented redistribution of resources,[14] and $\mathcal{T}_C \subseteq \{1, ..., \tau\}$ corresponds to the collection of periods in which Charlie protested during the game. Let Y denote the set of all

[10]To emphasize the role of redistribution in the analysis, our baseline model assumes that feasible offers are purely redistributive, i.e., it holds that $\sum_{i \in I} x_i = 0$ for all $x \in X$. This restriction is relaxed in Section 6.1, where we consider the possibility that an agreement between Ann and Bob may impact social welfare.

[11]The analysis would remain unchanged if, like x_C, also x_A and x_B were bounded below by a non-positive number in X. This follows since Ann and Bob have perfect veto power: No agreement can be reached without their consent. Thus, they cannot receive less than their status quo payoff in any equilibrium.

[12]To streamline the exposition, we assume that protests do not directly harm the bargainers. In Section 6.4, we show that our main results continue to hold if we relax this restriction.

[13]The assumption that $p < 1$ distinguishes our bargaining model with *protests* from those with (possibly costly) *veto rights*. See Cameron and McCarty (2004) for a survey of the veto bargaining literature.

[14]If $\tau = \infty$, we use the convention that $x = x_0$.

possible consequences. Agents are impatient expected utility maximizers with payoff functions $u_i : Y \to \mathbb{R}$ given by

$$u_i(\tau, x, \mathcal{T}_C) = \delta^\tau \cdot x_i$$

for $i \in \{A, B\}$, and

$$u_C(\tau, x, \mathcal{T}_C) = \delta^\tau \cdot x_C - f \cdot \sum_{\tau' \in \mathcal{T}_C} \delta^{\tau'}$$

for Charlie. In words, Ann and Bob only care about their own allocative well-being relative to the status quo, and discount future-period payoffs using the common discount factor $\delta \in (0, 1)$.[15] In addition to this, Charlie also dislikes paying the cost of protesting.

Solution concept. We adopt subgame perfect equilibrium (SPE) as the solution concept for our model. Our goal is to highlight the role *strategic delay*, i.e., the possibility that the responder makes his acceptance decision depend on whether Charlie chose to protest in the current period, can play in equilibrium. To do so, in Section 4, we first analyze symmetric Markov Perfect Equilibria, where strategic delay is impossible.[16] By definition, a Markov Perfect Equilibrium (MPE) is an SPE where agents play Markovian strategies, i.e., strategies depending only on payoff-relevant states (Maskin and Tirole, 2001). Since proposals expire when rejected and Charlie's protesting technology does not depend on the history of the game, the current-period proposal x^t is the unique payoff relevant state in our model. Therefore, an MPE consists of a profile of SPE strategies $\sigma = (\sigma_i)_{i \in I}$ such that the following holds:

- For $i \in \{A, B\}$, σ_i can be expressed as a profile of two (possibly random) behavior rules (β_i^P, β_i^R) such that

  $$\beta_i^P \in \Delta(X) \tag{1}$$

  is the offer agent i proposes every time he/she acts as the proposer in a given period, and

  $$\beta_i^R : X \to \Delta(\{yes, no\}) \tag{2}$$

  represents the decision to accept or reject the current-period proposal $x^t \in X$ when i acts as the responder; and

- σ_C can be expressed as a (possibly random) behavior rule

  $$\beta_C : X \to \Delta(\{protest, abstain\}) \tag{3}$$

  specifying Charlie's protesting decision as a function of the proposal $x^t \in X$ currently

[15] One useful interpretation of δ is to think of $(1 - \delta)$ as the probability that negotiations between Ann and Bob break down due to exogenous circumstances when moving across periods. As a result, the limit case $\delta = 0$ captures ultimatum bargaining.

[16] Markov Perfect Equilibria are often regarded as the benchmark solution concept for infinite dynamic games. As argued by Maskin and Tirole (2001), the reason is twofold: They impose minimal assumptions on agents' strategic sophistication and often induce unique equilibrium predictions. Section 4 shows that this is not the case in our model: A continuum of symmetric MPE outcomes exists for a parameter region of strictly positive Lebesgue measure.

under review.[17]

An MPE σ is *symmetric* if Ann and Bob behave symmetrically when exchanging the roles of proposer and responder across periods.[18] From now on, we refer to symmetric MPEs as *stationary equilibria*. Furthermore, when characterizing stationary equilibria we abuse notation and write $x = (x_P, x_R, x_C)$ instead of $x = (x_A, x_B, x_C)$ to represent generic offers $x \in X$, where x_P and x_R specify the allocation accruing to the proposer and responder in case x is implemented, respectively. The notations (v_P, v_R, v_C) for stationary equilibrium payoffs and $(\beta_P, \beta_R, \beta_C)$ for stationary equilibrium strategies are defined similarly. We denote the set of all SPEs of our model by $\mathcal{E}$, while the set of all stationary equilibria is denoted by $\mathcal{E}^{\mathrm{S}}$.

Model discussion. The model captures a setting where, every period, some resources are available to the agents for consumption, and split among them according to some convention or pre-existing rule. Ann and Bob represent two parties with a one-time opportunity to permanently change this rule: Their offers illustrate how each agent's net present consumption value would change relative to the prevailing agreement.

On the other hand, Charlie represents an opposition lacking formal representation: Despite any final agreement between Ann and Bob impacting his payoff, Charlie cannot bargain with the other two agents directly. Yet, because of his ability to stop negotiations through protests, Charlie holds some *de facto* bargaining power.

Charlie's protests represent any action aimed at terminating Ann and Bob's ongoing negotiations. Our model captures two key features of such actions: They are costly and not always successful. We abstract away from another important aspect of protests, namely, the free-riding problems they often generate. We do so because our goal is not to study how protests emerge, but to understand how they may influence a redistributive negotiation in equilibrium.

Whether an agreement between the bargainers has an impact on total welfare, the information available to the agents, the timing of their actions, and whether protests directly harm the bargainers, all play a role in determining the equilibrium outcome in our framework. For our baseline model, we assume that bargaining is *purely redistributive*, that Charlie observes proposals *perfectly*, that he can only protest *before* an offer is accepted, and that the bargainers are *not harmed* directly by a protest. In Section 6, we explain how our results change if we modify these assumptions.

3 Benchmark: No Protest

We start by analyzing a benchmark model where Charlie *cannot* protest, which we call the *no-protest benchmark*. Since Charlie is forced to abstain from protesting during the game, it is easy to see that this benchmark model is essentially identical to that of Rubinstein (1982),

[17] As is standard, $\Delta(X)$ denotes the set of all probability measures defined on the Borel σ-algebra of X, while the mappings β_A^R, β_B^R and β_C are assumed to be measurable.

[18] Formally, let $\zeta : X \to X$ denote the mapping that symmetrizes offers between Ann and Bob, i.e., $\zeta(x_A, x_B, x_C) = (x_B, x_A, x_C)$ for all $x \in X$. An MPE σ is symmetric if $\beta_A^P(E) = \beta_B^P(\zeta(E))$ and $\beta_A^R(\cdot|x) = \beta_B^R(\cdot|\zeta(x))$ for all measurable subsets $E \subseteq X$ and offers $x \in X$.

with the only exception that the surplus Ann and Bob can generate through an agreement is not exogenous, but equals the amount of resources they endogenously extract from Charlie. The following theorem applies Rubinstein's (1982) well-known results to characterize the SPE outcomes of the no-protest benchmark.

Theorem 0 (Rubinstein, 1982). *The no-protest benchmark admits a unique SPE outcome. This outcome – which we call the Rubinstein outcome – can be supported in stationary strategies and features the following sequence of events:*

(i) The proposer offers $x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right)$,

(ii) Charlie abstains from protesting,

(iii) The responder accepts x^* *with probability 1.*

Theorem 0 shows that if Charlie is not allowed to protest, our model is a reinterpretation of a standard alternating-offers bargaining game with an exogenous pie. Of course, the mere possibility of protesting will not affect the bargaining outcome if protests never succeed ($p = 0$) or are too costly relative to the resources at stake ($f > 1$). The following proposition shows that, as long as Charlie's protesting technology is relatively inefficient, this intuition extends: Even though Charlie's protesting decision is unconstrained, our model is equilibrium outcome-equivalent to the no-protest benchmark if and only if $f \geq p$.

Proposition 1. *The Rubinstein outcome is an SPE outcome of our model if and only if* $f \geq p$*. Furthermore, in this case, the Rubinstein outcome is the unique SPE outcome.*

Theoretically, protesting can help Charlie in two ways: It can increase his current-period payoff and, potentially, change his continuation payoff in equilibrium. If $f \geq p$, the first effect is weakly negative: The cost of protesting, $f > 0$, is larger than the maximum direct benefit Charlie could derive from protesting, namely, preventing with probability p an immediate agreement on a proposal x such that $x_C = -1$. Proposition 1 shows that no change in continuation payoffs due to protesting can overcome this weakly negative direct effect. As a result, Charlie never protests in equilibrium which, in turn, implies that the bargainers' equilibrium play is unaffected compared to the no-protest benchmark: They extract all resources from Charlie ($x_C^* = -1$), reach an agreement immediately, and split the generated surplus among themselves according to the standard Rubinstein shares.

Proposition 1 characterizes when Charlie's ability to protest is irrelevant in equilibrium. Therefore, in the remainder of the paper, we focus on situations where this is not the case, i.e., we assume that the following condition holds:

Assumption 1. *Charlie's protesting threat is credible, i.e.,* $f/p < 1$*.*

Despite the similarities emphasized in Theorem 0 and Proposition 1, there are two important distinctions between our model and the classic framework of Rubinstein (1982). First, agreement delay does not generate social inefficiency in our setting.[19] However, if only the bargainers'

[19] This follows from the "zero-sum" nature of feasible offers in our baseline framework. We relax this assumption in Section 6.1, where we extend our analysis beyond purely redistributive bargaining settings.

payoffs are considered, the standard logic applies: Ann and Bob dislike delay since they discount future-period profits. Second, a remarkable feature of Rubinstein's (1982) bargaining model is that it admits a unique SPE outcome, even though it is an infinite dynamic game. As the remainder of our analysis shows, this is not the case in our model if Charlie's protests are relevant, i.e., if Assumption 1 holds.

4 Stationary Equilibria

This section investigates the stationary equilibria of our model. If $f \geq p$, the restriction to stationary strategies is without loss of generality. We now show that this is no longer the case when Charlie's protesting threat is credible, i.e., when $f < p$. To do so, we proceed in two steps. First, in this section, we characterize the stationary equilibrium outcomes of our model and show that the bargainers cannot extract the full surplus. Then, in Section 5, we prove that the bargainers can do so if they depart from stationarity and use strategic delay.

4.1 Equilibrium Characterization

Let $\lambda := f/p < 1$ be the inverse of Charlie's protesting cost-effectiveness, and define the mapping $z \mapsto \varphi^{\mathrm{S}}(z)$ by

$$\varphi^{\mathrm{S}}(z) := z(1-z)\frac{(1+\delta)}{1+\delta(1-z)}. \tag{4}$$

In the following proposition, we establish the existence of stationary equilibria and completely characterize the equilibrium payoffs and outcomes they induce for generic model parameters.[20]

Proposition 2. *A stationary equilibrium always exists. In particular, the following holds:*

a) ***Accommodation:*** *If $\varphi^{\mathrm{S}}(p) < f < p$, all stationary equilibria induce the same equilibrium payoffs given by*

$$(v_P, v_R, v_C) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right).$$

Moreover, a continuum of stationary equilibrium outcomes exists. Those where the proposer does not randomize can be indexed by a scalar $\alpha \in [\lambda, 1]$. Fix the equilibrium outcome associated with α. The following sequence of events occurs:

(i) The proposer offers $x^* = (x^*_P, x^*_R, x^*_C) = \left(\alpha - \frac{\delta\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\alpha\right)$,

(ii) Charlie abstains from protesting,

(iii) The responder accepts x^* with probability $q_\alpha = \frac{\lambda-\delta\lambda}{\alpha-\delta\lambda} \in (0, 1]$.

b) ***Conflict:*** *If $0 < f < \varphi^{\mathrm{S}}(p)$, all stationary equilibria induce the same equilibrium payoffs given by*

$$(v_P, v_R, v_C) = \left(\frac{1-p}{1+\delta(1-p)}, \frac{\delta(1-p)^2}{1+\delta(1-p)}, -(1-p+f)\right).$$

[20] Proposition 2 does not cover the case $f = \varphi^{\mathrm{S}}(p)$, i.e., a Lebesgue measure-zero subset of model parameters. When $f = \varphi^{\mathrm{S}}(p)$, the proposer is indifferent between *Accommodation* and *Conflict*. As a result, any selection between these two outcomes can be supported in a stationary equilibrium. While this decision is payoff-irrelevant for the proposer, it impacts the other agents' utility, implying that equilibrium payoffs are not uniquely defined. See Section 4.2 for further discussion.

Moreover, the stationary equilibrium outcome is unique and features the following sequence of events:

(i) The proposer offers $x^* = (x_P^*, x_R^*, x_C^*) = \left(1 - \frac{(1-p)\delta}{1+\delta(1-p)}, \frac{(1-p)\delta}{1+\delta(1-p)}, -1\right)$,

(ii) Charlie protests,

(iii) If the protest fails, the responder accepts x^* *with probability 1.*

The proof of Proposition 2 consists of three parts. In Part 1, we prove by construction that a stationary equilibrium always exists. In Part 2, we show that whenever $f \neq \varphi^{\mathrm{S}}(p)$ (see footnote 20), the stationary equilibrium payoffs can be pinned down uniquely for all agents. We reach this conclusion in two main steps. First, we show that the proposer's equilibrium payoff set is a singleton, irrespective of model parameters. By the Markov property, agents' next-period continuation payoffs do not depend on the offer the proposer makes in the current period. Therefore, the proposer can always make an offer that leads to immediate acceptance by the responder and that induces Charlie to either protest or acquiesce on path. In equilibrium, the proposer's payoff must be weakly larger than choosing either of these options. However, it turns out that the proposer can also do no better than that: Delaying the agreement or inducing Charlie to randomize weakly lowers her payoff. This establishes the uniqueness of the proposer's payoff v_P in any stationary equilibrium. In the second step, we show that the same conclusion holds for the responder and Charlie's payoffs as long as $f \neq \varphi^{\mathrm{S}}(p)$. We do so by proving that all stationary strategies that result in a proposer's payoff of v_P also generate a unique equilibrium payoff for the other two agents. Finally, in Part 3 we use the uniqueness of the equilibrium payoffs established in Part 2 to describe the outcomes stationary equilibria can induce. In particular, as Figure 1 shows, depending on the model parameters, two different stationary equilibrium outcomes may emerge, which we now describe in detail.

If protesting were costless ($f = 0$), Charlie would protest against any proposal that worsens his position relative to the status quo. This implies that the only way the proposer can get a positive payoff in equilibrium is when Charlie protests unsuccessfully. Under these circumstances, the best strategy for the proposer is to offer the responder a share of the surplus that guarantees his immediate acceptance, while extracting as much as possible from Charlie. Proposition 2 shows that the optimality of this strategy extends to a range of positive protesting costs: As long as $f < \varphi^{\mathrm{S}}(p)$, the proposer strictly prefers being hostile towards Charlie, and *Conflict* is the unique stationary equilibrium outcome. Observe that the responder receives less than his standard Rubinstein share in the Conflict equilibrium. Intuitively, if he rejects the proposer's current-period offer, the responder will only receive a positive payoff if, next period, another round of protests is unsuccessful. As a result, the effective discount factor used to determine the responder share in equilibrium is lowered.[21]

For intermediate protesting costs, the proposer prefers to accommodate to Charlie. Charlie finds it optimal to abstain from protesting whenever his equilibrium payoff for reaching the

[21] While the proposer offers to extract one unit of resources from Charlie, she only allocates a share $x_R^* = \frac{(1-p)\delta}{1+(1-p)\delta} < \frac{\delta}{1+\delta}$ to the responder in the Conflict equilibrium. Therefore, relative to standard Rubinstein bargaining, it is *as if* the bargainers' discount factor was $\delta' = (1-p)\delta < \delta$ under Conflict.

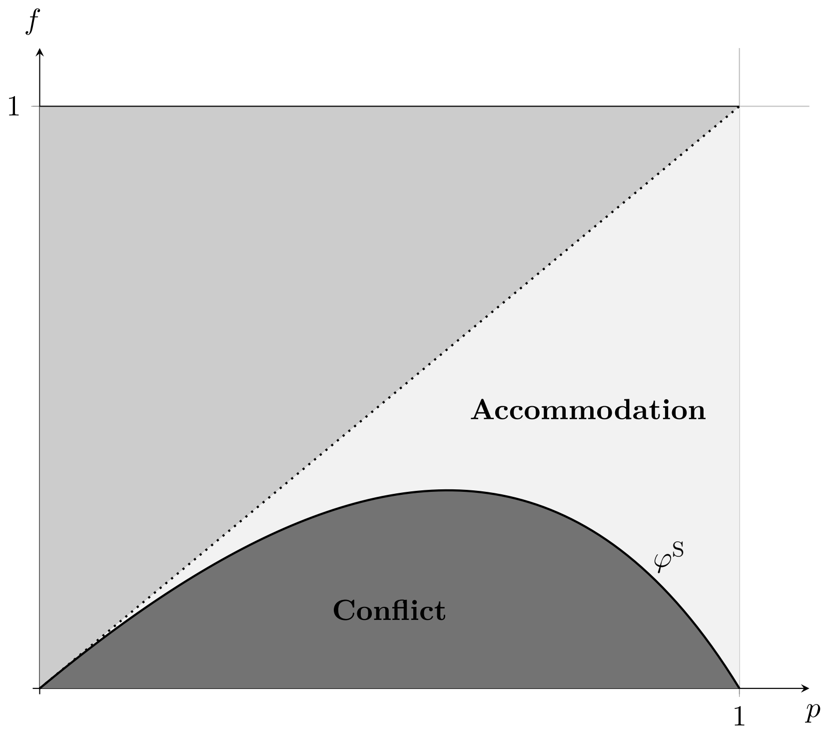

Figure 1: The stationary equilibrium outcomes as a function of f and p, with $f < p$, for a fixed $\delta \in (0, 1)$. The curve φ^{S} is defined in equation (4).

decision stage is at least $-\lambda$. When $\varphi^{\mathrm{S}}(p) < f < p$, conceding this payoff to him is more profitable to the proposer than triggering a risky protest. Therefore, all stationary equilibria in this parameter range feature *Accommodation.* While equilibrium payoffs are pinned down uniquely, a continuum of stationary equilibrium outcomes exists in this parameter region. Intuitively, there are infinitely many ways of raising Charlie's expected payoff: The proposer can offer to extract less from him or the responder can reject the agreement with positive probability, leading to a delayed extraction of his resources. Since agents are risk-neutral, they are indifferent between all of these possibilities.[22] As a result, anything ranging from offering to extract λ resources from Charlie followed by immediate acceptance ($\alpha = \lambda$) up to proposing to extract all of his resources followed by a corresponding decrease in the responder's acceptance probability ($\alpha = 1$), as well as arbitrary mixtures over these outcomes, constitutes a stationary equilibrium outcome.

While low values of f induce Conflict and higher values of f induce Accommodation, the corresponding comparative statics for p is *non-monotone.* In particular, intermediate values of p induce Conflict, while extreme values of p induce Accommodation (see Figure 1). The source of this asymmetry lies in the proposer's equilibrium incentives. To understand why, note that the curve φ^{S} is pinned down by the proposer's indifference between Accommodation and Conflict. While the protesting cost f affects the proposer's equilibrium payoff only under Accommodation through Charlie's incentive-compatibility constraint for acquiescence, the probability p governs protests' success rate and, therefore, impacts the proposer's equilibrium payoff both under Ac-

[22] In Section 6.1, we show that this indifference between outcomes breaks down when bargaining is not purely redistributive. In turn, which equilibrium outcomes exist depends on the sign and size of the social welfare change due to the agreement.

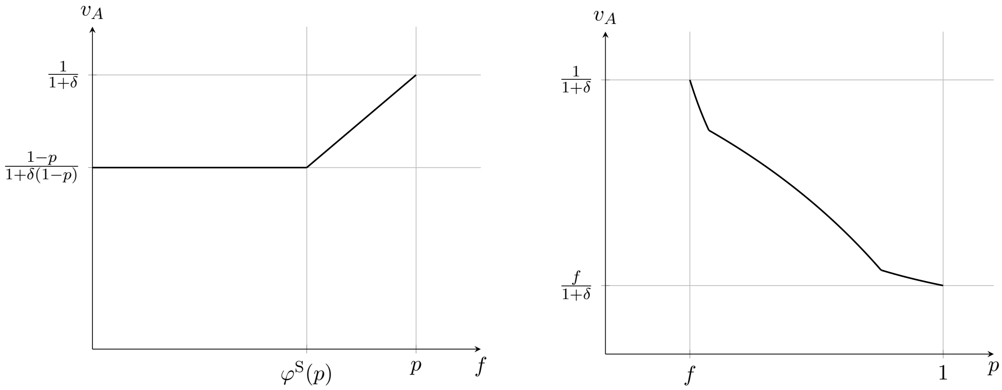

Figure 2: Ann's stationary equilibrium payoff as a function of the cost of protesting f (left panel) and protest success probability p (right panel).

commodation and under Conflict. In turn, this leads to the non-monotonicity of $p \mapsto \varphi^S(p)$.

Note that while equilibrium payoffs, and in the case of Conflict, even equilibrium outcomes, are pinned down uniquely, stationary equilibrium strategies are not. This is because, in subgames that are never reached, agents can choose arbitrarily which actions to play whenever indifferent. For example, off the equilibrium path, i.e., for offers $x \in X$ such that $x \neq x^*$, any acceptance probability $\beta_R(yes|x) \in [0,1]$ by the responder can be supported in a stationary equilibrium as long as $x_R = \delta v_P$. For this reason, in Proposition 2 we only characterize equilibrium outcomes and omit to spell out the corresponding equilibrium strategies.

Conflict and Accommodation outcomes can both be observed in reality. For example, consider two different mergers that involved the company Kraft Foods. In 2010, British workers protested against Kraft's proposed acquisition of Cadbury, a candy maker headquartered in the UK.[23] Despite this opposition, the deal was finalized quickly and resulted in the closure of a UK factory and the layoff of its 400 workers.[24] Thus, Kraft's acquisition of Cadbury resembles a Conflict equilibrium outcome with an unsuccessful protest. On the other hand, when Kraft merged with Heinz into the Kraft Heinz Food Company in 2015, workers acquiesced even though 5% of them were fired in the aftermath. As part of the deal, the affected workers received at least six months of severance pay, pointing toward Accommodation.[25] As Figure 1 illustrates, an explanation for the differences in the outcomes of these mergers could be variations in the strength of the unions between the UK and the US impacting the respective costs of protesting.

4.2 Welfare Analysis

How agents' equilibrium payoffs vary with the model parameters depends on their initial role in the bargaining protocol. Figure 2 displays the stationary equilibrium payoff of Ann, the initial proposer, as a function of Charlie's protesting cost $f > 0$ and protest success probability

[23] See NBC News article "Cadbury-Kraft deal approved amid protest" from February 2, 2010 (https://www.nbcnews.com/id/wbna35201579).

[24] See BBC article "Kraft and Cadbury: How is it working out?" from December 8, 2011 (https://www.bbc.com/news/uk-england-birmingham-16067571).

[25] See CNN Business article "Kraft Heinz cuts 2,500 jobs" from August 12, 2015 (https://money.cnn.com/2015/08/12/investing/kraft-heinz-job-cuts-layoffs).

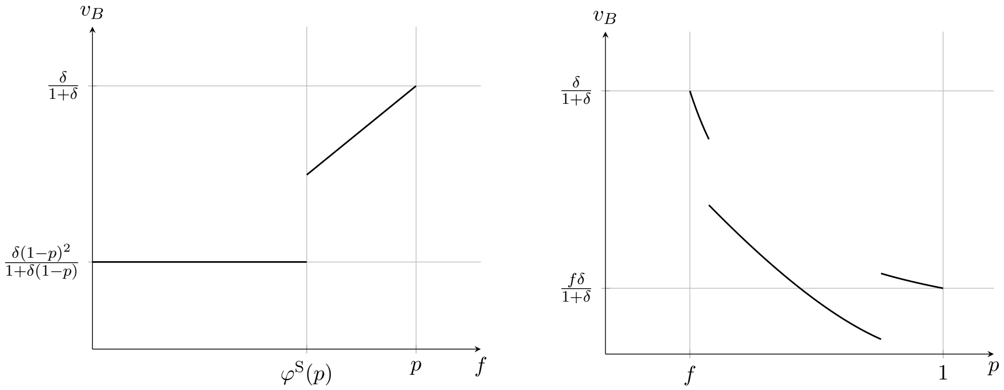

Figure 3: Bob's stationary equilibrium payoff as a function of the cost of protesting f (left panel) and protest success probability p (right panel).

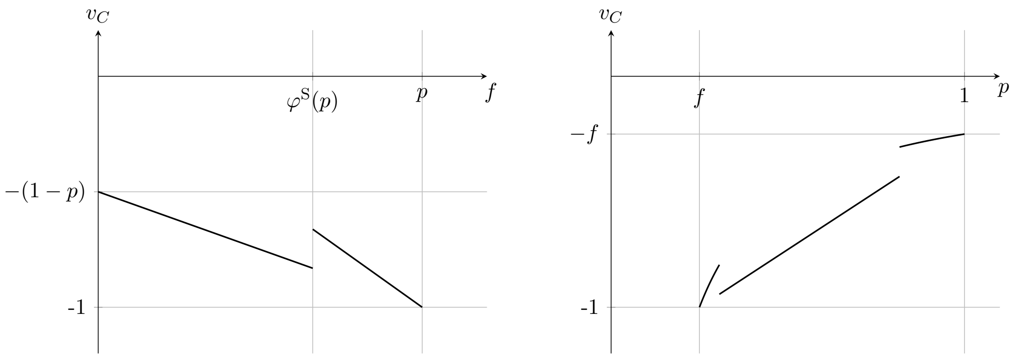

Figure 4: Charlie's stationary equilibrium payoff as a function of the cost of protesting f (left panel) and protest success probability p (right panel).

$p \in (0, 1)$. As these plots elucidate, Ann's equilibrium payoff is continuous in both f and p, increasing in f, and decreasing in p. In the Accommodation region, a decrease in the cost-effectiveness of Charlie's protesting technology increases the amount the bargainers can extract from him without triggering a protest. Instead, in the Conflict region, the proposed resource extraction does not change ($x_C^* = -1$), but a decrease in p increases the chances that an agreement is reached.

Bob and Charlie's stationary equilibrium payoffs, on the other hand, are neither continuous nor monotone in f and p. Figure 3 displays Bob's equilibrium payoff. Bob always benefits from an increase in the cost of protesting but may benefit or lose from an increase in the success probability of Charlie's protests. The discontinuities in Bob's payoff occur when the equilibrium outcome changes from Accommodation to Conflict. To see why, observe that in equilibrium Ann's offer makes Bob indifferent only conditional on reaching the decision stage. However, under Conflict, Charlie protests on path, implying that with probability p negotiations break down before reaching that stage. Observe that an increase in f can never change the equilibrium outcome from Accommodation to Conflict. As a result, Bob's equilibrium payoff is increasing in f, just like Ann's. On the other hand, an increase in p can move parameters both into and out of the Conflict region (see Figure 1). This explains why Bob's payoff is non-monotone in p.

Finally, as Figure 4 shows, Charlie's stationary equilibrium payoff is neither monotone in f

nor in p. In any stationary equilibrium, Charlie's payoff is equal to the negative of the sum of Ann and Bob's payoff minus the protesting fee. As a result, Charlie's payoff also drops when moving from the Accommodation to the Conflict parameter region. This leads to a surprising implication: Charlie may benefit from a weakening in his protesting technology through an increase in f or a decrease in p. Intuitively, a decrease in the cost-effectiveness of Charlie's protest makes Accommodation more attractive to Ann. In turn, this can lead her to make less aggressive proposals in equilibrium that Charlie is willing to endure without protests.

4.3 Impossibility of Full Surplus Extraction

As Proposition 1 shows, if $f \geq p$, protesting is too costly to affect the bargainers' equilibrium play, and the unique SPE outcome is the Rubinstein outcome. This outcome features two key properties: First, since Charlie does not protest on path, it satisfies *Pareto efficiency*:

$$v_A + v_B + v_C = 0. \tag{PE}$$

This means that what Charlie loses in equilibrium coincides with what Ann and Bob gain. Second, Charlie's equilibrium payoff corresponds to his single-period *min-max payoff* $\bar{u}_C$, i.e., it holds that

$$v_C = \bar{u}_C \tag{Min-Max}$$

where

$$\bar{u}_C := \min_{x \in X} \max\{x_C, (1-p)x_C - f\} = \max\{-1, -(1-p+f)\}.$$

From now on, whenever an equilibrium outcome satisfies properties (PE) and (Min-Max) jointly, we say that Ann and Bob *extract the full surplus.*[26] The following corollary shows that, unlike when $f \geq p$, full surplus extraction is impossible in any stationary equilibrium under Assumption 1.[27]

Corollary 1. *Suppose $f < p$. Then, Ann and Bob do not extract the full surplus in any stationary equilibrium.*

Under Accommodation, any stationary equilibrium outcome satisfies Pareto efficiency but Charlie's payoff is greater than his min-max payoff. Conversely, under Conflict, even though Charlie gets his min-max payoff, the stationary equilibrium outcome is not Pareto efficient: Since Charlie protests on path, some surplus gets wasted on costly protesting. As Figure 5 shows, this implies that the bargainers' joint payoff in any stationary equilibrium is always strictly below the negative of Charlie's single-period min-max payoff when $f < p$.

Corollary 1 suggests that, despite his lack of formal representation, the *de facto* bargaining power that credible protesting bestows upon Charlie is strong enough to prevent Ann and Bob from fully exploiting their opportunity for redistribution. The following section shows that this

[26] Full surplus extraction is a condition on agents' equilibrium payoffs and does not depend on other aspects of the game. In particular, the *amount of resources* the bargainers end up seizing from Charlie does not enter this definition: They could extract $x_A + x_B = -\bar{u}_C$ immediately or extract more with delay, as long as their joint (expected) equilibrium payoff $v_A + v_B$ equals $-\bar{u}_C$.

[27] Recall that when $f \geq p$, the unique SPE outcome can be supported in stationary strategies.

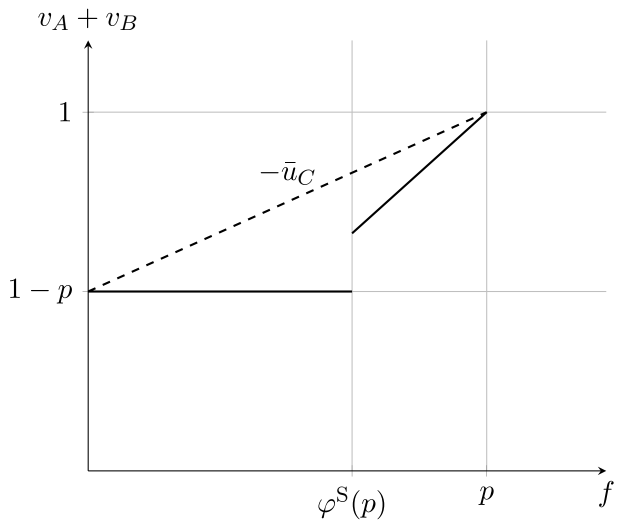

Figure 5: Ann and Bob's joint payoff in any stationary equilibrium (solid), and the negative of Charlie's min-max payoff (dashed) as a function of $f \in (0, p)$.

intuition relies heavily on stationarity and does not hold without it.

5 Strategic Delay

The restriction to stationary strategies of the previous section significantly impacts the equilibrium payoffs agents can achieve. In particular, without this restriction, the bargainers can extract the full surplus. Section 5.1 shows this by proving that a necessary and sufficient requirement for full surplus extraction in equilibrium is what we call *strategic delay*, i.e., the possibility that the responder makes his acceptance probability depend on Charlie's current-period protesting decision. Section 5.2 discusses the role that competition between the bargainers plays in enabling strategic delay.

5.1 Full Surplus Extraction

In this subsection, we state our main results: Strategic delay is necessary and sufficient to achieve full surplus extraction in equilibrium. To define strategic delay formally, the following notation is required. For any profile of strategies σ, let $\mathbb{P}_\sigma \in \Delta(Z)$ be the probability measure over terminal histories it induces. Moreover, for any terminal history $z \in Z$, denote by $H_C(z)$ the set of non-terminal histories preceding z where it is Charlie's turn to move. We say that the profile of strategies σ features *strategic delay* if

$$\mathbb{P}_\sigma \left(\left\{ z \in Z : \exists h \in H_C(z) \text{ with } \sigma_{R(h)}(accept|h, protest) \neq \sigma_{R(h)}(accept|h, abstain) \right\} \right) > 0, \quad (5)$$

where $R(h) \in \{A, B\}$ denotes the responder's identity at history h. In words, σ features strategic delay if it induces a path where the responder's acceptance probability depends on Charlie's current-period protesting decision with positive probability. The following theorem shows that any full surplus extraction SPE must feature strategic delay.

Theorem 1. *Suppose $f < p$, and let σ be an SPE where Ann and Bob extract the full surplus.*

Then, σ features strategic delay.

To understand Theorem 1, suppose the bargainers were restricted to using strategies that do not feature strategic delay. Then, Charlie could be punished for protests only through continuation play from the next period onward. However, this would imply that Charlie can guarantee a payoff strictly larger than his single-period min-max payoff $\bar{u}_C$ by protesting, if Bob rejects with positive probability Ann's offer in period $t = 0$. To see this, note that an unsuccessful protest still leads to a positive probability of delay (since delay must be independent of Charlie's action), and therefore to a continuation payoff to Charlie strictly larger than -1. On the other hand, if Bob accepts Ann's initial offer with certainty, the bargainers can do no better than to induce immediate Conflict or Accommodation, making full surplus extraction impossible.

While Theorem 1 shows that strategic delay is needed to achieve full surplus extraction in equilibrium, it leaves open the question of whether such an equilibrium exists. The following proposition shows that the answer is affirmative, even under the minimal relaxation of the Markov property that allows for strategic delay, i.e., *time-invariance*. We define time-invariant strategies as strategies where behavior may only depend on current-period events. This implies that the proposer and Charlie's behavior can still be described as in equations (1) and (3), respectively. However, the responder can now make his acceptance probability contingent on Charlie's current-period action, i.e.,

$$\beta_i^R : X \times \{protest, abstain\} \to \Delta(\{yes, no\})$$

instead of $\beta_i^R : X \to \Delta(\{yes, no\})$. We define a *time-invariant equilibrium* as an SPE where all agents use time-invariant strategies.

Let

$$\varphi^*(z) := z(1-z)\frac{\delta}{1+\delta(1-z)} = \frac{\delta}{1+\delta}\varphi^{\mathrm{S}}(z) \tag{6}$$

for all $z \in (0, 1)$. The following theorem shows that time-invariance is sufficient for the existence of a full surplus extraction equilibrium. Furthermore, it shows that, under symmetric strategies, the corresponding equilibrium outcome is unique.[28]

Theorem 2. *Suppose $f < p$. Then, a symmetric time-invariant equilibrium outcome where Ann and Bob extract the full surplus exists if and only if $\varphi^*(p) \leq f$. This outcome is unique and features the following sequence of events:*

(i) The proposer offers $x^* = (x_P^*, x_R^*, x_C^*) = (1 - \delta v_P, \delta v_P, -1)$, where $v_P = \frac{1-p+f}{1+\delta}$,

(ii) Charlie abstains from protesting,

(iii) The responder accepts x^* with probability

$$q^* := 1 - \frac{p - f}{1 - \delta(1 - p + f)} \in (0, 1). \tag{7}$$

[28] Like with Markovian strategies, we say that a profile of time-invariant strategies is symmetric if Ann and Bob behave symmetrically. Moreover, we employ the same convention used for stationary equilibria when denoting the proposer's and responder's allocation, payoff, and behavior rules.

Furthermore, in any equilibrium leading to this outcome, the bargainers use strategic delay. In particular, if Charlie protests against the equilibrium proposal x^*, the responder accepts x^* with certainty (instead of with probability $q^* < 1$) in case the protest fails.

The key feature that sustains full surplus extraction in the equilibrium outcome characterized in Theorem 2 is strategic delay: The responder accepts the proposed offer x^* with a probability q^* strictly below one if Charlie acquiesces, while he immediately accepts in case Charlie protests. Since Charlie discounts future losses, he enjoys agreement delay. However, the responder rejects the equilibrium offer with positive probability *only if* Charlie acquiesces. This contingent behavior lowers the value of protesting for Charlie. As a result, Ann and Bob are able to extract more resources from him without triggering a protest.

As equation (7) shows, the responder's equilibrium acceptance probability q^* is strictly increasing in the cost of protesting f, and strictly decreasing in the protest success rate p. This follows from the fact that q^* is pinned down by Charlie's indifference between acquiescence and protesting on path. In particular, since $x^*_C = -1$, the more likely the responder is to accept the equilibrium offer, the more tempting protesting becomes to Charlie. Thus, a decrease in f (or an increase in p), which makes protesting more effective, must be accompanied by a decrease in q^* to sustain Charlie's indifference. In turn, this implies that the expected time until an agreement is reached in equilibrium is decreasing in $(f - p)$.

In classical Rubinstein bargaining, agreement delay never occurs in equilibrium even though the responder is just indifferent between accepting and rejecting the proposer's offer: The proposer can improve upon allocations inducing delay by making a slightly better offer to the responder, since this allows the proposer to keep virtually all of the surplus that, otherwise, would be inefficiently wasted due to discounting. When $\varphi^*(p) \leq f < p$, this deviation is not profitable in the equilibrium outlined in Theorem 2. For an intuition, note that since $f < p$, Charlie would protest against any proposal $x \in X$ such that $x_C = -1$, unless he assigns a high enough probability that the responder will reject it. Given $x^*_R = \delta v_P$, such agreement delay is *credible* on path, since the responder is indifferent between accepting and rejecting the proposer's equilibrium offer. However, if the proposer were to deviate by offering slightly more to the responder, Charlie would anticipate an immediate agreement, in turn making protesting strictly optimal. As long as $f \geq \varphi^*(p)$, the proposer's benefit from extracting the full surplus more than outweighs her loss of agenda-setting power due to agreement delay.

Figure 6 shows the parameter region where the full surplus extraction equilibrium of Theorem 2 exists. Note that this is always the case when the stationary equilibrium outcome is Accommodation, i.e., when $\varphi^{\mathrm{S}}(p) < f < p$. In this region, strategic delay benefits Ann and Bob at the expense of Charlie, keeping aggregate welfare unchanged. However, full surplus extraction is also possible if $\varphi^*(p) < f < \varphi^{\mathrm{S}}(p)$. In this case, Conflict emerges in any stationary equilibrium, implying that Charlie receives his min-max payoff. Nevertheless, strategic delay helps to avoid inefficient protests on path, allowing the bargainers to do strictly better without further reducing Charlie's payoff.

Notably, the parameter region identified by Theorem 2 expands as the discount factor δ

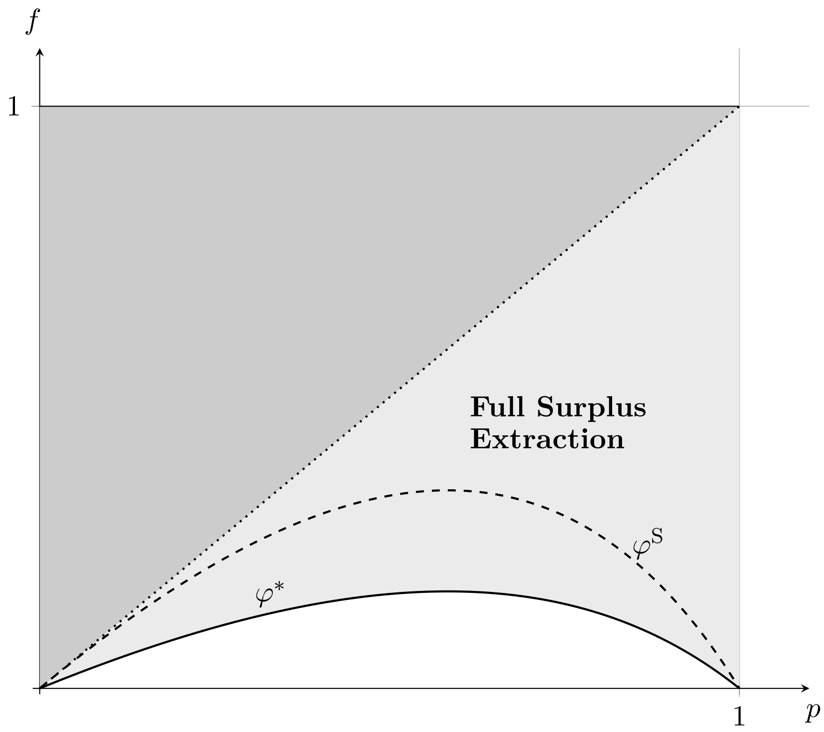

Figure 6: The parameter region in the (p, f)-space for which the symmetric time-invariant equilibrium outcome where Ann and Bob extract the full surplus exists, for a fixed $\delta \in (0, 1)$. The curves φ^{S} and φ^* are defined in equations (4) and (6), respectively.

decreases. Moreover, it holds that $\varphi^*(p) = \frac{\delta}{1+\delta}\varphi^{\mathrm{S}}(p) \to 0$ as $\delta \downarrow 0$. Thus, strategic delay is a punishment device that is easier to sustain in equilibrium the more *impatient* agents become. This is in stark contrast to standard Folk theorems in the repeated games literature where, instead, the threat of future punishments is typically more effective as δ increases. This difference emerges because, in our model, strategic delay punishes protests within the same period they occur rather than via future-period continuation play.

Agreement delays are often observed in international negotiations. Our redistributive bargaining framework offers a new explanation for this phenomenon. As a concrete example, consider the negotiations about Ukraine's admission to NATO. At its 2008 summit in Bucharest, NATO promised that Ukraine would be allowed to join the alliance eventually, but its members decided to delay handing a Membership Action Plan (MAP) to Ukraine to appease to Russia.[29] While, of course, our model does not capture all the intricate details that played a role in these negotiations, Theorems 1 and 2 show that our tractable framework already suffices to provide a rationale for why NATO may have postponed granting a MAP to Ukraine.[30]

[29] See BBC News article "Nato denies Georgia and Ukraine" from April 3, 2008 (`http://news.bbc.co.uk/2/hi/europe/7328276.stm`), and the Der Spiegel cover story from September 25, 2023 (`https://shorturl.at/QC4u2`).

[30] As Russia ended up invading Ukraine, the full surplus extraction equilibrium outcome characterized in Theorem 2 can be ruled out, as it prescribes perpetual acquiescence. Nevertheless, SPE outcomes with strategic delay followed by Conflict can exist. Details are available from the authors upon request.

5.2 The Role of Competition in Strategic Delay

To extract the full surplus, the bargainers need to be able to coordinate on an equilibrium featuring strategic delay. We now show that this is only possible if Ann and Bob's interests are misaligned. If, instead, their objective was to maximize their joint surplus, no equilibrium featuring strategic delay would exist.

To see this, consider the following variation of our redistributive bargaining model where Ann and Bob's interests are perfectly aligned. We call this variation the *collusive bargaining model.* Compared to the model of Section 2, the collusive bargaining model has the same game structure. The only difference is that Ann and Bob now care about their aggregate allocative performance, i.e., they both share the payoff function $u_{AB} : Y \to \mathbb{R}$ given by

$$u_{AB}(\tau, x, \mathcal{T}_C) := \sum_{i=A,B} u_i(\tau, x, \mathcal{T}_C) = \delta^\tau \cdot (x_A + x_B) .$$

Let $\mathcal{E}^{\mathrm{C}}$ be the set of SPEs of the collusive bargaining model, and denote by (v_{AB}, v_C) agents' equilibrium payoffs. Also, define the mapping $z \mapsto \varphi^{\mathrm{C}}(z)$ by

$$\varphi^{\mathrm{C}}(z) := z(1 - z). \tag{8}$$

The following proposition proves that an SPE of the collusive bargaining model always exists, characterizes its set of SPE payoffs, and describes the outcomes they induce for almost all model parameters.[31] Since the distribution of resources between Ann and Bob is payoff irrelevant, we describe offers by the total proposed allocation to the bargainers, $x_{AB} := x_A + x_B = -x_C$, in the proposition.

Proposition 3. *In the collusive bargaining model, an SPE always exists. In particular, the following holds:*

a) ***Accommodation:*** *If $\varphi^{\mathrm{C}}(p) < f < p$, all SPEs induce the same equilibrium payoffs given by*

$$(v_{AB}, v_C) = (\lambda, -\lambda) .$$

Moreover, the SPE outcome is unique, can be sustained in stationary strategies, and features the following sequence of events:

(i) The proposer offers x^* such that $x^*_{AB} = \lambda$,

(ii) Charlie abstains from protesting,

(iii) The responder accepts x^* with probability 1.

b) ***Conflict:*** *If $0 < f < \varphi^{\mathrm{C}}(p)$, all SPEs induce the same equilibrium payoffs given by*

$$(v_{AB}, v_C) = (1 - p, -f - (1 - p)) .$$

[31] Similarly to our baseline model, Charlie's equilibrium payoff is not uniquely identified when $f = \varphi^{\mathrm{C}}(p)$, since the proposer's indifference implies that she could select either Accommodation or Conflict in this case.

Moreover, the SPE outcome is unique, can be sustained in stationary strategies, and features the following sequence of events:

(i) The proposer offers x^* such that $x^*_{AB} = 1$,

(ii) Charlie protests,

(iii) If the protest fails, the responder accepts x^* with probability 1.

To characterize the SPE outcomes, we first show that in the bargainer-preferred equilibrium of the collusive bargaining model, i.e., the SPE inducing the highest payoff for the bargainers, agreement delay is impossible. To delay the agreement, the responder needs to weakly prefer receiving the discounted next-period's continuation value over implementing the current proposal. However, since the subgame starting next period is identical to the current one, no equilibrium with an even higher payoff for the bargainers exists, making delay in the bargainer-preferred equilibrium impossible. On the other hand, the proposer can always guarantee that the bargainers receive their no-delay equilibrium payoff. She does so by offering to extract all available resources from Charlie ($x^*_C = -1$) or just what makes Charlie indifferent between protesting and not ($x^*_C = -\lambda$). This proves that all SPE outcomes of the collusive bargaining model feature no delay, can be sustained in stationary strategies,[32] and yield the no-delay equilibrium payoff to the bargainers.

Under Assumption 1, i.e., when $f < p$, the collusive bargaining model admits two possible equilibrium outcomes, Accommodation and Conflict, analogous to those described in Proposition 2. In these outcomes, the bargainers' payoff is equal to what the bargainers jointly earn in the corresponding stationary equilibrium outcome of the baseline model. However, as the left panel of Figure 7 shows, the cutoff curve between these two equilibrium regions, φ^{C}, is strictly lower than its analog under stationarity in the baseline model, φ^{S}. This is the case since, under collusion, the proposer internalizes the negative externality protests have on the responder's payoff, making her more willing to accommodate to Charlie. This implies that, when restricting to stationary strategies, the bargainers' payoff is weakly larger under collusion than under competition.

This observation is reversed if we drop stationarity. To see this, observe that the right panel of Figure 7 implies that there always exists an equilibrium where the bargainers earn a (weakly) higher payoff under competition. In particular, whenever the bargainers are strictly better off colluding under stationarity, full surplus extraction can be sustained in our baseline model. Theorem 3 below formalizes this discussion.

For every profile of strategies σ, let $u_i(\sigma)$ be the payoff agent $i \in I$ obtains, and $u_{AB}(\sigma)$ be the payoff the bargainers with aligned interests obtain under σ. The following holds:

Theorem 3. *In equilibrium, competing bargainers can always do better than colluding ones. That is, there exists $\sigma^* \in \mathcal{E}$ such that*

$$u_A(\sigma^*) + u_B(\sigma^*) \geq u_{AB}(\sigma), \quad \forall \sigma \in \mathcal{E}^{\text{C}}. \tag{9}$$

[32] Since in any SPE the responder must accept the proposer's offer immediately (irrespective of Charlie's protesting decision), it is without loss to restrict attention to stationary strategies to characterize SPE outcomes.

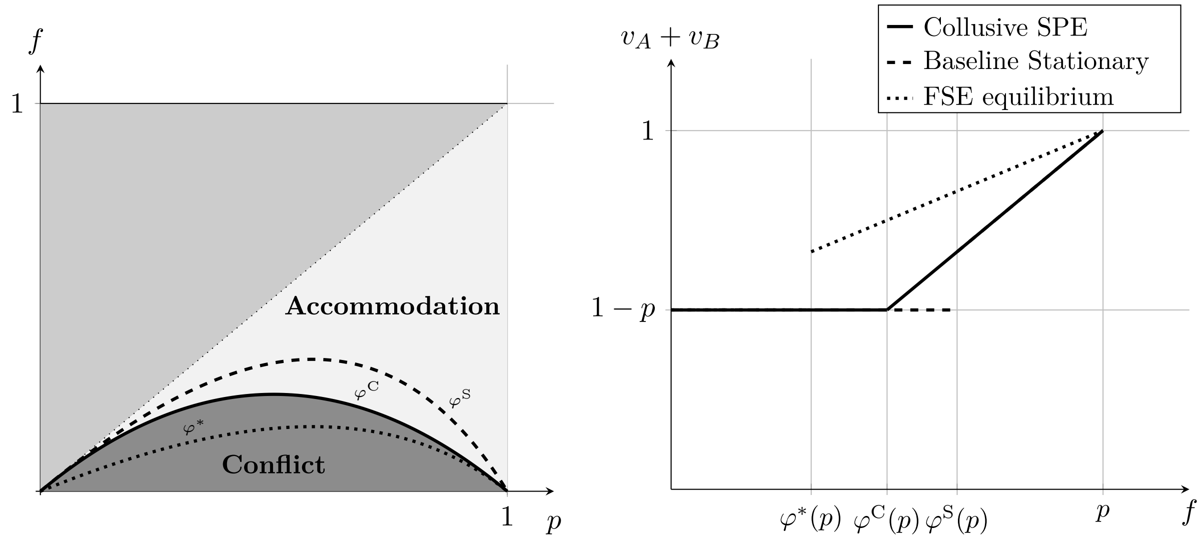

Figure 7: The SPE outcomes of the collusive bargaining model as a function of f and p, with $f < p$, for a fixed $\delta \in (0,1)$ (left panel), and the bargainers' joint payoff in the collusive equilibrium, the stationary equilibrium, and the full surplus extraction (FSE) equilibrium of Theorem 2 as a function of f for fixed $p \in (f,1)$ and $\delta \in (0,1)$ (right panel). The curves φ^{S}, φ^{*}, and φ^{C} are defined in equations (4), (6), and (8), respectively.

Furthermore, whenever the full surplus extraction equilibrium outcome described in Theorem 2 exists, i.e., whenever $\varphi^*(p) \leq f < p$, the statement above holds even if the weak inequality in (9) is replaced by a strict inequality.

When preferences are aligned, the bargainers would like to commit to strategic delay as full surplus extraction is impossible without it. However, once Charlie has refrained from protesting, it is always in the responder's best interest to accept the proposer's equilibrium offer immediately. Hence, strategic delay is impossible under collusion. This is not the case when the bargainers compete. Delay leads to payoffs being discounted, but also to a switch in the bargainers' roles of proposer and responder. Note that, when interests are misaligned, the proposer captures a larger share of the extracted surplus. Therefore, the responder dislikes agreement delay due to discounting but enjoys the increase in agenda-setting power he gains from it. This trade-off is precisely how opposing interests allow the bargainers to overcome the commitment problem they face due to impatience, ultimately enabling them to benefit from strategic delay. This result leads to a surprising finding: Charlie may be worse off when facing competing, rather than colluding bargainers.

6 Discussion

We conclude by discussing the role some of our modeling assumptions play in shaping our main results. In Section 6.1, we consider an extension where the focus on purely redistributive environments is relaxed. In Section 6.2, we examine the impact of a change in the timing of Charlie's protests. Section 6.3 investigates the consequences of dropping the assumption that Charlie can perfectly observe the proposer's offer. Finally, in Section 6.4, we allow for the possibility that, in addition to Charlie, the bargainers also incur a cost when protests are triggered.

6.1 Beyond Pure Redistribution

So far, our analysis focused on purely redistributive bargaining environments where an agreement between the bargainers does not create nor destroy social value. This restriction is reflected in our maintained assumption that feasible offers are "zero-sum" in nature, i.e., $\sum_{i \in I} x_i = 0$ for all $x \in X$. This assumption also implies that agreement delay does not impact social welfare: As long as Charlie does not protest on path, any agreement between the bargainers induces a Pareto efficient outcome, irrespective of when consensus is reached.[33] In some redistributive settings, however, the policy two bargainers ultimately agree upon can impact total welfare, implying that agreement delay is no longer welfare-neutral. For instance, a merger between two firms might generate efficiency gains that more than offset the welfare loss experienced by dismissed workers. Conversely, political parties may implement inefficient reforms just to favor their electoral base at the expense of opposing interest groups. This subsection examines how our main findings change when considering such possibilities.[34]

Formally, suppose that the set of feasible proposals is given by

$$X_\omega := \Big\{ x \in \mathbb{R}^I : \sum_{i \in I} x_i = \omega, \text{ and } x_C \geq -1 \Big\},$$

where $\omega \in \mathbb{R}$ is a parameter representing the social value of an agreement between the bargainers. The model specification of Section 2 corresponds to the special case of $\omega = 0$. Therefore, here we investigate the case where $\omega \neq 0$.

It turns out that whether ω is positive or negative can impact equilibrium predictions. When ω is negative, any agreement between the bargainers decreases social welfare, implying that agreement delay can be used to ameliorate such welfare losses. Conversely, when ω is positive, an agreement between the bargainers increases social welfare, implying that agreement delay is inefficient. We start by describing how these distinctions affect stationary equilibrium outcomes. We then discuss when strategic delay can still help the bargainers extract more resources from Charlie without triggering risky protests. Throughout this subsection, we impose Assumption 1, i.e., Charlie's protesting threat is credible.

Stationary equilibria: The case of $\omega < 0$. When $\omega < 0$, any agreement between the bargainers destroys aggregate resources. Therefore, we can interpret ω as a cost the bargainers need to pay to be able to redistribute resources across agents. It is easy to see that no redistribution will ever be implemented in equilibrium unless the resources the bargainers can extract from Charlie compensate them for paying such a cost. That is, if $\omega < -1$, the unique possible equilibrium outcome is *no agreement*. Thus, from now on, let us assume that $-1 \leq \omega < 0$. In this case, an Accommodation and a Conflict equilibrium region still exist. However, in contrast to the baseline model, they overlap. This follows since, although a continuum of accommoda-

[33] In fact, even never reaching an agreement is a Pareto efficient outcome in our baseline model.

[34] To keep our presentation short, our discussion in this subsection is relatively informal. The interested reader can find a formal analysis of the results mentioned below in Appendix B.

tion equilibrium outcomes of the form described in Proposition 2 can still exist, they no longer yield the same equilibrium payoff to the proposer and, consequently, to the responder. Instead, the bargainers' joint equilibrium payoff can be ranked according to the equilibrium agreement probability induced in each period: The smaller this probability, the larger their joint payoff in the corresponding equilibrium. In turn, this implies that there exists a parameter region where the Conflict outcome and *some* Accommodation outcomes can be sustained in equilibrium when $\omega < 0$.

To understand why the bargainers' payoffs under different Accommodation outcomes can be ranked, recall that under Accommodation, Charlie is always indifferent between protesting and acquiescing on path. Thus, while Charlie's equilibrium payoff is $v_C = -\lambda$ irrespective of the presence of delay, Accommodation outcomes associated with a smaller probability of agreement lead to strictly larger equilibrium payoffs to the bargainers since they postpone the payment of the redistribution cost $\omega < 0$. In particular, for a fixed Accommodation equilibrium outcome, it holds that

$$v_P + v_R = \lambda + \mathbb{E}\left[\delta^\tau\right] \cdot \omega \tag{10}$$

where the expectation is taken over the random period τ in which an agreement is reached.

Stationary equilibria: The case of $\omega > 0$. When $\omega > 0$, any redistributive agreement between the bargainers creates social value, implying that delaying an agreement is inefficient. In this case, unlike when $\omega \leq 0$, the Accommodation equilibrium outcome is unique and features no agreement delay. The intuition for this lack of lower semi-continuity of the equilibrium correspondence at $\omega = 0$ is as follows. When $\omega > 0$, delaying the agreement reduces the proposer's payoff since it postpones the moment she benefits from the extra surplus ω that the agreement generates. As a result, if there were a positive probability that the responder would reject her equilibrium offer, the proposer would find it optimal to induce immediate acceptance, instead. This is beneficial, even though it would require extracting fewer resources from Charlie to make sure he acquiesces. Conversely, when $\omega \leq 0$, the proposer benefits (at least weakly) from agreement delay, implying that it can be sustained as an Accommodation equilibrium outcome.[35]

Strategic delay. As Theorem 2 shows, strategic delay can be a powerful tool to extract more surplus from the third party in redistributive bargaining settings. How robust is this conclusion to departures from our maintained assumption $\omega = 0$? When $f < p$, full surplus extraction is impossible unless $\omega = 0$. To see this, note that the maximum surplus that the bargainers could hope to extract is given by $\bar{u}_C + \max\{\omega, 0\}$. However, in the case of $\omega < 0$, this would require delaying the agreement indefinitely, which makes it impossible to extract any payoff from Charlie. In the case of $\omega > 0$, this would require avoiding delay which, as we know, precludes the bargainers from extracting Charlie's min-max payoff.

[35] Observe that, unlike with immediate acceptance, the proposer cannot force "more" delay in equilibrium by making a unilateral deviation, since she cannot induce the responder to accept an offer with lower probability by making an alternative proposal. This explains why a continuum of Accommodation equilibrium outcomes with different agreement probabilities exists if $\omega < 0$ even though, according to equation (10), the bargainers' equilibrium payoffs vary with the degree of agreement delay.

Nevertheless, strategic delay can still help the bargainers to achieve a joint equilibrium payoff that is strictly larger than in any stationary equilibrium. Under symmetric time-invariant strategies with deterministic offers, this is the case if and only if $\varphi^*(p) \leq f < p$ and $-1 < \omega < \omega^*$, where $\omega^* := \frac{1-p}{p}$.[36] The proof of sufficiency is constructive. To understand necessity, observe that strategic delay enables the bargainers to extract more resources from Charlie but necessarily postpones the consumption of ω. If $\omega < 0$, both of these forces improve the bargainers' payoff. Conversely, if $\omega > 0$, agreement delay is costly. As a result, if the welfare gains of the agreement are too large, i.e., $\omega \geq \omega^*$, strategic delay must reduce the bargainers' payoff compared to the (unique) Accommodation equilibrium outcome. Finally, $\varphi^*(p) \leq f < p$ guarantees that the proposer does not have a strict incentive to break the responder's indifference by inducing inefficient Conflict on path.

6.2 The Timing of Protests

The timing of protests plays an important role in determining the bargaining outcome in equilibrium. In our baseline model, we focus on settings where protests are *informal*: They can terminate negotiations but cannot alter the bargaining outcome once an agreement is reached. For example, once a military alliance between two countries has formed, neighboring countries can do nothing but live with the new political reality, and once a merger has been finalized, it is very rarely reversed. *Formal* protests, on the other hand, typically occur *ex-post*, i.e., after an agreement is finalized. For example, in liberal democracies, citizens may challenge a law as being unconstitutional only after such a law has passed.

In this subsection, we discuss how our findings change if protests occur after, rather than before, an agreement between the bargainers is reached. Formally, we interchange the decision and protesting stages in every period. If the responder rejects the proposer's offer, the protesting stage is skipped, and the play continues with the proposing stage of the next period. If the responder approves the current-period proposal, Charlie has the opportunity to protest.

The key difference from our baseline model is that Charlie has no continuation game to consider when making his protesting decision: The game is guaranteed to end, either with a successful protest or with the current-period proposal being implemented. Importantly, this implies that delay can no longer be used to improve Charlie's continuation value. As a result, Charlie protests whenever the accepted proposal x^* satisfies $x^*_C < -\lambda$.

When $f < p$, the proposer therefore always makes one of two offers: She either proposes to extract everything from Charlie, i.e., $x_C = -1$, leading to an expected extraction of $1 - p$ resources, or she accommodates by proposing $x_C = -\lambda$. Since agreement delay no longer offers any benefit, and the responder can always prevent Conflict by rejecting an offer that would lead Charlie to protest ex-post, standard Rubinstein (1982) arguments show that agreement is reached immediately, and each bargainer receives their Rubinstein share of the extracted surplus.

Therefore, the equilibrium outcomes with ex-post protesting are identical to those under collusive bargaining, with the exception that the distribution of resources between proposer and

[36] See Proposition 7 in Appendix B for a formal statement and a proof of this result.

responder is pinned down uniquely. This leads to several implications. When protests can overturn a finalized agreement, the bargainers cannot use strategic delay to extract more resources from the third party. As Figure 7 shows, this leads to better protection of Charlie's resources when $\varphi^{\mathrm{C}}(p) < f < p$, but also to efficiency losses when $\varphi^*(p) < f < \varphi^{\mathrm{C}}(p)$, compared to the full surplus extraction equilibrium. Finally, the bargainers can achieve a higher equilibrium payoff in settings where protests may occur ex-interim, rather than ex-post.

6.3 Offer Observability

Our baseline model features public negotiations, i.e., in any period Charlie observes which offer is currently under review before deciding whether to protest or not. Of course, in many settings, the bargainers could keep proposals private. As we discuss in this subsection, it turns out that the bargainers may strictly prefer public over private negotiations.

To highlight the role of offer observability, we compare the outcomes of our baseline model to those of an alternative model where Charlie does not observe the proposals the bargainers discuss. In this set-up, Charlie's strategy cannot depend on the proposal under review, i.e., it can be represented by a protest probability $\beta_C^t(\mathit{protest}) \in [0,1]$ for every period. Therefore, the incentives that helped the proposer sustain the existence of the full surplus extraction equilibrium with observable offers are no longer present: The proposer could now break the responder's indifference by proposing to give him slightly more without affecting Charlie's protesting decision. Thus, the initial proposer could deviate and increase her payoff by almost $p - f > 0$, which precludes the existence of the equilibrium outcome of Theorem 2. To show this formally, we characterize the set of time-invariant equilibria under unobservable offers.[37] For Charlie, time-invariance implies that his protest probability is constant over time, i.e., $\beta_C^t(\mathit{protest}) = \beta_C(\mathit{protest})$ for every $t \geq 0$. From the bargainers' point of view, this makes the model identical to a Rubinstein (1982) bargaining game with an adjusted discount factor of $(1 - \beta_C(\mathit{protest}) \cdot p) \cdot \delta$. In particular, any time-invariant equilibrium is symmetric, features no delay, and involves an equilibrium proposal $x^* = (x_P^*, x_R^*, x_C^*)$ such that $x^* = (1 - \delta v_P, \delta v_P, -1)$, where v_P is the unique equilibrium payoff of the initial proposer. In turn, this implies that Charlie always protests on path ($\beta_C(\mathit{protest}) = 1$) whenever $f < p$: Without observable offers, the unique time-invariant equilibrium outcome features Conflict.

Therefore, under time-invariance, the bargainers always receive their Conflict equilibrium payoffs when offers are private while they could achieve higher equilibrium payoffs if they announced their offers publicly.[38] This shows that the bargainers may benefit from public, rather than private, bargaining. Intuitively, by announcing their offers publicly, the bargainers can exploit the fact that they move before Charlie, similar to how market leaders benefit from moving first in Stackelberg competition. If, instead, they bargain behind closed doors, it is *as if* the bargainers and Charlie move simultaneously, removing this timing advantage.

[37] If Charlie uses a time-invariant strategy, the restriction to time-invariance for the bargainers is without loss of generality. See Appendix D for details.

[38] For example, this would be the case if $\varphi^*(p) \leq f < p$, and with public negotiations the bargainers could coordinate on the full surplus extraction equilibrium of Theorem 2.

6.4 Harmful Protests

In the baseline model, we assume that a protest imposes a cost of f only on Charlie, but does not impact the bargainers unless it is successful. However, in certain contexts, protests can also harm the bargainers. For example, a strike could directly disrupt the operations of two merging firms, and a preventive war can impose significant burdens on the negotiating countries being attacked. In this subsection, we extend our model to account for such situations. In particular, we assume that each bargainer incurs a cost of $\psi > 0$ whenever Charlie protests, irrespective of its outcome.

Introducing the cost ψ produces two effects. First, it lowers the bargainers' payoffs in any equilibrium where Charlie protests on path. Second, since triggering a protest becomes less attractive to the bargainers, the parameter regions where the Accommodation and the full surplus extraction equilibrium exist both expand. Concretely, the condition for the existence of the Accommodation equilibrium under stationary strategies becomes $\varphi^{\mathrm{S}}_{\mathrm{harm}}(p) \leq f < p$, where

$$\varphi^{\mathrm{S}}_{\mathrm{harm}}(p) := p(1-p-\psi)\frac{(1+\delta)}{1+\delta(1-p)}.$$

Instead, the full surplus extraction equilibrium of Theorem 2 exists if and only if it holds that $\varphi^{*}_{\mathrm{harm}}(p) \leq f < p$, where

$$\varphi^{*}_{\mathrm{harm}}(p) := \frac{(1-p)p\delta - \psi(1+\delta)}{1+\delta(1-p)}.$$

Note that $\varphi^{\mathrm{S}}_{\mathrm{harm}}(p) < \varphi^{\mathrm{S}}(p)$ and $\varphi^{*}_{\mathrm{harm}}(p) < \varphi^{*}(p)$ since $\psi > 0$. Thus, when protests are harmful to the bargainers, they may no longer choose to be hostile. However, even when hostility is avoided, Charlie may not benefit, since the bargainers can extract the full surplus using strategic delay.

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Appendix

The Appendix is organized as follows. The omitted proofs of the results presented in the main text are collected in Appendix A. The remaining supplementary appendices pertain to the Discussion section. Specifically, Appendix B investigates the case where bargaining is not purely redistributive, and formalizes the discussion in Section 6.1. Appendix C explores a model where Charlie can only protest after a proposal is accepted, and contains the formal analysis discussed in Section 6.2. Finally, Appendix D analyzes a model variation where offers are unobservable to Charlie, and therefore pertains to the discussion in Section 6.3. Throughout, we use the notation δ_w to denote the Dirac measure on an element w of a measurable space W. Such notation should not be confused with the parameter $\delta \in (0, 1)$ measuring agents' degree of impatience.

A Omitted Proofs

Proof of Proposition 1

Sufficiency. Assume that $f \geq p$. The proof of sufficiency is divided into three parts. In Part 1, we show that a stationary equilibrium inducing the Rubinstein outcome exists. In Part 2, we introduce some useful preliminary notation. In Part 3, we show that any SPE outcome must coincide with the Rubinstein outcome outlined in Part 1.

Proof of Proposition 1, Part 1: Existence

The following lemma proves the existence of a stationary equilibrium inducing the Rubistein outcome.

Lemma 1. *A stationary equilibrium inducing the Rubinstein outcome exists.*

Proof. Consider the following stationary strategies:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta}{1+\delta} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta}{1+\delta} \end{cases}$$

  for all $x \in X$;
- β_C given by $\beta_C(\cdot|x) = \delta_{abstain}$ for all $x \in X$.

It is straightforward to verify that $(\beta_P, \beta_R, \beta_C)$ constitutes a stationary equilibrium and induces the Rubinstein outcome. Details are omitted. □

Proof of Proposition 1, Part 2: Preliminary notation

We introduce some useful preliminary notation. For every $i \in \{A, B\}$, let $H_{i,P}$ be the set of non-terminal histories where it is agent i's turn to move at the proposing stage. For example,

$\varnothing \in H_{A,P}$, while $(x, abstain, no) \in H_{B,P}$.[39] Note that for all $h, h' \in H_{i,P}$, the two subgames that start at h and h' are *strategically equivalent*: The sets of continuation strategies in these two subgames are isomorphic, while the agents' payoff functions across these two subgames are linearly dependent. For every $h \in H_{i,P}$, let Γ_i^h be the subgame starting at h where agents' payoffs are normalized so that if an allocation $x \in X$ is immediately implemented without protests, each agent gets a payoff of x_i. Observe that because of this payoff normalization, each subgame Γ_i^h does not actually depend on $h \in H_{i,P}$. Therefore, it makes sense to write Γ_i to indicate a generic subgame that starts with agent $i \in \{A, B\}$ making an offer during the proposing stage. Given this, let $\mathcal{E}(\Gamma_i)$ be the set of SPE strategy profiles in Γ_i, and for each $j \in I$, let $V_j(\Gamma_i)$ be the set of agent j's equilibrium payoffs. Observe that none of these sets are empty since Lemma 1 showed that stationary equilibria exist. Finally, let $\bar{v}_j(\Gamma_i)$ (resp., $\underline{v}_j(\Gamma_i)$) be the maximal (resp., minimal) equilibrium payoff for $j \in I$ in subgame Γ_i. It is easy to see that since Ann and Bob can always force an outcome with perpetual disagreement, it holds that $\underline{v}_j(\Gamma_i) \geq 0$ for all $i, j \in \{A, B\}$. Moreover, since the only resources that Ann and Bob can consume in equilibrium are those they manage to extract from Charlie, it also holds that $\bar{v}_j(\Gamma_i) \leq 1$ for all $i, j \in \{A, B\}$.

Proof of Proposition 1, Part 3: Uniqueness of the SPE outcome

Consider the strategic situation Ann faces at the start of Γ_A. We begin by characterizing a lower bound for $\underline{v}_A(\Gamma_A)$. Suppose Ann offers $x \in X$ such that $x_B > \delta\bar{v}_B(\Gamma_B)$ to Bob. Since this proposal gives Bob strictly more than his discounted best continuation-equilibrium payoff, Ann knows that Bob will accept x immediately if given the opportunity. Moreover, since $f \geq p$ and $(1-p)x_C - f < x_C$ if and only if $x_C > -f/p$, Ann also knows that Charlie would find it strictly sub-optimal to protest against x as long as $x_C > -1$. Recall that $0 \leq \underline{v}_B(\Gamma_B) \leq \bar{v}_B(\Gamma_B) \leq 1$. We conclude that, as long as $\varepsilon > 0$ is small enough, the proposal x^ε such that $x_B^\varepsilon = \delta\bar{v}_B(\Gamma_B) + \varepsilon$, $x_C^\varepsilon = -1 + \varepsilon$ and $x_A^\varepsilon = -(x_B^\varepsilon + x_C^\varepsilon)$ is feasible (i.e., $x^\varepsilon \in X$), and yields a payoff to Ann of $x_A^\varepsilon = 1 - \delta\bar{v}_B(\Gamma_B) - 2\varepsilon$. In any SPE, Ann's equilibrium payoff must be at least as large as when offering to Bob any of these proposals x^ε. This implies that

$$\underline{v}_A(\Gamma_A) \geq \sup_{\varepsilon > 0} x_A^\varepsilon = 1 - \delta\bar{v}_B(\Gamma_B), \tag{LB$_A$}$$

establishing a lower bound for $\underline{v}_A(\Gamma_A)$.

Let us now characterize an upper bound for $\bar{v}_A(\Gamma_A)$. As a first step, suppose that Ann offers $x \in X$ such that $x_B < \delta\underline{v}_B(\Gamma_B)$ to Bob. Since x_B is strictly lower than Bob's discounted minimal equilibrium payoff, it must be the case that Bob rejects x. This implies that, by offering x to Bob, Ann can earn at most $\delta \cdot (1 - \underline{v}_B(\Gamma_B))$ in equilibrium.[40] Conversely, suppose now that Ann offers $x \in X$ such that $x_B \geq \delta\underline{v}_B(\Gamma_B)$ to Bob. The best outcome for Ann would be that Charlie does not protest against x even though $x_C = -1$, and Bob accepts immediately. This would

[39] Here, $\varnothing$ denotes the empty-history, i.e., the start of the game.

[40] To see this, fix any continuation equilibrium $\sigma^* \in \mathcal{E}(\Gamma_B)$. By definition, it holds that $u_i(\sigma^*) \geq \underline{v}_i(\Gamma_B) \geq 0$, where $u_i(\sigma)$ denotes the payoff agent $i \in \{A, B\}$ gains when the strategy profile σ is played. Since the only resources that Ann and Bob can consume in equilibrium are those they extract from Charlie, it must be the case that $u_A(\sigma^*) + u_B(\sigma^*) \leq 1$, which implies that $u_A(\sigma^*) \leq 1 - \underline{v}_B(\Gamma_B)$. Therefore, by offering a proposal x such that $x_B < \delta\underline{v}_B(\Gamma_B)$ at the beginning of Γ_A, Ann can earn at most a payoff of $\delta \cdot (1 - \underline{v}_B(\Gamma_B))$ in equilibrium, as required.

yield Ann a payoff of $x_A = 1 - x_B \leq 1 - \delta \underline{v}_B(\Gamma_B)$. Since $\delta < 1$, this establishes the following upper bound for $\bar{v}_A(\Gamma_A)$:

$$\bar{v}_A(\Gamma_A) \leq 1 - \delta \underline{v}_B(\Gamma_B). \tag{UB$_A$}$$

Similar bounds can be found for $\underline{v}_B(\Gamma_B)$ and $\bar{v}_B(\Gamma_B)$ by considering the strategic situation of Bob at the start of Γ_B as follows:

$$\underline{v}_B(\Gamma_B) \geq 1 - \delta \bar{v}_A(\Gamma_A), \tag{LB$_B$}$$

$$\bar{v}_B(\Gamma_B) \leq 1 - \delta \underline{v}_A(\Gamma_A). \tag{UB$_B$}$$

Combining the inequalities (UB$_A$) and (LB$_B$) and re-arranging, we obtain that $\bar{v}_A(\Gamma_A) \leq \frac{1}{1+\delta}$, while combining the inequalities (LB$_A$) and (UB$_B$), we obtain that $\frac{1}{1+\delta} \leq \underline{v}_A(\Gamma_A)$. Therefore,

$$\underline{v}_A(\Gamma_A) = \bar{v}_A(\Gamma_A) = v_A^*(\Gamma_A) = \frac{1}{1+\delta}.$$

Analogous steps can be used to show that $\bar{v}_B(\Gamma_B) = \underline{v}_B(\Gamma_B) = v_B^*(\Gamma_B) = \frac{1}{1+\delta}$. We conclude that there exists a unique equilibrium payoff associated with being the proposer at the start of every period, namely

$$V_j(\Gamma_j) = \{v_P^*\} = \left\{\frac{1}{1+\delta}\right\}$$

for all $j \in \{A, B\}$.

Consider now the strategic situation Ann faces at the start of Γ_B. Since Ann can always reject any offer $x \in X$ she receives from Bob, and Bob would never induce Charlie to protest in equilibrium,[41] we conclude that $\underline{v}_A(\Gamma_B) \geq \delta v_P^* = \frac{\delta}{1+\delta}$. On the other hand, as argued above, the best outcome for Ann in Γ_B is when Bob extracts all resources from Charlie, Charlie does not protest, and Bob keeps only an amount $\underline{v}_B(\Gamma_B) = v_P^* = \frac{1}{1+\delta}$ of these resources for himself. Therefore, $\bar{v}_A(\Gamma_B) \leq 1 - \frac{1}{1+\delta} = \frac{\delta}{1+\delta}$, which implies $\underline{v}_A(\Gamma_B) = \bar{v}_A(\Gamma_B) = v_A^*(\Gamma_B) = \frac{\delta}{1+\delta}$. Since analogous steps can be used to show that $\bar{v}_B(\Gamma_A) = \underline{v}_B(\Gamma_A) = v_B^*(\Gamma_A) = \frac{\delta}{1+\delta}$, we conclude that there exists a unique equilibrium payoff associated with being the responder at the start of every period, namely

$$V_j(\Gamma_i) = \{v_R^*\} = \left\{\frac{\delta}{1+\delta}\right\}$$

for all $j \in \{A, B\}$ and $i \neq j$.

Because $V_j(\Gamma_j)$ and $V_j(\Gamma_i)$ for $j \in \{A, B\}$ and $i \neq j$ do not depend on the identity of the players and are singletons, we conclude that any SPE outcome can be sustained in stationary strategies. Moreover, since $v_P^* + v_R^* = 1$, we conclude that any SPE must satisfy the following three features: *(a)* the proposer offers to extract all the resources from Charlie, *(b)* Charlie never protests on path, and *(c)* the responder immediately accepts. Observe that there exists only one feasible offer that is consistent with these features, and with equilibrium payoffs $v_P^* = \frac{1}{1+\delta}$

[41] Since $f \geq p$, Charlie may protest on path only if $x_C = -1$ and Ann is going to accept Bob's offer immediately. Thus, in this case, Bob's equilibrium payoff is $(1-p)x_B = (1-p)(1-x_A)$. However, by offering $x' \in X$ such that $x'_A = x_A + \varepsilon$ and $x'_C = -1 + \varepsilon$, for some $\varepsilon > 0$ arbitrarily small, Bob would necessarily avoid inefficient protests while maintaining immediate acceptance. Thus, Bob's payoff by offering x' would be $x'_B = 1 - x_A - 2\varepsilon$ which is strictly greater than $(1-p)(1-x_A) = (1-p)x_B$ for $\varepsilon > 0$ sufficiently small. This contradicts the optimality of x.

and $v_R^* = \frac{\delta}{1+\delta}$: Namely, $x^* \in X$ such that

$$x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right).$$

We conclude that points *(i)*, *(ii)*, and *(iii)* of Proposition 1 hold, as required.

Necessity. The necessity part of Proposition 1 follows from Proposition 2.

This completes the proof of Proposition 1.

Q.E.D.

Proof of Proposition 2

Assume throughout that Assumption 1 holds, i.e., $f/p < 1$. Recall that $\lambda := f/p$ and that $\varphi^{\mathrm{S}}(z) := \frac{z(1-z)(1+\delta)}{1+\delta(1-z)}$ for all $z \in (0,1)$. This proof is divided into three parts. In Part 1, we show that a stationary equilibrium always exists. In Part 2, we characterize agents' equilibrium payoffs and establish their uniqueness for every parameter configuration (δ, p, f) such that $f \neq \varphi^{\mathrm{S}}(p)$. Finally, in Part 3, we use the results derived in Part 2 to characterize the stationary equilibrium outcomes whenever $f \neq \varphi^{\mathrm{S}}(p)$.

Proof of Proposition 2, Part 1: Existence

The following lemma establishes the existence of stationary equilibria under Assumption 1.

Lemma 2. *Suppose Assumption 1 holds. A stationary equilibrium always exists.*

Proof. As argued in Section 2, it is sufficient to specify a profile of three behavior rules $(\beta_P, \beta_R, \beta_C)$ to characterize a stationary equilibrium, where $\beta_P \in \Delta(X)$ is the behavior strategy played by the proposer, $\beta_R \in \Delta(\{yes, no\})^X$ is the behavior strategy of the responder, and $\beta_C \in \Delta(\{protest, abstain\})^X$ is the behavior strategy of Charlie.

To prove Lemma 2, we distinguish between two cases.

Case 1: Assume $\delta^2 + \delta p \leq 1$. We consider three different parameter configurations:

(i) Suppose $0 < f < p(1-p)\frac{\delta}{1-\delta p}$. The following behavior rules constitute a stationary equilibrium:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta(1-p)}, \frac{\delta(1-p)}{1+\delta(1-p)}, -1\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{abstain} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

for all $x \in X$.

(ii) Suppose $p(1-p)\frac{\delta}{1-\delta p} \leq f \leq \varphi^{\mathrm{S}}(p)$. The following behavior rules constitute a stationary equilibrium:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta(1-p)}, \frac{\delta(1-p)}{1+\delta(1-p)}, -1\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \end{cases}$$

for all $x \in X$.

(iii) Suppose $\varphi^{\mathrm{S}}(p) \leq f < p$. The following behavior rules constitute a stationary equilibrium:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta\lambda}{1+\delta} \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_R < \frac{\delta\lambda}{1+\delta} \end{cases}$$

for all $x \in X$.

Case 2: Assume $\delta^2 + \delta p > 1$. We consider two different parameter configurations:

(i) Suppose $0 < f \leq \varphi^{\mathrm{S}}(p)$. The following behavior rules constitute a stationary equilibrium:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{1}{1+\delta(1-p)}, \frac{\delta(1-p)}{1+\delta(1-p)}, -1\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \end{cases}$$

  for all $x \in X$;
- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R < \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{abstain} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

  for all $x \in X$.

(*ii*) Suppose $\varphi^{\mathrm{S}}(p) \leq f < p$. The following behavior rules constitute a stationary equilibrium:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta\lambda}{1+\delta} \end{cases}$$

  for all $x \in X$;
- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \geq \frac{\delta\lambda}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_R < \frac{\delta\lambda}{1+\delta} \end{cases}$$

  for all $x \in X$.

Since we considered all possible parameter configurations (δ, p, f) such that $f < p$, the above analysis proves that, under Assumption 1, a stationary equilibrium always exists, as required.

□

This concludes Part 1 of the proof of Proposition 2.

Proof of Proposition 2, Part 2: Stationary equilibrium payoffs

Assume throughout that Assumption 1 holds, i.e., $f < p$. In this part, we characterize agents' stationary equilibrium payoffs and establish their uniqueness for every parameter configuration (δ, p, f) such that $f \neq \varphi^{\mathrm{S}}(p)$. We proceed in four steps.

Step 1.

We establish a *lower bound* on the proposer's payoff in any stationary equilibrium.

Lemma 3. *Suppose Assumption 1 holds. Fix any stationary equilibrium $\sigma \in \mathcal{E}^{\mathrm{S}}$, and suppose that (v_P, v_R, v_C) is the profile of equilibrium payoffs associated with σ. It holds that:*

$$v_P \geq \max\Big\{\frac{\lambda}{1+\delta}, \frac{1-p}{1+\delta(1-p)}\Big\}. \tag{11}$$

Proof. Let $(\beta_P, \beta_R, \beta_C)$ be agents' behavior strategies associated with $\sigma \in \mathcal{E}^{\mathrm{S}}$. Suppose we are at the decision stage of an arbitrary period where the responder needs to decide whether to accept the current-period proposal $x \in X$. By subgame perfection, $\beta_R(\cdot|x) \in \Delta(\{yes, no\})$ must prescribe an optimal behavior. Since the value to the responder of rejecting x is δv_P, standard arguments imply that

$$\beta_R(yes|x) = \beta_R(x) \begin{cases} = 1 & \text{if} \quad x_R > \delta v_P \\ = 0 & \text{if} \quad x_R < \delta v_P \\ \in [0,1] & \text{if} \quad x_R = \delta v_P. \end{cases} \tag{12}$$

Similarly, suppose that we are at the protesting stage of an arbitrary period, where Charlie needs to decide whether to protest against the current-period proposal $x \in X$. Like before, subgame perfection imposes that $\beta_C(\cdot|x) \in \Delta(\{protest, abstain\})$ prescribes an optimal behavior. The value to Charlie of abstaining from protesting is $\pi_C(x) := \beta_R(x)x_C + (1-\beta_R(x))\delta v_C$. On the other hand, the value to Charlie of protesting is $-f + (1-p)\pi_C(x)$. Thus, the following holds:

$$\beta_C(protest|x) = \beta_C(x) \begin{cases} = 1 & \text{if} \quad \pi_C(x) < -\lambda \\ = 0 & \text{if} \quad \pi_C(x) > -\lambda \\ \in [0,1] & \text{if} \quad \pi_C(x) = -\lambda. \end{cases} \tag{13}$$

For $\varepsilon > 0$ sufficiently small, the following two offers x^ε and x'^ε are feasible proposals:

- $x^\varepsilon \in X$ given by $x^\varepsilon_R = \delta v_P + \varepsilon$, $x^\varepsilon_C = -\lambda + \varepsilon$ and $x^\varepsilon_P = \lambda - \delta v_P - 2\varepsilon$;
- $x'^\varepsilon \in X$ given by $x'^\varepsilon_R = \delta v_P + \varepsilon$, $x'^\varepsilon_C = -1$ and $x'^\varepsilon_P = 1 - \delta v_P - \varepsilon$.

Recall that, by assumption, it holds that $\lambda < 1$. Thus, from (12) and (13), we know that the responder would immediately accept both x^ε and x'^ε, while Charlie would protest against x'^ε but not against x^ε. Hence, by proposing x^ε, the proposer would get a payoff of $\lambda - \delta v_P - 2\varepsilon$, while by proposing x'^ε, the proposer would get a payoff of $(1-p)(1-\delta v_P - \varepsilon)$. In equilibrium, the proposer makes a payoff-maximizing offer. This implies that

$$v_P \geq \frac{\lambda}{1+\delta} - \frac{2\varepsilon}{1+\delta} \quad \text{and} \quad v_P \geq \frac{(1-p)(1-\varepsilon)}{1+\delta(1-p)}.$$

Since this is true for every $\varepsilon > 0$ sufficiently small, we conclude that $v_P \geq \max\left\{\frac{\lambda}{1+\delta}, \frac{1-p}{1+\delta(1-p)}\right\}$, as required. This concludes the proof of Lemma 3.

□

Step 2.

We show that in any stationary equilibrium, the proposer always makes offers that leave the receiver perfectly indifferent between acceptance and rejection.

Lemma 4. *Fix any $\sigma \in \mathcal{E}^{\mathrm{S}}$, and suppose that (v_P, v_R, v_C) is the profile of equilibrium payoffs associated with σ. Let β_P be the behavior rule played by the proposer under σ. It holds that:*

$$\beta_P\left(\left\{x \in X : x_R = \delta v_P\right\}\right) = 1.$$

Proof. Suppose by contradiction that $\beta_P\left(\left\{x \in X : x_R = \delta v_P\right\}\right) < 1$. Then, there must exist $x \in \mathrm{supp}(\beta_P)$ such that $x_R \neq \delta v_P$. Suppose first that $x_R > \delta v_P$. Consider the alternative proposal x' such that $x'_C = x_C + \varepsilon$, $x'_R = x_R - 2\varepsilon$ and $x'_P = -(x'_R + x'_C)$, where $\varepsilon > 0$ is so small that $x'_R > \delta v_P$. It is easy to see that x' is feasible, i.e., $x' \in X$. According to (12) and (13), the responder's behavior does not change while Charlie protests less often if the proposer offers x' instead of x. Therefore, offering x' would give the proposer a strictly higher payoff, implying that x cannot be offered in equilibrium, a contradiction.

Assume now that $x_R < \delta v_P$. By (12), we know that the responder would reject x when offered. Since x is offered in equilibrium by assumption, it must be optimal, i.e., it must induce the proposer to earn v_P. However, this means that $v_P \leq \delta v_R$. Now observe that the first part of this proof proved that in any stationary equilibrium, the responder is never offered more than δv_P. Therefore, $v_R \leq \delta v_P$. By combining the last two weak inequalities, we conclude that $v_P, v_R \leq 0$, a clear non-sense.[42] Thus, $x_R < \delta v_P$ is impossible. This concludes the proof of Lemma 4.

□

Step 3.

We establish an *upper bound* on the proposer's payoff v_P in any stationary equilibrium.

Lemma 5. *Suppose Assumption 1 holds. Fix any $\sigma \in \mathcal{E}^{\mathrm{S}}$, and suppose that (v_P, v_R, v_C) is the profile of equilibrium payoffs associated with σ. It holds that:*

$$v_P \leq \max\left\{\frac{\lambda}{1+\delta}, \frac{1-p}{1+\delta(1-p)}\right\}. \tag{14}$$

Proof. Assume throughout that Assumption 1 holds, i.e., $f/p = \lambda < 1$. Let $(\beta_P, \beta_R, \beta_C)$ be the agents' strategies associated with $\sigma \in \mathcal{E}^{\mathrm{S}}$, and fix any equilibrium offer $x \in \mathrm{supp}(\beta_P)$. From lemma 4, it holds that $x_R = \delta v_P$. We now consider two cases.

Recall from the proof of Lemma 3 that $\pi_C(x) = \beta_R(x) x_C + (1 - \beta_R(x))\delta v_C$. There are three possibilities:

[42] To see this, note that by offering $\tilde{x} = (\tilde{x}_P, \tilde{x}_R, \tilde{x}_C) = (\varepsilon, \varepsilon, -2\varepsilon)$ for $\varepsilon > 0$ sufficiently small, the proposer would obtain a strictly positive payoff, implying that v_P cannot be weakly negative.

(i) $\pi_C(x) > -\lambda$,

(ii) $\pi_C(x) = -\lambda$,

(iii) $\pi_C(x) < -\lambda$.

Possibility *(i)*. Consider possibility *(i)*. We know from (13) that Charlie would not protest against x if offered. Since by assumption x is offered in equilibrium, we obtain that

$$v_P = \beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta v_R.$$

By the redistributive nature of our bargaining game, agents' aggregate equilibrium welfare cannot be strictly positive, i.e., $\sum_{i \in I} v_i \leq 0$. Therefore,

$$v_P \leq \beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta(-v_C - v_P),$$

which is equivalent to

$$v_P \leq \frac{-\pi_C(x)}{1+\delta}.$$

Since $\pi_C(x) > -\lambda$ by assumption, we conclude that $v_P < \frac{\lambda}{1+\delta}$, contradicting Lemma 3. Hence, we must have $\pi_C(x) \leq -\lambda$ in equilibrium.

Possibility *(ii)*. Consider now possibility *(ii)*, i.e., $\pi_C(x) = -\lambda$. Let $\pi_P(x) = \beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta v_R$. Since by assumption x is offered in equilibrium, we have that

$$v_P = \beta_C(x)(1-p)\pi_P(x) + (1 - \beta_C(x))\pi_P(x).$$

It follows that

$$\begin{aligned} v_P &\leq \pi_P(x) \\ &= \beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta v_R \\ &\leq \beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta(-v_C - v_P) \\ &= -\pi_C(x) - \delta v_P \\ &= -\lambda - \delta v_P, \end{aligned}$$

where the first inequality follows from $\pi_P(x) \geq 0$,[43] the second inequality from $\sum_{i \in I} v_i \leq 0$, and the last equality holds by assumption. We conclude that if x is offered with positive probability and $\pi_C(x) = -\lambda$, then it holds that $v_P \leq \frac{\lambda}{1+\delta}$.

Possibility *(iii)*. Finally, consider possibility *(iii)*, i.e., $\pi_C(x) < -\lambda$. We know from (13) that Charlie would protest against x if offered. Since by assumption x is offered in equilibrium, we obtain that

$$v_P = (1-p)\Big(\beta_R(x)(-x_C - \delta v_P) + (1 - \beta_R(x))\delta v_R\Big).$$

[43] If $\pi_P(x) < 0$, then also $v_P < 0$ which is a non-sense.

By the redistributive nature of our bargaining game, $\sum_{i\in I} v_i \leq 0$. Therefore,

$$v_P \leq (1-p)\Big(\beta_R(x)(-x_C - \delta v_P) + (1-\beta_R(x))\delta(-v_C - v_P)\Big),$$

which is equivalent to

$$v_P \leq -\pi_C(x) \cdot \frac{1-p}{1+\delta(1-p)} \leq \frac{1-p}{1+\delta(1-p)}.$$

Thus, if x is offered with positive probability and $\pi_C(x) < -\lambda$, then it holds that $v_P \leq \frac{1-p}{1+\delta(1-p)}$.

Since, in equilibrium, the proposer must have chosen x to maximize her expected payoff, we conclude from the analysis of possibilities *(ii)* and *(iii)* that

$$v_P \leq \max\left\{\frac{\lambda}{1+\delta}, \frac{1-p}{1+\delta(1-p)}\right\}$$

as required. This concludes the proof of Lemma 5.

□

Corollary 2 follows immediately from Lemmas 3 and 5.

Corollary 2. *Suppose Assumption 1 holds. In any stationary equilibrium, it holds that*

$$v_P = \max\left\{\frac{\lambda}{1+\delta}, \frac{1-p}{1+\delta(1-p)}\right\}.$$

Step 4.

We conclude Part 2 of the proof of Proposition 2 by characterizing the receiver and Charlie's stationary equilibrium payoffs for every parameter configuration (δ, p, f) such that $f < p$ and $f \neq \varphi^{\mathrm{S}}(p)$. To do so, first observe that $f > \varphi^{\mathrm{S}}(p)$ if and only if $\frac{\lambda}{1+\delta} > \frac{1-p}{1+\delta(1-p)}$. We distinguish between two cases.

Case 1. Suppose $\varphi^{\mathrm{S}}(p) < f < p$. By Corollary 2, it must be that $v_P = \frac{\lambda}{1+\delta}$. Thus, all the weak inequalities we derived possibility *(ii)* of the proof of Lemma 5 to establish an upper bound on v_P must hold with equality. In particular, in equilibrium, we must have that $\beta_C(x) = 0$, i.e., Charlie does not protest on the equilibrium path. This implies that the equilibrium outcome is efficient, i.e., $\sum_{i\in I} v_i = 0$. Moreover, since the receiver only receives offers that make her indifferent between accepting and rejecting (Lemma 4), it also implies that $v_R = \delta v_P$. We conclude that $v_C = -\lambda$ and $v_R = \delta v_P = \frac{\delta\lambda}{1+\delta}$.

Case 2. Suppose $f < \varphi^{\mathrm{S}}(p)$. By Corollary 2, it must be that $v_P = \frac{1-p}{1+\delta(1-p)}$. Thus, all the weak inequalities we derived in possibility *(iii)* of the proof of Lemma 5 to establish an upper bound on v_P must hold with equality. In particular, in equilibrium, we must have that $\pi_C(x) = -1$. This is only possible if $x_C = -1$ and $\beta_R(x) = 1$, i.e., the receiver accepts immediately any equilibrium offer. Since Charlie protests on path, we conclude that $v_C = -f - (1-p)$ and $v_R = (1-p)\delta v_P = \frac{\delta(1-p)^2}{1+\delta(1-p)}$. This concludes the proof of Part 2 of Proposition 2.

Proof of Proposition 2, Part 3: Stationary equilibrium outcomes

We conclude the proof of Proposition 2 by proving the characterization of stationary equilibrium outcomes under Assumption 1 given in Proposition 2. As usual, we distinguish between two cases.

Case 1. Suppose $\varphi^{\mathrm{S}}(p) < f < p$. By Corollary 2, it must be that $v_P = \frac{\lambda}{1+\delta}$. Thus, all the weak inequalities we derived in possibility *(ii)* of the proof of Lemma 5 to establish an upper bound on v_P must hold with equality. In particular, in equilibrium, we must have that the proposer makes offers x such that $x_R = \delta v_P$, and $\beta_C(x) = 0$. Note that x_C and $\beta_R(x)$ are left undetermined. In particular, the proposer can proposer to extract strictly more than λ resources from Charlie in equilibrium (i.e., $x_C < -\lambda$) as long as $\beta_R(x) < 1$. In particular, $\beta_R(x)$ must leave Charlie exactly indifferent between protesting and abstaining from protesting. In turn, this implies that $\beta_R(x) = q_\alpha$ where $\alpha = -x_C$ as required by Proposition 2. The remaining points *a.(i)*, *a.(ii)* and *a.(iii)* of Proposition 2 now follow immediately from this discussion.

Case 2. Suppose $f < \varphi^{\mathrm{S}}(p)$. By Corollary 2, it must be that $v_P = \frac{1-p}{1+\delta(1-p)}$. Thus, all the weak inequalities we derived in possibility *(iii)* of the proof of Lemma 5 to establish an upper bound on v_P must hold with equality. In particular, in equilibrium, we must have that the proposer offers x such that $x_R = \delta v_P$, $x_C = -1$, $\beta_C(x) = 1$, and $\beta_R(x) = 1$. Thus, the stationary equilibrium outcome is unique and points *b.(i)*, *b.(ii)* and *b.(iii)* of Proposition 2 hold.

This concludes the proof of Proposition 2.

Q.E.D.

Proof of Corollary 1

If $\varphi^{\mathrm{S}}(p) < f < p$, i.e., if we are in the Accommodation equilibrium region, any stationary equilibrium outcome satisfies Pareto efficiency but Charlie's payoff is $v_C = -\lambda$, which is strictly greater than $\bar{u}_C = -(1-p+f)$. Conversely, if $f < \varphi^{\mathrm{S}}(p)$, the unique stationary equilibrium outcome is the Conflict equilibrium. Here, even though Charlie gets his min-max payoff, the equilibrium outcome is not Pareto efficient: Since Charlie protests on path, some surplus gets wasted due to the protesting cost. Therefore, $v_A + v_B < -v_C = -\bar{u}_C$.

Q.E.D.

Proof of Theorem 1

Assume throughout that Assumption 1 holds, i.e., $f < p$. Fix any SPE σ where Ann and Bob extract the full surplus, and denote with (v_A, v_B, v_C) the profile of equilibrium payoffs it induces. Let $\sigma_A(\cdot|\varnothing) \in \Delta(X)$ be Ann's possibly random equilibrium offer at the beginning of the game, and $\sigma_B(yes|x, a_C^0) \in [0,1]$ be Bob's equilibrium acceptance probability of any initial proposal x if Charlie's period-0 protesting decision is $a_C^0 \in \{protest, abstain\}$. Also, denote with $v_C(x, a_C^0, no) \in \mathbb{R}$ the normalized continuation equilibrium payoff Charlie would obtain after the period-0 history (x, a_C^0, no).[44] In what follows, we show that σ displays strategic delay in period $t = 0$ with certainty. In particular, we show that

$$\sigma_A(E|\varnothing) = 1, \tag{15}$$

where $E \subseteq X$ is given by

$$E := \Big\{x \in X : \sigma_B(yes|x, abstain) < \sigma_B(yes|x, protest)\Big\}.$$

Let $S^* = 1 - p + f = -\bar{u}_C$ be the negative of Charlie's min-max payoff. To prove equation (15), we make use of the following preliminary lemma.

Lemma 6. *The following statements hold:*

(i) Charlie never protests along the equilibrium path induced by σ, and $v_C = -S^*$.

(ii) We have that $\sigma_A(F|\varnothing) = 1$, where

$$F := \Big\{x : x_C \cdot \sigma_B(yes|x, abstain) + (1 - \sigma(yes|x, abstain)) \cdot \delta v_C(x, abstain, no) = -S^*\Big\}.$$

(iii) We have that $\sigma_A(\{x : x_C = -1\}|\varnothing) = 1$.

Proof of Lemma 6. Statement *(i)* holds by definition of full surplus extraction: If Charlie protests along the equilibrium path, the outcome would not be Pareto efficient. Similarly, v_C must equal Charlie's single-period min-max payoff $\bar{u}_C = -S^*$ in any full surplus extraction SPE.

We now prove statement *(ii)*. We begin by showing a weaker result. Namely, that

$$\sigma_A(F'|\varnothing) = 1, \tag{16}$$

where

$$F' := \Big\{x : x_C \cdot \sigma_B(yes|x, abstain) + (1 - \sigma(yes|x, abstain)) \cdot \delta v_C(x, abstain, no) \geq -S^*\Big\}.$$

Suppose by contradiction (16) does not hold, i.e., $\sigma_A(F'|\varnothing) < 1$. This means that $\sigma_A(\cdot|\varnothing)$ assigns strictly positive probability to allocations $x \in X$ such that

$$x_C \cdot \sigma_B(yes|x, abstain) + (1 - \sigma(yes|x, abstain)) \cdot \delta v_C(x, abstain, no) < -S^*.$$

Fix any such allocation $x \in X \setminus F'$. We show that Charlie strictly prefers to protest against x in period $t = 0$. To see this, let w_C be the payoff Charlie would get if he protests against x in

[44] We say that agents' payoffs in an arbitrary period are *normalized* if each agent receives a payoff of x_i when $x \in X$ is implemented in that period without protest. See also Part 2 of the proof of Proposition 1.

period $t = 0$, and follows the prescriptions of σ_C afterwards. This payoff is given by

$$w_C = -f + (1-p) \cdot \Big(x_C \sigma_B(yes|x, protest) + (1 - \sigma(yes|x, protest)) \cdot \delta v_C(x, protest, no) \Big).$$

Note that by definition of min-max, it holds that $v_C(x, protest, no) \geq -S^*$. Therefore,

$$w_C \geq -f + (1-p) \cdot \Big(x_C \sigma_B(yes|x, protest) + (1 - \sigma(yes|x, protest)) \cdot \delta(-S^*) \Big).$$

The RHS of the above inequality is linear in $\sigma_B(yes|x, protest) \in [0, 1]$. Thus, it can be minimized only by setting $\sigma_B(yes|x, protest) = 1$ or $\sigma_B(yes|x, protest) = 0$. However, the RHS would be weakly larger than $-S^*$ in both cases. Therefore $w_C \geq -S^*$, which implies that Charlie would find it strictly profitable to protest against x. Since this is true for all $x \in X \setminus F'$ and $\sigma_A(F'|\varnothing) < 1$ by assumption, we conclude that σ would induce Charlie to protest with strictly positive probability on path. However, this contradicts statement *(i)* and is therefore impossible.

To complete the proof of statement *(ii)*, note that

$$\begin{aligned} -S^* &\leq \mathbb{E}_{\sigma_A(\cdot|\varnothing)} \Big[x_C \cdot \sigma_B(yes|x, abstain) + (1 - \sigma(yes|x, abstain)) \cdot \delta v_C(x, abstain, no) \Big] \\ &= v_C \\ &= -S^* \end{aligned}$$

where the first inequality follows from $\sigma_A(F'|\varnothing) = 1$, and the last two equalities follow from statement *(i)*. Thus, $\sigma_A(F|\varnothing) = 1$, as required.

Finally, to prove statement *(iii)*, suppose by contradiction that $\sigma_A(\{x : x_C = -1\}) < 1$. Let $x \in \operatorname{supp} \sigma_A(\cdot|\varnothing)$ be such that $x_C \neq -1$. Since x must be feasible, this means that $x_C > -1$. From statements *(i)* and *(ii)*, we know that ($\sigma_A(\cdot|\varnothing)$-almost surely) the allocation x does not induce Charlie to protest on path and yields him a payoff of $v_C = -S^*$. Let w_C be the payoff Charlie would get if he protests against x in period $t = 0$, and follows the prescriptions of σ_C afterwards. Because $x_C > -1$ and $f < p$, it holds that

$$w_C = -f + (1-p) \cdot \Big(x_C \sigma_B(yes|x, protest) + (1 - \sigma(yes|x, protest)) \cdot \delta v_C(x, protest, no) \Big) > -S^* = v_C.$$

However this is absurd since σ is an SPE and, therefore, we must have $v_C \geq w_C$.

This completes the proof of Lemma 6.

$\square$

We are now ready to conclude the proof of Theorem 1. To do so, suppose that (15) does not hold, i.e., $\sigma_A(X \setminus E|\varnothing) > 0$. Fix any equilibrium proposal $x^* \in \operatorname{supp} \sigma_A(\cdot|\varnothing)$ such that $x \in X \setminus E$. This means that $q_B := \sigma_B(yes|x^*, abstain) \geq \sigma_B(yes|x^*, protest) =: r_B$, where $q_B, r_B \in [0, 1]$. From Lemma 6, we know that it is ($\sigma_A(\cdot|\varnothing)$-almost surely) the case that $x^*_C = -1$ and that

$$v_C = -1 \cdot q_B + (1 - q_B) \cdot \delta v_C(x^*, abstain, no) = -S^*. \tag{17}$$

If Charlie decides to protest against x^* in period $t = 0$ and follows σ_C's prescriptions afterwards, he obtains a payoff of

$$w_C = -f + (1-p) \cdot \Big((-1) \cdot r_B + (1-r_B) \cdot \delta \cdot v_C(x^*, protest, no) \Big) \geq -S^*.$$

If the above inequality were strict, Charlie would find it strictly profitable to protest against x^*, a contradiction. Therefore, the above inequality must hold with equality, which implies that $r_B = 1$. In turn, this implies that $q_B = 1$, because $q_B \geq r_B$ by assumption. But then, Charlie's equilibrium payoff *after* $x^* \in X$ is proposed is equal to -1, violating equation (17). This concludes the proof of Theorem 1.

Q.E.D.

Proof of Theorem 2

Suppose throughout that $f < p$. Our proof is divided into three parts: In Part 1, we show sufficiency, i.e., that if $\varphi^*(p) \leq f$, then a symmetric time-invariant equilibrium inducing the outcome described in Theorem 2 exists, and that Ann and Bob extract the full surplus in this equilibrium. In Part 2, we show that such an equilibrium outcome is unique, i.e., we prove that no other symmetric time-invariant equilibrium outcome can achieve full surplus extraction. Finally, in Part 3, we use the results established previously to prove the necessity of $\varphi^*(p) \leq f$.

Proof of Theorem 2, Part 1: Sufficiency

To prove sufficiency, suppose that $\varphi^*(p) \leq f$. We first describe an equilibrium candidate, i.e., a profile of symmetric time-invariant strategies. We then verify that such equilibrium candidate satisfies the desidered properties and is indeed an SPE.

Equilibrium candidate

We begin by describing the behavior rule of the proposer. Let $x^* = (x^*_P, x^*_R, x^*_C) \in X$ be given by

$$x^* = \left(1 - \frac{\delta}{1+\delta}(1+p-f), \frac{\delta}{1+\delta}(1-p+f), -1 \right).$$

We set β_P to be the Dirac measure on x^*, i.e., $\beta_P = \delta_{x^*}$. The proposer always proposes x^* with certainty in the candidate equilibrium.

We now describe the behavior strategy of the responder. Let q^* be given equation (7). Observe that $q^* \in (0,1)$ since $f < p$ by assumption. Suppose the proposer offered $x \in X$, and that Charlie did not protest against it in the protesting stage, i.e., $a_C = abstain$. In this case, we set

$$\beta_R(\cdot | x, abstain) = \begin{cases} \delta_{yes} & \text{if} \quad x_R > \frac{\delta(1-p+f)}{1+\delta} \\ \delta_{yes} \cdot q^* + (1-q^*) \cdot \delta_{no} & \text{if} \quad x_R = \frac{\delta(1-p+f)}{1+\delta} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta(1-p+f)}{1+\delta}. \end{cases}$$

Conversely, suppose that the proposer offered $x \in X$ and that Charlie did protest against it, i.e., $a_C = protest$. In this case, we set

$$\beta_R(\cdot|x, protest) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta(1-p+f)}{1+\delta} \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta(1-p+f)}{1+\delta}. \end{cases}$$

To describe Charlie's behavior rule, we distinguish between two cases:

- Case 1: Suppose $f < p(1-p)\frac{\delta}{1-\delta p}$. Then, we set

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R < \frac{\delta(1-p+f)}{1+\delta} \\ & \text{or} \quad x_R > \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R = \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C > -1 \\ \delta_{abstain} & \text{if} \quad x_R = \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C = -1 \\ & \text{or} \quad x_R > \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

  for all $x \in X$.

- Case 2: Suppose $f \geq p(1-p)\frac{\delta}{1-\delta p}$. Then, we set

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R > \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \leq \frac{\delta(1-p+f)}{1+\delta} \\ & \text{or} \quad x_R > \frac{\delta(1-p+f)}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

  for all $x \in X$.

As we prove below, we distinguish between Case 1 and Case 2 to make sure Charlie has no incentive to deviate in any subgame. However, these cases play no role in determining the equilibrium outcome of the game.

Equilibrium verification

We now verify that the equilibrium candidate specified above is indeed an SPE. This part of the proof proceeds in three steps. First, we characterize the outcome induced by the behavior rules of the candidate equilibrium described above. We then characterize the implied equilibrium payoffs. Finally, we analyze players' incentive compatibility (IC) constraints.

Outcome. Since x^* is such that $x^*_R = \frac{\delta(1-p+f)}{1+\delta}$, the outcome of the game is stochastic. In particular, it is given by the infinite repetition of the following sequence of events: In the proposing stage, the proposer proposes x^* with certainty. In the protesting stage, Charlie does not protest. In the decision stage, the responder accepts the proposer's offer with probability $q^* \in (0, 1)$. Note that this is precisely the equilibrium outcome described in the statement of Theorem 2.

Equilibrium payoffs. We begin by characterizing Charlie's equilibrium payoff (v_C). Because of time invariance, symmetry, and the fact that, under the equilibrium candidate, the play always reaches the next period with probability $1 - q^*$, we conclude that v_C satisfies

$$v_C = (-1)q^* + (1 - q^*)\delta v_C.$$

This implies that

$$v_C = \frac{-q^*}{1 - (1 - q^*)\delta} = -(1 - p + f) = \bar{u}_C. \tag{18}$$

Hence, Charlie receives his min-max payoff under our equilibrium candidate. We now turn to the payoffs obtained by the proposer (v_P) and the responder (v_R). First, observe that because Charlie never protests on-path, the outcome induced by our equilibrium candidate is Pareto efficient. Therefore, $v_P + v_R = -v_C$. Furthermore, since the responder is indifferent between accepting and rejecting, it holds that $v_R = \delta v_P$. Together, these conditions imply that

$$v_P = \frac{v_C}{1 + \delta} \quad \text{and} \quad v_R = \frac{\delta v_C}{1 + \delta}. \tag{19}$$

We now use equations (18) and (19) to prove that no agent has a strict incentive to deviate.

IC Responder: Combining equations (18) and (19), we conclude that δv_P – i.e., the value to the responder of declining the proposer's offer, thus moving the play to the next period – is $\frac{\delta(1-p+f)}{1+\delta}$. Now suppose the proposer offered x. Irrespective of what Charlie did, the responder strictly prefers to accept x if $x_R > \frac{\delta(1-p+f)}{1+\delta}$, he strictly prefers to reject x if $x_R < \frac{\delta(1-p+f)}{1+\delta}$, and he is indifferent between accepting and rejecting x if $x_R = \frac{\delta(1-p+f)}{1+\delta}$. Observe that β_R does not prescribe to take sub-optimal actions in any subgame. Therefore, the responder has no strict incentive to deviate from β_R.

IC Charlie: Suppose the proposer offered x. First, assume that x is such that $x_R > \frac{\delta(1-p+f)}{1+\delta}$. Then, Charlie knows that the responder will accept x immediately, irrespective of his decision to protest. As a result, Charlie finds it strictly optimal to protest if $x_C < -\lambda$. He finds it strictly sub-optimal to protest if $x_C > -\lambda$. Finally, he is indifferent if $x_C = -\lambda$. Note that β_C prescribes an optimal behavior for Charlie when $x_R > \frac{\delta(1-p+f)}{1+\delta}$.

Assume now that $x_R = \frac{\delta(1-p+f)}{1+\delta}$. If he does not protest, Charlie knows that x will be implemented immediately with probability q^*, whereas the play will reach the next period where Charlie will get δv_C in expectation with complementary probability. Conversely, if Charlie protests, the responder will accept x immediately. Therefore, the utility Charlie derives from protesting as a function of x_C is

$$u_C^1(x_C) := -f + (1 - p)x_C,$$

while the utility Charlie derives from not protesting is

$$u_C^0(x_C) := x_C q^* + (1 - q^*)\delta v_C.$$

Let

$$g(x_C) := u_C^0(x_C) - u_C^1(x_C).$$

Notice that $g(-1) = 0$, i.e., Charlie is indifferent between protesting or not when $x_C = -1$. Moreover, observe that $g(x_C)$ is linear in x_C.[45] Thus, whether Charlie has an incentive to protest when $x_C > -1$ depends on the sign of $g'(x_C) = q^* - (1-p)$: If $q^* \geq 1-p$, Charlie finds it weakly sub-optimal to protest when $x_C > -1$. Conversely, if $q^* < 1-p$, Charlie finds it strictly optimal to protest. Observe that $q^* \geq 1-p$ (resp., $q^* < 1-p$) if and only if Case 2 (resp., Case 1) holds. Thus, β_C prescribes an optimal behavior for Charlie when $x_R = \frac{\delta(1-p+f)}{1+\delta}$.

Finally, assume that $x_R < \frac{\delta(1-p+f)}{1+\delta}$. Charlie knows that the responder will reject x with certainty, irrespective of his decision to protest. Therefore, if he does not protest, the play will move to the next period where Charlie's expected payoff will be δv_C. Conversely, if he protests, Charlie gets an expected payoff of $-f + (1-p)\delta v_C$. Thus, protesting is weakly sub-optimal if $f \geq -p\delta v_C$. Given equation (18), this is equivalent to $f \geq p(1-p)\frac{\delta}{1-\delta p}$, i.e., Case 2. This proves that β_C prescribes an optimal behavior for Charlie also when $x_R < \frac{\delta(1-p+f)}{1+\delta}$. Therefore, Charlie has no strict incentive to deviate from β_C.

IC Proposer: As we argued above, by offering x^*, the proposer gets the payoff of $v_P = \frac{1-p+f}{1+\delta}$. In what follows, we show that the proposer has no strict incentive to deviate to any other proposal.

We begin by arguing that the following deviations are strictly sub-optimal:

(i) Proposing x such that $x_R < \frac{\delta(1-p+f)}{1+\delta}$.

(ii) Offering x such that $x_R = \frac{\delta(1-p+f)}{1+\delta}$ and $x_C > -1$.

In case (i), the responder will not accept x. Therefore, the proposer will get at most $\delta v_R < v_P$. In case (ii), the proposer is giving Charlie a chance to obtain strictly more than his min-max payoff in expectation. However, since the share x_R offered to the responder is x_R^*, this strategy must be strictly worse than offering x^* with certainty, i.e., playing β_P.

We are left to check that there are no profitable deviations among the proposals x such that $x_R > \frac{\delta(1-p+f)}{1+\delta}$. Given the behavior strategies played by the other agents, there are two best alternatives for the proposer in this case: The proposer is accommodating, i.e., $x_C = -\lambda$, or the proposer is hostile, i.e., $x_C = -1$. Therefore, we need to show that

$$v_P = \frac{1-p+f}{1+\delta} \geq \max\left\{\lambda - \frac{\delta(1-p+f)}{1+\delta}, (1-p)\left(1 - \frac{\delta(1-p+f)}{1+\delta}\right)\right\}. \tag{20}$$

One can verify that the above inequality holds if and only if $f \geq \varphi^*(p)$. Because $f \geq \varphi^*(p)$ holds by assumption, the proposer has no strict incentive to deviate. This concludes the proof of sufficiency.

[45] Indeed, $g(x_C)$ is given by $g(x_C) = \kappa + x_C[q^* - (1-p)]$, where $\kappa \in \mathbb{R}$ is a constant.

Proof of Theorem 2, Part 2: Uniqueness

The uniqueness of the symmetric time-invariant equilibrium stated in Theorem 2 follows from the proof of Theorem 1. In particular, statements *(i)*, *(ii)* and *(iii)* of Lemma 6 together imply that, in any symmetric time-invariant equilibrium where Ann and Bob extract the full surplus, the following holds:

(a) In every period, the proposer must make offers that prescribe to extract everything from Charlie, i.e., she must make offers $x \in X$ such that $x_C = -1$;

(b) If Charlie abstains from protesting, the responder must accept any of the proposer's equilibrium offers with probability $q^* \in (0, 1)$ given by equation (7).

Now, observe that conditions *(a)* and *(b)* jointly imply that the responder must always be indifferent between accepting and rejecting on path which, in turn, identifies a unique possible equilibrium offer $x^* \in X$ such that

$$x_C^* = -1, \quad x_P^* = 1 - x_R^*, \quad \text{and} \quad x_R^* = v_R = \delta v_P.$$

In particular, since $v_C = \bar{u}_C$ and $v_P + v_R = -v_C$ in any full surplus extraction equilibrium, we conclude that

$$x_R^* = \frac{\delta}{1+\delta}(1 - p + f).$$

Note that the proposal x^* described above coincides with the offer described in Theorem 2.

Finally, the off-path behavior of the responder following the hypothetical decision of Charlie to protest against x^* is also pinned down uniquely. In particular, in such a situation, the responder must accept x^* with probability 1 in case protests fail. Otherwise, Charlie would have a strict incentive to protest against x^*, contradicting Pareto efficiency, i.e., property (PE). This completes the proof of Theorem 2.

Proof of Theorem 2, part 3: Necessity

To see why $\varphi^*(p) \leq f$ is necessary for the existence of a symmmetric time-invariant equilibrium outcome where Ann and Bob extract the full surplus, notice that from the results established in Part 2, we know that in any such equilibrium, the following must hold:

$$v_C = \bar{u}_C, \quad v_R = \delta v_P, \quad \text{and} \quad v_P = \frac{1}{1+\delta}(1 - p + f).$$

However, if $f < \varphi^*(p)$, the proposer would be able to guarantee for herself a strictly higher payoff by inducing immediate Conflict, a contradiction. Concretely, by offering x such that $x_C = -1$ and $x_R = \delta v_P + \varepsilon$, the proposer would obtain a payoff of $u_P = (1 - p)(1 - \delta v_P - \varepsilon) > v_P$, as long as $\varepsilon > 0$ is small enough.

This concludes the proof of Theorem 2.

Q.E.D.

Proof of Proposition 3

Recall that $x_{AB} := x_A + x_B$, $\lambda := f/p$ and that $\varphi^{\mathrm{C}}(z) := z(1-z)$ for all $z \in (0,1)$. This proof is divided into three parts. In Part 1, we show that an SPE of the collusive bargaining model always exists for all possible parameter configurations $(\delta, p, f) \in \Theta := (0,1) \times (0,1) \times (0,\infty)$. In Part 2, we characterize the bargainers' joint equilibrium payoff v_{AB} and establish its uniqueness for every parameter configuration $(\delta, p, f) \in \Theta$. Finally, in Part 3, we use results derived in Part 2 to characterize Charlie's equilibrium payoff and the SPE outcomes whenever $f \neq \varphi^{\mathrm{C}}(p)$.

Proof of Proposition 3, Part 1: Existence

The following Lemma establishes the existence of SPEs for all possible parameter configurations $(\delta, p, f) \in \Theta := (0,1) \times (0,1) \times (0,\infty)$.

Lemma 7. *An SPE of the collusive bargaining model always exists: $\mathcal{E}^{\mathrm{C}} \neq \emptyset$ for all $(\delta, p, f) \in \Theta$.*

Proof of Lemma 10. It is sufficient to specify three behavior rules β_P, β_R and β_C to characterize an equilibrium, where $\beta_P \in \Delta(X)$ is the behavior strategy played by the proposer, $\beta_R \in \Delta(\{yes, no\})^X$ is the behavior strategy of the responder, and $\beta_C \in \Delta(\{protest, abstain\})^X$ is the behavior strategy of Charlie.

Case 1: Assume that $\delta + \delta p \leq 1$. We consider four different parameter configurations:

(i) Suppose $0 < f < p(1-p)\frac{\delta}{1-\delta p}$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (1, -1)$;
- β_R given by
$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta(1-p) \\ \delta_{no} & \text{if} \quad x_{AB} < \delta(1-p) \end{cases}$$
for all $x \in X$;
- β_C given by
$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_{AB} < \delta(1-p) \\ \delta_{abstain} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$
for all $x \in X$

(ii) Suppose $p(1-p)\frac{\delta}{1-\delta p} \leq f \leq \varphi^{\mathrm{C}}(p)$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (1, -1)$;
- β_R given by
$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta(1-p) \\ \delta_{no} & \text{if} \quad x_{AB} < \delta(1-p) \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_{AB} < \delta(1-p) \end{cases}$$

for all $x \in X$.

(iii) Suppose $\varphi^{\mathrm{C}}(p) \leq f < p$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (\lambda, -\lambda)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta\lambda \\ \delta_{no} & \text{if} \quad x_{AB} < \delta\lambda \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_{AB} \geq \delta\lambda \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_{AB} \geq \delta\lambda \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_{AB} < \delta\lambda \end{cases}$$

for all $x \in X$.

(iv) Suppose $p \leq f$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (1, -1)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta \\ \delta_{no} & \text{if} \quad x_{AB} < \delta \end{cases}$$

for all $x \in X$;

- β_C given by $\beta_C(\cdot|x) = \delta_{abstain}$, for all $x \in X$.

Case 2: Assume that $\delta + \delta p > 1$. We consider three different parameter configurations:

(i) Suppose $0 < f \leq \varphi^{\mathrm{C}}(p)$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta(1-p) \\ \delta_{no} & \text{if} \quad x_{AB} < \delta(1-p) \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_{AB} < \delta(1-p) \\ \delta_{abstain} & \text{if} \quad x_{AB} \geq \delta(1-p) \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

for all $x \in X$.

(ii) Suppose $\varphi^{\mathrm{C}}(p) \leq f < p$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (\lambda, -\lambda)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta\lambda \\ \delta_{no} & \text{if} \quad x_{AB} < \delta\lambda \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_{AB} \geq \delta\lambda \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_{AB} \geq \delta\lambda \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_{AB} < \delta\lambda \end{cases}$$

for all $x \in X$.

(iii) Suppose $p \leq f$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_{AB}, x^*_C) = (1, -1)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_{AB} \geq \delta \\ \delta_{no} & \text{if} \quad x_{AB} < \delta \end{cases}$$

for all $x \in X$;

- β_C given by $\beta_C(\cdot|x) = \delta_{abstain}$, for all $x \in X$.

Since we considered all possible parameter configurations $(\delta, p, f) \in \Theta$, this shows that the collusive bargaining model always admits an SPE, as required.

□

Proof of Proposition 3, Part 2: The bargainers' equilibrium payoff

In this part, we characterize the bargainers' joint SPE payoff and establish its uniqueness for almost every parameter configuration $(\delta, p, f) \in \Theta$. Let $\hat{\mathcal{V}}_{AB}$ denote the set of joint payoffs the bargainers can achieve in an SPE of the collusive bargaining model. By Part 1 of this proof, this set is nonempty. Let $\bar{v}_{AB} := \sup \hat{\mathcal{V}}_{AB}$ and $\underline{v}_{AB} := \inf \hat{\mathcal{V}}_{AB}$ denote the endpoints of $\hat{\mathcal{V}}_{AB}$. We now use these points to show that $\hat{\mathcal{V}}_{AB}$ is a singleton.

Lemma 8. *It holds that $\hat{\mathcal{V}}_{AB} = \{v^*_{AB}\}$, where*

$$v^*_{AB} := \begin{cases} \max\left\{\lambda, 1-p\right\} & \textit{if} \quad \lambda < 1, \\ 1 & \textit{if} \quad \lambda \geq 1. \end{cases}$$

Proof of Lemma 8. We proceed in two steps to show that $v^*_{AB} \leq \underline{v}_{AB} \leq \bar{v}_{AB} \leq v^*_{AB}$.

Step 1. Proof of $\bar{v}_{AB} \leq v^*_{AB}$: If $\bar{v}_{AB} \leq 0$, we are done. Otherwise, take any $v \in \hat{\mathcal{V}}_{AB}$ such that $v > \delta\bar{v}_{AB}$. Consider any equilibrium $\sigma \in \mathcal{E}^{\mathrm{C}}$ leading to a joint payoff of v to the bargainers. Let $x \in X$ be an offer made with positive probability by Ann to Bob in period 1. We now show that x must be such that $x_{AB} \geq v$. To see this, suppose by contradiction that $x_{AB} < v$. Since x is an equilibrium offer, it must be optimal. This means that v equals the joint equilibrium payoff that Ann and Bob earn *after* Ann offers x. Since this joint payoff cannot be larger than a convex combination between x_{AB} (in case Bob accepts), 0 (in case Charlie protests), and $\delta\bar{v}_{AB}$ (in case Bob rejects), we conclude that $v < v$, a contradiction. Thus, it holds that $x_{AB} \geq v > \delta\bar{v}_{AB}$. This implies that the initial responder immediately accepts. Therefore, Charlie knows that the responder accepts immediately, and protests if $x_{AB} > \lambda$. Thus, either $v \leq x_{AB} \leq \min\{\lambda, 1\}$, or $v \leq (1-p) \cdot x_{AB} \leq 1-p$. This proves Step 1.

Step 2. Proof of $\underline{v}_{AB} \geq v^*_{AB}$: Note that the proposer can guarantee the bargainers an equilibrium payoff of $1-p$ if $\lambda < 1$ and of 1 if $\lambda \geq 1$ by proposing $x_A + x_B = 1$, which is always accepted immediately.

Thus, it remains to show that $\underline{v}_{AB} \geq \lambda$ if $1 > \lambda > 1-p$. In that case, the proposer can offer $x_{AB} = \lambda - \varepsilon$, where $\varepsilon > 0$. As long as $\varepsilon < (1-\delta)\lambda \leq \lambda - \delta\bar{v}_{AB}$, the responder accepts immediately. Since $-(1-p)\cdot(\lambda-\varepsilon) - f < -(\lambda - \varepsilon)$, Charlie prefers to abstain from protesting, and this proposal results in a payoff of $\lambda - \varepsilon$ to the bargainers. Since ε can be chosen arbitrarily small, this implies that $\underline{v}_{AB} \geq \lambda$, completing the proof of Step 2. ☐

Proof of Proposition 3, Part 3: Charlie's equilibrium payoff and SPE outcomes

In this part of the proof, we characterize Charlie's equilibrium payoff v_C and the SPE outcomes when $f \neq \varphi^{\mathrm{C}}(p)$. We first show that irrespective of the previous history of the game, in any period, the responder always accepts the equilibrium offer of the proposer with probability 1. This is consistent with points $a.(iii)$, $b.(iii)$, and $c.(iii)$ of Proposition 3.

Lemma 9. *Let $\sigma \in \mathcal{E}^{\mathrm{C}}$ be an SPE of the collusive bargaining model, and fix any period t following any arbitrary non-terminal history. Consider the proposing stage of this period, and suppose the proposer offers x in equilibrium with positive probability. Then, the responder accepts x with probability 1.*

Proof. Since x is offered in equilibrium, by Lemma 8 we know that x must lead to a joint payoff of $v^*_{AB} > 0$ to the bargainers. Since the payoff to the bargainers is a convex combination of x_{AB}, δv^*_{AB}, and 0, it must be that $x_{AB} \geq v^*_{AB}$. This implies that $x_{AB} > \delta v^*_{AB}$. Therefore, the responder's best reply is to accept x with probability 1. □

By backward induction, Charlie protests against any equilibrium offer with $x_C < -\lambda$ and abstains from protesting against any equilibrium offer with $x_C > -\lambda$. We now describe the equilibrium outcome and Charlie's payoff by distinguishing between three cases.

Case 1. Suppose $\lambda \geq 1$. Then, $v^*_{AB} = 1$ (Lemma 8). The only way the bargainers can earn this join payoff in equilibrium is when the proposer offers $x^*_C = -1$, and Charlie does not protest on path, since any protest would lead to a payoff to the bargainers of less than 1. Therefore, $v_C = -1$, and points *a.(i)*, *a.(ii)* and *a.(iii)* of Proposition 3 hold.

Case 2. Suppose $\varphi^{\mathrm{C}}(p) < f < p$. Then $v^*_{AB} = \lambda$ and $\lambda > 1 - p$. Any equilibrium offer such that $x_C \neq -\lambda$ results in a bargainer payoff strictly less than λ. To see this, note that if $x_C < -\lambda$, Charlie protests and the responder immediately accepts (Lemma 9), resulting in a payoff of $v_{AB} = -x_C \cdot (1-p) \leq 1 - p < \lambda$. If, instead, $x_C > -\lambda$, then $v_{AB} \leq -x_C < \lambda$, since the responder immediately accepts while Charlie abstains from protesting. Thus, $x^*_C = -\lambda$, and Charlie abstains from protesting in equilibrium since, otherwise, the bargainers would receive a payoff strictly lower than v^*_{AB}. We conclude that $v_C = -\lambda$, and points *b.(i)*, *b.(ii)* and *b.(iii)* of Proposition 3 hold.

Case 3. Suppose $f < \varphi^{\mathrm{C}}(p)$. Then $v^*_{AB} = 1 - p$ and $1 - p > \lambda$. Any equilibrium offer x with $x_C > -1$ leads to a bargainer joint payoff strictly lower than $1 - p$. To see this, note that if $x_C \in (-1, -\lambda)$, Charlie protests, and the responder immediately accepts, resulting in a payoff of $v_{AB} = -x_C \cdot (1-p) < 1 - p$. If, instead, x is an equilibrium offer such that $x_C \geq -\lambda$, then $v_{AB} \leq -x_C \leq \lambda < 1 - p$, since the responder immediately accepts and Charlie abstains from protesting. Thus, $x^*_C = -1$, Charlie protests on path, and receives a payoff of $v_C = -f - (1-p)$. Moreover, points *c.(i)*, *c.(ii)* and *c.(iii)* of Proposition 3 hold.

This concludes the proof of Proposition 3.

Q.E.D.

Proof of Theorem 3

This Theorem follows from Theorem 2 and Proposition 3.

Q.E.D.

Supplementary Appendix

B Beyond Pure Redistribution

This Appendix formalizes the discussion of Section 6.1. Toward this goal, let

$$X_\omega := \left\{ x \in \mathbb{R}^I : \sum_{i \in I} x_i = \omega, \text{ and } x_C \geq -1 \right\}$$

be the set of feasible redistributions the bargainers can offer each other, where $\omega \in \mathbb{R}$ is a new model parameter indicating the social value (or cost) of an agreement. This Appendix is organized as follows. In Section B.1, we study the stationary equilibrium outcomes of this model extension. In Section B.2, we discuss whether the bargainers can still benefit from strategic delay. Throughout, we assume that $\lambda < 1$, i.e., Charlie's protesting threat is credible.

B.1 Stationary Equilibrium Outcomes

We study the stationary equilibrium outcomes of the model when $\omega \neq 0$. To enhance the clarity of our exposition, we analyze the cases $\omega < 0$ and $\omega > 0$ separately.

Case $\omega < 0$:

As discussed in Section 6.1, if $\omega < -1$, the unique possible equilibrium outcome is no agreement. Thus, from now on, let us assume that $\omega \in [-1, 0)$. For every $\alpha \in [\lambda, 1]$, let

$$w_\alpha := \frac{1}{1+\delta} \left(\lambda + \omega \cdot \frac{\lambda}{\alpha} \right).$$

The following proposition shows that, like in the baseline model, a continuum of accommodation equilibria exists where Charlie is just indifferent between protesting and not on path.

Proposition 4. *Let $\alpha \in [\lambda, 1]$, and suppose that*

$$w_\alpha \geq \frac{(1-p)(1+\omega)}{1+\delta(1-p)}. \tag{21}$$

Then, an accommodation equilibrium exists where agents' payoffs are given by

$$(v_P, v_R, v_C) = (w_\alpha, \delta w_\alpha, -\lambda).$$

In this equilibrium, the following sequence of events occurs:

(i) The proposer offers $x^* = (x^*_P, x^*_R, x^*_C) = (\alpha + \omega - \delta w_\alpha, \delta w_\alpha, -\alpha)$,

(ii) Charlie abstains from protesting,

(iii) The responder accepts x^* with probability $q_\alpha = \frac{\lambda - \delta\lambda}{\alpha - \delta\lambda} \in (0, 1]$.

Proof. Fix any $\alpha \in [\lambda, 1]$. Consider the following behavior rules:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = (\alpha + \omega - \delta w_\alpha, \delta w_\alpha, -\alpha)$;

- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R > \delta w_\alpha \\ q_\alpha \cdot \delta_{yes} + (1-q_\alpha)\cdot \delta_{no} & \text{if} \quad x_R = \delta w_\alpha \\ \delta_{no} & \text{if} \quad x_R < \delta w_\alpha \end{cases}$$

for all $x \in X_\omega$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R > \delta w_\alpha \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R = \delta w_\alpha \quad \text{and} \quad x_C < -\alpha \\ \delta_{abstain} & \text{if} \quad x_R > \delta w_\alpha \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_R = \delta w_\alpha \quad \text{and} \quad x_C \geq -\alpha \\ & \text{or} \quad x_R < \delta w_\alpha \end{cases}$$

for all $x \in X_\omega$.

Observe that $(\beta_P, \beta_R, \beta_C)$ constitutes a profile of symmetric Markovian strategies. In what follows, we verify that this strategy profile is indeed an SPE. This part of the proof proceeds in three steps. First, we characterize the outcome induced by these behavior rules. We then characterize the implied equilibrium payoffs. Finally, we analyze players' incentive compatibility (IC) constraints.

Outcome. Since x^* is such that $x^*_R = \delta w_\alpha$, the outcome of the game is stochastic. In particular, it is given by the infinite repetition of the following sequence of events: In the proposing stage, the proposer proposes x^* with certainty. In the protesting stage, Charlie does not protest. In the decision stage, the responder accepts the proposer's offer with probability $q_\alpha \in (0, 1]$.

Equilibrium payoffs. We begin by characterizing Charlie's equilibrium payoff (v_C). Because of stationarity, symmetry, and the fact that, under the equilibrium candidate, the play always reaches the next period with probability $1 - q_\alpha$, we conclude that v_C satisfies

$$v_C = (-\alpha) \cdot q_\alpha + (1 - q_\alpha) \cdot \delta v_C.$$

After substituting for q_α and re-arranging, one can verify that $v_C = -\lambda$. Hence, Charlie is indifferent between protesting and not on the candidate-equilibrium path. We now turn to the payoffs obtained by the proposer (v_P) and the responder (v_R). First, observe that since Charlie never protests on path, the outcome induced by our equilibrium candidate satisfies

$$\sum_{i\in I} v_i = \omega \cdot \mathbb{E}\left[\delta^{\tilde{\tau}}\right], \tag{22}$$

where $\tilde{\tau}$ represents the random period an agreement is reached along the candidate-equilibrium path.[46] In particular, since agreement is reached in every period with probability q_α, it holds

[46] Observe that since $v_C = -\lambda$, equation (22) is equivalent to equation (10) which we discussed in Section 6.1.

that

$$\mathbb{E}\left[\delta^{\tilde{\tau}}\right] = q_\alpha \sum_{t=0}^{\infty}[(1-q_\alpha)\delta]^t = \frac{q_\alpha}{1-(1-q_\alpha)\delta} = \lambda/\alpha,$$

and therefore

$$v_P + v_R = \lambda + \omega \cdot \frac{\lambda}{\alpha} = (1+\delta)\cdot w_\alpha.$$

Second, notice that since the responder is indifferent between accepting and rejecting on path, it holds that $v_R = \delta w_\alpha$. Together, these conditions imply that

$$v_P = w_\alpha, \quad v_R = \delta w_\alpha, \quad \text{and} \quad v_C = -\lambda. \tag{23}$$

We now use the equations in (23) to prove that no agent has a strict incentive to deviate.

IC Responder: From (23), we know that δv_P – i.e., the value to the responder of declining the proposer's offer, thus moving the play to the next period – is δw_α. Now suppose the proposer offered $x \in X_\omega$ to the responder. Irrespective of what Charlie did in the protesting stage, the responder strictly prefers to accept x if $x_R > \delta w_\alpha$, he strictly prefers to reject x if $x_R < \delta w_\alpha$, and he is indifferent between accepting and rejecting x if $x_R = \delta w_\alpha$. Observe that β_R does not prescribe to take sub-optimal actions in any subgame. Therefore, the responder has no strict incentive to deviate from β_R.

IC Charlie: Suppose the proposer offered $x \in X_\omega$. First, assume that x is such that $x_R > \delta w_\alpha$. Then, Charlie knows that the responder will accept x immediately, irrespective of his decision to protest. As a result, Charlie finds it strictly optimal to protest if $x_C < -\lambda$. He finds it strictly optimal to abstain if $x_C > -\lambda$. Finally, he is indifferent if $x_C = -\lambda$. Note that β_C prescribes an optimal behavior for Charlie when $x_R > \delta w_\alpha$.

Now, let us assume that x is such that $x_R = \delta w_\alpha$. Then, Charlie knows that the responder will accept x with probability $q_\alpha \in (0,1]$, irrespective of his decision to protest. As a result, Charlie finds it strictly optimal to protest if $x_C < -\alpha$. He finds it strictly optimal to abstain if $x_C > -\alpha$. Finally, he is indifferent if $x_C = -\alpha$. Note that β_C prescribes an optimal behavior for Charlie when $x_R = \delta w_\alpha$.

Finally, assume that $x_R < \delta w_\alpha$. Charlie knows that the responder will reject x with certainty, irrespective of his decision to protest. Therefore, if he does not protest, the play will move to the next period where Charlie's expected payoff will be δv_C. Conversely, if he protests, Charlie gets an expected payoff of $-f + (1-p)\delta v_C$. Thus, protesting is strictly sub-optimal if $f > -p\delta v_C$. Since $v_C = -\lambda$, we conclude that Charlie does not want to protest. Therefore, β_C prescribes an optimal behavior for Charlie also when $x_R < \delta w_\alpha$.

IC Proposer: As we argued above, by offering x^*, the proposer gets the payoff of $v_P = w_\alpha$. In what follows, we show that the proposer has no strict incentive to deviate to any other proposal.

We begin by arguing that the following deviations are strictly sub-optimal:

(i) Proposing x such that $x_R < \delta w_\alpha$.

(ii) Offering x such that $x_R = \delta w_\alpha$ and $x_C > -\alpha$.

In case (i), the responder will not accept x. Therefore, the proposer will get $\delta v_R < v_P$. In case (ii), the proposer gives a chance to Charlie to have less than α resources seized from him. However, since the share x_R offered to the responder is δw_α, this strategy must be strictly worse than offering x^* with certainty, i.e., playing β_P.

We are left to check that there are no profitable deviations among the proposals x that trigger Charlie's protests. It is easy to verify that $x \in X_\omega$ such that $x_C = -1$, $x_R = \delta w_\alpha + \varepsilon$ and $x_P = \omega - x_R - x_C$, where $\varepsilon > 0$ is arbitrarily small, approximate the best proposal among such deviations. Given the behavior strategies played by the other agents, this alternative proposal would guarantee a payoff of $(1-p)(1+\omega-\delta w_\alpha - \varepsilon)$ to the proposer. Therefore, the proposer does not want to deviate from x^* if and only if $w_\alpha \geq (1-p)(1+\omega-\delta w_\alpha-\varepsilon)$. Since this must be true for all $\varepsilon > 0$, we conclude that

$$w_\alpha \geq (1-p)(1+\omega-\delta w_\alpha). \tag{24}$$

Observe that (24) is equivalent to (21). This proves that also the proposer has no strict incentive to deviate, concluding the proof of the proposition.

□

Since w_α is strictly increasing in $\alpha \in [\lambda, 1]$, Proposition 4 implies that when $\omega < 0$, distinct accommodation equilibria yield different equilibrium payoffs to the proposer and, consequently, to the responder. In particular, these equilibrium payoffs strictly decrease with the equilibrium agreement probability q_α. It is easy to see that, unlike the case of $\omega = 0$, this fact implies that no mixture over accommodation equilibrium outcomes can constitute an equilibrium anymore. We now show that this also implies that the existence of parameter regions where the conflict equilibrium outcome and *some* accommodation equilibrium outcomes co-exist.

Proposition 5. *Suppose that*

$$w_\lambda < \frac{(1-p)(1+\omega)}{1+\delta(1-p)} < w_1.$$

Then, both an accommodation equilibrium and a conflict equilibrium exist.

Proof. Since $w_1 > \frac{(1-p)(1+\omega)}{1+\delta(1-p)}$, we know that the accommodation equilibrium associated with $\alpha = 1$ exists (Proposition 4). We now show that a conflict equilibrium also exists. To this goal, let $\bar{w} := \frac{(1-p)(1+\omega)}{1+\delta(1-p)}$, and consider the following two cases.[47]

Case 1: Assume $\frac{\delta p(1-p)}{1-\delta p} \leq p\cdot(\bar{w}(1+\delta)-\omega)$. We consider two different parameter configurations:

(i) Suppose $0 < f < p(1-p)\frac{\delta}{1-\delta p}$. Since $w_\lambda < \bar{w}$, one can verify that the following behavior rules constitute a stationary equilibrium with conflict. We omit the details.

[47] This case distinction is needed to guarantee that Charlie's strategy is optimal off-path. It plays no role in the determination of the equilibrium outcome.

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = (1 + \omega - \delta\bar{w}, \delta\bar{w}, -1)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \delta\bar{w} \\ \delta_{no} & \text{if} \quad x_R < \delta\bar{w} \end{cases}$$

  for all $x \in X_\omega$;
- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R < \delta\bar{w} \\ \delta_{abstain} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

  for all $x \in X_\omega$.

(ii) Suppose $p(1-p)\frac{\delta}{1-\delta p} \leq f \leq p \cdot (\bar{w}(1+\delta) - \omega)$. Since $w_\lambda < \bar{w}$, one can verify that the following behavior rules constitute a stationary equilibrium with conflict. We omit the details.

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = (1 + \omega - \delta\bar{w}, \delta\bar{w}, -1)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \delta\bar{w} \\ \delta_{no} & \text{if} \quad x_R < \delta\bar{w} \end{cases}$$

  for all $x \in X_\omega$;
- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C \geq -\lambda \\ & \text{or} \quad x_R < \delta\bar{w} \end{cases}$$

  for all $x \in X_\omega$.

Case 2: Assume $\frac{\delta p(1-p)}{1-\delta p} > p \cdot (\bar{w}(1+\delta) - \omega)$. Since $w_\lambda < \bar{w}$, one can verify that the following behavior rules constitute a stationary equilibrium with conflict. We omit the details.

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x_P^*, x_R^*, x_C^*) = (1 + \omega - \delta\bar{w}, \delta\bar{w}, -1)$;
- β_R given by

$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \delta\bar{w} \\ \delta_{no} & \text{if} \quad x_R < \delta\bar{w} \end{cases}$$

  for all $x \in X_\omega$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R < \delta\bar{w} \\ \delta_{abstain} & \text{if} \quad x_R \geq \delta\bar{w} \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

for all $x \in X_\omega$.

This concludes the proof of Proposition 5.

□

Case $\omega > 0$:

Suppose now that $\omega > 0$. One can easily verify that, as long as $\frac{\lambda+\omega}{1+\delta} \geq \frac{(1-p)(1+\omega)}{1+\delta(1-p)}$, an accommodation equilibrium with no agreement delay exists.[48] The following proposition shows that there cannot be other accommodation equilibria.

Proposition 6. *Fix any accommodation equilibrium. In this equilibrium, the bargainers reach an agreement immediately with probability 1.*

Proof. By contradiction, suppose there exists an accommodation equilibrium with an on-path acceptance probability below 1, and let (v_P, v_R, v_C) be the corresponding equilibrium payoffs. To reach a contradiction, we show the proposer could then deviate and follow a strategy that gives her a payoff strictly above v_P. Note that $v_C \geq -\lambda$, as otherwise, Charlie would protest on path, contradicting the accommodation assumption. Further, note that $v_R \geq \delta v_P$, since the responder always has the choice to reject, making him the next-period proposer. Finally, since delay occurs on path with positive probability and $\omega > 0$, $v_P + v_R + v_C = \mathbb{E}[\delta^{\tilde{\tau}}\omega] = (1-k)\cdot\omega$ for some $k \in (0,1)$, where $\tilde{\tau}$ represents the random period an agreement is reached along the candidate-equilibrium path. Combining these facts, we get $v_P \leq (1-k)\cdot\omega + \lambda - \delta v_P$.

Now, consider the offer $x = (x_P, x_R, x_C) = (\omega + \lambda - \delta v_P - 2\varepsilon, \delta v_P + \varepsilon, -\lambda + \varepsilon) \in X_\omega$, where $\varepsilon > 0$ is arbitrarily small. This offer is accepted immediately ($x_R > \delta v_P$) and does not trigger a protest ($x_C > -\lambda$). Moreover, it results in a payoff to the proposer of $\omega + \lambda - \delta v_P - 2\varepsilon$ which is strictly higher than v_P as long as $\varepsilon < k\omega/2$, a contradiction. □

B.2 Strategic Delay

In this subsection, we study whether the bargainers can still strictly benefit from strategic delay relative to stationary behavior even if $\omega \neq 0$. Compared to Theorem 2, Proposition 7 below shows that this is the case as long as ω is not too large.

Formally, let $\omega^* := \frac{1-p}{p} > 0$, and recall that $\varphi^*(z) = \frac{\delta}{1+\delta}\varphi^{\mathrm{S}}(z) = \frac{\delta z(1-z)}{1+\delta(1-z)}$ for all $z \in (0,1)$. If we restrict attention to symmetric time-invariant equilibria where the proposer does not randomize, the following holds:

[48] Use the behavior rules described in Proposition 4 for the case $\alpha = \lambda$.

Proposition 7. *A symmetric time-invariant equilibrium where the bargainers' joint payoff is strictly higher than what they can collectively achieve in any feasible stationary equilibrium exists if and only if $\varphi^*(p) \leq f < p$ and $-1 < \omega < \omega^*$.*

Proof. "If" direction: Suppose that $\varphi^*(p) \leq f < p$ and $-1 < \omega < \omega^*$. Let $S^* = 1 - p + f = -\bar{u}_C < 1$. One can verify that because $\varphi^*(p) \leq f < p$, the following profile of symmetric time-invariant strategies $(\beta_P, \beta_R, \beta_C)$ constitutes an SPE.[49]

We begin by describing the behavior rule of the proposer. Let $x^* = (x^*_P, x^*_R, x^*_C) \in X_\omega$ be given by

$$x^* = \left(1 + \omega - \frac{\delta}{1+\delta} S^*(1+\omega), \frac{\delta}{1+\delta} S^*(1+\omega), -1\right).$$

We set β_P to be the Dirac measure on x^*, i.e., $\beta_P = \delta_{x^*}$. The proposer always proposes x^* with certainty in equilibrium.

We now describe the behavior strategy of the responder. Let q^* be given equation (7). Observe that $q^* \in (0,1)$ since $f < p$ by assumption. Suppose the proposer offered $x \in X_\omega$, and that Charlie did not protest against it in the protesting stage, i.e., $a_C = abstain$. In this case, we set

$$\beta_R(\cdot|x, abstain) = \begin{cases} \delta_{yes} & \text{if} \quad x_R > \frac{\delta}{1+\delta} S^*(1+\omega) \\ \delta_{yes} \cdot q^* + (1-q^*) \cdot \delta_{no} & \text{if} \quad x_R = \frac{\delta}{1+\delta} S^*(1+\omega) \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta}{1+\delta} S^*(1+\omega). \end{cases}$$

Conversely, suppose that the proposer offered $x \in X$ and that Charlie did protest against it, i.e., $a_C = protest$. In this case, we set

$$\beta_R(\cdot|x, protest) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta}{1+\delta} S^*(1+\omega) \\ \delta_{no} & \text{if} \quad x_R < \frac{\delta}{1+\delta} S^*(1+\omega). \end{cases}$$

To describe Charlie's behavior rule, we distinguish between two cases:

- Case 1: Suppose $f < p(1-p)\frac{\delta}{1-\delta p}$. Then, we set

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R < \frac{\delta}{1+\delta} S^*(1+\omega) \\ & \text{or} \quad x_R > \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R = \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C > -1 \\ \delta_{abstain} & \text{if} \quad x_R = \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C = -1 \\ & \text{or} \quad x_R > \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

  for all $x \in X_\omega$.

[49] We omit the details since a straightforward adaptation of the arguments used in the proof of Theorem 2 suffices to prove this point.

- Case 2: Suppose $f \geq p(1-p)\frac{\delta}{1-\delta p}$. Then, we set

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_R > \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_R \leq \frac{\delta}{1+\delta} S^*(1+\omega) \\ & \text{or} \quad x_R > \frac{\delta}{1+\delta} S^*(1+\omega) \quad \text{and} \quad x_C \geq -\lambda \end{cases}$$

for all $x \in X_\omega$.

Arguments identical to those employed in the proof of Theorem 2 show that in this time-invariant equilibrium Charlie's payoff equals his min-max payoff, i.e., $v_C = -S^*$. Since there is no protesting on path, it follows that

$$v_A + v_B = S^* + \mathbb{E}\left[\delta^{\tilde{\tau}}\omega\right] = S^* + \omega q^* \sum_{t=0}^{\infty} [(1-q^*)\delta]^t = S^*(1+\omega),$$

where $\tilde{\tau}$ represents the random period an agreement is reached along the equilibrium path.

Now, let us consider the stationary equilibrium outcomes. Since $f < p$, these outcomes can be of two types: Either Conflict or Accommodation. The bargainers' joint payoff in under Conflict is $(1-p)(1+\omega)$, which is strictly smaller than $S^*(1+\omega)$ for all $\omega > -1$. When $\omega \in (-1, 0)$, the best Accommodation equilibrium for the bargainers is that associated with the maximal amount of agreement delay possible, i.e., the Accommodation equilibrium of Proposition 4 associated with $\alpha = 1$. In this equilibrium, the bargainers' joint payoff is $\lambda(1+\omega) < S^*(1+\omega)$. Instead, if $\omega > 0$, the unique Accommodation equilibrium features no delay and allow the bargainers to earn a joint payoff of $\lambda + \omega$. However, notice that since $\omega < \omega^*$, it still holds that $\lambda + \omega < S^*(1+\omega)$. This concludes the proof of the "if" direction.

"Only if" direction. We want to show that a symmetric time-invariant equilibrium where the bargainers' joint payoff is strictly higher than what they can collectively achieve under any feasible stationary equilibrium does not exist unless $\varphi^*(p) \leq f < p$ and $-1 < \omega < \omega^*$.

Necessity of $-1 < \omega$ and $f < p$. We begin by showing that $-1 < \omega$ and $f < p$ are necessary conditions. The reason why $-1 < \omega$ is necessary is obvious: Since any feasible proposal $x \in X_\omega$ must satisfy $x_C \geq -1$, if $\omega \leq -1$, the unique equilibrium payoff the bargainers can achieve is 0. To see why $f < p$ is also necessary, note that if $f \geq p$, the bargainers can earn $\omega + 1$ without delay by playing generalization of the Rubinstein equilibrium described in Theorem 0 that accommodates for the presence of $\omega > -1$. Since $\omega + 1$ is the highest payoff the bargainers can earn in equilibrium, this shows that time-invariant strategies cannot improve upon stationary strategies when $f \geq p$.

Necessity of $\omega < \omega^*$. We now show the necessity of $\omega < \omega^*$. To see this, suppose that a symmetric time-invariant equilibrium with strategic delay exists where Charlie does not protest on path.[50] Let Charlie's payoff in this equilibrium be given by $v_C = -k$ where $k \in [\lambda, S^*]$ is

[50] If Charlie protested on path, the bargainers could not be strictly better off relative to the Conflict equilibrium.

arbitrary for now, and denote by $x^* \in X_\omega$ the proposer's equilibrium offer. Clearly, it holds that $x^*_C \in [-1, -k]$. Finally, recall that $S^* = 1 - p + f < 1$.

Recall our restriction on equilibria where the proposer does not randomize his offer. On path, the responder must accept the proposer's equilibrium offer with a probability $q \in (0, 1]$ such that

$$-k = q \cdot x^*_C + (1 - q) \cdot \delta \cdot (-k),$$

which implies that

$$q = \frac{-k + \delta k}{x^*_C + \delta k}.$$

Note that q is strictly increasing in $x^*_C \in [-1, -k]$ for a fixed level of $k \in [\lambda, S^*]$. We will make use of this observation below.

For strategic delay to work as an incentive device, Charlie must find it optimal to abstain from protesting if he believes this would induce an immediate acceptance of the equilibrium offer x^*.[51] Therefore, it must be the case that

$$-k \geq -f + (1 - p)x^*_C,$$

which implies the following upper bound on x^*_C:[52]

$$x^*_C \leq \frac{-k + f}{1 - p}. \tag{25}$$

Now, if a time-invariant equilibrium with these features existed, the bargainers' joint equilibrium payoff would be

$$v_P + v_R = k + \mathbb{E}[\delta^{\tilde{\tau}}] \cdot \omega = k + \omega \cdot \frac{q}{1 - \delta(1 - q)} \tag{26}$$

where, as usual, $\tilde{\tau}$ denotes the random period where an agreement is reached on path. Observe that x^*_C enters equation (26) only through the equilibrium probability q. In what follows, we use this equation to characterize the highest possible joint equilibrium payoff the bargainers can achieve in a symmetric time-invariant equilibrium based solely on Charlie's IC constraint. To do so, we distinguish between two cases:

Case $\omega < 0$. When $\omega < 0$, according to equation (26), the bargainers' joint payoff is *decreasing* in q. Therefore, the highest possible joint payoff is attained when x^*_C is minimized, i.e., when it holds that $x^*_C = -1$. Plugging this value in equation (26) and re-arranging, we obtain that:

$$v_P + v_R = k(1 + \omega).$$

This expression is maximized at $k = S^*$. Observe that at $k = S^*$, it holds that $q = \frac{S^*(1-\delta)}{1-\delta S^*} < 1$. Thus, when $\omega < 0$, strategic delay can strictly benefit the bargainers and the highest possible equilibrium payoff the bargainers can attain is $S^*(1 + \omega)$.

[51] Indeed, this contingent behavior of the responder represents the worst possible punishment for protests in equilibrium since, by construction, it holds that $x^*_C < \delta(-k)$.

[52] Note that $-1 \leq \frac{-k+f}{1-p} \leq -k$ since $k \geq \lambda$ by assumption.

Case $\omega > 0$. When $\omega > 0$, according to equation (26), the joint payoff $v_P + v_R$ is increasing in q. Therefore, to maximize $v_P + v_R$, the bargainers should set x_C^* as large as possible which, under constraint (25), means setting $x_C^* = \frac{-k+f}{1-p}$. After implementing this substitution, equation (26) becomes:

$$v_R + v_P = k + \omega \cdot \frac{k(1-p)}{k-f}.$$

Some algebra shows that setting $k = \lambda$ maximizes $v_P + v_R$ if and only if $\omega \geq \omega^*$. However, when $k = \lambda$, we have that $q = 1$, i.e., no contingent delay. This proves that whenever $\omega \geq \omega^*$, strategic delay cannot help the bargainers gain a strictly higher payoff relative to stationary strategies.[53] Thus, $\omega < \omega^*$ is necessary.

Necessity of $f \geq \varphi^*(p)$. We are left to show that $f \geq \varphi^*(p)$ is a necessary condition as well. As we show below, the necessity of this condition comes from the IC constraint of the proposer. In particular, since in any equilibrium with strategic delay the responder must be indifferent, it holds that $v_R = \delta v_P$. Coupled with equation (26), this implies that:

$$v_P = \frac{1}{1+\delta}\left(k + \omega \cdot \frac{q}{1-\delta(1-q)}\right). \tag{27}$$

In equilibrium, the proposer should not have the strict incentive to deviate from x^* and induce Conflict. Note that the best deviation inducing Conflict would be to set $x_C = -1$ and $x_R = v_R$. This deviation would yield the proposer a payoff of

$$(1-p)\left[1 + \omega - \frac{\delta}{1+\delta}\left(k + \omega \cdot \frac{q}{1-\delta(1-q)}\right)\right]. \tag{28}$$

The RHS of (27) is weakly larger than the RHS of (28) if and only if

$$\frac{1+\delta(1-p)}{1+\delta}\left(k + \omega\frac{q}{1-\delta(1-q)}\right) \geq (1-p)(1+\omega). \tag{29}$$

Equation (29) represents a generalized IC constraint for the proposer. In what follows, we want to choose $x_C^* \in [-1, -k]$ and $k \in [\lambda, S^*]$ optimally to relax this constraint as much as possible. To do so, we distinguish between two cases.

Case $\omega < 0$. If $\omega < 0$, the LHS of (29) is *decreasing* in q. Therefore, to maximize this LHS, we need to optimally set $x_C^* = -1$. Given this substitution, the IC constraint becomes

$$\frac{1+\delta(1-p)}{1+\delta}k \geq 1-p$$

since $\omega > -1$. Notice that the LHS of the above inequality is *increasing* in k. Therefore, we set $k = S^*$ to obtain

$$S^* \geq \frac{(1+\delta)(1-p)}{1+\delta(1-p)},$$

[53]Conversely, the unique maximizer of $v_P + v_R$ is $k = S^*$ when $\omega < \omega^*$. Hence, in this case, strategic delay can help the bargainers.

which is equivalent to $f \geq \varphi^*(p)$. Thus, $f \geq \varphi^*(p)$ is necessary to sustain any time-invariant equilibrium with strategic delay when $\omega < 0$.

Case $\omega > 0$. When $\omega > 0$, the LHS of (29) is *increasing* in q. Therefore, to maximize this LHS, we need to optimally set $x_C^* = \frac{-k+f}{1-p}$, as prescribed by the upper bound given by (25). After this substitution, the LHS of (29) therefore becomes

$$\frac{1+\delta(1-p)}{1+\delta} k \left(1 + \omega \frac{1-p}{k-f}\right) \geq (1-p)(1+\omega).$$

For $\omega < \omega^*$, the LHS of this new inequality is maximized by setting $k = S^*$. After plugging in this value for k, the IC constraint (29) once again reads

$$S^* \geq \frac{(1+\delta)(1-p)}{1+\delta(1-p)},$$

which is equivalent to $f \geq \varphi^*(p)$. Thus, $f \geq \varphi^*(p)$ is necessary to sustain any time-invariant equilibrium with strategic delay also when $0 < \omega < \omega^*$.

This concludes the proof of Proposition 7.

□

C The Timing of Protests

This Appendix formalizes the discussion of Section 6.2. We show that if protests can occur *ex-post*, i.e., *after* an agreement between the bargainers has been reached, rather than *ex-ante*, i.e., when a proposal has not yet been accepted, then the equilibrium predictions of the model resemble those of collusive bargaining listed in Proposition 3, with the only exception that equilibrium offers are pinned down uniquely.

Formally, consider a variation of the baseline model of Section 2 where we interchange the decision and protesting stages in every period. If the responder rejects, the protesting stage is skipped, and the play continues with the proposing stage of the next period. If the responder approves the current-period proposal, Charlie has the opportunity to protest against it. We call this model the *ex-post protesting model.* Recall that $\varphi^{\mathrm{C}}(p) = p(1-p)$ for all $p \in (0,1)$. The following proposition characterizes the equilibrium outcomes of this model for almost every parameter configuration satisfying Assumption 1.[54]

Proposition 8. *Suppose $f < p$. Then, in the ex-post protesting model, an SPE always exists. In particular,*

a) ***Accommodation:*** *If $\varphi^{\mathrm{C}}(p) < f < p$, all SPEs induce the same equilibrium payoffs given by*

$$(v_A, v_B, v_C) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right).$$

Moreover, the SPE outcome is unique, can be sustained in stationary strategies, and features the following sequence of events:

(i) The proposer offers $x^* = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right)$,

(ii) The responder accepts x^* with probability 1,

(iii) Charlie abstains from protesting.

b) ***Conflict:*** *If $0 < f < \varphi^{\mathrm{C}}(p)$, all symmetric SPEs induce the same equilibrium payoffs given by*

$$(v_A, v_B, v_C) = \left(\frac{1-p}{1+\delta}, \frac{\delta(1-p)}{1+\delta}, -f-(1-p)\right).$$

Moreover, the SPE outcome is unique, can be sustained in stationary strategies, and features the following sequence of events:

(i) The proposer offers $x^* = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right)$,

(ii) The responder accepts x^* with probability 1,

(iii) Charlie protests ex-post.

[54] For the sake of brevity, we omit to consider the case where $f \geq p$, i.e., the case where Assumption 1 is violated. In this case, a version of Proposition 1 applies: The unique equilibrium outcome of the ex-post protesting model is a slight modification of the Rubinstein outcome defined in Theorem 0, where the only difference is in the timing of Charlie's decision to acquiesce. (This decision occurs *after* the responder accepts the equilibrium offer, rather than *before*).

Proof of Proposition 8

Assume throughout that $f < p$. This proof is divided into three parts. In Part 1, we show that an SPE of the ex-post protesting model exists for all possible parameter configurations $(\delta, p, f) \in \Theta := (0,1) \times (0,1) \times (0,\infty)$ such that $f < p$. In Part 2, we introduce some useful preliminary notation. In Part 3, we show that, as long as $f \neq \varphi^{\mathrm{C}}(p)$, the ex-post protesting model admits a unique SPE outcome, either Accommodation or Conflict, as described by Proposition 8.

Part 1: Existence

The following Lemma establishes the existence of SPEs for all possible parameter configurations (δ, p, f) such that $f < p$.

Lemma 10. *Suppose $f < p$. Then, an SPE of the ex-post protesting model always exists.*

Proof of Lemma 10. It is sufficient to specify three behavior rules β_P, β_R and β_C to characterize an equilibrium, where $\beta_P \in \Delta(X)$ is the behavior strategy played by the proposer, $\beta_R \in \Delta(\{yes, no\})^X$ is the behavior strategy of the responder, and $\beta_C \in \Delta(\{protest, abstain\})^X$ is the behavior strategy of Charlie. Recall that Charlie only moves if the bargainers reached an agreement.

Case 1: Assume that $f < \varphi^{\mathrm{C}}(p)$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right)$;
- β_R given by
$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta}{1+\delta} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R \geq \frac{\delta(1-p)}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \\ \delta_{no} & \text{otherwise} \end{cases}$$
for all $x \in X$;
- β_C given by
$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_C \geq -\lambda \end{cases}$$
for all $x \in X$.

Case 2: Assume that $\varphi^{\mathrm{C}}(p) \leq f < p$. The following behavior rules constitute an SPE:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right)$;
- β_R given by
$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta\lambda}{(1+\delta)(1-p)} \quad \text{and} \quad x_C < -\lambda \\ & \text{or} \quad x_R \geq \frac{\delta}{1+\delta} \quad \text{and} \quad x_C \geq -\lambda \\ \delta_{no} & \text{otherwise} \end{cases}$$

for all $x \in X$;

- β_C given by

$$\beta_C(\cdot|x) = \begin{cases} \delta_{protest} & \text{if} \quad x_C < -\lambda \\ \delta_{abstain} & \text{if} \quad x_C \geq -\lambda \end{cases}$$

for all $x \in X$.

Since we considered all possible parameter configurations $(\delta, p, f) \in \Theta$ such that $f < p$, this shows that under Assumption 1, the ex-post protesting model always admits an SPE.

□

Part 2: Preliminary notation

We introduce some useful preliminary notation. For every $i \in \{A, B\}$, let $H_{i,P}$ be the set of non-terminal histories where it is agent i's turn to move at the proposing stage. For example, $\varnothing \in H_{A,P}$, while $(x, no) \in H_{B,P}$.[55] Note that for all $h, h' \in H_{i,P}$, the two subgames that start at h and h' are *strategically equivalent*: The sets of continuation strategies in these two subgames are isomorphic, while the agents' payoff functions across these two subgames are linearly dependent. For every $h \in H_{i,P}$, let Γ_i be the subgame starting at h where agents' payoffs are normalized so that if an allocation $x \in X$ is immediately implemented without protests, each agent gets a payoff of x_i. Observe that because of this payoff normalization, each subgame Γ_i does not actually depend on $h \in H_{i,P}$. Therefore, it makes sense to write Γ_i to indicate a generic subgame that starts with agent $i \in \{A, B\}$ making an offer during the proposing stage. Given this, let $\mathcal{E}(\Gamma_i)$ be the set of SPE strategy profiles in Γ_i, and for each $j \in I$, let $V_j(\Gamma_i)$ be the set of agent j's equilibrium payoffs. Observe that none of these sets are empty since Lemma 8 showed that, under Assumption 1, the ex-post protesting model always admits an SPE. Finally, let $\bar{v}_j(\Gamma_i)$ (resp., $\underline{v}_j(\Gamma_i)$) be the maximal (resp., minimal) equilibrium payoff for $j \in I$ in subgame Γ_i. It is easy to see that since Ann and Bob can always force an outcome with perpetual disagreement, it holds that $\underline{v}_j(\Gamma_i) \geq 0$ for all $i, j \in \{A, B\}$. Moreover, since the only resources that Ann and Bob can consume in equilibrium are those they manage to seize from Charlie, it also holds that $\bar{v}_j(\Gamma_i) \leq 1$ for all $i, j \in \{A, B\}$.

Part 3: Uniqueness of the SPE outcome

Consider the strategic situation Ann faces at the start of Γ_A. We begin by characterizing a lower bound for $\underline{v}_A(\Gamma_A)$. Since in the ex-post protesting model Charlie can protest *after* an agreement between Ann and Bob is reached, it is easy to characterize his equilibrium protesting incentives. In particular, he has a strict incentive to protest against any accepted proposal x if $x_C < -\lambda$, while he has a strict incentive to abstain from protesting if $x_C > -\lambda$.

Suppose first that Ann offers $x \in X$ to Bob such that $x_C = -\lambda + \varepsilon$ and $x_B = \delta\bar{v}_B(\Gamma_B) + \varepsilon$, where $\varepsilon > 0$ is small. Since this proposal gives Bob strictly more than his discounted best continuation-equilibrium payoff and does not induce Charlie to protest, Ann knows that Bob

[55]Here, $\varnothing$ denotes the empty-history, i.e., the start of the game.

will accept x immediately if given the opportunity. This proposal yields a payoff to Ann of $x_A = 1 - \delta\bar{v}_B(\Gamma_B) - 2\varepsilon$. In any SPE, Ann's equilibrium payoff must be at least as large as when offering to Bob this proposal. This implies that $\underline{v}_A(\Gamma_A) \geq \sup_{\varepsilon>0} \lambda - \delta\bar{v}_B(\Gamma_B) - 2\varepsilon = \lambda - \delta\bar{v}_B(\Gamma_B)$.

Suppose now that Ann offers Bob a proposal $x \in X$ such that $x_B = \frac{\delta\bar{v}_B(\Gamma_B)+\varepsilon}{1-p}$ and $x_C = -1$, where $\varepsilon > 0$ is small. Since $x_C = -1 < -\lambda$, Ann and Bob know that if Bob accepts x then Charlie will protest against it. Yet, Bob is willing to accept Ann's proposal since, in expectation, he would receive strictly more than under his discounted best continuation-equilibrium payoff. Thus, this proposal yields to Ann a payoff of $(1-p) - \delta\bar{v}_B(\Gamma_B) - \varepsilon$. Once again, in any SPE, Ann's equilibrium payoff must be at least as large as when offering to Bob this proposal. This implies that $\underline{v}_A(\Gamma_A) \geq \sup_{\varepsilon>0}(1-p) - \delta\bar{v}_B(\Gamma_B) - \varepsilon = (1-p) - \delta\bar{v}_B(\Gamma_B)$. We conclude that

$$\underline{v}_A(\Gamma_A) \geq \max\Big\{\lambda - \delta\bar{v}_B(\Gamma_B), (1-p) - \delta\bar{v}_B(\Gamma_B)\Big\}. \tag{LB$_A$}$$

This establishes a lower bound for $\underline{v}_A(\Gamma_A)$.

Let us now characterize an upper bound for $\bar{v}_A(\Gamma_A)$. As a first step, suppose that Ann offers $x \in X$ such that Bob finds it strictly optimal to reject. In this case, Ann can receive at most an equilibrium payoff equal to

$$\delta \max\{\lambda - \underline{v}_B(\Gamma_B), (1-p) - \underline{v}_B(\Gamma_B)\}$$

since the play moves to next period, and Bob can always secure a payoff of $\underline{v}_B(\Gamma_B)$ for himself. Conversely, suppose that Ann proposes an offer $x \in X$ that Bob is willing to accept given some continuation equilibrium. There are two candidates for the best outcome for Ann in this case: Either having the proposal $x \in X$ such that $x_B = \delta\underline{v}_B(\Gamma_B)$ and $x_C = -\lambda$ immediately implemented without protests, or having the proposal $x \in X$ such that $x_B = \frac{\delta\underline{v}_B(\Gamma_B)}{1-p}$ and $x_C = -1$ immediately implemented with protests. The best alternative among these two options would yield Ann an equilibrium payoff of $\max\{\lambda - \delta\underline{v}_B(\Gamma_B), (1-p) - \delta\underline{v}_B(\Gamma_B)\}$. This establishes the following upper bound on $\bar{v}_A(\Gamma_A)$:

$$\bar{v}_A(\Gamma_A) \leq \max\Big\{\lambda - \delta\underline{v}_B(\Gamma_B), (1-p) - \delta\underline{v}_B(\Gamma_B)\Big\}. \tag{UB$_A$}$$

Similar bounds can be found for $\underline{v}_B(\Gamma_B)$ and $\bar{v}_B(\Gamma_B)$ by considering the strategic situation of Bob at the start of Γ_B as follows:

$$\underline{v}_B(\Gamma_B) \geq \max\Big\{\lambda - \delta\bar{v}_A(\Gamma_A), (1-p) - \delta\bar{v}_A(\Gamma_A)\Big\}, \tag{LB$_B$}$$

$$\bar{v}_B(\Gamma_B) \leq \max\Big\{\lambda - \delta\underline{v}_A(\Gamma_A), (1-p) - \delta\underline{v}_A(\Gamma_A)\Big\}. \tag{UB$_B$}$$

Case 1: Suppose $0 < f < \varphi^{\mathrm{C}}(p)$, so that $\lambda < (1-p)$. Combining the inequalities (UB$_A$) and (LB$_B$) and re-arranging, we obtain that $\bar{v}_A(\Gamma_A) \leq \frac{1-p}{1+\delta}$, while combining the inequalities (LB$_A$) and (UB$_B$), we obtain that $\frac{1-p}{1+\delta} \leq \underline{v}_A(\Gamma_A)$. Therefore,

$$\underline{v}_A(\Gamma_A) = \bar{v}_A(\Gamma_A) = v_A^*(\Gamma_A) = \frac{1-p}{1+\delta}.$$

Analogous steps can be used to show that $\bar{v}_B(\Gamma_B) = \underline{v}_B(\Gamma_B) = v_B^*(\Gamma_B) = \frac{1-p}{1+\delta}$. We conclude that there exists a unique equilibrium payoff associated with being the proposer at the start of every period, namely

$$V_j(\Gamma_j) = \{v_P^*\} = \left\{\frac{1-p}{1+\delta}\right\}$$

for all $j \in \{A, B\}$.

Consider now the strategic situation Ann faces at the start of Γ_B. Since Ann can always reject any offer $x \in X$ she receives from Bob, we conclude that $\underline{v}_A(\Gamma_B) \geq \delta v_P^* = \frac{\delta(1-p)}{1+\delta}$. On the other hand, since $1-p > f/p$ and because Bob cannot get a payoff lower than $v_P^* = \frac{1-p}{1+\delta}$ in Γ_B, the best outcome for Ann in Γ_B is when Bob proposes to extract all resources from Charlie (i.e., $x_C = -1$) even though this would lead to protests, and keeps only an amount $x_B = \frac{1}{1-p}\underline{v}_B(\Gamma_B) = \frac{1}{1-p}v_P^* = \frac{1}{1+\delta}$ of these resources for himself. Therefore, $\bar{v}_A(\Gamma_B) \leq (1-p)\left(1 - \frac{1}{1+\delta}\right) = \frac{\delta(1-p)}{1+\delta}$, which implies $\underline{v}_A(\Gamma_B) = \bar{v}_A(\Gamma_B) = v_A^*(\Gamma_B) = \frac{\delta(1-p)}{1+\delta}$. Since analogous steps can be used to show that $\bar{v}_B(\Gamma_A) = \underline{v}_B(\Gamma_A) = v_B^*(\Gamma_A) = \frac{\delta(1-p)}{1+\delta}$, we conclude that there exists a unique equilibrium payoff associated with being the responder at the start of every period, namely

$$V_j(\Gamma_i) = \{v_R^*\} = \left\{\frac{\delta(1-p)}{1+\delta}\right\}$$

for all $j \in \{A, B\}$ and $i \neq j$.

Because $V_j(\Gamma_j)$ and $V_j(\Gamma_i)$ for $j \in \{A, B\}$ and $i \neq j$ do not depend on the identity of the players and are singletons, we conclude that any SPE outcome can be sustained in stationary strategies. Moreover, since $v_P^* + v_R^* = 1 - p$ and $1 - p > f/p$, we conclude that any SPE must satisfy the following three features: *(a)* the proposer offers to extract all the resources from Charlie, i.e., $x_C^* = -1$ *(b)* the responder immediately accepts, and *(c)* Charlie protests ex-post. Observe that there exists only one feasible offer that is consistent with these features, and with equilibrium payoffs $v_P^* = \frac{1}{1+\delta}$ and $v_R^* = \frac{\delta}{1+\delta}$: Namely, $x^* \in X$ such that

$$x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}, -1\right).$$

We conclude that points *(i)*, *(ii)*, and *(iii)* of part *(b)* Proposition 8 hold, as required.

Case 2: Suppose $\varphi^{\mathrm{C}}(p) < f < p$, so that $\lambda > (1-p)$.

Combining the inequalities (UB_A) and (LB_B) and re-arranging, we obtain that $\bar{v}_A(\Gamma_A) \leq \frac{\lambda}{1+\delta}$, while combining the inequalities (LB_A) and (UB_B), we obtain that $\frac{\lambda}{1+\delta} \leq \underline{v}_A(\Gamma_A)$. Therefore,

$$\underline{v}_A(\Gamma_A) = \bar{v}_A(\Gamma_A) = v_A^*(\Gamma_A) = \frac{\lambda}{1+\delta}.$$

Analogous steps can be used to show that $\bar{v}_B(\Gamma_B) = \underline{v}_B(\Gamma_B) = v_B^*(\Gamma_B) = \frac{\lambda}{1+\delta}$. We conclude that there exists a unique equilibrium payoff associated with being the proposer at the start of every period, namely

$$V_j(\Gamma_j) = \{v_P^*\} = \left\{\frac{\lambda}{1+\delta}\right\}$$

for all $j \in \{A, B\}$.

Consider now the strategic situation Ann faces at the start of Γ_B. Since Ann can always reject any offer $x \in X$ she receives from Bob, we conclude that $\underline{v}_A(\Gamma_B) \geq \delta v_P^* = \frac{\delta\lambda}{1+\delta}$. On the other hand, since $1 - p < f/p$ and because Bob cannot get a payoff lower than $v_P^* = \frac{\lambda}{1+\delta}$ in Γ_B, the best outcome for Ann in Γ_B is when Bob proposes to extract λ from Charlie (i.e., $x_C = -\lambda$), and keeps only an amount $x_B = \underline{v}_B(\Gamma_B) = v_P^* = \frac{\lambda}{1+\delta}$ of these resources for himself. Therefore, $\bar{v}_A(\Gamma_B) \leq \lambda\left(1 - \frac{1}{1+\delta}\right) = \frac{\delta\lambda}{1+\delta}$, which implies $\underline{v}_A(\Gamma_B) = \bar{v}_A(\Gamma_B) = v_A^*(\Gamma_B) = \frac{\delta\lambda}{1+\delta}$. Since analogous steps can be used to show that $\bar{v}_B(\Gamma_A) = \underline{v}_B(\Gamma_A) = v_B^*(\Gamma_A) = \frac{\delta\lambda}{1+\delta}$, we conclude that there exists a unique equilibrium payoff associated with being the responder at the start of every period, namely

$$V_j(\Gamma_i) = \{v_R^*\} = \left\{\frac{\delta\lambda}{1+\delta}\right\}$$

for all $j \in \{A, B\}$ and $i \neq j$.

Because $V_j(\Gamma_j)$ and $V_j(\Gamma_i)$ for $j \in \{A, B\}$ and $i \neq j$ do not depend on the identity of the players and are singletons, we conclude that any SPE outcome can be sustained in stationary strategies. Moreover, since $v_P^* + v_R^* = \lambda$ and $\lambda > 1 - p$, we conclude that any SPE must satisfy the following three features: *(a)* the proposer offers to extract λ from Charlie, i.e., $x_C^* = -\lambda$ *(b)* the responder immediately accepts, and *(c)* Charlie abstains from protesting. Observe that there exists only one feasible offer that is consistent with these features, and with equilibrium payoffs $v_P^* = \frac{\lambda}{1+\delta}$ and $v_R^* = \frac{\delta\lambda}{1+\delta}$: Namely, $x^* \in X$ such that

$$x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{\lambda}{1+\delta}, \frac{\delta\lambda}{1+\delta}, -\lambda\right).$$

We conclude that points *(i)*, *(ii)*, and *(iii)* of part *(a)* Proposition 8 hold, as required.

This completes the proof of Proposition 8.

Q.E.D.

D Offer Observability

This Appendix formalizes the discussion of Section 6.3. We show that under Assumption 1, if Charlie cannot observe offers, the unique time-invariant equilibrium outcome resembles the Conflict equilibrium outcome described in Proposition 2 part *(b)*, even if $f > \varphi^{\mathrm{S}}(p)$.

Formally, consider a variation of the baseline model of Section 2 where Charlie cannot observe the offers Ann and Bob discuss. Suppose all other aspects of the model remain unchanged. We call this new model the *unobservable-offer model*. Since this model does not admit proper subgames anymore, the notion of SPE coincides with that of a *Nash Equilibrium* (NE). To obtain sharper predictions, the following definition refines NE by introducing a notion of time-invariance valid for the context of the unobservable-offer model.

Definition 1. *A time-invariant equilibrium of the unobservable-offer model is a NE σ such that:*

(i) Agents play time-invariant strategies. That is, for $i \in \{A, B\}$, σ_i can be represented by a profile $\beta_i = (\beta_i^P, \beta_i^R)$ such that

$$\beta_i^P \in \Delta(X) \quad \textit{and} \quad \beta_i^R \in \Delta(\{yes, no\})^{X \times \{protest, abstain\}}$$

while σ_C can be represented by $\beta_C \in \Delta(\{protest, abstain\})$;

(ii) For every $i \in \{A, B\}$, the strategy σ_i is sequentially rational given σ_{-i} at every decision node where agent i is active, both on and off path.

Note that condition *(ii)* of Definition 1 implies that, for a fixed strategy of Charlie β_C, the profile (σ_A, σ_B) constitutes an SPE of the perfect information bargaining game between Ann and Bob that results once we regard Charlie as an automaton that protests with probability $\beta_C(protest) \in [0, 1]$ in every period. We will make use of this observation below.

The following proposition characterizes the unique time-invariant equilibrium outcome of the unobservable-offer model under Assumption 1.

Proposition 9. *Suppose $f < p$. Then, in the unobservable-offer model, a time-invariant equilibrium always exists. In particular, all time-invariant equilibria induce the same equilibrium payoffs given by*

$$(v_A, v_B, v_C) = \left(\frac{1-p}{1+\delta(1-p)}, \frac{\delta(1-p)^2}{1+\delta(1-p)}, -f - (1-p) \right).$$

Moreover, the time-invariant equilibrium outcome is unique, can be sustained in symmetric strategies, and features the following sequence of events:

(i) The proposer offers $x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta(1-p)}, \frac{\delta(1-p)}{1+\delta(1-p)}, -1 \right)$ and Charlie simultaneously protests,

(ii) If the protest fails, the responder accepts x^* with probability 1.

Proof of Proposition 9

Assume throughout that $f < p$. This proof is divided into three parts. In Part 1, we show that a time-invariant equilibrium of the unobservable-offer model always exists for all possible parameter configurations (δ, p, f) such that $f < p$. In Part 2, we analyze an auxiliary game where Charlie's behavior rule is exogenous. In Part 3, we use the results established in Part 2 to show that the unobservable-offer model admits a unique time-invariant equilibrium outcome as described by Proposition 9.

Part 1: Existence

The following Lemma establishes the existence of SPEs for all possible parameter configurations (δ, p, f) such that $f < p$.

Lemma 11. *Suppose $f < p$. Then, a time-invariant equilibrium of the unobservable-offer model always exists.*

Proof of Lemma 11. We show that a symmetric time-invariant equilibrium in stationary strategies always exists. To do so, it is sufficient to specify three behavior rules β_P, β_R and β_C to characterize an equilibrium, where $\beta_P \in \Delta(X)$ is the behavior strategy played by the proposer, $\beta_R \in \Delta(\{yes, no\})^X$ is the behavior strategy of the responder, and $\beta_C \in \Delta(\{protest, abstain\})$ is the behavior strategy of Charlie. (Recall that Charlie cannot observe the bargainers' offers.)

Consider the following behavior rules:

- β_P given by $\beta_P = \delta_{x^*}$, where $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{1}{1+\delta(1-p)}, \frac{\delta(1-p)}{1+\delta(1-p)}, -1\right)$;
- β_R given by
$$\beta_R(\cdot|x) = \begin{cases} \delta_{yes} & \text{if} \quad x_R \geq \frac{\delta(1-p)}{1+\delta(1-p)} \\ \delta_{no} & \text{otherwise} \end{cases}$$
for all $x \in X$;
- β_C given by $\beta_C = \delta_{protest}$.

One can verify that profile $(\beta_P, \beta_R, \beta_C)$ satisfies conditions *(i)* and *(ii)* of Definition 1. This shows that under Assumption 1, the unobservable-offer model always admits a time-invariant equilibrium.

□

Part 2: The automaton game

In this part of the proof, suppose Charlie is an automaton that plays the action $a_C = protest$ with probability $\beta_C \in [0, 1]$ in every period. Note that, under this assumption, the model becomes a perfect information bargaining game between Ann and Bob. We call this model the *automaton game*. Our goal is to characterize the SPEs of this game. In particular, we want to prove the following lemma.

Lemma 12. *Fix $\beta_C \in [0,1]$. The automaton game always admits an SPE. Furthermore, the SPE outcome is unique, can be supported in symmetric time-invariant strategies, and features the following sequence of events:*

(i) The proposer offers $x^* = (x^*_P, x^*_R, x^*_C) = \left(\frac{1}{1+\delta m_C}, \frac{\delta m_C}{1+\delta m_C}, -1\right)$, where $m_C > 0$ is given by

$$m_C := 1 - \beta_C + \beta_C(1-p).$$

(ii) The responder accepts x^* with probability 1.

Proof of Lemma 12. For the sake of brevity, we omit to prove the existence of a SPE. One can easily verify that an equilibrium that is conceptually similar to the one characterized in Lemma 11 can always be found under Assumption 1. Instead, here we focus on the proof of the uniqueness of the SPE outcome and on the fact that time-invariant strategies are without loss. To do so, we introduce some useful preliminary notation. For every $i \in \{A, B\}$, let $H_{i,P}$ be the set of non-terminal histories where it is agent i's turn to move at the proposing stage. For example, $\varnothing \in H_{A,P}$, while $(x, abstain, no) \in H_{B,P}$.[56] Note that for all $h, h' \in H_{i,P}$, the two subgames that start at h and h' are *strategically equivalent*: The sets of continuation strategies in these two subgames are isomorphic, while the agents' payoff functions across these two subgames are linearly dependent. For every $h \in H_{i,P}$, let Γ_i be the subgame starting at h where agents' payoffs are normalized so that if an allocation $x \in X$ is immediately implemented without protests, each agent gets a payoff of x_i. Observe that because of this payoff normalization, each subgame Γ_i does not actually depend on $h \in H_{i,P}$. Therefore, it makes sense to write Γ_i to indicate a generic subgame that starts with agent $i \in \{A, B\}$ making an offer during the proposing stage. Given this, let $\mathcal{E}(\Gamma_i)$ be the set of SPE profiles in Γ_i, and for each $j \in \{A, B\}$, let $V_j(\Gamma_i)$ be the set of agent j's equilibrium payoffs. Observe that, because of the arguments given at the beginning of this proof, none of these sets are empty. Finally, let $\bar{v}_j(\Gamma_i)$ (resp., $\underline{v}_j(\Gamma_i)$) be the maximal (resp., minimal) SPE payoff for $j \in I$ in subgame Γ_i. It is easy to see that since Ann and Bob can always force an outcome with perpetual disagreement, it holds that $\underline{v}_j(\Gamma_i) \geq 0$ for all $i, j \in \{A, B\}$. Moreover, since the only resources that Ann and Bob can consume in equilibrium are those they manage to seize from Charlie, it also holds that $\bar{v}_j(\Gamma_i) \leq 1$ for all $i, j \in \{A, B\}$.

Consider the strategic situation Ann faces at the start of Γ_A. We begin by characterizing a lower bound for $\underline{v}_A(\Gamma_A)$. Suppose Ann offers Bob a proposal $x \in X$ such that $x_C = -1$ and $x_B = \delta \bar{v}_B(\Gamma_B) + \varepsilon$, for some $\varepsilon > 0$ small. Observe that because Ann is offering Bob more than he could get by inducing his best continuation-equilibrium from next period, Bob would accept x with probability 1 if given the opportunity. Thus, by offering x to Bob, Ann would obtain a payoff of

$$m_C(1 - \delta \bar{v}_B(\Gamma_B) - \varepsilon),$$

where $m_C = 1 - \beta_C + \beta_C(1-p) \in (0, 1]$ represents the probability that Charlie does not end the game via protests. In any equilibrium, Ann's payoff must be at least as large as when offering

[56] Here, $\varnothing$ denotes the empty-history, i.e., the start of the game.

to Bob this proposal. This implies that

$$\underline{v}_A(\Gamma_A) \geq m_C(1 - \delta \bar{v}_B(\Gamma_B)), \tag{LB$_A$}$$

establishing a lower bound for $\underline{v}_A(\Gamma_A)$.

Let us now characterize an upper bound for $\bar{v}_A(\Gamma_A)$. As a first step, suppose that Ann offers $x \in X$ that Bob finds it strictly optimal to reject. In this case, Ann can receive at most an equilibrium payoff of $\delta m_C(1-\underline{v}_B(\Gamma_B))$ since the play moves to next period. Conversely, suppose that Ann offers $x \in X$ such that Bob is willing to accept given some continuation equilibrium. There is a unique best equilibrium outcome for Ann in this case: Proposing $x \in X$ to Bob such that $x_B = \delta \underline{v}_B(\Gamma_B)$ and $x_C = -1$, and having this proposal immediately accepted by Bob. This proposal would yield Ann a payoff of $m_C(1 - \delta \underline{v}_B(\Gamma_B))$. This establishes the following upper bound for $\bar{v}_A(\Gamma_A)$:

$$\bar{v}_A(\Gamma_A) \leq m_C(1 - \delta \underline{v}_B(\Gamma_B)). \tag{UB$_A$}$$

Similar bounds can be found for $\underline{v}_B(\Gamma_B)$ and $\bar{v}_B(\Gamma_B)$ by considering the strategic situation of Bob at the start of Γ_B as follows:

$$\underline{v}_B(\Gamma_B) \geq m_C(1 - \delta \bar{v}_A(\Gamma_A)), \tag{LB$_B$}$$

$$\bar{v}_B(\Gamma_B) \leq m_C(1 - \delta \underline{v}_A(\Gamma_A)). \tag{UB$_B$}$$

Combining the inequalities (UB$_A$) and (LB$_B$) and re-arranging, we obtain that $\bar{v}_A(\Gamma_A) \leq \frac{m_C}{1+\delta m_C}$, while combining the inequalities (LB$_A$) and (UB$_B$), we obtain that $\frac{m_C}{1+\delta m_C} \leq \underline{v}_A(\Gamma_A)$. Therefore,

$$\underline{v}_A(\Gamma_A) = \bar{v}_A(\Gamma_A) = v_A^*(\Gamma_A) = \frac{m_C}{1 + \delta m_C}.$$

Analogous steps can be used to show that $\bar{v}_B(\Gamma_B) = \underline{v}_B(\Gamma_B) = v_B^*(\Gamma_B) = \frac{m_C}{1+\delta m_C}$. We conclude that there exists a unique equilibrium payoff associated with being the proposer at the start of every period, namely

$$V_j(\Gamma_j) = \{v_P^*\} = \left\{\frac{m_C}{1 + \delta m_C}\right\}$$

for all $j \in \{A, B\}$.

Consider now the strategic situation Ann faces at the start of Γ_B. Since Ann can always reject any offer $x \in X$ she receives from Bob with probability m_C, we conclude that $\underline{v}_A(\Gamma_B) \geq \delta m_C v_P^* = \frac{\delta m_C^2}{1+\delta m_C}$. On the other hand, because Bob cannot get a payoff lower than $v_P^* = \frac{m_C}{1+\delta m_C}$ in Γ_B, the best outcome for Ann in Γ_B is when Bob proposes to extract all resources from Charlie (i.e., $x_C = -1$), and keeps only an amount $x_B = \frac{1}{m_C} \underline{v}_B(\Gamma_B) = \frac{1}{m_C} v_P^* = \frac{1}{1+\delta m_C}$ of these resources for himself. Therefore, $\bar{v}_A(\Gamma_B) \leq m_C \left(1 - \frac{1}{1+\delta m_C}\right) = \frac{\delta m_C^2}{1+\delta m_C}$, which implies $\underline{v}_A(\Gamma_B) = \bar{v}_A(\Gamma_B) = v_A^*(\Gamma_B) = \frac{\delta m_C^2}{1+\delta m_C}$. Since analogous steps can be used to show that $\bar{v}_B(\Gamma_A) = \underline{v}_B(\Gamma_A) = v_B^*(\Gamma_A) = \frac{\delta m_C^2}{1+\delta m_C}$, we conclude that there exists a unique equilibrium payoff associated with being the responder at the start of every period, namely

$$V_j(\Gamma_i) = \{v_R^*\} = \left\{\frac{\delta m_C^2}{1+\delta m_C}\right\}$$

for all $j \in \{A, B\}$ and $i \neq j$.

Because $V_j(\Gamma_j)$ and $V_j(\Gamma_i)$ for $j \in \{A, B\}$ and $i \neq j$ do not depend on the identity of the players and are singletons, we conclude that any SPE outcome can be sustained in symmetric time-invariant strategies. Moreover, since $v_P^* + v_R^* = m_C$ and m_C represents the probability that (automaton) Charlie ends the game in every period, we conclude that any equilibrium must satisfy the following three features: *(a)* the proposer offers to extract all the resources from Charlie, i.e., $x_C^* = -1$ *(b)* the responder immediately accepts. Observe that there exists only one feasible offer that is consistent with these features, and with equilibrium payoffs $v_P^* = \frac{m_C}{1+\delta m_C}$ and $v_R^* = \frac{\delta m_C^2}{1+\delta m_C}$: Namely, $x^* \in X$ such that

$$x^* = (x_P^*, x_R^*, x_C^*) = \left(\frac{1}{1+\delta m_C}, \frac{\delta m_C}{1+\delta m_C}, -1\right).$$

We conclude that points *(i)*, *(ii)* of Lemma 12 hold, as required.

□

Part 3: Uniqueness of equilibrium outcome

To conclude the proof of Proposition 9, suppose $(\beta_A, \beta_B, \beta_C)$ is a time-invariant equilibrium of the unobservable-offer model. Then, fixing β_C, condition *(ii)* of Definition 1 requires profile (β_A, β_B) to be a time-invariant SPE of the automaton game induced by $\beta_C(protest) \in [0, 1]$. Note that, irrespective of the specific value of β_C, Lemma 12 implies that this game admits a unique time-invariant SPE where the proposer offers to seize all the resources from Charlie (i.e., $x_C^* = -1$), and the responder immediately accepts. Since $f < p$ and Charlie's conjecture must be correct in equilibrium, we conclude that Charlie must protest with probability 1 on path, i.e., he must set $\beta_C = \delta_{protest}$. This completes the proof of Proposition 9.

Q.E.D.