Economies of Density and Congestion in Equipment Capital*

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January 2024

ABSTRACT

Equipment rental markets are growing in developing countries as means to improve access to technology to small-scale producers, particularly in agriculture. Yet, the efficiency and distributional implications of these markets are not well understood. We propose a novel theory of these markets with two sided heterogeneity: demand varies by hours requested, location, and the returns from service; while supply varies by the technology for service provision. Profit maximizing providers prioritize large-scale demand, because the cost of moving equipment in space dilutes with scale; as well as small-scale demand in dense locations, because it maximizes machine-capacity utilization. We assess the merits of different market arrangements quantitatively, leveraging unique transaction level data from a rental market in India to calibrate our model. We show that deregulating service provision to induce providers to behave as profit maximizers can increase aggregate productivity by 2% while maintaining service finding rates for small-scale farmers in line with those obtained when dispatchers are induced to prioritize them. We also show that an increase in the supply of capital has non-linear effects on productivity gains, which taper off; and service finding rates for small-holder farmers, which accelerate in service capacity. Ultimately, efficiency gains depend on the joint spatial and productivity distribution of demand.

JEL codes: O13, O18, O47, R41.
Keywords: Economic Geography, Mechanization, Search, Rental Markets, Agriculture.

*We benefitted from discussion with Douglas Gollin (our discussant) as well as Kevin Donovan, Joe Kaboski, Philip Kircher, Diego Restuccia, Todd Schoellman, Will Strange and Shouyong Shi, as well as seminar participants at Penn State, Syracuse University, Minneapolis and St. Louis FED, Cornell University, SED, Y-RISE and BREAD annual meetings .

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1 Introduction

Equipment rental markets are becoming popular in developing countries as a means to improve access to technology to small-scale producers. Yet, the efficiency and distributional implications of these markets are not well understood. This paper fills the gap by answering the following questions: Is the expansion of equipment rental markets beneficial to aggregate productivity in agriculture and welfare? A key margin to answering this question is that agricultural production is highly time-sensitive and delays in service provision are costly for farm output. A related and equally important question is what are the distributional implications of alternative empirically-relevant market arrangements? Two margins are relevant to answering this question. First, equipment travels in space to provide service and hence, larger orders help dilute travel costs. Second, unused machine-service capacity is costly for providers, and hence, servicing demand of small magnitude in spatially dense areas, maximizes utilization. These features generate economies of density; and congestion in demand, because demand for equipment is synchronous.

We propose a frictional model of search and matching in the allocation of capital services. Such a model is the natural framework to study queueing, as well as sorting and rental rate dispersion, which are empirically relevant features of these market. As in the labor search tradition, the main friction built into the model is that it takes time for an agent to find an equipment provider willing to provide service at a price that is acceptable for both parties.[1] Formally, we model directed search with two sided heterogeneity: demand varies by hours requested, location, and the returns from service; while supply varies by the technology for service provision. Providers set prices with commitment and agents build expectations about the queue lengths when deciding where to stand in line. Providers can accommodate multiple services per period and face service capacity constraints in terms of machine hours, the two main generalization relative to Shi (2002). The first feature allows us to discuss compositional changes in serviced orders across demand size (hours) as well as to optimize service provision in space. The second feature, paired with discreteness in hours demanded, speaks directly to the role of small-scale orders in maximizing capacity utilization within a period. Farmers understand that providers offering lower equipment rental rates are those

[1] As highlighted by Lagos (2000) and Sattinger (2002), queueing models are powerful to micro-found a matching process between, in this case, farmers' orders and service providers.

where waiting times for service provision are longer, and they trade-off the probability of service, with the cost of service. Once they commit to a provider, they stand in line for a period.

We model providers' dispatch technologies following two prevalent methods in the markets that we study: profit-maximizing dispatchers ("market" providers), and first-come-first-served dispatchers ("fcfs" providers). While the former ranks orders by profitability, the latter does not and is closer to provisions often seen in markets subsidized by government funds, where distributional concerns are salient. The main predictions of the model are that when small- and large-scale farmers are equally distributed in space, small-scale farmers are more likely to approach the fcfs providers than the market provider. Travel costs imply that smaller orders are more costly to serve so a profit-maximizing dispatchers would shy away from them. Conversely, large-scale farmers are more likely to approach market providers. At the same time, small orders could be beneficial to providers, whenever located in dense locations, since they help maximize machine-capacity utilization. Sorting induces disparities in service finding rates between small and large-scale farmers but in equilibrium, rental rates adjust so that farmers face identical expected profits from either provider.

Salient features of the market that we study are delays in service provision, which the model rationalizes through endogenous heterogeneous service rates. In our own survey of 7000 households in the state of Karnataka, we show that farmers list delays in service provision as the most prevalent problem in accessing equipment rental services, a more prevalent barrier than lack of credit or financing. This data also reveals that delays are disproportionally borne by small-scale farmers, and that this disparity is partly driven by the spatial distribution of plots, i.e. farther away from service providers. This pattern of delays is fully rationalized by the model, through the density of demand channel. Delays in access are a relatively understudied barrier in accessing technology and particularly relevant in rental markets where returns are time-sensitive, as in agriculture.

We can use our model economy to study the equilibrium implications of alternative market arrangements for aggregate productivity and service findings rates to small-holder farmers. Before doing so, we augment our stylized theoretical framework to more heterogeneity in equipment demand, spatial location and productivity distributions, and work with simulated outcomes. We do so consistently with the empirical distribution of demand for rental services that we observe in the state of Karnataka India. This is a useful environment for our assessment because equipment ownership rates are low, small-scale producers are prevalent, and travel times could be large across locations. Perhaps most importantly, rental markets for equipment are active.

We discipline the model in two stages: in the first stage we target the queue lengths for small-scale farmers, as observed in the transaction level data from hubs; the share of large scale farmers in each market, as inferred from the Census data; and the observed average profitability of hub providers. Hub providers are have a technology for provision that resemble the FCFS. The reason is that the set up of these hubs was partially subsidized by the government with the intent of granting access to technologies to small-scale producers. With these targets, we jointly calibrate the composition of farmers in each market along large and small scales, the ratio of farmers per provider and the cost of service provision as parameterized by the wage of equipment drivers. The main outcome of this calibration exercise are the endogenous queues by provider and farmer type, and the equilibrium rental prices per hour of equipment.

In the second stage, we augment the model to acommodate further heterogeneity. We bootstrap queues from the empirical joint distribution of service-hours demanded, productivity and plot location, and the assess service finding rates, and equilibrium productivity costs. In the simulated economy, we also test the implications for efficiency and distributional outcomes of alternative market arrangements, as well as technologies to optimize service routes given the realization of queues.

First, we ask how does the current equilibrium where FCFS and market providers coexist compare to allocations where only equipment owners are allowed to supply rental services. One could interpret this counterfactual as measuring the effect of a government induced increase in supply consistent with the creation of service hubs in the locations under analysis. We find that service capacity increased dramatically relative to ownership rates, and that lead to two-to-three fold increase in service findings rates, and declines in the cost of service provision of more than 20% per hour serviced.

Second, we investigate alternative market deregulation scenarios, where we allow providers in hub to maximize profits, irrespective of its distributional consequences. The short run response of the economy (with no endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the FCFS provision. However, service finding rates for large-scale farmers are below those of small scale farmers, suggesting no detrimental effect on market access to small scale access. This small scale producers are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers in the short run.

In the long run, once the number of providers can adjusts through entry and exit, rental costs fall again towards their baseline levels, and service findings rates are still higher for small holder farmers than large holder farmers. Importantly, we show that aggregate productivity costs from delays are 2% lower than in the benchmark equilibrium, suggesting that the deregulation improves allocations. To model entry and exit into the market, we assume that in the deregulated markets providers pay an operating cost equal to an annuity of the expected profitability of a provider. Hence, in equilibrium, the measure of providers in the market is such that they exactly cover the operating cost.

Third, we ask what happens to allocations as we increase the overall supply of equipment services. We can show that service finding rates for small scale farmers are non-linear in the supply of services and that they demand on the joint spatial and size distribution of demand. Only large enough increases in supply benefit small-holder farmers relatively to large-scale farmers. Indeed, we show that market suppliers have comparable wait times to a fcfs service dispatch for plots that entail lower travel costs, irrespective of their size. The largest differences in service wait times, and productivity costs associated to delays across providers are concentrated in farms operating farther away from service hubs, i.e. where spatial density is lower.

Literature Review Delays are often overlooked as a barrier to technology adoption, yet they are a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of activities.[2] The role of barriers to technology adoption in agriculture as a source of low productivity in poor countries has received extensive attention (see Suri and Udry, 2022 for a review). We contribute to a growing literature that highlights

[2]Other markets with similar features include the market for perishable products, or the allocation of personal shoppers. There is an extensive literature that studies taxi markets which also feature frictional meetings between customers and providers Frechette et al. (2019), however, costs bared by consumers from delays in service provision are typically abstracted away.

the importance of adopting mechanized practices, Caunedo and Keller (2020); Caunedo and Kala (2021), by drawing attention to rental markets, which could be disruptive in lowering adoption costs relative to labor, (Yang et al., 2013; Manuelli and Seshadri, 2014; Yamauchi, 2016). We also contribute to the literature that emphasizes geography, as a driver of productivity disparities in agricultural production, e.g. Adamopoulos and Restuccia (2021).

To our knowledge, this is the first paper to study the distributional consequences of rental markets for productive capital. In particular, we focus on heterogeneous access by scale of production and spatial allocation. A novel dimension of our problem is that service capacity constraints bring value to small-scale service orders because they improve capacity utilization. The value of small-scale producers in bringing density to demand has been mostly overlooked in the literature. We propose a tractable model that expands the seminal work of Shi (2002), along two relevant dimensions: multiple service provision within a period and service capacity constraints.

The notion that there might be scale economies associated to concentrating production in certain locations goes back to Marshall (1890). Holmes and Lee (2012) explore it in the context of crop choices of adjacent plots, where agglomeration economies rely on economies of scale in output. In our application, agglomeration economies stem from lower transport costs for service provision as in models of trade, Rossi-Hansberg (2005), as well as from the indivisibility of capital purchases, which generates incentives for sharing services through rental markets. Duranton and Puga (2004) review the micro-foundations for agglomeration economies and classifies them into three mechanisms: "sharing", "matching" and "learning". In our framework the first two mechanisms are at play. A paper that studies the "sharing" mechanism is Bassi et al. (2022), with an application to rental markets for door producers in urban Uganda, where they argue frictions are relatively limited. In contrast, we document substantial price dispersion in rental rates paired with unused service capacity, a common symptom of matching frictions. We document higher price dispersion for farmers operating lower scales, consistently with our theory, where smaller farms face more variation on the probability of service across providers. We are explicit about the role of "matching" in generating service transactions between providers and input demanders, and how they affect rental prices and queueing behaviour.

Finally, our quantitative results are of relevance to rationalize the seemingly contradicting and heterogeneous impact of mechanization policies throughout Africa, LatinAmerica and Asia. Pingali (2007) argues that "public sector run (...) tractor-hire operations have neither been successful nor equitable". He suggests that mechanization attempts have failed because market infrastructure and economic incentives that induce production response were not there. Our theory and quantitative assessment suggests that the success of increases in equipment supply in improving accessibility to capital and ultimately, productivity, depend on the joint spatial and size distribution of farms.

2 A model of capital rental services in space

We build a model of capital rental services where farmers of different plot sizes and locations search for equipment providers with different technologies for service provision. Some providers prioritize high value requests whereas others simply use a first-come-first-serve dispatch system. The latter, simpler dispatch system, may service requests that would otherwise be rationed out from provision. Formally, we model a two-sided heterogeneity directed-search framework, Shi (2002), where farmers request different hours of service, and where providers use alternative technologies for service provision. We further extend this framework along two dimensions. First, we allow for multiple orders to be served within each period; enabling the study of optimal service routes and the role of travel time in assessing value across orders. Second, we build-in providers' service-hours capacity constraints, enabling the study of congestion and equilibrium service delays as a function of the composition of the service queue along the size and geographical location of demand.

2.1 Environment

Consider an economy populated by F farmers, heterogeneous in their service-hours demand and location; and H service providers (machines), heterogeneous in their dispatch technology and location. A market is defined as a catchment area around any of these providers, and locations are exogenously given. A fraction s of farmers are large-scale farmers and demand k_s hours, while the remaining $(1-s)$ fraction are small-scale farmers, and demand k_{s^-} hours.

Hours demanded are determined by land holdings, and plots are either mechanized or not, bringing discreteness in demand.[3] A fraction h of providers use a first-come-first-served (*fcfs*) dispatch technology, while the remaining fraction $1-h$ has access to a selection technology that allows them to prioritize high value service requests (*mkt*).[4] For simplicity, we assume no depreciation or capital accumulation and no maintenance costs. Service providers have machine-hours capacity constraint $\bar{k}$ per day, which has implications for the number of orders that can be served within a day.

Denote the ratio of farmers to service providers, $f = \frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \to \infty$ and neither side is infinitely larger than the other, $f \in (0, \infty)$. Providers post prices r_{ij} indexed by the scale of demand i and the type of provider j; and a selection criteria (with commitment) $\chi_j \in [0, 1]$ simultaneously at the beginning of each period. The selection rule is a technology only available to the *mkt* provider, and applies whenever he receives requests from both types of farmers. The provider prefers the large scale farm if $\chi_j = 1$, prefers a small scale farm if $\chi_j = 0$, and he is indifferent between them for $\chi_j \in (0, 1)$. When the *mkt* provider receives requests from a single farmer type, he randomly selects one farmer for service. The *fcfs* provider serves orders as they arrive in the queue.

Geographical considerations for service provision are included into the opportunity cost of moving equipment from a provider to the plot, which includes the value of time for the equipment driver, i.e. his wage; as well as the value of the foregone services that could have been provided if the equipment would have not travelled, i.e. the shadow value of time as per the capacity constraint of the provider.

Farmers decide whether and which provider to approach, with commitment, generating queues for each available provider. Providers decide which orders to serve given their selection criteria and capacity constraint. Service provision takes place and farmers produce. Delays in provision occur in equilibrium inducing productivity costs for certain farmers. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize

[3] The capital demand for a farmer is given exogenously. We could trivially model the link between landholdings and capital demand through a Leontief production function between capital and land.

[4] Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

over the set of preferable providers.

A type i-farmer's strategy is a vector of probabilities $P_i \equiv \left(p_{i,\text{fcfs}}, ...; p_{i\text{mkt}}, ...\right)$ where p_{ij} is the probability of applying to each type j-provider. Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service.

2.2 Queue lengths as strategies.

A convenient object for analysis is the *queue length*, i.e., the expected number of farmers requesting a service from a given provider.[5] Let q_{ij} be the queue length of type i farmers that apply to a type j provider, where $i \in \{s, s^-\}$ and $j \in \{\text{fcfs}, \text{mkt}\}$. Then, $q_{sj} = sFp_{sj}$ and $q_{s^-j} = (1-s)Fp_{s^-j}$. The farmer type i is determined by its service-hours demanded and location.

Assumption 1: *Service-hours demanded satisfy $k_s > k_{s^-}$. The expected travel time to servicing small-holder farmers is weakly higher than that for large-scale farmers, $d_s \leq d_{s^-}$.*

The probability of approaching different providers for a single farmer should add up to one, which leads to the following feasibility constraints

$$H\left(hq_{s,\text{fcfs}} + (1-h)q_{s,\text{mkt}}\right) = Fs \tag{1}$$

$$H\left(hq_{s^-,\text{fcfs}} + (1-h)q_{s^-,\text{mkt}}\right) = F(1-s) \tag{2}$$

A farmer of scale i that requests service from provider j gets served with probability Δ_{ij}. This conditional probability depends on the provider's selection criteria, its capacity, machine-hours demanded k_i and the expected travel time for service d_i. Hence, Δ_{ij} is the sum across all possible number of orders of type i being served, $\bar{o}_i$, of the probability of servicing $\bar{o}_i$ type i farmers , $\phi_{ij}(\bar{o}_i)$, times the probability that a certain farmer of type i is chosen, $\tilde{\Delta}_{ij}(\bar{o}_i)$,

$$\Delta_{ij} = \sum_{\bar{o}_i \in \{1,2,3\}} \phi_{ij}(\bar{o}_i)\tilde{\Delta}_{ij}(\bar{o}_i).^{6} \tag{3}$$

[5] From a theory standpoint, when the number of providers and firms grow large, the probability of requesting a service to a given provider approaches zero and it is inconvenient to work with. From an empirical standpoint, queues are observable in our administrative data, while probabilities are not.

[6] The full derivation can be found in Appendix A.

The probability of type i being served (weakly) declines in the queue length of type $i' \neq i$ farmers. For the first-come-first-served provider the result is straightforward because service probabilities decline with the number of machine-hours in the queue, irrespective of their type. For the market provider with a selection criteria that favours type i' farmers, the decline in the probability of service for type $i \neq i'$ is strict as the number of type i' farmers in the queue increases. The service probability for type i' farmers is independent of the queue length of type $i \neq i'$ due to the selection criteria.

2.3 Farmer's decisions.

We follow Burdett et al. (2001) and describe a farmer's decision as a function of the market price it would get for the rental service, r_{ij}, which in turn determines its expected "market" profit, U_i. Farmers take the value of the market profit as given when the number of agents in the economy is large, $F, H \to \infty$. Each farmer chooses a service provider to minimize costs given U_i and the production technology:

$$\min_j r_{ij} k_i$$

subject to

$$\pi_{ij}(z_{ij}, k_i, r_{ij}) \equiv \Delta_{ij} \left(z_{ij} k_i^{\alpha} - r_{ij} k_i \right) \geq U_i,$$

where π_{ij} are the expected profits of the farm when requesting service from provider j and $z_{ij} \equiv E(z(\Delta_{ij}))$ is the expected productivity in the farm, which is a function of expected service probability, through equilibrium delays.

Farm's productivity depends on the realization of a random shock that yields the timing of agricultural activities. We summarize the optimal timing for agricultural activities by the optimal "land preparation" date, $\theta^{\star}$, and relate deviations from this optimal timing to productivity costs. The realization of the land preparation date is a random draw, θ, from a known distribution $G(\bar{\theta}(\Delta_{ij}))$ with mean $\bar{\theta}(\Delta_{ij})$ that depends on provider j's probability of service. We assume that $\frac{\partial \bar{\theta}(\Delta_{ij})}{\partial \Delta} < 0$ so that a high probability of service induces shorter wait times. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date as follows,

$z = \bar{z}(1 - \eta(\theta - \theta^\star)I_{\theta^\star \leq \theta})$, where η is the productivity cost per delayed service day in percentage points. Expected productivity is

$$z_{ij}(z(\Delta_{ij})) = \bar{z}(1 - \eta(\bar{\theta}(\Delta_{ij}) - \theta^\star)I_{\theta^\star \leq \bar{\theta}(\Delta_{ij})}).$$

The expected productivity is independent of the choice of provider whenever the expected wait time is relatively low, i.e. the probability of service is high. Finally, because the draw of the service provision is idiosyncratic, there is no aggregate uncertainty in the economy and factor prices are time independent.

A type i farmer requests a service from a type j firm with positive probability if the expected profits are weakly larger than U_i. The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1, yielding $q_{ij} \to \infty$ as the number of farmers grows large. Then, $\Delta_{ij} \to 0$ contradicting that $\pi_{ij}(z_{ij}, r_{ij}, k_i) > \tilde{U}_i$.

The farmers' strategy is

$$\begin{aligned} q_{ij} \in (0, \infty) \quad & \text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) = \tilde{U}_i \\ q_{ij} = 0 \quad & \text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) < \tilde{U}_i \end{aligned} \tag{4}$$

This expression summarizes the tradeoff between lower provision cost and higher farming profits; and a lower probability of service. Given the shape of the probability function (which enters into expected profits, π) there exist a unique queue $q(r_{ij}, U_i)$ that satisfies the problem of the farmer. The farmer decides his queueing strategy as a function of his capital demand, k_i, expected productivity z_{ij} and market prices r_{ij}.

2.4 Service provider's decisions.

A service provider j maximizes expected returns. The cost of servicing a farmer depends on its location relative to the provider. Providers choose the cost of service r_{ij} taking the the machine-hours demanded by each type of farmer and their locations as given. For a vector $\{U_i, k_i, d_i\}_{i=s,s^-}$, he chooses the queue lengths by picking the cost of service and service strategy. The queue length is reset at the end of each period and therefore the service provision problem is static.[7]

The service capacity of providers satisfies Assumption 2.

Assumption 2: *Providers hold capacity that satisfies*

$$o(k_{s^-} + d_{s^-}) <= \bar{k} \quad \text{and} \quad o(k_s + d_s) > \bar{k},$$

$$(o-1)(k_s + d_s) + k_{s^-} + d_{s^-} <= \bar{k},$$

$$(o-1)(k_{s^-} + d_{s^-}) + k_s + d_s <= \bar{k}.$$

Hence, if the provider serves only large-scale orders, it can serve $(o-1)$ orders, or it can instead combine those $(o-1)$ orders with one-small scale order. Service capacity is also enough to serve $o-1$ small scale orders and one large scale order. In either case, the provider serves up to o orders within a period. In computing service probabilities, we assume an empirically relevant upper bound for the number of orders per machine of $o = 3$.[8]

The cost of travel time includes the foregone services that could have been provided if the equipment was not traveling, as well as the opportunity cost of the driver, which commands a wage w per hour.

First-come-first-served provider. Consider the problem of a *fcfs* provider. His value is the expected return from servicing at most $o = 3$ orders within each period. Let $\bar{o}_i \leq o$ be the number of orders of type i being served within the period. The per period return $\tilde{V}$ from facing queue q_{fcfs} depends on the number of orders of each type being served, $\{\bar{o}_s, \bar{o}_{s^-}\}$ and the revenue per type net of labor and transportation costs, $\{r_{i,\text{fcfs}} k_i - w k_i - w d_i\}$.[9] The value for a first-come-first-served provider is

$$V_{\text{fcfs}}(\bar{k}) = \max_{\{r_{i,\text{fcfs}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{q_{\text{fcfs}}}, \{r_{i,\text{fcfs}} k_i - w k_i - w d_i\}_{i=s,s^-}\right), \tag{5}$$

[7] This feature allow us to handle the high dimensionality of the combinatorics problem when providers are allowed to prioritize certain farmer types.

[8] This value is consistent with the median number of orders served within a day in our administrative data as we show in Section XXXX.

[9] We assume the revenue is separable in the number of orders and relax this assumption in the quantitative exercise, when the provider minimizes transportation cost across orders.

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{fcfs}}} k_i + d_i \leq \bar{k}. \tag{6}$$

Market provider. Consider now the problem of a *mkt* provider who, in addition to choosing the cost of provision, $r_{i\text{mkt}}$, chooses a selection criteria χ. This choice in turn determines the type of orders being served and their quantity, given service capacity. The value of a market provider is

$$V_{\text{mkt}}(\bar{k}) = \max_{\chi, \{r_{i,\text{mkt}}\}_{i=s,s^-}} \tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{(q_{\text{mkt}},\chi)}, \{r_{i,\text{mkt}} k_i - wk_i - wd_i\}_{i=s,s^-}\right), \tag{7}$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{mkt}}} k_i + d_i \leq \bar{k}. \tag{8}$$

The full description of the value of these providers, $\tilde{V}$, can be found in Appendix B.

Entry. Providers pay an operating cost I_j. Free entry assures that their expected profitability equals their operating cost.

$$I_j = V_j(\bar{k}). \tag{9}$$

3 Symmetric Equilibrium

A symmetric equilibrium consists of farmers expected profits U_s, U_{s^-}, provider strategies r_{ij}, χ, and farmer strategies, q_{ij} for $i = \{s, s^-\}$ and $j = \{\text{fcfs}, \text{mkt}\}$, that satisfy:

1. given U_s, U_{s^-} and other providers' strategies, each type provider maximizes value, equations 5 and 7;

2. observing the providers' decisions, farmers choose who to queue with, equation 4;

3. the values U_s, U_{s^-}, through q_{ij}, are consistent with feasibility, equations 1 and 2; and

4. providers values $V_{\text{fcfs}}, V_{\text{mkt}}$ satisfy free-entry.

Proposition 1. *In all symmetric equilibria where providers serve both types of farmers, the selection process is $\chi = 1$ and the per period profit of servicing farmers of type i is V_i^j:*

$$V_i^j = \gamma_{1i}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) + \gamma_{2i}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}),$$

where $\gamma_{1,i}^j, \gamma_{2,i}^j$ are non-linear functions of the queue lengths and the elasticity of the service probabilities with respect to the length of the queue and $\tilde{z} \equiv \bar{z}(1-\eta)$.[10]

The expected per period value of servicing large-scale farmers is higher than for low-scale farmers, $V_s^j > V_{s^-}^j$. If the surplus from large-scale orders is sufficiently larger than from small-scale orders, the expected profit for large-scale farmers is greater than for small-holder farmers, $U_s > U_{s^-}$.[11]

A few characteristics are worth highlighting. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. For two plots located at the same distance to the provider, the marginal cost of service is lower for larger scale farmers. Second, small-scale farmers are useful in terms of capacity utilization (Assumption 1) and therefore, even providers that prioritize large-scale farmers have incentives to serve them. Third, the *fcfs* provider manages to attract some large-scale farmers by lowering their rental costs relative to the *mkt* provider. These lower costs for both large and small farmers compensate them for higher expected queues at the *fcfs* provider. Finally, the farmers' expected profit from equipment services depends on the return to his own demand for services and on the equilibrium rental rates. In equilibrium, farmers that are served by both providers shall be indifferent between them. Hence, the product between the probability of service, conditional on machine-hours demanded, and the cost of service should equalize across providers.

[10] The expected productivity $\tilde{z}$ is a log-linear function of an exogenous component $\bar{z}$ and an endogenous component $1 + \frac{\partial z_{ij}}{\partial \Delta_{ij}} \frac{\Delta_{ij}}{z_{ij}}$ which depends on the elasticity of productivity to the probability of service. Under Assumption 3 in Appendix B this elasticity is constant an equal to $-\eta$. This assumption guaranties that the surplus from transactions is independent of the probability of service.

[11] The ratio of the surpluses $\frac{\tilde{z}k_s^\alpha - wk_s - wd_s}{\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}}$ must be larger than a constant that depends on the elasticity of the probability of service, see equation 24 in Appendix B.

4 Equipment rental markets: empirical evidence.

We are now ready to discuss the empirical features of the rental market for equipment. To do so, we bring in novel data from the state of Karnataka in India, a state where equipment ownership is low and small-holder farmers are prevalent. Due to these features, the potential disruptive effect of equipment rental markets could be large, a question that we further study in Section 5. Most importantly, this is an interesting market to study because its development, partially aided by government incentives, is such that market and first-come-first-serve providers coexist in service provision. The latter constitutes an attempt to ensure smallholder access to mechanized services, whose effectiveness and costs, we also study in Section 5.

4.1 Data description

We combine four sources of data. First, we use transaction level data from the universe of equipment rentals engaged through a public-private platform, within the context of a mechanization program in the state of Karnataka that started in 2016. Our data corresponds to all recorded transactions during the Kharif season of 2018 (May-October). The data contains information on number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Second, we use our own census of farming households covering 40,000 households across 150 villages, including information on equipment ownership and rental market engagement. Third, we use detailed survey data that we collected over 5500 farming households with information on ownership and rental market engagement, including information on equipment rental pricing, delays in service provisions, as well as output and input expenses, crop choices and land ownership.

4.2 Equipment supply

Farmers in the area rely on informal rental markets in the village. These transactions are usually on short-term credit (1-3 weeks), and the rental price of the equipment varies across the season, with prices increasing during peak cultivation times (which are times of higher demand) and falling when demand is low. Most operators of equipment are farmers who own the equipment and rent it out to utilize the slack capacity. Equipment ownership is relatively low and so is the supply of services from this source.

Equipment is also available through custom-hiring-centers (CHCs, or hubs). The median hub provides a menu of equipment that ranges from sprayers to rotavators. We focus our analysis on rentals of rotavators and cultivators, which are the pieces of equipment most commonly used at the land-preparation stage. Land-preparation is the process where mechanization is prevalent in our sample (Caunedo and Kala, 2021).

LEFT HERE We start by reporting patterns of ownership (service capacity by farmers) and rentals of equipment across the farmers in our survey (see Figure 1).[12] Most farmers report owning hand tools and animal pulled equipment. Less than 10% of the farmers report owning larger equipment such as tractors, or rotavators and cultivators. At the same time, tractors and cultivators are among the pieces of equipment with the highest service-hours rented. The average hours rented in a season per farmer is 12 hours for tractors and 10 hours for cultivators. These rental transactions mostly entail relational contracts. We collect information on the typical customer for a farmer that rents out his/her equipment. We find that 72% of owners report to renting out to people they know from the village or with whom they have worked with in the past.

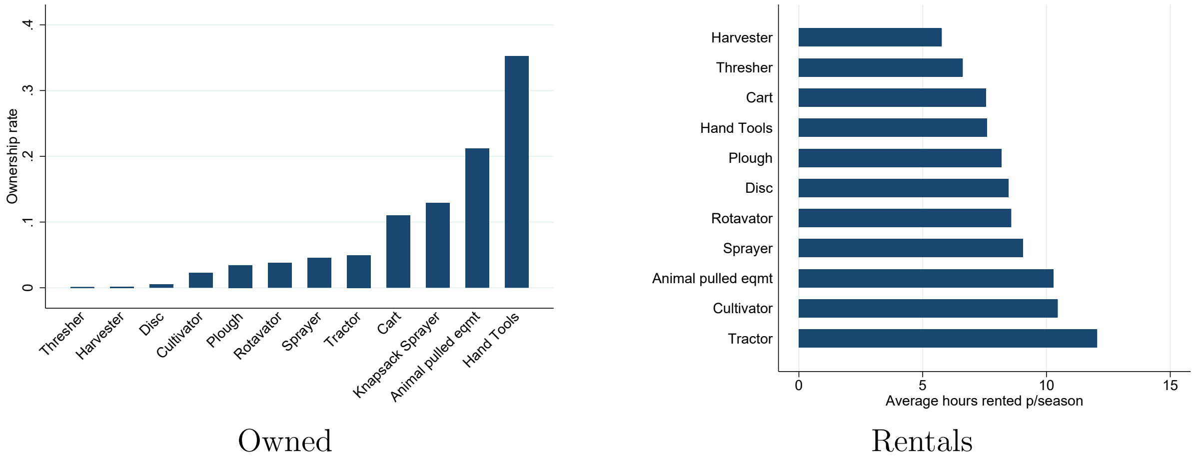
Delays are the most common issue faced by farmers when renting equipment, with 78% of farmers reporting it as an issue. Importantly, larger farmers (cultivating at the 75th percentile of the land size distribution) are nearly 5 percentage points less likely to report delays as an issue. Hence, delays in accessing mechanization are more pervasive among smaller farmers.

Given the disparities in value of agricultural implements as well as their contribution to production, it is useful to construct a measure of equipment services from rentals and owned equipment. We measure these services as the product of average hours of usage during a season h_i, market rental rates, r_i and the number of implements i owned or rented, N_i. Hence, equipment services in a farm k are

$$k = \sum_i N_i r_i h_i$$

[12] Appendix D reports similar statistics using data from the Census.

Figure 1: Ownership and rentals by implement.

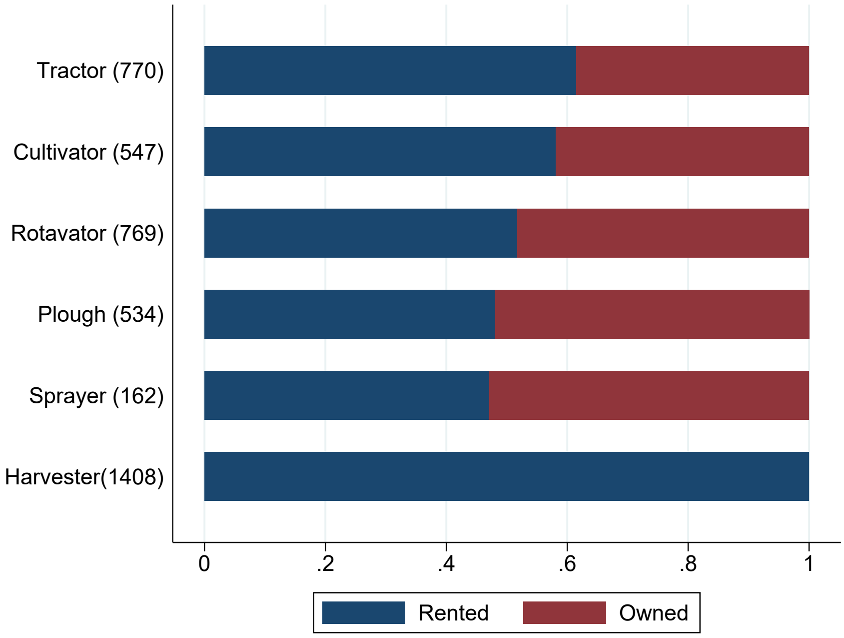


The ownership rate is the share of farmers that report owning a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season.

The main hypothesis behind this measure is that differences in rental rates across implements shall reflect differences in the services they provide, and that therefore, more expensive equipment provides higher services to production. The main challenge in constructing such a measure is the availability of data on market rental rates. We exploit our transaction level dataset to construct mean rental rates per implement at the village level. Figure 2 displays log owned and rented services. Harvesters (the most expensive implement in our bundle) is reported to be only rented. For those farmers using tractors, more than 60% of the services available in the farming sector come from rentals whereas the remaining 40% stem from ownership. Services associated with smaller and cheaper equipment, such as sprayers, are equally accounted for by rentals and ownership.

It is worth noting that given land holdings, ownership of equipment is not cost-effective for most farmers. For instance, the rental price of a rotavator is between ₹750 and ₹1,000 per hour (including tractor, a driver and fuel) and the average farmer demands about 6 hours of rotavator services in the season or between ₹4500 and ₹6000 in services. The purchase price of a new rotavator is over ₹110,000 which means that, absent maintenance costs (which are certainly non-negligible), the average farmer needs 19 years to amortize the investment. The rental rate for an inferior technology that serves a similar purpose, i.e. a harrow, is half of the rental rate of the rotavator (₹360) and the cost of purchase is about ₹50000. Overall, these price differentials are consistent with the observed extensive engagement in rental markets for equipment.

Figure 2: Capital services from ownership and rentals, by implement.



Shares of log capital services by implement and ownership/rental status. Average rental rates for an hour of service (in ₹) are reported next to each implement.

4.3 Equipment demand

Table 1 reports the 10 most commonly rented implements from the platform in years 2017 and 2018, the number of transactions recorded for each implement, their per-hour rental price, and month where the implement is most commonly rented (has the highest number of transactions).

4.4 Equilibrium outcomes

Unfortunately, we cannot directly assess the cost of these delays in service provision with our data, because such an exercise would require high-frequency (daily) information on agricultural activities and outcomes. To assess these costs, we bring in high-frequency data from International Crops Research Institute for the Semi-Arid Tropics (ICRISAT), covering

Table 1: Summary Statistics of Commonly Rented Implements from Rental Database

	Commonly Rented Implements		
	Number of Transactions	Median Price Per Hour	Peak Month
Rotavator 6 Feet	11,239	770	July
Cultivator Duckfoot	7,287	550	April
Cultivator 9 Tyne	5,245	525	May
Plough 2MB Hydraulic Reversible	3,716	450	February
Trolley 2 WD	2,436	250	January
Harvester Tangential Axial Flow (TAF)-Trac	2,048	1800	May
Rotavator 5 Feet	1,811	700	September
Blade Harrow Cross	1,793	360	March
Knapsack Sprayer 20 Litres	1,688	22.5	October
Blade Harrow 5 Blade	1,600	360	June

6,200 plots in 18 villages in India during 2009-2014, with daily detailed measures of inputs and output in farming. We exploit data from villages with similar crop choices as those in the state in Karnataka.

5 Quantitative implications

In this section, we bring the model to the data to characterize how allocations change with the presence of a *fcfs* dispatch system relative to the market dispatch system, both in space and across farmers of different production scales. The key outcomes of interest are the selection of farmer types across providers, the equilibrium delays and therefore farming productivity costs, as well as provider profitability.

We then ask whether small scale farmers are hurt by this deregulation despite efficiency improvements in allocations. Our market deregulation consists of (i) allowing first-come-first-serve providers to have access to a service selection technology, and (ii) allowing providers' entry and exit in the market mimicking the "long-run" equilibrium for the deregulated market. We show that the increase in capital supply from the subsidy is large enough to generate a disproportionally higher increase in service probability for small-holder farmers relative to large-scale ones. The relatively higher service probability for small-holder farmers is however not warranted for every level of the subsidy: only large enough subsidies benefit small-holder farmers relatively more.

5.1 Motivating facts

We start by describing the characteristics of the service demand and farmers equipment supply. Then, we focus on a handful of outcomes that are informative to the theory that we describe in Section 2. First, because agricultural activities are highly time sensitive, the timing of demand is synchronous leading to endogenous waiting times as a function of service capacity. The service capacity includes farmers' ownership as well as CHCs capacity. Second, because equipment needs to travel for transactions to take place, the joint distribution between travel time and the scale of demand, i.e. service-hours per request, is a key input when optimizing service provision. Third, we document substantial price dispersion in rental rates after controlling for observable household characteristics and village/market characteristics, consistent with frictional rental markets. Fourth, delays in service provision are costly to farmers, because they affect field productivity. In what follows we document each of these features.

5.1.1 Service Capacity and Service Demand.

5.1.2 Heterogeneous Queuing by Production Scale

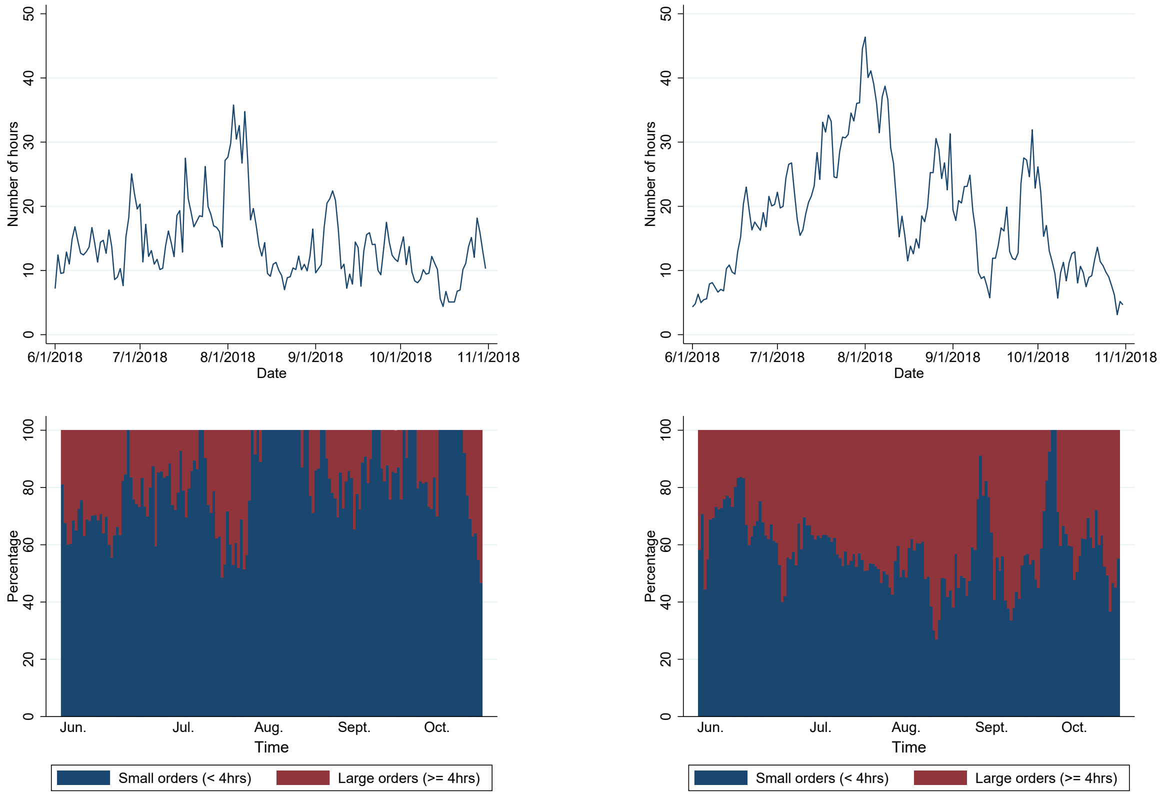
The demand for equipment rental services vary by agricultural process and therefore throughout the agricultural season. The synchronous nature of many of these processes across farmers induces queuing in the market. Our transaction level data allows us to measure demand fluctuations by computing hours outstanding for service at a daily frequency. We focus on two commonly rented implements for land preparation, rotavators and cultivators. Indeed, our survey data indicates that farmers are most likely to engage in the rental market for land preparation.

Figure 3 shows hours of unfulfilled orders for each of these implements over the 2018 kharif season. Queueing peaks by the end of July for rotavators and beginning of August for cultivators. At the peak of the season, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders on average at a point in time.

Demand moves distinctively between large and small requests, measured in service hours (Figure 3). A large portion of hours outstanding are accounted for by small orders (less than

Figure 3: Hours outstanding in the queue.

Cultivator Rotavator



by size by size

Notes: Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

4 hours of service), although at peak time the share of hours accounted for by large farmers increases.

5.1.3 Delays in rental services

As demand fluctuates over the season in a somehow predictable manner, it is expected that service supply may adjust. If supply expands proportionally to the increase in demand, any delays in service supply could be constant across the season. We find that service rates fluctuate during the season, and that they positively correlate with hours serviced suggesting some adjustment in supply (see Appendix Figure 14). The relationship between hours in the queue and service rates is non-linear, increasing for low service rates and declining for high-service rates, suggesting longer delays then. At peak queue hours, service rates in the CHC are 40% on average, suggesting that it takes 2.5 days to go through a hub's queue.

Because our queue measures and service rates exploit data from the administrative platform only, we complement this analysis with delays reported by the farmers in our survey data. Service delays are negatively associated with cultivated area suggesting that even if the productivity costs of delays are of same magnitude between small- and large-holder farmers, the incidence of those delays is disproportionally borne by those with small plot sizes, columns (1) and (3) in Table 2. It is possible that these delays are explained by the geographical location of plots since equipment needs to travel to generate services. Columns (2) and (4) in Table 2 show that delays have an important spatial dimension, because adding village fixed effects substantially attenuates the coefficient on the log of land size, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays, but if this clustering is not accounted for, smaller farmers face longer delays.

Table 2: Delays as a Function of Land Area and Location Fixed Effects

	Delays (Sum of Average Delays Over the Season)			
Log(Area)	-0.215*	-0.144	-0.319**	-0.128
	(0.115)	(0.0926)	(0.145)	(0.108)
Observations	5,615	5,615	4,345	4,345
R-squared	0.002	0.182	0.003	0.252
Village Fixed Effects	No	Yes	No	Yes
Mean Delays	2.158	2.158	2.789	2.789

Estimated coefficients from a regression of reported delays in service provision and the log(area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

5.1.4 How costly are these delays?

We define an optimal planting time as the date that maximizes the profits per acre in a given village year.ICRISAT's high frequency data is particularly suitable for this exercise. Then, we define the cost of the delay as the difference in average value added per acre or profit per acre (depending on the variable of interest) as we move away from the optimal planting date. Formally, we estimate

$$Y_{i,year} = \beta_0 + \beta_1^+(\text{Planting Date-Optimal})_{>0} + \beta_1^-(\text{Planting Date-Optimal})_{\leq 0} + \alpha X_{i,year} + \epsilon_{i,year}$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Standard errors are clustered at the village level. Our estimates for the costs in value added per acre are reported in Table 9. They indicate that within a 5-day windows, each additional day away from the profit maximizing date entails a cost of 3.4% in terms of value added per acre. In the 5-day window, for farmers that plant too early relative to the optimal date, moving *closer* to the optimal date by one day increases value added per acre by INR 391 per acre. Conversely, for farmers that plant too late relative to the optimal date, moving *away* from the optimal date by one day reduces value added per acre by INR 215 per acre. Therefore, moving closer to optimal date increases returns. This result is robust to enlarging the window around the optimal planting date, with an estimate cost of delay in the planting date of 8.5% per day.

As a robustness check, we also estimate the cost of deviation with sowing time estimated at the weekly rather than daily level. These estimates are reported in Table **??**. They indicate a cost of delay of between 5.6% to 11.5% for an additional week's delay in terms of value added per acre, or between 0.8% and 1.6% per day of delay (depending on whether we restrict delay to be within 5 or 3 weeks of optimal sowing time, respectively). These estimates are also consistent with agronomic estimates that estimate yield losses from deviating from the optimal planting time in other contexts - for instance, Liu et al. (2023) estimate yield losses of about 13% for a 10-day deviation from the optimal sowing time for winter wheat in China, or about 1.3% per day.[13]

5.1.5 Frictional rental markets

But why are there delays to begin with? Is this a consequence of low ownership rates and service capacity, or rather the consequence of frictions in the rental market that prevent

[13]ICRISAT's sample sizes after controlling for crops are too small to reliably estimate the effects on yields. The point estimates for yield costs in crops such as pea are 5.3% over a five day window of the optimal planting date.

farmers and providers to contract services when desired? There are two features of the market that indicate the presence of frictions in the rental market.

The first features is that the current supply of equipment seem adequate to serve market demand. To compute supply we turn to a Census of 150 villages from the same area, which includes information on over 40,000 farmers. We assume that the equipment has a catchment area of about 10km, since transporting equipment over large distances is time-consuming and expensive, particularly for farmers whose main activity is not equipment rentals.[14] We also include machines available in the CHCs within each relevant catchment area.

On average, the number of available cultivators can serve up to 2016 orders per season, while average demand is 1190 orders. The number of available rotavators can serve up to 1008 orders in the season while market demand is 450 orders. In these computations we assume a six-week plant preparation season and that each piece of equipment serves three orders a day. The latter is consistent with serviced orders per equipment per driver at peak utilization in our transactions dataset.[15] Hence, these machine-hours supply and demand estimates within each geographical market suggests that congestion may not be related to supply shortages. This supply shortages may exist if farmers attempt to access equipment within a shorter span than the overall plant-preparation season, i.e. all in the same week. While there is certainly evidence of coordination in timing of demand in late july equipment demand is widespread throughout Kharif, a five-month span between June and October (see Figure 3).

The second, and perhaps most important feature, is the presence of price dispersion in rental prices of equipment within a 10km catchment area of each village. As part of our survey we ask farmers how much did they pay for land-preparation equipment rentals during the season prior to Kharif 2018. Plant-preparation equipment includes mostly rotavators and cultivators, implements that rent out for similar hourly rates at the CHC. Figure 4 panel A shows the distribution of rental rates paid per hour, controlling for village fixed effects. That is, the variation in rental rates per hour serviced across farmers within a village.[16]

[14]This 10km cutoff was decided based on conversations with our data partner but results are robust to enlarging the catchment area to 20km radius.

[15]Even if we shorten the plant preparation season to a time-span of four weeks, supply would account for 1344 orders for cultivators and 672 orders for rotavators, well over our estimated equipment demand.

[16]Observed dispersion is similar even after we account for total area cropped. Indeed, on average hourly rental rates are lower for farmers with larger cropped area. These results are available upon request.

The interquartile range is 1.71 while the coefficient of variation is 0.67. Importantly, we find systematic disparities in average prices paid as well as price disparities between large and small farmers (i.e. above and below the mean area cropped in our sample of 3.3 acres), Figure 4 panel B. We also find disparities in the dispersion in prices, which small farmers facing more dispersion in prices, consistently with higher queuing risk.

Burdett and Judd (1983) were the first to show that price dispersion could arise in an environment with identical agents where consumers/farmers found it costly to search for providers. Price dispersion can also be related to informational asymmetries (Varian, 1980) or to consumer preferences for certain providers over others (Rosenthal, 1980). Overall, the exchange of identical goods for heterogeneous prices is typically a sign of frictions in the market. Importantly to our application, mean hourly rental rates paid for identical services are distinct across farmers of different land-holdings (and therefore demand size), which we entertain through the structural model that we study next.

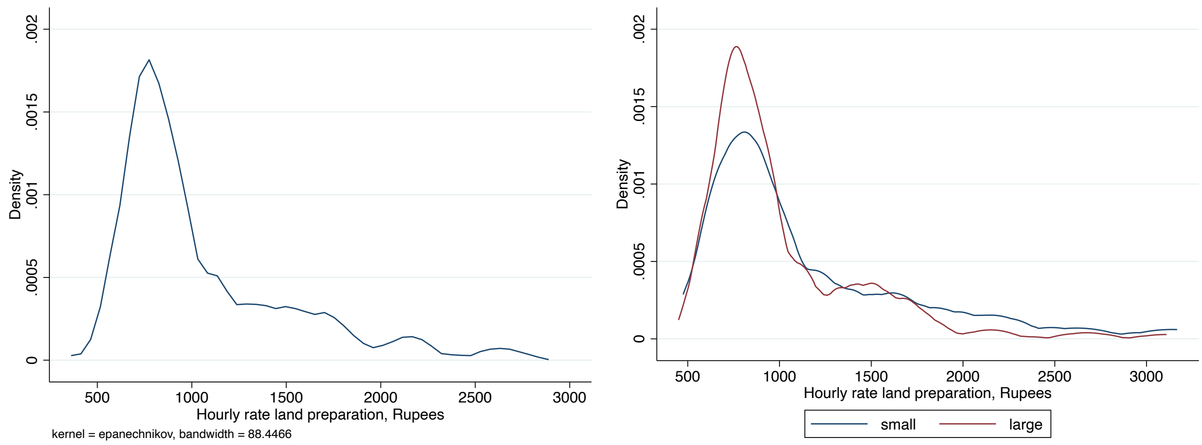


Figure 4: Rental rates

Notes: Hourly rental rates for equipment at land-preparation, including Cultivators and Rotavators. Panel (a) displays the residual rental rate dispersion after controlling for village fixed effects. Panel (b) presents these residuals for farmers with less than the average area cropped (small, < 3.3 acres); and for farmers with more than that area cropped (large, $>= 3.3$). Source: Own survey data.

5.2 Bringing the model to the data

The quantitative assessment of the impact of the government intervention in the rental markets for equipment consists of two blocks. The first block solves the model in Section 2 for the equilibrium market rental rates and queue lengths, given the empirical supply and demand for equipment services. The second block simulates queues and service provision strategies for farmers with different scales and geographical locations.

Solving the first block requires taking a stand on the heterogeneity in machine-hours demanded. We construct two groups of farmers following their average machine-hours requests in the transaction data: those with requests of more than 3.5 machine-hours per order are denominated large-scale while those with requests of less than 3.5 machine-hours are denominated small-scale. Then, we solve for an equilibrium in which both types of farmers are served by both types of providers, as in the data. We call this equilibrium the "status quo".

The second block involves finding the expected delay and subsequent productivity costs as well as provider profitability under alternative dispatch systems using equilibrium rental rates and queue lengths from the first block. In theory, the queue length itself yields the expected wait time by farm type. However, we recognize that empirically, farmer heterogeneity is richer than the one accommodated by the stylized theoretical model both in terms of machine-hours demanded and in the spatial allocation of demand. We simulate 1000 paths of queues of length $q^\star$ and composition $(q^\star_s, q^\star_{s^-})$ as dictated by the equilibrium of the selection model. The sample paths for queues $(q^\star_s, q^\star_{s^-})$ are drawn from the joint empirical distribution of machine-hours and geographical location. Then, given the equilibrium rental rates and the technology for dispatch, we let the provider optimize service delivery. The optimization of service provision in space is effectively the solution to a traveling salesman problem, conditional on the set of orders in the queue.

5.2.1 Parameterization

There are 10 parameters per hub that need to be calibrated, as shown in Table 3. Eight of these parameters are calibrated directly from the data while the remaining two are calibrated internally by solving the model. Consistently with the evidence in Section 4 we use data for the Kharif season (June to October) in year 2018. We exploit four sources of data: (1) detailed transaction data from the government subsidized service provider, (2) our own survey of farmers, (3) our own census of farming households in the catchment area of the subsidized service providers and (4) high-frequency data from ICRISAT.

From those parameters measured in the data, four of them are common across hubs: the providers' discount factor β, and their opportunity cost of moving equipment in space w, the productivity cost of delays η, and the curvature of the farming profit function α. The remaining 6 parameters are hub specific and include the share of first-come-first-serve providers relative to the total supply of equipment in the catchment area of a hub h, the parameters characterizing the joint distribution of productivity and machine-hours demand within the catchment area of the hub (i.e., mean and standard deviation of productivity and the correlation between productivity and machine-hours), the ratio of farmers demanding service to the providers in the catchment area of each hub f, as well as the share of large farmers in the population of farmers demanding equipment in the catchment area of the hub s. The latter two model-calibrated parameters are chosen to match the queue length of small-holder farmers at first-come-first-served providers, and to make sure the equilibrium displays positive queues of small and large-scale farmers with both providers, as we observe in the data. In addition to these 10 parameters, we feed the distribution of plots in space (and their corresponding travel-time) as measured from the platform data.

We set the discount factor to $\beta = 0.999$ with an implied daily discount rate of 0.1%. The opportunity cost of travel time equals the hourly wage of a driver which is directly observed from the platform data, at $w =$₹75. The curvature of the profit function is set to 0.6, as estimated from our own survey data on farm profitability. We exploit the fact that farming profits are proportional to this parameter, i.e. $\pi_i = (1-\alpha)y_i$ and estimate α from the average ratio of profits to value added as reported by farming households.

To discipline the productivity costs of delays, $\eta = 3.4\%$, we use high frequency data from ICRISAT as described in Section 4. We also need to calibrate the optimal planting date, $\theta^{\star}$. We assume enough service capacity such that farmers choose providers in a manner that on expectation, there are no productivity losses from using mechanized services, i.e. $E(z(\Delta_{ij})) = \bar{z}$.[17] Once queues are realized, productivity costs realize as a feature of the service provision process. Finally, we need a mapping between the probability of service and the realization of the service date, $\theta(\Delta_{ij})$. We set it to be a strictly monotonic function, i.e. $\theta(\Delta_{ij}) = -\ln(\Delta_{it})$. Its logarithmic shape brings tractability to the problem because

[17] This is analogous to an outside option that entails no usage of mechanization services and that is large enough so that farmers participate in this market only if on expectation, they face no productivity costs.

Table 3: Parameterization

Parameter	Description	Value	Source/Moment
Measured directly in the data			
common across hubs			
α	Curvature of the profits function	0.6	Survey data
β	Discount factor	0.999	Interest rate
w	Travel/op. cost (INR/hr)	75	Platform data
η	Productivity loss/day	3.4%	ICRISAT sample
hub specific		method/value	
h	Share of fcfs providers	Table 4	Census data
μ	Log-normal mean of productivity	MLE	Survey data
σ	Log-normal s.d. of productivity	MLE	Survey data
ρ	Correlation order size and productivity	Table 4	Survey + Platform data
k_i, d_i	Joint-distribution of order size and travel time	B-splines	Platform data
$\bar{k}$	hub-capacity (hours)	Table 4	Platform data, peak
Calibrated using the model (hub-specific)			
s	Share of large farmers	Table 4	Census data
f	No. of farmers/No. of equipment	Table 4	Small-scale queue, fcfs

Notes: Benchmark model parameterization. Productivity is measured as output per acre. Hub-capacity corresponds to the hours serviced per machine within a day at the peak of service demand, i.e. the maximum number of hours outstanding in the queue during the season.

it implies a constant elasticity of the delay time to the probability of service equal to the productivity costs of delays η, see Appendix B.

Then, we calibrate hub-specific parameters. We use our census, to compute the share of machinery available from government-subsidized hubs and that available from machine-owners (i.e. we count inventory of implements per hub and implements owned by farmers within the catchment area of each hub). To characterize the productivity of farmers requesting different machine-hours we use the subsample of transactions that overlaps with the survey data (approximately, 1,300 observations) and compute the underlying correlation between farm productivity, measured as output per acre, and machine-hours requested. Their correlation ranges from -0.28 to 0.35 displaying the wide-heterogeneity in demand characteristics across hubs, column (5) in Table 4. When machine-hours requested are proportional to plot sizes a negative correlation between output per acre and machine-hours follows from the negative correlation between productivity and farm size, as has been documented by others in the literature, e.g. Foster and Rosenzweig (2022). A positive correlation is consistent with more productive mechanized farms. Our data is rich enough to display both patterns. We assume that the distribution of productivity is log-normal, $\ln(\bar{z}) \sim \mathcal{N}(\mu, \sigma)$ and fit the empirical distribution of value-added per acre for survey farmers in the catchment area of each hub via maximum likelihood. The estimated mean of productivity suggests differences in log productivity across hubs of 36% (from 7.4 to 9.3) on average, and a log-variance ranging from 1.1 to 2.9, columns (3-4) in Table 4. Finally, we fit the joint distribution of machine-hours demanded and travel time to services from the platform data for each hub using B-splines, akin to a non-parametric estimation of the distribution, see Figure **??** in Appendix D. On the travel dimension, the distribution is typically bimodal, with orders bunching at less than 10-minutes travel time from the hub and 30-minutes travel time.[18]

Table 4: Hub specific characteristics.

	Measured Directly					Calibrated	
	Supply		Demand				*Farmers*
Hub	sh. fcfs	capacity	Productivity		Correlation	*sh. large*	*per provider*
	h	$\bar{k}$	mean	variance	prod - hours	s	f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	0.80	4	9.83	1.12	-0.01	*0.3*	*5.0*
2	0.86	4	8.89	1.33	-0.12	*0.3*	*4.6*
3	0.86	5	8.67	1.49	0.07	*0.3*	*4.4*
4	0.86	5	9.08	1.52	0.07	*0.3*	*5.2*
5	0.86	6	9.35	1.08	-0.20	*0.5*	*3.3*
6	0.86	6	9.08	1.52	0.07	*0.5*	*3.3*
7	0.86	7	9.08	1.52	0.07	*0.4*	*5.3*
8	0.63	8	8.85	2.56	0.16	*0.4*	*4.1*
9	0.75	11	8.85	2.56	0.16	*0.4*	*3.4*
10	0.67	11	8.15	2.89	0.01	*0.4*	*4.7*
11	0.75	14	8.85	2.56	0.16	*0.3*	*3.4*
average	0.79	7	8.97	1.83	0.04	*0.4*	*4.3*

Notes: Hub-specific parameters for each hub-implement combination, "Hub" in Column (1). Hubs labeled 3, 4 and 8 correspond to Cultivators while the remaining hubs contain information for Rotavators. Information for hubs labeled 4-5 correspond to different implements in a single government subsidized hub, and therefore demand characteristics are the same. Column (2) reports the share of first-come-first-serve providers in the total equipment supply within each catchment area. Columns (3)-(5) report demand characteristics for each hub, including the characteristics of the distribution of productivity across farmers and its correlation between hours demanded. Columns (6)-(7) report parameters calibrated jointly in the model.

[18]We could have alternatively calibrated a joint distribution of productivity, machine-hours requested and travel time. However, the overlap of the survey data and platform accounts for 20% of the survey data and we therefore benefitted including the fullness of the distribution of machine-hours and travel time. The latter is a key input into the costs of service and therefore the incentives to service large and small scale farmers.

Table 5: Moments

	Share of large scale s		Queue q_{s-fcfs}		Queue *untargeted* q_{sfcfs}/q_{s-fcfs}	
	data	model	data	model	data	model
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.29	0.45	1.5	3.0	0.5	1.4
2	0.25	0.30	2.3	3.5	1.2	3.2
3	0.19	0.30	3.3	3.3	3.3	2.9
4	0.28	0.30	4.0	4.0	4.0	3.4
5	0.12	0.40	1.3	2.0	0.7	1.8
6	0.35	0.40	2.0	2.0	2.0	1.8
7	0.19	0.25	7.3	4.3	1.5	3.8
8	0.35	0.50	1.0	2.5	0.3	2.2
9	0.39	0.40	0.7	2.0	0.2	1.9
10	0.31	0.30	2.0	4.0	0.5	4.1
11	0.28	0.35	1.0	2.0	1.0	2.0

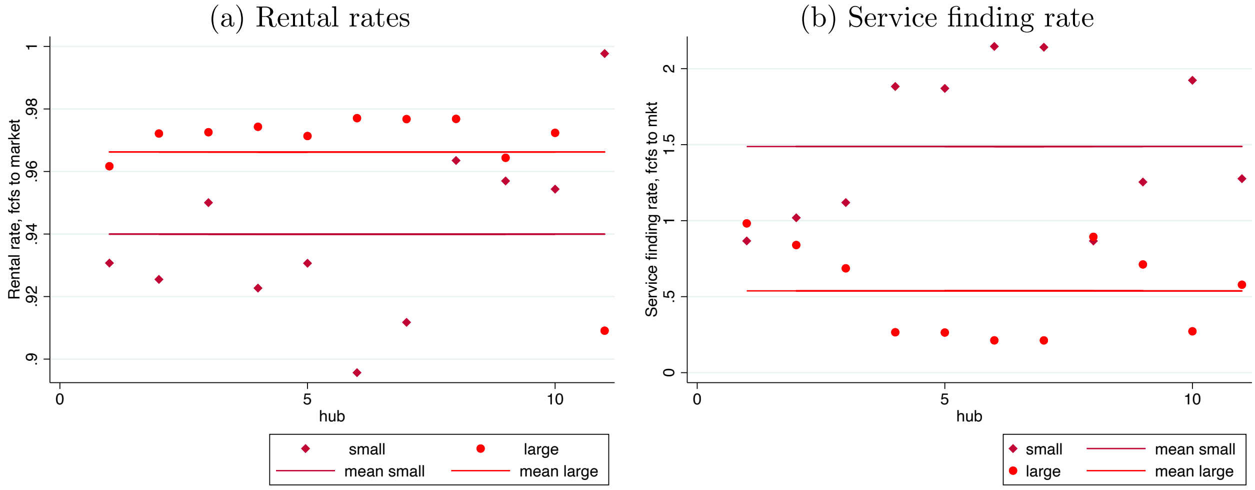
Notes: Calibration moments, data and model counterparts, Columns (2-5). Untargeted queue length for large-scale farmers relative to small-holder farmers, Columns (6-7).

Parameters calibrated jointly include the ratio of farmers to providers in the catchment area of each hub, as well as the the share of large-scale requests in that catchment area. The ratio of providers to farmers minimizes the distance between the model predicted queue of small-holder farmers at the fcfs provider and the data. We pick the share of large-scale requests that its closest to its empirical counterpart while generating an equilibrium allocation that displays service request from both types of farmers to both type of providers (as in the data). To do so, we take the stand that small-holder farmers are those with requests of up to 3.5 machine-hours per order, and large-scale farmers are those with orders of more than 3.5 machine-hours per order.

The calibrated ratio of farmers to providers ranges from 3.3 to 5.3, where a provider should be interpreted as a piece of equipment, column (8) in Table 4. The calibrated shares of large-scale farmers are higher than in the data (0.4 in the model vs. 0.27 in the data on average across hubs).[19] When there are few large-scale requests, the model generates queue lengths for small farmers that are broadly in line in the data, Table 5. Queues that are

[19] Alternatively, we could have targeted the queue of large-scale farmers in the first-come-first-serve providers which we currently report as an untargeted moment. Results are qualitatively similar to those reported here and available upon request.

Figure 5: Equilibrium



Notes: Equilibrium rental rates and service finding rates by hub for the calibrated economy. Hubs are ordered by increasing service capacity as in Table 4.

"too short" (less than 2 orders) fail to generate equilibria where both type of farmers request service from both providers because a queue length of 1 order for the market provider implies that small farmers are served with probability one there, given capacity. If the queue length is instead "too long" (more than 5 orders) the model benefits an equilibrium where small-farmers only request service from fcfs providers, which is inconsistent with the engagement of farmers across both types of providers, which we observe in the data.

For completeness, we report the (untargeted) ratio of queue lengths of large-scale and small-holder farmers. On average, this ratio is lower in the data than in the model, i.e. small scale farmers are more strongly sorted into fcfs providers than predicted by the model. This difference is in part driven by a larger share of large-scale farmers in the model than observed in the data, to be able to sustain equilibria where both types of farmers reach out to both providers.

5.3 Status quo equilibrium

We solve for the rental rates and queue lengths when both types of farmers have access to both types of providers, i.e. the status quo equilibrium. In the remainder of the analysis we order hubs by service capacity $\bar{k}$ in increasing order. The rental rates for both types of farmers are lower at the fcfs provider than at the mkt provider to compensate for the longer queues, Figure 5 panel (a). Rental rates are particularly lower for small farmers queueing with fcfs providers, which makes these providers attractive. Consistently, the service finding rate for small-scale farmers is larger with the fcfs than with the market providers, while the opposite is true for large-scale farmers, as shown in Figure 5 panel (b). Service finding rates are defined analogously to findings rates in the search literature (Barnichon and Figura, 2015), i.e. the ratio between the number of serviced orders per period and the number of farms searching for a service, $\frac{q_{ij}\Delta_{ij}H}{F} = \frac{q_{ij}\Delta_{it}}{f}$.

The level of the rental rates are higher for small-scale than for large-scale farmers due do the higher cost of service per machine-hour rented (e.g. travel costs), see Table **??** panel (b) in Appendix D. At the same time, rental rates weigh differences in the probability of service across providers, with lower rental rates for the fcfs provider irrespective of scale. This is a consequence of lower conditional probabilities of service with the fcfs on average, Δ_{ij}, particularly for large-scale farmers. The flip side of this feature are higher queues of small farmers with the fcfs and higher queues of large-scale farmers with the market provider, see Table **??** panel (a) in Appendix D. It is important to highlight that despite the fcfs has higher service probability for small-scale farmers, the conditional probabilities of service are lower than with the market providers. In other words, a higher service probability is driven by queue lengths. Figure **??** in Appendix D decomposes service finding rates into differences in queue lengths and service probabilities and compares them across providers.

5.4 Accommodating empirically relevant heterogeneity

The stylized equilibrium queueing model does not capture for full extent of the observed heterogeneity in location and machine-hours demand. We accommodate this heterogeneity through simulation exercises where service queues to each provider are drawn from the empirical joint distribution of location and machine-hours observed in the catchment area of each hub. In other words, the queue lengths of small and large scale orders are the equilibrium ones, but their composition is allowed to vary following the empirical distribution of machine-hours and location observed in the data. We sample, with replacement, 1000 queues per provider. Each order in the queue is a three dimensional object that includes the machine-hours demanded, the location of the plot, and the productivity of the farmer that requested service. This productivity level (output per acre) is then used to compute the cost of equilibrium delays in service provision.[20]

On the supply side, we solve the service dispatch system through two possible delivery routes. One follows a "hub and spoke" pattern, under which the equipment must return to the CHC between two consecutive orders. The other allows for a solution to a "Traveling Salesman Problem (TSP)", where the implement travels optimally from order to order within the day. Under fcfs, the provider follows the route that minimizes the travel time within a given day for a given set of requests (and their order) in the queue. Under the market allocation, the type of requests being served are jointly determined with the best service route. The value of an order in the market allocation depends on the density of orders around them, and the size of the order relative to serving capacity.[21] We then estimate the value function for each provider, i.e. a function that maps any queue of orders to their service value, conditional on the dispatch system and the delivery route. As expected the value of service when we solve the TSP problem is always higher than without it, at the same time, the value of service provision is higher for market providers, and particularly so, for relatively close and larger orders, see Figure **??**.

5.4.1 Farmer allocation across providers

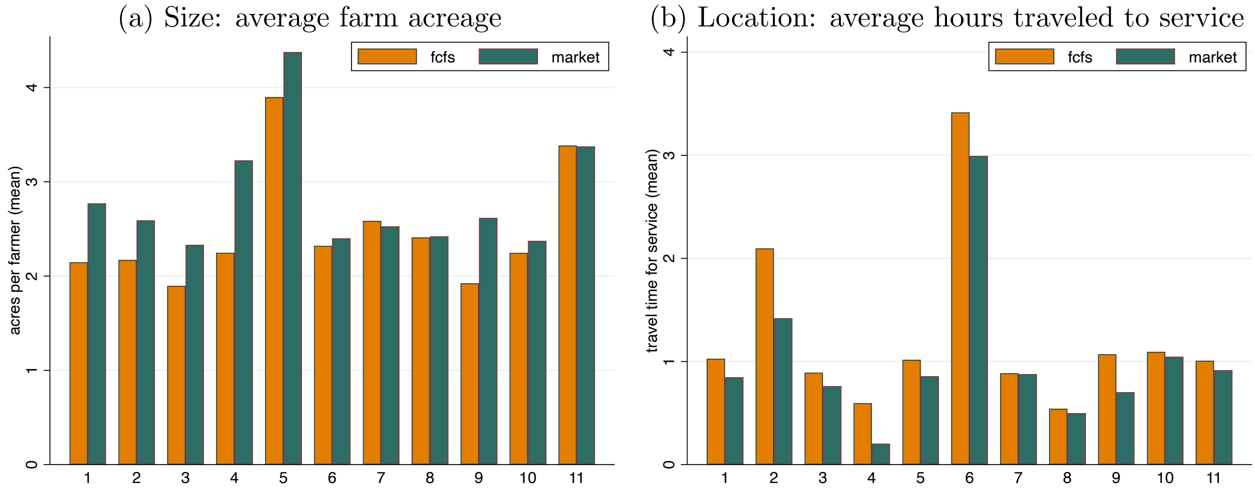
We start by describing the sorting of farmers into different providers, classifying farmers by order size, i.e. the acreage of the plot for service; and by location, i.e. the travel time for service, see Figure 6.

The average order size served by a market provider is 3 acres while the average order size served by a fcfs provider is 2.2 acres. This differential is a consequence of the disparities in the queue composition discussed before. At the same time, there are systematic differences in the travel time to locations. During a service day, market providers travel on average .8 hours (48 minutes), while fcfs providers travel twice as much. This differential travel

[20] As robustness, we simulate outcomes when we assume no correlation between farm productivity and order sizes within the catchment area of a hub. These results are available upon request.

[21] As we explain in Appendix C, this is a high dimensional problem, and the number of possible combinations of orders to be served within a period grows exponentially with the number of orders in the queue and its characteristics (including hours serviced and location, i.e., latitude and longitude).

Figure 6: Demand characteristics by provider



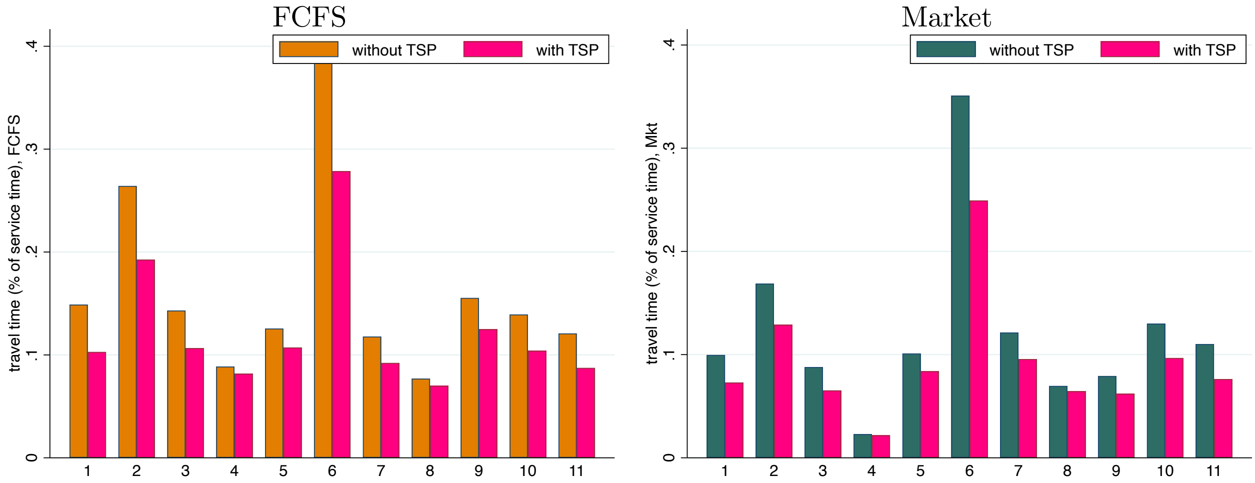
Notes: Panel (a) shows the sorting of farmers across providers by average farm acreage per order and Panel (b) shows the travel time to the service location, in hours. Hubs are ordered by increasing service capacity as in Table 4.

time reflects not only differences in the queue composition along the location dimension, but also disparities in the ability to prioritize orders. While equal-access-concerns may favour a fcfs service arrangement, it is possible to improve upon the baseline allocation by allowing government subsidized hubs to optimize service delivery within a day. Figure 7 displays travel times when providers are allowed to solve a TSP among the orders served within each day. This option is particularly beneficial to fcfs providers that are not allowed to prioritize order sizes. The average travel time (as % of service time) is 11% for the market provider and it declines to 9% once optimizing routes. The average travel time (as % of service time) is 15% for the fcfs provider and it declines to 11.5% once solving the TSP.

5.4.2 Effectiveness in service delivery

Figure 8 panel (a) compares waiting times across the bootstrapped samples for different dispatch system and demand characteristics. On average, the mean wait time faced by farmers queueing with market providers is longer that with fcfs. This feature is in part due to differences in the composition of the queue. Market providers have higher service rates for large-scale farmers, but those farmers are slightly more than 30% of the population of farmers in the economy on average. For the remainder of the farmers, market providers have

Figure 7: Travel times



Notes: Travel time by provider and dispatch system, "with TSP" correspond to allocations where providers are allowed to solve a traveling salesman problem for the orders serviced within a day. Travel time is expressed as a ratio of the total service within a day. Hubs are ordered by increasing service capacity as in Table 4.

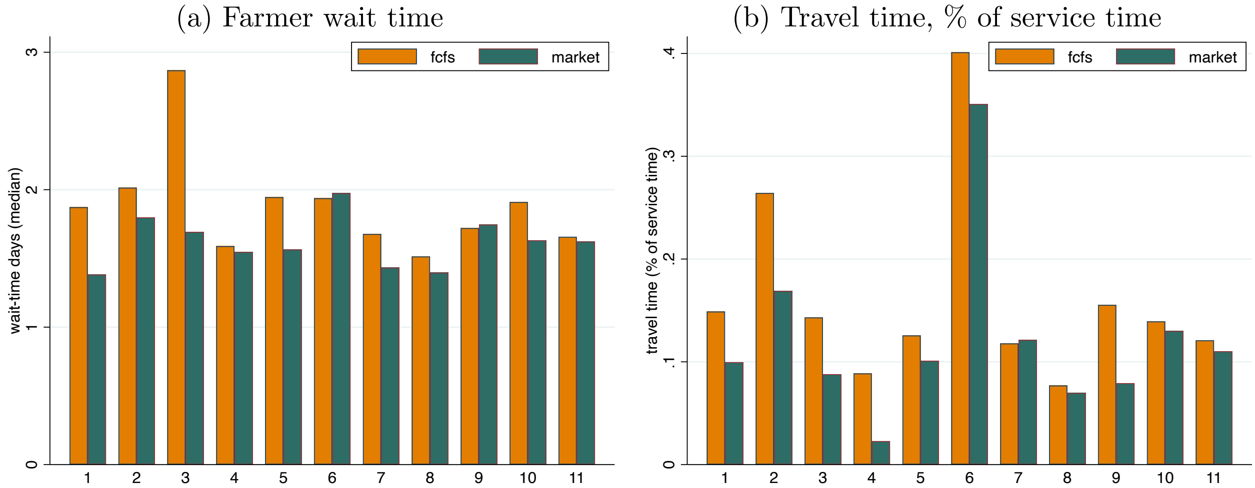
lower service rates than fcfs providers (Figure 5), which is reflected in higher waiting time among those queueing with market providers.

Despite the better waiting time for farmers queueing with fcfs providers, these providers face equipment transportation costs (in terms of the opportunity cost of time) that can double those of the market providers. In other words, their inability to prioritize order sizes also affects their ability to service demand in space, traveling "too much" relative to their market counterparts, see Figure 8 panel (b). It is not surprising then that the value of optimizing service routes is higher for fcfs providers as we showed before. Finally, notice that hubs are ordered in terms of service capacity with hub 1 holding less than a third of the service capacity of hub 11. On average, travel time as % of service time increases as service capacity increases, and this correlation is stronger for the market provider than it is for the fcfs provider. We need to enlarge here what we have to say about the TSP problem, Ref1.

5.4.3 The cost of delays and of service provision in space

Delays in service provision are costly for farmers. Delays are an equilibrium outcome in our economy given the nature of search in the market for rental equipments, the disparities

Figure 8: Time management by hub

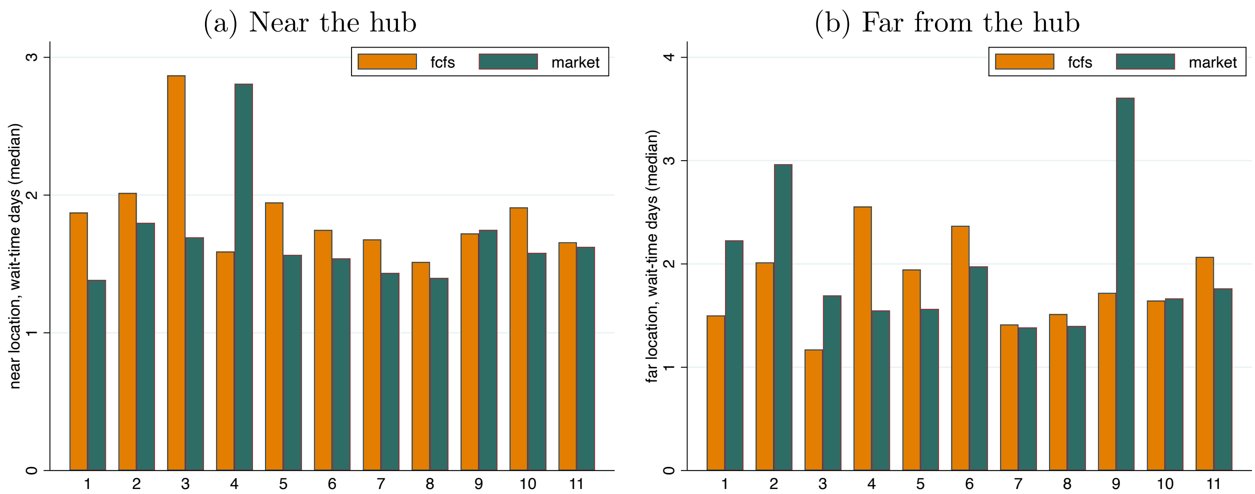


Notes: Panel (a) shows average days waiting for service across providers for the average farm in the catchment area of each hub. Panel (b) shows the average travel time as a ratio of the service time by provider and hub. Hubs are ordered by increasing service capacity as in Table 4.

in service provision across providers and the spatial distribution of service demand. Figure 9 displays the distribution of waiting time for service time across farmers located near the catchment area of a hub (less than 30 minutes of travel time, left panel) and those located far (more than 30 minutes of travel time, right panel). Most of the excess delay in near locations observed for market providers has to do with the sorting of demand by size, i.e. the market providers attract relatively more large-scale farmers, which induces delays in service provision toward smaller farmers. The difference in delays between providers is maximized for far away locations, because market providers systematically avoid servicing those.

These delays induce costs in revenue per acre that depend on the joint distribution of location, size and productivity. Figure 10 reports the productivity cost per acre across providers for the bootstrapped samples. These are measured as the decline in revenue per acre relative to the average revenue per acre in the catchment area of a hub. Despite documented disparities in delays across providers in near locations, productivity costs per acre are similar across dispatch systems. For example, in hub 4, the market provider induces a delay in provision of roughly an additional day relative to the fcfs provider (from 2.5 to 4 days). In the sample, farmers that wait longer are on average less productive and therefore the cost per acre is similar across providers, 8.4% of average revenue per acre

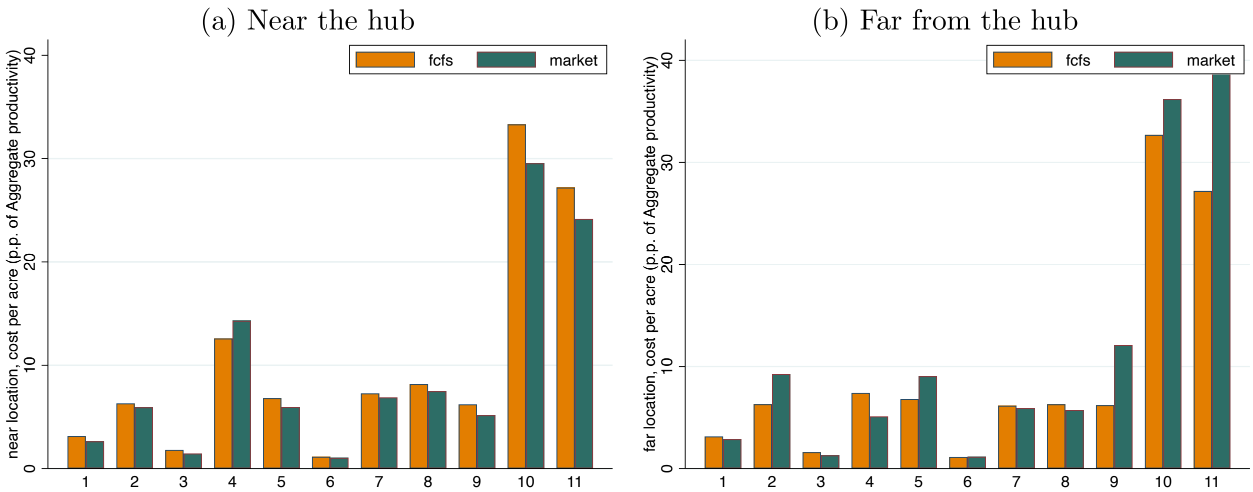
Figure 9: Delays in space



Notes: Panel (a) shows the average days waiting for service across providers for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. Hubs are ordered by increasing service capacity as in Table 4.

for the fcfs provider and 9% for the market provider. Conditional on servicing far away locations, the disparities in delays are so large across providers, that those delays induce stronger productivity costs for those queueing with the market providers. Still, as in nearby locations, the disparities in productivity are smaller than the disparities in delays across providers.

Figure 10: Productivity costs in space



Notes: Panel (a) shows the cost in revenue per acre for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. The cost is measured in p.p. of the average revenue per acre in the catchment area of each hub. Hubs are ordered by increasing service capacity as in Table 4.

6 Evaluating dispatch systems

It is possible to rank allocations based on the costs in profitability incurred by farmers, as well as the travel costs incurred by providers to service those slots.

$$\iota_j = \int_{k_i,d_i} \Delta_{ij}(\bar{z}_i\eta(\theta_i - \theta^\star)k_i^\alpha + d_i w)\zeta_i$$

where k_i indexes the size of the order (in acres/machine-hours), d_i indexes the travel time to a location and ζ_i is the joint distribution of farms in space and size as calibrated in the model. This cost ι can also be computed per acre

$$\iota_j = \int_{k_i,d_i} \Delta_{ij}\frac{\bar{z}_i\eta(\theta_i - \theta^\star)k_i^\alpha + d_i w}{k_i}\zeta_i$$

7 The value of increased supply

7.1 The market prior to CHCs.

We now study how the status quo equilibrium compares to the equilibrium prior to the intervention. While data for the pre-subsidy market is unavailable, we can account for this effect by running a counterfactual analysis where we shut down the supply of equipment stemming from fcfs, i.e. the government subsidized supply. This counterfactual is valid under the assumption that market participants accommodated the entry of new supply in the market. This is a plausible scenario due to the low ownership of equipment in the population and the desire of most farmers to engage in rentals.

The substantial increase in equipment supply due to the subsidy implies an increase in the service finding rates, moving from 15% prior to the subsidy to between 40% and 55% after the subsidy depending on the provider, see Table 6. Gains in service findings rates are mostly attributed to small farmers, which prior to the subsidy faced a service finding rate of 6% and after the subsidy face a service finding rate of 56%, with a bit more than half of it attributed to the fcfs providers. Market providers' service probability also increases in equilibrium because the cost of service declines in response to stronger competition. Rental rates fall by 18% for small scale farmers and by 5% for large farmers. The differential effect is a consequence of the implicit priority given to small-holder farmers by the fcfs subsidized provider.

Table 6: Effect of the subsidy

	prior to policy	post subsidy fcfs	mkt
Service Finding Rate			
average farmer	0.15	0.41	0.56
small-scale	0.05	0.28	0.22
large-scale	0.1	0.13	0.34
Rental Rate			
small-scale	142	121	125
large-scale	94	93	94

Notes: This table shows the service findings rates and hourly rental rates (in rupees) across different equilibrium allocations. These results are presented by farmer scale of production and by provider. The "prior to policy" equilibrium corresponds to an allocation where only the market suppliers are available. The "post subsidy" equilibrium corresponds to our benchmark economy.

7.2 Market deregulation

One of the findings in Section 5.4.2 is that fcfs providers could benefit from operating a technology that allows them to optimize equipment in space as well as optimize the type of orders being served. We study the effect of a market deregulation through counterfactuals. Because prioritizing large-scale farmers is costless and the fcfs providers are at least as well off as before (i.e. they can now prioritize the high marginal return orders), a profit driven fcfs provider would choose to adopt the technology, leading to a shift in provider composition towards $h = 0$.[22] In other words, there is no longer any differentiation between these two types of providers. The nature of the equilibrium may however change due to the endogenous farmer sorting, and pricing of services.

Table 7: Effect of Market Deregulation

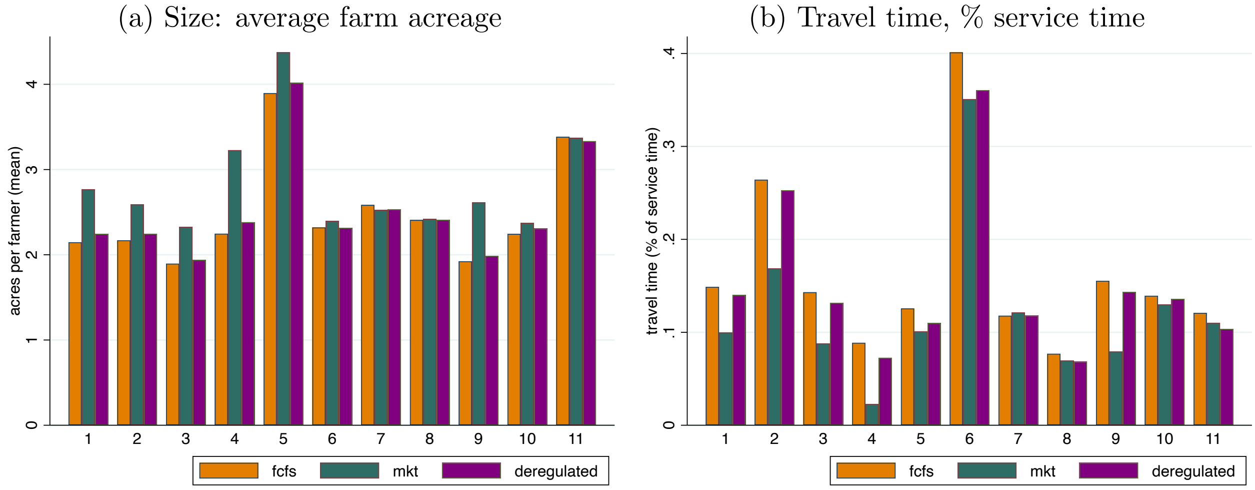
	Benchmark		Deregulation	
	fcfs	mkt	short-run	long-run
Service Finding Rate				
average farmer	0.41	0.56	0.49	0.5
small-scale	0.28	0.22	0.29	0.3
large-scale	0.13	0.34	0.2	0.21
Rental Rate				
small-scale	121	125	122	120
large-scale	93	94	92	92

Notes: This table shows the service findings rates and hourly rental rates (in rupees) across different equilibrium allocations. These results are presented by farmer scale of production and by provider. The "benchmark allocation" corresponds to the calibrate economy, whereas the "deregulated" economy corresponds to economies where fcfs providers are allowed to prioritize orders as their market counterparts. The "short-run" is an equilibrium without providers' entry-and-exit whereas the "long-run" allows for it.

The short run response of the economy (without endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistently with the profit maximizing strategy of market providers (see Table 7). However, their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers. The long run response of the economy (with endogenous entry and exit) restores rental costs

[22] As we pointed out before, the value of service for market providers is always above the one for first-come-first-served providers, and therefore each provider has incentives to adopt a dispatch system that prioritizes large orders.

Figure 11: Impact of Deregulation, demand characteristics



Notes: Panel (a) shows the sorting of farmers by average farm acreage per order. Panel (b) shows the average travel time to the service location as a ratio of the average service time. Hubs are ordered by increasing service capacity as in Table 4.

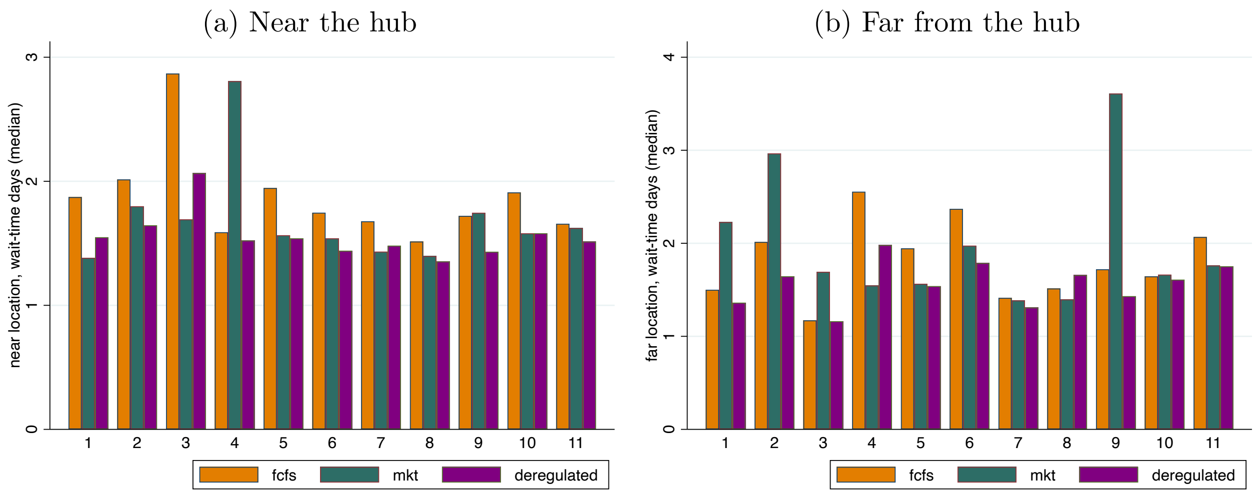
to their baseline levels, and generates allocations where service findings rates are higher for small holder farmers than large holder farmers, despite market providers prioritizing the latter. The reason is that small-scale requests are valuable when there are service capacity constraints because they improve capacity utilization.

Figure 11 plots the change in average farm sizes served and the travel time for service, i.e. the change in the distribution of served location, as the market deregulates. Exit of providers induces an increase in the average size of the farm served by each provider, and a reduction in the travel time to service, consistently with providers prioritizing services with low marginal cost of provision. Travel time as a proportion of the service time declines by 20%, on average across hubs.

7.3 Small-holder access and equipment supply.

An extensive literature in agricultural economies has studied the effect of government subsidies for equipment. Two conclusions arise: (a) equipment subsidies are regressive because they benefit relatively wealthier farmers; (b) the impact of these subsidies in mechanization is widely heterogeneous, with only a handful of successful cases (see review in Pingali, 2007).

Figure 12: Impact of Deregulation, delays



Notes: Panel (a) shows the average days waiting for service across providers for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. In purple we include outcomes for the deregulated economy, i.e. where fcfs providers can prioritize orders. Hubs are ordered by increasing service capacity as in Table 4.

Our results suggest a different interpretation of the heterogeneity in the effectiveness of subsidies. First, small-holder farmers can benefit disproportionally more from these subsidies than large-scale farmers (at least in terms of service findings rates), as they do in our benchmark result. Their ability to do so depends on their value in terms of capacity utilization, or the density of their location in space; and the generosity of the subsidy, or the increase in the supply of machine-hours. Indeed, if the supply increase is relatively low, service finding rates for small-holder farmers are below that of large-scale farmers as we show in Table 8. One could erroneously interpret this results as the subsidy being regressive.

Table 8 also highlights that heterogeneity in the success of these government programs for generating long-term shifts in mechanization may have been related to heterogeneity in the subsidy amounts, and the joint spatial and size distribution of farms.

Table 8: Service finding rates across supply of machine-hours, H.

	H_{ps}	H relative to H_{ps}			
	1	1.4	2	3.3	5
average farm	0.15	0.21	0.28	0.39	0.47
small-scale	0.05	0.08	0.12	0.19	0.26
large-scale	0.10	0.13	0.16	0.19	0.20

Note: Column H_{ps} corresponds to the level of pre-subsidy supply of machine-hours. The pre-subsidy supply level is estimated as the equipment supply absent CHC's machine hours, see Section 7.1. Service finding rates are computed as described in the text.

8 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spatial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, poses important policy-relevant trade offs for service provision. The returns to these rental markets depend crucially on factors such as spatial density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize traveling equipment time. In this paper, we document and quantify how these factors determine the distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment for rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. When equipment owners are allowed to maximize their profits by prioritizing larger scale orders, the equilibrium allocation may induce higher finding rates for small-scale producers. The reason is that small-scale orders are valuable in terms of service capacity-utilization, particularly when located in high-density areas.

We provide a parsimonious framework to study the allocation of shared services across heterogenous demand and spatial allocations. The framework can be readily extended to study similar markets where capacity utilization and geography are important determinants in service provision. Finally, while we take the location of service providers as given, a natural step forward would be to study the properties of the endogenous location of providers in space, as in Oberfield et al. (2020).

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A Characterization of the probability of service.

A.1 Probability of service for a particular farmer

The probability that a provider chooses a particular farmer given that he chooses one of type i is $\tilde{\Delta}_{ij}(1)$, i.e.

$$\tilde{\Delta}_{ij}(1) = \sum_{n=0}^{f_i-1} \binom{f_i - 1}{n} p_{ij}^n (1 - p_{ij})^{f_i-1-n} \frac{1}{n+1},$$

where f_i is the number of farmers of type i searching for a provider, $f_s = sF$ and $f_{s^-} = (1-s)F$ and $\binom{f_i-1}{n} = \frac{f_i-1!}{n!(f_i-1-n)!}$. Hence,

$$\tilde{\Delta}_{ij}(1) = \frac{1 - (1 - p_{ij})^{f_i}}{f_i p_{ij}}.$$

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$\tilde{\Delta}_{ij}(1) = \frac{1 - e^{-q_{ij}}}{q_{ij}}.$$

That is, the probability that at least one farmer of type i has requested a service, $1 - e^{-q_{ij}}$, divided by the number of requests of a given type, q_{ij}.

Next, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 2$ orders of type i, $\tilde{\Delta}_{ij}(2)$. Similar computations to those above yield a service probability as follows

$$\tilde{\Delta}_{ij}(2) = 2(\frac{1 - e^{-q_{ij}}}{q_{ij}}) - e^{-q_{ij}}.$$

Finally, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 3$ orders of type i, $\tilde{\Delta}_{ij}(3)$, which follows

$$\tilde{\Delta}_{ij}(3) = 3(\frac{1 - e^{-q_{ij}}}{q_{ij}}) - 2e^{-q_{ij}} - e^{-q_{ij}} q_{ij}.$$

A.2 Probability of service of a farmer of type i

Next, we characterize the probability that a provider of type j services a farmer of type i given that a farmer of type i is standing in the queue.

First-come-first-served. The fcfs provider only considers feasibility and the position of the farmer in the queue. Let the probability of serving $\bar{o}$ farmersof type i be $\phi_{i,fcfs}(\bar{o})$.

Given the queue lengths at this provider, there are ${}^{q_s+q_{s^-}}P_o = \frac{q_s+q_{s^-}!}{(q_s+q_{s^-}-o)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a fcfs provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue, which occurs with probability $\hat{\phi}_{s,fcfs}(1) \equiv 3q_s \frac{{}^{q_{s^-}}P_2}{{}^{q_s+q_{s^-}}P_3}$; and at least one has applied.

$$\phi_{s,fcfs}(1) = \psi_{s,fcfs}(1)\hat{\phi}_{s,fcfs}(1),$$ [23]

where $\psi_{s,fcfs}(1) \equiv (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$ is the probability of having at least three orders in the queue of which at least two are of type s^-, when a single farmer of type s has requested service. To this probability we should add the probability of service when less than 3 farmers apply for service. The latter is $\hat{\psi}_{s,fcfs}(1) \equiv \left(e^{-q_{s,fcfs}}(e^{-q_{s^-,fcfs}} + q_{s^-,fcfs}e^{-q_{s^-,fcfs}})\right)$, i.e. the probability of service of large scale order when there are no other service request or there is exactly one additional order requested.

A fcfs provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have requested service. Let the first probability be $\hat{\phi}_{s,fcfs}(2) \equiv 3q_s(q_s-1)\frac{{}^{q_s-2+q_{s^-}}P_1}{{}^{q_s+q_{s^-}}P_3}$

$$\phi_{s,fcfs}(2) = \psi_{s,fcfs}(2)\hat{\phi}_{s,fcfs}(2),$$

where $\psi_{s,fcfs}(2) = (1 - e^{-q_{s,fcfs}} - e^{-q_{s^-,fcfs}}q_s e^{-q_{s,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s requesting service, of which at least two are of type s (including the one requesting service).[24] To this probability we should add the probability that there are only two large-scale farmers in the queue $\hat{\psi}_{s,fcfs}(2) \equiv \left(q_{s,fcfs}e^{-q_{s,fcfs}}e^{-q_{s^-,fcfs}}\right)$.

Given feasibility, the fcfs provider never serves 3 large-scale orders, $\phi_{s,fcfs}(3) = 0$.

A fcfs provider serves a single small-holder farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders,

$$\phi_{s^-,fcfs}(1) = \psi_{s^-,fcfs}(1)\hat{\phi}_{s^-,fcfs}(1),$$

and adding the probability $\hat{\psi}_{s^-,fcfs}(1)$ when there are less than three orders.

A fcfs provider services 2 small-holder farmers if at least two small-scale orders in the

[23]Note that $\hat{\phi}_{s,fcfs}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \to \infty$ these two are arbitrarily close.

[24]This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

first o positions of the queue, $\hat{\phi}_{s^-,fcfs}(2) \equiv 3\frac{q_{s^-}(q_{s^-}-1)q_s}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(2) = \psi_{s^-,fcfs}(2)\hat{\phi}_{s^-,fcfs}(2),$$

where $\psi_{s^-,fcfs}(2) = (1-e^{-q_{s,fcfs}})(1-e^{-q_{s^-,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s^- requesting service, of which at least one is of type s and at least two are of type s^- (including the one requesting the service). To this probability we should add the probability that there are only two orders in the queue, $\hat{\psi}_{s^-,fcfs}(2)$ defined analogously than for large-scale farmers.

A fcfs provider services 3 small-holder farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^-,fcfs}(3) = \frac{q_{s^-}P_3}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(3) = \psi_{s^-,fcfs}(3)\hat{\phi}_{s^-,fcfs}(3),$$

where $\psi_{s^-,fcfs}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^-,fcfs}(3) = (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$.

The general form for the probability of service is,

$$\Delta_{i,fcfs} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i,fcfs}(\bar{o}) + \phi_{i,fcfs}(\bar{o})\tilde{\Delta}_{i,fcfs}(\bar{o}), \tag{10}$$

where we have defined $\hat{\psi}_{s,fcfs}(3) \equiv 0$ to ease notation.

The main difference in the probability of service for large and small relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^{3} \phi_{s^-,fcfs}(\bar{o}) = \sum_{\bar{o}=2}^{3} \phi_{s,fcfs}(\bar{o})$.

Market. The market provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly $\bar{o}$ farmers of type i are served conditional on the farmer under consideration having applied.

Conditional on a large farmer having applied, a single large-scale farmer is served by a market provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first o available positions, which happens with probability $(1-\chi)\tilde{\phi}_{s,mkt}(1) = (1-\chi)\phi_{s,fcfs}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi\psi_{s,mkt}(1) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$. These service probabilities add up to,

$$\phi_{s,mkt}(1) = \chi(\psi_{s,mkt}(1)) + (1-\chi)\tilde{\phi}_{s,mkt}(1),$$

the latter assumes that at least three orders have been requested. We should add to these the event that there are less than three orders, which happens with probability $\hat{\psi}_{i,mkt}(1) = \hat{\psi}_{i,fcfs}(1)$ for any i=s, s^- by definition.

Two large-scale farmers are served by a market provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability $(1-\chi)\tilde{\phi}_{s,mkt}(2) = (1-\chi)\phi_{s,fcfs}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\chi\psi_{s,mkt}(2) = \chi(1 - e^{-q_{s,mkt}} - e^{-q_{s^-,mkt}} q_s e^{-q_{s,mkt}})$.[25] These service probabilities add up to

$$\phi_{s,mkt}(2) = \chi(\psi_{s,mkt}(2)) + (1-\chi)\tilde{\phi}_{s,mkt}(2).$$

We then add the probability when less than three orders are in the queue, $\hat{\psi}_{s,mkt}(2)$ defined analogously to the fcfs provider.

Feasibility prevents three large-scale orders to be served within the period and therefore, $\phi_{s,mkt}(3) = 0$.

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-holder farmer is always served by a market provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi\psi_{s^-,mkt}(1) = \chi(1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}}$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi)\tilde{\phi}_{s^-,mkt}(1)$, where $\tilde{\phi}_{s^-,mkt}(1) = \phi_{s^-,fcfs}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to serve at most $o-1$ orders leaving always and idle slot. Finally, we add the probability that there are less than two large-scale orders in the queue.

$$\phi_{s^-,mkt}(1) = \chi(\psi_{s^-,mkt}(1)) + (1-\chi)\tilde{\phi}_{s^-,mkt}(1),$$

and $\hat{\psi}_{s^-,mkt}(1)$.

Two small-holder farmers are served by a market provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi\psi_{s^-,mkt}(2) = \chi q_{s,mkt}e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}})$. Alternatively, two small-holder farmers are served if the provider does not prioritize large-scale orders and there are two small-scale orders among the first three orders in

[25] If there are more large-scale orders the provider still serves two because of its capacity constraints.

the queue.

$$\phi_{s^-,mkt}(2) = \chi(\psi_{s^-,mkt}(2)) + (1-\chi)\tilde{\phi}_{s^-,mkt}(2).$$

To these probabilities we add those associated to the event when there are strictly less than two orders in the queue, $\hat{\psi}_{s^-,mkt}(2)$.

Three small-holder farmers are served by the market provider if it prioritizes high-scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi\psi_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$\phi_{s^-,mkt}(3) = \chi\psi_{s^-,mkt}(3) + (1-\chi)\tilde{\phi}_{s^-,mkt}(3).$$

In sum, the probability of service for a market provider follows

$$\Delta_{i,mkt} = \sum_{\bar{o}=1}^{3} \hat{\psi}_{i,mkt}(\bar{o}) + \phi_{i,mkt}(\bar{o})\tilde{\Delta}_{i,mkt}(\bar{o}). \tag{11}$$

Following equations 10 and 11, the probability of being served for a given type i (weakly) declines in the queue length of the other type of farmers. In the first-come-first-served provider the result is straightforward. For the market provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

A.3 Unconditional probabilities of service.

The unconditional probabilities of service are important in characterizing the value of service for each provider. We consider alternative scenarios, i.e. when the provider serves at capacity ($o = 3$ orders) and when the provider serves less than capacity.

The first-come-first-served provider can serve three orders of small scale (given feasibility) with a service probability of

$$\Phi_{s^-,fcfs}(3) = \left(1 - e^{-q_{s^-,j}}(1 + q_{s^-,j} + \frac{1}{2}q_{s^-,j}^2)\right) \frac{q_{s^-,j}P_3}{q_{s,j}+q_{s^-,j}P_3};$$

or to serve two orders of one type and one of another, with probability

$$\bar{\Phi}_{i,fcfs}(1) = (1 - e^{-q_{i,j}})(1 - e^{-q_{i',j}} - q_{i',j}e^{-q_{i',j}})\frac{3q_{i,j}q_{i',j}(q_{i',j}-1)}{q_{i,j}+q_{i',j}P_3},$$

and $\bar{\Phi}_{i',fcfs}(2) = \bar{\Phi}_{i,fcfs}(1)$ for $i' \neq i$.

The provider can also serve two orders of large size, (either because he received only two orders, or because all orders in the queue are of large scale)

$$\tilde{\Phi}_{s,fcfs}(2) = \left(1 - e^{-q_{s,fcfs}} - q_{s,fcfs}e^{-q_{s,fcfs}}\right) e^{-q_{s-,fcfs}},$$

or it can receive exactly two orders of small size and serve those,

$$\tilde{\Phi}_{s^-,fcfs}(2) = \frac{1}{2}q_{s^-,j}^2 e^{-q_{s^-,j}}(e^{-q_{s,j}}).$$

Finally, the provider can serve two orders, one of each type

$$\tilde{\Phi}_{i,fcfs}(1_2) = \left(q_{i,j}e^{-q_{i,j}}q_{i',j}e^{-q_{i',j}}\right).$$

or only one order, with occurs with probability

$$\tilde{\Phi}_{i,fcfs}(1_1) = \left(q_{i,j}e^{-q_{i,j}}e^{-q_{i',j}}\right).$$

The probabilities for the market provider are similar to the ones above, except that we need to account for the market provider's ability to select large scale orders.

The market provider can serve three orders of small scale,

$$\bar{\Phi}_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}} - \frac{1}{2}q_{s^-,mkt}^2 e^{-q_{s^-,mkt}}) + (1-\chi)\Phi_{s^-,fcfs}(3), \tag{12}$$

or the orders of large scale and one small,

$$\bar{\Phi}_{s,mkt}(2) = \chi((1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}})(1 - e^{-q_{s^-,mkt}}) + (1-\chi)\bar{\Phi}_{s,fcfs}(2), \tag{13}$$

or two orders of small scale and on large,

$$\bar{\Phi}_{s^-,mkt}(2) = \chi q_{s,mkt}e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}}) + (1-\chi)\bar{\Phi}_{s^-,fcfs}(2). \tag{14}$$

When there are less than three orders in the queue there is no need to prioritize orders, and therefore the probabilities of service are identical to those characterized for the FCFS problem, i.e. $\tilde{\Phi}_{i,mkt} = \tilde{\Phi}_{i,fcfs}$.

Expected value of service provision The characterization of the probability allows us to compute the expected value of service provision:

$$
\begin{aligned}
&\tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{\mathbf{q}_{\text{fcfs}}}, \{(r_{i,\text{fcfs}} - w)k_i - wd_i\}_{i=s,s^-}\right) \equiv \\
\sum_{i=s,s^-} &\bar{\Phi}_{i,fcfs}(2)\left[2\left((r_{i,fcfs} - w)k_i - wd_i\right) + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right] + \\
&\tilde{\Phi}_{i,fcfs}(1_1)\left((r_{i,fcfs} - w)k_i - wd_i\right) + \\
&+\tilde{\Phi}_{i,fcfs}(1_2)\left((r_{i,fcfs} - w)k_i - wd_i + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right) + \\
&\tilde{\Phi}_{i,fcfs}(2)2\left((r_{i,fcfs} - w)k_i - wd_i\right) \\
&\Phi_{s^-,fcfs}(3)3((r_{s^-,fcfs} - w)k_{s^-} - wd_{s^-}),
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&\tilde{V}\left(\{\bar{o}_s, \bar{o}_{s^-}\}_{(\mathbf{q}_{\text{mkt}},\chi)}, \{(r_{i,\text{mkt}} - w)k_i - wd_i\}_{i=s,s^-}\right) \equiv \\
\sum_{i=s,s^-} &\bar{\Phi}_{i,mkt}(2)\left[2\left((r_{i,mkt} - w)k_i - wd_i\right) + (r_{i',mkt} - w)k_{i'} - wE(d_{i'})\right] + \\
&\tilde{\Phi}_{i,mkt}(1_1)\left((r_{i,mkt} - w)k_i - wd_i\right) + \\
&+\tilde{\Phi}_{i,mkt}(1_2)\left((r_{i,mkt} - w)k_i - wd_i + (r_{i',fcfs} - w)k_{i'} - wE(d_{i'})\right) + \\
&\tilde{\Phi}_{i,mkt}(2)2\left((r_{i,mkt} - w)k_i - wd_i\right) \\
&\Phi_{s^-,mkt}(3)3((r_{s^-,mkt} - w)k_{s^-} - wd_{s^-}).
\end{aligned} \tag{16}
$$

where the first two terms in either expression correspond to the expected value of serving three orders or different types, while the remaining terms correspond to the expected returns of serving strictly less than three orders.

B Proofs

B.1 Proposition 1

First, we solve for the equilibrium value of service when both farmers queue with both providers. Then, we show the value when the service provider serves a single type of farmers. Then we show that the guess that the selection criteria for the market provider should be to prioritize large scale orders. Finally, we show that the expected value of service is higher for large scale farmers.

Value of service when serving both type of farmers.

Proof. Using the definition of the expected value of service provision (equations 15 and 16) and rearranging terms, as well as the participation constraint for the farmers, equation 4, the problem of the provider is

$$\max_{q_{s,j},q_{s^-,j}} \sum_{i=s,s^-} \Phi_{i,j}(2)[z_{ij}k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i+d_i)] +$$
$$\sum_{i=s,s^-} \Phi_{i,j}(1)[z_{ij}k_{i'}^\alpha - \frac{U_{i'}}{\Delta_{i'j}} - w(k_{i'}+E(d_{i'}))] +$$
$$\Phi_{s^-,j}(3)3[z_{ij}k_{s^-}^\alpha - \frac{U_{s^-}}{\Delta_{s^-j}} - w(k_{s^-}+d_{s^-})],$$

where $\Phi_{i,j}(2) = (\bar{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(1_1) + \tilde{\Phi}_{i,j}(1_2))$ and $\Phi_{i,j}(1) = (\bar{\Phi}_{i,j}(2) + \tilde{\Phi}_{i,j}(1_2))$.

Let $\bar{V}_{ij}$ be the profit per order of type i for provider j, i.e. $\bar{V}_{ij} \equiv \big((r_{i,j}-w)k_i - wd_i\big)$. The optimality condition swith respect to the queue length of large-scale and small-holder farmers are

$$\sum_{i=s,s^-}\left(\frac{\partial\Phi_{i,j}(2)}{\partial q_{s,j}}\bar{V}_i + \frac{\partial\Phi_{i,j}(1)}{\partial q_{s,j}}\bar{V}_{i'}\right) + 3\frac{\partial\Phi_{s^-,j}(3)}{\partial q_{s,j}}\bar{V}_{s^-} +$$
$$\sum_{i=s,s^-}\left(\Phi_{i,j}(2)\frac{\partial\bar{V}_i}{\partial q_{s,j}} + \Phi_{i,j}(1)\frac{\partial\bar{V}_{i'}}{\partial q_{s,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial\bar{V}_{s^-}}{\partial q_{s,j}} = 0, \tag{17}$$

$$\sum_{i=s,s^-}\left(\frac{\partial\Phi_{i,j}(2)}{\partial q_{s^-,j}}\bar{V}_i + \frac{\partial\Phi_{i,j}(1)}{\partial q_{s^-,j}}\bar{V}_{i'}\right) + 3\frac{\partial\Phi_{s^-,j}(3)}{\partial q_{s^-,j}}\bar{V}_{s^-} +$$
$$\sum_{i=s,s^-}\left(\Phi_{i,j}(2)\frac{\partial\bar{V}_i}{\partial q_{s^-,j}} + \Phi_{i,j}(1)\frac{\partial\bar{V}_{i'}}{\partial q_{s^-,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial\bar{V}_{s^-}}{\partial q_{s^-,j}} = 0, \tag{18}$$

where

$$\frac{\partial\bar{V}_i}{\partial q_{i,j}} = -\left[\frac{\bar{V}_i + w(k_i+d_i) - z_{ij}k_i^\alpha(1+\frac{\partial z_{ij}}{\partial\Delta_{ij}}\frac{\Delta_{ij}}{z_{ij}})}{\Delta_{ij}}\right]\left(\frac{\partial\Delta_{i,j}}{\partial q_{ij}}\right).$$

Assumption 3. *Let $\bar{\theta}(\Delta_{ij}) = -\ln(\Delta_{ij})$, a strictly decreasing function of the probability of service.*

Given Assumption 3 elasticity of productivity to the probability of service equals $-\eta$. We can define

$$\tilde{z} \equiv \bar{z}(1-\eta),$$

as the "adjusted" productivity, and the envelope condition reads

$$\frac{\partial \bar{V}_i}{\partial q_{i,j}} = -(\bar{V}_i + w(k_i + d_i) - \tilde{z}k_i^{\alpha}(1-\eta))\frac{1}{\Delta_{ij}}\left(\frac{\partial \Delta_{i,j}}{\partial q_{ij}}\right).$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q}\frac{q_{sj}}{\Delta_{sj}(o)}$, let the elasticity of the value of service to the queue length be $\zeta_{\bar{V}q} \equiv \frac{\partial \bar{V}}{\partial q_{ij}}\frac{q_{ij}}{\bar{V}}$ and that of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q}\frac{q}{\psi(o)}$. The envelope condition indicates that the elasticity of the value of service to the queue length is an inversely proportional function of the elasticity of the probability of service to the queue length, $\zeta_{\Delta ij}$.

$$\zeta_{\bar{V}_i} = \left(1 + \frac{w(k_i + d_i) - \tilde{z}k_i^{\alpha}}{\bar{V}_i}\right)\zeta_{\Delta ij}.$$

Equations 17 and 18 form a system of linear equations that can be solved for the two unknowns $\bar{V}_{s^-}, \bar{V}_s$ as a function of the queue lengths.[26]

$$\Gamma\begin{bmatrix}\bar{V}_s\\ \bar{V}_{s^-}\end{bmatrix} = a\begin{bmatrix}\tilde{z}k_s^{\alpha} - w(k_s + d_s)\\ \tilde{z}k_{s^-}^{\alpha} - w(k_{s^-} + d_{s^-})\end{bmatrix}$$

where $\Gamma = \begin{bmatrix}\Gamma_1\Gamma_2\\ \Gamma_3\Gamma_4\end{bmatrix}$, for

$$\Gamma_1 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s,j}} + \zeta_{\Delta_s q_s}\left(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}}\right)$$

$$\begin{aligned}\Gamma_2 &= \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}} + \\ &\quad \zeta_{\Delta_{s-} q_s}\left(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}}\right)\end{aligned}$$

$$\Gamma_3 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s^-,j}} + \zeta_{\Delta_s q_{s-}}\left(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}}\right)$$

$$\begin{aligned}\Gamma_4 &= \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s^-,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}} + \\ &\quad \zeta_{\Delta_{s-} q_{s-}}\left(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}}\right)\end{aligned}$$

[26]Note that equation 17 reduces to 21 when there are no small-scale orders.

and in the LHS

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \zeta_{\Delta_s q_s}(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}}) & \zeta_{\Delta_{s^-} q_s}(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}}) \\ \zeta_{\Delta_s q_{s^-}}(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}}) & \zeta_{\Delta_{s^-} q_{s^-}}(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}}) \end{bmatrix}$$

The last vector on the LHS corresponds to the surplus from trade for each farmer type.

Standard matrix algebra implies that expected value to the providers satisfies

$$\bar{V}_s^j = \gamma_{1s}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) + \gamma_{2s}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}) \tag{19}$$

$$\bar{V}_{s^-}^j = \gamma_{1s^-}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - d_{s^-}) + \gamma_{2s^-}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) \tag{20}$$

for $\gamma_{1s^-}^j = \frac{\Gamma_1 a_{22} - a_{12}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2,s^-}^j = \frac{a_{21}\Gamma_1 - a_{11}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ while $\gamma_{1,s}^j = \frac{a_{11}\Gamma_4 - \Gamma_2 a_{21}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$ and $\gamma_{2s}^j = \frac{a_{12}\Gamma_4 - \Gamma_2 a_{22}}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}$. Notice that the denominator of each of the γ parameters shifts depending on the provider as a function of the probability of service. This heterogeneity changes the value for the derivatives in Γ. □

Value of service when serving only large scale farmers.

If a ***provider j attracts only large-scale farmers****, i.e.* $q_{s^-j} = 0$*, then the expected per period profit of the provider satisfies*

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(\tilde{z}k_s^\alpha - w(k_s + d_s))$$

where the second term corresponds to the surplus associated to the transaction and $\gamma \in (0,1)$ *is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue,* ζ*, and the share of capital in farming production.*

Proof. The problem of the supplier when it only receives large scale orders is

$$\max_{q_{sj}, r_{sj}} \psi \bar{V}_s$$

subject to

$$\Delta_{sj}\pi_s(r_{sj}, k_s) \geq U_s$$

$$\sum_{i \in \hat{q}_j} k_s(i) + E(d_s(i)) \leq \bar{k}$$

where $\psi = 2\left(1 - e^{-q_{s,\text{mkt}}}(1 + q_{s,\text{mkt}})\right) + e^{-q_{s,\text{mkt}}} q_{s,\text{mkt}}$ because there are no small-scale orders and either the supplier serves one or two orders of large scale.

Using the definition of profits to the farmers, equation 4, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$\max_{q_{sj}} \psi \left[\bar{z} k_s^\alpha - \frac{U_s}{\Delta_{sj}} - w(k_s + d_s) \right]$$

Note that the properties of the probabilities ψ and Δ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$\frac{\partial \psi}{\partial q} \bar{V}_s - \psi \left[\frac{\bar{V}_s + w(k_s + d_s) - \tilde{z} k_s^\alpha}{\tilde{\Delta}_{sj}(2)} \right] \frac{\partial \tilde{\Delta}_{sj}(2)}{\partial q} = 0. \tag{21}$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q} \frac{q_{sj}}{\Delta_{sj}(o)}$ and let the elasticity of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. Finally, let $\gamma(q_{s,j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) \equiv \frac{\zeta_{q\Delta}}{\zeta_{q\psi} + \zeta_{q\Delta}}$,

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(\tilde{z} k_s^\alpha - w(k_s + d_s)) \tag{22}$$

which proves the result. For the value to be positive we require $\gamma > 0$ which is true by construction. The provider takes a fraction of the surplus from the transaction.

□

*If a **provider j attracts only small-holder farmers**, i.e. $q_{s,j} = 0$, then the expected per period profit of the provider satisfies*

$$\bar{V}_{s^-} = \gamma(q_{s^-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha)(\tilde{z} k_{s^-}^\alpha - w(k_{s^-} + d_{s^-}))$$

where $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ, and the share of capital in farming production.

The derivations when the provider serves only small-scale providers follow the same steps as the ones above, so we omit them for brevity.

The market provider wants to prioritize large scale orders: Compute $\frac{\partial \Pi_{\text{mkt}}}{\partial \chi}$, which are strictly positive, given the definition for the unconditional probabilities of service, 12 to 14, and the value of the provider, equations 7 and 16. Then, the optimal selection rule is at the corner, $\chi = 1$.

Expected profits to the farmers The expected profits to the farmers depend on the equilibrium being realized, i.e. whether providers serve both type of farmers or providers specialize in a single type. The reason is that the expected profits to the farmer depend on the cost of service, which can be in turn expressed as a function of the value of service using the definition of the value per period, $\tilde{z}k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + d_i) = \bar{V}_i^j$

$$U_i = (\tilde{z}k_i^\alpha - w(k_i + d_i) - \bar{V}_i^j)\Delta_{ij} \tag{23}$$

Replacing the values of expected profits for the providers we obtain

1. If providers serve both type of farms,

$$U_s = \Delta_{sj}((1 - \gamma_{1s}^j)(\tilde{z}k_s^\alpha - wk_s + wd_s) - \gamma_{2s}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-}))$$

$$U_{s^-} = \Delta_{s^-j}((1 - \gamma_{1s^-}^j)(\tilde{z}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-}) - \gamma_{2s^-}^j(\tilde{z}k_s^\alpha - wk_s + wd_s))$$

2. If a provider serves only large scale farmers,

$$U_s = \tilde{\Delta}_{sj}(1 - \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_s^\alpha - wk_s + wd_s)$$

3. If a provider serves only small scale farmers,

$$U_{s^-} = \tilde{\Delta}_{s^-j}(1 - \gamma(q_{s^-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_{s^-}^\alpha - wk_{s^-} + wd_{s^-})$$

When the providers specialize in service provision, they determine the expected profits to the farmer.

Equilibrium queue lengths. In an equilibrium where farmers reach out to both providers, they should be indifferent across them and the feasibility constraints of the economy should be satisfied.[27] We describe the indifference condition for large scale farmers, the ones for small scale farmers are analogous.

$$\frac{\Delta_{s\text{mkt}}}{\Delta_{s\text{fcfs}}} = \frac{(1 - \gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(1 - \gamma_{1s}^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{mkt}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}$$

[27] If they choose to reach out to a single provider, then the equilibrium queue length is determined by feasibility only, which in turn determines the expected value for farmers.

When small scale farmers queue only with fcfs providers, the indifference condition for the large farmer is

$$\frac{\Delta_{\text{smkt}}}{\Delta_{\text{sfcfs}}} = \frac{(1-\gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(1-\gamma_s^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s)}$$

These indifference conditions jointly with the feasibility constraints of the economy, equations 1 and 2, yield the optimal queue lengths by provider and type.

Rental rates The rental rates can be computed from the definition of U once the optimal queues have been solved for.

Value of service for farmers $U_s \geq U_{s^-}$ whenever the different in the surplus of service for large-scale providers is large enough. Because in equilibrium the market value of service is the same irrespective of the provider, it is w.l.o.g. to use the values from the fcfs providers.

$$\begin{aligned} U_s - U_{s^-} \geq 0 \\ (\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)(\tilde{z}k_s^\alpha - wk_s - wd_s) - \\ (\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})) \geq 0 \end{aligned}$$

If the surplus is weakly higher for large scale farmers, $\tilde{z}k_s^\alpha - wk_s - wd_s \geq \tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}$, then it is sufficient that

$$\frac{(\Delta_{sj}(1-\gamma_{1s}^j) + \Delta_{s^-j}\gamma_{2s^-}^j)}{(\Delta_{sj}\gamma_{2s}^j + \Delta_{s^-j}(1-\gamma_{1s^-}^j))} \geq \frac{(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-})}{(\tilde{z}k_s^\alpha - wk_s - wd_s)}. \tag{24}$$

The above is a condition on the elasticities of the probability of service to the queue lengths (in γ) relative to the values of the surplus.

Value of service for providers The value of serving large farmers is higher than the value of serving small-farmers for any provider For $\bar{V}_s > \bar{V}_{s^-}$ it is sufficient that $(\gamma_{1s} - \gamma_{2s^-}) > 0$ and $(\gamma_{2s} - \gamma_{1s^-}) > 0$, see equations 19 and . This is the same as

$$a_{11}(\Gamma_4 + \Gamma_3) - a_{21}(\Gamma_2 + \Gamma_1)$$

$$a_{12}(\Gamma_4 + \Gamma_3) - a_{22}(\Gamma_2 + \Gamma_1)$$

If the queue lengths are the same $\Gamma_4 + \Gamma_3 = \Gamma_2 + \Gamma_1$ and also $a_{11} > a_{21}$ and $a_{12} > a_{22}$ because the elasticity of the probability of service to the queue large farmers is higher than for the queue of small farmers. By continuity, if the queue lengths are not too different the above result holds. Intuitively, the reason is that if there are no systematic differences in travel time across farmers, then the provider's marginal cost of provision is higher for the smaller farmers and therefore the provider finds them less valuable.

C Numerical Solution and Output

C.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order i in the queue comprises two dimensions: h_i, the number of hours demanded discretized to 6 bins, and d_i the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3, the value function is a mapping from R^6 to R^1.

$$V(\{(k_1, \nu_1), (k_2, \nu_2), (k_3, \nu_3)\}) : R^6 \to [0, \infty]$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see Judd et al. (2014) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2. Integration is carried out using Newton-Cotes quadrature.

C.2 Simulations

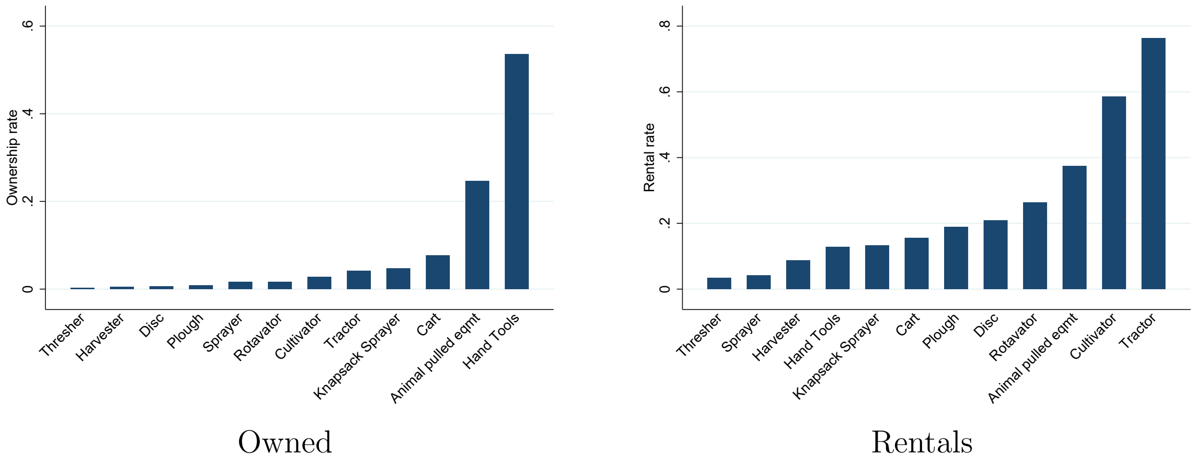
We simulate the expected wait time and productivity cost under the fcfs arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a log normal distribution where the parameters are obtained by fitting the actual productivity information to a log normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the fcfs arrangement and the market arrangement respectively. In any period if one or more out of these three orders are not served, the queue is filled going through the bootstrap sample. The productivity cost is calculated by multiplying the simulated productivity by the wait-time and the percentage productivity loss per day as described in the table 3.

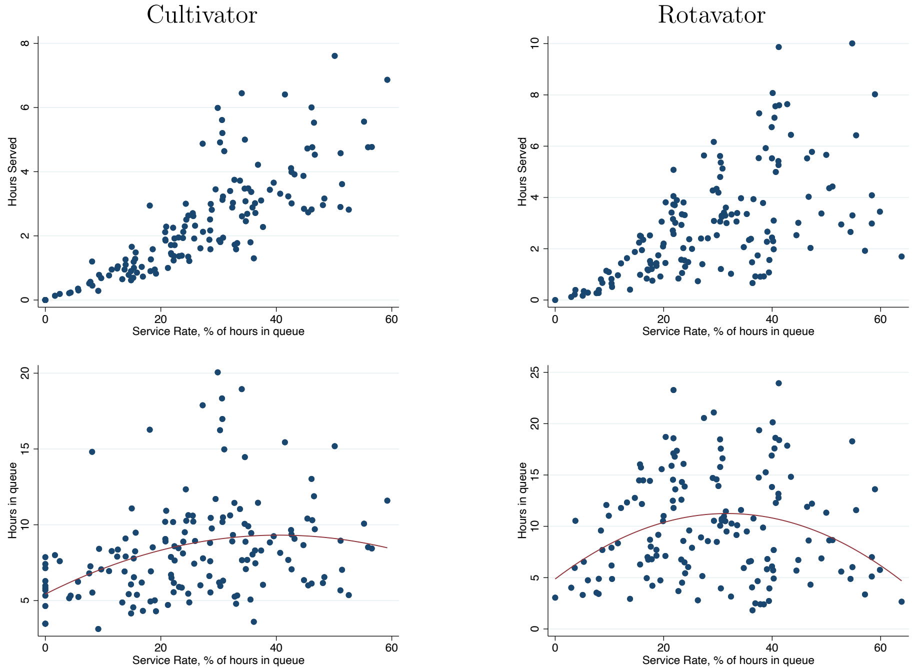
D Additional Tables and Figures

Figure 13: Ownership and rentals by implement.



Notes: The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed.

Figure 14: Service rates.



Notes: Service rates and queueing behavior in CHCs, Kharif 2018. Each data point corresponds to the average queue or hours served across hubs per day in the season. The bottom panels include a polinomial fit of the series. Source: Own computations and CHC admin data.

Table 9: Costs of Delays Relative to Optimal Planting Time, Value Added per Acre

	Cost per day, value added per acre				
	Whole Sample	5-day around optimal		10-day around optimal	
		Before	After	Before	After
β_1	-41.97	391.1**	-215.7	1,166***	-931.1***
	(26.33)	(140.2)	(146.9)	(338.7)	(298.5)
Observations	6,034	1,461	1,882	1,010	1,221
R-squared	0.408	0.625	0.584	0.706	0.659
Mean of Value Added per acre	10228	11425	10694	11921	10998

Notes: The optimal date is a village-year measure, and is the sowing date that maximized mean value added per acre in the village in a given year. Standard errors clustered at the village-level in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, **** $p < 0.001$.