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Journal of Economic Theory 163 (2016) 110–140

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Labor supply with job assignment under balanced growth [☆]

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Received 22 January 2013; final version received 20 July 2015; accepted 9 January 2016

Available online 18 January 2016

Abstract

We consider a competitive equilibrium growth model where technological progress is embodied into new jobs which are assigned to workers of different skills. In every period workers decide whether to actively participate in the labor market and if so how many hours to work on the job. Balanced growth requires that the job technology is complementary with the worker's total labor input on the job, which is jointly determined by his skill and his working hours. Since lower skilled workers can supply longer hours, we show that the equilibrium features positive assortative matching (higher skilled workers are assigned to better jobs) only if differences in consumption are small relative to differences in worker skills. When the pace of technological progress accelerates, wage inequality increases and workers participate less often in the labor market but supply longer hours on the job. This mechanism can explain why, as male wage

☆ The research reported in the paper is not the result of any for-pay consulting relationship. We are indebted to Ricardo Lagos (the Editor) and two anonymous referees whose insightful suggestions have contributed to significantly improving the paper. We have also benefited from comments by Michael Elsby, Boyan Jovanovic, Lars Ljungqvist, Víctor Ríos-Rull, and seminar participants at Goethe University, Federal Reserve Banks of Minneapolis, Philadelphia and Richmond, Stockholm School of Economics, Universitat Autonòma de Barcelona, University of Bonn, Universidad Carlos III, University of Edinburgh, University of Oxford, University of Southampton, University of Toronto, the EEA Annual Meetings in Barcelona, the CEPR ESSIM in Tarragona, the SED Meetings in Cyprus, the IV REDg DGEM Workshop in Barcelona, and the "Frontier Research on Markets with Frictions" conference in Montreal. Funding from Fundación Ramón Areces (VI Concurso) is gratefully acknowledged. Claudio Michelacci also acknowledges the financial support of the European Research Council (ERC Advanced Grant 293692).

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http://dx.doi.org/10.1016/j.jet.2016.01.004

inequality has increased in the US, labor force participation of male workers of different skills has fallen while their working hours have increased.

JEL classification: G31; J31; E24

Keywords: Working hours; Participation; Job heterogeneity; Wage inequality

1. Introduction

The idea that new technologies come embodied into a limited supply of new capital vintages dates back at least to Solow (1960). If the production technology requires each worker to be assigned to a specific capital unit, technological progress also leads to heterogeneity in jobs. In the words of Akerlof (1981), good jobs become a scarce resource, which the economy should assign to workers with potentially different skills. This assignment friction has been widely studied, see Sattinger (1993) for a literature review. But existing assignment models have typically abstracted away from labor supply decisions either at the intensive margin (how many hours to work on the job) or at the extensive margin (whether to actively participate in the labor market). This is an interesting issue because, in assignment models, standard income and substitution effects in labor supply lead to a non-trivial allocation problem between the number of hours worked on the job, which determines the output each job produces, and labor force participation, which determines the number and quality of operating jobs. Income and substitution effects also play a non-trivial role in determining whether the equilibrium features positive assortative matching—i.e. whether higher skilled workers are assigned to better jobs. This is because the amount of labor input supplied by a worker on the job is determined by his skill as well as by his working hours. So a low skilled worker can supply greater working hours to compensate for his lower skill level, which implies that standard conditions for assortative matching based on capital-skill complementarity (Becker, 1973) are directly affected by labor supply.

To study labor supply in an assignment model, we consider a simple neoclassical growth model with perfectly competitive labor markets and vintage capital as in Jovanovic (1998). Technological progress is embodied into new jobs, which are slowly created over time. Hence in equilibrium there is dispersion in job technologies. Workers differ in skills and they can be employed in at most one job. This leads to a simple assignment problem in the spirit of Becker (1973) and Sattinger (1975). But in our framework labor supply is endogenous because in every period each worker decides whether to actively participate in the labor market, which involves a fixed utility cost, and how many hours to work on the job he is assigned to. To guarantee the existence of a balanced growth path, we assume log preferences in consumption (so that in the long run income and substitution effects cancel out) and a production technology on the job that features unitary elasticity of substitution between the job technology and worker's total labor input, which is jointly determined by the worker's skill and his working hours. In equilibrium, the model endogenously generates inequality in jobs, wages, and labor supply, but all workers of the same skill consume the same amount—which is a natural implication of the permanent income hypothesis. Subject to the assignment friction, the competitive equilibrium is efficient and its allocation coincides with the solution chosen by a social planner who gives (potentially) different Pareto weights to workers of different skills.

When labor supply is exogenous, complementarity between the job technology and worker skill ensures that the equilibrium features positive assortative matching (see for instance Becker, 1973). But in our framework the amount of labor input supplied by a worker in a job is function both of his skill and his working hours. Since working hours depend positively on the job technology (due to the substitution effect) and negatively on the worker's wealth (due to the income effect), the total labor input supplied by a poor low skilled workers assigned to a high technology job could be higher than the analogous amount supplied by a wealthy high skilled worker in the same job. This could make profitable assigning a low skilled worker to a high technology job. We show that positive assortative matching requires that workers consumption differences are small relative to their skill differences. This ensures that a low skilled worker assigned to a high technology job faces a small substitution effect relative to the income effect, which in turn guarantees that his total labor input on the job is smaller than the analogous amount supplied by a high skilled worker in the same job. In the social planner problem consumption differences just reflect differences in Pareto weights. But in the decentralized economy, consumption differences arise endogenously as the result of differences in wage income and non-labor income of workers. In the absence of differences in non-labor income, the condition for positive assortative matching requires that workers skill differences are large enough compared to differences in job technologies. If this is not the case, positive assortative matching still arises in equilibrium if low skilled workers enjoy a sufficiently large amount of non-labor income.

When capital embodied technological change accelerates older jobs become more obsolete relative to the technological frontier. It is therefore optimal to reallocate working time from older to newer jobs, which leads to a fall in the participation rate and an increase in average hours per worker. Participation rate falls because older jobs are scrapped earlier. Average hours per worker increase because the income effect on average dominates the substitution effect. This is because the income effect is driven by aggregate detrended consumption, which falls both because the relative technology of all jobs that remain in operation worsen and because aggregate labor force participation falls, while the substitution effect is driven by the average difference in technology between operating jobs and new jobs, which increases relatively little since only the jobs close to the technological frontier remain in operation. The assignment friction is essential for generating opposite movements in the intensive and extensive margins of labor supply in response to faster technological progress. When workers can operate any amount of capital units, so that the assignment friction is absent, income and substitution effect cancel out, and hence hours per worker and labor force participation never move in opposite directions.

In principle, this mechanism can explain why in the US since the 70's, as wage inequality has increased, labor force participation of male workers has fallen while hours per employed worker have increased. To study the quantitative relevance of the mechanism, we parameterize the model to account for differences in employment rates, hours per worker, labor income, and consumption across educational groups in the 1970's. The calibrated model implies that, in the 70's, 75 percent of the hourly wage premium between college graduates and workers with no high school degree was due to skill differences, while the remaining 25 percent was due to differences in job technologies. We then follow Greenwood and Yorokoglu (1997), Greenwood et al. (1997) and Violante (2002) in arguing that the speed of technological progress embodied in new jobs has increased over the period 1970–2000. In the model, the acceleration in the speed of technological progress accounts for 40% of the observed fall in the labor income of workers with no high school degree relative to college graduates. More importantly, the model generates a fall of 8 percentage points in the participation rate and an increase of 1.2 hours worked per week by an average employed worker. This is in line with the data, which show an 8 percent fall in the aggregate participation rate and an increase of 1.5 weekly hours. Finally, the model accounts reasonably well for the observed variation across educational groups. In particular the fact that highly educated workers have experienced a larger increase in hours per worker and a less pronounced fall in participation rates.

Our findings are related to Elsby and Shapiro (2012) who argue that the fall in productivity growth in the US since the 70's has caused a decrease in the return to labor market experience, which can explain why male employment rates for different skill groups have fallen. Our model provides a novel alternative mechanism whereby changes in the long run rate of growth affect labor supply in models with balanced growth preferences. According to our model, employment rates have fallen because of an acceleration in technological progress which has exacerbated technological differences across jobs.

The remaining of the article is organized as follows. In Section 2 we characterize the economic environment and solve for the social planner problem. In Section 3 we discuss how to decentralize the social planner allocation. Section 4 discusses our quantitative results. Section 5 concludes. The Appendix contains proofs and computational details.

2. Model

We analyze the trade-off between labor force participation and working longer hours in an economy with a job assignment problem and worker heterogeneity. For expositional purposes we focus the analysis on the social planner problem and postpone the discussion on how to decentralize the planner allocation to Section 3. After describing the economic environment, we characterize the properties of the frictionless economy. We then turn to the economy with assignment frictions. We study first the case where all workers are identical and then characterize the conditions under which the allocation features positive assortative matching when workers differ in their skills. The analysis focuses on the effects of technological progress on labor supply under balanced growth, which is the topic of the quantitative exercise of Section 4. All propositions are fully proved in the Appendix.

2.1. Assumptions

The economy is in continuous time. There is a representative household with subjective discount rate ρ. The consumption good is the numeraire and we assume log preferences in consumption to guarantee the existence of a balanced growth path with constant aggregate labor supply.[2] The household consists of an entrepreneur and a measure one of infinitely lived workers who differ in their type i. There are N types of workers with skill level $h_i > h_{i+1} > 0$ for $i = 1, 2, \ldots, N-1$. The mass of type i workers is $z_i \in (0, 1)$ with $\sum_{i=i}^{N} z_i = 1$. We assume that a worker with human capital h_i working n hours supplies

$$e = h_i^{1-\theta} n^{\theta} \tag{1}$$

efficiency units of labor with $\theta \in (0, 1)$. For any worker's type, worker's disutility from working n hours in the period is equal to

$$v(n) = \begin{cases} \lambda_0 + \lambda_1 \frac{n^{1+\eta}}{1+\eta} & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases} \tag{2}$$

[2] Since for convenience we assume separability between consumption utility and the disutility of working, we do not allow for the more general specification of preferences discussed in King et al. (1988).

where $\lambda_0 > 0$ measures the fixed cost of going to work, $\lambda_1 > 0$ governs the magnitude of the variable component and $\eta \geq 0$ regulates the Frisch elasticity. To allow consumption levels to vary by worker's type, we assume that the social planner gives different Pareto weights ν_i to workers of different type. We impose $\nu_i \geq \nu_{i+1}$ for all $i = 1, 2, \ldots, N-1$ with $\sum_{i=1}^{N} \nu_i z_i = 1$. Strict equality implies equality of consumption across workers skill types. Strict inequality means that more skilled workers enjoy higher consumption, which is the empirically relevant case. This justifies why we disregard the case $\nu_i < \nu_{i+1}$.

To produce consumption units a worker has to be matched with a job. We will think of a job as a machine and we will use the term job and machine interchangeably. As in Jovanovic (1998), at every instant in time t, $m < \infty$ new jobs of quality e^{qt} become available, with $q > 0$ measuring the speed of embodied technical change. Jobs are in excess supply because the number of potential workers is fixed to one while new jobs of relatively better quality become continuously available. At each point in time we rank jobs by their age τ and we denote by τ^* the (endogenously determined) critical age such that all jobs older than τ^* are scrapped. Then the age distribution of operating jobs is uniform with support $\left[0, \tau^*\right]$, which implies a probability density equal to $1/\tau^*$. Let p_i denote the participation rate for workers of type i, i.e. the fraction of workers of type i who actively participate to the labor market. We focus the analysis on a balanced growth path equilibrium, where the aggregate participation rate $p = \sum_{i=1}^{N} p_i z_i$, the critical age threshold τ^*, and the mass of newly created jobs m are constant over time. For simplicity, we assume $p_i \in (0, 1)\ \forall i$. Since every worker is paired with a job, the number of jobs in operation is equal to the number of employed workers so that

$$\int_0^{\tau^*} m\, d\tau = p$$

which implies that $\tau^* = \frac{p}{m}$. This means that the age distribution of operating jobs has support over the interval $[0, \frac{p}{m}]$ and density m/p.

The quality of a job depreciates at constant rate δ, so a job of age τ at time t has quality $\tilde{k}_t^\tau = e^{q(t-\tau)-\delta\tau}$. In the rest of the paper we work with detrended job qualities defined as equal to

$$k^\tau \equiv \tilde{k}_t^\tau e^{-qt} = e^{-(q+\delta)\tau} \tag{3}$$

This implies that the detrended quality of the best job in operation is equal to $k^0 = 1$ while the (detrended) quality of the marginal job is $k^* = e^{-\frac{(q+\delta)p}{m}}$. We assume that, at a time, workers cannot work in more than one job and that a job cannot be matched with more than one worker. This is the key friction of our economy, which arises because workers and jobs are indivisible. To guarantee the existence of a balanced growth path with constant growth, we assume that a job of quality k when matched with a worker of type $i = 1, 2, \ldots, N$ who supplies n hours of work on the job produces an amount of consumption units given by the following homogenous of degree one Cobb–Douglas function

$$f(k, h_i, n) = k^\alpha \left(h_i^{1-\theta} n^\theta\right)^{1-\alpha}, \quad \alpha \in (0, 1).$$

Jobs are created by entrepreneurs. Creating a job has a cost κ in utility terms. This guarantees the existence of a balanced growth with constant job creation, since both the cost and the value of a newly created job, once evaluated in utils, remain constant over time.[3] The social planner instantaneous utility can then be expressed as equal to

$$\sum_{i=1}^{N} \nu_i \left[z_i \log c_i - p_i \int_{z_{i-1}}^{z_{i-1}+z_i} v\left(n(j)\right) dj \right] - \kappa m \tag{4}$$

with $z_0 = 0$ and $n(j)$ denoting the hours worked by worker $j \in [0, 1]$, where workers are indexed over the unit interval and ranked in (weakly) decreasing order of skill.

2.2. No assignment friction

We start characterizing the economy where workers can operate any amount of machines, so that no assignment friction is present. In this economy aggregate consumption is obtained by combining aggregate capital K with aggregate labor L according to the Cobb–Douglas production function $K^\alpha L^{1-\alpha}$. Detrended capital is equal to

$$K = m \int_0^\infty k^\tau d\tau = m \int_0^\infty e^{-(q+\delta)\tau} d\tau = \frac{m}{q+\delta} \tag{5}$$

which means that all capital available in the economy is used in production. The aggregate supply of labor is equal to $L = \sum_{i=1}^{N} p_i z_i e_i$, where $e_i = h_i^{1-\theta} n_i^\theta$ are the efficiency units supplied by an employed worker of type i and p_i is the participation rate of workers of type i. The social planner cares equally for all individuals of the same type i and gives them the same level of (detrended) per capita consumption c_i. The problem of choosing consumption c_i, labor force participation p_i, and hours worked n_i for given m is intrinsically static and it amounts to maximizing

$$\max_{c_i, p_i, n_i} \left\{ \sum_{i=1}^{N} \nu_i z_i \left[\log c_i - p_i v\left(n_i\right)\right] - \kappa m \right\} \tag{6}$$

subject to the aggregate resource constraint $C \equiv \sum_{i=1}^{N} z_i c_i = K^\alpha L^{1-\alpha}$ with associated Lagrange multiplier μ. The first order conditions for consumption c_i leads to

$$\mu = \frac{\nu_i}{c_i} \tag{7}$$

which implies that the relative consumption of different worker types is equal to their relative Pareto weights. We can now multiply equation (7) by z_i, and then add up over all i's. After remembering that $\sum_{i=1}^{N} \nu_i z_i = 1$, we obtain that

$$\mu = \frac{1}{C}. \tag{8}$$

[3] There are alternative ways of modelling the cost of job creation. For example we could assume that the cost of creating a new job at time t is in consumption units and equal to $\overline{\kappa} e^{\alpha q t}$, which would also guarantee the existence of a balanced growth path with constant job creation. This alternative formulation would have the advantage of mimicking more closely what is typically assumed in the neoclassical growth model, but at the cost of complicating substantially the theoretical analysis. This is because changes in the job creation rate m would affect aggregate consumption via the aggregate resource constraint, which thereby affects labor supply through income effects. We study this alternative formulation in the quantitative analysis of Appendix B, where we find that these additional income effects are quantitatively unimportant.

Taking the first order conditions with respect to n_i and p_i we obtain the two conditions:

$$(1-\alpha)\,\theta \frac{e_i}{Ln_i} = \nu_i\, v'(n_i) \quad \text{and} \quad (1-\alpha)\frac{e_i}{L} = \nu_i\, v(n_i), \quad \forall i = 1, 2, \ldots, N \tag{9}$$

which implies that n_i and p_i are independent of q for any worker type i.[4] This follows from consumption log-preferences that make income and substitution effects cancel out exactly. We then prove:

Proposition 1. *In the absence of a job assignment problem, (i) labor force participation and average hours worked on the job for any worker type $i = 1, 2, \ldots, N$ are unaffected by the pace of technological progress q; and (ii) the optimal rate of job creation is equal to*

$$m = \frac{\alpha}{\kappa} \cdot \frac{q+\delta}{q+\delta+\rho}$$

Proposition 1 implies that the rate of job creation m is weakly increasing in q. With higher q the stock of detrended capital is lower, see (5), and its marginal product is larger, which increases the value of new machines. But a higher q also increases the obsolescence rate of detrended capital, which makes new machines less valuable. With $\rho > 0$ the former effect dominates, and m is strictly increasing in q. With $\rho = 0$ the two effects cancel out exactly and the rate of creation of new machines m is independent of q.[5]

2.3. Solving the economy with homogeneous workers

We now go back to the economy where jobs have to be assigned to workers. We start characterizing the economy where all workers are identical, $h_i = \nu_i = 1$, $\forall i = 1, 2, \ldots, N$. In this economy the social planner equalizes detrended consumption c across all individuals. In every period, the planner also chooses how many workers p should actively participate in the labor market, the age of the job τ each of them should be matched with and their working hours n^τ in the match. As workers are homogenous, the exact identity of workers is irrelevant and thereby indeterminate, therefore we simply index workers by the age of the machine they are paired with. We start characterizing the economy for constant job creation rate m, but we later show that the results of the analysis generally survive when m is endogenized. For given rate of job creation m, the planner's problem is intrinsically static and consists in maximizing the sum of instantaneous utilities

$$\max_{c,p,n^\tau} \left\{ \log c - m \int_0^{\frac{p}{m}} v\left(n^\tau\right) d\tau - \kappa m \right\} \tag{10}$$

subject to the aggregate resource constraint for detrended consumption

[4] For simplicity we are assuming that $p_i \in (0,1)$. If instead the participation rate of type i is at a corner, $p_i \in \{0,1\}$, it must be that $\left[(1-\alpha)\frac{e_i}{L} - \nu_i v(n_i)\right](1-2p_i) \le 0$.

[5] One can also notice that with $\rho = 0$, the optimal value of m is set to maximize the steady state utility of the representative household in (6). But when ρ is positive, the planner chooses a lower rate of job creation because increasing steady state utility is costly today. This is the traditional distinction between the golden rule proposed by Phelps (1961), and the modified golden rule emphasized in the neoclassical growth model by Ramsey (1928), Cass (1965), and Koopmans (1965).

$$c = m \int_0^{\frac{p}{m}} f\left(e^{-(q+\delta)\tau}, n^\tau\right) d\tau \tag{11}$$

In the Appendix (see Lemma 3), we show that n^τ decline with job quality $e^{-(q+\delta)\tau}$ and increase with the Lagrange multiplier $\mu = 1/c$ of the aggregate resource constraint in (11), which measures the marginal value of income. The former is a conventional substitution effect in labor supply. The latter characterizes the income effect.

Clearly an acceleration in the pace of technological progress (an increase in q) leads to an increase in welfare. But, when focusing on detrended quantities, the increase in q is equivalent to a higher depreciation rate of capital. To see this notice that after detrending, the quality of a newly created job is always equal to one, $k^0 = 1$, while the quality of a job of age $\tau \geq 0$ is equal to $k^\tau = e^{-(q+\delta)\tau}$, which falls with q and the more so the larger is τ. This makes the levels of detrended output and consumption fall, while differences in job technologies, as measured by the ratio between the quality of a newly created job and a marginal job, equal to $1/k^* = e^{\frac{(q+\delta)p}{m}}$, increase. In general we can prove that

Lemma 1. *For given rate of job creation m, when technological progress accelerates (q goes up), we have that: (i) Detrended consumption c falls; (ii) The ratio of the quality between the top and the marginal job, $\frac{1}{k^*}$ increases; (iii) Hours worked in newly created jobs n^0 increase; (iv) Hours worked in the marginal job n^* remain unchanged.*

When technological change accelerates (q goes up), older jobs become more technologically obsolete and so they are scrapped earlier, which immediately implies a fall in the participation rate p. Instead average hours per employed worker, which are equal to

$$\overline{n} = \int_0^{\frac{p}{m}} \psi(\tau, \mu) \frac{m}{p} d\tau \tag{12}$$

increase, because, on average, the income effect dominates the substitution effect. The income effect is measured by the value of income $\mu = 1/c$, and thereby by aggregate (detrended) consumption c, which falls both because the detrended quality of all jobs that remain in operation worsen and because aggregate labor force participation p falls. In the aggregate, the substitution effect is instead measured by the average difference in technology between operating jobs and new jobs, which increases relatively little since old jobs are now scrapped earlier. As a result, the intensive and the extensive margin of labor supply move in opposite directions in response to an increase in q:

Proposition 2. *For given rate of job creation m, when technological progress accelerates (q increases) the participation rate p falls, while average hours per employed worker $\overline{n}$ increase.*

So far we have analyzed the properties of the economy for a given rate of job creation m. We now show that Proposition 2 holds true also when m is endogenized, as long as the subjective discount rate of agents ρ is small enough, which in the quantitative exercise of Section 4 appears to be the relevant case.[6]

Proposition 3. *If ρ is close enough to zero, when technological progress accelerates (q increases) we have that: (i) The rate of job creation m increases; (ii) The participation rate p falls; (iii) Average hours per employed worker $\overline{n}$ increase.*

2.4. Solving the economy with worker heterogeneity

We now characterize the economy with worker heterogeneity. Let n_i^τ denote the working hours of workers of type i when matched with a machine of age τ. Again let μ denote the Lagrange multiplier of the aggregate resource constraint, which measures the marginal value of income to the social planner. Then the utility flow value of matching a job of age τ with a worker of type i is equal to:

$$\widetilde{s}_i(\tau) = \max(0, s_i(\tau)) \tag{13}$$

where $s_i(\tau)$ measures the flow value when supplying positive working hours on the job:

$$s_i(\tau) = \max_{n>0} \left\{ \mu f\left(e^{-(q+\delta)\tau}, h_i, n\right) - \nu_i v(n) \right\}. \tag{14}$$

This is equal to the difference between the value of the income the worker produces on the job and the disutility cost of working to the social planner. Notice that this expression is independent of t. The zero value in (13) simply reflects the option value of not producing with the worker. For given participation rates p_i, it is optimal to assign higher skilled workers to higher quality jobs, if and only if higher skilled workers are relatively more valuable in newer than in older jobs, which is equivalent to requiring that $\frac{\partial[s_i(\tau)-s_{i+1}(\tau)]}{\partial\tau} \le 0$. By solving for n in (14) we immediately obtain that a worker of type i in a job of age τ should supply an amount of hours equal to

$$n_i^\tau = \left[\frac{(1-\alpha)\theta h_i^{(1-\alpha)(1-\theta)}}{\lambda_1 \nu_i}\right]^{\frac{A}{1+\eta}} e^{-\frac{\alpha(q+\delta)A}{1+\eta}\tau} \mu^{\frac{A}{1+\eta}}, \tag{15}$$

where $A = \frac{(1+\eta)}{1+\eta-(1-\alpha)\theta} > 1$. By applying the envelope theorem in (14) it also follows that

$$\frac{\partial s_i(\tau)}{\partial \tau} = -\mu(q+\delta)\alpha f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) < 0.$$

This implies that $\frac{\partial[s_i(\tau)-s_{i+1}(\tau)]}{\partial\tau} \le 0$ holds, if and only if job output is increasing in worker's type, $f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) \ge f\left(e^{-(q+\delta)\tau}, h_{i+1}, n_{i+1}^\tau\right)$, which, given (15) and the definition of output, happens if and only if the following condition holds:

$$\frac{h_i}{h_{i+1}} \ge \left(\frac{\nu_i}{\nu_{i+1}}\right)^{\frac{\theta}{(1-\theta)(1+\eta)}}, \quad \forall i < N. \tag{A1}$$

This immediately leads to the following proposition:

[6] We were not able to prove the result for an arbitrary value of ρ. But, for plausible calibrations of the model we have always found that in response to an increase in q the participation rate p falls and average hours per employed worker $\overline{n}$ increase. We further discuss this issue in Section 4 and Appendix B.

Proposition 4. *The optimal allocation features positive assortative matching if and only if job output is increasing in worker's type, which requires that condition (A1) holds true.*

Condition (A1) states that human capital differences are large relative to Pareto weights. The condition is more likely to hold when θ is low, which implies that hours matter less for the total labor input supplied on the job, or when η is large, which means that substitution effects have small impact on working hours on the job.

We now write the social planner problem under assumption (A1), so that the optimal allocation features positive assortative matching. Then the minimal age of the jobs where workers of type $i \geq 1$ are employed is τ^*_{i-1}, while the maximal age is τ^*_i, where

$$\tau^*_i = \tau^*_{i-1} + \frac{p_i z_i}{m}, \qquad \forall i = 1, 2, \ldots, N. \tag{16}$$

with the convention $\tau^*_0 = 0$. For given job creation m, the social planner then solves the following (static) problem

$$\max_{c_i, p_i, n_i^\tau} \left\{ \sum_{i=1}^{N} \nu_i \left[z_i \log c_i - \int_{\tau^*_{i-1}}^{\tau^*_i} v\left(n_i^\tau\right) m \, d\tau \right] - \kappa m \right\} \tag{17}$$

subject to the resource constraint

$$\sum_{i=1}^{N} z_i c_i = C \equiv \sum_{i=1}^{N} \int_{\tau^*_{i-1}}^{\tau^*_i} f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) m \, d\tau \tag{18}$$

with associated Lagrange multiplier μ. The first order condition with respect c_i immediately yields (7), which implies that Lagrange multiplier μ still satisfies (8). To write the first order condition with respect to p_i, notice that (16) implies that $\frac{d\tau^*_j}{dp_i} = \frac{z_i}{m}$, $\forall j \geq i$ and zero otherwise. This is because, as the participation of type i workers increases, all workers of lower type, $j > i$, are displaced to marginally older jobs, while workers of higher types, $j < i$, are left unaffected. Let's start assuming for simplicity that $p_i \in (0, 1)$, $\forall i$. The first order condition with respect to p_N immediately leads to

$$s_N\left(\tau^*_N\right) = 0, \tag{19}$$

which means that the worst job operated by the lowest skill workers must have zero value to the social planner. The analogous condition for p_i $i < N$ can be expressed as

$$s_i\left(\tau^*_i\right) - \sum_{j=i+1}^{N} \left[s_j\left(\tau^*_{j-1}\right) - s_j\left(\tau^*_j\right) \right] = 0, \tag{20}$$

which emphasizes that assigning a job to a worker of type i has an opportunity cost, because other, less skilled, workers can no longer operate the job. So employing one more worker of type i to operate a job of age τ^*_i gives less than $s_i\left(\tau^*_i\right)$ to the social planner, since this job was already operated by a type $i+1$ worker. The net increase in social value is then measured by the left hand side in (20), which takes into account that, as the mass of type i workers used in production increase, all employed workers of type $j > i$ are displaced to marginally older jobs. Condition (20) can be solved recursively using (19) and starting from $i = N - 1$ to obtain

$$s_i\left(\tau_i^*\right) = s_{i+1}\left(\tau_i^*\right), \quad \forall i < N \tag{21}$$

which says that at τ_i^* using a type i or a type $i+1$ worker gives the same return.

The working hours of type i workers n_i^τ are given by (15). Exactly as in the one-type model, hours worked decrease with job age τ and increase with the marginal value of income μ. For given τ and μ, hours worked are increasing in the worker's skill h_i and decreasing in the worker's Pareto weight ν_i, which, given (7) determine worker's consumption. Since higher skilled workers have both higher skill h_i and higher Pareto weight ν_i, it is generally unclear whether working hours are increasing in workers skills. By evaluating (15) for workers of different type i, we can characterize the conditions under which n_i^τ decreases with i:

Lemma 2. *For given marginal value of income μ and job quality k^τ, working hours are increasing in the skill type of workers if and only if*

$$\frac{h_i}{h_{i+1}} > \left(\frac{\nu_i}{\nu_{i+1}}\right)^{\frac{1}{(1-\alpha)(1-\theta)}} \quad \forall i < N \tag{22}$$

This condition says that working hours are increasing in the skill type of workers when the skill advantage of more productive workers is large relative to their disutility cost of working, as measured by their Pareto weights. Of course, if (22) holds, also output in a job is increasing in skill type but the converse is not necessarily true. For output to be increasing in the skill type of a worker, (A1) should hold, which is less restrictive than (22).

A particular case arises when Pareto weights are independent of worker skills, $\nu_i = \nu\ \forall i$. In this case consumption is equalized across workers, see (7), and, since (A1) is satisfied, the optimal allocation features positive assortative matching. By Lemma 2 we also have that working hours are increasing in the skill type of workers. Finally, since the value of labor market participation is higher for higher skilled workers at all machine ages, $s_i(\tau) > s_j(\tau)$, $\forall i < j, \forall \tau$, condition (21) can never hold as an equality, leading to corner solutions in participation rates. In particular, there will be an i^* such that $p_i = 1, \forall i < i^*$ and $p_i = 0, \forall i > i^*$, which implies that higher skilled workers participate more in the labor market.

Finally the planner chooses the optimal job creation rate m so as to equate the cost of a newly created job to its present value V, which is equal to the discounted value of the utility flow values generated by all workers who produce on the job over its entire production life. So in the optimal allocation it must be that

$$\kappa = V = \sum_{i=1}^{N} \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} s_i(\tau)\, d\tau. \tag{23}$$

3. Decentralization

We now discuss how the social planner allocation can be decentralized through prices using employment lotteries in the same spirit as Hansen (1985) and Rogerson (1988). We focus on a steady state equilibrium, where the labor market is characterized by a wage function $w_i(n)$, that specifies the (detrended) income paid to workers of type i when supplying $n > 0$ hours in a job, and by an assignment function $\varphi_i(\tau)$ that specifies the probability density at which a worker of type i actively participating in the labor market in any period t is assigned to a job of age τ. Given the functions w_i and φ_i, jobs can freely choose their demand for working hours while workers choose whether to actively participate in the labor market and how many hours to supply on the job. A stable assignment requires that no job and no worker should find optimal to deviate from the allocation prescribed by the assignment function $\varphi_i(\tau)$. We conjecture and later verify that the equilibrium features positive assortative matching.

3.1. Households

There is a representative household of size z_i for each type i of workers.[7] To guarantee that (23) holds in the decentralized equilibrium, we assume that entrepreneurs are randomly assigned across households, with a mass z_i of entrepreneurs being part of the household of type i. Creating a new job requires the effort of all entrepreneurs in the economy, with each of them incurring a utility cost κ. The household of type i chooses the fraction p_{it} of its members that go to work in period t as well as how many hours n_{it}^{τ} each worker should supply on the job τ she is assigned to. This yields per capita detrended labor income to the household equal to

$$W_{it} = p_{it} \int_{R_+} w_i\left(n_{it}^{\tau}\right) \varphi_i(\tau)\, d\tau. \tag{24}$$

All workers in the household obtain the same consumption level. The household chooses per capita consumption $\tilde{c}_{it}$ and assets $\tilde{b}_{it}$, labor supply p_{it} and n_{it}^{τ}, and entrepreneurial effort m_t to maximize

$$\max_{\tilde{c}_{it}, \tilde{b}_{it}, p_{it}, n_{it}^{\tau}, m_t} \int_0^{\infty} e^{-\rho t} z_i \left[\log \tilde{c}_{it} - p_{it} \int_{R_+} v\left(n_{it}^{\tau}\right) \varphi_i(\tau)\, d\tau - \kappa m_t \right] dt \tag{25}$$

subject to the sequence of budget constraints:

$$\dot{\tilde{b}}_{it} = e^{\alpha q t} W_{it} + e^{\alpha q t} \phi_i m_t - \tilde{c}_{it} + r_t \tilde{b}_{it}, \tag{26}$$

where a dot denotes a time derivative while ϕ_i denotes the detrended compensation that each entrepreneur in the household receives for creating a new job.

3.2. Jobs and assignment

A job of age τ matched with a worker of type i chooses hours to maximize detrended profits equal to

$$\pi_i(\tau) = \max_{n_{it}^{\tau}} \left\{ f\left(k^{\tau}, h_i, n_{it}^{\tau}\right) - w_i\left(n_{it}^{\tau}\right) \right\}. \tag{27}$$

In an equilibrium with positive assortative matching, the assignment function should be given by:

$$\varphi_i(\tau) = \begin{cases} \frac{1}{\tau_i^* - \tau_{i-1}^*}, & \text{if } \tau \in \left[\tau_{i-1}^*, \tau_i^*\right] \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

[7] The existence of a representative household for workers of the same type i requires that all workers of the same type are initially endowed with the same amount of wealth b_{i0}. Workers can then achieve perfect consumption insurance because they can freely borrow and save and there is no aggregate uncertainty.

where the job age thresholds τ_i^* satisfy (16). Since there is an excess supply of jobs, it must also be that at τ_N^* the jobs make zero profits

$$\pi_N\left(\tau_N^*\right)=0, \tag{29}$$

while $\forall\tau\in\left[\tau_{i-1}^*,\tau_i^*\right]$ jobs should prefer to hire workers of type i rather than any other worker type and should at least break even:

$$\pi_i(\tau)=\max_j \pi_j(\tau)\geq 0, \quad \forall\tau\in\left[\tau_{i-1}^*,\tau_i^*\right], \ \forall i\geq 1. \tag{30}$$

Clearly at τ_i^*, (30) should hold both for i and $i+1$, which implies that at the marginal job, higher skilled workers capture all rents so that

$$\pi_i\left(\tau_i^*\right)=\pi_{i+1}\left(\tau_i^*\right), \quad \forall i<N. \tag{31}$$

In an equilibrium with positive job creation $m>0$, the cost of creating a new job at time t, $\sum_i z_i e^{\alpha q t}\phi_i$, should be equal to its value, $e^{\alpha q t}P_t$, hence

$$\sum_{i=1}^{N} z_i\phi_i=P_t \tag{32}$$

where P_t is the detrended present value of all profit generated by the job:

$$P_t=\sum_{i=1}^{N}\int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau}\pi_i(\tau)\,d\tau. \tag{33}$$

In writing this expression for P_t we used the fact that in steady state equilibrium the modified golden rule applies so that the market interest rate satisfies $r=\rho+\alpha q$.

3.3. Equilibrium

Market clearing in the goods market implies that aggregate consumption is equal to the total amount of consumption units produced in the economy so that

$$C=\sum z_i c_i=\sum_{i=1}^{N} z_i W_i+\Pi \tag{34}$$

where Π denotes steady state aggregate detrended profits equal to

$$\Pi=\sum_{i=1}^{N}\int_{\tau_{i-1}^*}^{\tau_i^*}\pi_i(\tau)\,m\,d\tau=\sum_{i=1}^{N} z_i\left(\rho b_{i0}+\phi_i m\right), \tag{35}$$

where the last equality follows from the fact that aggregate profits should be equal to the sum of all non-labor income earned by the household in the economy. Hence, non-labor income of a type i household can be written as a constant fraction σ_i of aggregate profits

$$\rho b_{i0}+\phi_i m=\sigma_i\Pi, \tag{36}$$

where $\sum_{i=1}^{N} z_i\sigma_i=1$. This means that differences in non-labor income across households' types can be parameterized indifferently in terms of either σ_i's or b_{i0}'s.

Let c_i and b_i denote detrended consumption and detrended assets, respectively. After defining the tuple

$$x = \left[c_i, b_i, \Pi, w_i(n), \pi_i(\tau), \varphi_i(\tau), p_i, n_i^\tau, \phi_i, m, r, P\right]$$

we can then state the following definition for a steady state equilibrium:

Definition. A steady state equilibrium is a tuple x such that (i) each representative household maximizes her utility in (25); (ii) jobs maximize their profits in (27); (iii) the labor market clears, so (16) and (28) hold; (iv) the conditions for stable assignment (29)–(30) are satisfied; (v) the free-entry condition for job creation in (32) holds; and (vi) the goods market clears so (34)–(36) are satisfied.

The following proposition clarifies the relation between the equilibrium of the decentralized economy and the allocation chosen by the social planner:

Proposition 5. *If the shares of type i households on aggregate non-labor income, σ_i's, are such that*

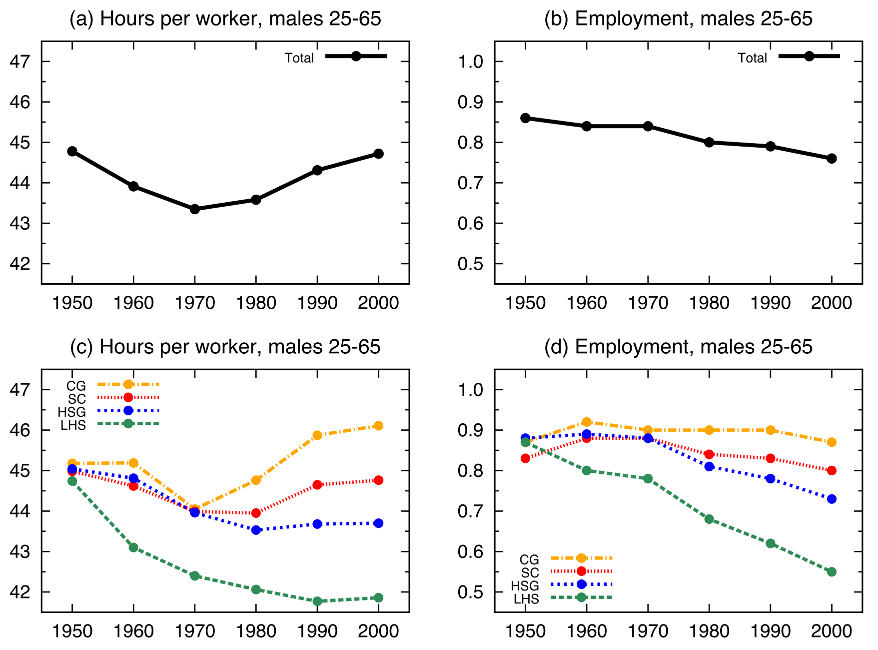
$$\frac{h_i}{h_{i+1}} \geq \left(\frac{c_i}{c_{i+1}}\right)^{\frac{\theta}{(1-\theta)(1+\eta)}}, \quad \forall i < N, \tag{A2}$$

then the equilibrium of the decentralized economy features positive assortative matching and the equilibrium allocation solves the social planner problem in (17) with the set of Pareto weights ν_i satisfying $\nu_i = \frac{c_i}{C}, \forall i \geq 1$.

The difference between assumptions (A1) and (A2) is that in the decentralized allocation, consumption differences are an equilibrium outcome, while in the social planner problem they reflect differences in Pareto weights. Generally (A2) requires that skill differences are large relative to consumption differences, which is more likely to hold when technological differences across jobs are smaller and when the share of non-labor income on total income of low skilled workers is greater.

4. A quantitative exercise

Our theory states that an increase in the speed of embodied technical change rises wage inequality, rises hours per employed worker, and diminishes participation. According to Greenwood and Yorokoglu (1997) and Greenwood et al. (1997) investment specific technological progress has actually accelerated since the 1970, and it has been argued—see for instance Violante (2002)—that this is the cause of the increase in US wage inequality documented among others by Katz and Autor (1999) and Heathcote et al. (2010). In this Section we study the quantitative role of the observed change in the pace of investment specific technological progress q in accounting for some important changes in male labor supply observed in the US over the 1970–2000 period. To analyze this question we consider the decentralized economy studied in Section 3, with $N = 4$ household types corresponding to 4 education groups in the data: college



Note. Employment rates and hours per worker are for male workers of age 25–65 from the US Census. CG refers to college graduates, SC to high school graduates with some college education, HSD to high school graduates, and LHS to workers with less than high school degree.

Fig. 1. Hours and participation.

graduates, workers with some college education but no college degree, high school graduates, and workers without a high school degree.[8]

Male labor supply has indeed changed substantially in the US between 1970 and 2000. As documented by Juhn (1992), Aaronson et al. (2006) and Michelacci and Pijoan-Mas (2012) the participation rate of US male workers has fallen substantially while average hours worked per employed worker have increased. Fig. 1 documents these facts using the 1 percent sample of the decennial Census, as provided by the Integrated Public Use Microdata Series (IPUMS) at the University of Minnesota (www.ipums.org). We focus the analysis on a sample of male workers aged between 25 and 64 years old. Panels (a) and (b) describe the evolution of average hours per employed worker and of the employment rate, respectively. It is also well known that these changes have varied depending on the skill level of workers, here identified by their educational level. As shown in Panels (c) and (d), high skilled workers have experienced a larger increase in hours per worker and a smaller fall in employment rates.

4.1. Calibration in 1970

Calibrating a version of the model with $N = 4$ involves choosing 21 parameters, of which 3 are set using one normalization condition and two add-up constraints. Of the remaining 18 pa-

[8] Alternatively we could have used a version of the social planner problem studied in Section 2. But in this model, for given Pareto weights, a change in technological progress would have no effects on relative consumption by skill groups, which as shown in Table 3 would be highly counterfactual.

Table 1
Parameter values and calibration targets, 1970.

Symbol	Model		Calibration target	
	Benchmark	Alternative	Statistic	Value
Preferences				
ρ	0.04	0.04	–	–
η	2	2	–	–
λ_0	0.65	0.92	Average employment to population ratio	0.84
λ_1	6.97	9.82	Average hours per employed person	43.3
Technology				
δ	0.06	0.06	–	–
q (%)	5.51	5.51	Rate of fall of price of investment goods	0.02
$\kappa, \bar{\kappa}$	3.87	1.39	Average machine age	5.6
α	0.64	0.64	Capital share	0.33
θ	0.98	0.98	Labor income ratio between group 4 and 1, $\overline{w}_4/\overline{w}_1$	0.54
Population				
z_1	0.15	0.15	Add-up constraint $\sum z_i = 1$	–
z_2	0.11	0.11	Population share of group 2	0.11
z_3	0.31	0.31	Population share of group 3	0.31
z_4	0.43	0.43	Population share of group 4	0.43
h_1	1	1	Normalization	–
h_2	0.76	0.76	Consumption of group 2 relative to group 1, c_2/c_1	0.84
h_3	0.64	0.64	Consumption of group 3 relative to group 1, c_3/c_1	0.76
h_4	0.51	0.51	Consumption of group 4 relative to group 1, c_4/c_1	0.68
σ_1	1.00	−1.12	Add-up constraint, $\sum \sigma_i z_i = 1$	–
σ_2	0.92	−0.31	Participation rate for group 2, p_2	0.88
σ_3	0.90	0.25	Participation rate for group 3, p_3	0.88
σ_4	1.09	2.62	Participation rate for group 4, p_4	0.78

Note. *Benchmark* refers to the benchmark model and *Alternative* refers to the model described in Appendix B, where job creation costs are specified in consumption units. Groups 1 to 4 refer to college graduates, high school graduates with some college education, high school graduates and workers with no high school degree, respectively. All statistics are for male workers of age 25–65. Population shares, employment rates, hours per worker and income differences are from the 1970 US Census. Consumption levels are from 1980 CEX.

rameters, 6 are set directly while 12 are set by requiring that the model matches 12 moments from the data. Table 1 summarizes the resulting parameter values and the corresponding calibration targets.

We set the annual discount rate ρ to 4%, and the Frisch labor elasticity parameter η to 2. The depreciation rate δ is set to 6%, which is taken from Nadiri and Prucha (1996). The shares of workers of different skill type i, z_i, are chosen to match the corresponding values in 1970. We choose λ_0 and λ_1 to match the average male employment rate and average hours per employed male worker in the US in 1970, which are equal to 0.84 and 43.3 weekly hours, respectively.[9]

[9] Hours in the model are calibrated to 43.3/112, where 112 corresponds to the amount of non-sleeping hours in a week available to the worker (7 days a week times 16 hours a day). Hours in the model are then multiplied by 112 to report the results in tables.

The cost of job creation κ determines the mass of newly created machines m, which affects the average machine age in the model economy, equal to $\frac{p}{2m}$. We then use as calibration target the average age of capital equipment as reported by the Bureau of Economic Analysis (BEA), which in the mid 60's was equal to 5.6 years.[10]

We set α to match a labor share of one third which in the model is calculated as

$$\text{Labor share} = \frac{\sum_{i=1}^{4} z_i W_i}{C},$$

which follows Cooley and Prescott (1995) in splitting entrepreneurial income between capital and labor income in the same proportion as in the rest of the economy.

To determine the rate of growth of capital-embodied technical change q we follow Hornstein et al. (2007) in reproducing the 2% rate of fall of the relative price of investment goods in the 70's reported by Greenwood et al. (1997). As in Hornstein et al. (2007) the rate of decline of the price of a new efficiency unit of capital is equal to $(1-\alpha)q$, which leads to a choice of q equal to 5.51%.[11]

We normalize h_1 to one. The remaining three values for h_i, together with the three independent values for σ_i and the value for θ are chosen to match (i) the employment rate for the three educational types (the fourth is then matched since the aggregate participation rate is a target), (ii) the consumption level of each type of worker relative to the consumption of workers with a college degree, and (iii) the average labor income per employed worker of the lowest skill group relative to the highest skill group. Relative consumption comes from CEX in 1980, which is the first wave available. Average labor incomes by skill group are calculated using the 1970 Census.

The calibrated economy satisfies the condition (A2) for assortative matching. Column 1 in Panel (A) of Table 2 reports the value of the employment rates and hours worked in 1970 in the data. The corresponding values for the calibrated economy are in column 1 of Panel (B). Since the aggregate employment rate, the average hours per worker, and the employment rates by educational groups are calibration targets, the model matches their values perfectly.

In the calibration we do not target hours per employed worker by educational group. The model rightly predicts that better educated workers work longer hours, although it slightly over-predicts differences by educational group. Column 1 in Table 3 reports the relative labor income and the relative consumption by skill group in the data (in Panel (A)) and in the model (in Panel (B)). Relative consumption patterns are matched by construction and so is the labor income of workers with no high school degree relative to college graduates. But the relative labor income of the two other educational groups was not targeted, still the model matches their value quite accurately.

Finally, we can use the calibrated model to measure how much of the wage return to education in 1970 was due to worker skill differences and how much was due to job differences. To do so, we solve the calibrated model for a very small value of κ such that in equilibrium m is very large and hence all jobs are identical. We find that 75% of the hourly wage ratio between college workers and workers with no high school degree is still present in the low κ economy. This implies that, in 1970, 3/4 of the college premium was due to differences in worker skills with the remaining 1/4 was due to job differences.

[10] See Table 2.10 at http://www.bea.gov/National/FAweb/AllFATables.asp.

[11] The value of creating a new capital unit is $e^{\alpha q t} P_t m$ and the capital unit embodies e^{qt} units of capital. Hence, the relative price of an efficiency unit of capital is $P_t e^{-(1-\alpha)qt}$, which falls at rate $(1-\alpha)q$.

Table 2
Labor supply.

Statistic	(A) Data		(B) Benchmark			(C) Same σ_i			(D) Alternative		
	1970 (1)	Δ_{00-70} (2)	1970 (1)	$\Delta q, \bar{m}$ (2)	Δq (3)	1970 (1)	$\Delta q, \bar{m}$ (2)	Δq (3)	1970 (1)	$\Delta q, \bar{m}$ (2)	Δq (3)
Participation rate											
Average	0.84	−0.08	0.84	−0.14	−0.08	0.84	−0.14	−0.08	0.84	−0.06	−0.04
CG	0.90	−0.03	0.90	−0.11	−0.07	0.90	−0.11	−0.07	0.90	−0.05	−0.04
SC	0.88	−0.08	0.88	−0.12	−0.07	0.85	−0.13	−0.08	0.88	−0.05	−0.04
HSG	0.88	−0.15	0.88	−0.12	−0.08	0.83	−0.14	−0.08	0.88	−0.04	−0.03
LHS	0.78	−0.23	0.78	−0.16	−0.10	0.82	−0.15	−0.09	0.78	−0.08	−0.04
Hours per worker											
Average	43.4	1.5	43.3	2.0	1.2	43.3	2.1	1.2	43.3	3.0	2.2
CG	44.1	2.5	47.3	3.6	2.1	47.3	3.6	2.1	47.3	5.2	3.8
SC	44.0	1.0	46.6	3.3	2.0	46.6	3.3	2.0	46.6	4.9	3.6
HSG	44.0	−0.2	44.2	2.3	1.4	44.4	2.4	1.4	44.2	3.4	2.6
LHS	42.3	−0.5	40.0	0.7	0.4	40.1	0.9	0.5	40.0	1.1	0.9

Note. Panel (B) refers to the benchmark model; Panel (C) refers to the same model with the same parameters, but where we impose $\sigma_i = 1, \forall i$; Panel (D) refers to the model described in Appendix B, where job creation costs are in consumption units rather than in utils. Employment rates and hours per worker in the data are for male workers of age 25–65 from the 1970 US Census. Changes in columns 2 and 3 of each panel are differences in participation rates and weekly hours, respectively. See footnote in Fig. 1 for education labels.

Table 3
Labor income and consumption.

Statistic	(A) Data		(B) Benchmark		
	1980 (1)	2000 (2)	1980 (1)	$\Delta q, \bar{m}$ (2)	Δq (3)
Average labor income					
CG	1.00	1.00	1.00	1.00	1.00
SC	0.79	0.66	0.81	0.77	0.79
HSG	0.69	0.54	0.71	0.65	0.68
LHS	0.54	0.39	0.54	0.44	0.48
Average consumption					
CG	1.00	1.00	1.00	1.00	1.00
SC	0.84	0.77	0.84	0.81	0.83
HSG	0.76	0.68	0.76	0.72	0.74
LHS	0.68	0.54	0.68	0.64	0.66

Note. All statistics are for male workers of age 25–65. Labor income comes from the 1970 US Census; Consumption from the 1980 CEX. See footnote in Fig. 1 for education labels.

4.2. *Our economy in 2000*

We now increase the value of q from 5.51% to 12.39%. This matches the evidence in Greenwood et al. (1997) that the rate of fall in the relative price of investment goods has increased from 2% to 4.5%. All other parameters are left unchanged.

Table 4
Labor income shares.

	Total	CG	SC	HSG	LHS
Benchmark model	0.67	0.74	0.72	0.70	0.59
Model with $\sigma_i = 1$	0.67	0.75	0.70	0.67	0.63

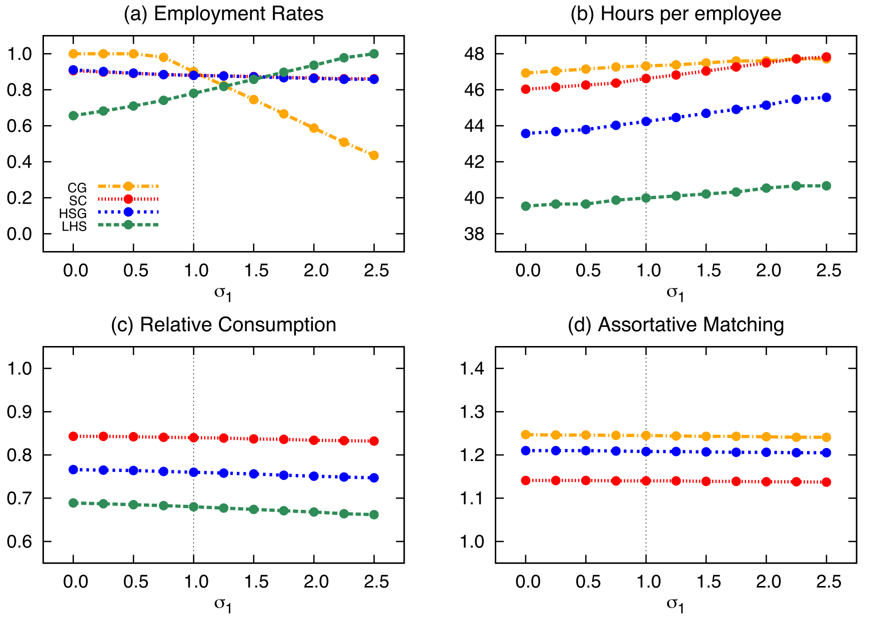
Note. See footnote in Fig. 1 for education labels.

Column 2 of Panel (B) in Table 2 reports the implied changes in the economy when we keep m constant. The model predicts a fall in the aggregate participation rate of 14 percentage points, which is larger than the 8 percentage fall observed in the data. The model also predicts an increase of 2 weekly hours, in line with the observed increase of 1.5 hours. Finally, the model is also consistent with the increase in the dispersion of employment rates and hours per worker between education groups. In particular, the drop in participation rates is larger for the less educated, while the increase in hours per worker is larger for the more educated. But when q increases the value of new jobs increases. In equilibrium this leads to an increase in m, which reduces the increase in the dispersion of job technologies. As shown in column 3 of Panel (B), this leads to slightly more muted responses in the labor market: the aggregate employment rate now declines by 8 percentage points while weekly hours increase by 1.2 hours. These numbers are still in line with the actual changes in the data. Regarding the differences by education groups, they are qualitatively identical to the model with fixed m but quantitatively slightly less pronounced. In the model m increases by 20%, from 0.075 to 0.090, and the average age of a machine falls by 1.4 years. In the data, the average age of capital equipment as reported by the Bureau of Economic Analysis (BEA) fell only by 0.7 years. This might suggest that the model tends to slightly overpredict the response of job creation in the data.

Finally, in columns 2 and 3 of Panel (B) in Table 3 we report the model implications for relative labor income and relative consumption across educational groups for constant m and once allowing m to respond. With constant m, the increase in q generates a substantial increase in labor income inequality: the labor income of workers with no high school degree relative to college graduates falls from 0.54 to 0.44 in the model. This is around two thirds of the fall observed in the data (see column 2 in Panel (A)). The consumption of workers with no high school degree relative to college graduates falls from 0.68 to 0.64. Once allowing m to respond, the increase in q makes the labor income of workers with no high school degree relative to college graduates fall from 0.54 to 0.48 and the consumption of workers with no high school degree relative to college graduates fall from 0.68 to 0.66 (see column 5). Overall the differences between the two specifications are quantitatively small.

4.3. Changing the non-labor income shares

As shown in Table 1, the calibrated differences in the shares of non-labor income by educational group σ_i are small: the share is 9% higher for workers with no high school degree ($i = 4$) than for workers in the reference group (college graduates, $i = 1$), 10% lower for high school graduates ($i = 3$), and 8% lower for high school graduates with some college ($i = 2$). As a result, forcing the shares σ_i to be identical across household types changes little the quantitative results, see Panel (C) in Table 2. This is because the labor income shares for the baseline calibration and for the calibration with $\sigma_i = 1$ remain increasing in workers skill (see Table 4), which is a feature of the calibration consistent with the US evidence by Budría et al. (2002) who report that in the



Note. The share of non-labor income σ_1 is on the x-axis. Panel (a) characterizes employment rates; Panel (b) average hours worked per employee; Panel (c) consumption relative to college graduates; Panel (d) the condition (A2) for positive assortative matching, which is plotted as $\left(\frac{h_i}{h_{i+1}}\right)^{1-\theta} / \left(\frac{c_i}{c_{i+1}}\right)^{\frac{\theta}{(1+\eta)}}$ for $i = 1, 2, 3$.

Fig. 2. The effect of changing the non-labor income shares σ_i's.

Survey of Consumer Finances of 1998 the share of labor income in total income is 67.2% for college graduates and 64.1% for workers with no high school degree.

The distribution of non-labor income is important for the model implications on labor supply, but it is not critical for having an equilibrium with positive assortative matching. To see this, we now change the share of the economy's non-labor income accruing to the most skilled workers σ_1 while we adjust the share of the economy's non-labor income accruing to the least skilled workers σ_4 to guarantee that $\sum_{i=1}^{N} z_i\sigma_i = 1$. In Panel (a) of Fig. 2 we plot the participation rate p_i of each type $i = 1, 2, 3, 4$ against σ_1. The point $\sigma_1 = 1$ corresponds to the benchmark economy. Panel (a) shows that as the most (least) educated individuals get more (less) non-labor income they participate less (more) in the labor market. The participation rates of the intermediate types decline monotonically as we increase σ_1 due to general equilibrium effects. In Panel (b), we plot hours worked per employee, which increase for all worker types as we increase σ_1. As the participation rate of the most skilled type declines, the average machine quality used by workers of all other types improves and thereby working longer hours becomes more profitable. In Panel (c) we report the consumption of each worker type relative to workers with a college degree, $i = 1$. Relative consumption falls for all types, since workers of type $i = 1$ receive a higher amount of non-labor income and so consume more. But the effects are quantitatively small. This is because, as non-labor income of type 1 increases with σ_1, the labor income of type 1 workers falls due to the reduction in the participation rate. This pattern guarantees that the condition for an equilibrium with positive assortative matching is always satisfied in our numerical exercises. Positive assortative matching requires that condition (A2) holds, which implies that the quantities

$$\left(\frac{h_i}{h_{i+1}}\right)^{1-\theta} / \left(\frac{c_i}{c_{i+1}}\right)^{\frac{\theta}{(1+\eta)}}$$

must be larger than one, for $i = 1, 2, 3$. In Panel (d), we plot these ratios for different values of σ_1. We observe that they are larger than one for every type i, and that they change little when σ_1 changes. This follows directly from the previous discussion: h_i/h_{i+1} is exogenous and hence independent of σ_1, while c_i/c_{i+1} changes very little because increases in non-labor income are partly offset by reductions in labor income, due to a reduction in labor force participation, see Panel (a) of Fig. 2.

5. Conclusions

We have studied labor supply decisions in an assignment model with balanced growth. In the model, technological progress is embodied into new jobs which are slowly created over time. Hence there is dispersion in job technologies. Workers differ in skills and they can be employed in at most one job. This leads to a simple assignment problem in the spirit of Becker (1973) and Sattinger (1975). But in our framework labor supply is endogenous because in every period each worker decides whether to actively participate in the labor market, and how many hours to work on the job he is assigned to. Since lower skilled workers can supply longer hours, we have shown that the equilibrium features positive assortative matching (higher skilled workers are assigned to better jobs) only if differences in consumption are small relative to differences in workers skills, which guarantees that low skilled workers do not compensate their lower skill level with much greater working hours. In equilibrium, the model endogenously generates inequality in jobs, wages, and labor supply, but all workers of the same skill consume the same amount. When the pace of technological progress accelerates, differences in job technologies widen, wage inequality increases and workers participate less often in the labor market but supply longer hours on the job. We have shown quantitatively that this mechanism can explain why, as male wage inequality has increased in the US, labor force participation of male workers of different skills has fallen while their working hours have increased. The model also matches reasonably well the observed variation by skill groups.

Our analysis could be extended along several dimensions. In particular, in our model skill differences are perfectly observable, constant over time, and exogenously given. This simplifies the analysis, but it neglects some important features of the labor market, such as worker types learning, as in Eeckhout and Weng (2011) and Groes et al. (2015), or human capital accumulation as in Eeckhout and Jovanovic (2002), Imai and Keane (2004), and Michelacci and Pijoan-Mas (2012). Introducing dynamic elements into the analysis would make the return to labor supply intertemporal, which would affect the incentive to participate in the labor market, working hours decisions and the value of being matched to a specific job. Following Jovanovic (1998) we have also assumed that different jobs produce perfectly substitutable goods. But as emphasized by Costinot and Vogel (2010), different vintages could produce different goods and it would be worth characterizing how the elasticity of substitution across these goods affects the conditions under which the equilibrium features positive assortative matching as well as the response of labor supply to changes in the pace of technological progress. Additionally, in our model machines and workers are combined in a fixed proportion which is exogenously given. As in Eeckhout and Kircher (2011), it would be interesting to have a richer theory of the firm where not only the skill level but also the number of workers matched with each machine is endogenously determined.

Appendix A. Proofs

A.1. *Proof that* $\frac{\mathbf{e}^{\gamma_0 \mathbf{q}}-\mathbf{1}}{\mathbf{q}}$ *is increasing in* $\mathbf{q}$ *when* $\gamma_{\mathbf{0}} > \mathbf{0}$ *and* $\mathbf{q} \geq 0$

Let $\gamma_0 > 0$. The derivative of the function

$$z(q) = \frac{e^{\gamma_0 q} - 1}{q} \tag{37}$$

has the same sign as

$$g(q) = \gamma_0 e^{\gamma_0 q} q - e^{\gamma_0 q} + 1,$$

which is positive for $q \geq 0$, since $g(0) = 0$ and $g(q)$ is increasing in q for all $q > 0$ which follows from

$$g'(q) = \gamma_0^2 e^{-\gamma_0 q} q > 0.$$

A.2. *Proof of Proposition 1*

Proof. Part (i) follows directly from (9). The optimal choice for the number of newly created machines m is determined by equalizing the cost of a machine κ to its value. Let $U_t(\tau)$ denote the value in utils of a unit of capital of age τ at time t. This value solves the asset type equation

$$\rho U_t(\tau) = \frac{1}{\tilde{c}_t} \alpha \frac{\tilde{c}_t}{K_t} e^{-\delta\tau} + \frac{\partial U_t(\tau)}{\partial t} + \frac{\partial U_t(\tau)}{\partial \tau}$$

where $K_t \equiv e^{q_t} K$ and $\tilde{c}_t \equiv e^{\alpha q_t} c$. The first term in the right hand side measures the instantaneous value in utils of one unit of capital of age τ at time t—equal to the marginal value in utils of one unit of capital times the number of units of capital of the machine of age τ, while the last two terms measure capital gains due to the change of time. After using (5), it is easy to check that $U_t(\tau) = U e^{-qt} e^{-\delta\tau}$ with

$$U = \frac{\alpha}{m} \cdot \frac{q+\delta}{q+\delta+\rho}. \tag{38}$$

Since the creation of a new job at time t involves the creation of e^{qt} units of capital, we have that the social planner will choose m so that $\kappa = U$ which, given (38), implies that

$$m = \frac{\alpha}{\kappa} \cdot \frac{q+\delta}{q+\delta+\rho}. \qquad \square$$

A.3. *Proof of Lemma 3*

Lemma 3. *Hours worked are decreasing in the age of the job the worker is matched with and increasing in the marginal utility of consumption. Hours worked in the marginal job* n^* *depend just on preferences and the output elasticity to labor.*

Proof. Let μ denote the Lagrange multiplier of the resource constraint in (11). Then, by maximizing (10) with respect to c we immediately obtain that μ is equal to the marginal utility of consumption: $\mu = 1/c$. By writing the first order conditions with respect to p and n^τ we obtain:

$$v\left(n^*\right) = \mu\, f\left(k^*, n^*\right) \tag{39}$$
$$v'\left(n^\tau\right) = \mu\, f_2\left(k^\tau, n^\tau\right) \tag{40}$$

where $n^* \equiv n^{\tau^*}$ denotes hours worked in the marginal job. Equation (39) implicitly determines the participation rate by equating the disutility of sending the marginal individual to work to the value of output in the marginal job. Equation (40) determines working hours in jobs of age τ, n^τ, by equating the marginal disutility of a working hour to the marginal value of hours in production. This condition determines n^τ as a function of job age τ and the marginal value of income μ:

$$n^\tau = \psi\left(\tau, \mu\right) = \left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1}{\eta+\alpha}} e^{-\frac{\alpha}{\eta+\alpha}(q+\delta)\tau} \mu^{\frac{1}{\eta+\alpha}} \tag{41}$$

This implies that hours are increasing in job quality so decreasing in job age, $\psi_1 < 0$, which characterizes the substitution effect. Hours are also increasing in the marginal value of income μ, $\psi_2 > 0$, which characterizes the income effect. The amount of hours in the marginal job can be characterized by evaluating (40) at $k^\tau = k^*$ and $n^\tau = n^*$, and then dividing the resulting expression side by side by (39). After rearranging this yields

$$\frac{n^* v'\left(n^*\right)}{v\left(n^*\right)} = 1 - \alpha \tag{42}$$

which determines n^* just as a function of preferences and the output elasticity to labor, which is constant under a Cobb–Douglas production function. □

A.4. *Proof of Lemma 1*

Proof. Points (iii) and (iv) follow from Lemma 3, see equations (41) and (42) and the result that $\mu = 1/c$. To prove (i), we first use (41) to totally differentiate (11):

$$\left(1 + \frac{m \int_0^{\frac{p}{m}} f_2 \psi_2 d\tau}{c^2}\right) dc = f\left(k^*, n^*\right) dp - \left\{ \int_0^{\frac{p}{m}} m\tau \left[e^{-(q+\delta)\tau} f_1 + \frac{\alpha}{\eta+\alpha} f_2 \psi \right] d\tau \right\} dq \tag{43}$$

Similarly by taking logs in (39), and then totally differentiating, after remembering that (42) implies that n^* is independent of q, we obtain:

$$\frac{dc}{c} = -\frac{\alpha}{m}\left[pdq + (q+\delta)dp\right]. \tag{44}$$

After solving for dp in (44) we obtain:

$$dp = -\frac{m}{\alpha(q+\delta)c} dc - \frac{p}{q+\delta} dq$$

which substituted into (43), and after some rearranging, leads to:

$$\frac{dc}{dq} = -\frac{f\left(k^*, n^*\right) \alpha c p + \alpha(q+\delta) c m \int_0^{\frac{p}{m}} \tau \left[e^{-(q+\delta)\tau} f_1 + \frac{\alpha}{\eta+\alpha} f_2 \psi \right] d\tau}{\alpha(q+\delta)c + \frac{\alpha(q+\delta)m}{c} \int_0^{\frac{p}{m}} f_2 \psi_2 d\tau + m f\left(k^*, n^*\right)} < 0$$

To prove (ii) just notice that (39), for given n^*, implies that when μ goes up (which happens when c falls), k^* falls. □

A.5. Proof of Proposition 2

Proof. The resource constraint in (11) can be written as

$$c=\left(\mu\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}}\int_0^{\frac{p}{m}} me^{-\frac{(1+\eta)\alpha}{\eta+\alpha}(q+\delta)\tau}d\tau$$

After solving the integral, remembering that $\mu=1/c$, and some rearranging we obtain

$$c^{\frac{1+\eta}{\eta+\alpha}}=\left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}}\frac{(\eta+\alpha)\,m}{(1+\eta)\,\alpha(q+\delta)}\left[1-e^{-\frac{(1+\eta)\alpha(q+\delta)p}{(\eta+\alpha)m}}\right]. \tag{45}$$

By rewriting (39) and then solving for consumption we obtain

$$c=e^{-\alpha(q+\delta)\frac{p}{m}}\frac{(n^*)^{1-\alpha}}{v\,(n^*)}, \tag{46}$$

which can be used to replace c in (45) to yield

$$\left[\frac{(n^*)^{1-\alpha}}{v\,(n^*)}\right]^{\frac{1+\eta}{\eta+\alpha}}=\left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}}\frac{(\eta+\alpha)\,m}{(1+\eta)\,\alpha}\frac{e^{\frac{(1+\eta)\alpha}{(\eta+\alpha)}(q+\delta)\frac{p}{m}}-1}{q+\delta}.$$

The left hand side is independent of q by point (iii) in Lemma 1. The right hand side is increasing both in p and in q since we have shown above that the function $\frac{e^{\gamma_0 x}-1}{x}$ is increasing in x when $\gamma_0>0$. This proves that $\frac{dp}{dq}<0$.

Integrating (12) after using (41) yields

$$\overline{n}=\frac{\eta+\alpha}{\alpha}\left(\frac{1-\alpha}{\lambda_1 c}\right)^{\frac{1}{\eta+\alpha}}\frac{1-e^{-\frac{\alpha}{\eta+\alpha}(q+\delta)\frac{p}{m}}}{(q+\delta)\frac{p}{m}}$$

After using (46) to replace consumption we finally obtain

$$\overline{n}=\frac{\eta+\alpha}{\alpha}\left[\frac{(1-\alpha)v\,(n^*)}{\lambda_1\,(n^*)^{1-\alpha}}\right]^{\frac{1}{\eta+\alpha}}\frac{e^{\frac{\alpha}{\eta+\alpha}\cdot(q+\delta)\frac{p}{m}}-1}{(q+\delta)\frac{p}{m}}, \tag{47}$$

which implies that average hours per worker $\overline{n}$ are increasing in $(q+\delta)p$, due again to the properties of the function $\frac{e^{\gamma_0 x}-1}{x}$. This concludes the proof, since point (ii) in Lemma 1 states that $k^*=e^{-(q+\delta)\frac{p}{m}}$ is decreasing in q. □

A.6. Proof of Proposition 3

Proof. Let $s(t,\tau)$ denote the current net flow value in utils of a job of age $\tau\in[0,\tau^*)$ at time t. This is equal to the difference between the utility value of the job output and the disutility cost of working on the job:

$$s(t,\tau)=\frac{1}{\tilde{c}_t}f\left(e^{q_t}k^\tau,n^\tau\right)-v\left(n^\tau\right)$$

where $\tilde{c}_t=e^{\alpha q_t}c$ denotes consumption at time t, so $1/\tilde{c}_t$ denotes the corresponding value of income. After using (41) to replace n^τ we obtain that $s(t,\tau)$ is independent of t and equal to

$$s(t,\tau) = s(\tau) = \bar{s}\mu^{\frac{1+\eta}{\eta+\alpha}} e^{-\frac{(1+\eta)\alpha(q+\delta)}{\eta+\alpha}\tau} - \lambda_0$$

where μ is the steady state value of detrended income while $\bar{s}$ is a constant equal to

$$\bar{s} = \left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}} \frac{\eta+\alpha}{1+\eta}.$$

The value of a newly created job is equal to the discounted sum of all the $s(\tau)$'s generated during its production life:

$$V = \int_0^{\tau^*} e^{-\rho\tau} s(\tau) d\tau = \frac{\bar{s}\mu^{\frac{1+\eta}{\eta+\alpha}}}{\frac{(1+\eta)\alpha(q+\delta)}{\eta+\alpha} + \rho} \left\{1 - e^{-\left[\frac{(1+\eta)\alpha(q+\delta)}{\eta+\alpha} + \rho\right]\frac{p}{m}}\right\} - \frac{\lambda_0}{\rho}\left(1 - e^{-\frac{\rho p}{m}}\right). \quad (48)$$

The optimal level of job creation satisfies the condition

$$\kappa = V, \quad (49)$$

which says that jobs are created up to the exhaustion of any surplus from job creation.

We use continuity arguments and prove the result for $\rho = 0$. When $\rho = 0$, the optimal allocation is characterized by the following system of three equations in the three unknowns μ, m and p:

$$\kappa = \bar{s}\mu^{\frac{1+\eta}{\eta+\alpha}} \frac{1 - e^{-\frac{p}{m}\gamma(q)}}{\gamma(q)} - \lambda_0 \frac{p}{m} \quad (50)$$

$$\mu^{-\frac{1+\eta}{\eta+\alpha}} = \left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}} m \cdot \frac{1 - e^{-\frac{p}{m}\gamma(q)}}{\gamma(q)} \quad (51)$$

$$\mu = e^{\alpha(q+\delta)\frac{p}{m}} \frac{v(n^*)}{(n^*)^{1-\alpha}} \quad (52)$$

where

$$\gamma(q) = \frac{(1+\eta)\alpha}{\eta+\alpha}(q+\delta)$$

which is linear in q. Equation (50) is (49) evaluated at $\rho = 0$, equation (51) corresponds to (45) after using the definition of $\gamma(q)$ and the fact that $c = 1/\mu$, equation (52) corresponds to (46). Notice that n^* is constant and determined by (42). To simplify this system we get rid of μ by substituting equation (51) into (50) and (52). This delivers the following system in m and p:

$$m = \frac{\bar{s}}{\kappa}\left(\frac{\lambda_1}{1-\alpha}\right)^{\frac{1-\alpha}{\eta+\alpha}} - \lambda_0 \frac{p}{\kappa} \quad (53)$$

$$\left[\frac{(n^*)^{1-\alpha}}{v(n^*)}\right]^{\frac{1+\eta}{\eta+\alpha}} = \left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}} \cdot \frac{e^{p\frac{\gamma(q)}{m}} - 1}{\frac{\gamma(q)}{m}} \quad (54)$$

The first equation establishes a negative (linear) relation between m and p. The right hand side of the second equation is decreasing in m (again due to the properties of the function $\frac{e^{\gamma_0 x}-1}{x}$, which is increasing in x when $\gamma_0 > 0$) and increasing in p. This means that (54) establishes a positive relation between p and m. This implies that the system (53) and (54) yields a unique solution for m and p. We can also notice that (53) is independent of q while the right hand side of (54) is increasing in q. So (54) implies that, when q goes up, p should fall for given m. But since (53) establishes a negative relation between m and p, we immediately have that an increase in q leads to an increase in m ($dm/dq > 0$) and a decrease in p ($dp/dq < 0$), which proves points (i) and (ii) of the proposition. Now multiply and divide by p the right hand side of (54) to obtain

$$\left[\frac{(n^*)^{1-\alpha}}{v\,(n^*)}\right]^{\frac{1+\eta}{\eta+\alpha}} = \left(\frac{1-\alpha}{\lambda_1}\right)^{\frac{1-\alpha}{\eta+\alpha}} \cdot p\,\frac{e^{\frac{p\gamma(q)}{m}}-1}{\frac{p\gamma(q)}{m}}$$

We know that p falls and that the last fraction in the above expression is increasing in $\frac{p\gamma(q)}{m}$. This implies that $\frac{qp}{m}$ increases when q goes up. To see that average hours per worker increases (point (iii)), we can then just use (47) to notice that $\overline{n}$ is an increasing function of $\frac{qp}{m}$. □

A.7. Proof of Proposition 5

Proof. By solving the problem in (25) with respect to consumption we obtain

$$\frac{\dot{c}_{it}}{c_{it}} = r - \rho - \alpha q \tag{55}$$

where $c_{it} = e^{-\alpha q t}\tilde{c}_{it}$ is detrended per capita consumption of workers of type i. This condition can be used to integrate forward (26), which, together with the transversality condition, yields

$$\int_0^\infty e^{-(r-\alpha q)t} c_{it} dt = b_{i0} + \int_0^\infty e^{-(r-\alpha q)t}\left(W_{it} + \phi_i m_t\right) dt. \tag{56}$$

By solving (56) and after using the fact that (55) implies that in steady state

$$r = \rho + \alpha q \tag{57}$$

we obtain that

$$c_i = W_i + \rho b_{i0} + \phi_i m \tag{58}$$

which says that per capita consumption of type i household is equal to permanent income. The first order condition for n^{τ}_{it} in the household problem (25) reads as

$$v'\left(n^{\tau}_{it}\right) = \frac{1}{c_{it}} w'_i\left(n^{\tau}_{it}\right). \tag{59}$$

We assume that the first order condition of the household problem (25) for labor market participation p_{it}, holds as an equality, $p_{it} \in (0, 1)$.[12] This yields

$$v\left(n^{\tau}_{it}\right) = \frac{1}{c_{it}} w_i\left(n^{\tau}_{it}\right). \tag{60}$$

The first order condition for m_t in the household problem (25) yields

[12] If instead participation rates are at a corner, $p_i \in \{0, 1\}$, we should have that

$$\left[w_{it}\left(n^{\tau}_{it}\right) - c_{it} v\left(n^{\tau}_{it}\right)\right](1 - 2p_i) \le 0$$

which says that the value of participating in the labor market is negative (positive) if $p_i = 0$ ($p_i = 1$).

$$\phi_i = c_{it}\kappa, \tag{61}$$

which implies that (32) can be written as

$$\kappa = \frac{P_t}{C_t} \tag{62}$$

where $C_t = \sum z_i c_{it}$.

By solving the firm problem in (27) we obtain

$$f_3\left(k^\tau, h_i, n_{it}^\tau\right) = w_i'\left(n_{it}^\tau\right). \tag{63}$$

We conjecture that, if $\forall i \;\; p_i \in (0, 1)$, the equilibrium features the wage function

$$w_i(n) = \begin{cases} c_i\lambda_0 + c_i\lambda_1\frac{n^{1+\eta}}{1+\eta}, & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases} \tag{64}$$

which implies that (59) and (60) in the household problem hold as an identity.[13] As in Prescott et al. (2009), this means that, in every period, households are just indifferent about whether to participate in the labor market and about how many hours to supply in the job. In equilibrium the aggregate use of labor is determined by firms demand for labor.

By comparing (14) with (27), we also have that, under (64), the value of a job to the social planner is equal to the firm's private value, $Cs_i(\tau) = \pi_i(\tau)$, $\forall\tau \geq 0$ and $i \geq 1$, if and only if type i households consume the same in the two economies. Given (7), this requires that the share of type i household on aggregate non-labor income, σ_i, is such that the consumption c_i that solves (58) is equal to a fraction ν_i of the aggregate consumption units C, $c_i = \nu_i C$, $\forall i \geq 1$.

When $\nu_i = \frac{c_i}{C}$ holds, it can be easily checked that the equilibrium conditions of the decentralized economy are identical to the conditions that characterize the solution of the social planner problem. For example by comparing (19) and (21) with (29) and (31), we immediately see that the critical age thresholds τ_i^*'s are equal, while by comparing (15) with (63) we obtain the same working hours decisions n_i^τ. We can also notice that $V = \frac{P}{C}$, where V is given in (48) and P is given in (33). After comparing (23) with (62) this implies that the job creation rates m are also equal. To analyze under which conditions the decentralized equilibrium features positive assortative matching we can use (30) and apply the same logic that allowed us to prove Proposition 4. □

Appendix B. Job creation costs in consumption units

The model discussed in the paper assumes that jobs are created by entrepreneurs who incur a utility cost κ for each newly created job. We now assume that job creation involves a cost in terms of consumption units equal to $\overline{\kappa}e^{\alpha q t}$. This cost is paid by a representative firm that decides how many jobs to create to maximize its profits. At the end of the period the firm rebates back to households the amount $(\Pi - \overline{\kappa}m)\, e^{\alpha q t}$ as dividend payments. In this new set-up, all the equilibrium equations of the model remain unchanged except for the determination of type i consumption in (58) and the optimal job creation condition in (62). Type i (detrended) consumption per capita is now given by

[13] If some p_i's are at a corner, then the fixed terms in the wage compensation schedule $w_i(n)$ in (64), call it a_{0i}, will have to be modified slightly. Generally the a_{0i}'s are pinned down by the conditions (29) and (31) leading to $a_{0i} < c_i\lambda_0$ if $p_i = 0$, to $a_{0i} > c_i\lambda_0$ if $p_i = 1$, and to $a_{0i} = c_i\lambda_0$ if $p_i \in (0, 1)$.

$$c_i = W_i + \sigma_i \left(\Pi - \overline{\kappa} m\right), \tag{65}$$

where the key difference relative to the utility model is that now job creation costs affect the non-labor income transferred to households, so changes in the job creation rate m will affect labor supply through income effects. The optimal choice for m is now governed by

$$\overline{\kappa} = P, \tag{66}$$

which says that the cost and the value of a newly created job, both measured in consumption units, should be equalized.

We calibrate this economy to the same targets as before and we change q to match the observed increase in the relative price of investment. In Table 1 we report the parameters for this economy. In comparing this economy with the baseline economy, we see that there are two groups of parameters that change substantially: the preference parameters in the utility function λ_0 and λ_1, and the households' shares in aggregate non-labor income σ_i. These differences can be explained by comparing (58) with (65): for given m, in the specification with creation costs in consumption units, less non-labor income is transferred to households, so consumption is lower, which makes workers more willing to work. To compensate for this the utility cost parameters λ_0 and λ_1 should increase. Similarly, the lower non-labor income implies that the dispersion in the shares σ_i's should increase to match the same dispersion in participation rates.

Panel (C) in Table 2 (which corresponds to Panel (B) for the baseline economy) shows that the properties of this economy are qualitatively similar to those of the benchmark model. The acceleration in the pace of technological progress leads to a fall in the aggregate participation rate and an increase in weekly hours. But now the participation rate falls less while weekly hours increase more. For example, when holding m constant, the participation rate falls by 6 percentage points, compared with 14 percentage points in the baseline model, while hours increase by 3.0 hours per week, compared with the increase of 2.0 hours obtained in the benchmark model. Differences in the response of hours by educational groups are also now more pronounced than in the baseline economy. When allowing m to increase, changes in participation and in hours are smaller but the quantitative effect of endogenizing m is less important than in the benchmark model.

Appendix C. Numerical solution

To solve the model, we use the conditions for stable assignment (30) and (31), the households intertemporal budget constraints (58), and the job creation condition (62). This yields a system of $2N + 1$ non-linear equations that we solve for c_i, p_i, and m with a Gauss–Seidel algorithm that uses a bisection method for each equation. To calibrate the economy we write a system of 12 non-linear equations (the 12 model statistics described in Section 4.1) in the 12 unknown model parameters. This system is solved exactly with the Broyden's method.

In this appendix we start deriving the expressions for firm profits (see expression (68) below), which are needed to write the conditions for stable assignment that form the first set of N equations. Then we rewrite the household budget constraints, which represent the second set of N equations (see expression (72) below). Finally we obtain the expression for the optimal job creation condition, which completes the system (see equation (73) below).

C.1. Firm profits

Firms profits $\pi_i(\tau)$ can be written as

$$\pi_i(\tau) = f\left(k^\tau, h_i, n_i(\tau)\right) - c_i\lambda_0 - c_i\lambda_1 \frac{n_i(\tau)^{1+\eta}}{1+\eta}$$

where

$$n_i(\tau) = \left[\frac{(1-\alpha)\theta}{c_i\lambda_1} f\left(k^\tau, h_i, n_i(\tau)\right)\right]^{\frac{1}{1+\eta}},$$

which can be substituted in the expression for profits to obtain

$$\pi_i(\tau) = \frac{1}{A} f\left(k^\tau, h_i, n_i(\tau)\right) - c_i\lambda_0. \tag{67}$$

The optimal demand for labor by firms comes from (63) which implies that

$$n_i(\tau) = \left[\frac{(1-\alpha)\theta\,(k^\tau)^\alpha\, h_i^{(1-\alpha)(1-\theta)}}{c_i\lambda_1}\right]^{\frac{A}{1+\eta}}.$$

Substituting this expression into output we obtain that

$$f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) = \left[\frac{(1-\alpha)\theta}{c_i\lambda_1}\right]^{A-1} e^{-\alpha A(q+\delta)\tau} h_i^{(1-\alpha)(1-\theta)A}$$

which can be plugged into (67) to obtain

$$\pi_i(\tau) = \frac{1}{A}\left[\frac{(1-\alpha)\theta}{c_i\lambda_1}\right]^{A-1} e^{-\alpha A(q+\delta)\tau} h_i^{(1-\alpha)(1-\theta)A} - c_i\lambda_0. \tag{68}$$

Note that $\pi_i(\tau)$ is function of model parameters and of the endogenous variable c_i, and once evaluated at τ_i^* it will also be function of $p_j\ \forall j \leq i$ and m. Substituting (68) into the conditions for stable assignment (30) and (31) gives us the first set of N equations in the $2N+1$ unknowns.

C.2. The household budget constraint

The intertemporal budget constraint of household i implies that

$$c_i = W_i + \sigma_i \Pi \tag{69}$$

where Π, which is defined in (35), can be written as

$$\Pi = \sum_{i=1}^{N} z_i \Pi_i$$

where W_i and Π_i are average labor income and profits generated by workers of type i

$$W_i = \frac{1}{z_i}\int_{\tau_{i-1}^*}^{\tau_i^*} w_i\left(n_i^\tau\right) m\, d\tau \quad \text{and} \quad \Pi_i = \frac{1}{z_i}\int_{\tau_{i-1}^*}^{\tau_i^*} \pi_i\left(n_i^\tau\right) m\, d\tau.$$

Let F_i denote the average output generated by jobs assigned to workers of type i:

$$F_i = \frac{1}{z_i}\int_{\tau_{i-1}^*}^{\tau_i^*} f\left(e^{-(q+\delta)\tau}, h_i, n_i^\tau\right) m\, d\tau \tag{70}$$

Using (67) and the fact that $F_i = W_i + \Pi_i$ we write average profits and average wages as:

$$W_i = \left(1 - \frac{1}{A}\right) F_i + \lambda_0 c_i p_i \qquad \text{and} \qquad \Pi_i = \frac{1}{A} F_i - \lambda_0 c_i p_i \tag{71}$$

After integrating (70), average output F_i can be expressed as equal to

$$F_i = \frac{m}{z_i} \left[\frac{(1-\alpha)\theta}{\lambda_1 c_i}\right]^{A-1} h_i^{(1-\alpha)(1-\theta)A} \frac{1}{\alpha A (q+\delta)} e^{-\alpha A (q+\delta) \tau_{i-1}^*} \left[1 - e^{-\alpha A (q+\delta) \frac{p_i z_i}{m}}\right]$$

The additional set of N equations is then obtained by substituting (71) into (69) to obtain

$$c_i = \left(1 - \frac{1}{A}\right) F_i + \lambda_0 c_i p_i + \sigma_i \sum_{i=1}^{N} z_i \left(\frac{1}{A} F_i - \lambda_0 c_i p_i\right), \tag{72}$$

where F_i is defined in (70). The equation in (72) again depends on c_i, p_i, and through F_i, on m.

C.3. Job creation

To write the condition (62) for the optimal choice of m, we need an expression for P. From (33) we have:

$$P = \sum_{i=1}^{N} \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} \pi_i(\tau)\, d\tau = \sum_{i=1}^{N} \int_{\tau_{i-1}^*}^{\tau_i^*} e^{-\rho\tau} \left[\frac{1}{A} f\left(k^\tau, h_i, n_i(\tau)\right) - \lambda_0 c_i\right] d\tau$$

After integrating we obtain

$$P = \frac{1}{A} \frac{[(1-\alpha)\theta]^{A-1}}{\left[\alpha A (q+\delta) + \rho\right]} \sum_{i=1}^{N} \frac{h_i^{(1-\alpha)(1-\theta)A}}{(\lambda_1 c_i)^{A-1}} \left\{e^{-\left[\alpha A (q+\delta)+\rho\right]\tau_{i-1}^*} - e^{-\left[\alpha A (q+\delta)+\rho\right]\tau_i^*}\right\}$$
$$- \sum_{i=1}^{N} \frac{\lambda_0 c_i}{\rho} \left(e^{-\rho \tau_{i-1}^*} - e^{-\rho \tau_i^*}\right) \tag{73}$$

This is the final equation that completes the system of $2N+1$ equations that we solve at the computer.

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