

# The Social Cost of Near-Rational Investment\*

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## Abstract

We show that the stock market may fail to aggregate information even if it appears to be efficient; the resulting collapse in the dissemination of information may drastically reduce welfare. We solve a macroeconomic model in which information about fundamentals is dispersed and households make small, correlated errors around their optimal investment policies. As information aggregates in the market, these errors amplify and crowd out the information content of stock prices. When stock prices reflect less information, the perceived and the actual volatility of stock returns rise. This increase in financial risk makes holding stocks unattractive, distorts the long-run level of capital accumulation, and causes costly (first-order) distortions in the long-run level of consumption.

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# 1 Introduction

An important function of financial markets is to aggregate information that is dispersed across market participants. Market prices should reflect the information held by countless investors and direct resources to their most efficient use. If stock prices reflect information, investors have an incentive to learn from equilibrium prices and to update their expectations accordingly. But if investors learn from equilibrium prices, *anything* that moves prices has an impact on the expectations held by *all* market participants. We explore the implications of this basic dynamic in a world in which people are less than perfect – a world in which they make small mistakes when investing their wealth.

We solve a real business cycle model in which information is dispersed across market participants. Households observe the equilibrium stock price as well as a private signal about aggregate productivity in the next period. Based on this information they trade in stocks and bonds. As households place their trades, the equilibrium stock price aggregates the information in the market and becomes informative about future productivity. Because households optimize when they decide how to allocate their portfolios, small deviations from their optimal policy have little impact on their individual welfare. However, if these deviations are correlated across households (say households are on average just a little bit too optimistic in some states of the world and a little bit too pessimistic in others), they affect the equilibrium price and hence may have a large external effect on the equilibrium expectations held by all market participants.

The first main insight from our model is that if information is dispersed, small errors in households' investment decisions may result in a large rise in the volatility of equilibrium stock returns. Consider a state of the world in which households are on average just a little bit too optimistic about future productivity. If the average investor is slightly too optimistic, the stock price must rise. Households who observe this higher stock price may interpret it in one of two ways: It may either be due to errors made by their peers or, with some probability, it may reflect more positive information about future productivity received by other market participants. Rational households should thus revise their expectations of future productivity upwards whenever they see a rise in the stock price. As households revise their expectations upwards, the stock price must rise further, triggering yet another revision in expectations, and so on. Small errors in the investment decision of the average household may thus lead to much larger deviations in equilibrium stock prices. This amplified noise in stock prices crowds out the information content of prices and raises the volatility of equilibrium stock returns. Small errors in households' investment decisions may thus result in a deterioration of the market's ability to aggregate information and in an increase in the financial risk associated with investing in the stock market.

The second main insight from our model is that the level of financial risk determines the

amount of capital that is accumulated in the economy. If the equilibrium variance of stock returns rises, stocks become a riskier asset to hold and households demand a higher risk premium for holding stocks rather than bonds. This risk premium determines the marginal product of capital in the long run (at the stochastic steady state). Changes in the (conditional) variance of stock returns thus change the *level* of capital accumulation, output, and consumption. A rise in the volatility of stock returns may therefore cause large aggregate welfare losses by distorting the level of consumption at the stochastic steady state. Interestingly, this is true even if the capital stock responds very little to any given change in stock prices and there is an observed disconnect between the stock market and the real economy.

The combination of these two insights produces a surprising result: A model in which near-rational behavior causes a rise in the volatility of stock returns and large aggregate welfare losses, although there are no opportunities for earning abnormal returns in financial markets and all households are arbitrarily close to their rational behavior.

**The Model** Our model is a standard real business cycle model in which a consumption good is produced from capital and labor. Households supply labor to a representative firm and invest their wealth by trading claims to capital (‘stocks’) and bonds. The consumption good can be transformed into capital, and vice versa, by incurring a convex adjustment cost. The accumulation of capital is thus governed by its price relative to the consumption good (Tobin’s  $Q$ ). The only source of real risk in the economy are shocks to total factor productivity. We extend this standard setup by assuming that each household receives a private signal about productivity in the next period and solve for equilibrium expectations.

As a useful benchmark, we first examine two extreme cases in which the stock market has no role in aggregating information. In the first case, the private signal is perfectly accurate such that all households know next period’s productivity without having to extract any information from the equilibrium price. In this case, our model is very close to the “News Shocks” model of Jaimovich and Rebelo (2009), in which all information about the future is common (everybody knows everything there is to know from the outset). The opposite extreme is the case in which the private signal is perfectly inaccurate (it contains no information at all and consists only of noise). In this case our model resembles the standard real business cycle model in which no one in the economy has any information about the future and there is consequently nothing to learn from the equilibrium stock price. The first result we show is that households face less financial risk in the former case than in the latter: The more households know about the future, the more information is reflected in the equilibrium price, and the lower is the volatility of equilibrium stock returns.

The paper centers on the more interesting case in which households’ private signals are neither perfectly accurate nor perfectly inaccurate: private signals contain both information about future productivity and some idiosyncratic noise (information is dispersed). In this case,

households' optimal behavior is to look at the equilibrium stock price and to use it to learn about the the future. When information is dispersed, the stock market thus serves to aggregate information.

We call the situation in which all households behave perfectly rationally the “rational expectations equilibrium”. If households are perfectly rational in making their investment decisions, the stock market is very effective at aggregating information: As long as the noise in the private signal is purely idiosyncratic, the equilibrium stock price becomes perfectly revealing about productivity in the next period. (This is the well-known result in Grossman (1976).) Since the equilibrium stock price in the rational expectations equilibrium reflects all information about tomorrow's productivity, the equilibrium volatility of stock returns is just as low as it was in the case in which the private signal was perfectly accurate. Loosely speaking, the level of financial risk depends on how much information is in the equilibrium stock price and not on how it got there.

We then show that the rational expectations equilibrium is unstable in the sense that the economy behaves very differently if we allow households to make small, correlated errors around their optimal investment policy. We refer to this as the “near-rational expectations equilibrium” to emphasize that the expected utility cost accruing to an individual household due to deviations from its optimal policy must be economically small.

In the near-rational expectations equilibrium the average household is slightly too optimistic in some periods and slightly too pessimistic in others. These small errors in the expectations of the average household must impact the equilibrium price. But when households try to learn about future productivity from the equilibrium price, they cannot infer whether a given change in the stock price is attributable to information about productivity or to near-rational errors made by their peers. The small error in the expectation of the average household thus feeds from the stock price into households' expectations and back into the stock price. The more dispersed information is across households the stronger is this feedback effect, because households rely more heavily on the stock price when their private signal is relatively uninformative. In fact, we show that arbitrarily small near-rational errors in the investment behavior of households may completely destroy the stock markets capacity to aggregate information if information is sufficiently disperse. In other words, the stock market's ability to aggregate information is most likely to break down precisely when it is most valuable, i.e. when information is highly dispersed. As near-rational errors may drastically reduce the amount of information reflected in the equilibrium stock price, they may lead to large increases in the volatility of equilibrium stock returns, and thus to large increases in the amount of financial risk faced by households.

We remain agnostic about the exact mechanism prompting households to make small, correlated errors in their investment decisions. We may think of some form of behavioral bias as in Dumas, Kurshev, and Uppal (2006), where households falsely believe that an uninformative

public signal contains a tiny amount of information about future productivity.<sup>1</sup> Alternatively, we may think of "animal spirits" or of a world in which investors must incur a small menu cost in order to eliminate small correlated errors from their investment decisions (Mankiw (1985)).<sup>2</sup> The point is that the private gain from avoiding near-rational errors is low, while the social gain from avoiding the resulting rise in the volatility of stock returns may be large.

This is easiest to see for the example of a small open economy in which households can borrow and lend at an exogenous international interest rate. Risk-averse investors demand a higher risk premium for holding stocks when stock returns are more volatile. In the near-rational expectations equilibrium the marginal unit of capital installed must therefore yield a higher expected return than in the rational expectations equilibrium, in order to compensate investors for the additional risk they bear. It follows that an increase in the volatility of stock returns depresses the equilibrium level of capital installed at the stochastic steady state and consequently lowers the level of output and consumption in the long run.<sup>3</sup> Moreover, returns to capital rise while wages fall.<sup>4</sup>

Because welfare losses are driven mainly by a distortion in the stochastic steady state rather than by an intertemporal misallocation of capital, rises in the volatility of stock returns may cause large welfare losses even if the capital stock responds little to any given change in stock prices. In our model, the observed sensitivity of the capital stock with respect to stock prices is therefore uninformative about the welfare consequences of rises in the volatility in stock returns. This contrasts with a widely held view among macroeconomists that pathologies in the stock market may not matter for the real economy if there is an observed disconnect between stock prices and changes in the capital stock (Morck, Shleifer, and Vishny (1990)).

**Calibration** We quantify the aggregate welfare losses attributable to near-rational behavior as the percentage rise in consumption that would make households indifferent between remaining in an equilibrium in which the volatility of stock returns is high (the near-rational expectations equilibrium) and transitioning to the stochastic steady state of an economy in which all households behave fully rationally until the end of time (the rational expectations equilibrium). Our baseline results are for the case of a small open economy. In our preferred

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<sup>1</sup>A large literature in behavioral finance has developed psychologically founded mechanisms that prompt households to make correlated mistakes in their investment decisions. Some examples are Odean (1998); Odean (1999); Daniel, Hirshleifer, and Subrahmanyam (2001); Barberis, Shleifer, and Vishny (1998); Bikhchandani, Hirshleifer, and Welch (1998); Hong and Stein (1999) and Allen and Gale (2001).

<sup>2</sup>For example, we may think of a world in which there are two computer programs for pricing stocks; a free program which prices stocks with a small error and another version which is available at a menu cost and prices stocks accurately.

<sup>3</sup>The stochastic steady-state is the vector of capital, bonds, and prices at which those quantities do not change in unconditional expectation.

<sup>4</sup>In a closed economy the fact remains that any distortion in the level of output and consumption is associated with first-order welfare losses. However, the effects are slightly more complicated (due to the precautionary savings motive), such that rises in the volatility of stock returns may drive consumption at the stochastic steady state up or down.

calibration the conditional variance of stock returns in the rational expectations equilibrium is 25% lower than in the near-rational expectations equilibrium. Aggregate welfare losses due to this “excess volatility” amount to 3.76% of consumption. Almost all of this loss is attributable to distortions in capital accumulation. The results for a closed economy are quantitatively and qualitatively similar.

An important caveat with respect to our quantitative results is that we use the standard real business cycle model as our model of the stock market. This model has a well-known deficiency, which is that it cannot simultaneously match the volatility of output and the volatility of asset prices. A large literature in macroeconomics and finance has developed a range of remedies for this deficiency (e.g. Campbell and Cochrane (1999), Bansal and Yaron (2004), and Barro (2009)). In order to keep the analysis as simple as possible we chose not to incorporate these remedies into our model. Instead, we calibrate the model to match the volatility of stock returns observed in the data and make appropriate adjustments to our welfare calculations to ensure that they are not driven by a counterfactually high standard deviation of consumption.

**Related Literature** This paper is to our knowledge the first to address the welfare costs of pathologies in information aggregation within a full-fledged dynamic stochastic general equilibrium model. In a related paper, Mertens (2009) derives welfare improving policies for economies in which distorted beliefs create too much volatility in asset markets. He shows that the stabilization of asset prices enhances welfare and that history-dependent policies may improve the information content of asset prices.

Our work relates to a literature that studies the welfare cost of pathologies in stock markets, including Stein (1987) and Lansing (2008). Most closely related are DeLong, Shleifer, Summers, and Waldmann (1989) who analyze the general equilibrium effects of noise-trader risk in an overlapping generations model with endogenous capital accumulation. A large literature in macroeconomics and in corporate finance focuses on the sensitivity of firms’ investment to a given mispricing in the stock market. Some representative papers in this area are Morck, Shleifer, and Vishny (1990); Blanchard, Rhee, and Summers (1993); Baker, Stein, and Wurgler (2003); Gilchrist, Himmelberg, and Huberman (2005); and Farhi and Panageas (2006).<sup>5</sup> While most of these papers find that investment responds moderately to mispricings in the stock market, our model suggests that welfare losses due to a rise in the volatility of stock returns may be large regardless of how responsive investment is to mispricings in the stock market.

Moreover, this paper relates to a large literature on the costs of business cycles in two ways:<sup>6</sup> First, we demonstrate that macroeconomic fluctuations affect the *level* of consumption if they create financial risk. This level effect is not captured in standard cost-of-business cycles

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<sup>5</sup>Also see Galeotti and Schiantarelli (1994); Polk and Sapienza (2003); Panageas (2005); and Chirinko and Schaller (2006)

<sup>6</sup>See Barlevy (2004) for an excellent survey.

calculations in the spirit of Lucas (1987). Second, our model suggests that this level effect may cause economically large welfare losses if near-rational investor behavior causes a substantial amount of financial risk.

The notion of near-rationality is due to Akerlof and Yellen (1985) and Mankiw (1985). In their models near-rational behavior amplifies business cycles. Our application is closest to Cochrane (1989) and Chetty (2009) who use the utility cost of small deviations around an optimal policy to derive "economic standard errors".<sup>7</sup>

In our application, we argue that the aggregation of information in financial markets may break down because households have little incentive to avoid small correlated errors in their investment decisions: the lack of incentives to individuals adversely affects the quality of public information. In this sense our paper relates closely to an emerging literature which is concerned with the social value of public information. Recent work in this area includes Morris and Shin (2002), Amador and Weill (2007), Angeletos et al. (2007), and Angeletos and La'O (2008). While this literature focuses on pathologies that arise from strategic complementarities in households' actions, the mechanism in the model does not depend on such strategic complementarities.<sup>8</sup>

Another difference to existing work in this area is that our model requires solving for equilibrium expectations under dispersed information in a non-linear (general equilibrium) framework. We are able to do so due to recent advances in computational economics. We follow the solution method in Mertens (2009) to solve for the equilibrium. This method builds on Judd (1998) and Judd and Guu (2000) in using a higher-order expansion in all state variables around the deterministic steady state of the model in combination with a nonlinear change of variables (Judd (2002)).<sup>9</sup>

In the main part of the paper we concentrate on the slightly more tractable small open economy version of the model (alternatively we may think of it as a closed economy in which households have access to a certain type of storage technology). After setting up the model in section 2 we discuss equilibrium expectations and how near-rational behavior may lead to a collapse of information aggregation and to a rise in financial risk (section 3). In section 4 we build intuition for the macroeconomic implications of a rise in financial risk by presenting a simplified version of the model which allows us to show all the main results with pen and paper. In this simplified version of the model households consist of two specialized agents: a "capitalist" who has access to the stock and bond markets and a "worker" who provides labor services but is excluded from trading in the stock market. We then solve the full model computationally in section 5. Section 6 gives the results of our calibrations and also gives results for a closed economy version of the model.

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<sup>7</sup>Other recent applications include Woodford (2005) and Dupor (2005).

<sup>8</sup>The provision of public information thus always raises welfare in our framework (see Appendix C).

<sup>9</sup>See Devereux and Sutherland (2006), Tille and van Wincoop (2007), and Fernández-Villaverde and Rubio-Ramírez (2006) for other recent applications based on perturbation.

## 2 Setup of the Model

The model is a de-centralization of the standard Mendoza (1991) framework: A continuum of households work and trade in stocks and bonds. A representative firm produces a homogeneous consumption good by renting capital and labor services from households. Total factor productivity is random in every period and the firm adjusts factor demand accordingly. An investment goods sector has the ability to transform units of the consumption good into units of capital, while incurring convex adjustment costs.<sup>10</sup> All households and the representative firm are price takers and plan for infinite horizons.

At the beginning of each period, households receive a private signal about productivity in the next period. Given this signal and their knowledge of prices and the state of the economy, they form expectations of future returns. Households make correlated near-rational errors when forming expectations about future productivity.

### 2.1 Economic Environment

Technology is characterized by a linear homogeneous production function that uses capital,  $K_t$ , and labor,  $L$  as inputs

$$Y_t = e^{\eta_t} F(K_t, L), \quad (1)$$

where  $Y_t$  stands for output of the consumption good. Total factor productivity,  $\eta_t$ , is normally distributed with a mean of  $-\frac{1}{2}\sigma_\eta^2$  and a variance of  $\sigma_\eta^2$ . The equation of motion of the capital stock is

$$K_{t+1} = K_t(1 - \delta) + I_t, \quad (2)$$

where  $I_t$  denotes aggregate investment and  $\delta$  is the rate of depreciation. Furthermore, there are convex adjustment costs to capital,

$$AC = \frac{1}{2}\chi \frac{I_t^2}{K_t}, \quad (3)$$

where  $\chi$  is a positive constant. There is costless trade in the consumption good at the world price, which we normalize to one. All households can borrow and lend abroad at rate  $r$ . Foreign direct investment and international contracts contingent on  $\eta$  are not permitted.

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<sup>10</sup>The alternative to introducing an investment goods sector is to incorporate the investment decision into the firm's problem. The two modeling devices are equivalent as long as there are no frictions in contracting between management and shareholders.

## 2.2 Households

There is a continuum of identical households indexed by  $i \in [0, 1]$ . At the beginning of every period each household receives a private signal about tomorrow's productivity:

$$s_t(i) = \eta_{t+1} + \nu_t(i), \quad (4)$$

where  $\nu_t(i)$  represents *i.i.d.* draws from a normal distribution with zero mean and variance  $\sigma_\nu^2$ . Given this information and their knowledge about the economy, households maximize lifetime utility by choosing an intertemporal allocation of consumption,  $\{C_t(i)\}_{t=0}^\infty$ , and by weighting their portfolios between stocks and bonds at every point in time,  $\{\omega_t(i)\}_{t=0}^\infty$ , where  $\omega$  represents the share of equity in their portfolio. Formally, an individual household's problem is

$$\max_{\{C_t(i)\}_{t=0}^\infty, \{\omega_t(i)\}_{t=0}^\infty} U_t(i) = \mathcal{E}_{it} \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \log(C_s(i)) \right\} \quad (5)$$

subject to

$$W_{t+1}(i) = [(1 - \omega_t(i))(1 + r) + \omega_t(i)(1 + \tilde{r}_{t+1})](W_t(i) + w_t L - C_t(i)) \quad \forall t, \quad (6)$$

where  $\mathcal{E}_{it}$  stands for household  $i$ 's conditional expectations operator,  $W_t(i)$  stands for financial wealth of household  $i$  at time  $t$  and  $\tilde{r}_{t+1}$  is the equilibrium return on stocks. We denote the market price of capital with  $Q_t$  and dividends with  $D_t$ :<sup>11</sup>

$$1 + \tilde{r}_{t+1} = \frac{Q_{t+1}(1 - \delta) + D_{t+1}}{Q_t}. \quad (7)$$

Households have rational expectations, but they make small mistakes when forming their expectation of  $\eta_{t+1}$ : The expectations operator  $\mathcal{E}$  is the rational expectations operator with the only exception that the conditional probability density function of  $\eta_{t+1}$  is shifted by  $\tilde{\epsilon}_t$ :<sup>12</sup>

$$\mathcal{E}_{it}(\eta_{t+1}) \equiv E_{it}(\eta_{t+1}) + \tilde{\epsilon}_t, \quad (8)$$

where,  $E_{it}$  denotes the rational expectations operator, conditional on all information available

<sup>11</sup>We implicitly assume here that stocks split proportionally to the percentage change in aggregate capital stock at the end of each period. The stock price is then always equal to the price of a claim to one unit of capital.

<sup>12</sup>This implies that households have the correct perception of all higher moments of the conditional distribution of  $\eta_{t+1}$ :

$$\mathcal{E} \left[ (\eta_{t+1} - \mathcal{E}(\eta_{t+1}|s_t(i), Q_t))^k | s_t(i), Q_t \right] = E \left[ (\eta_{t+1} - E(\eta_{t+1}|s_t(i), Q_t))^k | s_t(i), Q_t \right] \text{ for all } k \neq 1.$$

to household  $i$  at time  $t$ ,

$$E_{it}(\cdot) = E(\cdot | Q_t, s_t(i), K_t, B_{t-1}, \eta_t). \quad (9)$$

For simplicity we assume that all households make the same small mistake. Alternatively, we may think of  $\tilde{\epsilon}_t$  as the average mistake made by households trading in the stock market. The deviation caused by  $\tilde{\epsilon}_t$  is zero in expectation and its variance,  $\sigma_{\tilde{\epsilon}}^2$ , is small enough such that the expected utility loss from making this mistake is below some threshold level.<sup>13</sup> Our favorite interpretation of this error is that households observe an uninformative public signal and falsely believe that it contains a small amount of information about  $\eta_{t+1}$  (Dumas et al. (2006)). However, we may think of a number of other interpretations involving animal spirits, menu costs, behavioral biases, or even an evolutionary regime under which households invest by rules of thumb and change these rules only if they expect a significant utility gain from doing so.

In order to avoid having to keep track of the wealth distribution across households, we assume that they can insure against idiosyncratic risk which is due to their private signal: At the beginning of each period (and before receiving their private signal), households can buy claims that are contingent on the state of the economy and on their individual idiosyncratic shock  $\nu_t(i)$ . These claims are in zero net supply and pay off at the beginning of the next period. Contingent claims trading thus completes markets between periods and leads all households, in equilibrium, to enter each period with the same amount of wealth. In order to keep the exposition simple, we suppress the notation relating to contingent claims except for when we define the equilibrium and relegate details to Appendix A.

## 2.3 Firms

A representative firm purchases capital and labor services from households. As it rents services from an existing capital stock, its maximization collapses to a period-by-period problem.<sup>14</sup> The firm's problem is to maximize profits

$$\max_{K_t^d, L_t^d} e^{\eta_t} F(K_t^d, L_t^d) - w_t L_t^d - D_t K_t^d, \quad (10)$$

where  $K_t^d$  and  $L_t^d$  denote factor demands for capital and labor respectively. First order conditions with respect to capital and labor pin down the fair wage and the dividend. Both factors receive

<sup>13</sup>More precisely,  $\tilde{\epsilon}_t(i)$  has a mean of  $-\frac{1}{2}\sigma_{\tilde{\epsilon}}^2$  such that agents hold the correct expectation of log returns in expectation.

<sup>14</sup>Note that by choosing a structure in which firms rent capital services from households, we abstract from all principal agent problems between managers and stockholders. Managers therefore cannot prevent errors in stock prices from impacting investment decisions, as in Blanchard, Rhee, and Summers (1993). On the other hand, they do not amplify shocks or overinvest as in Albuquerque and Wang (2005).

their marginal product:

$$e^{\eta_t} F_K \left( K_t^d, L_t^d \right) = D_t \quad (11)$$

and

$$e^{\eta_t} F_L \left( K_t^d, L_t^d \right) = w_t. \quad (12)$$

As the production function is linear homogeneous, the representative firm makes zero economic profits.

## 2.4 Investment Goods Sector

The representative firm owns an investment goods sector which converts the consumption good into units of capital, while incurring adjustment costs. It takes the price of capital as given and then performs instant arbitrage:

$$\max_{I_t} Q_t I_t - I_t - \frac{1}{2} \chi \frac{I_t^2}{K_t}, \quad (13)$$

where the first term is the revenue from selling  $I_t$  units of capital and the second and third terms are the cost of acquiring the necessary units of consumption goods (recall the price of the consumption good is normalized to one) and the adjustment costs respectively. Since there are decreasing returns to scale in converting consumption goods to capital, the investment goods sector makes positive profits in each period. Profits are paid to shareholders as a part of dividends.<sup>15</sup>

Taking the first order condition of (13), gives us equilibrium investment as a function of the market price of capital:

$$I_t = \frac{K_t}{\chi} (Q_t - 1) \quad (14)$$

Whenever the market price of capital is above one, investment is positive, raising the capital stock in the following period. When it is below one the investment goods sector buys units of capital and transforms them back into the consumption good. Note that the parameter  $\chi$  scales the adjustment costs and can be used to calibrate the sensitivity of capital investment with respect to the stock price.

## 2.5 Definition of Equilibrium

### Definition 2.1

*Given a time path of shocks  $\{\eta_t, \tilde{\epsilon}_t, \{\nu_t(i) : i \in [0, 1]\}\}_{t=0}^{\infty}$  an equilibrium in this economy is a time path of quantities  $\{\{C_t(i), B_t(i), W_t(i), \omega_t(i), \varphi(i; n) : i \in [0, 1]\}, C_t, B_t, W_t, \omega_t, K_t^d, L_t^d, Y_t, K_t$*

<sup>15</sup>Alternatively, profits may be paid to individuals as a lump-sum transfer; this assumption matters little for the results of the model.

,  $I_t\}_{t=0}^\infty$ , signals  $\{s_t(i) : i \in [0, 1]\}_{t=0}^\infty$  and prices  $\{Q_t, r, D_t, w_t, \rho_t(n)\}_{t=0}^\infty$  with the following properties:

1.  $\{C_t(i), \omega_t(i), \varphi(i; n)\}_{t=0}^\infty$  solve the households' maximization problem (5) given the vector of prices, initial wealth, and the random sequences  $\{\tilde{\varepsilon}_t, \{\tilde{\nu}_t(i)\}_{t=0}^\infty\}$ ;
2.  $\{K_t^d, L_t^d\}_{t=0}^\infty$  solve the representative firm's maximization problem (10) given the vector of prices;
3.  $\{I_t\}_{t=0}^\infty$  is the investment goods sector's optimal policy (14) given the vector of prices;
4.  $\{w_t\}_{t=0}^\infty$  clears the labor market,  $\{Q_t\}_{t=0}^\infty$  clears the stock market,  $\rho_t(n)$  clears the contingent claims market, and  $\{D_t\}_{t=0}^\infty$  clears the market for capital services;
5. There is a perfectly elastic supply of the consumption good and of bonds in world markets. Bonds pay the rate  $r$  and the price of the consumption is normalized to one;
6.  $\{Y_t\}_{t=0}^\infty$  is determined by the production function (1),  $\{K_t\}_{t=0}^\infty$  evolves according to (2),  $\{W_t(i)\}_{t=0}^\infty$  evolve according to the budget constraints (6), and  $\{s_t(i)\}_{t=0}^\infty$  is determined by (4);
7.  $\{B_t(i), C_t, B_t, W_t, \omega_t\}_{t=0}^\infty$  are given by the identities

$$B_t(i) = (1 - \omega_t(i)) (W_t(i) - C_t(i)), \quad (15)$$

$$X_t = \int_0^1 X_t(i) di, \quad X = C, B, W \quad (16)$$

and

$$\omega_t = \frac{Q_t K_{t+1}}{W_t - C_t}, \quad (17)$$

where  $n$  is a realization of the vector  $[\eta_{t+1}, \tilde{\varepsilon}_t, \nu_t]$ ,  $\varphi_t(i; n)$  is the quantity of contingent claims bought by household  $i$  that pay off one unit of consumption at time  $t+1$  if state  $n$  occurs and  $\nu_t$  coincides with  $\nu_t(i)$ , and  $\rho_t(n)$  is the time  $t$  price of these claims.

In the *rational expectations equilibrium* agents do not make mistakes,  $\sigma_{\tilde{\varepsilon}} = 0$ , such that the expectations operator  $\mathcal{E}$  in equation (5) coincides with the rational expectation in (9). The *near-rational expectations equilibrium* posits that  $\sigma_{\tilde{\varepsilon}} > 0$ ; households make small errors around their optimal policy, as given in (8). The idea behind the near-rational expectations equilibrium is that small errors in households' policies result in minor welfare losses for the individual household. The following definition formalizes what it means for near-rational households to suffer only "economically small" losses:

**Definition 2.2**

A near-rational expectations equilibrium is  $k$ -percent stable if the welfare gain to an individual household of obtaining rational expectations is less than  $k\%$  of consumption.

**3 Equilibrium Expectations**

In this section we explore how small near-rational errors in households' investment behavior may result in much larger errors in market expectations and in a loss of the market's capacity to aggregate information. To fix ideas, let us define the error in market expectations of  $\eta_{t+1}$  as the difference between the average expectation held by households in the near-rational expectations equilibrium and the average expectation they would hold if  $\tilde{\epsilon}_t$  happened to be zero in this period. We call the error in the market expectations

$$\epsilon_t = \gamma \tilde{\epsilon}_t$$

and solve for  $\gamma$  below.<sup>16</sup> The main insight is that the multiplier  $\gamma$  may be large. This amplification of errors is a result of households learning from equilibrium prices: a rise in prices causes households to revise their expectations upwards; and when households act on their revised expectations, the price rises further. Trades that are correlated with the average error made by investors thus represent an externality on other households' expectations.

**3.1 Solving for Expectations in General Equilibrium**

In order to say more about the relationship between  $\tilde{\epsilon}_t$  and  $\epsilon_t$  we need to solve for equilibrium expectations. This is a challenge because our model is non-linear, and in particular because the market price of capital ( $Q_t$ ) is a non-linear function of  $\eta_{t+1}$ . Households' optimal behavior is characterized by two Euler equations which take the form

$$\begin{aligned} \mathcal{E}_{it} \left( C_t [\kappa_t(i)]^{-1} - \beta \left[ C_{t+1} [\kappa_{t+1}(i)]^{-1} (1 + \tilde{r}_{t+1} [\kappa_{t+1}(i)]) \right] \right) &= 0 \\ \mathcal{E}_{it} \left( C_t^{-1} [\kappa_t(i)] - \beta \left[ C_{t+1}^{-1} [\kappa_{t+1}(i)] (1 + r) \right] \right) &= 0 \end{aligned} \tag{18}$$

where  $\kappa_t(i) = (K_t, B_{t-1}, \eta_t, \eta_{t+1}, \tilde{\epsilon}_t, \nu_t(i))$  is a vector of state variables, the productivity shocks at time  $t$  and  $t + 1$ , the near-rational error, and the idiosyncratic noise in the private signal.

Solving for equilibrium behavior thus poses two difficulties: First, households care about the payoff they receive from stocks and about their future consumption, but they receive information about  $\eta_{t+1}$ , and there is a complicated non-linear relationship between these variables. Second, households learn from  $Q_t$  about  $\eta_{t+1}$ , but  $Q_t$  is again a non-linear function of  $\eta_{t+1}$ .

<sup>16</sup>More formally,  $\epsilon_t = \int (\mathcal{E}_{it}(\eta_{t+1}) + \tilde{\epsilon}_t) di|_{\tilde{\epsilon}_t > 0, \sigma_{\tilde{\epsilon}} > 0} - [\int \mathcal{E}_{it}(\eta_{t+1}) di|_{\tilde{\epsilon}_t = 0, \sigma_{\tilde{\epsilon}} > 0}]$ .

We use two tricks developed in Mertens (2009) to transform (18) into a form which we can solve with standard techniques: First, we use perturbation methods to show that given the households' information sets, their conditional expectation of  $\eta_{t+1}$  is a sufficient statistic for their expectation of both future consumption and of future stock returns; i.e. there is a deterministic relationship between households' expectations of tomorrow's productivity and what they expect to happen in the future more generally. Moreover,  $K_t, B_{t-1}$  and  $\eta_t$  have no predictive power over and above the information contained in  $Q_t$  and  $s_t(i)$ . This reduces the problem to solving for  $\int \mathcal{E}(\eta_{t+1}|Q_t, s_t(i)) di$ . Second, we use a nonlinear change of variables to obtain a transformation of the equilibrium stock price which is linear in households' average expectation of tomorrow's productivity. This linear transformation, we call it  $\hat{q}_t$ , is a linear function of  $\eta_{t+1}$ , but has the same information content as  $Q_t$  (i.e. both variables span the same  $\sigma$ -algebra). The basic intuition is that  $Q_t$  is a monotonic function of  $\eta_{t+1}$ , such that learning from  $Q_t$  is just as good as learning from its linear transformation. Framed in terms of this  $\hat{q}_t$ , the equilibrium boils down to computing prices and expectations such that the following equation is satisfied:

$$\hat{q}_t = \int E(\eta_{t+1}|\hat{q}_t, s_t(i)) di + \tilde{\epsilon}_t, \quad (19)$$

where  $\hat{q}_t$  is a function of the state variables and shocks known at time  $t$ . Equation (19) is the familiar linear equilibrium condition of a standard noisy rational expectations model. We can now apply standard methods to solve for equilibrium expectations in terms of  $\hat{q}_t$  (Hellwig (1980)) and then transform the system back to recover the equilibrium  $Q_t$ . Technical details are given in Appendix B.1.

### 3.2 Amplification of Small Errors

We now obtain equilibrium expectations by solving for  $\hat{q}$ . As it turns out we are able to show all the main qualitative results on the aggregation of information in this linear form. In section 6, we map the solution back into its non-linear form to show the quantitative implications for the equilibrium stock price and for stock returns.

Since  $\hat{q}_t$  equals the market expectation of  $\eta_{t+1}$  in (19), we may guess that the solution for  $\hat{q}_t$  is some linear function of  $\eta_{t+1}$  and  $\tilde{\epsilon}_t$ :

$$\hat{q}_t = \pi_0 + \pi_1 \eta_{t+1} + \gamma \tilde{\epsilon}_t. \quad (20)$$

This guess formally defines the multiplier  $\gamma$ . Our task is to solve for the coefficients in this equation. Assuming that our guess for  $\hat{q}_t$  is correct, the rational expectation of  $\eta_{t+1}$  given the private signal and  $\hat{q}_t$  is

$$E_{it}(\eta_{t+1}) = A_0 + A_1 s_t(i) + A_2 \hat{q}_t, \quad (21)$$

where the constants  $A_0$ ,  $A_1$  and  $A_2$  are the weights that households give to the prior, the private signal and the market price of capital respectively. We get market expectations by adding the near-rational error and summing up across households. Combining this expression with our guess (20) yields

$$\int E(\eta_{t+1}|\hat{q}_t, s_t(i)) di + \tilde{\epsilon}_t = (A_0 + A_2\pi_0) + (A_1 + A_2\pi_1)\eta_{t+1} + A_2\gamma\tilde{\epsilon}_t + \tilde{\epsilon}_t, \quad (22)$$

where we have used the fact that  $\int s_t(i)di = \eta_{t+1}$ . This expression reflects all the different ways in which  $\tilde{\epsilon}_t$  affects market expectations: The last term on the right hand side is the direct effect of the near-rational error on individual expectations. If we introduced a fully rational household into the economy and gave it the same private signal as one of the near-rational households, the two households' expectations of  $\eta_{t+1}$  would differ exactly by  $\tilde{\epsilon}_t$ . The third term on the right hand side represents the deviation in market expectations that results from the fact that the market price transmits the average error as well as information about future fundamentals. The extent of this amplification depends on how much weight the market price has in the rational expectation (21) and on how sensitive  $\hat{q}_t$  is to  $\tilde{\epsilon}_t$  in (20). Finally, the second term on the right hand side tells us that the mere fact that households make near-rational errors may reduce the extent to which the market can predict  $\eta_{t+1}$  by changing the coefficients  $A_1$  and  $A_2$ .

Plugging (22) into (19) and matching coefficients with (19) allows us to show solve for the amplification of  $\tilde{\epsilon}_t$ :

**Proposition 3.1**

*Through its effect on the market price of capital, the near-rational error,  $\tilde{\epsilon}_t$ , feeds back into the rational expectation of  $\eta_{t+1}$ . The more weight households place on the market price of capital when forming their expectations about  $\eta_{t+1}$ , the larger is the error in market expectations relative to  $\tilde{\epsilon}_t$ . We have that*

$$\gamma = \frac{1}{1 - A_2}. \quad (23)$$

**Proof.** See appendix B. ■

It follows that the larger the weight on the market price of capital in the rational expectation,  $A_2$ , the larger is the variance in  $\epsilon_t$  relative to the variance in  $\tilde{\epsilon}$ . Small, near-rational errors may thus generate large deviations in the equilibrium stock price if households rely heavily on it when forming their expectations about the future.

The same matching coefficients algorithm also gives us the coefficient determining the amount of information reflected in the market price of capital:  $\pi_1 = \frac{A_1}{1 - A_2}$ . We can solve for the weights  $A_1$ ,  $A_2$  by applying the projection theorem. With explicit solutions in hand, we can show that:

**Proposition 3.2**

The absolute amount of information aggregated in the stock price decreases with  $\sigma_{\tilde{\epsilon}}$ ,

$$\frac{\partial \pi_1}{\partial \sigma_{\tilde{\epsilon}}} < 0$$

**Proof.** See appendix B. ■

While near-rational errors amplify and lead to potentially large deviations in the stock price, they simultaneously hamper the capacity of the stock market to transmit and aggregate information. The conditional variance of  $\eta_{t+1}$  in the near-rational expectations equilibrium therefore exceeds the conditional variance in the rational expectations equilibrium for two reasons: First, because the stock price becomes noisy and second because it contains less information about the future.<sup>17</sup>

When information is highly dispersed in the economy, households rely relatively more on the stock price when forming their expectations. But when households pay a lot of attention to the stock price ( $A_2$  is large), near-rational errors are amplified most, and the information content of prices is most vulnerable to near-rational behavior. The following proposition takes this insight to its logical conclusion:

**Proposition 3.3**

For any given level of  $\sigma_{\tilde{\epsilon}}$ , the noise to signal ratio in the market price of capital becomes arbitrarily large as the precision of the private signal goes to zero,

$$\lim_{\sigma_{\nu} \rightarrow \infty} \frac{\sqrt{\text{var}(\gamma \tilde{\epsilon}_t)}}{\sqrt{\text{var}(\pi_1 \eta_{t+1})}} = \infty.$$

**Proof.** See appendix B. ■

As information becomes more dispersed across households, the private signal becomes less informative relative to the stock price. Households adjust by paying relatively more attention to the public signal. If households put less weight on their private signal, less information enters the equilibrium price; and the more attention they pay to the market price, the larger is the amplification of  $\tilde{\epsilon}_t$ . Both effects result in a rising noise to signal ratio in equilibrium stock prices. The implication of this finding is that if the private signal received by households is sufficiently noisy, *arbitrarily small* correlated errors in investor behavior may completely destroy the market's capacity to aggregate information.

Figure 1 illustrates this point. It plots the ratio of the conditional variance of the productivity shock to its unconditional variance over the level of dispersion of information,  $\sigma_{\nu}/\sigma_{\eta}$ . A

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<sup>17</sup>See Appendix B.4 for an analytical solution for the conditional variance of  $\eta_{t+1}$ .

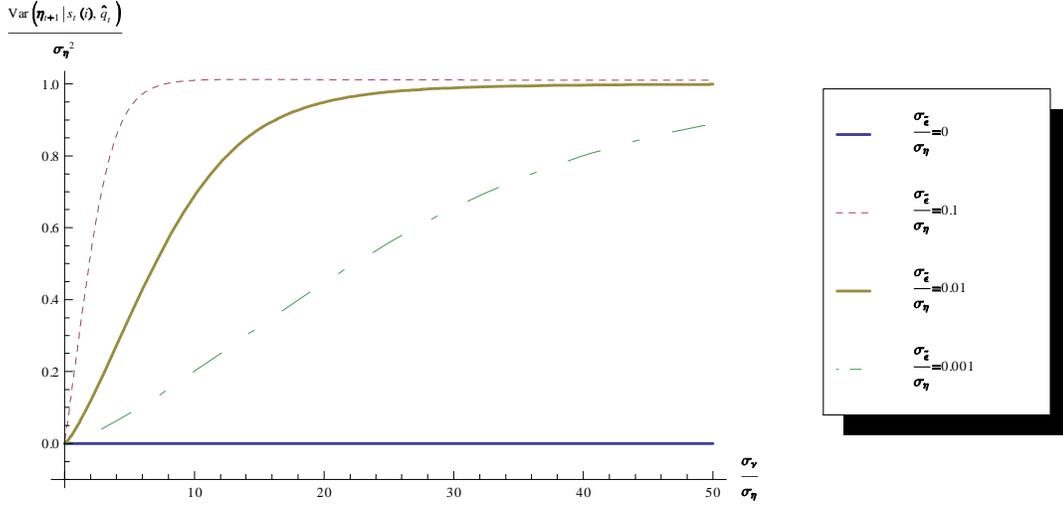


Figure 1: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information,  $\sigma_\nu/\sigma_\eta$ .

value of zero on the vertical axis indicates that households can perfectly predict tomorrow's realization of  $\eta_{t+1}$ , whereas a value of 1 indicates that  $\eta_{t+1}$  is completely unpredictable from the perspective of a household in the economy. The thick blue line shows that in the rational expectations equilibrium ( $\frac{\sigma_\varepsilon}{\sigma_\eta} = 0$ ), productivity is perfectly predictable, regardless of how dispersed information is in the economy. If all households are perfectly rational, the conditional variance of  $\eta_{t+1}$  is always zero, because the market price of capital perfectly aggregates the information in the economy. This situation changes drastically when  $\frac{\sigma_\varepsilon}{\sigma_\eta} > 0$ : The solid line plots the results for the case in which the standard deviation of the near-rational error is 1% of the standard deviation of the productivity shock. The curve rises steeply and quickly converges to one. When information is highly dispersed and we allow for near-rational behavior, the aggregation of information collapses and productivity becomes completely unpredictable.

The implication of Proposition 3.3 is that this qualitative result does not depend on *how* near-rational households are. Figure 1 plots the results for near-rational errors that are an order of magnitude larger ( $\frac{\sigma_\varepsilon}{\sigma_\eta} = 0.1$ ) and an order of magnitude smaller ( $\frac{\sigma_\varepsilon}{\sigma_\eta} = 0.001$ ) for comparison. In each case, the productivity shock becomes completely unpredictable if information is sufficiently dispersed.

One important feature of such a breakdown in the aggregation of information is that it affects everyone in the economy: If we placed a fully rational household into our economy, this fully rational household would do only a marginally better job at predicting  $\eta_{t+1}$  than a near rational household. In fact, the conditional variance we plotted in Figure 1, is the conditional variance

of  $\eta_{t+1}$  from the perspective of such a fully rational household. We can write it as

$$\frac{\text{Var}(\eta_{t+1}|s_t(i), \hat{q}_t)}{\sigma_\eta^2} = \frac{1}{\sigma_\eta^2} \left( A_1^2 \sigma_\nu^2 + (1 - \pi_t)^2 \sigma_\eta^2 + (\gamma - 1)^2 \sigma_\xi^2 \right). \quad (24)$$

The conditional variance from the perspective of a near-rational households is identical, except that the third term in brackets is then given by  $\gamma^2 \sigma_\xi^2$ .

Figure 2 decomposes the conditional variance into its three components. The solid line in Figure 2 is the same as the solid line in Figure 1, it plots the ratio of the conditional variance of the productivity shock to its unconditional variance over the level of dispersion of information for the case in which  $\frac{\sigma_\xi}{\sigma_\eta} = 0.01$ . The dotted line plots the first term on the right hand side of (24), which is the error that households make in their forecast of  $\eta_{t+1}$  due to the error in their private signal. It is close to zero throughout, reflecting the fact that households downweight their private signal when it contains more noise, such that differences of opinion remain small in equilibrium. The broken green line plots the second term, which is the error that households make in their forecast because the stock price does not reflect all information about  $\eta_{t+1}$ , and the third component is the error that they make due to amplified near-rational errors in the stock price.

At low levels of  $\sigma_\nu$ , amplified near-rational errors are the main source of households' forecast errors. As information becomes more dispersed, the amplification rises and eventually peaks as households, confronted with noisy private signals and a noisy stock price begin to rely more on their priors. At the same time, the information content of the stock price begins to fall rapidly. In the region in which the broken line approaches one, small near-rational errors result in a complete collapse of information aggregation.

Note that the basic logic of these results is not particular to the exact information structure we choose. For example, we may think of a situation in which the noise in the private signal is correlated across agents, such that  $\int s_t(i) di \neq \eta_{t+1}$ , in which case the stock price would not be fully revealing in the rational expectations equilibrium (the thick line in Figure 1 and the intercept of the solid and broken lines would shift upwards); or we may think of a situation in which households receive a public as well as a private signal about  $\eta_{t+1}$ , in which case the information contained in the public signal would survive in the near-rational expectations equilibrium (the solid and broken lines in Figure 1 would converge a value less than one). In each case, near-rational errors impede the aggregation of the part of the information which is dispersed across households. (See Appendix C for details.)

Now that we understand the aggregation of information in our model we can ask how near-rational behavior impacts the economy as a whole. Intuitively, the less information is reflected in the stock price, the higher is the conditional variance of stock returns and the more financial risk households face in equilibrium. It follows that the conditional variance of stock returns must

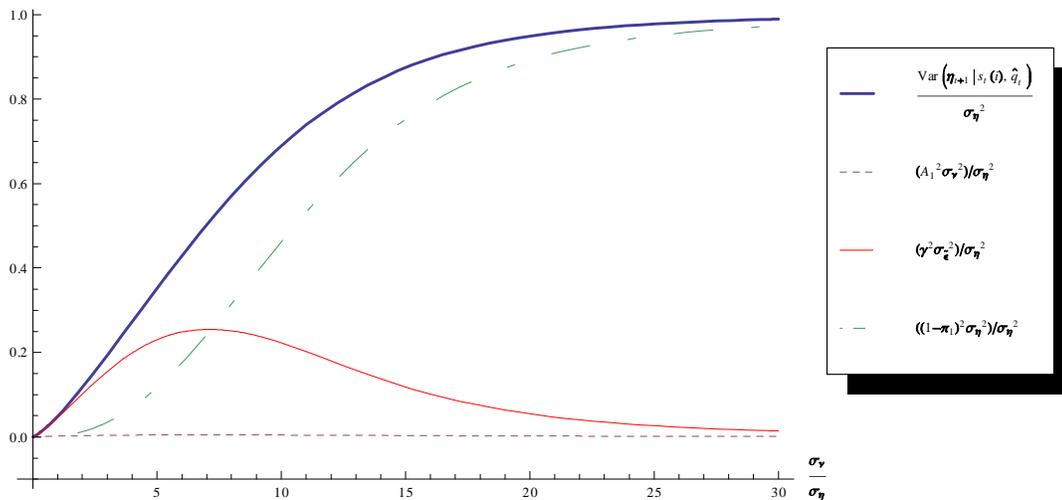


Figure 2: Decomposition of the ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information,  $\sigma_\nu/\sigma_\eta$ .

be strictly higher in the near-rational expectations equilibrium than in the rational expectations equilibrium. For the purposes of our discussion below, we define this difference in financial risk as “excess volatility”:

**Definition 3.4**

*Excess volatility in stock returns is the percentage amount by which the conditional standard deviation of stock returns in the near-rational expectations equilibrium,  $\sigma$ , exceeds the conditional standard deviation of stock returns in the rational expectations equilibrium,  $\sigma^*$ ,*

$$\frac{\sigma - \sigma^*}{\sigma} 100.$$

The amount of excess volatility in stock returns that may arise due to near-rational errors depends on the non-linearities of the model. Before we turn to quantifying these effects we first build some intuition for the impact that this particular pathology in financial markets may have on the macroeconomy.

## 4 Intuition: The Macroeconomic Effects of Financial Risk

In this section we turn to the effect that near-rational behavior has on the macroeconomic equilibrium. To provide a maximum of intuition for the mechanisms at work, the exposition focuses on a simplified version of the model for which we are able to derive the main results

analytically. In section 6 we show computationally that the relevant implications of the simplified model carry over to the full model.

Assume that households consist of two specialized agents, a "capitalist" who trades in the stock and bond markets and a "worker" who provides labor services, receives wages and the profits from the investment goods sector, but is excluded from trading in financial markets. This division eliminates labor income from the capitalist's portfolio choice problem such that we can solve it with pen and paper. A capitalist's budget constraint is

$$W_{t+1}(i) = ((1 - \omega_t(i))(1 + r) + \omega_t(i)(1 + \tilde{r}_{t+1}))(W_t(i) - C_t(i)) \quad \forall t. \quad (25)$$

Taking as given that the distribution of equilibrium asset returns is approximately log-normal (this is true to a first-order approximation), we can solve for the capitalist's optimal consumption and portfolio allocation.<sup>18</sup>

**Lemma 4.1**

*Capitalists' optimal consumption is a constant fraction of financial wealth*

$$C_t(i) = (1 - \beta)W_t(i) \quad (26)$$

*and the optimal portfolio share of stocks is the expected excess return divided by the conditional variance of stock returns,  $\sigma^2$*

$$\omega_t(i) = \frac{\mathcal{E}_{it}(1 + \tilde{r}_{t+1}) - (1 + r)}{\sigma^2}. \quad (27)$$

**Proof.** Appendix D gives a detailed derivation which proceeds analogous to Samuelson (1969).

■

In this simplified version of the model, only the capitalist, rather than the entire household, makes small mistakes as defined in (8) when investing in the stock market. The stock market clears when the value of shares demanded equals the value of shares in circulation:

$$\int_0^1 \beta \frac{\mathcal{E}_{it}(1 + \tilde{r}_{t+1}) - (1 + r)}{\sigma^2} W_t(i) di = Q_t K_{t+1}. \quad (28)$$

It is this condition that links the stock market to the real economy. We can apply the definition (7), as well as (26) and use the fact that all capitalists hold the same beginning of period wealth in equilibrium to get

$$\int_0^1 \mathcal{E}_{it} \left( \frac{Q_{t+1}(1 - \delta) + D_{t+1}}{Q_t} \right) di = 1 + r + \omega_t \sigma^2, \quad (29)$$

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<sup>18</sup>We require approximate log-normality for the analytical solution below but not for the computational results.

where  $\omega_t$  is defined in equation (17) and represents the aggregate degree of leverage required in order to finance the domestic capital stock. In equilibrium, the average capitalist holds a share  $\omega_t$  of her wealth in stocks. The left hand side of (29) is the market expectation of stock returns; the right hand side is the required return that investors demand given the risk that they are exposed to. The equity premium,  $\omega_t \sigma^2$ , rises with the conditional variance of stock returns and with the amount of leverage required to hold the domestic capital stock.

Any error in market expectations has two important channels through which it affects the real side of the model. First, it causes a temporary misallocation of capital by distorting  $Q_t$  and aggregate investment (14). Second, a rise in the conditional variance of returns raises the equity premium and with it the expected dividend demanded by capitalists in general equilibrium. While the former channel mainly influences the dynamics of the model, the latter channel has a direct effect on the stochastic steady state. We discuss each in turn.

#### 4.1 Distortion of Capital Accumulation

##### Definition 4.2

*The stochastic steady-state is the level of capital, bonds, and prices at which those quantities do not change in unconditional expectation.*

In the simplified version of the model we are able to obtain a closed form solution for the stochastic steady state and thus analytically show the following result:

##### Proposition 4.3

*The equilibrium has a unique stochastic steady state iff  $\beta \leq \frac{1}{1+r}$ . At the stochastic steady state the aggregate degree of leverage is*

$$\omega_o = \sqrt{\frac{1}{\sigma^2} \left( \frac{1-\beta}{\beta} - r \right)}; \quad (30)$$

*and the stochastic steady state capital stock is characterized by*

$$(1 + \delta\chi) (r + \omega_o \sigma^2 + \delta) = F_K (K_o, L). \quad (31)$$

**Proof.** See Appendix E. ■

The intuition for the first result is simple: If the time discount factor is larger than  $\frac{1}{1+r}$ , investors are so patient that even those holding a perfectly riskless portfolio containing only bonds would accumulate wealth indefinitely. In that case, no stochastic steady state can exist. However, if  $\beta \leq \frac{1}{1+r}$ , there exists a unique value  $\omega_o$  at which the average capitalist has an expected portfolio return that exactly matches his time discount factor:  $\beta = (1 + r + \omega_o^2 \sigma^2)^{-1}$ .

At this value, there is no expected growth in consumption and the economy is at its stochastic steady state.<sup>19</sup>

The second result, (31), follows directly from applying the steady state to equation (29). On the left hand side,  $1 + \delta\chi$  is the market price of a unit of capital at the stochastic steady state. This is multiplied with the required return to capital: the risk free rate plus the risk premium and the rate of depreciation. At the stochastic steady state, the required return on one unit of capital must equal the expected dividend, which is precisely the expected marginal product of capital (on the right hand side of the equation). This brings us to one of the main results of this paper:

**Proposition 4.4**

*A rise in the conditional variance of stock returns unambiguously depresses the stochastic steady state level of capital stock and output.*

$$\frac{\partial K_o}{\partial \sigma} < 0$$

**Proof.** We use (30) to eliminate  $\omega_o$  in (31) and take the total differential, see Appendix E for details. ■

The higher the risk of investing in stocks, the higher is the risk premium demanded by capitalists. A higher risk premium requires higher dividends at the stochastic steady state and, with a neoclassical production function, a lower level of capital stock. The conditional variance of stock returns thus has a *level* effect on the amount of capital accumulated at the stochastic steady state, and less installed capital in turns implies lower production.

Interestingly, this level effect may operate even if the stock market seems to have little influence on the allocation of capital in the economy:

**Corollary 4.5**

*A rise in the conditional variance of stock returns depresses the stochastic steady state level of output even if the sensitivity of the capital stock with respect to stock prices is low.*

**Proof.** From (14) we have that  $\frac{\partial(I_t/K_t)}{\partial Q_t} = \frac{1}{\chi}$ . The sensitivity of physical investment as a share of the existing capital stock with respect to the stock price is fully determined by the adjustment cost parameter  $\chi$ . From (30) and (31) we have that  $\frac{\partial^2 F_K(K_o, L)}{\partial \sigma^2 \partial \chi} = \delta \sqrt{\frac{1}{\sigma^2} \left( \frac{1-\beta}{\beta} - r \right)} > 0$ . ■

If the adjustment cost parameter  $\chi$  is sufficiently large, the stock market in this economy may appear as a “sideshow” (Morck, Shleifer, and Vishny (1990)) in the sense that a given change in the stock price has little influence on investment. To the casual observer it may therefore seem as though pathologies in the stock market should not have much influence on the real economy.

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<sup>19</sup>Conversely we can determine the wealth of our economy relative to the value of its capital stock at the stochastic steady state by choosing an appropriate time discount factor. We shall make use of this feature when we calibrate the model in section 5.

However, a low responsiveness of physical investment to the stock price is uninformative about the impact that excess volatility has on the stochastic steady state. Excess volatility in stock returns may cause a large depression of output at the stochastic steady state while leaving virtually no evidence to the econometrician. Since our model does not exempt replacement investments from capital adjustment costs, the impact of an incremental rise in stock market volatility on the stochastic steady state level of capital actually *rises* with  $\chi$ , implying that excess volatility may actually have a larger effect on the stochastic steady state in economies in which the stock market appears to be a “sideshow”.

Finally, the volatility of stock returns has an important implication for the distribution of income in the economy:

**Corollary 4.6**

*A rise in the conditional variance of stock returns unambiguously lowers wages and raises dividends at the stochastic steady state.*

**Proof.** The result follows directly from (11), (12) and proposition 4.4. ■

Excess volatility may paradoxically raise the incomes of stock market investors: At lower levels of  $K$ , dividends rise relative to wages, increasing the return to each unit of capital. Over some range, such a rationing raises the total payments to capital. As the conditional variance of stock returns rises, it pushes the economy towards higher dividends, compensating capital for the loss of aggregate output at the expense of payments to labor.

## 4.2 Dynamics of the Model

Solving the dynamics of the model requires a computational algorithm that we discuss in section 5. However, we can gain some intuition from the simplified version of our model. Equations (2), (11), (14), (29), and the standard transversality condition jointly determine the market price of capital. Every vector of state variables and shocks is therefore associated with a unique stock price.

Regardless of initial conditions, the economy transitions to a unique stochastic steady state in expectation. To understand this, imagine an economy that is at its stochastic steady state and receives a positive productivity shock. Capitalists will save a fraction of the currently high dividends and are now on average richer than they were before. This implies that the aggregate portfolio share required to finance the domestic capital stock in the following period falls,  $\omega_{o+1} < \omega_o$ . As capitalists are now less leveraged, they require a lower risk premium in the next period. Expected returns therefore tend to be lower following a positive shock and higher following a negative shock: Equilibrium returns exhibit negative autocorrelation and thereby

generate stationary dynamics and a unique ergodic distribution.<sup>20</sup>

In the rational expectations equilibrium, the market price of capital reflects households' knowledge about productivity in the next period. In the near-rational expectations equilibrium, the market price contains amplified noise and less information about the future, resulting in potentially large errors in market expectations about future productivity. These errors in market expectations increase the conditional variance of stock returns in the near-rational expectations equilibrium relative to the rational expectations equilibrium. Near-rational behavior thus results in an increase in financial risk and a depression in the stochastic steady state level of capital accumulation and output. Moreover, each error in market expectations passes into physical investment through the arbitrage performed by the investment goods sector, causing a temporary misallocation of capital.

To summarize, the near-rational expectations equilibrium of the simplified version of our model exhibits a higher volatility of returns around a lower stochastic steady state level of capital and output. Expected returns to capital are higher and expected wages are lower than in the rational expectations equilibrium. As we show below, these conclusions carry over to the full version of the model.

## 5 Quantifying Welfare Cost

In this section we return to the full version of our model and quantify the welfare cost of the near-rational behavior. To this end, we first derive a standard welfare metric, based on a simple experiment in which near-rational behavior is purged from financial markets and the economy transitions to the stochastic steady state of the rational expectations equilibrium. We then briefly describe the computational algorithm used to solve this problem and calibrate the model to the data.

### 5.1 Welfare Calculations

Consider an economy that is at the stochastic steady state of the near-rational expectations equilibrium and suppose that at time 0, there is a credible announcement that all households henceforth commit to fully rational behavior until the end of time. Immediately after the announcement, the conditional variance of stock returns falls and households require a lower risk-premium for holding stocks. The stochastic steady state levels of capital and output rise.

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<sup>20</sup>There is a large body of literature discussing the non-stationarity of small open economy models (see for example Schmitt-Grohé and Uribe (2003)). The issue of non-stationarity is, however, a consequence of the linearization techniques typically employed to solve these models and *not* an inherent feature of the small open economy setup. Since we solve our model using higher order expansions we obtain stationary dynamics.

Although the economy does not jump to the new stochastic steady state immediately, it accumulates capital over time and converges to it in expectation. Over the adjustment process, output rises, wages rise and returns to capital fall. The level of consumption increases and due to the reduction in uncertainty about future productivity the variance of consumption may fall as well.

Formally, we ask by what fraction  $\lambda$  we would have to raise the average household's consumption in order to make it indifferent between remaining in the near-rational expectations equilibrium and transitioning to the stochastic steady state of the rational expectations equilibrium.  $\lambda$  then indicates the magnitude of the welfare loss attributable to excess volatility as a fraction of lifetime consumption. It is defined as follows:

$$E \int_0^1 \sum_{t=0}^{\infty} \beta^t \log((1 + \lambda)C_t(i)) di \equiv E \int_0^1 \sum_{t=0}^{\infty} \beta^t \log(C_t^*(i)) di, \quad (32)$$

where we denote variables pertaining to the rational expectations equilibrium with an asterisk. From (32) we can see that welfare losses may result either from a lower *level* of consumption or from a higher *volatility* of consumption. In appendix F we decompose  $\lambda$  into two components,  $\lambda^\Delta$ , measures the change in welfare due to a change in the level of consumption and  $\lambda^\sigma$  measures the change in welfare due to a change in the volatility of consumption, where  $1 + \lambda = (1 + \lambda^\Delta)(1 + \lambda^\sigma)$ .

## 5.2 Numerical Solution

The numerical solution of our model employs perturbation methods in combination with a non-linear change of variables. It proceeds in three stages. First, we expand the conditions of optimality around the deterministic steady state. Second, we employ the non-linear change of variables described in section 3.1 in order to bring the equilibrium conditions of the model into a form which allows us to solve for conditional expectations in closed form. Finally, we make a natural guess for the equilibrium price function, solve for conditional expectations taking equilibrium prices as given, and verify the validity of the guess as described in section 3.2.

For the first step, we obtain the two conditions of optimality (18) and plug in for the households' budget constraint, stock returns, optimal investment, wages and dividends. Ultimately, we obtain two functions of known and unknown state variables and shocks  $K_t$ ,  $B_{t-1}$ ,  $K_t$ ,  $\int \mathcal{E}_{it}(\eta_{t+1}) di$ , and  $\nu_t(i)$  which characterize the optimal behavior of the individual.

We solve the Euler equations (18) for the optimal policies, impose market clearing, and solve for the deterministic steady state of the model. We then begin with a higher-order expansion in state variables and shocks around this point. We use a fourth order expansion to generate the results below. All variances reported are calculated at the deterministic steady state of the

system by analytically integrating over the second order expansion.

The crucial step which gets us back to a stochastic economy is to build at least a second-order expansion in the standard deviation of  $\eta$  and in the standard deviation of the conditional expectation of  $\eta$ . Financial risk thus affects the economy through the second moments of shocks. For details on perturbation methods see Judd (2002).

### 5.3 Calibration

Our main objective in this paper is to explore the fragile interaction between the aggregation of information in financial markets and the macroeconomy. We have therefore refrained from complicating the analysis by adding state of the art features of calibrated real business cycle models and of calibrated macro-finance models. The calibrations below should therefore not be viewed primarily as a moment matching exercise but as a first attempt to quantify the link between information aggregation, financial risk, and welfare.

In particular, a well known issue with the standard real business cycle model is that it cannot simultaneously match the volatility of output and the volatility of asset prices. Rather than complicating the analysis by incorporating habit formation, long-run risk, or rare disasters into our model, we side-step the issue by choosing  $\sigma_\eta$  to match the standard deviation of stock returns in the data and then adjust our welfare calculations to ensure that they are not driven by a counterfactually high standard deviation of consumption.<sup>21</sup>

In our preferred calibration we set the standard deviation of  $\tilde{\epsilon}$  to a very low level to ensure that the losses accruing to individual households due to their near-rational errors remain economically small; we set  $\frac{\sigma_{\tilde{\epsilon}}}{\sigma_\eta} = 0.01$ . We choose an adjustment cost parameter of  $\chi = 2$ , a risk free rate of  $r = 0.04$ , and a rate of depreciation of  $\delta = 0.15$ . We pick the time discount factor  $\beta$  such that the entire capital stock is owned by domestic households at the stochastic steady state of the near-rational expectations equilibrium,  $\omega_o = 1$ . Finally, we choose a Cobb-Douglas production technology with a capital share of  $\frac{1}{3}$ . Since our economy is scale-independent, we can normalize labor supply to one without loss of generality. Finally, we choose  $\sigma_\eta$  to match the conditional standard deviation of stock returns in the data for a given dispersion of information  $\frac{\sigma_\nu}{\sigma_\eta} = 35$ . We begin by presenting comparative statics with respect to  $\sigma_\nu$  and then calibrate it to match various moments in the data that reflect the information content of stock prices.

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<sup>21</sup>However, we suspect that our results would look similar if instead of learning about productivity shocks, households learned about growth rates (Bansal and Yaron (2004)) or about disaster probabilities (Barro (2009), Gabaix (2010), and Gourio (2010))

## 6 Results

Figure 3 relates the standard deviation of stock returns to the level of dispersion of information in the economy. The thick line plots the conditional standard deviation of stock returns in the rational expectations equilibrium ( $\sigma^*$ ). The line is horizontal, as the stock market perfectly aggregates the information held by all households, regardless of how dispersed information is in the economy. However, the fact that the stock price is perfectly informative about next period's productivity does not mean that households do not face financial risk. In the next period, they learn about productivity in the period after that, and the stock price adjusts such that stock returns remain uncertain. This is why the solid line intercepts the vertical axis at a positive value. Learning about tomorrow's productivity can reduce the variance of stock returns to 0.13, but not to zero. The solid upward sloping line gives the conditional variance of stock returns in the near-rational expectations equilibrium ( $\sigma$ ). When information is dispersed and we allow for near-rational errors, the aggregation of information in the economy deteriorates and eventually collapses. The result is a higher volatility of stock returns and thus more financial risk. If information is sufficiently dispersed,  $\sigma$  converges to 0.17.

The conditional standard deviation of stock returns is the standard deviation of stock returns from the perspective of a household that knows the state variables of the economy  $K_t, B_{t-1}, \eta_t$ , and extracts information about future productivity from  $Q_t$  and  $s_t(i)$ . Figure 3 also plots the (unconditional) standard deviation of stock returns which is not conditional on any information about future productivity (i.e. from the perspective of a household that knows the  $K_t, B_{t-1}, \eta_t$ , but does not receive a private signal and does not know the equilibrium stock price). In the rational expectations equilibrium the conditional and unconditional standard deviation are identical, because all information is common in equilibrium and there remain no differences of opinion about tomorrow's productivity. The equilibrium stock price thus always adjusts such that the expected stock returns equals the return required by investors. The dashed line in Figure 3 is the unconditional standard deviation of stock returns in the near-rational expectations equilibrium. It is almost identical with the conditional standard deviation as even in the near-rational expectations equilibrium differences of opinion remain small in equilibrium regardless of the level of dispersion of information in the economy (compare this to Figure 2 where the term  $A_1^2 \sigma_\nu^2$  remained small throughout).

The vertical distance between the solid and the thick line in Figure 3 reflects the amount of excess volatility in stock returns which is attributable to near-rational behavior. Figure 4 plots this excess volatility as a percentage amount. It shows that if information is sufficiently dispersed, the conditional standard deviation of stock returns is up to 25.67% lower in the rational expectations equilibrium than in the near-rational expectations equilibrium.<sup>22</sup>

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<sup>22</sup>There is nothing particular about the 25.67% we get in this calibration. With other parameters this value can be 50% or higher.

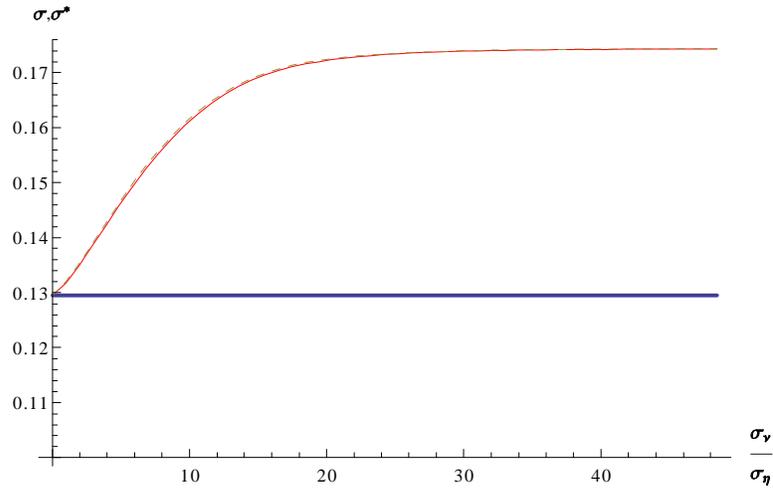


Figure 3: Solid line: Conditional standard deviation of stock returns in the near-rational expectations equilibrium. Dashed line: Unconditional standard deviation of stock returns in the near-rational expectations equilibrium. Thick line: Conditional and unconditional standard deviation of stock returns in the rational expectations equilibrium.

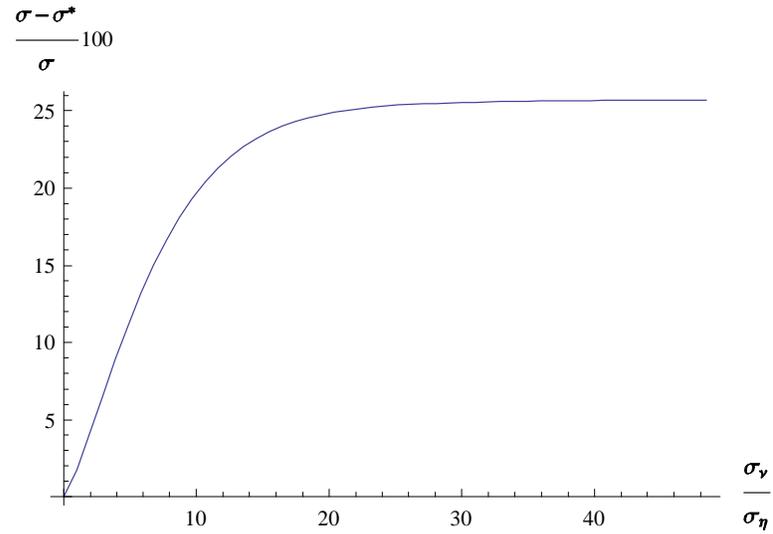


Figure 4: Excess volatility in stock returns for  $\sigma_{\varepsilon}/\sigma_{\eta} = 0.01$  plotted over a range of  $\sigma_{\nu}/\sigma_{\eta}$ .

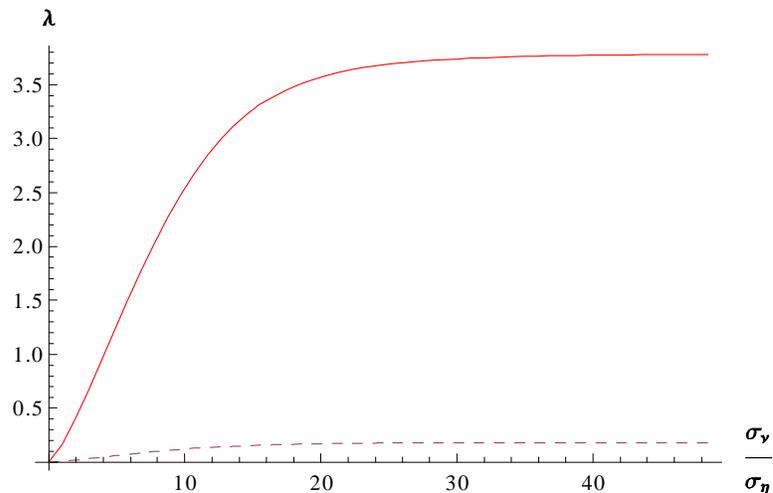


Figure 5: Solid line: Compensating variation for eliminating all present and future near-rational behavior and transitioning to the rational expectations equilibrium. Dashed line: Upper bound for the amount of the total compensating variation that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium

The most striking result of our simulations is that the welfare cost of near-rational behavior is very large, even for moderate levels of excess volatility in stock returns. The solid line in Figure 5 plots the compensating variation for households over a range of  $\sigma_\nu/\sigma_\eta$ . At the low end, for example, take  $\sigma_\nu/\sigma_\eta = 5$ , excess volatility accounts for 12.5% of the conditional standard deviation of stock returns and aggregate welfare losses amount to 1.27% of consumption. For our preferred calibration in which  $\sigma_\nu/\sigma_\eta = 35$  excess volatility in stock returns is 25%, and the compensating variation amounts to 3.76% of consumption. Households would thus be willing to give up 3.76% of their consumption if they could get all other households in the economy to behave fully rationally and thereby lower the conditional standard deviation of stock returns by 25%.

Our estimates for the welfare losses which are attributable to near-rational behavior thus exceed even relatively high estimates of the costs of business cycles (see for example Alvarez and Jermann (2005)). The reason is that standard costs of business cycles calculations calculate the welfare cost of fluctuations around a given mean, and these are typically small (Lucas (1987)). In our model, near-rational behavior affects both the volatility and the level of consumption, as near-rational behavior induces financial risk, and the level of financial risk determines the level of capital accumulation. The dashed line in Figure 5 shows an upper bound for the share of the overall welfare costs that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium,  $\lambda^\sigma$ . (It is the

willingness to pay of the average household for eliminating all of the variability in consumption which is due to productivity shocks and near-rational errors, while keeping the path and the level of capital accumulation the same as in the near-rational expectations equilibrium.) Throughout, this upper bound is less than 0.2% of consumption, indicating that the vast majority of the welfare loss caused by near-rational behavior is attributable to the distortions it causes in capital accumulation.

We now explore how our model fares in explaining key financial and macroeconomic data. The table below gives details for our preferred calibration and contrasts it with useful benchmarks. Column 1 gives the four moments of the data which we attempt to match. These are: the conditional standard deviation of stock returns, the correlation of stock price growth with GDP growth one year ahead, the standard deviation of the average forecast of GDP growth, and the standard deviation of the average error in forecasting GDP growth (the latter two are normalized with the standard deviation of GDP growth).<sup>23</sup>

Column 2 gives our preferred calibration in which the standard deviation of the error in the private signal is 35 times the standard deviation of the productivity shock. Columns 3 and 4 contrast it with two limiting cases in which the stock market has no role in the aggregation of information in the economy. The calibration in column 3 is the case in which the private signal is perfectly accurate ( $\frac{\sigma_\nu}{\sigma_\eta} = 0$ ) such that all households know next period's productivity without having to extract any information from the equilibrium stock price (we call this the "News Shocks" calibration). The calibration in column 4 gives the other extreme in which the private signal is perfectly inaccurate ( $\frac{\sigma_\nu}{\sigma_\eta} = \infty$ ) such that no one in the economy has any information about the future and there is consequently nothing to learn from the equilibrium stock price (we call this the "RBC" calibration). Column 5 gives the results for the rational expectations equilibrium. It uses our preferred calibration and imposes perfectly rational behavior ( $\frac{\sigma_\varepsilon}{\sigma_\eta} = 0$ ).

In our preferred calibration, we match the conditional variance of stock returns perfectly, and come close to matching the three other moments: For the correlation of stock price growth with future GDP growth we get 0.42 (around 0.5 in the data). The standard deviation of the average forecast comes in slightly too high (0.73 rather than 0.62 in the data), and the standard deviation of the average forecast error comes in slightly too low (0.7 rather than 0.88 in the data). The last two lines show that our preferred calibration implies an excess volatility of 25.67% of the conditional standard deviation of stock returns and a welfare loss attributable to near-rational errors of 3.76% of consumption

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<sup>23</sup>All moments are taken from US data as they are not readily available for other countries. The unconditional standard deviation of stock returns is around 0.18 (Campbell (2003)). Regressing stock returns on lagged price dividend and price earnings ratios (both variables available to the households in our model) yields an  $R^2$  of about 4% (Cochrane, 2005, p.393), suggesting that the conditional standard deviation of stock returns in the data is about 0.17. The correlation of stock price growth with one year ahead GDP growth is from Backus et al. (2007), and the remaining moments are from the author's calculations based on data provided by the Philadelphia Federal Reserve.

By construction, the standard deviation of stock returns is lower in the News Shocks calibration, and all three variables reflecting the information content of stock prices reflect (almost) perfect aggregation of information, as households are perfectly informed about future productivity from the outset. In this case, near-rational behavior is of almost no consequence: Excess volatility is minimal (0.01%) as near-rational errors are not amplified when the stock market has no role in aggregating information.

In the RBC calibration the conditional standard deviation of stock returns is the same as in the near-rational expectations equilibrium (actually it is minimally higher in the third digit), indicating that in our preferred calibration, a near rational error of 1% of the standard deviation of the productivity shock wipes out most of the aggregation of information. This is reflected in the moments concerning the standard deviation of expectations, which are also (almost) the same in the two calibrations. However, the RBC calibration implies a correlation between stock price growth and future GDP growth which is an order of magnitude too low (0.07 versus about 0.5 in the data).<sup>24</sup>

Table 1: Calibration

	(1)	(2)	(3)	(4)	(5)
	Data	N-REE	“News Shocks”	“RBC”	REE
Near-rational error, $\frac{\sigma_{\varepsilon}}{\sigma_{\eta}}$		0.01	0.01	0.01	0
Dispersion of information, $\frac{\sigma_{\nu}}{\sigma_{\eta}}$		35	0	$\infty$	35
Conditional standard deviation of stock returns, $\sigma$	$\approx 0.17$	0.17	0.13	0.17	0.13
Corr.(growth in stock prices, future GDP growth)	$\approx 0.50$	0.42	0.99	0.07	0.99
Std.(average forecast of GDP)/Std.(GDP growth)	0.62	0.73	1.00	0.73	1.00
Std.(average forecast error of GDP)/Std.(GDP growth)	0.88	0.70	0.01	0.70	0.00
Excess Volatility, $\frac{\sigma - \sigma^*}{\sigma}$		25.67	0.01		
Welfare loss attributable to near-rational errors, $\lambda$		3.76	0.00		

In summary, our calibrations suggest that near-rational behavior may result in an economically significant rise in financial risk and that such a rise in financial risk may result in economically large welfare losses.

<sup>24</sup>As there is no information to aggregate in the RBC calibration there is no benchmark against which to calculate  $\frac{\sigma - \sigma^*}{\sigma}$  and  $\lambda$ .

## 6.1 Closed Economy

In the closed economy version of our model the interest rate  $r$  becomes an endogenous variable and bonds are in zero net supply,  $B_t = 0$ . The dynamics of the model are slightly more involved in the closed economy case as the capital stock at the stochastic steady state of the rational expectations equilibrium may be either higher or lower. This is due to the precautionary savings motive which may or may not dominate the effect of a higher risk-premium. Nevertheless, the basic economic intuition holds: Any distortion in capital accumulation causes a distortion in the level of consumption; and any distortion in the level of consumption causes first-order welfare losses.

We calibrate the closed economy version to the parameters of our preferred calibration above. The compensating variation for eliminating near-rational behavior in this specification is 3.32% of consumption, which is very close to the result we get for the small open economy.

## 7 Conclusion

This paper showed that financial markets may fail to aggregate information even if they appear to be efficient; and that a decrease in the information content of asset prices may drastically reduce welfare. In our model, each household has some information about future productivity. If all households behave perfectly rationally, the equilibrium stock price reflects the information held by all market participants and directs resources to their most efficient use. We showed that this core function of financial markets may break down if we relax the assumption of full rationality and allow for the possibility that households do not respond to incentives which are economically small. In particular, we allow for households to make small errors in their investment decisions. If these errors are correlated they collectively move the equilibrium price and generate potentially large errors in market expectations of future productivity, as households rationally inform on the equilibrium price when forming their expectations. If information is sufficiently dispersed, arbitrarily small errors in households' investment decisions may cause a complete collapse of the information content of stock prices. Such a collapse of the information content of stock prices increases the amount of financial risk faced by households and thus induces them to demand higher risk premia for holding stocks. Higher risk premia in turn distort the level of capital accumulation, output and consumption in the long run.

We have argued that near-rational behavior is likely to inhibit information aggregation in financial markets precisely when it is most socially valuable – when information is highly dispersed in the economy. The core of the pathology we describe is that individuals are concerned with the return they receive on their investments and not with the impact that their investment behavior has on the expectations that others hold about the future. It is this externality that

makes the social return to diligent investor behavior larger than the private return; and we have argued that this basic result is likely to hold in a large range of macroeconomic models with dispersed information.

## References

- Akerlof, G. and J. Yellen (1985). A near-rational model of the business cycle with wage and price inertia. *Quarterly Journal of Economics*, 823–838.
- Albuquerque, R. and N. Wang (2005). An agency-based asset pricing model. *mimeo Columbia Business School*.
- Allen, F. and D. M. Gale (2001). Asset price bubbles and monetary policy. *Wharton School Center for Financial Institutions Working Paper 01-26*.
- Alvarez, F. and U. J. Jermann (2005). Using asset prices to measure the cost of business cycles. *Econometrica* 73, 1977–2016.
- Amador, M. and P.-O. Weill (2007). Learning from private and public observations of others' actions. *mimeo, Stanford University*.
- Angeletos, G.-M. and J. La'O (2008). Dispersed information over the business cycle: Optimal fiscal and monetary policy. *mimeo, Massachusetts Institute of Technology*.
- Angeletos, G.-M., G. Lorenzoni, and A. Pavan (2007, October). Wall street and silicon valley: A delicate interaction. (13475).
- Backus, D. K., B. R. Routledge, and S. E. Zin (2007). Asset prices in business cycle analysis. *mimeo New York University*.
- Baker, M., J. C. Stein, and J. Wurgler (2003). When does the market matter? stock prices and the investment of equity-dependent firms. *Quarterly Journal of Economics*, 969–1005.
- Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, 1481–1509.
- Barberis, N., A. Shleifer, and R. Vishny (1998). A model of investor sentiment. *Journal of Financial Economics* 49, 307–343.
- Barlevy, G. (2004). The cost of business cycles and the benefits of stabilization: A survey. *NBER Working Paper w10926*.
- Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. *American Economic Review* 99, 1, 243–264.
- Bikhchandani, S., D. Hirshleifer, and I. Welch (1998). Learning from the behavior of others: Conformity, fads, and informational cascades. *Journal of Economic Perspectives* 12, 151–170.
- Blanchard, O. J., C. Rhee, and L. H. Summers (1993). The stock market and investment. *Quarterly Journal of Economics* 108, 115–136.

- Campbell, J. Y. (2003). Consumption-based asset pricing. *in George Constantinides, Milton Harris, and Rene Stulz eds. Handbook of the Economics of Finance Vol. IB, North-Holland, Amsterdam*, 803–887.
- Campbell, J. Y. and J. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *The Journal of Political Economy* 107, 2, 205–251.
- Chetty, R. (2009). Bounds on elasticities with optimization frictions: A reconciliation of micro and macro labor supply elasticities. *mimeo Harvard University*.
- Chirinko, R. S. and H. Schaller (2006). Fundamentals, misvaluation and investment: The real story. *Institute for Advanced Studies, Economic Series 200*.
- Cochrane, J. H. (1989). The sensitivity of tests of the intertemporal allocation of consumption to near-rational alternatives. *The American Economic Review* 79(3), 319–337.
- Cochrane, J. H. (2005). *Asset Pricing*. Princeton University Press, Princeton.
- Daniel, K. D., D. Hirshleifer, and A. Subrahmanyam (2001). Overconfidence, arbitrage, and equilibrium asset pricing. *Journal of Finance* 56, 921–965.
- DeLong, B. J., A. Shleifer, L. H. Summers, and R. J. Waldmann (1989). The size and incidence of the losses from noise trading. *Journal of Finance* 44, 681–696.
- Devereux, M. B. and A. Sutherland (2006). Solving for country portfolios in open-economy macro models. *mimeo University of British Columbia*.
- Dumas, B., A. Kurshev, and R. Uppal (2006). What can rational investors do about excessive volatility and sentiment fluctuations? *Swiss Finance Institute Research Paper 06-19*.
- Dupor, B. (2005). Stabilizing non-fundamental asset price movements under discretion and limited information. *Journal of Monetary Economics* 52, 727–747.
- Farhi, E. and S. Panageas (2006). The real effects of stock market mispricing at the aggregate: Theory and empirical evidence. *mimeo Harvard University*.
- Fernández-Villaverde, J. and J. F. Rubio-Ramírez (2006). Solving dsge models with perturbation methods and a change of variables. *Journal of Economic Dynamics and Control* 30, 12, 2509–2531.
- Gabaix, X. (2010). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *Mimeo NYU Stern*.
- Galeotti, M. and F. Schiantarelli (1994). Stock market volatility and investment: Do only fundamentals matter? *Economica (New Series)* 61, 147–165.

- Gilchrist, S., C. P. Himmelberg, and G. Huberman (2005). Do stock price bubbles influence corporate investment? *Journal of Monetary Economics* 52, 805–827.
- Gourio, F. (2010). Disaster risk and business cycles. *mimeo Boston University*.
- Grossman, S. (1976). On the efficiency of competitive stock markets where trades have diverse information. *The Journal of Finance* 31(2), 573–585.
- Hellwig, M. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory* 22, 477–498.
- Hong, H. and J. C. Stein (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance* 54, 2143–2184.
- Jaimovich, N. and S. Rebelo (2009). Can news about the future drive the business cycle? *The American Economic Review* 99, 4, 1097–1118.
- Judd, K. L. (1998). *Numerical Methods in Economics*. The MIT Press.
- Judd, K. L. (2002). Perturbation methods with nonlinear change of variables. *mimeo, Hoover Institution, Stanford University*.
- Judd, K. L. and S.-M. Guu (2000). The economic effects of new assets: An asymptotic approach. *mimeo, Hoover Institution*.
- Lansing, K. (2008). Speculative growth and overreaction to technology shocks. *Federal Reserve Bank of San Francisco Working Paper 2008-08*.
- Lucas, R. E. (1987). *Models of business cycles*. Basil Blackwell.
- Mankiw, N. G. (1985). Small menu costs and large business cycles: A macroeconomic model of monopoly. *The Quarterly Journal of Economics* 100(2), 529–537.
- Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review* 81 (4), 797–818.
- Mertens, T. (2009). Excessively volatile stock markets: Equilibrium computation and policy analysis. *mimeo New York University*.
- Morck, R., A. Shleifer, and R. W. Vishny (1990). The stock market and investment: Is the market a sideshow? *Brookings Papers on Economic Activity* 2, 157–215.
- Morris, S. and H. S. Shin (2002). Social value of public information. *The American Economic Review* 92(5), 1521–1534.

- Odean, T. (1998). Volume, volatility, price, and profit: When all traders are above average. *Journal of Finance* 53, 1887–1934.
- Odean, T. (1999). Do investors trade too much? *American Economic Review* 89, 1279–1298.
- Panageas, S. (2005). Neoclassical theory of investment in speculative markets. *mimeo the Wharton School*.
- Polk, C. and P. Sapienza (2003). The real effects of investor sentiment. *CEPR Discussion Paper 3826*.
- Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *Review of Economics and Statistics* 51, 239–246.
- Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics* 61 (1), 163–185.
- Stein, J. C. (1987). Informational externalities and welfare-reducing speculation. *Journal of Political Economy* 95 (6), 1123–1145.
- Tille, C. and E. van Wincoop (2007). International capital flows. *NBER Working Paper w12856*.
- Woodford, M. (2005). Robustly optimal monetary policy with near-rational expectations. *NBER Working Paper No. W11896*.

# Appendix

## A Details on Contingent Claims

This appendix gives formal details on the contingent claims which ensure that all households hold the same amount of wealth in equilibrium. Each period is divided in two subperiods. In the first subperiod the productivity shock realizes, contingent claims pay off and households buy state contingent claims for next period. In the second subperiod they consume, receive their private signal of next period's productivity shock, choose their consumption and a their portfolio. We can write equations (5) and (6) as

$$\max_{\{\varphi_t(i;n)\}} \mathcal{E} \left[ \max_{C_t(i), \omega_t(i)} U_t(i) = \mathcal{E}_{it} \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \log(C_s(i)) \right\} \middle| K_t, B_{t-1}, \eta_t \right] \quad (33)$$

and for all  $t$

$$W_{t+1}(i) = [(1-\omega_t(i))(1+r) + \omega_t(i)(1+\tilde{r}_{t+1})] \left( W_t(i) + w_t L - C_t(i) + \int \varphi_t(i;n) (\Phi_{t+1}(i;n) - \rho_t(n)) dn \right), \quad (34)$$

where  $n$  is a realization of the vector  $[\eta_{t+1}, \tilde{\varepsilon}_t, \nu_t]$ ,  $\varphi_t(i;n)$  is the quantity of contingent claims bought by household  $i$  that pay off in state  $n$ ,  $\Phi_{t+1}(i;n)$  is an indicator function that is one if state  $n$  occurs at time  $t+1$  and  $\nu$  coincides with  $\nu_t(i)$  and zero otherwise, and  $\rho_t(n)$  is the time  $t$  price of a claim that pays off one in state  $n$  at time  $t+1$  and zero otherwise.

The crucial assumption is that households trade state contingent claims in the first subperiod, during which they have homogeneous information. Contingent claims are in zero net supply ( $\int \varphi_t(i;n) di = 0 \forall t$ ), such that, in equilibrium households use these claims to insure against the idiosyncratic risk arising from the heterogeneity in the private signal they receive.

Recall that the errors in private signals,  $\nu_t(i)$ , are by definition uncorrelated and independent of the realization of aggregate shocks in the next period. Since households do not know their own realization of  $\nu_t(i)$ , they cannot predict the payoff they receive from the state-contingent securities and this payoff is uncorrelated and independent of any of the other variables influencing their decisions. It follows that trading in state contingent claims does not distort the portfolio and consumption decisions of the household, while ensuring that the wealth distribution collapses to its average at the beginning of each period.

## B Equilibrium Expectations

### B.1 Non-linear Change in Variables

We start with the optimality conditions in (18) in which we plug in the definition of returns (7), rearrange terms, and integrate over individuals. The rearrangements result in the form

$$Q_t = \int \mathcal{E}_{it} \left( \beta \frac{C_t[\kappa_t(i)]}{C_{t+1}[\kappa_{t+1}(i)]} ((1 - \delta)Q_{t+1} + D_{t+1}) \right) di \quad (35)$$

$$C_t(i) = \mathcal{E}_{it} (\beta C_{t+1}[\kappa_{t+1}(i)]^{-1} (1 + r))^{-1} \quad (36)$$

For a more concise exposition, we only demonstrate how to solve for the stock price in (35). However, the method applies to the consumption function analogously.

#### Lemma B.1

The average expectation of the rewritten optimality condition (35) can be written in the following form<sup>25</sup>

$$Q_t = \int \mathcal{E}_{it} \left( \beta \frac{C_t[\kappa_t(i)]}{C_{t+1}[\kappa_{t+1}(i)]} ((1 - \delta)Q_{t+1} + D_{t+1}) \right) di = h_{S_t^k}^1 \left( \int (E_{it}[\eta_{t+1}|s_t(i), Q_t] di + \tilde{\epsilon}_t) di \right)$$

where  $h_{S_t^k}(\cdot)$  depends solely on a vector of known state variables,  $S_t^k = [K_t, B_{t-1}, \eta_t]$ , and moments, as well as on the market expectation of next period's productivity conditional on the information set at time  $t$ .<sup>26</sup>

To see the result in the lemma, take an infinite order Taylor series expansion of the product of next periods marginal utility with the asset return in  $K_{t+1}$ ,  $B_t$ ,  $\eta_{t+1}$ ,  $E[\eta_{t+2}|s_t(i), \log(Q_t)]$ ,  $\sigma_\eta$ , and take the expectation conditional on  $s_t(i)$  and  $Q_t$ . This gives us a series of terms depending on  $K_{t+1}$ ,  $B_t$ , and  $\sigma_\eta$ , which are known at time  $t$ . Moreover, we get a series of terms depending on the conditional expectation of  $\eta_{t+2}$ . Since  $\eta_{t+2}$  is unpredictable for an investor at time  $t$ , the first-order term is 0, and all the higher-order terms depending on  $E[\eta_{t+2}|s_t(i), Q_t]$  are just cumulants of the unconditional distributions of  $\eta$  and  $\tilde{\epsilon}$ . The only interesting terms are then

<sup>25</sup>In the simplified version of our model in which households consist of specialized capitalists and workers we can solve for the consumption policy in closed form. The optimal behavior of households (18) and market clearing in the stock market : The equation to be inverted is then

$$\left( (1 + r) Q_t + \frac{Q_t^2 K_t \left( 1 - \delta + \frac{1}{\chi} (Q_t - 1) \right)}{\beta (B_{t-1} (1 + r) + Q_t K_t (1 - \delta) + e^{\eta_t} F_K(K_t, L) K_t) \sigma^2} \right) = \int \mathcal{E}_{it} (Q_{t+1} (1 - \delta) + D_{t+1}) di. \quad (37)$$

<sup>26</sup>For computational purposes, it turns out that we can reduce the state space by replacing  $\eta_{t+1}$  and  $\tilde{\epsilon}_t$  with the average expectation of productivity in the next period. The reason is that households cannot distinguish between productivity and near-rational errors and hence the coefficient on either shock in the perturbation is identical.

those depending on  $\eta_{t+1}$ . We can write

$$\mathcal{E}_{it} \left[ \beta \left[ C_{t+1} [\kappa_{t+1} (i)]^{-1} (1 + \tilde{r}_{t+1} [\kappa_{t+1} (i)]) \right] \right] = \sum_{j=0}^{\infty} c_j(K_{t+1}, B_t) \mathcal{E}[(\eta_{t+1} - E[\eta_{t+1}])^j | s_t(i), Q_t],$$

where the coefficients  $c_j(K_{t+1}, B_t)$  involve all the terms depending on the  $K_{t+1}$ ,  $B_t$ ,  $\sigma_\eta$ , and the higher cumulants of  $\eta$  and  $\tilde{\epsilon}$ .

Next, take the term in the expectations operator on the right hand side and expand it to get

$$\begin{aligned} & \mathcal{E}[(\eta_{t+1} - E[\eta_{t+1}])^j | s_t(i), Q_t] \\ &= \mathcal{E}[(\eta_{t+1} - \mathcal{E}[\eta_{t+1} | s_t(i), Q_t]) + (\mathcal{E}[\eta_{t+1} | s_t(i), Q_t] - E[\eta_{t+1}])^j | s_t(i), Q_t] \\ &= \sum_{k=0}^j \binom{j}{k} \mathcal{E} \left[ (\eta_{t+1} - \mathcal{E}[\eta_{t+1} | s_t(i), Q_t])^k (\mathcal{E}[\eta_{t+1} | s_t(i), Q_t] - E[\eta_{t+1}])^{j-k} | s_t(i), Q_t \right] \\ &= \sum_{k=0}^j \binom{j}{k} \mathcal{E} \left[ (\eta_{t+1} - \mathcal{E}[\eta_{t+1} | s_t(i), Q_t])^k | s_t(i), Q_t \right] (\mathcal{E}[\eta_{t+1} | s_t(i), Q_t] - E[\eta_{t+1}])^{j-k} \\ &= \sum_{k=0}^j \binom{j}{k} m(k) (\mathcal{E}[\eta_{t+1} | s_t(i), Q_t] - E[\eta_{t+1}])^{j-k}, \end{aligned}$$

where  $m(k) = \mathcal{E} [(\eta_{t+1} - \mathcal{E}[\eta_{t+1} | s_t(i), Q_t])^k | s_t(i), Q_t]$ . Now we can use the fact that the operator  $\mathcal{E}$  is a rational expectations operator in which the probability density function of  $\eta$  has been shifted by  $\tilde{\epsilon}$ . This means that we can replace

$$\mathcal{E} \left[ (\eta_{t+1} - \mathcal{E}[\eta_{t+1} | s_t(i), Q_t])^k | s_t(i), Q_t \right] = E \left[ (\eta_{t+1} - E[\eta_{t+1} | s_t(i), Q_t])^k | s_t(i), Q_t \right] \text{ for all } k,$$

where for  $k = 1$ , the expression collapses to zero.  $m(k)$  is then just the  $k$ -th moment of the conditional distribution of  $\eta$ .

The conditional expectation that households hold of all higher moments of  $\eta_{t+1}$  is thus a non-linear function of their conditional expectation (the first moment) of  $\eta_{t+1}$  and of all higher conditional moments,  $m(k)$ . However, since  $\eta_{t+1}$  is normally distributed, we know that its conditional distribution must also be normal. Therefore all the higher conditional moments depend only on the conditional variance and on known parameters. Moreover, the conditional variance is constant.

We can now collect terms in the expression above and integrate to get

$$\int \mathcal{E}_{it} \left( \beta \left[ C_{t+1} [\kappa_{t+1}(i)]^{-1} (1 + \tilde{r}_{t+1} [\kappa_{t+1}(i)]) \right] \right) di \quad (38)$$

$$= \int \sum_{j=0}^{\infty} c_j(K_{t+1}, B_t) \left( \sum_{k=0}^j \binom{j}{k} m(k) (\mathcal{E}[\eta_{t+1}|s_t(i), Q_t] - E[\eta_{t+1}])^{j-k} \right) di \quad (39)$$

The last step is to use (8) and (21) in combination with (4) and integrate over households to write

$$\mathcal{E}[\eta_{t+1}|s_t(i), Q_t] = A_1 \nu_t(i) + \int \mathcal{E}[\eta_{t+1}|s_t(i), Q_t] di,$$

where  $A_1 \nu_t(i)$  is the weight households put on their private signal multiplied with the error they receive in their private signal. This term represents the only source of idiosyncratic variation in household expectations. We then substitute this expression into (38) and expand the sum in its polynomial terms. In the resulting expression, all terms containing  $\nu_t(i)$  give us the unconditional moments of the distribution of  $\nu$ , which is known. Finally, we can define the resulting expression on the right hand side as  $h_{S_t^k}^1(\int \mathcal{E}_{it}[\eta_{t+1}] di)$ .

The only remaining piece of the puzzle is then to obtain the conditional expectation and the conditional variance of  $\eta_{t+1}$ , as well as the coefficient  $A_1$ . See section 3.2 for a derivation of the conditional expectation and of  $A_1$ . Appendix B.4 gives the conditional variance. ■

Moreover, we can show computationally that  $h_{S_t^k}^i(\cdot)$ ,  $i = 1, 2$ , is invertible with

$$h_{S_t^k}^i(0) = 0 \quad (h_{S_t^k}^i)'(\cdot) > 0 \quad h_{S_t^k}^i(\infty) = \infty. \quad (40)$$

Using lemma B.1, we can re-write equation (35) in the linear form

$$\hat{q} = \int E(\eta_{t+1}|\hat{q}_t, s_t(i), K_t, B_t, \eta_t) di + \tilde{\epsilon}_t,$$

where  $\hat{q} \equiv (h_{S_t^k}^1)^{-1}(Q_t)$ . Analogously, we solve for the consumption function. See Mertens (2009) for a more detailed derivation of these results.

## B.2 Proof of Proposition 3.1

Matching coefficients between (22) and (20) yields three equations:  $A_0 + A_2\pi_0 = \pi_0$ ,  $A_1 + A_2\pi_1 = \pi_1$ , and  $1 + A_2\gamma = \gamma$ . Solving the three equations and three unknowns yields

$$\pi_0 = \frac{A_0}{1 - A_2}, \quad (41)$$

$$\pi_1 = \frac{A_1}{1 - A_2}, \quad (42)$$

and

$$\gamma = \frac{1}{1 - A_2}. \quad (43)$$

## B.3 Proof of Proposition 3.3

The vector  $(\eta_{t+1}, s_t(i), \hat{q}_t)$  has the following variance covariance matrix:

$$\Sigma = \begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1 \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1 \sigma_\eta^2 \\ \pi_1 \sigma_\eta^2 & \pi_1 \sigma_\eta^2 & \pi_1^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \end{pmatrix}$$

Applying the projection theorem yields the coefficients  $A_1$  and  $A_2$  that correspond to the rational expectation of  $\eta_{t+1}$  given  $s_t(i)$  and  $\hat{q}_t$  in (21):

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \sigma_\eta^2 & \pi_1 \sigma_\eta^2 \\ \sigma_\eta^2 & \pi_1 \sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1 \sigma_\eta^2 \\ \pi_1 \sigma_\eta^2 & \pi_1^2 \sigma_\eta^2 + \gamma^2 \sigma_\varepsilon^2 \end{pmatrix}^{-1},$$

yielding

$$A_1 = \frac{\gamma^2 \sigma_\eta^2 \sigma_\varepsilon^2}{\gamma^2 \sigma_\nu^2 \sigma_\varepsilon^2 + \sigma_\eta^2 (\pi_1^2 \sigma_\nu^2 + \gamma^2 \sigma_\varepsilon^2)}, \quad A_2 = \frac{\pi_1 \sigma_\eta^2 \sigma_\nu^2}{\gamma^2 \sigma_\nu^2 \sigma_\varepsilon^2 + \sigma_\eta^2 (\pi_1^2 \sigma_\nu^2 + \gamma^2 \sigma_\varepsilon^2)}. \quad (44)$$

These coefficients are still functions of endogenous variables  $\pi_1$  and  $\gamma$ . Combining them with equations (42) and (43) yields the following closed-form solutions:

$$\gamma = \frac{1}{6\sigma_\eta^4} \left[ \frac{2\sigma_\eta^2 (\sigma_\eta^2 - 2\sigma_\nu^2) + \frac{22^{1/3} \sigma_\eta^4 (\sigma_\eta^2 + \sigma_\nu^2)^2 \sigma_\varepsilon^2}{(27\sigma_\eta^{12} \sigma_\nu^2 \sigma_\varepsilon^4 + 2\sigma_\eta^6 (\sigma_\eta^2 + \sigma_\nu^2)^3 \sigma_\varepsilon^6 + 3\sqrt{3} \sqrt{\sigma_\eta^{18} \sigma_\nu^2 \sigma_\varepsilon^8 (27\sigma_\eta^6 \sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3 \sigma_\varepsilon^2)})^{1/3}}}{+ \frac{2^{2/3} (27\sigma_\eta^{12} \sigma_\nu^2 \sigma_\varepsilon^4 + 2\sigma_\eta^6 (\sigma_\eta^2 + \sigma_\nu^2)^3 \sigma_\varepsilon^6 + 3\sqrt{3} \sqrt{\sigma_\eta^{18} \sigma_\nu^2 \sigma_\varepsilon^8 (27\sigma_\eta^6 \sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3 \sigma_\varepsilon^2)})^{1/3}}{\sigma_\varepsilon^2}} \right]$$

and

$$\begin{aligned}\pi_1 = & (92^{2/3}\sigma_\eta^{16}\sigma_\nu^2\sigma_\epsilon^6 + 92^{2/3}\sigma_\eta^{14}\sigma_\nu^4\sigma_\epsilon^6 + 2^{2/3}\sqrt{3}\sigma_\eta^4\sigma_\epsilon^2\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_\epsilon^8(27\sigma_\eta^6\sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3\sigma_\epsilon^2)}) \\ & + 2^{2/3}\sqrt{3}\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_\epsilon^8(27\sigma_\eta^6\sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3\sigma_\epsilon^2)} - 92^{1/3}\sigma_\eta^{12}\sigma_\nu^2\sigma_\epsilon^4(\Psi)^{1/3} \\ & - 2^{1/3}\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_\epsilon^8(27\sigma_\eta^6\sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3\sigma_\epsilon^2)}(\Psi)^{1/3} + 6\sigma_\eta^{10}\sigma_\epsilon^2(\Psi)^{2/3} / (6\sigma_\eta^{10}\sigma_\epsilon^2(\Psi)^{2/3}),\end{aligned}$$

where  $\Psi = 27\sigma_\eta^{12}\sigma_\nu^2\sigma_\epsilon^4 + 2\sigma_\eta^6(\sigma_\eta^2 + \sigma_\nu^2)^3\sigma_\epsilon^6 + 3\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_\epsilon^8(27\sigma_\eta^6\sigma_\nu^2 + 4(\sigma_\eta^2 + \sigma_\nu^2)^3\sigma_\epsilon^2)}$ . Given these results

$$\lim_{\sigma_\nu \rightarrow \infty} \frac{\text{var}(\gamma\tilde{\epsilon}_t)}{\text{var}(\pi_1\eta_{t+1})} = \infty$$

can easily be calculated using a mathematical software package.

## B.4 Conditional Variance

The projection theorem also gives us the conditional variance of  $\eta_{t+1}$  as

$$\text{var}(\eta_{t+1}|\hat{q}_t, s_t(i)) = \sigma_\eta^2 - \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 \end{pmatrix} \quad (45)$$

$$= \frac{\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2}{\gamma^2\sigma_\nu^2\sigma_\epsilon^2 + \sigma_\eta^2(\pi_1^2\sigma_\nu^2 + \gamma^2\sigma_\epsilon^2)}. \quad (46)$$

A closed form solution follows from combining this expression with equations (42) and (43).

## B.5 Proof of Proposition 3.2

The derivative  $\frac{\partial\pi_1}{\partial\sigma_\epsilon}$  can easily be calculated from (42). However, the resulting expression is too complex to be reproduced here. The fact that  $\frac{\partial\pi_1}{\partial\sigma_\epsilon} < 0$  can be verified using a mathematical software package.

## C Alternative Information Structures

This appendix discusses the case of more complex information environments.

### C.1 Public Signal

Assume that households observe a public signal about future productivity in addition to the private signal they receive,

$$g_t = \eta_{t+1} + \zeta_t,$$

where  $\zeta_t$  represents i.i.d. draws from a normal distribution with zero mean and variance  $\sigma_\zeta^2$ . We may then guess that the solution for  $\hat{q}_t$  is some linear function of  $\eta_{t+1}$ ,  $\zeta_t$ , and  $\tilde{\epsilon}_t$ :

$$\hat{q}_t = \pi_0 + \pi_1\eta_{t+1} + \pi_2\zeta_t + \gamma\tilde{\epsilon}_t,$$

where the rational expectation of  $\eta_{t+1}$  given  $\hat{q}_t$  and the private and public signals is

$$E_{it}(\eta_{t+1}) = A_0 + A_1s_t(i) + A_2\hat{q}_t + A_3g_t.$$

A matching coefficients algorithm parallel to that in Appendix B.2 gives

$$\pi_1 = \frac{A_1 + A_3}{1 - A_2}, \quad \pi_2 = \frac{A_3}{1 - A_2}, \quad \gamma = \frac{1}{1 - A_2}$$

The amplification of near-rational errors is thus influenced only in so far as that the presence of public information may induce households to put less weight on the market price of capital when forming their expectations.

The vector  $(\eta_{t+1}, s_t(i), \hat{q}_t, g_t)$  has the following variance covariance matrix:

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_2^2\sigma_\zeta^2 + \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 & \pi_2\sigma_\zeta^2 + \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 & \pi_2\sigma_\zeta^2 + \pi_1\sigma_\eta^2 & \sigma_\zeta^2 + \sigma_\eta^2 \end{pmatrix}$$

Solving the signal extraction problem returns

$$\begin{aligned} A_1 &= \frac{\gamma^2\sigma_\zeta^2\sigma_\eta^2\sigma_\epsilon^2}{\sigma_\zeta^2(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2 + (\pi_1 - \pi_2)^2\sigma_\nu^2) + \gamma^2\sigma_\nu^2\sigma_\epsilon^2) + \gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ A_2 &= \frac{(\pi_1 - \pi_2)\sigma_\zeta^2\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2 + (\pi_1 - \pi_2)^2\sigma_\nu^2) + \gamma^2\sigma_\nu^2\sigma_\epsilon^2) + \gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ A_3 &= \frac{\sigma_\eta^2\sigma_\nu^2(\gamma^2\sigma_\epsilon^2 + \pi_2(\pi_2 - \pi_1)\sigma_\zeta^2)}{\sigma_\zeta^2(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2 + (\pi_1 - \pi_2)^2\sigma_\nu^2) + \gamma^2\sigma_\nu^2\sigma_\epsilon^2) + \gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \end{aligned}$$

Based on these results Figure 6 plots the conditional variance of  $\eta_{t+1}$  for the rational and near-rational expectations equilibrium and for varying levels of precision of the public signal .

In the rational expectations equilibrium the provision of public information makes no difference, as households are already fully informed from the outset. In the near-rational expectations equilibrium the presence of the public signal is relevant only insofar as a collapse of information aggregation affects only the subset of information that is dispersed across households and not the information that is publicly available. If the public information provided is relatively precise, the conditional variance of stock returns now converges to lower values as  $\sigma_\nu$  increases.

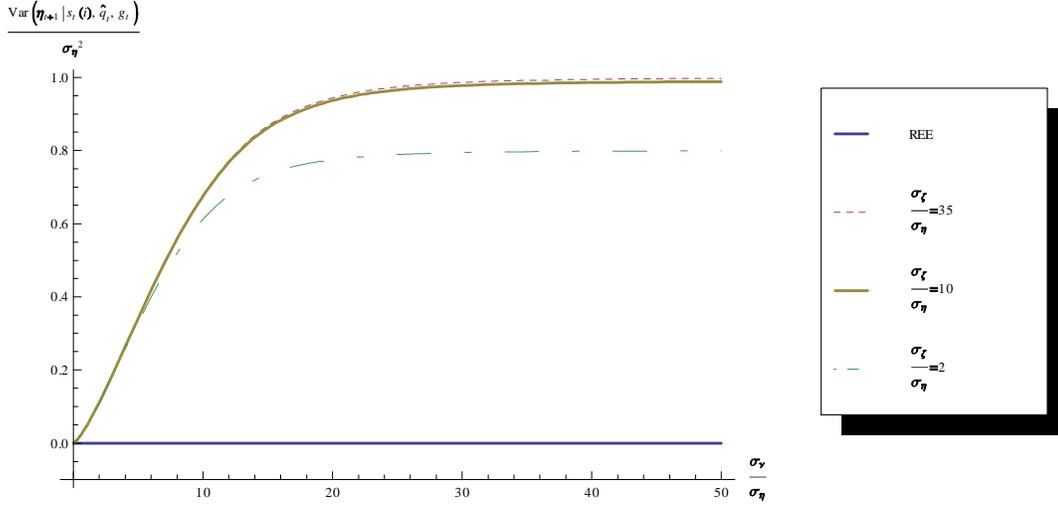


Figure 6: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information,  $\sigma_\nu/\sigma_\eta$ , and for varying precisions of the public signal. In each case,  $\sigma_{\tilde{\varepsilon}}/\sigma_\eta$  is set to 0.01.

## C.2 Aggregate Noise in Private Signals

Alternatively, we may consider a situation in which that the private signal received by households contains some aggregate noise:

$$s_t(i) = \eta_{t+1} + \nu_t(i) + \zeta_t$$

In this case we may guess that

$$\hat{q}_t = \pi_0 + \pi_1 (\eta_{t+1} + \zeta_t) + \gamma \tilde{\varepsilon}_t,$$

where both the rational expectation (21), and the coefficients  $\pi_1$  and  $\gamma$  are the same as these given in the main text. However, the variance covariance matrix of the vector  $(\eta_{t+1}, s_t(i), \hat{q}_t)$  changes to

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1 \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\zeta^2 + \sigma_\eta^2 + \sigma_\nu^2 & \pi_1 (\sigma_\zeta^2 + \sigma_\eta^2) \\ \pi_1 \sigma_\eta^2 & \pi_1 (\sigma_\zeta^2 + \sigma_\eta^2) & (\sigma_\zeta^2 + \sigma_\eta^2) \pi_1^2 + \gamma^2 \sigma_{\tilde{\varepsilon}}^2 \end{pmatrix},$$

and we get

$$A_1 = \frac{\gamma^2 \sigma_\eta^2 \sigma_{\tilde{\varepsilon}}^2}{\sigma_\zeta^2 (\gamma^2 \sigma_{\tilde{\varepsilon}}^2 + \pi_1^2 \sigma_\nu^2) + \sigma_\eta^2 (\gamma^2 \sigma_{\tilde{\varepsilon}}^2 + \pi_1^2 \sigma_\nu^2) + \gamma^2 \sigma_\nu^2 \sigma_{\tilde{\varepsilon}}^2}$$

$$A_2 = \frac{\pi_1 \sigma_\eta^2 \sigma_{\tilde{\varepsilon}}^2}{\sigma_\zeta^2 (\gamma^2 \sigma_{\tilde{\varepsilon}}^2 + \pi_1^2 \sigma_\nu^2) + \sigma_\eta^2 (\gamma^2 \sigma_{\tilde{\varepsilon}}^2 + \pi_1^2 \sigma_\nu^2) + \gamma^2 \sigma_\nu^2 \sigma_{\tilde{\varepsilon}}^2}.$$

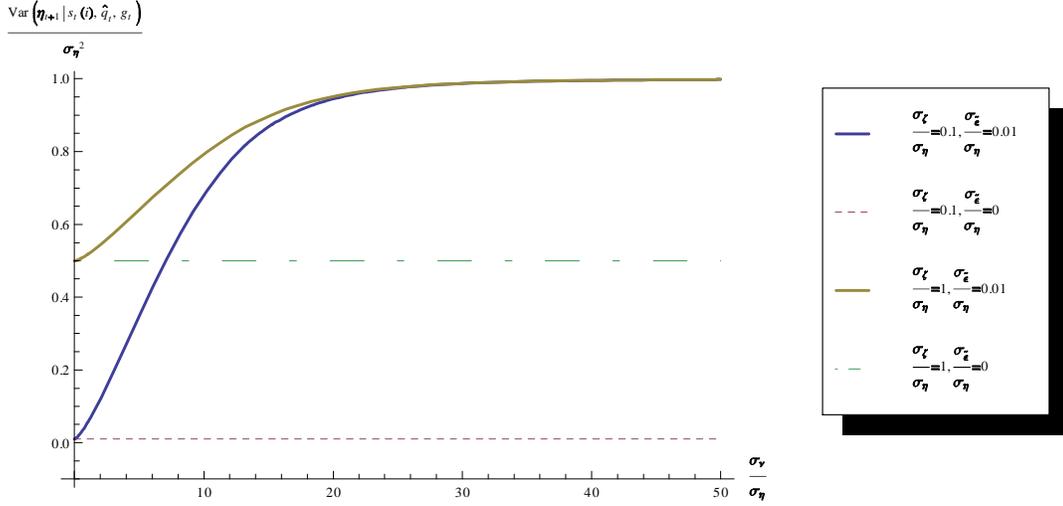


Figure 7: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information,  $\sigma_\nu/\sigma_\eta$ , and for varying amounts of aggregate noise in the private signal.

Based on these calculations Figure 7 plots the conditional variance of  $\eta_{t+1}$  for the rational and near-rational expectations equilibrium and for varying levels of aggregate noise in the private signal.

The more aggregate noise there is in the private signal the less information is there to aggregate, and the intercept of the curves in Figure 7 shift upwards.

## D Proof of Lemma 4.1

We can re-write (5) in Bellman form:

$$V(W_t(i), \pi_t(i)) = \max_{C_t(i), \omega_t(i)} \log(C_t(i)) + \beta \mathcal{E}_{it} [V(W_{t+1}(i), \pi_{t+1}(i))],$$

where we abbreviate  $\pi_t(i) = E_{it}(1 + \tilde{r}_{t+1}) - (1 + r)$ . The conditions of optimality are:

$$\frac{1}{C_t(i)} = \beta \mathcal{E}_{it} \left[ R_{i,t+1}^p V'(W_{t+1}(i), \pi_{t+1}(i)) \right], \quad (47)$$

$$\mathcal{E}_{it} \left( (\tilde{r}_{t+1} - r)(W_t(i) - C_t(i)) V'(R_{i,t+1}^p(W_t(i) - C_t(i)), \pi_{t+1}(i)) \right) = 0, \quad (48)$$

and

$$V'(W_t(i), \pi_t(i)) = \beta \mathcal{E}_{it} \left( R_{i,t+1}^p V'(W_{t+1}(i), \pi_{t+1}(i)) \right),$$

where  $R_{i,t+1}^p \equiv ((1 - \omega_t(i))(1 + r) + \omega_t(i)(1 + \tilde{r}_{t+1}))$  and  $V'$  denotes  $\frac{\partial V}{\partial W}$ . It follows immediately that

$$\frac{1}{C_t(i)} = V'(W_t(i)). \quad (49)$$

Guess the value function:

$$V_t(W_t(i)) = \kappa_1 \log(W_t(i)) + \kappa_2(\pi_t(i)) + \kappa_3 \quad (50)$$

Verification yields:

$$\begin{aligned} \kappa_1 &= \frac{1}{1 - \beta} \\ \kappa_2 &= \frac{1}{1 - \beta} \mathcal{E}_{it} \left\{ \sum_{s=1}^{\infty} \beta^s \log(R_{t+s}^{p*}(i)) \right\} \\ \kappa_3 &= \frac{1}{1 - \beta} \log(1 - \beta) + \frac{\beta}{(1 - \beta)^2} \log(\beta), \end{aligned}$$

where  $R_t^{p*}$  is the optimized portfolio return. Furthermore, the transversality condition has to hold:

$$\lim_{s \rightarrow \infty} \beta^s \kappa_2(R_{t+s}^{p*}(i)) = 0$$

The first result in Proposition 4.1 follows directly from taking the derivative with respect to  $W_t(i)$  in (50) and combining it with (49). For the second result, combine (48) with (50) to obtain

$$(1 + r) \mathcal{E}_{it} (R_{t+1}^p(i))^{-1} = \mathcal{E}_{it} \left( (1 + \tilde{r}_{t+1}) (R_{t+1}^p(i))^{-1} \right),$$

take logs on both sides, use the fact that

$$\log \mathcal{E}_{it}(\cdot) = \mathcal{E}_{it} \log(\cdot) + \frac{1}{2} \text{var}(\log(\cdot)),$$

and re-arrange the resulting expression to recover (27).

## E Solving for the stochastic steady state

### E.1 Proof of Proposition 4.3

If at any time  $o$  the economy is at its stochastic steady state, we can write  $E_o B_{o+1} = B_o$ ,  $E_o K_{o+1} = K_o$  and  $I_o = \delta K_o$ , where  $E_o$  is the unconditional expectations operator, which conditions only on public information available at time  $o$ ,  $E_o(\cdot) = E(\cdot | Q_o, K_o, B_o, \eta_o)$ . From equation (14) it immediately follows that  $Q_o = E_o Q_{o+1} = 1 + \delta \chi$ . We first calculate the steady state dividend, from which we then back out the steady state capital stock. Finally we derive

the steady state value of  $\omega$ .

From equation (11),

$$D_{t+1} = e^{\eta_{t+1}} F_K(K_{t+1}, L),$$

At the steady state:

$$E_o D_{o+1} = F_K(K_o, L)$$

Taking the unconditional expectation of (29) and plugging in yields

$$r + \omega_o \sigma^2 = -\delta + \frac{1}{1 + \delta\chi} (F_K(K_o, L))$$

and

$$(1 + \delta\chi) (r + \omega_o \sigma^2 + \delta) = F_K(K_o, L).$$

This proves the second statement in Proposition 4.3.<sup>27</sup>

We now turn to solving for  $\omega_o$ . The first step is to derive the equilibrium resource constraint for capitalists from (2), (11), (14), (25) and (17): From (17) we get that  $W_t - C_t = Q_t K_{t+1} + B_t$  plugging this into (25) yields

$$Q_t K_{t+1} + B_t + C_t = (1 + r)B_{t-1} + (Q_t(1 - \delta) + D_t) K_t.$$

Now we can use (2) to eliminate  $K_{t+1}$ :

$$Q_t(1 - \delta) K_t + Q_t I_t + B_t + C_t = (1 + r)B_{t-1} + (Q_t(1 - \delta) + D_t) K_t.$$

This simplifies to

$$Q_t I_t + B_t + C_t = (1 + r)B_{t-1} + D_t K_t. \quad (52)$$

The next step is to re-write (52) in terms of  $K_o$  and  $\omega_o$ . For this purpose note that

$$C_o = (1 - \beta)W_o,$$

$$\beta W_o = K_o(1 + \delta\chi) + B_o,$$

$$B_o = \beta W_o(1 - \omega_o),$$

and

$$(1 + \delta\chi)K_o = \beta W_o \omega_o$$

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<sup>27</sup>With a Cobb-Douglas specification and a capital share of  $\alpha$  we can further write

$$\left( \frac{(1 + \delta\chi)(r + \omega_o \sigma^2 + \delta)}{\alpha L^{1-\alpha}} \right)^{\frac{1}{\alpha-1}} = K_o. \quad (51)$$

$$\rightarrow B_o = \frac{1 - \omega_o}{\omega_o} (1 + \delta\chi) K_o$$

Plugging these conditions into (52) and simplifying yields

$$(1 + \delta\chi) \left( \delta + \frac{1 - \beta}{\beta} + \frac{1 - \omega_o}{\omega_o} \left( \frac{1 - \beta}{\beta} - r \right) \right) = F_K(K_o, L) \quad (53)$$

We can eliminate  $K_o$  from this equation by substituting in (31). Some manipulations yield

$$\omega_o = \sqrt{\frac{1}{\sigma^2} \left( \frac{1 - \beta}{\beta} - r \right)},$$

proving the first statement in Proposition 4.3.

## E.2 Proof of Proposition 4.4

Combining (30) and (31) and taking the total differential gives

$$\frac{dK_o}{d\sigma} = \frac{1 + \delta\chi}{F_{KK}(K_o, L)} \left( \frac{1 - \beta}{\beta} - r \right)^5.$$

Proposition 4.3 states that a stochastic steady state exists iff  $\beta \leq \frac{1}{1+r}$ . Proposition 4.4 then follows directly from the fact that  $F_{KK}(K_t, L) < 0$ .

## F Decomposition of welfare losses

This section decomposes households' total welfare loss into components attributable to additional variability of consumption and a distortion in the capital accumulation. Given the parameters of the model and initial conditions  $K_o, \omega_o, B_o$  (see Appendix E), define the expected utility level of the average household in the near-rational expectations equilibrium  $U$  as

$$U = E_o \int_0^1 \sum_{t=0}^{\infty} \beta^t \log(C_t(i)) di,$$

where  $E_o$  is the unconditional expectations operator, which conditions only on public information available at time  $o$ ,  $E_o(\cdot) = E(\cdot | K_o, B_o, \eta_o, Q_o)$ . Similarly, given the same parameters and initial conditions define the expected utility level  $U^*$  of transitioning to the stochastic steady state of the rational expectations equilibrium as

$$U^* = E_o \int_0^1 \sum_{t=0}^{\infty} \beta^t \log(C_t^*(i)) di.$$

We can solve (32) for  $\lambda$  to obtain

$$1 + \lambda = \exp [(E_o U^* - E_o U) (1 - \beta)]. \quad (54)$$

We now define a reference level of utility,  $C^\sigma$ . In this scenario, the path and the level of capital accumulation remain the same as in the near-rational expectations equilibrium, but households exogenously receive compensation for the variability in consumption which is due to productivity shocks and near-rational errors. In technical terms, we calculate the lifetime utility of households which consume  $C_t^\sigma = C(K_t, B_{t-1}, 1, 1, 1, 0)$ , rather than  $C(K_t, B_{t-1}, \eta_t, \eta_{t+1}, \tilde{\varepsilon}_t, \nu_t(i))$ : Households are exogenously given the level of consumption they would have received had productivity shocks and near rational errors been at their mean.

$$U^\sigma = E_o \int_0^1 \sum_{t=0}^{\infty} \beta^t \log(C_t^\sigma(i)) di.$$

This reference level of utility allows us to calculate an upper bound for the welfare costs that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium. The remainder of the difference between the reference utility and welfare in the rational expectations equilibrium must thus be due to a distortion in the accumulation of capital.<sup>28</sup> We can write

$$U^\Delta = U^* - U^\sigma$$

We can now apply these definitions in (32):

$$1 + \lambda = \exp [(U^* - U^\sigma + U^\sigma - U) (1 - \beta)]$$

and

$$\begin{aligned} 1 + \lambda &= \exp [(U^* - U^\sigma) (1 - \beta)] \\ &\cdot \exp [(U^\sigma - U) (1 - \beta)]. \end{aligned}$$

This implies that

$$1 + \lambda = (1 + \lambda^\Delta) (1 + \lambda^\sigma).$$

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<sup>28</sup>We subsume the second order effect due to the variability of the capital stock in this category.