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Abstract

We compute the impulse response of output to an aggregate monetary shock in a general equilibrium when firms set prices subject to a costly observation of the state and a menu cost. We study how the aggregate effects of a monetary shock depend on the relative size of these costs. We find that empirically reasonable observations costs increase the impact and the persistence of the output response to monetary shocks compared to models with menu cost only, flattening the shape of the impulse response function. Moreover we show that if the shocks are not large the results are independent of the assumption of whether firms know the realization of the monetary shock on impact.

JEL Classification Numbers: E5

Key Words: sticky prices, inattentiveness, monetary shocks, impulse responses.

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1 Introduction

A substantial disagreement exists on the role of price rigidity for the transmission of aggregate shocks. An important reason behind this disagreement lies in the microeconomic cause for price stickiness. Many authors, such as Caplin and Spulber (1987) and Golosov and Lucas (2007), showed that when the price rigidity arises from the presence of fixed non-convex cost of price adjustment the real effects of monetary shocks tend to be small and short lived. Other authors, such as Mankiw and Reis (2006), have argued that models where the price rigidity arises from information frictions are more successful at generating a large output response to monetary shocks.[1]

This paper bridges the gap between the two literatures in a model where firms set prices under two frictions: a standard fixed cost of adjusting the price, inducing infrequent price adjustments, and a fixed cost of observing the state, inducing infrequent information acquisition. In the model each firm plans about two related choices: observing the state and adjusting the price. Our model is a general equilibrium version of the price setting problem studied in Alvarez, Lippi, and Paciello (2011), and embeds as special cases the "menu cost" model (e.g. Barro (1972); Dixit (1991)) as well as the "observation cost" model (e.g. Caballero (1989); Bonomo and Carvalho (2004); Reis (2006)). The menu cost model aggregates similarly to Golosov and Lucas (2007) and provides a useful benchmark of comparison since the predictions of this model have been extensively studied in the literature. The observation cost model is a general equilibrium version of Reis's (2006) inattentive producers model which, with a constant fixed cost of observing the state, features reviews at approximately uniformly distributed times, and therefore behaves similarly to Taylor's (1980) staggered price model.

We derive the aggregate equilibrium responses to nominal shocks and compare the predictions about the aggregate effects of a one time unexpected permanent increase in money

[1] See also Midrigan (2011), Kehoe and Midrigan (2010), Nakamura and Steinsson (2010), Gertler and Leahy (2008) for a review of the debate on the ability of menu cost models to generate substantial money non-neutrality. See Mankiw and Reis (2010) for a review of the costly-information literature.

supply produced by models with different combinations of observation and menu costs, including the two special cases mentioned above (where one of the costs is zero). For comparison we also consider a model where the times of adjustments and observations coincide and are assumed exponentially distributed.[2] We find this case interesting because the adjustment rule resembles the one in *Calvo* or Mankiw and Reis (2002), a widely used assumption in the literature that typically predicts that monetary shocks have a large and persistent output effect.

Moreover, we explore the role of the assumptions on the observability of monetary shocks considering two alternative setups: the first one assumes that the firms are unaware that a monetary shock occurred until the first costly observation of the state at which point both the monetary and the idiosyncratic shocks are learned, in the spirit of Mankiw and Reis (2002). The second setup assumes that the firms immediately and freely learn the monetary shock, independently of the time(s) at which they acquire information about their state. We find the case of immediately observed monetary shocks more realistic: these shocks are likely to be analyzed and covered in the press, while idiosyncratic shocks may require accounting, management, and marketing resources of the firm. The belief that monetary shocks are easily observable has been identified as a potential weakness of the imperfect information hypothesis, see e.g. Woodford (2001).

The paper delivers several results. First, we show that economies that are observationally equivalent with respect to the average frequency and size of price adjustments are characterized by different responses to nominal shocks, depending on the ratio of observation to menu costs. We find that the larger the observation cost relative to the menu cost, the larger and the more persistent the output response to a monetary shock. For example, calibrating the model to reproduce frequency and size of price adjustments for the US shows that the cumulated output response to a 1% unexpected increase of the money supply is about 0.4% with a Calvo mechanism, about 0.2% in the pure "observation cost" model and about 0.1%

[2] We model those exogenous adjustments as product substitutions.

in the menu cost model, while an intermediate value of 0.15% occurs in the model with both frictions.

Second, and somewhat surprisingly, the shape of the impulse response is not just a "simple average" of the shapes in the models with menu cost only and observation cost only: over a wide range of values for observation and menu costs, the shape largely resembles the profile of the impulse response in a model with observation cost only. At our baseline parametrization, the time-dependent component of the adjustment rule dominates over the state-dependent component of the adjustment rule in determining the shape of the output response to the monetary shock, so that the shape of the impulse response is roughly linear. This linearity, akin to the one in *Taylor* type price adjustments, implies that the effects on output are not very persistent, which contrast with the larger effect of monetary shocks displayed by *Calvo* type of price adjustment, whose impulse response decays exponentially.[3]

Third we find that, for monetary shocks of moderate size, the assumptions about the information structure regarding the monetary shocks have negligible consequences for the predictions of the model about the real effects of such shocks. For instance, for a once and for all monetary shock of up to 2% the output response of the model with perfectly observed monetary shocks is almost identical to the one of the model with unobserved monetary shocks. We view this property as a sign of robustness of the approach used by Mankiw and Reis (2002) discussed above. Instead, when monetary shocks are perfectly observed but are large enough, the model displays considerable price flexibility. For instance, for a once and for all monetary shock larger than 5%, the model with both observation and menu costs, and perfectly observed monetary shock, exhibits more price flexibility than the pure menu cost model.

[3]See Chari, Kehoe, and McGrattan (2000) for a comparison of the persistence of Taylor vs. Calvo type of adjustment.

Setup and related literature

The setup of our model is similar to Golosov and Lucas (2007), where in addition to the menu cost that firms pay to change prices, we introduce costly observation of their cost, as a way to model rational inattention, as we did in Alvarez, Lippi, and Paciello (2011). Each firm is a monopolistically producer of a single product, and maximizes the expected discounted value of future profits. Firms in the economy are subject to permanent idiosyncratic productivity shock z, for each product. Additionally we assume that the life of each product is exponentially distributed, with average duration $1/\lambda$. All new products have productivity $z_t = 1$. This assumption allows our model to nest the Mankiw and Reis's (2002) model where observations and adjustments are exponentially distributed. When the product is exogenously replaced, the firm knows the productivity of the new product and has to set its price. There is an idiosyncratic preference shock associated to each product which acts as a multiplicative shifter of the product demand. For easier exposition, we assume that productivity and demand shocks are perfectly correlated, so that the cross-section distribution of outputs is stationary even in absence of product replacement.[4] After time zero the log of the money supply is assumed to grow with drift parameter μ. In each period, conditional on survival, the firm faces two different choices: the first choice is whether to pay the observation cost and observe the level of productivity; the second choice is whether and to what level to adjust its price upon payment of the menu cost. Both observation and menu costs are assumed to be proportional to firm's steady state profits. The decision rule for price adjustments upon observations is characterized by an inaction range, so that the posted price is adjusted upon observation only if it is outside the inaction region. Upon each observation of the state, the firm also decides the next observation date, so that the time of next observation depends on the state of the last observation.

This paper builds on results we obtained in Alvarez, Lippi, and Paciello (2011) to identify the relevant parameters of the model from available data. In particular, we use that the ratio

[4] In Appendix F we show that the assumption on preference shocks is irrelevant for the quantitative predictions of our benchmark economy.

of menu to observation cost can be identified from the ratio of the frequency of observations to price changes. For given ratio of menu to observation costs, then the frequency and size of price changes can be used to identify the level of the two fixed costs as well as the volatility of innovations to productivity. Applying those results to survey evidence on U.S. firms from Blinder et al. (1998) we obtain that observation costs are twice as large as adjustment costs.

The analysis of our paper relates to a few recent contributions that study the price setting of firms in the presence of both observation and menu costs, which we first explored in Alvarez, Lippi, and Paciello (2011). A similar setup is considered by Demery (2012) to examine the effect of monetary shocks. However the method he uses to compute the equilibrium (imposing that the output response must have the shape of an AR(1) process) precludes finding different shapes of the impulse responses for different combinations of observation and menu costs, as well as finding different shapes as a function of the size of the monetary shock, two of the main results of our paper.[5] Moreover we study the differential impact of observed and unobserved *aggregate* shocks.

Our finding that the effect of monetary shocks displayed by *Calvo* type exogenous adjustments are much larger than the ones of models with any combination of fixed observation or menu costs contrasts with the conclusions obtained by Mankiw and Reis (2002). They conclude instead that a model with infrequent reviews can generate output responses to monetary shocks that are comparable to *Calvo*. In fact, our observation cost only model aggregates differently from what is used in Mankiw and Reis's (2002) based upon Reis (2006).[6] In Alvarez, Lippi, and Paciello (2012) we discuss this comparison at length.

The rest of the paper is organized as follows. In Section 2 we describe the model, and characterize the firm's optimal decision rules as well as the general equilibrium. Section 3 discusses the choice of parameters, in particular of observation and menu costs, and computes the response of output to a monetary shock under different assumptions about the information

[5]See Appendix G for a detailed comparison of the two set-ups as well as an explanation of the difference in the methods.

[6]Another difference is that their model features price plans, i.e. price changes taking place between review dates as opposed to a constant nominal price.

structure regarding such shocks. Section 4 discusses the sensitivity of the results to alternative parametrizations of observation and menu costs. Section 5 concludes.

2 The model

The economy is a variant of the general equilibrium model of Golosov and Lucas (2007), which offers a relatively simple framework to study the effects of monetary shocks in general equilibrium with non-convex costs of price adjustments. The key deviation of our economy from Golosov and Lucas (2007) is that we allow for disjoint costs of price reviews and price adjustments as we did in Alvarez, Lippi, and Paciello (2011). We are interested in studying the aggregate effects of monetary shocks as a function of the relative size of these two costs. The monetary shock takes the form of an unexpected (unforeseen) one time increase in money supply, m_t, at period $t = 0$. For simplicity, there are no other aggregate shocks hitting the economy, so that after period $t = 0$, the log of the money supply is assumed to grow with drift parameter μ:

$$d\ log(m_t) = \mu\ dt. \tag{1}$$

There are two types of agents making decisions in this economy, a representative household and a unitary mass of monopolistically competitive firms. Firms are subject to persistent idiosyncratic productivity shock for each product, $z(i)$, evolving according to

$$d\ log(z_t(i)) = \gamma\ dt + \sigma\ dB_t(i) \tag{2}$$

where $B_t(i)$ is a standard brownian motion with zero drift and unit variance, the realizations of which are independent across firms. Firm i's output at time t is given by $z_t(i)\, l_t(i)$, where $l_t(i)$ is the labor employed by firm i in production. With probability λ per unit of time a product is dismissed and is replaced by a new one, i.e. product life is exponentially distributed. We will refer to this event as a product replacement or substitution. Once a product is replaced, the price of the new product is chosen under complete information. Without loss of generality, all new products have productivity $z_t = 1$. The economic motivation for replacement is that allows our model to nest the *Calvo* model of random times of price adjustments. The technical motivation for replacement is to insure stationarity of the cross-section distribution of prices and outputs. The next sections describe the problems of the household and firms in detail.

2.1 Household Problem

We assume that (real) aggregate consumption c_t is given by the Spence-Dixit-Stiglitz consumption aggregate

$$c_t = \left[\int_0^1 (A_t(i)\ C_t(i))^{(\eta-1)/\eta}\, di \right]^{\eta/(\eta-1)} \quad \text{with} \quad \eta > 1 \ ,$$

where $C_t(i)$ denotes the consumption of variety i at time t, and all the varieties enter symmetrically. There is a preference shock $A_t(i)$ associated to good i at time t, which acts as a multiplicative shifter of the demand of good i. We assume that $A_t(i) = 1/z_t(i)$, so the (log) of the marginal cost and the demand shock are perfectly correlated. We introduce this assumption for several reasons, mostly related to easier exposition. In this setup where demand and productivity shocks are perfectly negatively correlated, the relative demand of different products is stationary, and the frictionless profits of the different firms are constant across different productivity levels. This property is appealing for two reasons. First, we will show that the assumption that observation and menu costs are a fraction of frictionless profit is then equivalent to the assumption of constant observation and menu costs. Second, the cross-section distribution of outputs is stationary even in absence of product replacement, i.e. $\lambda = 0$. Finally, this version of the model with correlated demand and cost shocks has been analyzed in the literature by several authors (see Woodford (2009), Bonomo, Carvalho, and Garcia (2010) Midrigan (2011), Alvarez and Lippi (2012)), so it makes the results for our benchmark case comparable to the existing literature.[7]

Household's preferences over time are given by

$$\int_0^\infty e^{-\rho t}\left[\frac{c_t^{1-\epsilon}}{1-\epsilon} - \xi\; L_t + log\left(\frac{\hat{m}_t}{P_t}\right)\right] dt \quad \text{with} \quad \rho > 0\;, \tag{3}$$

where period t utility depends on consumption, c_t, on labor supply, L_t, and on cash holdings $\hat{m}_t$ deflated by the price index P_t:

$$P_t = \left[\int_0^1 \left(A_t(i)^{-1}\; p(i)\right)^{(1-\eta)} di\right]^{1/(1-\eta)} . \tag{4}$$

The household has perfect foresight on the path of money, nominal wages, nominal interest rates, nominal lump-sum subsidies and aggregate nominal profits. Financial markets are complete, in the sense that all profits of firms are held in a diversified mutual fund. Since all aggregate quantities are deterministic, the budget constraint of the representative agent is a simple present value constraint:

$$\hat{m}_0 \;\; \geq \;\; \int_0^\infty Q_t \left[\int_0^1 p_t(i)\, C_t(i)\, di + R_t \hat{m}_t - \mu m_t - W_t L_t - D_t\right] dt, \tag{5}$$
$$\text{where } Q_t = \exp\left(-\int_0^t R(s) ds\right)\;,$$

and where R_t is the instantaneous risk-free net nominal interest rate −and hence the opportunity cost of holding money−, Q_t is the implied time zero price of a dollar delivered at time t, W_t is time t nominal wage, and D_t is aggregate nominal net profits rebated from all firms to households at time t. The household chooses the buying strategy, $C_t(\cdot)$, labor supply, L_t, and money-holding, $\hat{m}_t$, so to maximize equation (3), subject to equation (5), and taking prices $Q_t,\; P_t,\; R_t,\; W_t$, and initial money holdings, $\hat{m}_0$, as given.

After using the equilibrium condition in the money market, $\hat{m}_t = m_t$, the first order

[7] Nevertheless in the Online Appendix F we solve the model without preference shocks, i.e. $A_t(i) = 1$ for all i and t, and conclude that the assumption on preference shocks is irrelevant for the quantitative predictions of our benchmark economy.

condition for money holdings reads

$$e^{-\rho t}/m_t = \zeta\ Q_t R_t, \tag{6}$$

where ζ is the Lagrange multiplier of equation (5). After setting $A_t(i) = 1/z_t(i)$ the first order conditions for consumption and labor supply are given by

$$e^{-\rho t}\, c_t^{1/\eta-\epsilon}\, C_t(i)^{-1/\eta}\ z_t(i)^{-1+1/\eta} = \zeta\ Q_t\ p_t(i), \tag{7}$$

$$e^{-\rho t}\xi = \zeta\ Q_t\ W_t. \tag{8}$$

Taking logs and differentiating w.r.t. time equation (6) one obtains the following o.d.e.:

$$\dot{R}(t) = R(t)\,(R(t) - \mu - \rho) \tag{9}$$

which has two steady states, zero and $\rho + \mu > 0$. The steady state $\rho + \mu$ is unstable: if $0 < R(0) < \rho + \mu$ then it converges to zero, and if $R(0) > \rho + \mu$ it diverges to $+\infty$. Thus, there exists an equilibrium where, regardless of the initial condition for individual prices we have:

$$R_t = R = \rho + \mu \quad , \quad W_t = \xi\ R\ m_t\ , \quad \text{and} \quad \zeta = \frac{1}{m_0 R}\ . \tag{10}$$

It is clear from the analysis that different goods i that at time t have the same price $p = p_t(i)$ and the same productivity $z = z_t(i)$ enter in a symmetric way in the utility and first order conditions. Thus, from now on we index goods by the pair (p, z), rather than the index i. For instance, we write $C_t(p, z)$ for the demand of the corresponding consumption variety that is produced with productivity z and sold at price p.

2.2 The Individual Firm Problem

In this section we set up and analyze the price setting problem of a firm until its product is replaced. We first set up the notation for the period profits, then we describe the information structure and the adjustment technology, and finally we set up the dynamic programming problem of the firm.

The firm's nominal per period profits, scaled by the economy wide money supply, are given by

$$C_t(p,z)\left(\frac{p}{m_t}-\frac{W_t/z}{m_t}\right)\ . \tag{11}$$

After substituting the first order conditions to the household problem in equations (7)-(10) into equation (11), the static profits maximizing price is given by

$$p_t^* = \frac{\eta}{\eta-1}\frac{m_t\ R\ \xi}{z}\ , \tag{12}$$

and, abusing notation, the per period profits evaluated at a generic price level, p, and at p^*, scaled by the money supply, can be expressed respectively as

$$c_t^{1-\epsilon\eta} R\ z^{1-\eta}\left(\frac{p}{Rm_t}\right)^{-\eta}\left(\frac{p}{Rm_t}-\frac{\xi}{z}\right) = c_t^{1-\epsilon\eta}\ F(p/p_t^*) \equiv \Pi(\,p/p_t^*\,,\,c_t\,)\ , \tag{13}$$

$$c_t^{1-\epsilon\eta} R\ \ z^{1-\eta}\left(\frac{\eta}{\eta-1}\frac{\xi}{z}\right)^{-\eta}\left(\frac{\eta}{\eta-1}\frac{\xi}{z}-\frac{\xi}{z}\right) = c_t^{1-\epsilon\eta}\ F(1) \equiv \Pi^*(c_t)\ , \tag{14}$$

where $F(\cdot)$ is given by

$$F(e^{\,g}) \equiv R\ \xi^{1-\eta}\left(e^{\,g}\ \frac{\eta}{\eta-1}\right)^{-\eta}\left(e^{\,g}\ \frac{\eta}{\eta-1}-1\right)\ ,\ \text{and where}\ \ g\equiv\log p-\log p^*\ . \tag{15}$$

We denote $\bar{\Pi}$ be the nominal profits relative to money supply evaluated at the steady state consumption $\bar{c}$, i.e.

$$\bar{\Pi} \equiv \Pi^*(\bar{c}) = \bar{c}^{1-\epsilon\eta}\,F(1)\,.$$

We refer to log-difference g of the nominal price relative to the current value of the optimal static price for the monopolist as to the "price gap". A value of 1 indicates that the current sale price is the optimal (static) one. It follows from the definition of g, and from the laws of motion of W and z, that the dynamics of g are given by

$$dg(t) = (\gamma - \mu)\ dt + \sigma dB(t)\ . \tag{16}$$

Next we describe the information structure. In the spirit of the *inattentiveness* literature that followed Caballero (1989) and Reis (2006), we assume that paying attention to economic variables that are relevant for the price setting decision is costly. We model this framework by assuming that the firms do not observe their productivity z_t, or other variables informative about the the firms' relevant state, unless they decide to undertake a costly action, which we refer to as an observation. As standard in sticky price models, we assume that between observation dates firms post a nominal price and produce the amount that is demanded. We furthermore assume that the decision maker in the firm does not observe profits, quantities, or labor between observation dates. An observation requires a fixed amount of labor. We express the value of the observation cost as a fraction of steady state flexible price profits: $\theta\ \bar{\Pi}$, where $\theta > 0$ is a parameter. After paying the observation cost the firm learns perfectly the current value of z.[8] Firms have no information on the realizations of idiosyncratic productivity shocks until the next observation date. We assume that a firm that has just paid the observation cost can adjust its price subject to a menu cost. Each price change requires a fixed amount of labor. We express the value of this cost as a fraction of steady state frictionless profits: $\psi\ \bar{\Pi}$, where $\psi > 0$ is a parameter. As in the case of the observation cost, the adjustment cost is a constant across firms and over time.

We start the economy in a steady state corresponding to the old level of m_t and the aggregate shock is "unexpected", i.e. firms assigned probability zero prior to this event.[9] The monetary shock at $t = 0$ implies a once and for all permanent increase in nominal

[8] Given the process of z, information about past values of z is redundant.

[9] As standard, this is interpreted as an approximation to the case where the "shock" occurs with very low probability.

marginal cost. We consider two polar cases with respect to the information available to firms about the aggregate state, m_t, with the intention to imitate results in the literature, and compare the two extreme cases.[10] In one case we assume that firms only learn about the (unforeseen) aggregate shock to m_t after their first observation of the state, and after that date they have perfect foresight on the aggregate quantities, in particular on nominal wages, aggregate nominal prices, and aggregate real consumption. This is equivalent to assume that firms don't learn about the change in nominal wages, real aggregate consumption, the aggregate price level, or their own profits unless they paid the observation cost. In the second case we assume that firms learn about the exact realization of the aggregate shock to m_t immediately at the time of the monetary shock and that have perfect foresight as households with respect to aggregate variables.

In Alvarez, Lippi, and Paciello (2011) we show that, in absence of aggregate shocks, firms only adjust their price upon observation of the state, if the menu cost ψ is large enough. This is the case for the parameter values considered in this paper: *price plans* or *indexation* of the type proposed by Mankiw and Reis (2002) would not be optimal in the economy we study in this paper.[11] Moreover, this prediction is consistent with available empirical evidence on the average frequency of price reviews and adjustments.[12] Thus, we set up the firms' problem so that in absence of aggregate shocks, i.e. for all $t \neq 0$, no price adjustment occurs between observation dates.

Upon the realization of the aggregate shock at $t = 0$, it may instead be optimal for some firms to pay the menu cost and adjust prices to the aggregate shock without necessarily observing the idiosyncratic state in the same period. Given the form of the shock and by the

[10] See Mankiw and Reis (2002) and Mankiw and Reis (2006).

[11] One of the economies we study will feature strictly positive observation cost, $\theta > 0$, but zero adjustment cost, $\psi = 0$. If menu cost is zero then price plans, i.e. adjustments between observations, will be optimal. Yet, when we report the solution for the case with zero adjustment cost we do not allow for price plans. We do so since we include this case only to compare it with the one of non-negligible menu cost, which is our benchmark.

[12] In the Online Appendix A we emphasize that firms reviewing prices more frequently than adjusting them is a robust pattern of the data. If price plans were instead used pervasively, then there would be more adjustments than reviews.

same arguments discussed above, at most one price adjustment will be optimal between the aggregate shock and the next observation date, and such adjustment will take place at $t = 0$. Moreover, after the first observation of the state, the firms' problem resembles the problem they face in steady state. Thus we setup the firm's problem so that we can distinguish two possibilities at $t = 0$: (i) it pays the menu cost, adjusts the price without observing the state, and decides to observe the state in τ_0 periods; (ii) it does not adjust the price without observing the state, and decides to observe the state in $\tau_0 \geq 0$ periods.

2.3 Characterization of the Individual Firm Problem

We are now ready to describe the firms' problem in detail. Because of the "unexpected" monetary shock, the firms' problem is qualitatively different at times t after the first observation, from the firms' problem at time $t = 0$. We start, as in backward induction, by considering the problem after the first observation. In this case the firm already knows that a monetary shock has occurred, that nominal wages had jump up, and that aggregate consumption, and hence real interest rates and its own demand, will change for all future periods. Then, with this problem characterized, we move to the description of the firm's problem at time $t = 0$.

The firm's problem after the first observation

We first describe the problem for a firm upon any observation taking place at time $t \geq 0$. Given that no new information arrives between observation dates, we can fully characterize the solution to the firms' problem at any $t \geq 0$ by studying the optimal policy upon observation dates τ_i. The stopping times $\{\tau_i\}$ denote the dates where the subsequent observations will take place, conditional on the product not being replaced. Here the subindex i denotes the i^{th} observation date. These stopping times satisfy $\tau_0 \leq \tau_1 \leq \tau_2 \leq \cdots$, where $\tau_0 \geq 0$ is the first observation date after the aggregate shock.

The timing of the firm's decision problem is as follows. At an observation time τ_i, the firm learns z_{τ_i}, so that the information available at time τ_i is given by $\{z_{\tau_0}, z_{\tau_1}, \cdots, z_{\tau_i}\}$.

Given the structure of our productivity shock, this is summarized by z_{τ_i}. Until the next observation date, no new information arrives. Thus, the next time to gather information, τ_{i+1}, is decided with the information obtained at the previous observation date τ_i, namely z_{τ_i}. Thus, without loss of generality, τ_{i+1} is decided at $t = \tau_i$. Immediately after an observation at time τ_i, the firm decides whether to pay the menu cost and change its nominal price, using all the information available at that time. Thus a price adjustment would only take place at observation date, τ_i. No action takes place between τ_i and τ_{i+1}.

The state of the firm's problem upon an observation is a tridimensional object, as it includes the price p at which the firm is selling its product, as well as the idiosyncratic productivity level z and the aggregate state. It turns out that we can substantially reduce the curse of dimensionality of the firm's problem and express it in a recursive formulation where the relevant state is unidimensional and expressed in terms of the price gap $g = p/p^*$ defined in equation (15). Two assumptions yield this result: i) observation and adjustment costs proportional to frictionless profits Π_t^*, and ii) permanent shocks to productivity z_t. This result is stated in Proposition 1.

PROPOSITION 1. Consider the firm's problem evaluated at any observation date $t = \tau_i \geq 0$. The firm maximizes the single-state value function:

$$
\begin{aligned}
v_t(g) \quad &= \quad \max\left\{\hat{v}_t \;,\; \bar{v}_t(g)\right\} \quad \text{where} \qquad\qquad (17)\\
\hat{v}_t \quad &= \quad -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{C_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta} f(\hat{g},s)\, ds \;+ \\
&+ \quad e^{-(\rho+\lambda)T} \int_{-\infty}^{\infty} v_{t+T}\left(e^{\;\hat{g}+(\gamma-\mu)T+\sigma\sqrt{T}\,x}\right)\, dN(x)\;, \\
\bar{v}_t(g) \quad &= \quad -\theta + \max_{T} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{C_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta} f(g,s)\, ds \;+ \\
&+ \quad e^{-(\rho+\lambda)T} \int_{-\infty}^{\infty} v_{t+T}\left(e^{\;g+(\gamma-\mu)T+\sigma\sqrt{T}\,x}\right)\, dN(x)\;, \\
&\text{where} \quad f(g',s) \equiv \eta\, e^{\left((1-\eta)g' + (\eta-1)(\mu-\gamma) + \frac{\sigma^2}{2}(\eta-1)^2\right)s} - (\eta-1)\, e^{\left(-\eta g' + \eta(\mu-\gamma) + \frac{\sigma^2}{2}\eta^2\right)s}\;,
\end{aligned}
$$

and where $N(\cdot)$ is the CDF of a standard normal.

See the Online Appendix B for details. The decision of when to review next, and whether to adjust the current price, depends only on the price gap upon observation, g. The function $f(g,s) = \mathbb{E}_t[\Pi(g_{t+s}, \bar{c})/\bar{\Pi}]$ gives the expected real static profits evaluated s periods after re-setting the price to a gap g at time t, and scaled by the frictionless steady state profits. The first term of $f(g,s)$ depends on the expected growth in real revenues between t and $t+s$. The second term of $f(g,s)$ depends on the expected growth in real marginal cost. The discount rate in equation (17) is $\rho + \lambda$, reflecting the steady state real rate as well as the probability that the good disappears. The term involving $\{c_{t+s}/\bar{c}\}$ reflects the impact of aggregate consumption on discounted profits both through the impact on the real interest rate at which future profits are discounted, as well as through the impact on demand and therefore profits. The value function v_t that solves equation (17) depends on time *only* because of the effect of the path of $\{c_{t+s}/\bar{c}\}$. In a steady state, i.e. when $c_t = \bar{c}$ the solution of the stationary value function characterizes completely the dynamics of price changes across firms. We study a version of the steady-state problem of equation (17) in Alvarez, Lippi, and Paciello (2011). In that paper the period profit was assumed to be a quadratic function of g, which can be derived as a second order approximation to $\Pi(\cdot, \bar{c})$ of equation (13).

The optimal decision rules for each time t are described by three values for the price gap upon observation, $\underline{g}_t < \hat{g}_t < \bar{g}_t$, and a function $\mathrm{T}_t(\cdot)$. After observing its price gap g at t the firm leaves its price unchanged if $g \in (\underline{g}_t\,, \bar{g}_t)$, it otherwise changes its price gap to $\hat{g}_t$. The function $\mathrm{T}_t(g)$ gives the time elapsed until the next observation as a function of the price gap after the adjustment decision. This form of the decision rules assumes that $v_t(\cdot)$ is single peaked. In Alvarez, Lippi, and Paciello (2011) we show that this is the case, and we provide an analytical characterization for the decision rules. In Section 2.4 we argue that such analytical characterization is a good approximation of the optimal decision rules in the model of this paper.

The firm's problem at $t = 0$

Next we describe the firms' problem at the time of the aggregate shock $t = 0$. The time of the aggregate shock, $t = 0$, is a somewhat special date because due to the aggregate shock the cost, and hence the price gap, changes discretely but the exact values of the price gap is not known, given that most firms are between observation dates. Thus, we need to characterize the firm' problem as a function of beliefs about its price gap. We use that the belief about g is normal with parameters $(\tilde{g}, \tilde{\sigma}^2)$, while beliefs about aggregate output are denoted by $\{\tilde{c}_t\}_{t \geq 0}$.

Definition 1. A firm that at time $t = 0$ has beliefs that its own price gap g is normally distributed with parameters $(\tilde{g}, \tilde{\sigma}^2)$, and beliefs on the path of aggregate consumption given by $\{\tilde{c}_t\}_{t \geq 0}$, solves:

$$\begin{aligned} \max \quad & \left\{ \max_{\tau_0 \geq 0} \int_0^{\tau_0} e^{-(\rho+\lambda)s} \left(\frac{\tilde{c}_s}{\bar{c}} \right)^{1-\epsilon\eta} f_0\left(\tilde{g}, s, \tilde{\sigma}^2\right) ds + \right. \\ & \quad + e^{-(\rho+\lambda)\tau_0} \int_{-\infty}^{\infty} v_{\tau_0} \left(e^{\tilde{g}+(\gamma-\mu)\tau_0 + \sigma\sqrt{\tau_0 + \frac{\tilde{\sigma}^2}{\sigma^2}}\, x} \right) dN(x) \,, \\ & -\psi + \max_{\hat{g}_0, \tau_0 \geq 0} \int_0^{\tau_0} e^{-(\rho+\lambda)s} \left(\frac{\tilde{c}_s}{\bar{c}} \right)^{1-\epsilon\eta} f_0\left(\hat{g}_0, s, \tilde{\sigma}^2\right) ds \; + \\ & \quad \left. + e^{-(\rho+\lambda)\tau_0} \int_{-\infty}^{\infty} v_{\tau_0} \left(e^{\hat{g}_0+(\gamma-\mu)\tau_0 + \sigma\sqrt{\tau_0 + \frac{\tilde{\sigma}^2}{\sigma^2}}\, x} \right) dN(x) \right\} \\ & \text{where } f_0(\tilde{g}, s, \tilde{\sigma}^2) \equiv \int_{-\infty}^{\infty} \frac{F\left(e^{\tilde{g}+(\gamma-\mu)s + \sigma\sqrt{s + \frac{\tilde{\sigma}^2}{\sigma^2}}\, x} \right)}{F(1)} dN(x). \end{aligned} \tag{18}$$

The first term in equation (18) refers to the case where only the first observation date is chosen, while the second term refers to the case where both the first observation date is chosen and the price is adjusted. Notice that the problem in equation (18) differs from the problem in equation (17) because $\tilde{\sigma}^2 \neq 0$. This has two consequences. First, the function $f_0(\cdot)$ replaces the function $f(\cdot)$. Second, log productivity will accumulate normal innovations with variance σ^2 per unit of time, starting from a variance equal to $\tilde{\sigma}^2$.[13] After the first observation

[13]For instance, the term $\sigma\sqrt{\tau_0 + \tilde{\sigma}^2/\sigma^2}$ is the standard deviation of the distribution of the log of the price gap after T periods, which includes both the cumulative effect of the innovations, as well as the uncertain

date occurring τ_0 periods from $t = 0$, the firms' problem is described by Proposition 1 so the continuation value is given by the function $v_{\tau_0}(\cdot)$.

In equation (18) the firm has two possibilities at $t = 0$: (i) it pays the menu cost ψ, adjusts the price without observing the state, and decides to observe the state in τ_0 periods; (ii) it does not adjust the price without observing the state, and decides to observe the state in τ_0 periods. Notice that when the firm chooses $\tau_0 = 0$, it is choosing to observe the state immediately at $t = 0$, which means that at $t = 0$ the firm solves the problem described in Proposition 1, and can eventually adjust the price after observing the state.

The firm' s optimal decision rule is described by two functions determining the inaction region as a function of the beliefs, defined by $\underline{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2) < \bar{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2)$, a function determining the optimal return point for the belief of the price gap when adjusting prices without observing the state, defined by $\hat{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2)$, and a function for the time to the first observation depending on the beliefs about the price gap, defined by $\mathrm{T}_0(\tilde{g}, \tilde{\sigma}^2)$. At $t = 0$, a firm that has beliefs about its price gap equal to $(\tilde{g},\, \tilde{\sigma}^2)$ leaves its price unchanged, if $\tilde{g} \in (\underline{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2)\,,\, \bar{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2))$, and otherwise changes its price so that the beliefs about its price gap are given by $(\hat{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2), \tilde{\sigma}^2)$.

As anticipated above, we consider two extreme versions of the beliefs of firms at time $t = 0$ with respect to the aggregate "shock", i.e. the beliefs about initial value of the economy wide money supply m_0 and the path of aggregate consumption $\{c_t\}_{t \geq 0}$. In the first case we assume that firms do not learn that there is a new value for the money supply or aggregate output until they make their first observation at same date $\tau_0 \geq 0$: $\tilde{c}_t = \bar{c}$ for all $t < \tau_0$, and $\tilde{c}_t = c_t$ for all $t \geq \tau_0$. Moreover, given that no new information arrives at the time of the aggregate shocks, beliefs about firms' own price gap $(\tilde{g}, \tilde{\sigma}^2)$ are unaffected. Thus, in this case, the firms use the steady state decision rules, which they incorrectly believe to describe the optimal action until τ_0. This first case represents an environment where firms are mostly concerned with idiosyncratic shocks, so they only learn about aggregates as a consequence of observing their own cost and profits. This case corresponds closely to the assumptions in

initial level of the price gap.

the inattentiveness modeling by Mankiw and Reis (2006).

In the second case we assume that all firms learn at time $t = 0$ the new level of the money supply and the correct path of aggregate output: $\tilde{c}_t = c_t$ for all $t \geq 0$. In such a case, firms' beliefs about their own price gap are affected by an equal shift of size δ to the expected price gap of all firms, i.e. $\tilde{g}_0 = \tilde{g} - \delta$, where $\tilde{g}$ denotes the expected logarithm of the price gap at $t = 0$ just before the monetary shock hits. The uncertainty $\tilde{\sigma}^2$ is unaffected by the monetary shock. This case represents an environment where firms continually pay attention to information about aggregates and where this information is readily and freely available.

2.4 Equilibrium

In this section we define an equilibrium, and specify the mapping from the set of paths of aggregate consumption into itself, so that the fixed point characterizes an equilibrium.

Proposition 1 and Definition 1 describe a maximization problem, given a path for aggregate consumption $\{c_t\}_{t\geq 0}$, the solution of which is described by the policies $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$ and $\{\underline{\mathbf{g}}_{\mathbf{0}}(\cdot), \hat{\mathbf{g}}_{\mathbf{0}}(\cdot), \bar{\mathbf{g}}_{\mathbf{0}}(\cdot), \mathrm{T}_0(\cdot,\cdot)\}$ respectively. Now we describe a consistency condition implied by equilibrium, i.e. a mapping from policies to a path of aggregate consumption $\{c_t\}_{t\geq 0}$. From Proposition 1 and Definition 1, the state of firms' price setting decision at any point in time t is given by the pair $(\tilde{g}, \tilde{\sigma}^2)$. We let $\phi_t(g, \tilde{g}, \tilde{\sigma}^2)$ be the distribution of the actual price gaps, g, and of firms' beliefs about the state, $(\tilde{g}, \tilde{\sigma}^2)$, at any point in time. Given an initial distribution ϕ_0 and policies $\{\underline{\mathbf{g}}_{\mathbf{0}}(\cdot), \hat{\mathbf{g}}_{\mathbf{0}}(\cdot), \bar{\mathbf{g}}_{\mathbf{0}}(\cdot), \mathrm{T}_0(\cdot,\cdot)\}$ and $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$ we will get a path for the distributions $\{\phi_t\}_{t>0}$. These distributions determine the path of consumption which satisfies

$$c_t = \left(\int_{g,\tilde{g},\tilde{\sigma}^2} \left(\xi \, \frac{\eta}{\eta - 1} \, e^{\,g} \right)^{1-\eta} \phi_t(dg, d\tilde{g}, d\tilde{\sigma}^2) \right)^{\frac{1}{\epsilon\,(\eta-1)}} , \tag{19}$$

as a consequence of the first order conditions in equations (6)-(7), of the definition of g_t in equation (15), and of the definition of c_t. An equilibrium consists of a fixed point in the sequence $\{c_t\}_{t\geq 0}$. In fact, in order to obtain the equilibrium path of $\{c_t\}_{t\geq 0}$, one need to have the path for g_t. The the path of g_t is characterized by the law of motion of the triplet $(g_t, \tilde{g}_t, \tilde{\sigma}_t^2)$, given policies $\{\underline{\mathbf{g}}_\mathbf{0}(\cdot), \hat{\mathbf{g}}_\mathbf{0}(\cdot), \bar{\mathbf{g}}_\mathbf{0}(\cdot), \mathrm{T}_0(\cdot,\cdot)\}$ and $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$ which in turn depend on the path of $\{c_t\}_{t\geq 0}$.[14]

In the special case where $\eta\epsilon = 1$ the equilibrium fixed point simplifies vastly.[15] In particular, the problem for the firm described in Proposition 1 and Definition 1 is independent of the path for aggregate consumption and associated beliefs. Hence in this case the aggregate shock affects the path of aggregate variables only through the aggregation of time invariant firms' decision rules. In the more general case where $\eta\epsilon \neq 1$, the general equilibrium feedback effect from aggregate consumption changes firms's decision rules in response to aggregate shocks. However, while our definition of equilibrium and our numerical solution take this general equilibrium feedback into consideration (i.e. the fixed point on paths for $\{c_t\}_{t\geq 0}$), its effects on the decision rules are very small for realistic monetary shocks. The result that these type of effects are negligible for small monetary shocks is formally established in closely related set-ups in Gertler and Leahy (2008) and Alvarez and Lippi (2012). In the Online Appendix E we present a proposition which establishes this result for the models at hand, and we document the small size of these effects for the calibrated version of our model.

3 Impulse responses to nominal shocks

In this section we solve the model numerically, and compute aggregate output and price responses to a one time, unforeseen, permanent increase in money supply, m_t, of size δ. We solve numerically for the equilibrium by discretizing the model to a one week period (see the Online Appendix D for a detailed description of our algorithm). In Section 3.1 we describe

[14] See the Online Appendix C for more details.

[15] The assumption that $\eta\epsilon = 1$ means that the substitution elasticity of a firm product is the same across varieties than it is across time, so that the consumption of each variety enters in the utility in an additively separable way. As can be seen from the consumers' first order conditions (equation (7)) the additive separability implies that the demand faced by each firm is independent of the aggregate consumption at that date. Instead, when varieties are more substitutable across them than aggregate output across time, i.e. when $\eta > 1/\epsilon$, an increase of the aggregate consumption c_t reduces the household demand for good $C_t(p, z)$, so that the demand for the firms producing this good shifts down.

the parametrization of the model. In Section 3.2 we show results for the case where firms are unaware of the monetary shock until they make a costly observation of the state. We compare our results for the calibrated model with both adjustment and observation costs to impulse responses in the three polar cases where firms face either adjustment cost only, or observation cost only, or substitutions only. The comparison of impulse responses predicted by these three extreme cases is particularly interesting to illustrate the different implications of the different types of frictions for aggregate responses. In Section 3.3 we show results for the case in which all firms are instead perfectly informed about the realization and size of the unexpected monetary shock.

3.1 Choosing the parameters of the model

We calibrate the parameters of our model to match salient statistics from the U.S. economy. We set $\eta = 4$ so that the average price markup is roughly one third, i.e. between the values used by Midrigan (2011) and Golosov and Lucas (2007). Following Golosov and Lucas (2007), we set $\epsilon = 2$ so to have an intertemporal elasticity of substitution of $1/2$, and $\xi = 6$ so that households allocate approximately $1/3$ of the unit time endowment to work in steady state. We set the yearly discount rate $\rho = 0.02$. We choose the growth rate of money supply, μ, and the drift in the cost process, γ, so that the second order approximation of the firm problem is similar to the problem studied in the closely related paper Alvarez, Lippi, and Paciello (2011). Appendix E shows that this implies $\mu = 0.00$ and $\gamma = (2\eta - 1)\sigma^2/2$. The values of μ and γ are irrelevant for the quantitative predictions of this paper, for an empirically reasonable range of values. In order to focus on the propagation of monetary shocks originated from adjustment and observation costs we set $\lambda = 0$, so that no substitution takes place in the baseline calibration of our model.

The remaining parameters, θ, ψ and σ are key to determine firms's pricing decisions and, therefore, to determine the aggregate effects of nominal shocks. In Alvarez, Lippi, and Paciello (2011) we show that these parameters can be uniquely identified through the model implied statistics about (i) the average number of price adjustments per year (denoted by n_a), (ii) the ratio of the average number of price reviews to the average number of adjustments per year (denoted by n_r/n_a), and (iii) the average size of absolute price changes conditional on a price change (denoted by $s_p \equiv \mathbb{E}\left[||\log(p_{\tau_i}/p_{\tau_{i-1}})|| \,\middle|\, p_{\tau_i} \neq p_{\tau_{i-1}}\right]$). In particular, the ratio n_r/n_a identifies the relative size of the two frictions, i.e. ψ/θ. For given estimate of ψ/θ, the observables n_a and s_p identify the *levels* of θ and ψ, as well as the volatility of the state, σ.

We use estimates of n_a and s_p obtained by Nakamura and Steinsson (2008) on U.S. CPI data from 1998 to 2005. In particular, we target the average number of price adjustments per year in our model to be $n_a = 1.3$, and the average size of price changes conditional on adjustment to be $s_p = 0.085$.[16] We match the ratio of the average number of price reviews to the average number of price adjustments estimated by Blinder et al. (1998) on U.S. data, which is equal to $n_r/n_a = 1.4$. In Appendix A we discuss further available evidence on the ratio n_r/n_a across different countries and sectors. In Section 4 we evaluate the sensitivity of our results to matching different values of n_r/n_a.

Table 1: Parameters vary as a function of the ratio of the average frequencies of reviews to adjustments

Ratio of the average frequency of reviews to adjustments, n_r/n_a	ψ	θ	σ	λ
$n_r/n_a = 1.0$ (*Calvo*)	∞	∞	0.095	1.30
$n_r/n_a = 1.0$ (*Caballero-Reis-Bonomo-Carvalho*)	0.00%	2.60%	0.125	0.00
$n_r/n_a = 1.4$ (Baseline with Both Costs)	0.29%	0.77%	0.105	0.00
$n_r/n_a = \infty$ (*Golosov-Lucas*)	0.50%	0.00%	0.097	0.00

The statistics about the average frequencies of reviews and adjustments are obtained from simulating the model in steady state. In all parameterizations the average frequency of price adjustment is equal to 1.3 adjustments per year, while the average size of price changes is equal to 0.085.

The third row of Table 1 reports the values of ψ, θ and σ corresponding to $n_a = 1.3$, $s_p = 0.085$ and $n_r/n_a = 1.4$. The estimated value $\theta = 0.77\%$ implies that each observation

[16] Nakamura and Steinsson (2008) report the median frequency of price changes across different products to be 8.7% per month, when excluding sales but including substitutions, where the frequency of price changes for each product is computed as an average over the period 1998-2005. Similarly, the average size of price changes refer to the median product for the same period.

costs to 0.19% of steady state revenues, which is close to estimates by Zbaracki et al. (2004) who find that managerial cost of price changes amount to 0.28% of yearly revenues. The estimated value $\psi = 0.29\%$ implies that each price adjustment costs 0.07% of yearly revenues, which is in between the menu costs estimated directly by Zbaracki et al. (2004) and Levy et al. (1997), who find that menu costs amount to 0.04% and 0.70% of revenues respectively.[17]

Moreover, as we showed in Alvarez, Lippi, and Paciello (2011), the ratio of the two costs, ψ/θ, can also be identified from moments of the distribution of price changes: for given average size of price changes, a measure of the variability in the size of price changes identifies the ratio of menu to observation costs. In fact, the ratio ψ/θ is over-identified. We can therefore use moments from the distribution of price changes to assess the reliability of our estimate of ψ/θ. For instance, Eichenbaum et al. (2012) find that the fraction of price changes smaller than 5% in absolute value is equal to 24.4% of all price changes.[18] Our model with parameters chosen to match $n_r/n_a = 1.4$ predicts the fraction of price changes smaller than 5% in absolute value to be equal to 25% of all price changes.

Next we discuss the parametrization of three alternative and popular models nested by our framework: the model with only menu cost, the model with only observation cost and the model with random adjustments and observations. When firms only face observations costs, i.e. $\theta > 0$, $\psi/\theta = 0$ and $\lambda = 0$, the price adjustment rule is a time-dependent one, so that price adjustments coincide with observations. We refer to this model as the *Caballero-Reis-Bonomo-Carvalho* model since Caballero (1989) first proposed and analyzed it, and Reis's (2006) sections 2-4 extended and worked it out in detail, while Bonomo and Carvalho (2004) studied a version where the observation cost is also an adjustment cost.[19] When instead firms only face menu costs and observe the state continuously, i.e. $\psi > 0$, $\psi/\theta = \infty$ and $\lambda = 0$, the posted price is adjusted whenever it is far enough from the optimal

[17] Revenues are measured in steady state and at the profit-maximizing price in absence of frictions in price setting, i.e. $\Pi^*(z)$, evaluated at $z = 1$. We have used the fact that revenues are equal to $\eta\, \Pi^*(z)$.

[18] See Table 1 in Eichenbaum et al. (2012).

[19] Note that this model differs from the version in Section 5-6 of Reis (2006) which assumes that successive reviews dates are exponential. See Alvarez, Lippi, and Paciello (2012) for an explanation of this difference.

price, so that the price adjustment rule is a state-dependent one. The firm's problem in this case has been analyzed in the seminal papers by Barro (1972) and Dixit (1991), and its aggregate consequences in Danziger (1999) and Golosov and Lucas (2007) among others, so sometimes we refer to this model simply as *Golosov-Lucas*. Finally, for comparison, we consider another class of models characterized by both infinite observation cost and menu cost, so that adjustments and observations only take place at exogenously exponentially distributed times, as in *Calvo*'s paper. In our set-up it corresponds to have all the price changes due to what we call substitutions, i.e. $\theta = \psi = \infty$ and $\lambda > 0$, so we also refer to this model as the one with "substitutions only".

As a discipline device in comparing the different economies, we choose θ and σ in the observation cost only model, and ψ and σ in the menu cost only model to match the same average frequency and size of price changes of our baseline parametrization, i.e. $n_a = 1.3$ and $s_p = 0.085$. Obviously, these two polar cases cannot match the ratio of the average frequency of price changes and reviews, as such ratio is determined by the nature of the model to be $n_r/n_a = 1$ in the observation cost only model and $n_r/n_a = \infty$ in the menu cost only model. Similarly, the ratio ψ/θ is not well defined in the Calvo model, and n_r/n_a can be taken to be one, so that we choose $\lambda = 1.3$ and σ so that the average frequency and size of price changes is the same of the other models. We think this is an interesting exercise because it allows us to compare predictions of models that are observationally equivalent in terms of two popular statistics summarizing pricing behavior, frequency and size of price changes, but that can yet deliver very different predictions about the effects of monetary shocks because of the different nature of the friction behind sticky prices.

3.2 Impulse responses for unobserved monetary shocks

We start with the economy in a steady state, which includes a distribution of firms' beliefs about their price gaps and the dates at which they will make the next observation. In this section we assume that the monetary shock is only learned by the firms upon the first time the firm makes an observation. This first observation dates are distributed across firms according to pre-shock steady state. This case is equivalent to the assumption typically made in the *inattentiveness* literature that followed Mankiw and Reis (2002) where agents are oblivious about the realization of the monetary shock until the first observation. We recall that in our model a once and for all increase in the money supply leaves the nominal interest rate unchanged, and increases nominal wages permanently on impact.

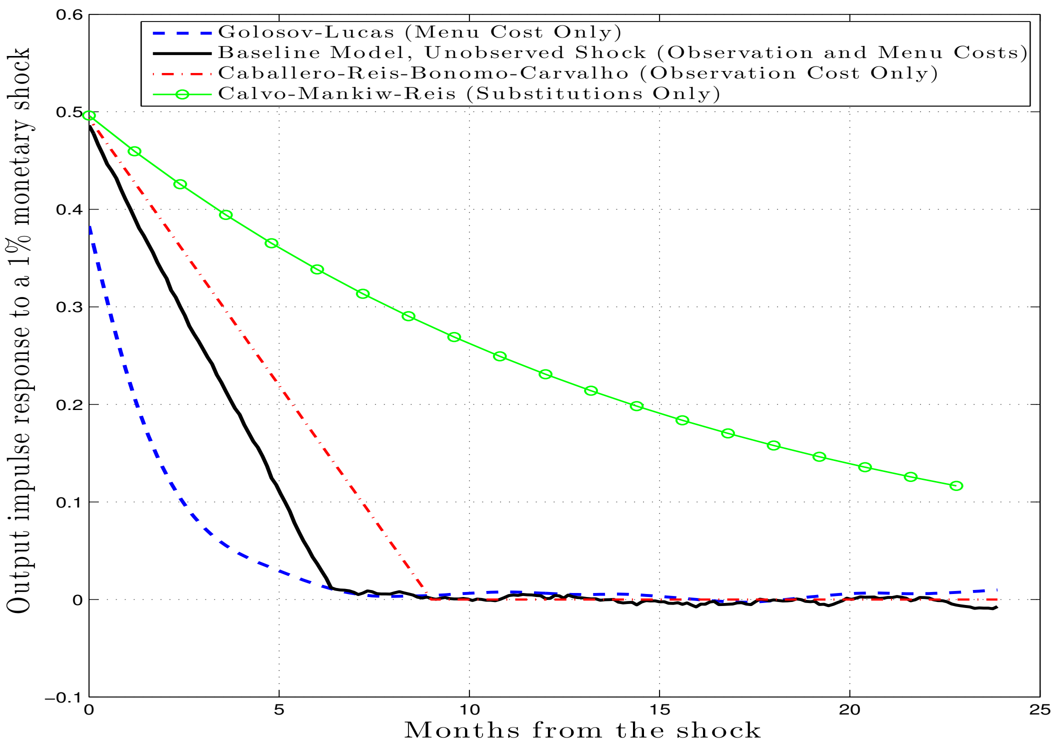
Figure 1 plots output impulse responses to a one time unexpected permanent increase in money supply equal to $\delta = 1\%$ for the four different economies described in Table 1. Figure 2 plots the cumulated output response as a function of the size of the monetary shock, δ, for the same four economies. The cumulated output response, $\mathcal{M}(\delta)$ is defined as

$$\mathcal{M}(\delta) = \int_0^\infty (\log(c_t(\delta)) - \log(\bar{c}))\ dt\ , \tag{20}$$

where $c_t(\delta)$ is the equilibrium level of aggregate output in period $t \geq 0$, after a monetary shock of size δ realized in period t_0, and $\bar{c}$ is the equilibrium level of aggregate output in the steady state.

Despite being characterized by the same frequency and size of price adjustments, these different models have substantially different predictions about the aggregate output response to the same monetary shock. First, compare the predictions of the two polar cases with observation cost only (dashed red line) and menu cost only (dashed blue line). The output response to the monetary shock is larger on impact in the model with observation cost only than in the model with adjustment cost only. The output response is linear in the time elapsed since the shock in the model with observation cost only, while it is exponentially decreasing in the time elapsed since the shock in the model with adjustment cost only. For all sizes of monetary shocks δ, the cumulated output response $\mathcal{M}(\delta)$ is larger in the model with observation cost only than in the model with menu cost only. In addition, $\mathcal{M}(\delta)$ increases linearly in the size of the monetary shock in the model with observation cost only, while it is characterized by an inverse-U shape in the model with menu cost only, with a peak around

Figure 1: Impulse response of $\log c_t$ to an unexpected 1% increase in m_t, $\delta = 1\%$



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.3$, and to the average size of price changes equal to $s_p = 8.5\%$.

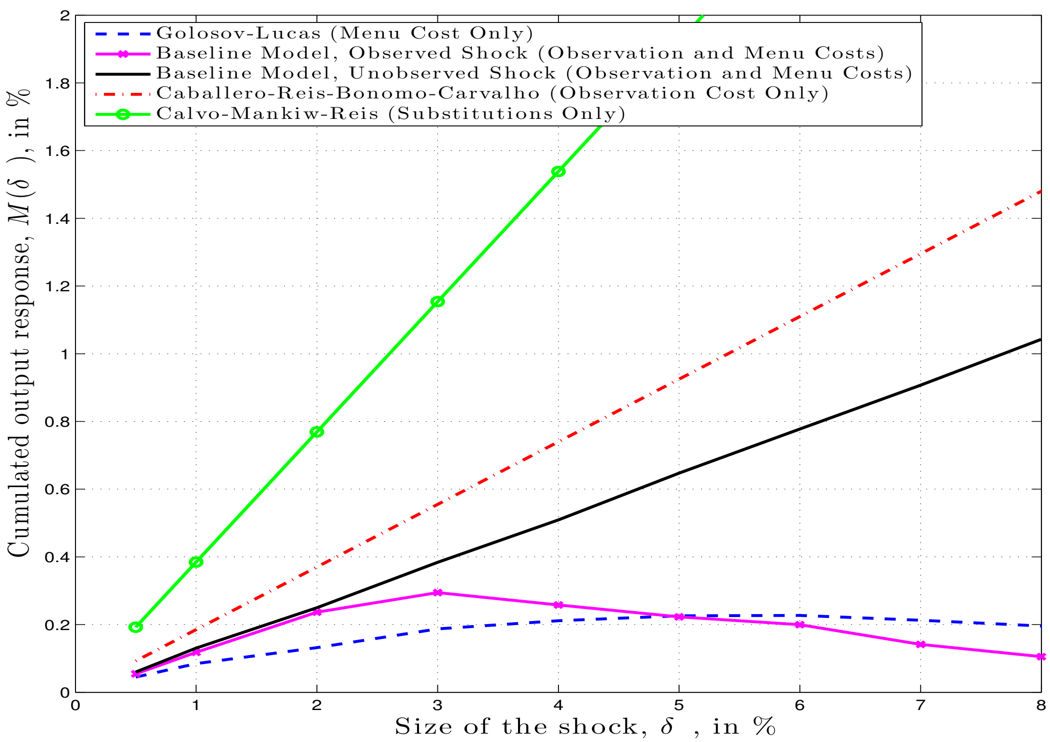
$\delta = 5\%$. As a result, with monetary shocks of size $\delta = 1\%$ and $\delta = 5\%$ the cumulated output response in the observation cost only model is about twice as large and four times as large as in the menu cost only model respectively.

The reasons for the different qualitative and quantitative predictions of these two different models are the following. First, recall that the different models match the same average frequency and size of price changes, so that the different predictions of such models arise from the different combinations of frictions in price setting achieving those same observables. In the model with adjustment cost only, the firms' price setting rule is state-dependent: firms adjust prices whenever the price gap g_t crosses the thresholds $\{\underline{g}_t, \bar{g}_t\}$. As emphasized by Golosov and Lucas (2007), a positive monetary shock at $t = 0$ reduces all price gaps on impact by the same amount, making firms closer to the threshold $\underline{g}_0$ to exit the inaction region and adjust their price. The larger the mass of firms that is moved outside of the inaction region at $t = 0$, the larger the increase of the aggregate price on impact, and the smaller the output response to the monetary shock. After the response on impact, the distribution of price gaps diffuses back to the invariant, implying the exponentially decaying response of output.In the model with observation cost only, approximately a constant fraction of firms makes their first observation in a given interval of time after the monetary shock. The model with observation cost only implies that firms plan to observe the state at equal and constant intervals of time. Given that firms only adjust prices once they observe the state, this implies that the price setting will follow a time-dependent rule.[20] This time-dependent pricing rule determines the linear-looking shape of the impulse response and of the cumulated output response. Moreover, given the absence of a menu cost, all firms adjust their price after the observation, so that no selection effect of the type present in menu cost models is present here.

Next, consider our baseline model with both observation and menu costs (solid black

[20]The invariant distribution of the times until the next observation for the cross section of firms is truncated exponential, with arrival rate equal to λ, and truncation given by the optimal time to the next observation. See Alvarez, Lippi, and Paciello (2012) for more details.

Figure 2: Cumulated output response as a function of the size of the shock δ



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.3$, and to the average size of price changes equal to $s_p = 8.5\%$.

line). This model predicts an impulse response of output that is, not surprisingly, in between the impulse responses obtained in the model with observation cost only and in the model with menu cost only. More surprisingly, the shape of the output response is roughly linear, as in the observation cost only model, for about seven months, which is the time it takes before all firms have observed the state of the economy, i.e. the maximum time between observations in steady state. After such period, the shape of the impulse response displays more non-linearity inherited from the state-dependent component of the firm adjustment policy, but such component of the impulse response accounts for a negligible fraction of the cumulated output response as output is close to its steady state level. Therefore, at our baseline parametrization, the time-dependent component of the adjustment rule dominates over the state-dependent component of the adjustment rule in determining the shape of the output response. As a result, also the cumulated output response in Figure 2 is roughly an increasing linear function of the size of the shock δ.

The predictions of the model with both observation and menu costs are determined by the interaction of the two types of frictions and corresponding observation and adjustment decisions. As we show in Alvarez, Lippi, and Paciello (2011), for given average frequency and size of price changes, the complementarity between the observation and adjustment decisions implies that the inaction region conditional on an observation is smaller than the inaction region in the model with menu cost only. Thus, the invariant distribution of price gaps conditional on observing the state displays a relatively large mass of firms close to the adjustment thresholds $\underline{g}_t$ and $\bar{g}_t$ so that a 1% monetary shock causes a relatively large fraction of observing firms to adjust the price. Thus, the shape of the impulse response is roughly linear as the model behave similarly to the observation cost only model: a constant fraction of firms observe the state every period, and a large fraction of these firms adjust the price after the monetary shock. While characterized by a similar shape, the model with both costs predicts smaller real effects of the observation cost only model. In fact, given not all observations lead to a price adjustment in steady state, the fraction of firms observing the state every period has to be larger than in the the observation cost only model, in order for the two models to match the same average frequency of price adjustments. On the other side the model with both costs predicts larger real effects than the model with menu cost only, despite the smaller inaction region, because not all firms make an observation in a given period of time.

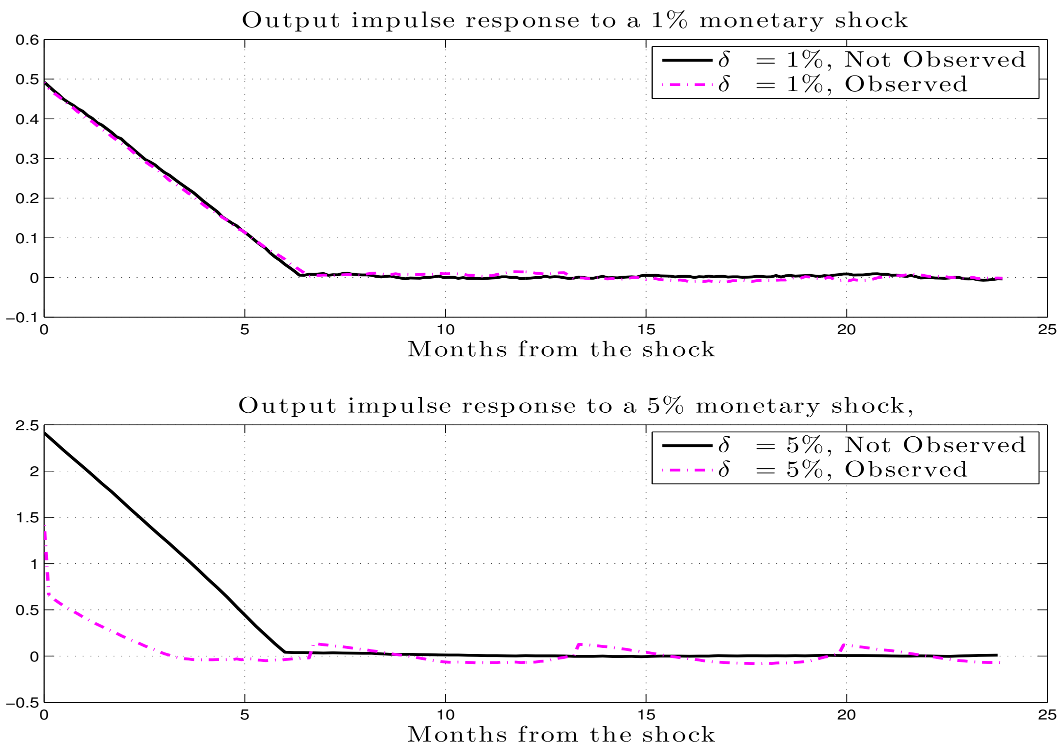
Finally, an interesting benchmark of comparison is given by the model with substitutions only (green line with circles), which is equivalent to a model with *Calvo* type exponentially distributed times between adjustments, or with *Mankiw-Reis* exponentially distributed times between observations. Such model predicts much more persistent real effects of the monetary shock than the other three models. In particular, as we show in more detail in Alvarez, Lippi, and Paciello (2012), the cumulated output response in the substitution only model is twice as large as in the model with observation cost only.

3.3 Impulse responses for observed monetary shocks

In this section we compute impulse responses in the model with both observation and menu costs under the assumption that the monetary shock is immediately observed upon impact. This is a conservative assumption as one may think of a more general framework where the cost of acquiring information about the aggregate state is lower than the idiosyncratic state but not necessarily zero. Mackowiak and Wiederholt (2009) propose a different mechanism to address what information firms decide to pay attention to, relying on firms facing a constraint in the amount of information they can process every period, and optimally allocating attention between idiosyncratic and monetary shocks. While that paper addresses the allocation of attention decision by firms, it cannot address the relationship between frequency of observations, adjustments and predictions about the real effects of monetary shocks, which is instead the focus of our paper.

In our economy, upon realization of the monetary shock at $t = 0$, all firms in the economy will revise their beliefs about the new level of nominal wages and about the path of real aggregate consumption. Figure 2 plots the cumulated output response as a function of the size of the monetary shock, δ, for the case of observed monetary shock (magenta solid line) against the other cases. Figure 3 plots the consumption impulse responses to monetary shocks of size $\delta = 1\%$ (top panel) and $\delta = 5\%$ (bottom panel), in the cases of such shocks being immediately observed by all firms (dashed line) or not being observed until the first observation date (solid line).

Figure 3: Impulse response of $\log c_t$ to an unexpected increase in m_t



Note: impulse responses of $\log c_t$ to a one-time permanent increase in m_t, in % deviation from steady state in our baseline model, and in the case where monetary shocks are perfectly observed upon impact. All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.3$, and to the average size of price changes equal to $s_p = 8.5\%$.

The output impulse response to a monetary shock of size $\delta = 1\%$ for the case of freely observed monetary shock is roughly identical to the case of unobserved monetary shock. In contrast, for a monetary shock of size $\delta = 5\%$, the output response is much smaller on impact and much less persistent under the assumption that the monetary shock is freely and immediately observed than under the assumption that the monetary shock is unobserved. Similarly to the menu cost only model, the cumulated output response $\mathcal{M}(\delta)$ is hump-shaped in δ: the value of $\mathcal{M}(\delta)$ is larger than in the menu cost only model for $\delta \leq 5\%$, and smaller for values of $\delta > 5\%$. In addition, the cumulated output response has a peak at at $\delta = 3\%$, thus before the peak of the menu cost only model. The cumulated output response in the case of observed monetary shocks is always smaller than in the case of unobserved shocks. However, the difference between the predictions of $\mathcal{M}(\delta)$ under observed and unobserved monetary shocks is negligible for values of $\delta \leq 2\%$. As δ increases further, the cumulated output response with observed monetary shocks departs more and more from the case of unobserved monetary shocks.

The next proposition showcases the different behavior of the impulse responses to a monetary shock comparing the two assumptions about the acquisition of the information about the aggregate monetary shock that occur at date $t = 0$. In the first case we assume that information about the aggregate state of the economy is known only after the first observation at times $s \geq t$. In the second case we assume that the aggregate shock is costlessly known at time $t = 0$.

Proposition 2. Let $t = 0$ be the time at which an aggregate monetary shock $\delta > 0$ occurs.

1. Assume that the aggregate shock is not observed at $t = 0$. All firms make their *first* observation at the date that was planned before $t = 0$. Moreover, all firms that will adjust at time $s \geq t$ do so immediately after observing their idiosyncratic state at time s.

2. Assume that the aggregate shock is observed at $t = 0$. At time $t = 0$ firms can be divided into two groups:

    (a) Firms that *immediately* increase their prices without observing their idiosyncratic state, or

(b) Firms that change the date of their *first* observation relative to the date that was planned before $t = 0$, including a strictly positive fraction that observes immediately. Firms that have negative expected price change will postpone their *first* observation. Moreover, for small δ, almost all firms that have a positive expected price change will anticipate their *first* observation.

See Appendix A for the proof. When the information of the monetary shock is costlessly observed at $t = 0$, the model responds to a monetary shock in a way that is closer to the menu cost model on two dimensions. First, the monetary shock has an *impact* effect on the price level, i.e. it causes a jump in the fraction of firms that adjust immediately, some of them adjusting without observing the idiosyncratic state and some of them anticipating their observation of the state and adjusting immediately after.

Second the model displays a *selection* effect: right after the unexpected increase in money supply, the fraction of firms that increase prices is larger than the fraction that decrease prices. The last effect originates from the different behavior of firms with positive and negative expected price gap with respect to the planning of their first observation date (and possibly adjustment) after the monetary shock. In fact, firms with positive expected price change (i.e. negative price gap) before the monetary shock anticipate their first observation as the positive monetary shock increases the probability of the actual price gap being outside of the inaction region. Firms with negative expected price change (i.e. positive price gap) postpone their first observation as the positive monetary shock decreases the probability of the actual price gap being outside of the inaction region.

In contrast, the model where the monetary shock is only learned after the idiosyncratic shock is observed predicts no immediate jump in the price level and no selection of the type described above, as the fraction of firms adjusting largely depends on the fraction of firms observing, which has not changed with respect to steady state. As however Figure 2 and Figure 3 show, the quantitative implications for the real effects of monetary shocks of such *impact* and *selection* effects depend on the size of the monetary shock. When the monetary shock is smaller, fewer firms find optimal to adjust the price in response to the monetary shock without observing, and less asymmetry will be present in the decision of when to observe next between firms positive and negative expected price gaps. Thus, unless the monetary shock is large, for instance larger than 2%, the impact of such effects on the impulse responses is negligible, so that the assumptions about the availability of information on monetary shocks is quantitatively irrelevant.

4 Sensitivity of impulse responses to ψ/θ

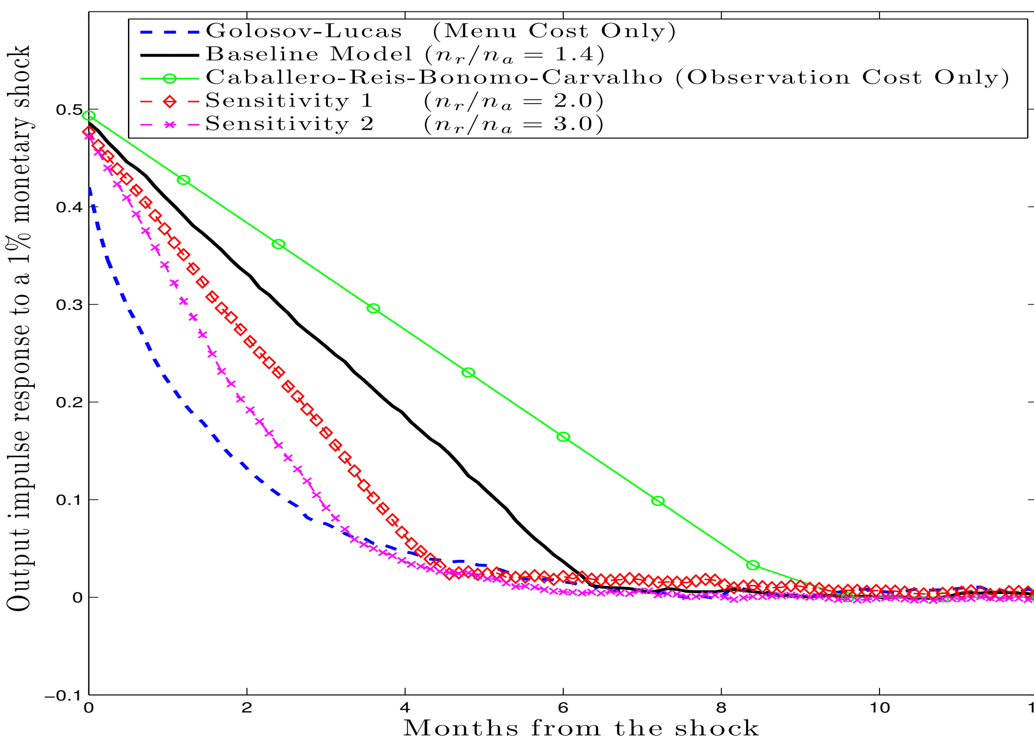
In this section we compute impulse responses for alternative parametrizations of the relative size of observation and menu costs, and compare them to predictions obtained under our baseline parametrization. We compute impulse responses in economies characterized by the same average frequency and size of price changes, i.e. $n_a = 1.3$ and $s_p = 0.085$, and by different ratios of the average frequency of price reviews to price adjustments, i.e. $n_r/n_a = 2$ and $n_r/n_a = 3$. These choices of n_r/n_a imply ratios of adjustment to observation costs that are much larger than in our baseline parametrization.[21] We find these alternative parametrizations interesting for two reasons. First, these values of n_r/n_a are consistent, for instance, with firms' behavior is some European countries. See Appendix A for details. Second, they are a conservative parametrization of the relative size of observation cost.

Figure 4 plots impulse responses of output to a $\delta = 1\%$ increase in m_t in our baseline model, and in two alternative economies where the average frequency of price reviews is respectively $n_r/n_a = 2$ and $n_r/n_a = 3$. The larger the ratio of frequencies of reviews to adjustments, the larger the implied ratio of menu cost to observation cost, ψ/θ. The larger ψ/θ, the smaller the impact effect of the monetary shock on output. Remarkably, despite the large variation in ψ/θ, the shape of the impulse response in all cases with both observation and adjustment cost is well described by a function that is approximately linear, as in the

[21] For given $n_a = 1.3$ and $s_p = 0.085$, we have that $n_r/n_a = 2$ implies $\theta = 0.29\%$, $\psi/\theta = 1.5$ and $\sigma = 0.101$, while $n_r/n_a = 3$ implies $\theta = 0.11\%$, $\psi/\theta = 4.2$, and $\sigma = 0.097$.

observation cost only model, up to a given point, and exponentially decaying as in the menu cost only model after. The point where the shape of the impulse response changes is approximately given by the maximum time between consecutive observations. In addition, most of the real effects of the monetary shock are accounted by the linear piece of the impulse response. Thus, the shape of the impulse response is not just a "simple average" of the shapes in the models with menu cost only and observation cost only, but in a wide and sensible range of values for ψ/θ, the shape of the impulse response in the model with both observation and menu cost largely inherits the shape of the impulse response of the model with observation cost only.

Figure 4: Response of $\log c_t$ to an unexpected 1% increase in m_t, for different values of ψ/θ



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. $n_a = 1.3$, and to the average size of price changes equal to $s_p = 8.5\%$.

5 Concluding remarks

We conclude that empirically reasonable observation costs increase the power of monetary shocks relative to models with menu cost only for economies that are observationally equivalent in terms of the frequency and size distribution of price changes. We find that the output effect of a 1% monetary shock with a Calvo mechanism is 4 times larger than in a menu cost model, while it is 2 times larger in an economy with observation costs only, and about 1.5 larger in an economy with both observation and menu costs.

Interestingly, while the decision rules in the model with both costs combine both "state-dependent" and "time-dependent" elements, the shape of the impulse responses inherits a great deal of the linearity implied by the time-dependent rules. Thus the time-dependent component of the decision rule dominates the selection of firms adjusting prices in response to the monetary shock.

Moreover, for monetary shocks smaller than 2%, results in the model with both observation and menu costs are roughly independent of the assumption of whether firms know the realization of the monetary shock on impact. We see this property as a sign of robustness of the approach proposed by Mankiw and Reis (2002), and as a response to the common criticism that easily observables monetary shocks would substantially weaken the imperfect information hypothesis.

The underlying framework of our analysis is the economy studied by Golosov and Lucas (2007). We used this economy as a laboratory to evaluate the impact different microeconomic mechanisms behind price rigidity, and relate to the menu cost model as a benchmark of comparison. Thus, we emphasized the impact of different combinations of sizes of menu and observation costs for the output effects to monetary shocks *relatively* to the menu cost model. Other authors (e.g. Nakamura and Steinsson (2010), Midrigan (2011) and Alvarez and Lippi (2012)) have emphasized different mechanism affecting the *level* of the output effects to monetary shocks for given microeconomic friction in price changes, such as decreasing return to scale in production, different assumptions about the distribution of idiosyncratic shocks and firms setting prices for multiple products. Studying how such assumptions would affect the output responses for given combination of observation and menu costs is an interesting avenue for future research.

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A Proofs

Proof. (of Proposition 2)

The proof of point 1 follows immediately from the assumption that all firms are unaware about the realization of the aggregate shock until their first observation. Given they are unaware of the shock, their decision rule will not be affected by the shock until then. After the first observation, firms' beliefs about the evolution of the price gap are identical to the ones in steady state, so that price adjustments occur only upon observation. The proof of point 2.a follows from the fact that there is a non-zero mass of firms that observed the state at least an instant before the realization of the aggregate shock, and have uncertainty about the price gap small enough that it is not worth observing the state. Among those firms, a fraction of them found their price gap upon the last observation of the state to be inside the inaction region, with the distance from the adjustment threshold $\underline{g}_t$ being smaller than the size of the monetary shock δ. Thus the monetary shock causes the price gap of these firms to exit the inaction region, i.e. $g < \underline{g}_t$, leading to a price increase without observation. On the other side, another fraction of firms found their price gap close to the adjustment threshold $\bar{g}_t$ at their last observation, so to update their beliefs of price gap further to the left of such threshold. Thus there will be no firm decreasing price without observing the state after a positive monetary shock. The proof of point 2.b follows from the fact that the monetary shock causes an equal shift to the expected price gaps of all firms. If the monetary shock is not too large, not all firms will adjust their price on impact. Among those firms that do not adjust their price immediately, their next observation time will be the same that would have been chosen at the last observation date if the price gap was smaller by an amount δ. Thus, given the inverse-U shape of the optimal time to the next observation, those firms that had a negative price gap (thus a positive expected price change) before the monetary shock, and do not adjust their price immediately, will anticipate the next observation date. At least a fraction of those firms that had a positive price gap upon the last observation (thus a positive expected price change) before the monetary shock, and do not adjust their price immediately, will postpone their next observation date. For instance, those firms that upon the last observation date found their price gap equal to $\bar{g}_t$.

Online Appendix

Monetary Shocks with Observation and Menu Costs

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A Evidence on price adjustments vs. price reviews

Several recent studies measure two distinct dimensions of the firm's price management: the frequency of price reviews, or the decision of assessing the appropriateness of the price currently charged, and the frequency of price changes, i.e. the decision to adjust the price. The typical survey question asks firms: "In general, how often do you review the price of your main product (without necessarily changing it)?"; with possible choices yearly, semi-yearly, quarterly, monthly, weekly and daily. The same surveys contain questions on frequency of price changes too. Fabiani et al. (2007) survey evidence on frequencies of reviews and adjustments for different countries in the Euro area, and Blinder et al. (1998), Amirault, Kwan, and Wilkinson (2006), and Greenslade and Parker (2008) present similar evidence for US, Canada and UK. This section uses this survey data to document that the frequency of price reviews is larger than the frequency of price changes. We believe that the level of both frequencies, especially the one for reviews, are measured very imprecisely. Yet, importantly for the theory presented in this paper, we have found that in all countries, and in almost all industries in each country, and for almost all the firms in several countries, the frequency of review is consistently higher than the frequency of adjustment. We will argue that, given our understanding of the precision of the different surveys, the most accurate measures of the ratio of price reviews to price adjustments per year are between 1 and 2. In the rest of this section we document this fact .

Table II : Price-reviews and price-changes per year

	AT	BE	FR	GE	IT	NL	PT	SP	EURO	CAN	UK	US
						Medians						
Review	4	1	4	3	1	4	2	1	2.7	12	4	2
Change	1	1	1	1	1	1	1	1	1	4	2	1.4
						Mass of firms (%) with at least 4 reviews/changes						
Review	54	12	53	47	43	56	28	14	43	78	52	40
Change	11	8	9	21	11	11	12	14	14	44	35	15

Number of price changes and reviews per year. The sources for the medians are Fabiani et al. (2007) 2003 Euro area survey, Amirault, Kwan, and Wilkinson (2006) 2003 Canadian survey, Greenslade and Parker (2008) 2008 UK survey.

The upper panel of Table II reports the median frequency of price reviews and the median frequency of price adjustments across all firms in surveys taken from various countries. The median firm in the Euro area reviews its price a bit less than three times a year, but changes its price only about once a year, and similar for UK and US.[22]

These surveys collect a wealth of information on many dimensions of price setting, well beyond the ones studied in this paper. Yet, for the questions that we are interested in, the survey data from several countries have some drawbacks. We think that, mostly due to the design of these surveys, the level of the frequencies of price review and price adjustments

[22]This evidence about the frequency of price adjustment is roughly consistent with previous studies at the retail level. See Alvarez et al. (2006) for more details.

are likely subject to a large amount of measurement error. One reason is that in most of the surveys firms were given the following choices for the frequency of price reviews: yearly, quarterly, monthly and weekly (in some also semi-yearly and less than a year). It turns out that these bins are too coarse for a precise measurement, given where the medians of the responses are. For example, consider the case where in the population the median number of price reviews is exactly one per year, but where the median number of price changes is strictly larger than one per year. Then, in a small sample, the median for reviews will be likely 1 or 2 reviews a year, with similar likelihood. Instead the sample median for number of adjustments per year is likely to be one. From this example we remark that the median for price reviews is imprecisely measured, as its estimates fluctuates between two values that are one hundred percent apart. The configuration described in this example is likely to describe several of the countries in our surveys.[23] Another reason is that in some cases the sample size is small. While most surveys are above one thousand firms, the surveys for Italy has less than 300 firms and the one for the US has about 200 firms. Yet another difficulty with these measures is that several surveys use different bins to classify the frequency of price reviews and that one price changes. For instance in France and Italy firms are asked the average number of changes, instead of being given a set of bins, as is the case for the frequency of reviews.[24]

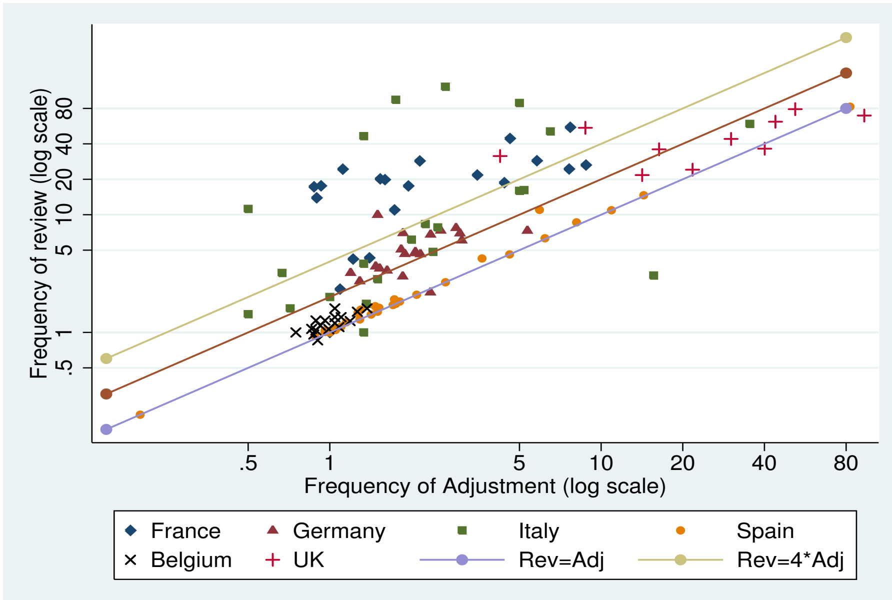
The bottom panel of Table II reports another statistic that is informative on the relative frequency of reviews and adjustments: the fraction of firms reviewing and changing respectively their price at least four times a year. We see this statistic as informative, and less subject to measurement error, because this frequency bin appears in the questionnaire for both the review and the adjustment decision for almost all countries. It shows that the mass of firms reviewing prices at least four times a year is substantially larger than the corresponding one for price changes.

Figure V plots the average number of price reviews against the average number of price adjustments across a number of industries in six countries. This figure shows that in the six countries the vast majority of the industry observations lies above the 45 degree line, where the two frequencies coincide. Most of the industries for Belgium, Spain and the UK have a ratio of number of reviews per adjustment between 1 and 2 (i.e. lies between the two lower straight lines). The data for France, Italy and Germany has much higher dispersion in this ratio. We believe that the reason of the higher dispersion for Italy, Germany and France is due to the measurement error discussed above. Our belief is based on the fact that the questionnaire in the surveys for Belgium and Spain treat price reviews and price changes symmetrically and they record the average frequencies as an integer as opposed to a coarse bin.

[23]For example in Portugal the median frequency of review is 2, but the fraction that reviews at one year or less is 47%, while for price adjustment the median is one and the fraction of firms adjusting exactly once a year is 49.5%, see Martins (2005). In the UK for 1995 the median price review is 12 times a year, but the fraction that reviews at most 4 times a years is about 46%, while for price adjustment the median is 2 and the fraction of firms adjusting 2 times or less a year is 66%, see Hall, Walsh, and Yates (2000). Indeed, consistent with our hypothesis of measurement error, in a similar survey for the UK for year 2007-2008, the median price review is 4 and the median price adjustment is 4, see Greenslade and Parker (2008).

[24]Furthermore for Germany firms where asked whether or not they adjusted the price in each of the preceding 12 months; this places an upper bound of 12 on the frequency of adjustments, while no such restriction applies to the number of reviews.

Figure V: Average industry frequency of price changes vs. adjustments



Note: data for each dot are the mean number of price changes and reviews in industry j in country i.

Table III : Frequency of Price Changes and Reviews at Firm Level

	Belgium	France	Germany	Italy	Spain*
Percentage of Firms with:					
1) Change > Review	3	5	19	16	0
2) Change = Review	80	38	11	38	89
3) Change < Review	17	57	70	46	11
N of Observations (firms)	890	1126	835	141	194

* For Spain is only for firms that review four or more times a year. Sources: Table 17 in Aucremanne and Druant (2005) for Belgium, and our calculations based on the individual data described in Loupias and Ricart (2004), Stahl (2009), and Fabiani, Gattulli, and Sabbatini (2004) for France, Germany, and Italy. For Spain from section 4.4 of Alvarez and Hernando (2005).

For four countries Table III classifies the answers of each firm on three mutually exclusive categories: 1) those that change their prices more frequently than they review them, 2) those that change and review their prices at the same frequency, 3) and those that change their prices less frequently than they adjust them. Table III shows that most of the firms respond that they review their prices at frequencies greater or equal than the one in which change their prices. We conjecture that the percentage of firms in category 1, i.e. those changing the price more frequently than reviewing it, is actually even smaller than what is displayed in the table due to measurement error.

B From the sequence to the recursive firm problem

We let p_{τ_i} be the price chosen in the case of an adjustment. We let $\chi_{\tau_i} = 1$ be an indicator of price adjustment and let $\chi_{\tau_i} = 0$ be an indicator of no price adjustment at tau_i. The optimal value for a firm that maximizes expected discounted profits net of observation and menu costs at the first observation date after the aggregate shock, $t = \tau_0$, and up to the time the product is replaced is

$$V_t(p, z, m) = \max_{\{\tau_{i+1},\ p_{\tau_i},\ \chi_{\tau_i}\}_{i=0}^{\infty}} \mathcal{V}_t\left(\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty};\, p, z, m\right) \ , \tag{21}$$

where

$$\begin{aligned} &\mathcal{V}_t\left(\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty};\, p, z, m\right) \equiv \mathbb{E}\Bigg\{ \sum_{i=0}^{\infty} e^{-(\mu+\rho+\lambda)(\tau_i - t)} \Big[m_{\tau_i}\ \theta\, \bar{\Pi} \quad + \\ &(1-\chi_{\tau_i}) \int_{\tau_i}^{\tau_{i+1}} e^{-(\mu+\rho+\lambda)(s-\tau_i)}\ \mathbb{E}\left[m_s\ \Pi\left(p_{\tau_{i-1}}/p_s^*, c_s\right) \big| z_{\tau_i}\right] ds \quad + \\ &\chi_{\tau_i} \left(m_{\tau_i}\ \psi\, \bar{\Pi} + \int_{\tau_i}^{\tau_{i+1}} e^{-(\mu+\rho+\lambda)(s-\tau_i)}\ \mathbb{E}\left[m_s\ \Pi\big(p_{\tau_i}/p_s^*, c_s\big) \big| z_{\tau_i}\right]\ ds \right) \Big] \ \Big| z_t = z \Bigg\} \ , \end{aligned} \tag{22}$$

where $m_t = m$, $p_{-1} = p$ and $\tau_0 = t \geq 0$. The firm price at all dates $s \geq t$ is thus given by

$$\begin{aligned} p_s &= p_{\tau_{i-1}} \text{ for } s \in [\tau_i, \tau_{i+1}) \text{ if } \chi_{\tau_i} = 0 \ \text{ no adjustment takes place at } \tau_i \ , \\ p_s &= p_{\tau_i} \quad \text{ for } s \in [\tau_i, \tau_{i+1}) \text{ if } \chi_{\tau_i} = 1 \ \text{ an adjustment takes place at } \tau_i \ , \end{aligned}$$

where p_s^* is given by equation (12), z_s follows equation (2) and where m_s follows equation (1). Profits, observation cost and menu cost are scaled by the stock of money supply and hence the value function V_t is expressed in nominal terms. The discounting uses the result that the nominal interest rate equals $\rho + \mu$ as well as the fact that products die at rate λ. Since no information is gathered between observation dates, expectations of profits are conditional on the information gathered at the last observation date. No pricing decision takes place between observation dates, and hence the nominal price is constant between observation dates.

The optimal value of the sequence problem in equation (21) solves the following equations:

$$
\begin{aligned}
\frac{V_t(p,z,m)}{m} &= \max\left\{ \frac{\hat{V}_t(z,m)}{m} \; , \; \frac{\bar{V}_t(p,z,m)}{m} \right\} , \\
\frac{\hat{V}_t(z,m)}{m} &= -\theta\;\bar{\Pi} - \psi\;\bar{\Pi} + \max_{T,\hat{p}} \int_0^T e^{-(\rho+\lambda)s}\; c_{t+s}^{1-\epsilon\eta}\;\mathbb{E}\left[\;F\left(\frac{\hat{p}}{p^*_{t+s}}\right)\big| z_0 = z\right] ds + \\
&+ \; e^{-(\rho+\lambda)T}\;\mathbb{E}\left[\frac{V_{t+T}\left(\hat{p}, z_T, m_T\right)}{m\,e^{\mu T}}\;\big| z_0 = z\right] , \\
\frac{\bar{V}_t(p,z,m)}{m} &= -\theta\;\bar{\Pi} + \max_{T} \int_0^T e^{-(\rho+\lambda)s}\; c_{t+s}^{1-\epsilon\eta}\;\mathbb{E}\left[\;F\left(\frac{p}{p^*_{t+s}}\right)\;\big| z_0 = z\right] ds + \\
&+ \; e^{-(\rho+\lambda)T}\;\mathbb{E}\left[\frac{V_{t+T}\left(p, z_T, m_T\right)}{m\,e^{\mu T}}\;\big| z_0 = z\right] ,
\end{aligned}
$$

From this recursion we can show the following homogeneity for the nominal expected discounted net profits:

$$
V_t(p,z,m) \;=\; m\;\; V_t\left(\frac{p\;z}{m}, 1, 1\right) \quad \text{for all} \;\; p,z,m,t \geq 0 \tag{23}
$$

and hence we define the expected discounted profits relative to the economy money supply, for a price relative to the flexible optimal price. This value function, scaled by an index of real aggregate demand, turns out to be equal to the ratio of the nominal value function to the nominal flexible price profits, which gives an economic interpretation to the normalization:

$$
v_t(g) \;\equiv\; \frac{V_t(p,z,m)}{m\;\bar{\Pi}} = \frac{V_t\left(R\alpha^{\frac{\eta}{\eta-1}}\;e^{\;g}, 1, 1\right)}{\bar{c}^{1-\epsilon\eta}\;F(1)} \tag{24}
$$

We can write a one state Bellman equation associated to the real net profits immediately after the time of an observation as:

$$
\begin{aligned}
v_t(g) &= \max\left\{\hat{v}_t \;,\; \bar{v}_t(g)\right\} \;\; \text{where} \\
\hat{v}_t &= -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta}\mathbb{E}\left[\frac{F\left(e^{g_s}\right)}{F(1)}\;\big|\; g_0 = \hat{g}\right] ds + \\
&+ \; e^{-(\rho+\lambda)T}\mathbb{E}\left[v_{t+T}\left(g_T\right)\;\big|\; g_0 = \hat{g}\right] \; , \\
\bar{v}_t(g) &= -\theta + \max_{T} \int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_{t+s}}{\bar{c}}\right)^{1-\epsilon\eta}\mathbb{E}\left[\frac{F\left(e^{g_s}\right)}{F(1)}\;\big|\; g_0 = g\right] ds + \\
&+ \; e^{-(\rho+\lambda)T}\mathbb{E}\left[v_{t+T}\left(g_T\right)\;\big|\; g_0 = g\right] \; ,
\end{aligned}
$$

The dynamics of the state g in the inaction region are given by $dg = (\gamma-\mu)dt + \sigma dB$. Thus for each s the distribution of g_s is given by:

$$
g_s \;=\; g_0 + \;(\gamma-\mu)s\; + \sigma\sqrt{s}\,x
$$

where x is a standard normal random variable. Finally, we define the function $f(\cdot)$ as

$$
\begin{aligned}
f(g', s) &\equiv \mathbb{E}\left[\frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = g'\right] \\
&= \int_{-\infty}^{\infty} \frac{F\left(\, e^{g' + (\gamma - \mu)s + \sigma\sqrt{s}\,x}\right)}{F(1)} dN(x),
\end{aligned}
$$

which after simple algebra is given by

$$
f(g', s) = \eta\, e^{\left((1-\eta)g'\,(\eta-1)(\mu-\gamma) + \frac{\sigma^2}{2}(\eta-1)^2\right)s} - (\eta - 1)\, e^{\left(-\eta g' + \eta(\mu-\gamma) + \frac{\sigma^2}{2}\eta^2\right)s} \, . \tag{25}
$$

C Beliefs dynamics

There are four cases. First, if $t = 0$ is not an observation date and the product has not been replaced, then given initial beliefs $(\tilde{g}, \tilde{\sigma}^2)$ we have

$$
(g_0 \ , \ \tilde{g}_0 \ , \ \tilde{\sigma}_0^2) = \begin{cases} (g_0 \ + \hat{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2) - \ \tilde{g} \ , \ \hat{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2) \ , \ \tilde{\sigma}^2) & \text{if } \tilde{g} \notin (\underline{\mathbf{g}_{\mathbf{0}}}(\tilde{\sigma}^2) \ , \ \bar{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2)) \\ (g_0 \ , \ \tilde{g} \ , \ \tilde{\sigma}^2) & \text{if } \tilde{g} \in (\underline{\mathbf{g}_{\mathbf{0}}}(\tilde{\sigma}^2) \ , \ \bar{\mathbf{g}}_{\mathbf{0}}(\tilde{\sigma}^2)) \end{cases} \tag{26}
$$

and the first observation date takes place in $\tau_0 = \mathrm{T}_0(\tilde{g}_0, \tilde{\sigma}_0^2)$ periods. Second, if t is an observation date, i.e. $t = \tau_i$, then $\tau_{i+1} = \mathrm{T}_t(g_t)$, and

$$
(g_t \ , \ \tilde{g}_t \ , \ \tilde{\sigma}_t^2) = \begin{cases} (\hat{g}_t \ , \ \hat{g}_t \ , \ 0) & \text{if } g_t \notin (\underline{g}_t \ , \ \bar{g}_t) \\ (g_t \ , \ g_t \ , \ 0) & \text{if } g_t \in (\underline{g}_t \ , \ \bar{g}_t) \end{cases} \tag{27}
$$

Third, if $t > 0$ is not an observation date and the product has not been replaced, so that $\tau_i < t < \tau_{i+1}$, then $\tau_{i+1} = \mathrm{T}_{\tau_i}(g_{\tau_i})$ and

$$
(g_t \ , \ \tilde{g}_t \ , \ \tilde{\sigma}_t^2) = \left(g_{\tau_i} - \mu(t - \tau_i) \ + \log(z_t/z_{\tau_i}) \ , \ g_{\tau_i} + (\gamma - \mu)(t - \tau_i) \ , \ \sigma^2(t - \tau_i)\right) \tag{28}
$$

Fourth, if t is not an observation date, but the product has been replaced at date $\tau_0 < t < \tau_1$, then $\tau_1 = \mathrm{T}_{\tau_0}(g_{\tau_0})$ and

$$
(g_t \ , \ \tilde{g}_t \ , \ \tilde{\sigma}_t^2) = \left(g_{\tau_0} - \mu(t - \tau_0) \ + \log(z_t/z_{\tau_0}) \ , \ g_{\tau_0} + \gamma(t - \tau_0) \ , \ \sigma^2(t - \tau_0)\right) \tag{29}
$$

D Computation of Impulse Response

We discretize time into intervals of length Δ, so now calendar dates, and hence choices of times to review $t \in \mathbb{T} \equiv \{0, \Delta, 2\Delta, ...\}$. Let ϕ_0 be the initial distribution, which is set to the stationary distribution for constant money growth rate, and whose computation is described below.

The numerical procedure to compute the path for equilibrium quantities consists of iterating on the following two steps until convergence.

1. Solve for the path for aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$ assuming the economy starts with distribution ϕ_0, receives a monetary shock ϵ_m at $t=0$, and the firms use decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$.

2. Solve the problem of the firm for all times $t \in \mathbb{T}$ to obtain the decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$ given a path for the aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$.

The initial condition for this process is given by $c_t = \bar{c}$ for all $t \in \mathbb{T}$. Also we impose that the equilibrium path of consumption $c_t = \bar{c}$ for $t \geq \hat{T}$ where $\hat{T}$ is a large number. Below we give more details about steps 1 and 2.

D.1 Computation of the Stationary Equilibrium

First we discuss the computation of the decision rules. We use standard value function iterations in the problem described in equation (17), for $c_t = \bar{c}$, with the restriction that time elapsed between reviews $T \in \mathbb{T}$. In this case, the decision rules and value function do not depend on calendar time. In each of the value function iterations we solve the value function in a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives the time independent decision rules $\underline{g}, \hat{g}, \bar{g}, \mathrm{T}(\cdot)$ and the corresponding, time independent, value function $v(\cdot)$.

Using the time independent optimal decision rules we compute the stationary distribution ϕ_0 by simulating the discrete time analog of the triplets $(g, \tilde{g}, \tilde{\sigma}^2)$ for N firms for $\bar{T}/\Delta$ model periods. For each firm i we simulate:

$$g_{t+\Delta,i} = \begin{cases} g_{t,i} + \gamma\Delta + \sigma\sqrt{\Delta}\ x & \text{with prob. } e^{-\Delta\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

where x is a standard normal and where the starting date is $t = -\bar{T}/\Delta$ up to $t = 0$. Using this realization for g_{ti} and the times at which products have been replaced, we can find the sequence for the triplet $(g_{it}, \tilde{g}_{it}, \tilde{\sigma}^2_{it})$ using the decision rules as described in Section 2.4 in equation (27) - equation (29). The outcome of this process is to have N time series, each of length $\bar{T}/\Delta$ of the triplets $(g_{it}, \tilde{g}_{it}, \tilde{\sigma}^2_{it})$. The number of firms N and the number of periods $\bar{T}$ are chosen to be large enough so that the cross section of this triplets at time $t = 0$ stabilizes, and hence represents ϕ_0 accurately.

D.2 Computing the consumption path implied by the decision rules

We use the time dependent decision rules to simulate the price gaps, productivity g_{it} and the triplets $\{g_{it}, \tilde{g}_{it}, \tilde{\sigma}^2_{it}\}$ for each of the N firms for $\hat{T}$ periods, as explained in Section 2.4, and equation (27)-equation (29). Each of the N firms is initialized from the sample of initial distribution ϕ_0 of the triplets $(g, \tilde{g}, \tilde{\sigma}^2)$, $(g_{i0}, \tilde{g}_{i0}, \tilde{\sigma}^2_{i0})$ described above. Then for each cross section at times $t \geq 0$ we compute the implied aggregate consumption as:

$$c_t = \left(\sum_{i=1}^{N} \left(\frac{\eta \, \xi}{\eta - 1} \; e^{g_{it}} \right)^{1-\eta} \right)^{\frac{1}{\epsilon \, (\eta - 1)}} \tag{31}$$

D.2.1 Computing the decision rules implied by the consumption path

The decision rules are the solution of the dynamic programming problem described in equation (17) where time elapsed between reviews are restricted to $T \in \mathbb{T}$. The value function and decision rules are computed by backward induction, taking as given the stationary value function v at $t = \hat{T}$. In each of the value function iterations we solve for the value function on a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives a path of decisions rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t \in \mathbb{T}, t \leq \hat{T}}$.

E General equilibrium feedback effects

In this section we show that the steady state decision rules are a very good approximation to the actual optimal firms' decision rule when the monetary shock is unobserved at $t = 0$. When instead the monetary shock is observed at $t = 0$, the steady state decision rules are still a very good approximation to the actual optimal firms' decision rule for all periods following the first observation date after the aggregate shock.

The next proposition states precisely the sense in which the general equilibrium feedback effect of the sequence $\{c_t\}$ has a negligible effect on the firms decisions. Let (p, m, z) be arbitrary initial conditions at time t, let $\{c_{t+s}\}_{s \geq 0}$ be an arbitrary path of aggregate consumption starting at t and let $\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty}$ be an arbitrary policy for the firm for observation times, price change decision and optimal price setting at observation times.

Proposition 3. The objective function of the firm can be written as:

$$\begin{aligned} &\mathcal{V}_t\left(\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty} \, ; \, p, z, m\right) = \mathcal{V}^Q\left(\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty} \, ; \, p, z, m\right) \\ &+ \int_t^{\infty} e^{-r(s-t)} o\left(||(g_s, \; c_s - \bar{c})||^2\right) ds + \Gamma\left(\{c_{t+s}\}_{s \geq 0}\right) \; , \end{aligned}$$

where $\mathcal{V}^Q$ is the objective function one obtains from equation (13) replacing $\Pi(g, c)$ by $(g-1)^2 \, \eta(\eta-1)/2$ at each time $s \geq t$, where o denote a function of smaller order, and where Γ is a function which does not depend on the decisions $\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty}$.

The main idea of the proof is to use a quadratic approximation of $\Pi(g, c)$ around $(g, c) = (0, \bar{c})$. This approximation has a zero derivative w.r.t. g, i.e. $\Pi_g(0, \bar{c}) = 0$, by definition of the price gap. The approximation also has a zero cross derivative, i.e. $\Pi_{gc}(0, \bar{c}) = 0$ as a consequence that c enters in a multiplicative way in the profit function in equation (13). In this sense there is no strategic complementary between the pricing decision of other firms, which will affect c, and the pricing decision of a representative firm.

To understand the implications of this proposition note that $\mathcal{V}^Q$ does not depend on the path of $(\{c_{t+s}\}_{s \geq 0})$, and hence it is the solution of a steady state problem. Moreover, it is a relatively tractable problem since the period return is quadratic. Also note that if the costs θ and ψ are small, the deviation from the price gaps will be small, i.e. g will be close to 0 all the time. Furthermore, note that if the monetary shocks are small, i.e. δ is close to zero, then $c - \bar{c}$ will be small, and hence the deviation from the objective function of g and c will be of third or higher order. Hence, the policy that is obtained from maximizing the steady state problem based on the quadratic approximation return in $\mathcal{V}^Q$ has to give a good approximation to the solution of the original firm problem as long as the cost are small and the monetary shocks are not too large. This is useful because in Alvarez, Lippi, and Paciello (2011) we derive an analytical characterization of the optimal decision rule that maximizes $\mathcal{V}^Q$. This analytical characterization helps guiding the numerical solution to the optimal decision rule maximizing $\mathcal{V}_t$.

To compare the accuracy in the two cases, we plot in Figure VI the optimal decision rules in steady state, described by three values for the price gap $\underline{g} < \hat{g} < \bar{g}$, and a function $\mathrm{T}(\cdot)$, computed through the analytical solution that maximizes $\mathcal{V}^Q$ against the the optimal decision rule maximizing $\mathcal{V}_t$. The function $\mathrm{T}(g)$ is inverse-U shaped, as firms that find themselves closer to the inaction region are more likely to get a shock that will push them outside of such region, so observing sooner is optimal. Figure VI shows that the analytical solution to the approximated problem only slightly understates the time to the next observation and the width of the inaction region.[25]

Next, we show that aggregate output impulse response to a relatively small monetary shock obtained with the analytical approximation of the optimal decision rule in steady state is roughy identical to the impulse response obtained by solving numerically for the optimal decision rule. Thus not only the analytical solution to the approximated problem is a good characterization to the optimal decision rule in steady state, but also provides a good characterization of the economy response to small monetary shocks. First we show that a second order approximation of the profit function has a representation that is similar to the problem we studied in Alvarez, Lippi, and Paciello (2011). The period return function of the recursive problem described in Proposition 1 is given by $f(g', s)$. The second order approximation of $f(g, s)$ around $(g, s) = (0, 0)$ is given by

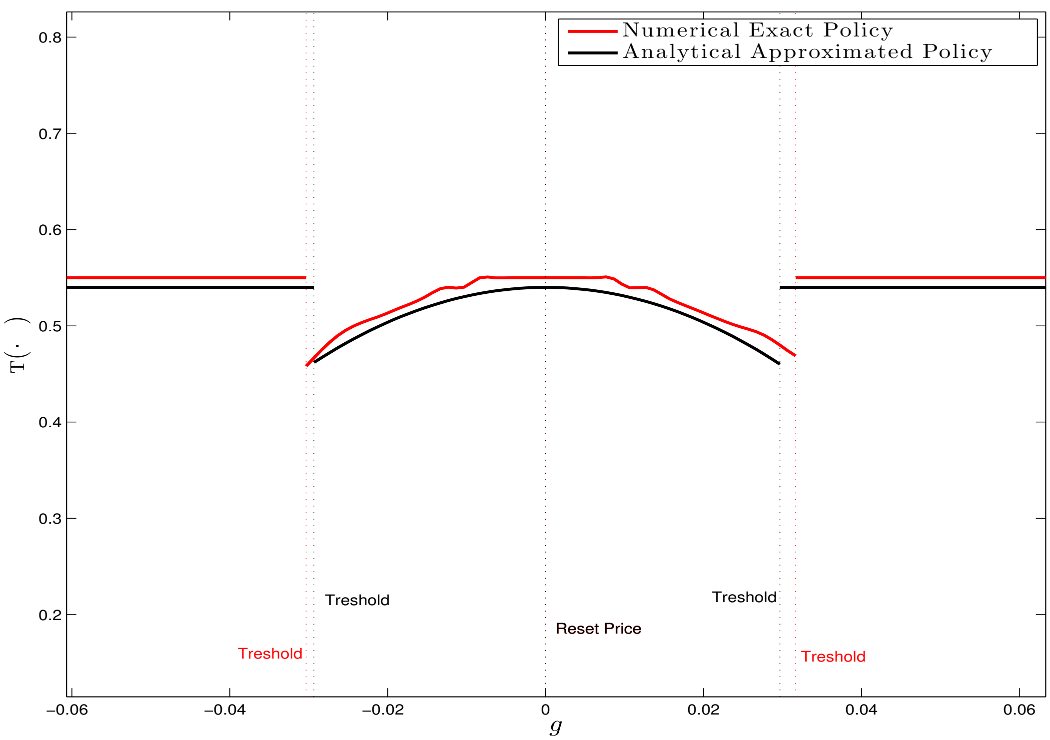
$$f(g,s) = 1 + f_s(0,0)\sigma^2 \; s + \frac{1}{2} \; f_{gg}(0,0) \; (g)^2 + \frac{1}{2} \; f_{ss}(0,0) \; s^2 + f_{gs}(0,0) \; g \; s + o(||\sigma^4 s^2, \sigma^2 \; g^2||),$$

where

$$\begin{aligned}
f_g(0,0) = 0, \quad f_s(0,0) &= -\frac{\eta(\eta-1)}{2} \, \sigma^2, \quad f_{gg}(0,0) = -\eta(\eta-1), \\
f_{ss}(0,0) &= -\eta(\eta-1) \; \left[(\mu-\gamma)^2 + (2\eta-1)(\mu-\gamma)\sigma^2 + (3\eta^2 - 3\eta + 1)\frac{\sigma^4}{4}\right], \\
f_{g,s}(0,0) &= \; \eta(\eta-1)^2 \; \left[(\mu-\gamma) + (2\eta-1) \; \frac{\sigma^2}{2}\right].
\end{aligned}$$

[25] In this exercise we use parameter values that are representative of the U.S. economy, and that we describe in detail in Section 3.1. See Appendix E for a systematic comparison of the analytical approximation and numerical exact solution, and for the equations describing the analytical solution.

Figure VI: Steady state decision rules from approximated vs numerical solution method



Note: Parameters are such that the model matches the average frequency of price changes, i.e. 1.3 adjustments per year, to the ratio of average number of price reviews to price adjustments per year equal to 1.4, and to the average size of absolute price changes equal to 8.5%. See Section 3.1 for more details on parameters choices.

In the case in which $\mu - \gamma = -(2\eta - 1)\ \frac{\sigma^2}{2}$, it follows that $f_{g,s}(0,0) = 0$ and $f_{ss}(0,0) = (f_s(0,0))^2$. In this case the problem is similar to the one we studied in Alvarez, Lippi, and Paciello (2011), in the sense that the value function for the firm problem with zero inflation (equation 7 on page 1928 in our paper) is obtained if the function $f(g,s)$ is approximated by $\sigma^2 s$ which is obviously a good approximation for small values of t around the optimal return point $g = 0$.[26]

Next we describe the analytical characterization of the quadratic problem. In Alvarez, Lippi, and Paciello (2011) we show that, as $\rho \downarrow 0$, the solution to firm's problem of maximizing the value function $\mathcal{V}^{\mathcal{Q}}\left(\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty}; p, z, m\right)$ with quadratic period return $\frac{1}{2}\ f_{gg}(0,0)\ g^2$ defined in Proposition 3 is given by:

$$\mathrm{T}(g) = \tau\ - \left(\frac{g}{\sigma}\right)^2 + o(|g^3|)\ \text{ for } g \in (-\bar{g},\ \bar{g}), \quad \text{and} \quad \mathrm{T}(g) = \tau \ \text{ otherwise.} \tag{32}$$

Define $\alpha \equiv \psi/\theta$ and $B = \eta(\eta - 1)/2$. The optimal values for the time until the next revision after an adjustment, τ, and the width of the range of inaction, $\bar{g}$, are given by

$$\tau = \sqrt{\frac{\theta}{\sigma^2\ B}}\ \frac{\sqrt{\alpha}}{\varphi(\alpha)}\ , \tag{33}$$

$$\bar{g} = \left[\sigma^2 \frac{\psi}{B}\right]^{\frac{1}{4}}\ \sqrt{\varphi(\alpha)}\ , \tag{34}$$

where $\varphi(\alpha)$ solves

$$1 = \varphi(\alpha)^2 \left(\frac{2}{\alpha} + 4\left[1 - N\left(\varphi(\alpha)\right)\right]\right)\ . \tag{35}$$

Finally, we compare impulse responses to a 1% increase in money supply obtained by approximating firm's decision rule according to equations (32)-(35), against the predictions of the model where firms' decision rule is solved numerically as described in Appendix D. Figure VII plots impulse responses for the two cases just described.

F The model without preference shocks

This appendix considers the problem where preferences shocks are shut down so that the Spence-Dixit-Stiglitz consumption aggregate is given by

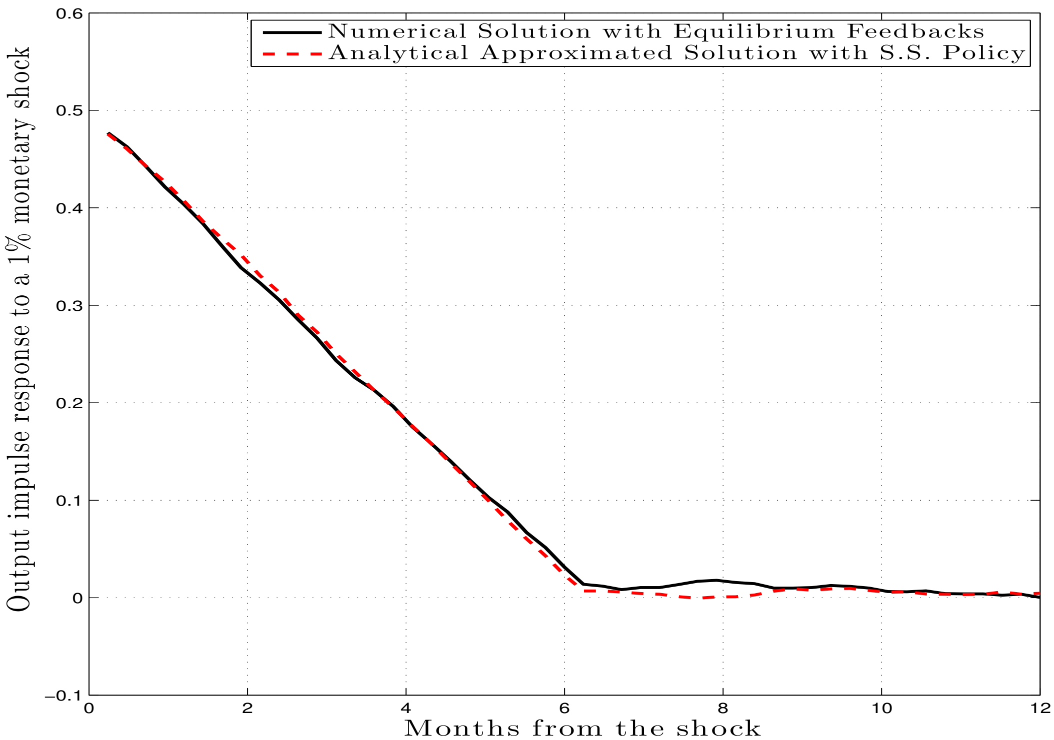
$$c_t = \left[\int_0^1 (C_t(i))^{(\eta-1)/\eta} di\right]^{\eta/(\eta-1)} \quad \text{with} \ \ \eta > 1\ .$$

The price setting equation is the same one as in equation (12), but profits are now given by

$$\Pi_t(p/p^*, z) \ = \ c_t^{1-\epsilon\eta} z^{\eta-1}\ F(p/p_t^*) \qquad \text{and} \qquad \Pi_t^*(z) = c_t^{1-\epsilon\eta}\ z^{\eta-1}\ F(1) \tag{36}$$

[26] For clarity of comparison note that the value function in Alvarez, Lippi, and Paciello (2011) is expressed as a difference from the maximized frictionless profits, i.e. $f(g,s) - 1$.

Figure VII: Impulse response of $\log c_t$ to an unexpected 1% increase in m_t, $\delta = 1\%$



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. 1.3 adjustments per year, and to the average size of price changes equal to 8.5%.

where, letting $g \equiv \log(p/p^*)$, the function $F(e^g) \equiv \; R\; \xi^{1-\eta} \; \left(e^g \; \frac{\eta}{\eta-1}\right)^{-\eta} \left(e^g \; \frac{\eta}{\eta-1} - 1\right)$. We assume that the observation cost is and the adjustment cost are a constant fraction of flexible price profits: $\theta_t(z) = \theta \; \Pi_t^*(z)$ and $\psi_t(z) = \psi \; \Pi_t^*(z)$. Here, however, $\Pi_t^*(z)$ is a function of the level of productivity of each firm. Technically, the purpose of having the observation (and menu) cost to scale up with profits is to induce stationarity in the firm's problem, which is a common feature in many models. Finally, λ large enough ensures that there exists an invariant distribution of profits, output and productivities.

Proposition 4. Consider the normalized value function: $v_t(g) \equiv \frac{V_t(p,z,m)}{m\,\Pi_t^*(z)}$ where g is the price gap defined in equation (15). This normalized, single-state, value function solves:

$$\begin{aligned}
v_t(g) \quad &= \quad \max\left\{\hat{v}_t \;,\; \bar{v}_t(g)\right\} \;\; \text{where} \qquad (37)\\
\hat{v}_t \quad &= \quad -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,\hat{g})\, ds \;+ \\
&+ \quad e^{-(\rho+\lambda)T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\,x\right)} \; v_{t+T}\left(e^{\hat{g}\; + (\gamma-\mu)T + \sigma\sqrt{T}\,x}\right)\; dN(x)\;,\\
\bar{v}_t(g) \quad &= \quad -\theta + \max_{T} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,g)\, ds \;+ \\
&+ \quad e^{-(\rho+\lambda)T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\,x\right)} \; v_{t+T}\left(e^{g\; + (\gamma-\mu)T + \sigma\sqrt{T}\,x}\right)\; dN(x)\;,\\
&\text{where} \quad f(s,g') \equiv \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma s + \sigma\sqrt{s}\,x\right)} \; \frac{F\left(e^{g'\; + (\gamma-\mu)s + \sigma\sqrt{s}\,x}\right)}{F(1)} dN(x)\;,
\end{aligned}$$

and where $N(\cdot)$ is the CDF of a standard normal.

Proof. (of Proposition 4) The optimal value of the sequence problem in equation (21) solves the following functional equation:

$$\begin{aligned}
\frac{V_t(p,z,m)}{m} \quad &= \quad \max\left\{\frac{\hat{V}_t(z,m)}{m} \;,\; \frac{\bar{V}_t(p,z,m)}{m}\right\}\\
\frac{\hat{V}_t(z,m)}{m} \quad &= \quad -\theta_t(z) - \psi_t(z) + \max_{T,\hat{p}} \int_0^T e^{-(\rho+\lambda)s} \; c_{t+s}^{1-\epsilon\eta} \; \mathbb{E}\left[z_s^{\eta-1} \; F\left(\frac{\hat{p}}{p^*_{t+s}}\right) |z_0 = z\right] ds \;+\\
&+ \quad e^{-(\rho+\lambda)T} \; c_{t+T}^{1-\epsilon\eta} \; \mathbb{E}\left[\frac{V_{t+T}\left(\hat{p}, z_T\right)}{m\, e^{\mu T}} \; |z_0 = z\right] \;,\\
\frac{\bar{V}_t(p,z,m)}{m} \quad &= \quad -\theta_t(z) + \max_{T} \int_0^T e^{-(\rho+\lambda)s} \; c_{t+s}^{1-\epsilon\eta} \; \mathbb{E}\left[z_s^{\eta-1} \; F\left(\frac{p}{p^*_{t+s}}\right) \; |z_0 = z\right] ds \;+\\
&+ \quad e^{-(\rho+\lambda)T} \; c_{t+T}^{1-\epsilon\eta} \; \mathbb{E}\left[\frac{V_{t+T}\left(p, z_T\right)}{m\, e^{\mu T}} \; |z_0 = z\right] \;,
\end{aligned}$$

From this recursion we can show the following homogeneity for the nominal expected discounted net profits:

$$V_t(p,z,m) \;\; = \;\; m\ z^{\eta-1}\ V_t\left(\frac{p\ z}{m},1,1\right) \;\; \text{for all} \;\; p,z,m,t \geq 0 \tag{38}$$

and hence we define the expected discounted profits relative to the economy money supply, for a normalized shock and a price relative to the flexible optimal price. This value function, scaled by an index of real aggregate demand, turns out to be equal to the ratio of the nominal value function to the nominal flexible price profits, which gives an economic interpretation to the normalization:

$$v_t(g) \;\; \equiv \;\; \frac{V_t\left(R\alpha \frac{\eta}{\eta-1}\ e^{g},1,1\right)}{c_t^{1-\epsilon\eta}\ F(1)} = \frac{V_t(p,z,m)}{m\ \Pi_t^*(z)} \tag{39}$$

We can write a one state Bellman equation associated to the real net profits immediately after the time of an observation as:

$$\begin{aligned}
v_t(g) \;\; &= \;\; \max\left\{\hat{v}_t\ ,\ \bar{v}_t(g)\right\} \;\; \text{where} \\
\hat{v}_t \;\; &= \;\; -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = \hat{g}\right] ds \; + \\
&+ \;\; e^{-(\rho+\lambda)T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1} v_{t+T}\left(g_T\right) \mid g_0 = \hat{g}\right] \;, \\
\bar{v}_t(g) \;\; &= \;\; -\theta + \max_{T} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = g\right] ds \; + \\
&+ \;\; e^{-(\rho+\lambda)T} \left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1} v_{t+T}\left(g_T\right) \mid g_0 = g\right] \;,
\end{aligned}$$

The dynamics of the state g in the inaction region are given by $dg = (\gamma - \mu)dt + \sigma dB$.

Finally, we define the function $f(\cdot)$ as

$$\begin{aligned}
f(s,g') &\equiv \mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1} \frac{F\left(e^{g_s}\right)}{F(1)} \mid g_0 = g'\right] \\
&= \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma s+\sigma\sqrt{s}\,x\right)}\ \frac{F\left(e^{g'\ +(\gamma-\mu)s+\sigma\sqrt{s}\,x}\right)}{F(1)} dN(x),
\end{aligned}$$

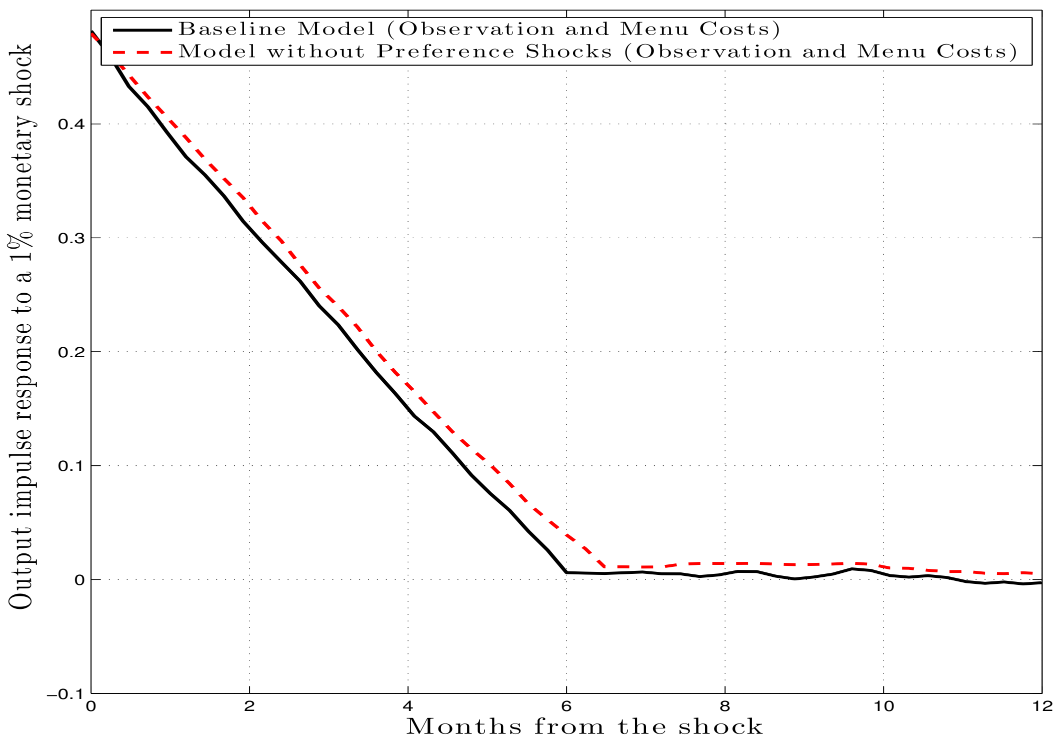
which after simple algebra is given by

$$f(s,g') = \eta\ e^{\ (1-\eta)g'+(\eta-1)\mu s} - (\eta-1)\ e^{-\eta g'+(\eta\mu-\gamma+0.5\sigma^2)s}. \tag{40}$$

$\square$

Finally, we solve numerically for the equilibrium dynamics following an unexpected $\delta = 1\%$ permanent increase in m_t at $t = 0$, in the model of this section and we compare it to the equilibrium dynamics predicted by our baseline model of Section 3.2. We assume that the monetary shock is not observed until the firm pays the observation cost. Figure VIII displays output response to the monetary shock in the two models.

Figure VIII: Impulse response of $\log c_t$ to an unexpected 1% increase in m_t, $\delta = 1\%$



Note: All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, i.e. 1.3 adjustments per year, and to the average size of price changes equal to 8.5%.

G On the relation with Demery (2012)

This appendix discusses some differences with respect to Demery's (2012) model and results. The assumptions in the two papers are quite similar (a presentation of the assumptions is given at the end of the appendix). We thus start by discussing the different results which are mostly due to the different solution methods used by the 2 papers. A key difference with our paper is that we compute the exact output response to monetary shocks, i.e. we do not make any parametric assumption on the dynamics of aggregate output when solving for the consumption or output impulse response. After discussing Demery's (2012) parametric assumptions we divide the analysis in two parts. First, we discuss those differences in results that follow directly from the parametric assumptions, and that relate to the shape and magnitude of output impulse responses to monetary shocks of different sizes. Second, we discuss those differences in results that cannot be explained by Demery's (2012) parametric assumptions, and that relate to the magnitude and persistence of output impulse responses as a function of different combinations of observation and menu costs. We will argue that Demery's (2012) results about output impulse response analysis are biased by the computation procedure he followed. Our argument builds on comparing Demery's (2012) estimates for the consumption and "output-gap" processes. Given the absence of capital, consumption is equal to output in this model. Given that, without nominal frictions, monetary shocks have no real effects on output, output and output-gap dynamics only differ by a constant. Thus the estimated parameters for the consumption and output-gap processes should be fully consistent with each other, predicting the same impulse responses. Demery's (2012) estimates fail this simple consistency test. Details follow.

Demery (2012) obtains the impulse responses presented in his Figure 2 from a regression of the "output-gap" on its own lag and on the growth rate in nominal wages, i.e. $\log(w_t/w_{t-1}) = \mu + v_t$. While the equation is not reported in the paper, the text suggests the following process for the (log-) output-gap

$$\hat{g}_t = \beta_1 \hat{g}_{t-1} + \beta_2 \log(w_t/w_{t-1}) + u_t, \tag{41}$$

where $\hat{g}_t \equiv \log(y_t) - \log(\tilde{y}_t)$ is the (log-) output-gap, with y_t denoting aggregate output, and $\tilde{y}_t$ being its counterpart in a benchmark economy with no adjustment and observation costs. Notice that, since nominal shocks are the only aggregate shocks affecting the economy, output is constant in the benchmark economy as monetary shocks have no effect on output in absence of nominal rigidities, i.e. $\tilde{y}_t = \tilde{y}$ for all t.[27] We claim that Demery's parametric assumptions on the path of aggregate output and consumption have important consequences for the predictions of the model about the real effects of monetary shocks.

First, according to Demery's (2012) Figure 2, the *shape* of the output response is exponentially decaying in the time elapsed since the monetary shock, independently of the combination of observation and menu costs. This exponential profile is a direct consequence of the assumed functional form in equation (41). In contrast, we find that the shape of the output response decays at an approximate linear rate for a large range of combinations of observation and menu costs, and that it is exactly linear for the observation cost only model. Furthermore, we find that the results about the output effects of monetary shocks depend on the size of the monetary shock, with larger shocks featuring smaller real effect per unit of increase in money, since more firms pay the menu cost. Instead, the impulse response in equation (41) assumes that shocks of any size have a proportional effect on aggregate consumption, a feature that we find at odds with models where the decision rules are state dependent.

Second, the findings about the size of the output effect, and how it relates to the relative size of menu and observation costs, differ between the two papers.[28] We find that the real

[27] Demery estimates equation (41) with a regression on model simulated data. Estimates of β_1 and β_2 are reported at the bottom of his Table 3, while an estimate of β_0 (a constant in the regression) is missing. Different parameters of the model, corresponding to different menu vs observation costs, imply different values of $\beta' s$.

[28] Both Demery's and our paper analyze the output effect of a monetary shock in economies that differ in the ratio of menu to observation cost, and are characterized by the *same* average size and frequency of price changes.

effects of monetary shocks are decreasing in the ratio of menu to observation cost, with the observation-cost-only model predicting the largest effects. Thus, our baseline model with both menu and observation costs predicts real effects that are in between the two polar cases with menu-cost-only and observation-cost-only. In contrast Demery shows that the model with both menu and observation costs has larger and more persistent real effects than the observation cost only model (see his Figure 2).

We suspect that Demery's (2012) results about output impulse response analysis are biased by the computation procedure he followed. First, notice that Demery reports estimates of the following AR(1) process for log-consumption at the top of Table 3,

$$\log(c_t) = \lambda_0 + \lambda_1 \log(c_{t-1}) + \lambda_2 v_t + e_t, \tag{42}$$

which he uses to approximate firm's beliefs about the path of aggregate consumption (but not to construct the impulse response function, which is instead computed with equation (41)). Demery estimates the $\lambda's$ by simulating the model, just like he does to estimate the $\beta's$. Next we argue that these 2 equations are completely equivalent so their estimates should deliver a similar message, which however is not the case.

Using $y_t = c_t$ (the model has no capital), the process in equation (41) can be rewritten as: $\log(c_t) = (1 - \beta_1)\log(\tilde{c}) + \beta_1 \log(c_{t-1}) + \beta_2(\mu + v_t) + u_t$.[29] In order for the process in equation (41) to give an unbiased estimates of the output response to the monetary shock, and in particular of β_2, the econometrician would need to add a constant to its estimate. Let's denote such constant as β_0 and further rewrite the process in equation (41) as: $\log(c_t) = \beta_0 + (1-\beta_1)\log(\tilde{c}) + \beta_1 \log(c_{t-1}) + \beta_2(\mu + v_t) + u_t$. The latter and equation (42) are equivalent, requiring $\beta_1 = \lambda_1$, $\beta_2 = \lambda_2$ and $\beta_0 = \lambda_0 - (1 - \lambda_1)\log(\tilde{c}) - \lambda_2\mu$. No estimate of β_0 is given (or is mentioned) in Demery's paper (nor in Table 3), though he explained to us in private correspondence that he used one. Yet, the comparison between the estimates of λ_i and β_i allows to test for a bias in the estimates: λ_2 and β_2, as well as λ_1 and β_1, are indeed substantially different (that is, taking into account the estimates' standard errors), suggesting that the estimates may indeed be biased (given the presence of the constant, the source of the bias is unclear to us). As a consequence, impulse responses predicted by the process estimated in equation (42) differ substantially from the impulse responses predicted by the process estimated in equation (41). Interestingly, Demery's estimates of equation (42) predict, as we do, that the model with both menu and observation costs has real effects on consumption that are in between the polar cases with menu cost only and observation cost only, thus contradicting his own Figure 2.

Another important difference with our model is that Demery (2012) only studies the case where aggregate shocks are not observed (until the first observation of the state), while we also allow for the possibility that aggregate shocks are freely observed on impact. Thus, he does not investigate the role of the information structure about monetary shocks for predictions

[29] Another small difference, concerning the definition of output, is inconsequential for the difference in results. In particular, while we define aggregate consumption as the CES aggregator over the consumption of the different varieties $c_t = (\int_0^1 (C_t(i))^{\frac{\eta-1}{\eta}}\, di)^{\frac{\eta}{\eta-1}}$, Demery defines aggregate output based on the linear aggregator: $\hat{y}_t \equiv \int_0^1 C_t(i)\, di$, instead of $y_t = c_t$, to estimate equation (41). Nevertheless, $\hat{y}_t$ and y_t (or $\hat{c}_t$ and c_t) are equivalent at a first order approximation, so that we can still use the estimates of equation (42) to construct impulse responses and verify equation (41).

about the real effects such shocks. We show how such assumption matters as a function of the size of the monetary shock. We find that such assumption matters for monetary shocks to the growth rate of money supply larger than 2%, above which the model with unobserved monetary shocks substantially overstates the real effect of monetary shocks relatively to the model with observed monetary shocks.

There are a few differences in assumptions in our model relatively to Demery (2012). These differences do not have major quantitative or qualitative implications, but in some cases the assumptions we make may favor exposition and comparison to the existing literature. In particular, the tight relationship with the model we solved in Alvarez, Lippi, and Paciello (2011) provides reliable analytical approximations to the firms' optimal policy which help both the interpretation of results and make the numerical solution more reliable. First, Demery assumes that menu and observation costs are set as a constant number of labor inputs. As a result, the scale of such costs relatively to firms' profits varies as a function of the productivity of each firm, with more productive firms facing relatively lower observation and adjustment costs. In our model menu and observation cost are a number of labor inputs proportional to each own firms' frictionless profit, so that the scale of such costs relatively to firms' profits does not vary across firms as a function of productivity. In addition, in our baseline model we have perfectly negatively correlated idiosyncratic productivity and demand shocks so that steady state frictionless profit is constant. Thus, in our baseline model menu and observation costs are a constant number of labor inputs, as well as being a constant fraction of frictionless profit. Second, in Demery's (2012) model idiosyncratic productivity shocks follow a mean reverting AR(1). Instead in our baseline model productivity shocks follow a continuous time random walk. Nevertheless, we also consider in Appendix F a case of mean reverting shocks, where products are reset at exponentially distributed times, and show that quantitative results are similar. Third, Demery assumes that time is discrete, with the time period being a month, while we assume continuous time, and use weekly time periods for computational purposes. Apart from modeling convenience, this assumption is inconsequential for the results. Finally, Demery assumes that aggregate (log) nominal wages are a random walk, while we consider a once and for all increase in (log) nominal wages.