Credibility For Sale*

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Abstract

We develop a model where a sovereign's incentive to repay its debt depends on the identity of its creditors. Higher exposure to official lenders improves incentives and thus credibility, for instance, because default would jeopardize the benefits from membership in a club (such as EU or EMU). But higher exposure also carries costs, because of reduced flexibility ex post and because official lenders may collude to extract rents. We characterize the equilibrium composition of debt across creditor groups as well as equilibrium debt prices. Our model can account for an important feature of sovereign debt crises: Official lending to sovereigns takes place only in times of debt distress and carries a favorable rate. It also offers a novel perspective on the interaction between deficits, debt overhang and the availability of official funds in determining default risk.

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1 Introduction

The recent sovereign debt crisis in the EU, like many other crises before, has exhibited two features: Sovereign borrowers with large borrowing requirements who face high interest rates on credit markets; and official lenders (such as the IMF and EU governments) who step in to provide funds at a cheaper rate than private creditors.[1] With variations, this pattern has repeated itself many times during the last decades. But to the best of our

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[1]Another common feature, which our analysis will abstract from, is the implementation of adjustment policies (fiscal adjustment, currency devaluation, structural reforms etc.) insisted upon as a condition for the provision of official funds.

knowledge, no formal theory except work by Boz (2011) (discussed below) exists that can account for the observed change in the composition of sovereign debt in response to heightened levels of sovereign debt stress.[2]

In this paper, we argue that this change arises because the incentives of a sovereign borrower to repay its debt depend on the identity of its creditors. In particular, a borrower that defaults may suffer larger losses when it defaults against official creditors—the "enforcer"—than against private creditors. Borrowing from the enforcer therefore enhances credibility and improves access to funding.[3] But it also carries costs, because it reduces the borrower's flexibility ex post and because official creditors may collude and extract rents or the provision of funds is costly to them. The equilibrium debt composition during "normal" periods and periods of debt stress is a reflection of the relative size of these benefits and costs.

For concreteness, consider a specific pair: A borrowing country, "Greece," and an enforcer "(other) countries from the Euro-EU group" (henceforth, EU). Greece lacks commitment to repay its debt, as in Eaton and Gersovitz (1981), and this lack of commitment creates social losses. In particular, when choosing whether to repay maturing debt or rather default on it, the government trades off the cost of debt service and the output losses triggered by default. This ex post optimal default choice does not internalize the effect on equilibrium prices ex ante. The size of the resulting welfare losses rises with the amount of debt being issued.

The lack of commitment problem is alleviated when Greece enters into a credit relationship with the EU since such a relationship increases the cost of defaulting. In particular, a Greek default on official EU loans jeopardizes the benefits of club membership in the European Union as Greece might be expelled from the Euro area or structural fund payments and other transfers might be cut. More generally, a Greek default on official EU loans might lead the other EU countries to enact policies that are less favorable to Greek interests than the policies that would have been enacted in the absence of a default.[4]

While mitigating Greece's commitment problems and improving its access to funding, the credit relationship with EU also carries costs. On the one hand, as mentioned above, EU may charge a premium in order to extract rents from Greece or to cover funding costs that are specific to official loans. The latter may arise ex ante, pertaining to the design and creation of official institutions and mechanisms that manage the EU-Greece credit relationship. Or, they may arise ex post if default occurs in spite of the mitigated commitment problem.[5] On the other hand, Greece's gains from credibility ex ante come

[2]We associate sovereign debt stress with large borrowing requirements, high interest rates on credit markets for sovereign debt and weak economic conditions in the borrowing country.

[3]Arguably, borrowing from the enforcer may also improve risk sharing. In particular, having the enforcer monitor the sovereign may allow lenders to differentiate between unwillingness to pay on the one hand and inability to pay on the other. This, in turn, may allow to structure debt repayment in a more explicitly state contingent manner.

[4]To ensure broad political support for enforcement ex post it might be advantageous for EU members to secure club (union-wide) participation in the official lending operations even if most funding is provided by a small set of countries.

[5]In particular, exposure of EU to Greece may cause disruptions to EU fiscal policy or its financial institutions in the wake of default, and expulsion of Greece from EU (or lesser "punishments") may hurt

at the cost of reduced flexibility ex post in those states where a default is attractive.[6]

Our model embeds the benefits and costs of official lending in a standard sovereign debt framework. We assume that defaulting against official creditors triggers income losses for the borrowing country that go beyond those that are suffered when all debt is held privately. Official lending therefore strengthens incentives, and because the repayment rates on official and private debt are linked it can improve the sovereign's funding conditions in official and private debt markets.

The extent to which the borrowing country exploits this option to gain credibility mainly depends on three factors. First, the marginal increase in default costs and thus, gain in credibility due to official lending. Second, the premium charged by official lenders. And third, the intensity of the borrowing country's financing needs as determined by the country's relative impatience and its output profile. While we consider general specifications for the additional default costs associated with official lending and the official finance premium we also work with specific examples that reflect different potential causes for this finance premium. We show that the model can account for the stylized facts mentioned above, that is, the shift away from private to official lending during periods of debt distress, combined with interest rates considerably below what the market rate would have been in the absence of official lending.

We also examine how long-term debt overhang affects default risk. We find that debt overhang affects a sovereign's propensity to default depending on the source of new funding available to the borrowing country. When debt overhang is large and refinancing needs high then the sovereign is more likely to default on outstanding debt when the new funding is provided by official rather than private sources. This holds true although, by assumption, default does not trigger exclusion from private funding markets. Interestingly, since the sovereign's demand for new official funds may be higher after a default than otherwise, and since official lenders profit from such higher demand, official creditors may actually encourage the sovereign to default under these circumstances. When the sovereign's funding needs are limited, in contrast, then the borrowing country is more likely to tap private credit markets and default risk is lower.

The literature on the composition of sovereign debt by type of creditor is scant. Boz (2011) reviews the literature on IMF lending, summarizes empirical evidence and presents a quantitative model of a sovereign that may borrow from private lenders and the IMF. She assumes that private lending is subject to default risk, IMF lending is default risk free, and the cost of IMF funds exceeds the risk free rate by an exogenous surcharge that increases with the amount of IMF lending. She also assumes that IMF lending triggers an increase in the sovereign's discount factor (in a reduced form way, this is meant to capture conditionality). Her model predicts modest, countercyclical and intermittent IMF lending.

EU itself if club membership had been chosen optimally in the first place. The presence of default costs *for EU* may create a time inconsistency problem. The enforcer might want to bail out the borrowing country rather than letting a costly default happen or, if enforcement power derives from the enforcer's ability to punish, the enforcer might want to avoid such costly punishment ex post. Concern for reputation could help EU overcome this problem, for example if it extracts rents from the lending relationship and there is a possibility of repeat business with other club members or, more generally, if interactions within EU are multidimensional and interconnected.

[6]See Zame (1993) for a discussion of the insurance benefits of implicitly state contingent debt.

In the model proposed here, official lending does not change the sovereign's discount factor; the borrower's objective and the costs of official funding therefore are disconnected. Also in contrast to the setup in Boz (2011), we assume that the repayment rate on official and private funds is uniform.[7] We believe that this assumption is reasonable for episodes like the current European sovereign debt crisis where official lenders cannot afford to crowd out private funding over a longer period. This view is supported by the conditions of the Greek debt exchange in Spring 2012 and by the more recent discussions about financial support for Spain.[8]

Bolton and Jeanne (2011) analyze the interaction between multiple sovereigns of different credit quality and the banking system in a financially integrated area. They argue that a country issuing 'safe haven' government debt may derive rents from exploiting its position as monopolistic supplier of this safe asset. In the model proposed here, in contrast, official lenders may exploit their position as monopolistic suppliers of credibility. Niepelt (2011) analyzes the composition of sovereign debt across maturities rather than lenders, as considered here, and Diamond and He (2012) analyze the implications of the maturity structure of debt overhang on investment decisions. Finally, Tirole (2012) distinguishes between ex-post bailouts with the aim to avoid collateral damage and ex-ante risk-sharing, for example joint-and-several liability, among sovereigns.

The rest of the paper is organized as follows. We set up the model in Section 2 and characterize equilibrium in Section 3. Section 4 contains tractable examples that help build intuition. In Section 5, we present an extension with long-term debt that sheds light on the interaction between default and refinancing decisions. Section 6 concludes.

2 The Model

The economy lasts for two periods, $t = 1, 2$. It is inhabited by a representative taxpayer, a government and foreign investors.

Taxpayers neither save nor borrow.[9] They have time- and state-additive preferences over consumption with strictly increasing and concave felicity function $u(\cdot)$ and discount

[7] Boz (2011) rationalizes her assumption of default risk free official lending by the fact that historically, very few IMF loans went sour (p. 75).

[8] Zettelmeyer, Trebesch and Gulati (2012) report that the Greek debt exchange put private and official lenders (the EFSF) on an equal footing. "Greece and the remaining signatories of the agreement committed to a payment schedule in which the EFSF and bondholders would be repaid pro-rata and on the same day. In the event of a shortfall in payments by Greece, the common paying agent committed to distributing allocating this shortfall pro rata between the EFSF and the bondholders. Hence, the co-financing agreement makes it difficult for Greece to default on its bondholders without also defaulting on the EFSF" (p. 25).

Regarding the financial support for Spain, The Wall Street Journal (June 29, 2012, Investors Cheer Europe Deal) writes that Merkel's agreement "to make ESM loans to Spain equal to Spanish bonds in creditors' pecking order was largely a recognition by Germany that this was necessary to protect Spain's ability to sell bonds ...."

[9] Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with "savers" and "spenders."

factor $\delta \in (0,1)$. Welfare of taxpayers in period t is given by

$$\mathbb{E}\left[\sum_{j\geq t}^{2}\delta^{j-t}u(y_j^p-\tau_j)|s_t,\pi_t\right],$$

where y_t^p denotes pre-tax income, τ_t taxes, s_t the state (to be specified below) and π_t the policy choice in period t. We often write $\mathbb{E}_t[\cdot]$ instead of $\mathbb{E}[\cdot|s_t,\pi_t]$.

The government maximizes the welfare of taxpayers. It chooses the repayment rate on maturing debt, r_t, issues zero-coupon one-period debt, b_{t+1}, and (residually) levies taxes. Without loss of generality, public spending other than debt repayment is normalized to zero. Crucially, the government cannot commit its successors (or future selves).

Foreign investors are risk neutral, require a risk free gross interest rate $\beta^{-1}>1$ and hold all government debt (since taxpayers do not save).[10] To guarantee positive debt positions, we assume $\delta \ll \beta$ as is standard in the sovereign debt literature (see, for example, Aguiar and Gopinath (2006) or Arellano (2008)). Foreign investors are composed of private and official lenders. Private lenders are competitive. Official lenders—we refer to them as "the enforcer"—coordinate amongst themselves and may behave non-competitively vis-a-vis the borrowing country. The amount of zero coupon debt held by the enforcer is given by b_{t+1}^e; and that held by private lenders by $b_{t+1}-b_{t+1}^e$. Short-selling is not permitted, $0\leq b_{t+1}^e\leq b_{t+1}$.

A sovereign default—a situation where the repayment rate falls short of unity—triggers income losses for taxpayers (cf. Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Aguiar and Gopinath, 2006; Arellano, 2008). More specifically, a default in period t triggers an income loss $L_t\geq 0$ where L_t is the realization of an i.i.d. random variable with cumulative distribution function $F_t(\cdot)$ and associated density function $f_t(\cdot)$, $f_t(L)>0$ for all $L_t\geq 0$. In the presence of official lending, default triggers additional income losses for the borrowing country, given by $\mathcal{L}(b_2^e)$ with $\mathcal{L}(0)=0$ and $\mathcal{L}'(b_2^e)\geq 0$ for all $b_2^e>0$. As discussed in the introduction, default occurs uniformly across privately and officially held debt.

The sequence of events in each period is as follows. In the beginning of the period, L_t and the realization of the exogenous stochastic output process in period t, y_t, become known. The state is given by $s_t=(y_t,L_t,b_t,b_t^e)$. Conditional on s_t, the government chooses its policy instruments, $\pi_1=(r_1,b_2,b_2^e)$ or $\pi_2=r_2$, taking as given the equilibrium relationship between these choices and the asset prices in that period.

Let $q_1(s_1,\pi_1)$ and $p_1(s_1,\pi_1)$ denote the price in period $t=1$ and state s_1 of debt issued to private and official lenders, respectively, if the government implements policy π_1. When choosing its policy, the government takes the price functions $q_1(s_1,\cdot)$ and $p_1(s_1,\cdot)$ as given. Define the borrowing country's deficit in period $t=1$ and state s_1 under policy π_1 as the funds raised, $d_1(s_1,\pi_1)\equiv(b_2-b_2^e)\,q_1(s_1,\pi_1)+b_2^e p_1(s_1,\pi_1)$. The budget constraint of the government is $\tau_1=b_1r_1-d_1(s_1,\pi_1)$ and pre-tax income of taxpayers equals $y_1^p=y_1-\mathbf{1}_{[r_1<1]}L_1$ and $y_2^p=y_2-\mathbf{1}_{[r_2<1]}(L_2+\mathcal{L}(b_2^e))$ where $\mathbf{1}_{[x]}$ denotes the indicator function for

[10] The assumption that the sets of taxpayers and investors do not "overlap" simplifies the analysis and does not matter for the main results.

event x. Taxpayers' consumption therefore is given by $c_1 = y_1 - b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1 + d_1(s_1, \pi_1)$ in the first period and $c_2 = y_2 - b_2 r_2 - \mathbf{1}_{[r_2<1]}(L_2 + \mathcal{L}(b_2^e))$ in the second period.

Let $G_1(s_1)$ denote the value of the government's objective function conditional on state s_1. We have

$$\begin{aligned} G_1(s_1) &= \max_{r_1 \in [0,1],\ 0 \le b_2^e \le b_2} u(y_1 - b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1 + d_1(s_1, \pi_1)) + \delta \mathbb{E}_1 \left[G_2(s_2) \right] \\ &\qquad \text{s.t. } p_1(s_1, \cdot), q_1(s_1, \cdot), \\ G_2(s_2) &= \max_{r_2 \in [0,1]} u(y_2 - b_2 r_2 - \mathbf{1}_{[r_2<1]}(L_2 + \mathcal{L}(b_2^e))). \end{aligned}$$

As reflected by the first value function, the government chooses the repayment rate on maturing debt as well as debt issuance in period $t = 1$ in order to maximize the sum of the flow utility from consumption in that period and the discounted expected continuation value. The latter represents the maximized flow utility from consumption in period $t = 2$, as reflected by the second value function. Importantly, the default rate in period $t = 2$ is chosen by the government in that period alone, due to lack of commitment.

An *equilibrium* conditional on the official-funds price function $p_1(\cdot, \cdot)$ then consists of value and policy functions in periods $t = 1$ and $t = 2$ and a private-funds price function $q_1(\cdot, \cdot)$ such that

i. conditional on s_1 as well as the price functions, the policy choices are optimal for the borrowing country,

$$\pi_t(s_t) \text{ solves } G_t(s_t),\ t = 1, 2;$$

ii. the private-funds price function reflects rational expectations as well as the participation constraint of competitive private lenders (i.e., investors earn the expected, competitive rate of return),

$$q_1(s_1, \pi_1) = \beta\ \mathbb{E}_1 \left[r_2(s_2) \right]. \tag{1}$$

Note that in the definition of equilibrium we use a general specification for the price function of official funds, $p_1(\cdot, \cdot)$. This allows us to study debt policy under alternative assumptions about the institutional environment in place and the enforcer's cost of capital. Consider for example the case in which the risk-neutral enforcer has negligible bargaining power vis-a-vis the borrowing country. In equilibrium, the price $p_1(s_1, \pi_1)$ then is set such that the enforcer attains no more than its outside value. If exposure to the borrowing country after a default generates some costs $\mathcal{C}(b_2^e)$ (beyond capital losses) to the enforcer then the enforcer's binding participation constraint implies

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e\ \mathbb{E}_1[r_2(s_2)] - \beta\ \text{Prob}[r_2(s_2) < 1]\ \mathcal{C}(b_2^e). \tag{2}$$

As another example, consider the case where the enforcer has sufficient bargaining power vis-a-vis the borrowing country to negotiate a fixed "mark-down" relative to the price on private markets. The equilibrium price of official funds then equals

$$p_1(s_1, \pi_1) = \kappa\ q_1(s_1, \pi_1),\ 0 < \kappa < 1. \tag{3}$$

Note that in both examples, $p_1(s_1, \pi_1) \leq q_1(s_1, \pi_1)$. That is, credibility not only is for sale, but it comes at a price.

To simplify notation, we define the normalized price difference, $\beta\Delta_1(s_1, \pi_1) \equiv q_1(s_1, \pi_1) - p_1(s_1, \pi_1)$. The deficit therefore can be expressed as

$$\begin{aligned} d_1(s_1, \pi_1) &= (b_2 - b_2^e)q_1(s_1, \pi_1) + b_2^e p_1(s_1, \pi_1) \\ &= b_2 q_1(s_1, \pi_1) - b_2^e \beta\Delta_1(s_1, \pi_1). \end{aligned} \tag{4}$$

We proceed under the assumption that the government's program is well behaved and gives rise to smooth policy functions. In the examples considered below, we verify that this is indeed the case.

3 Analysis

Choice of Repayment Rate Consider first the government's choice of repayment rate in the last period, r_2. Since the marginal cost of reducing r_2 equals zero for $r_2 < 1$, the optimal repayment rate equals either zero or unity. In particular,

$$r_2(s_2) = \begin{cases} 1 & \text{if } L_2 \geq b_2 - \mathcal{L}(b_2^e) \\ 0 & \text{if } L_2 < b_2 - \mathcal{L}(b_2^e) \end{cases} . \tag{5}$$

Condition (5) states that the government chooses to default when the resulting income losses, $L_2 + \mathcal{L}(b_2^e)$, are smaller than the amount of debt coming due.[11] Condition (5) is consistent with the notion that governments tend to default when the political costs—specifically income losses of pivotal pressure groups—are low. Governments also tend to default when economic activity is depressed (Borensztein, Levy Yeyati and Panizza, 2006; Tomz and Wright, 2007). The model is consistent with this fact as well when it is slightly extended to include direct default costs for the government in addition to the income losses for taxpayers. Note that corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those invoked here.

Equation (5) pins down the expected repayment rate. From (1), the equilibrium price of private funds equals

$$q_1(s_1, \pi_1) = \beta(1 - F_2(b_2 - \mathcal{L}(b_2^e))). \tag{6}$$

This price is purely forward looking and independent of the government's current choice of repayment rate, r_1. If the same holds true for the price of official funds, $p_1(s_1, \pi_1)$, (and

[11] If selective default were possible privately held debt would seem to be more likely to be exposed to default risk than official debt. Letting b_2^p denote privately held debt and r_2^p, r_2^e the repayment rates on b_2^p and b_2^e, respectively, a reasonable specification allowing for selective default may be as follows:

$$c_2 = y_2 - b_2^e r_2^e - b_2^p r_2^p - \mathbf{1}_{[r_2^e \text{ or } r_2^p < 1]} L_2 - \mathbf{1}_{[r_2^e < 1]} \mathcal{L}(b_2^e).$$

Clearly, selective default against official lenders would never be optimal in this case. But selective default against private lenders could be optimal as long as $b_2^e < \mathcal{L}(b_2^e)$. Boz (2011) completely rules out default against the enforcer.

thus, the price difference $\Delta_1(s_1, \pi_1)$) then the deficit in period $t = 1$ also is independent of r_1 and the equilibrium repayment rate satisfies

$$r_1(s_1) = \begin{cases} 1 & \text{if } L_1 \geq b_1 \\ 0 & \text{if } L_1 < b_1 \end{cases} . \tag{7}$$

Independence of $p_1(s_1, \cdot)$ and r_1 reflects the assumption that the trade-offs present in the lending relationship between the enforcer and the borrowing country from period $t = 1$ onwards are independent of the repayment rate on initially outstanding debt. For the particular cases discussed above (see equations (2) and (3)) this assumption is clearly satisfied. In other settings it is not. For example, in section 5 where we introduce long-term debt, the price of newly issued debt depends on the default decision of the government because this decision affects the extent of debt overhang.

Another reason for $p_1(s_1, \cdot)$ to vary with r_1 could be that the enforcer's participation constraint holds "before" r_1 is chosen. Equation (2) then would be replaced by the condition

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e \, \mathbb{E}_1[r_2(s_2)] - \beta \, \text{Prob}[r_2(s_2) < 1] \, \mathcal{C}(b_2^e) + b_1^e r_1$$

and the equilibrium price of newly issued debt purchased by the enforcer would depend on the repayment rate in the initial period. In this case, the enforcer would be indifferent between lowering r_1 by an amount ϵ and increasing p_1 by the amount $\epsilon b_1^e / b_2^e$. Such a combination of changes in r_1 and p_1 could strictly increase the welfare of the borrowing country if $b_1^e < b_1$, that is, if there were another group of investors that could be "burned."[12] Consequently, a default in the first period could be in the joint interest of the borrowing country and the enforcer.[13] We do not pursue this variation of the model here further.

Choice of Debt Issued to Private Lenders Issuing debt to private lenders has two effects on the deficit. On the one hand, it raises funds from the marginal unit of debt, in proportion to its price. On the other hand, it reduces the funds raised from inframarginal units of private and official lending, by changing the price of these units. This latter effect is a direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent repayment decisions. Formally, from (4) and (6),

$$\begin{aligned} \frac{\partial d_1(s_1, \pi_1)}{\partial b_2} &= q_1(s_1, \pi_1) + b_2 \frac{\partial q_1(s_1, \pi_1)}{\partial b_2} - b_2^e \beta \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2} \\ &= q_1(s_1, \pi_1) - b_2 \beta f_2(b_2 - \mathcal{L}(b_2^e)) - b_2^e \beta \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2}. \end{aligned}$$

[12] Naturally, a proper specification of the problem would require that such incentives are recognized and priced ex ante.

[13] Broner, Martin and Ventura (2010) argue that secondary markets undermine the ability of a sovereign to discriminate between groups of lenders. The above argument suggests that the borrowing country may collude with lenders rolling over its debt and discriminate against other holders of outstanding debt by choosing r_1 and p_1 appropriately.

Funding from private sources is maximized at the peak of the "debt-Laffer curve" which is reached when the above marginal effect equals zero. A completely myopic government ($\delta = 0$) maximizes the deficit in each period and attains the maximum of the debt-Laffer curve. A non-myopic government ($\delta > 0$), in contrast, does not maximize the deficit because each additional unit of debt strictly reduces the continuation value. Both in the myopic and the non-myopic case, the equilibrium value of b_2 therefore is (weakly) smaller than the value that attains the maximum of the debt-Laffer curve. Moreover, this equilibrium value (weakly) exceeds b_2^e, due to the short-selling constraint vis-a-vis private investors. In the following, we refer to the range of b_2 values defined by the lower bound of b_2^e and the upper bound of the maximizer of the debt-Laffer curve as the "relevant range" for b_2.

Let λ and μ denote the multipliers associated with the short-selling constraints $0 \leq b_2^e$ and $b_2^e \leq b_2$, respectively. The effect of a marginal increase in debt issued to private lenders on the government's program is given by

$$\frac{\partial G_1(s_1;\pi_1)}{\partial b_2} = u'(c_1)\frac{\partial d_1(s_1,\pi_1)}{\partial b_2} + \delta\frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} + \mu$$

which can be expressed as[14]

$$\begin{aligned}(1 - F_2(b_2 - \mathcal{L}(b_2^e)))(\beta u'(c_1) - \delta\mathbb{E}_1[u'(y_2 - b_2)]) \\ - u'(c_1)\beta\left(b_2 f_2(b_2 - \mathcal{L}(b_2^e)) + b_2^e\frac{\partial \Delta_1(s_1,\pi_1)}{\partial b_2}\right) + \mu. \qquad (8)\end{aligned}$$

The first part of this marginal effect represents the consumption smoothing benefit from the marginal unit of debt. It differs from the corresponding expression in the case without default risk because the price of debt equals $\beta(1 - F_2(b_2 - \mathcal{L}(b_2^e)))$ rather than β and because debt repayment occurs with probability $(1 - F_2(b_2 - \mathcal{L}(b_2^e)))$ rather than always.[15] The marginal rate of substitution between current and future consumption and thus, the profile of output, as well as the relative price between current and future consumption determine the strength of the consumption smoothing benefit.

The second part of the marginal effect arises because the repayment probability and thus, the price of debt depends on the quantity issued. This second part would be absent in a model with commitment. Because of the negative effect on the repayment probability, each extra unit of debt issued raises the interest rate on inframarginal units of debt. This increase in the interest rate makes first period consumption more expensive. As

[14] We use the fact that

$$\begin{aligned}&\frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} = \\ &\quad \frac{\partial}{\partial b_2}\int_{L_2 \leq b_2 - \mathcal{L}(b_2^e)} \mathbb{E}_1[u(y_2 - L_2 - \mathcal{L}(b_2^e))|L_2]\mathrm{d}F_2(L_2) + \frac{\partial}{\partial b_2}\int_{L_2 > b_2 - \mathcal{L}(b_2^e)} \mathbb{E}_1[u(y_2 - b_2)]\mathrm{d}F_2(L_2) \\ &= \mathbb{E}_1[u(y_2 - b_2)]f_2(b_2 - \mathcal{L}(b_2^e)) - \mathbb{E}_1[u(y_2 - b_2)]f_2(b_2 - \mathcal{L}(b_2^e)) - (1 - F_2(b_2 - \mathcal{L}(b_2^e)))\mathbb{E}_1[u'(y_2 - b_2)].\end{aligned}$$

[15] With risk free debt, the marginal effect would reduce to $\beta u'(c_1) - \delta\mathbb{E}_1[u'(y_2 - b_2)]$.

a consequence, the equilibrium amount of debt issued (conditional on b_2^e) generally is smaller than that under commitment. The final part of the marginal effect, the multiplier μ, is strictly positive if the short-selling constraint $b_2^e \leq b_2$ is binding, and equals zero otherwise.

It may seem surprising that the negative welfare effect associated with the reduction of funds raised from inframarginal units of debt (the second part discussed above) is not balanced by a positive welfare effect from the reduced repayment probability of these inframarginal units in the future. In fact, this effect is present. However, it does not appear in (8) because it is equal in absolute value to a third welfare effect of opposite sign, reflecting the increased risk of future social losses in the wake of default.[16] It is these social losses that are the source of the reduced incentive (relative to the commitment case) for the government to issue debt. Niepelt (2011) contains a detailed discussion in the context of a model with multiple maturities.

Choice of Debt Issued to Official Lenders Issuing debt to official lenders while holding total debt constant (that is, substituting official for private debt) affects the deficit threefold. First, by raising the output losses of the borrowing country in case of future default, it reduces default risk and increases the price of debt. This has a positive effect on the deficit. Second, it reduces the deficit at the margin by the amount $\beta\Delta_1(s_1, \pi_1)$ if private debt is cheaper than official debt. And third, through $\Delta_1(s_1, \pi_1)$, it changes the price discount on inframarginal units of debt issued to the enforcer. Formally, from (4) and (6),

$$\frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} = b_2 \beta f_2(b_2 - \mathcal{L}(b_2^e))\mathcal{L}'(b_2^e) - \beta\Delta_1(s_1, \pi_1) - b_2^e \beta \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2^e}.$$

The effect of substituting official for private funds on the government objective function is given by

$$\frac{\partial G_1(s_1; \pi_1)}{\partial b_2^e} = u'(c_1)\frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} + \delta \frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2^e} + \lambda - \mu$$

where the multipliers reflect the two short-selling constraints. This can be expressed as[17]

$$\mathcal{L}'(b_2^e)\left(u'(c_1)\beta f_2(b_2 - \mathcal{L}(b_2^e))b_2 - \delta \mathbb{E}_1\left[\int_0^{b_2 - \mathcal{L}(b_2^e)} u'(y_2 - L_2 - \mathcal{L}(b_2^e))\mathrm{d}F_2(L_2)\right]\right)$$
$$-u'(c_1)\beta\left(\Delta_1(s_1, \pi_1) + b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2^e}\right) + \lambda - \mu. \tag{9}$$

The first part of this marginal effect reflects the consumption smoothing benefit of issuing more b_2^e. On the one hand, a larger share of official debt generates stronger

[16] Higher debt issuance increases subsequent default risk and thus, the risk of future output losses in the wake of default. The corresponding first-order welfare effects that operate through the continuation value are zero. This is a consequence of an envelope condition—the subsequent government is indifferent at the margin between bearing the costs of debt repayment on the one hand or income losses in the wake of default on the other (see footnote 14).

[17] Note that $\partial \mathbb{E}_1[G_2(s_2)]/\partial b_2^e = -\mathcal{L}'(b_2^e)\mathbb{E}_1\left[\int_0^{b_2 - \mathcal{L}(b_2^e)} u'(y_2 - L_2 - \mathcal{L}(b_2^e))\mathrm{d}F_2(L_2)\right]$ (see footnote 14).

repayment incentives and hence lower default risk. This has a positive effect on $q_1(s_1, \pi_1)$ and the deficit, and it allows the country to consume more in the first period. On the other hand, the larger share of official debt inflicts additional income losses in case default actually occurs subsequently (which happens for low realizations of L_2). The second part of the marginal effect reflects the price difference between the marginal units of private and official lending, $\beta\Delta_1(s_1, \pi_1)$, and it also reflects the fact that changing the debt composition may affect the price discount applied to inframarginal units of official funds.

Types of Equilibria The equilibrium composition of debt is determined by equations (8) and (9). Four types of equilibria may emerge. Letting $\mathcal{M}(b_2, b_2^e)$ denote the marginal effect in (8) without the multiplier and $\mathcal{M}^e(b_2, b_2^e)$ the marginal effect in (9) without the multipliers, these four types can be summarized as follows:

i. $\mu = \lambda = 0$. b_2, b_2^e interior with $\mathcal{M}(b_2, b_2^e) = \mathcal{M}^e(b_2, b_2^e) = 0$.

ii. $\mu = 0, \lambda > 0$. b_2 interior, $b_2^e = 0$ with $\mathcal{M}(b_2, 0) = 0$, $\mathcal{M}^e(b_2, 0) < 0$.

iii. $\mu > 0, \lambda = 0$. $b_2 = b_2^e > 0$ with $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) = 0$.

iv. $\mu > 0, \lambda > 0$. $b_2 = b_2^e = 0$ with $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) < 0$.

Which type of equilibrium emerges depends on the intensity of the borrowing needs as manifested by the ratio β/δ and the steepness of the output profile; the distribution function of output losses $F_2(\cdot)$; preferences $u(\cdot)$; the enforcement technology $\mathcal{L}(\cdot)$; and the price discount $\Delta(\cdot)$. In the general case, the exact contribution of these factors is difficult to isolate, due to the potential non-linearity of the functions $F_2(\cdot)$, $u(\cdot)$ or $\mathcal{L}(\cdot)$. This potential non-linearity appears to be non-essential for the central questions of interest although it might be expected to contribute to interior solutions. In the interest of characterizing equilibrium in closed form and presenting solutions that highlight the first-order determinants of the debt composition, we therefore abstract from such non-essential features.

4 Analytical Examples

We eliminate all sources of non-linearity from the model that appear to be non-essential. In particular, we assume that the utility function is linear, $u'(c) = 1$, the loss function is linear, $\mathcal{L}(b_2^e) = \mathcal{L}' \cdot b_2^e$ with $0 \leq \mathcal{L}' < 1$, and the distribution function of L_2 is uniform, $F_2(L_2) = f_2 \cdot L_2$ over the relevant range.[18] The government's value function in period

[18]The restriction $\mathcal{L}' < 1$ is required for a debt-Laffer curve to exist. Without it, official lending could completely eliminate default risk. With $\mathcal{L}' = 1$ and $\kappa = 1$, the country could attain the commitment outcome.

$t = 1$ then reduces to (net of some constants)

$$G_1(s_1) = \max_{r_1 \in [0,1],\ 0 \le b_2^e \le b_2} -b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1 + \beta[(1 - f_2 \cdot (b_2 - \mathcal{L}' b_2^e))b_2 - \Delta_1(s_1, \pi_1) b_2^e] - \delta \left\{ \int_0^{b_2 - \mathcal{L}' b_2^e} (L_2 + \mathcal{L}' b_2^e) f_2 \mathrm{d}L_2 + (1 - f_2 \cdot (b_2 - \mathcal{L}' b_2^e))b_2 \right\}. \quad (10)$$

We highlight the roles played by δ/β, $\mathcal{L}(\cdot)$ and $\Delta(\cdot)$ by working through a series of examples. These examples show that the model can account for the issuance of official debt in periods of debt distress at yields that appear favorable to the borrower compared with the yields that would have to be paid on private markets.

Exogenous Price Discount Suppose that funds provided by the enforcer carry an exogenous, constant price discount or "mark-down" relative to funds obtained from private investors, $p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1)$ with $\kappa \le 1$. The marginal effects defined above then equal

$$\begin{aligned} \mathcal{M}(b_2, b_2^e) &= (1 - F_2)(\beta - \delta) - \beta f_2 (b_2 - b_2^e(1 - \kappa)), \\ \mathcal{M}^e(b_2, b_2^e) &= \mathcal{L}'(\beta f_2 b_2 - \delta F_2) - \beta \left((1 - F_2)(1 - \kappa) + b_2^e f_2 \mathcal{L}'(1 - \kappa)\right) \end{aligned}$$

where $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$ denotes the probability of default. Holding b_2^e constant, G_1 is concave in b_2. Holding b_2 constant, G_1 can be either concave or convex in b_2^e. The determinant of the Hessian is negative and thus, the Hessian is negative definite.[19] This implies that any interior critical points of (10) represent saddle points and the equilibrium is in a corner. We consider the two corner equilibria—one with private debt and the other with official debt—in turn.[20]

If sovereign debt is exclusively funded from private sources then $\mathcal{M}(b_2, 0) = 0$. Solving for the equilibrium debt level and the associated welfare value gives[21]

$$b_2^{\mathrm{PR}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta - \delta}, \quad b_2^{e\,\mathrm{PR}} = 0, \quad G_1^{\mathrm{PR}} = \frac{1}{2 f_2} \frac{(\beta - \delta)^2}{2\beta - \delta}.$$

Note that this equilibrium is valid only when $\delta \le \beta$ due to the short-selling constraint $b_2 \ge 0$. The maximum of the debt-Laffer curve is achieved at the debt level $(2f_2)^{-1}$, the level chosen by a myopic government with $\delta = 0$, and is associated with a default probability of $1/2$.

Suppose instead that all sovereign debt is funded by official sources. Setting $\mathcal{M}(b_2, b_2^e) + \mathcal{M}^e(b_2, b_2^e)$ equal to zero and solving for the level of debt yields

$$b_2^{\mathrm{OF}} = \frac{1}{f_2} \frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad b_2^{e\,\mathrm{OF}} = b_2^{\mathrm{OF}}, \quad G_1^{\mathrm{OF}} = \frac{1}{2 f_2} \frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

[19] Except for the special case of $1 - \kappa = \mathcal{L}'$ in which the determinant is zero and thus, the Hessian indeterminate. See, for example, Simon and Blume (1994, Theorem 16.1).

[20] We assume that δ/β is sufficiently small for the sovereign to be a borrower.

[21] In the examples in this section, we abstract from the terms $-b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1$ when reporting $G_1(s_1; \pi_1)$ since, as discussed earlier, the default decision does not interact with the choice of debt instruments.

Due to the short-selling constraint $b_2^e \geq 0$ this solution is valid only when $\delta \leq \beta\kappa$. The maximum of the debt-Laffer curve now is obtained at a debt level $(2f_2(1-\mathcal{L}'))^{-1}$ (the level chosen by a myopic government) which is again associated with a default probability of 1/2. As long as $\mathcal{L}' > 0$, the debt level attaining the maximum of the debt-Laffer curve is higher in the corner with official than with private debt.

Comparing the outcomes in the two cases, note that $G_1^{\mathrm{OF}} > G_1^{\mathrm{PR}}$ whenever $b_2^{\mathrm{OF}}(\beta\kappa - \delta) > b_2^{\mathrm{PR}}(\beta - \delta)$. This follows from the fact that $G_1^{\mathrm{OF}} = b_2^{\mathrm{OF}}(\beta\kappa - \delta)/2$ and $G_1^{\mathrm{PR}} = b_2^{\mathrm{PR}}(\beta - \delta)/2$. Consequently, $G_1^{\mathrm{OF}} > G_1^{\mathrm{PR}}$ implies $b_2^{\mathrm{OF}} > b_2^{\mathrm{PR}}$ and thus, countries that borrow official funds tend to be more heavily indebted than countries borrowing private funds. This prediction of the model is consistent with the stylized fact that official debt is more likely to be observed when debt levels are high.

When studying the country's choice of debt instrument it is useful to first consider the case of $\delta = 0$. We showed earlier that the debt level corresponding to the maximum of the debt-Laffer curve is higher in the corner with official than in the corner with private debt. But this does not imply that a myopic government, unconcerned about the size of future debt liabilities, necessarily chooses official over private debt. In fact, comparing G_1^{PR} with G_1^{OF} for $\delta = 0$ reveals that this government will opt for official debt if and only if $1 - \kappa < \mathcal{L}'$. Intuitively, a myopic government aims at maximizing the value of funds obtained at present. By opting for official rather than private funds it decreases the probability of a future default (because of the additional output losses, $\mathcal{L}'(b_2^e)$) and thus the default premium paid on current borrowing. At the margin, this effect is proportional to $\mathcal{L}'$. But at the same time, it reduces the funds raised from the marginal unit of debt because official funds are available at a premium relative to private funds. This effect is proportional to $1-\kappa$. In the following, we posit that the condition $1-\kappa < \mathcal{L}'$ is met such that a myopic government favors issuing debt to official creditors.

When $\delta > 0$ the criterion for the choice of debt instrument is

$$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}} = \frac{1}{2f_2}\left[\frac{(\beta\kappa-\delta)^2}{2\beta\kappa - \delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'} - \frac{(\beta-\delta)^2}{2\beta-\delta}\right].$$

$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ is convex in δ and strictly negative at $\delta = \beta$. It follows that there exists a unique threshold value $\delta^\star$ such that for $\delta \leq \delta^\star$ (high borrowing needs) official funding is preferred while for $\delta > \delta^\star$ (low borrowing needs) private funding is preferred. The model thus predicts, in line with the stylized facts sought to explain, that episodes of high borrowing needs (as captured by a low δ/β ratio) are associated with borrowing from official rather than private sources.

Figure 1 displays a particular numerical example. The figure plots the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ for different values of δ, when $\beta = 0.9$ and $f_2 = 0.1$. The solid curve corresponds to intermediate values of enforcement power ($\mathcal{L}' = 0.25$) and price discount ($\kappa = 0.9$). Holding δ fixed, the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ increases if $\mathcal{L}'$ is raised (dashed curve for $\mathcal{L}' = 0.4$) and decreases if κ is lowered (dotted curve for $\kappa = 0.8$). Stronger enforcement power therefore raises $\delta^\star$ and renders official funding more likely while higher price discounts lower $\delta^\star$ and increase the relative advantage of private funding. These intuitive comparative statics results hold for arbitrary parameter combinations (under the maintained assumptions).

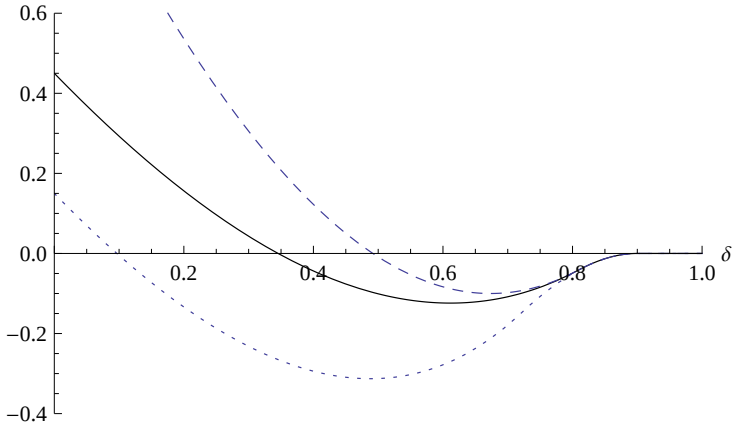

Figure 1: $G_1^{\text{OF}} - G_1^{\text{PR}}$ as function of δ. Higher $\mathcal{L}'$ shifts the curve up (dashed line), lower κ shifts the curve down (dotted line).

Finally, consider the price of debt. A given amount of debt, b_2, carries the price $\beta(1 - f_2 \cdot b_2)$ when issued to private lenders and $\kappa\beta(1 - f_2 \cdot b_2(1 - \mathcal{L}'))$ when issued to official lenders. A given amount of debt therefore is cheaper when financed from official sources than from private sources if and only if

$$\mathcal{L}' \geq \frac{1 - f_2 b_2}{f_2 b_2} \frac{1 - \kappa}{\kappa}.$$

This inequality suggests that strong enforcement power, large levels of debt and a small mark-down on official funds (a large value for κ) all contribute to making official debt attractive relative to private debt.

Endogenous Price Discount Consider next the case where the price discount is determined endogenously as the outcome of bargaining between the sovereign and the enforcer. In the simplest case, all bargaining power lies with the sovereign and default generates a cost $\mathcal{C}(b_2^e)$ to the enforcer (in addition to the capital loss). The binding participation constraint of the enforcer (2) then reads

$$b_2^e p_1(s_1, \pi_1) = b_2^e \beta(1 - F_2) - \beta F_2 \, \mathcal{C}(b_2^e),$$

where, as before, we let $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$. If the cost is linear, $\mathcal{C}(b_2^e) = \mathcal{C}' \cdot b^e$ with $\mathcal{C}' \geq 0$, then this participation constraint simplifies to

$$p_1(s_1, \pi_1) = q_1(s_1, \pi_1) - \beta F_2 \mathcal{C}' = \beta(1 - F_2)(1 + \mathcal{C}') - \beta \mathcal{C}'. \tag{11}$$

The properties of the equilibrium in this example are similar to those obtained previously. The equilibrium is in a corner. If sovereign debt is exclusively funded from private sources, the level of debt and the value of the government's objective remain unchanged relative to the previous example. But if all debt is funded from official sources then the equilibrium is characterized by

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad G_1^{\text{OF}} = \frac{1}{2 f_2} \frac{(\beta - \delta)^2}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

The maximum of the debt-Laffer curve now is at the debt level $(2f_2(1+\mathcal{C}')(1-\mathcal{L}'))^{-1}$, the level chosen by a myopic government, and yields a default probability of $1/2(1+\mathcal{C}')$. Consequently, as long as $(1+\mathcal{C}')(1-\mathcal{L}') \leq 1$ and $\delta = 0$, more debt is issued when the source is official rather than private.

As far as the choice of the debt instrument is concerned, the desirability of official relative to private funds is determined by

$$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}} = \frac{1}{2f_2}(\beta-\delta)^2 \left[\frac{1}{2\beta(1+\mathcal{C}') - \delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'} - \frac{1}{2\beta-\delta}\right] = \frac{1}{2f_2}(\beta-\delta)^2(b_2^{\mathrm{OF}} - b_2^{\mathrm{PR}})$$

and official funding is preferred if and only if $b_2^{\mathrm{OF}} \geq b_2^{\mathrm{PR}}$.

What are the properties of $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as a function of δ? First, it is positive at $\delta = 0$ if $(1+\mathcal{C}')(1-\mathcal{L}') \leq 1$. Second, it attains a zero in the interval $[0, \beta)$ if $\mathcal{L}'(2\mathcal{C}' + \mathcal{L}') < 2\mathcal{C}'$. Finally, it always attains a zero at $\delta = \beta$. Hence, if the first two conditions are satisfied, there exists a unique threshold value $\delta^\star$ such that for $\delta \leq \delta^\star$ (high borrowing needs) official funding is preferred while the opposite holds for $\delta > \delta^\star$. The threshold value increases with $\mathcal{L}'$, as in the previous example, and falls with $\mathcal{C}'$. This is intuitive since a higher $\mathcal{C}'$ increases the expected costs (beyond capital losses) that the enforcer bears in case of default; in order to compensate for these expected costs, the enforcer requires a premium relative to the rate charged by private debt buyers. An increase of $\mathcal{C}'$ therefore has the same qualitative effect on $\delta^\star$ as a decrease of κ in the previous example.

As far as the price of funds is concerned, a fixed quantity of debt b_2 carries a higher interest rate when raised from private sources. The price for such debt equals $\beta(1 - f_2 b_2)$ while the price for the same quantity of debt issued to official creditors equals $\beta(1 - f_2 b_2(1-\mathcal{L}'))(1+\mathcal{C}') - \beta\mathcal{C}'$ (from (11)) which is larger than $\beta(1 - f_2 b_2)$ under the first condition described above.

These findings are robust to changing the specification of the cost function $\mathcal{C}(\cdot)$. Suppose, for example, that costs are not proportional but contain a fixed component so that $\mathcal{C}(b_2^e) = c > 0$ if $b_2^e > 0$ and $\mathcal{C}(b_2^e) = 0$ if $b_2^e = 0$. The enforcer's participation constraint (2) satisfied at equality then reads

$$p_1(s_1, \pi_1) = q_1(s_1, \pi_1) - \beta f_2 \cdot (b_2 - \mathcal{L}' b_2^e)c/b_2^e$$

and equilibrium again is at a corner. It can easily be verified that the properties of the equilibrium are qualitatively identical to those obtained under the assumption of proportional costs.[22]

5 Long-Term Debt

We now relax the assumption that all initially outstanding debt is maturing in the first period and instead allow for the presence of long term debt that matures in the second

[22] We have

$$b_2^{\mathrm{OF}} = \frac{1}{f_2}\frac{\beta(1-cf_2(1-\mathcal{L}')) - \delta}{2\beta - \delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}, \quad G_1^{\mathrm{OF}} = \frac{1}{2f_2}\frac{(\beta(1-cf_2(1-\mathcal{L}')) - \delta)^2}{2\beta - \delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}.$$

where, due to the short-selling constraint $b_2^e \geq 0$, these values only apply for $\delta \leq \beta(1 - cf_2(1-\mathcal{L}'))$.

period. This modification permits to investigate the effects of debt overhang on the default decision as well as the amount and composition of debt issued in the first period. We find that with outstanding long-term debt, the default decision and the choice of debt issuance are linked, in contrast to the examples discussed in section 4. We also find that high levels of outstanding long-term, private debt favor a sovereign's choice to seek new financing from official rather than private sources and to default on the outstanding debt.

Let b_{02} denote the stock of debt issued to private investors in the past that is due in period 2. Define $\tilde{b}_2 \equiv b_{02}\xi_1 + b_2$, where ξ_1 is a variable linked to the default decision in the first period. In particular, if default in the first period affects both the debt maturing in that period and the outstanding long-term debt, then $\xi_1 \equiv r_1$. If, instead, default in the first period does not directly affect the repayment rate on long-term debt, then $\xi_1 \equiv 1$. While the latter specification is consistent with a strict notion of lack of commitment, the former often seems plausible on legal and economic grounds and generates more closely intertwined default and refinancing choices. We solve the model under both specifications.

Except for the introduction of b_{02} and ξ_1, we maintain the assumptions of example 1 in section 4. The value function of the government therefore equals

$$\begin{aligned} G_1(s_1) \;=\; \max_{r_1\in[0,1],\; 0\le b_2^e, b_2-b_2^e+b_{02}\xi_1} & -b_1 r_1 - \mathbf{1}_{[r_1<1]}L_1 + \beta(1-f_2\cdot(\tilde{b}_2-\mathcal{L}'b_2^e))(b_2+(\kappa-1)b_2^e) \\ & - \delta\left\{\int_0^{\tilde{b}_2-\mathcal{L}'b_2^e}(L_2+\mathcal{L}'b_2^e)f_2\mathrm{d}L_2 + (1-f_2\cdot(\tilde{b}_2-\mathcal{L}'b_2^e))\tilde{b}_2\right\}. \end{aligned}$$

The central difference from the government's program in (10) is in the fact that the amount of debt owed in the second period equals $\tilde{b}_2$ rather than b_2. As a consequence, the repayment probability and the price of newly issued debt depend on $\tilde{b}_2$.[23]

We will first analyze the choice of debt issuance in period 1 conditional on r_1 (and thus ξ_1) and then consider the equilibrium default choice in the first period. Throughout, we assume that $f_2 b_{02}\xi_1 < 1$ so that the probability of default is smaller than one and new debt issuance depresses the price of debt.

If debt is issued to private investors, its quantity is given by[24]

$$b_2^{\mathrm{PR}} = \frac{1}{f_2}\frac{\beta-\delta}{2\beta-\delta}(1-f_2 b_{02}\xi_1).$$

Less debt—by a factor of $(1-f_2 b_{02}\xi_1)$—is issued relative to the case without outstanding debt and fewer funds are acquired in period $t=1$. This is due to the fact that outstanding long-term debt increases the losses on inframarginal units due to the adverse effect on the price, relative to the consumption smoothing benefit from the marginal unit of debt. Stated differently, debt overhang pushes the country closer to the peak of the debt-Laffer curve and as a consequence, it makes the issuance of new debt less beneficial. The modified short-selling constraint only binds for values of δ that exceed $\beta\kappa$.

[23] We allow for premature redemption of outstanding private debt (that is, repayment in period $t=1$ of long-term debt due in period $t=2$) but still rule out short positions. The short-selling constraint therefore is modified and takes the form $b_2 \ge -b_{02}\xi_1$ rather than $b_2 \ge 0$.

[24] Again this applies when $\delta \le \beta\kappa$.

If debt is issued to the enforcer, the optimal quantity is given by

$$b_2^{\mathrm{OF}} = \frac{1}{f_2}\frac{\beta\kappa-\delta}{2\beta\kappa-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}(1-f_2 b_{02}\xi_1) - \frac{1}{f_2}\frac{\delta\mathcal{L}' f_2 b_{02}\xi_1}{2\beta\kappa-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}.$$

As in the privately-held debt case, the incentive to issue debt is weakened by outstanding debt (giving rise to the wedge $(1-f_2 b_{02}\xi_1)$ in the first term). Unlike in the privately-held case, however, outstanding debt also depresses official debt issuance through a second channel that is reflected in the second term of b_2^{OF}. To understand this second channel which is not present in the privately-held case, recall that the benefit of official debt to the borrower comes from the reduced risk premium while the cost (apart from the mark-down $1-\kappa$) comes from the higher income losses suffered by the debtor in case of default (represented by the term $-\delta F\mathcal{L}'$ in $\mathcal{M}^e(b_2, b_2^e)$). This cost increases with debt overhang because debt overhang increases the probability of default, F_2. In contrast, the benefit is not affected by the amount of b_{02}. Larger debt overhang therefore reduces the incentive to issue new debt, in particular for large values of $\mathcal{L}'$.[25]

Figure 2 plots the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as a function of δ for different values of $b_{02}\xi_1$. The parameter values are as in example 1 in the previous section. We also assume that $b_1 = L_1 = 0$. The solid line corresponds to $b_{02}\xi_1 = 0$, the dashed line to $b_{02}\xi_1 = 1$ and the dotted line to $b_{02}\xi_1 = 3$. The figure shows that official debt becomes less desirable (the threshold value of $\delta^\star$ for the choice of official debt becomes smaller) as outstanding debt increases.

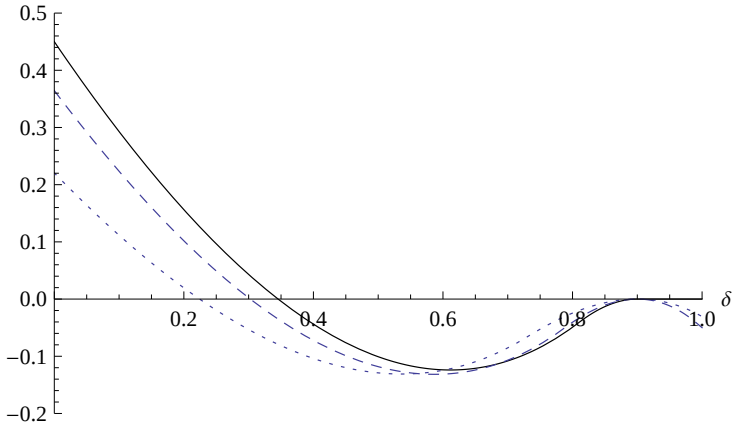

Figure 2: $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as function of δ. Higher $b_{02}\xi_1$ reduces $\delta^\star$.

Turn next to the default decision in the first period. Note that if the government can only default on currently maturing debt ($\xi_1 = 1$) then the threshold value $\hat{L}_1$ that makes it optimal to default is the same as in the case without outstanding debt, namely $\hat{L}_1 = b_1$. In contrast, if default applies to both maturing and outstanding debt ($\xi_1 = r_1$), then the default trade-off is dynamic and the threshold value $\hat{L}_1$ exceeds b_1 whenever

[25] We again impose the short-sales constraint $b_2^e \geq 0$.

$b_{02} > 0$ because the defaulting country gets rid of both b_1 and b_{02}.[26] Holding the *source* of new funding constant, larger debt overhang therefore increases a sovereign's incentive to default. At the same time, for a given *debt overhang*, stronger refinancing needs (reflected in low values of δ) also increase the incentive to default because the elimination of outstanding debt moves the borrower away from the top of the debt-Laffer curve. And holding the *default decision* constant, larger debt overhang increases a sovereign's incentive to refinance privately (see the above equations for debt in the two corners).

Figure 3 illustrates how these trade-offs interact. The figure displays $\hat{L}_1^p(\delta)$, the default threshold that makes the sovereign indifferent between either defaulting (on b_1 and b_{02}) or not defaulting in period 1, when fresh funds are provided by private investors; and $\hat{L}_1^o(\delta)$, the threshold value for the same decision when fresh funds are provided by official lenders. Default occurs for realizations of L_1 below the relevant loci. To build intuition, note that for $b_{02} = 0$, the two loci would coincide and be flat at level b_1. If $b_{02} > 0$, as in the example illustrated in the figure (with $b_1 = 0, b_{02} = 3$) the two loci are upward sloping because a more patient borrower puts higher value on getting rid of debt that is due in the future.

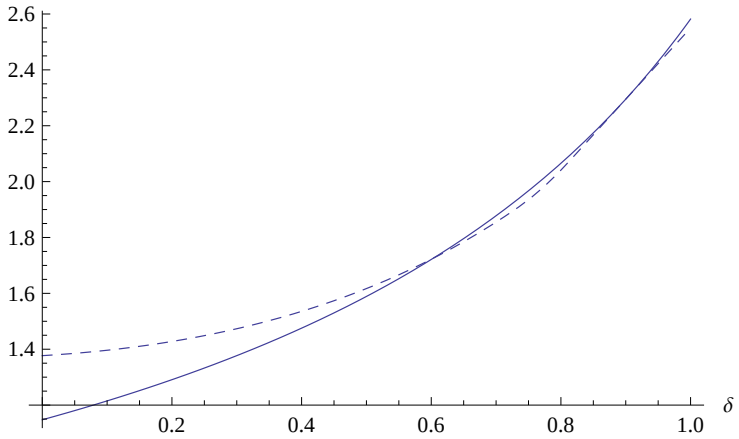

Figure 3: $\hat{L}_1^{\mathrm{PR}}$ (solid), $\hat{L}_1^{\mathrm{OF}}$ (dashed) as function of δ.

The figure shows that for low values of δ (less than 0.62 in this particular example), intermediate realizations of L_1 ($L_1 = 1.3$ for $\delta = 0$ say) induce the sovereign to default if refinancing is provided by official sources but not if it is provided privately. Intuitively, official funds are particularly valuable when borrowing needs are high (δ is low) because they reduce risk premia and hence the amount of funds obtained in the present. But at the same time, the borrower is discouraged from pursuing them by the fact that they increase the cost of future default (recall the cost term $-\delta F_2 \mathcal{L}'$). This makes the elimination of outstanding debt particularly attractive when official creditors are willing to make funds available. For high values of δ, in contrast, the sovereign's borrowing needs are low and refinancing from official sources compares unfavorably with private sources *even*

[26] In an environment where $\xi_1 = r_1$, the cost of defaulting plausibly exceeds the cost of defaulting in an environment where $\xi_1 = 1$. In the text, we do not distinguish the cost L_1 across environments but this is irrelevant since we do not compare outcomes across environments.

abstracting from the cost term $-\delta F_2 \mathcal{L}'$. In this case, the positive effect due to strengthened credibility is not sufficient to offset the negative effect of the mark down $1-\kappa$.[27] When refinancing through official funds does not appear to be an attractive option, defaulting on b_{02} (besides b_1) looks less attractive.

The analysis shows that the availability of official sources of finance increases default risk when borrowing needs are high. Interestingly, it is not only the borrowing country that might favor default in these circumstances, but also the official creditors. To see this, note that private investors are collectively indifferent to how much debt they buy (due to risk neutrality and perfect substitutability of debt issuers). In contrast, risk neutral official lenders profit from the debt they buy as long as $\kappa < 1$ and, as a consequence, they prefer the debtor to default in order to increase the demand for official funds.

6 Concluding Remarks

In recent decades, the usual course of events following a sovereign debt crisis has been for an external official party (the IMF or a foreign government) to step in and provide funds—often in large amounts—at a favorable rate to the affected country. This is also the course followed during the recent crisis in the Euro zone, with the European countries together with the IMF providing funds to meet Greece's, Ireland's, Portugal's and Spain's short term financing needs at below-market rates. While one can think informally of reasons that could justify these actions, the literature does not provide much coherent, formal theory that does this.[28]

In this paper, we have rationalized foreign official lending during a debt crisis. Our main argument is that official foreign entities may possess superior enforcement power relative to private credit markets when lending to certain countries. To the extent that this superior enforcement power is costly to apply the model has the potential to match the stylized fact that official lending only takes place during periods of sovereign debt stress. If, in addition, the borrower has much bargaining power vis-a-vis official creditors, then the model also predicts that the interest rate charged on official loans is low relative to what private markets would charge for comparable amounts of debt.

Our analysis also offers insights on the role played by outstanding long-term debt for default decisions. The model predicts that the combination of strong borrowing needs (a large deficit) and large outstanding long-term debt makes it more likely that a sovereign will default on these obligations, and particularly so if official funds are available for refinancing.

Naturally, our analysis is quite general and applies equally well to credit relationships that do not involve sovereign debt. What is important is the existence of different classes of creditors that differ both in terms of the punishment they can inflict on delinquent debtors and the cost they themselves suffer in this process (beyond the capital losses

[27] A lower value of κ shifts the $\hat{L}_1^{\mathrm{OF}}$ curve down that is, an increase in the premium demanded on official funds shrinks the region of values for δ for which $\hat{L}_1^{\mathrm{OF}}$ exceeds $\hat{L}_1^{\mathrm{PR}}$.

[28] For a recent view related to the one proposed in this paper, see "The eurozone's journey to defaults," *Financial Times*, March 11, 2011.

suffered).

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