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Persistence of Civil Wars

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Abstract

A notable feature of post-World War II civil wars is their very long average duration. We provide a theory of the persistence of civil wars. The civilian government can successfully defeat rebellious factions only by creating a relatively strong army. In weakly-institutionalized polities this opens the way for excessive influence or coups by the military. Civilian governments whose rents are largely unaffected by civil wars then choose small and weak armies that are incapable of ending insurrections. Our framework also shows that when civilian governments need to take more decisive action against rebels, they may be forced to build over-sized armies, beyond the size necessary for fighting the insurrection, as a commitment to not reforming the military in the future.

Keywords: civil wars, commitment, coups, military, political transitions, political economy.

JEL Classification Numbers: H2, N10, N40, P16.

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1 Introduction

One of the most striking facts of the post-World War II international politics is the unusually long average duration of civil wars.[1] Some scholars (e.g., Hironaka, 2008, Kalyvas, 2006) argue that this is largely due to the proliferation of politically weak states since World War II and the onset of decolonization.[2] While the link between politically weak states, which lack the Weberian monopoly of violence, and persistence of civil wars is compelling, it raises another major question: why has the political weakness of many post-World War II states persisted?[3]

In this paper, we provide an explanation for why civil wars may persist in weakly-institutionalized polities. Central to our explanation is the *political moral hazard problem* generated by a strong military (Acemoglu, Ticchi and Vindigni, 2010a). In weakly-institutionalized polities, the checks that would prevent a strong military from intervening in domestic politics are absent. This makes the building of a strong army a double-edge sword for many civilian governments, even if such an army is necessary for defeating rebels and establishing the monopoly of violence over their territory.

We formalize these ideas using a simple dynamic game. The civilian government is controlled by an elite, which derives rents from holding power. It faces armed rebellion from an opposition group (e.g., a group of different ethnicity or religion). The minimum scale of the army is insufficient for ending this armed rebellion and establishing the monopoly of violence. The elite can instead choose a larger size army, which will end the civil war, but this will also increase the role of the military in domestic politics. The civilian government-military interaction is complicated by the fact that the elite cannot credibly commit to not reforming and downsizing the military once the civil war is over. Consequently, a stronger military, which is necessary for defeating the rebels, may also attempt a coup. Thus the elite often face a choice

[1] Civil wars during the nineteenth and early twentieth century were usually relatively short; the average length of a civil war between 1900 and 1944 was one and half years. After World War II, the average duration of civil wars has tripled to over four years. The number of ongoing civil wars has also increased dramatically since 1945. For example, an average of about twenty civil wars per year were ongoing concurrently in the 1990s, corresponding to a rate approximately ten times the historical average since the nineteenth century. The surge in the number of ongoing civil wars has been mainly due to the increase in average duration rather than in the rate of outbreak of new conflicts. See, e.g., Hironaka (2008).

[2] Many political scientists point out that decolonization increased the number of independent states, but many new states lacked the monopoly of coercion and the political capacity common among Western states (see, e.g., Herbst, 2004, Centeno, 2002).

[3] Acemoglu, Robinson and Santos-Villagran (2009) argue that weakness of central governments may arise as an equilibrium outcome when non-state armed actors provide support to one of the factions competing for control of the central government.

between a persistent civil war versus the risk of a coup. Our framework also points out to another strategy for the elite: to build an over-sized army as a commitment to not reforming the military after the end of the civil war (since the over-sized army is strong enough to resist any attempt to reform). This suggests that in weakly-institutionalized polities both the persistence of civil wars and the emergence of over-sized armies with excessive influence on domestic politics are possible equilibrium outcomes.[4]

Our analysis shows that when the elite's rents are relatively unaffected by its lack of monopoly of violence, for example, because the civil war is in a remote area or it does not interfere with their control of natural resources, then the elite will be unwilling to build a strong army. In contrast, when the rebels pose a more costly threat to their rents, the elite is more likely to build a strong army, either risking the possibility of a coup after the end of the civil war or accepting excessive concessions to an over-sized army.

Our framework also generates a novel substitutability between fiscal and political capacity of the state. While these capacities are generally thought to be complements (e.g., Besley and Persson, 2009), in our model higher fiscal capacity raises the equilibrium cost of building strong armies (because it makes military dictatorships both more likely and more costly to the elite) and via this channel, it contributes to the persistence of civil wars.

Our work is related to several different literatures in comparative politics. The large literature on the causes of civil wars is surveyed in Blattman and Miguel (2009). Fearon and Laitin (2003) and Herbst (2004), among others, emphasize the role of weak states in the emergence of civil wars, while the duration of civil wars is studied in Collier, Hoeffler, and Söderbom (2004), de Rouen and Sobek (2004), Hegre (2004), Fearon (2007), Powell (2004, 2009) and Yared (2009). Our paper is also related to the small economics literature on weakly-institutionalized polities, the problems of weak states, and the analysis of state formation, including Acemoglu, Robinson and Verdier (2004), Acemoglu (2005), Acemoglu, Ticchi and Vindigni (2010b), and Besley and Persson (2009), and to the political economy literature on regime transitions (see, e.g., Acemoglu and Robinson, 2006, Acemoglu, Egorov and Sonin, 2009). Our analysis of the political moral hazard problem between the civilian government and the military builds

[4] An illustrative example of a regime unwilling to build a strong army despite ongoing civil wars, most likely because of fear of increasing the power of the military in the future, is Zaire (Congo) under Mobutu (e.g., Snyder, 1992). An example of a regime building an over-sized army is Egypt under Mubarak in his fight against Muslim Brotherhood (Owen, 2004). An example of a regime building a strong army to fight communist rebels and then facing a coup is the Philippines under Marcos.

on Acemoglu, Ticchi and Vindigni (2010a). The closely related and complementary work by Besley and Robinson (2010) also emphasizes the cost of concessions that the civilian government must make to the military and analyze the choice between strong armies and "tinpot" militaries.

The rest of the paper is organized as follows. Section 2 presents our basic model. Section 3 contains some preliminary results and Section 4 characterizes the equilibrium and present our main results. Section 5 concludes.

2 The Model

We consider a society consisting of four social groups, the elite, E, the citizens, L, the rebels, $\Re$, and the military, M. Each agent j at time $t = 0$ maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} + r_{j,t} \right), \tag{1}$$

where $\mathbb{E}_0$ is the expectation at time $t = 0$, $\beta \in (0, 1)$ is the discount factor, $c_{j,t} \geq 0$ denotes the consumption of the final good (equal to disposable income), and $r_{j,t} \geq 0$ is a rent appropriated by each individual whose group is in power at time t, representing non-monetary payoffs from holding power or returns from natural resources or other income sources.

The size of the elite is normalized to 1. The size of the citizens is equal to n, while the size of the military, which will be determined endogenously, is x_t at time t. For simplicity, we assume that only the citizens are recruited into the army, and that $x \in \{x_\ell, x_m, x_h\}$, where $x_\ell < x_m < x_h < n$. The minimum size of the army, x_ℓ, is necessary for maintaining law and order and national defence. An army of size larger than the minimum level x_ℓ can be chosen to deal with the rebels as explained further below. For reasons that will become obvious shortly, we refer to x_h as an "over-sized army". Each elite agent has productivity a, and each citizen has a productivity $A < a$. Citizens recruited into the military do not produce any income.

There are three political states $s_t \in \{W, D, M\}$;[5] W corresponds to a civilian regime with civil war (rebellion); D is a civilian regime (democracy) without rebellion; and M is a military dictatorship. The civilian government, with or without rebellion, is ruled by the elite and can either represent a democracy (including a captured democracy) or a non-democratic regime ruled by an oligarchy. Instead, in a military dictatorship, the military commander (or a group

[5] Payoff-relevant states will be given by elements of $\{W, D, M\} \times \{x_\ell, x_m, x_h\}$.

of officers) is in power. Since our focus is on the persistence of civil wars, we assume that the initial political state is $s_0 = W$—a civilian regime under a rebellion. If the rebellion is defeated, there will be a transition to $s = D$, but the military can attempt a coup against democracy and cause a transition to $s = M$. To simplify the analysis, we assume that the military dictatorship is absorbing: once we have $s = M$, this will apply in all future dates. We also assume that a military coup is not possible starting from $s = W$. Thus possible transitions are $W \to D \to M$.

Both civil war and coups cause economic inefficiencies. Civil war disrupts economic transactions and reduces all incomes by a factor $\delta \in [0, 1]$, so that the income of each elite becomes $(1-\delta)a$ and that of each citizen is $(1-\delta)A$. Similarly, the military is not equipped to run the economy, and thus under a military dictatorship, all incomes are reduced by a factor $\phi \in [0, 1]$.

The government collects revenues with proportional taxation $\tau_t \in [0, 1]$, and these revenues are used to pay the salaries of soldiers. We model tax distortions in a simple way, assuming that there are no costs of taxation until some rate $\hat{\tau} > 0$, and after $\tau = \hat{\tau}$, taxation is prohibitively costly (this makes $\hat{\tau}$ the peak of the Laffer curve). The government budget constraint, which must be satisfied at each period, thus takes the form

$$w(x_t|s_t)x_t \leq \tau(x_t|s_t)(a_t + (n - x_t)A_t), \tag{2}$$

where $w(x_t|s_t)$ and $\tau(x_t|s_t)$ denote the military wage and the tax rate with an army of size x_t in the political state s_t.

We next describe transitions in greater detail. As noted above, we start in $s_0 = W$. There is a transition to $s = D$ when the rebellion (civil war) is defeated, and for simplicity, there is no further possibility of another rebellion. The probability that the rebellion will be defeated is a function of the strength of the state (the size of the army). In particular, we assume that the civil war ends with probability $p(x) \in [0, 1]$ in each period, where

$$p(x_\ell) = p < p(x_m) = p(x_h) = 1.$$

This implies that when $x_t = x_\ell$, there is a "high likelihood," probability $1-p$, that the civil war will persist because of the weakness of the state. In contrast, a moderate or an over-sized army is sufficient to end the civil war immediately. In addition, however, strong armies, $x \in \{x_m, x_h\}$, can undertake a coup against the civilian government once the civil war is defeated. This makes them a double-edge sword for the incumbent civilian government, for they defeat the rebels, but may attempt a coup after the end of the conflict. The difference between intermediate-sized strong army, x_m, and the over-sized strong army, x_h, is that the former can be downsized by the civilian government, and the probability that civilian government can do this within any given period is equal to $\lambda \in [0, 1]$. In contrast, an over-sized army, $x = x_h$, is strong enough to withstand any attempt to reform and can thus never be reformed and downsized by a civilian government. The initial size of the military, x_0, is decided by the civilian government at the beginning of time $t = 0$.

We also assume that each soldier has to put effort, which costs $h > 0$, in fighting the rebels. If he does not do so, he is caught with probability $q \in (0, 1)$, and is punished by losing his wage for one period. This imperfect monitoring technology will lead to "efficiency wages" for soldiers during times of civil war.[6]

We represent the economy described so far as a dynamic game between the soldiers and the elite. The rebels and the citizens do not play an active role because of our simplifying assumptions, and there is no conflict within groups, so that we can suppose that decisions are taken by a representative agent from each group (e.g., the commander of the army and a representative elite agent in a civilian government).

More formally, the timing of events starting in $s_t = W$ or D is as follows:

1. The civilian government chooses the size x_t of the army, sets taxes τ_t and military wages w_t subject to the constraint that $x_t = x_{t-1}$, either if $x_{t-1} = x_h$ or if the state of the world at time t is such that the army cannot be reformed, and subject to the budget constraint (2).

Then if $s_t = W$:

2. The rebels are defeated with probability $p(x_t)$ and the civil war ends permanently, inducing a transition to $s_{t+1} = D$. Otherwise, $s_{t+1} = W$, and the same sequence of events is repeated.

If $s_t = D$:

2. If $x_{t-1} = x_m$ and the state of the world is such that the military can be reformed, the civilian government decides whether or not to reform it (if there is no reform, then $x_t = x_m$). If $x_t = x_m$ or $x_t = x_h$, the military decides whether to attempt a coup against the civilian government. A coup succeeds with probability 1, inducing a transition to $s_t = M$.

[6] The imperfect monitoring assumption and the resulting efficiency wage simplify the exposition. Without this feature, the participation constraint of soldiers would be binding during the civil war, and thus wages would depend on expectations of future coups and military wages. Allowing for perfect monitoring or for more severe punishments do not affect our general results.

In state $s_t = M$, which is absorbing, the military government chooses taxes and military wages subject to the government budget constraint (2).

In the following, we characterize the Markov Perfect Equilibrium (MPE) of the dynamic political game between the elite and the military. As a first step in this characterization, we write the values (discounted present value) of the players as functions of payoff-relevant state variables (s_t, x_t), where $s_t \in \{W, D, M\}$ and $x_t \in \{x_\ell, x_m, x_h\}$.

3 Values and Strategies of the Military

Let us start in the political state $s = W$. Since fighting against the rebels requires an effort cost h for each soldier and shirking is detected only with probability q, the incentive compatible equilibrium military wage during the civil war needs to be at least h/q (see Acemoglu, Ticchi and Vindigni, 2010b). We assume that this wage also satisfies the participation constraint ensuring that citizens are weakly better off as soldiers than as producers (for example, $h/q \geq (1 - \bar{\tau}_\ell)A$, with $\bar{\tau}_\ell$ defined in (3), would be sufficient for this). Taking into account the income disruption generated by the civil war, the tax rate that satisfies the government budget constraint (2) must be

$$\bar{\tau}_i = \frac{x_i}{(1-\delta)(a + (n - x_i) A)} \frac{h}{q} \quad \text{for } i \in \{\ell, m, h\}, \tag{3}$$

provided that this tax rate is less than the maximum feasible rate, $\hat{\tau}$.

Next consider the political state $s = D$. If $x = x_\ell$, then coups are not feasible and effort is no longer necessary, thus military wages will be determined by the participation constraint, which makes a soldier indifferent between working as a civilian and working as a soldier, i.e., $w_\ell = (1 - \tau_\ell) A$, where τ_ℓ is the equilibrium tax rate in this case. Consequently, the value of a soldier and the value of a civilian under democracy and $x = x_\ell$ are

$$V^M(D, x_\ell) = V^L(D, x_\ell) = \frac{(1-\tau_\ell) A}{1-\beta}, \tag{4}$$

where the tax rate τ_ℓ balancing the government budget (2) is $\tau_\ell = x_\ell A/(a + nA)$.

When $x \in \{x_m, x_h\}$, the army may attempt a coup against the democratic government in the state $s_t = D$, that is, after the rebels have been defeated. Consequently, in these cases the elite need to take into account the strategy of the military to set fiscal policy. In particular, as in Acemoglu, Ticchi and Vindigni (2010a), there will be a *no coup constraint* of the form:

$$V^M(D, x_i|\text{coup}) \leq V^M(D, x_i|\text{no coup}) \quad \text{for } i \in \{m, h\}, \tag{5}$$

which the elite must satisfy if they wish to prevent coups; $V^M(D, x_i|\text{coup})$ and $V^M(D, x_i|\text{no coup})$ denote the values of soldiers with an army size of x_i when they undertake a coup and when they choose not to do so. To derive the implications of the no coup constraint, first consider a military regime, and let R denote the rents that soldiers receive in such a regime. Recall that this regime is absorbing and since there are no costs of taxation until $\hat{\tau}$, soldiers will set this tax rate and redistribute the proceeds as wages to themselves. Therefore,

$$V^M(M, x_i) = \frac{R + \hat{\tau}(1-\phi)(a + (n - x_i)A)/x_i}{1-\beta} \quad \text{for } i \in \{m, h\}, \tag{6}$$

which takes into account that incomes are reduced by a fraction ϕ, because the military is running the economy, and only $n - x_i$ citizens are working in production. The proceeds from taxation are distributed equally among the soldiers, thus the division by x_i in the denominator.

Consider next the case where $x = x_h$ (with $s = D$). If the elite do not prevent coups, the value to the military is $V^M(D, x_h|\text{coup}) = V^M(M, x_h)$ as given by (6) for $i = h$. Alternatively, the elite could pay an "efficiency wage" to the soldiers, w_h^P, to make it worth for them not to attempt coups—i.e., to satisfy the no coup constraint, (5). When $x = x_h$, the expression for the efficiency wage is straightforward to derive, since there is no possibility of reforming the military. Therefore, the value to the military when the elite pay such a wage is $V^M(D, x_h|\text{no coup}) = w_h^P + \beta V^M(D, x_h|\text{coup})$, where w_h^P is the level of the efficiency wage that makes (5) hold as equality, and this expression takes into account that in the next period the military must receive the value that it can get with a coup (either by undertaking a coup, or because the elite will pay them the necessary efficiency wage). This implies that the efficiency wage w_h^P will be given by

$$w_h^P = \frac{\hat{\tau}(1-\phi)(a + (n - x_h)A)}{x_h} + R, \tag{7}$$

and the tax rate that satisfies the government budget constraint in this case is

$$\tau_h^P = \hat{\tau}(1-\phi) + \frac{x_h R}{a + (n - x_h)A}. \tag{8}$$

However, it may not be feasible for the civilian government to pay such high wages to soldiers because in the government budget constraint, (2), we need to have $\tau \leq \hat{\tau}$. Hence, coup prevention with an army of size x_h is feasible only if $w_h^P x_h \leq \hat{\tau}(a + (n - x_h)A)$. Thus from (7), we obtain that coups starting with $x = x_h$ can be prevented provided that

$$\phi \geq \frac{x_h R}{\hat{\tau}(a + (n - x_h)A)} \equiv \phi_h^*. \tag{9}$$

Let us next consider the case where $x = x_m$ (again with $s = D$). If the elite prevent coups by paying an efficiency wage w_m^P, then the value to each soldier is:[7]

$$V^M(D, x_m|\text{no coup}) = w_m^P + \beta[\lambda V^L(D, x_\ell) + (1-\lambda)V^M(D, x_m|\text{coup})],$$

which now takes into account that with probability λ, there will be an opportunity to reform and downsize the military, and the civilian government will use this opportunity, and thereafter, soldiers will receive the value $V^L(D, x_\ell)$ as given by (4). If there is no opportunity to reform, then the soldiers will receive the value from a coup (either because they will undertake a coup or because the no coup constraint, (5), will be satisfied with equality). Using (4) and (5), we can compute $V^M(D, x_m|\text{no coup}) = [w_m^P + \beta\lambda(1-\tau_\ell)A/(1-\beta)]/(1-\beta(1-\lambda))$. The value from a coup is given by (6). Repeating the same analysis as above, we find that with an army of size $x = x_m$, it will be feasible to satisfy the no coup constraint, (5) only when:[8]

$$\phi \geq \frac{\beta\lambda}{1-\beta(1-\lambda)}\left[1 - \frac{(1-\tau_\ell)x_m A}{\hat{\tau}(a+(n-x_m)A)}\right] + \frac{x_m R}{\hat{\tau}(a+(n-x_m)A)} \equiv \phi_m^*. \qquad (10)$$

To save space, in the remainder, we impose the following assumption, which allows us to focus on the more novel and economically interesting cases.

Assumption 1 *(1)* $R_m < R \leq \bar{R}_h$, *where* $\bar{R}_h \equiv \hat{\tau}(a+(n-x_h)A)/x_h$ *and* $R_m \equiv \beta A(1 - x_\ell A/(a+nA)) + (1-\beta)\hat{\tau}(a+(n-x_m)A)/x_m$.

(2) $\phi \in [\phi_h^*, 1]$ *and* $\lambda \in (\lambda^*, 1]$.

(3) $\beta > \beta^*$, *where* $\beta^* < 1$ *is implicitly defined by the following equation* $\beta^* A(1 - x_\ell A/(a+nA)) + (1-\beta^*)\hat{\tau}(a+(n-x_m)A)/x_m = \hat{\tau}(a+(n-x_h)A)/x_h$.

The first part of Assumption 1 states that military rents in military dictatorship are intermediate, so that military dictatorships are not desirable when soldiers know that they will have sufficient influence in the civilian regime, that is, they will receive efficiency wages without any risk of downsizing, but are worthwhile when they do not receive efficiency wages. More specifically, $R \leq \bar{R}_h$ ensures that $\phi_h^* \leq 1$ so that for values of $\phi \in [\phi_h^*, 1]$ it will be feasible to satisfy (5) and to prevent coups with an over-sized army ($x = x_h$). In contrast, $R_m < R$

[7] This is the value of soldiers *after* the realization of the state of nature that the military cannot be reformed.

[8] In this case, the efficiency wage w_m^P necessary for prevention is $w_m^P = (1-\beta(1-\lambda))[\hat{\tau}(1-\phi)(a+(n-x_m)A)/x_m + R]/(1-\beta) - \beta\lambda(1-\tau_\ell)A/(1-\beta)$, and the tax rate balancing the government budget is $\tau_m^P = (1-\beta(1-\lambda))[\hat{\tau}(1-\phi) + x_m R/(a+(n-x_m)A)]/(1-\beta) - \beta\lambda[x_m/(a+(n-x_m)A)](1-\tau_\ell)A/(1-\beta)$.

ensures that preventing coups with an intermediate-sized army ($x = x_m$) is not feasible when the probability of potential reform, λ, is sufficiently high (i.e., $\phi_m^* > 1$ as λ approaches to 1). In particular, let λ^* be defined as the value of λ such that $\phi_m^* = 1$. Then this assumption implies that when $\lambda \in (\lambda^*, 1]$, (10) can never be satisfied and coups cannot be prevented with intermediate-sized army. The second part of the assumption then imposes that $\phi \in [\phi_h^*, 1]$ so that prevention of coups with over-sized military is indeed feasible, and $\lambda \in (\lambda^*, 1]$ so that coup prevention with an intermediate-sized military is never feasible. Finally, the third part of the assumption ensures that $R_m < \bar{R}_h$, so that the first part of the assumption is meaningful.

4 Characterization of the MPE

In this section we characterize the MPE of the dynamic political game by determining what type of army the elite will choose as a response to the ongoing civil war.

The expected value to the elite when there is a civil war and the size of the military is $x = x_\ell$ can be written as

$$V^E(W, x_\ell) = (1 - \bar{\tau}_\ell)(1 - \delta)a + \bar{r} + \beta \left[pV^E(D, x_\ell) + (1 - p) V^E(W, x_\ell) \right],$$

where $\bar{\tau}_\ell$ is given by (3) and $\bar{r}$ is the rent accruing to the elite when they are in power but there is an ongoing civil war. This expression incorporates the fact that the rebels are defeated with probability $p = p(x_\ell)$ in each period, and subsequently the continuation value to the elite is $V^E(D, x_\ell) = ((1 - \tau_\ell)a + r)/(1 - \beta)$, where r is the exogenous rent of being in power without a civil war (since an army of size x_ℓ cannot attempt a coup, $s = D$ with $x = x_\ell$ is an absorbing state). Therefore, the value to the elite of choosing a small army, in the midst of a civil war, is

$$V^E(W, x_\ell) = \frac{(1 - \beta)((1 - \bar{\tau}_\ell)(1 - \delta)a + \bar{r}) + \beta p((1 - \tau_\ell)a + r)}{(1 - \beta)(1 - \beta(1 - p))}. \tag{11}$$

Given Assumption 1, when the elite choose an army of size $x = x_m$, then coups cannot be prevented, and thus their value can be written as

$$V^E(W, x_m) = (1 - \bar{\tau}_m)(1 - \delta)a + \bar{r} + \beta \left[\frac{\lambda((1 - \tau_\ell)a + r)}{1 - \beta} + \frac{(1 - \lambda)(1 - \hat{\tau})(1 - \phi)a}{1 - \beta} \right] \tag{12}$$

where $\bar{\tau}_m$ is given by (3). This expression takes into account that rebels are defeated in one period and, in the following period, the army is reformed with probability λ, while reforms are not possible with the complementary probability and the military undertakes a coup.

Finally, if the elite choose $x = x_h$, their value depends on whether coups will be prevented in the subgame starting after the defeat of the rebels. Given Assumption 1, such prevention is feasible, and clearly optimal. Hence, the value to the elite in this case is

$$V^E(W, x_h) = (1 - \bar{\tau}_h)(1 - \delta)a + \bar{r} + \beta \frac{(1 - \tau_h^P)a + r}{1 - \beta}, \tag{13}$$

where τ_h^P is defined in (8).[9]

In light of this discussion, the potential strategies for the elite are: (1) form an over-sized military (x_h), defeat the rebels, and prevent coups, thus remaining in power but with a very influential military; (2) form an intermediate army (x_m), defeat the rebels, but face the risk of military takeover; (3) choose a small army (x_ℓ), and thus allow for persistent civil war.

To compare these three options, note that $V^E(W, x_\ell) = V^E(W, x_\ell|p)$ defined in (11) is a strictly increasing function of the probability p that a small army (x_ℓ) will defeat the rebels (hence the explicit conditioning on p), while $V^E(W, x_h)$ and $V^E(W, x_m)$ defined in (13) and (12) are independent of p. This implies that there exists a threshold $\hat{p} \in [0, 1]$ such that $V^E(W, x_h) \gtreqless V^E(W, x_\ell|p = \hat{p})$ whenever $p \lesseqgtr \hat{p}$, and a threshold $p^* \in [0, 1]$ such that $V^E(W, x_m) \gtreqless V^E(W, x_\ell|p = p^*)$ whenever $p \lesseqgtr p^*$. It can be verified that both thresholds are always smaller than 1, because the value to the elite when $x = x_\ell$ and $p = 1$ is always greater than their value when choosing x_h and x_m. However, these thresholds need not be positive. In particular, $\hat{p} > 0$ only when $V^E(W, x_h) > V^E(W, x_\ell|p = 0)$, that is, when

$$(1 - \bar{\tau}_\ell)(1 - \delta)a < (1 - \beta)(1 - \bar{\tau}_h)(1 - \delta)a + \beta\left(1 - \tau_h^P\right)a + \beta(r - \bar{r}). \tag{14}$$

Otherwise $V^E(W, x_h) < V^E(W, x_\ell)$ for all $p \in [0, 1]$, and in this case, a small army (x_ℓ) will always be preferred by the elite to an over-sized one (x_h), and by convention, in this case we set $\hat{p} = 0$. Similarly, $p^* > 0$ when

$$(1-\bar{\tau}_\ell)(1-\delta)a < (1-\beta)(1-\bar{\tau}_m)(1-\delta)a+\beta\lambda(1-\tau_\ell)a+\beta(1-\lambda)(1-\hat{\tau})(1-\phi)a+\beta(\lambda r-\bar{r}), \tag{15}$$

and thus when this condition is not satisfied, the elite always prefer x_ℓ to x_m. In what follows, the reader should bear in mind that both thresholds, $\hat{p}$ and p^*, can be zero.

Let us finally introduce the following condition

$$(\bar{\tau}_h - \bar{\tau}_m)(1 - \delta)a \leq \frac{\beta}{1 - \beta}\left[(1 - \lambda)(r + \phi a) + (\tau_\ell - \hat{\tau}(1 - \phi))\lambda a - \frac{x_h a}{a + (n - x_h)A} R\right]. \tag{16}$$

[9] If the elite chose not to prevent a coup, they would receive $V^E(W, x_h) = (1 - \bar{\tau}_h)(1 - \delta)a + \bar{r} + \beta(1 - \hat{\tau})(1 - \phi)a/(1 - \beta)$, which can be verified to be less than (13).

It can be verified that when this condition is satisfied, $V^E(W, x_h) > V^E(W, x_m)$, and the elite prefer an over-sized army to an intermediate one.

We now provide a characterization of the MPE in this dynamic economy.[10]

Proposition 1 *The political game above has a unique MPE with the following structure.*

1. *Suppose that (16) is satisfied and $p \in [\hat{p}, 1]$ or that (16) is not satisfied and $p \in [p^*, 1]$. Then the elite choose a small army, $x = x_\ell$, and there is persistence of civil war. After (or if) the civil war ends, the civilian government (the elite) remains in power.*

2. *Suppose that (16) does not hold and $p \in [0, p^*)$, then the elite choose an intermediate-sized army, $x = x_m$, and the civil war ends immediately, but there is possibility of a military coup and the formation of a military dictatorship.*

3. *Suppose that condition (16) is satisfied and $p \in [0, \hat{p})$, then the elite choose an over-sized army, $x = x_h$, the civil war ends immediately, and civilian government remains in power, but with high wages and concessions for the military.*

The following corollary provides comparative statics of the key thresholds.

Corollary 1 *The threshold $\hat{p}$ is nondecreasing in r, δ, ϕ, and it is nonincreasing in $\bar{r}$, $\hat{\tau}$, x_h, R.*

The threshold p^* is nonincreasing in $\bar{r}$, $\hat{\tau}$, ϕ, x_m, is independent of R, and is nondecreasing in δ, and also in r if λ is high enough and nonincreasing in r otherwise.

Proposition 1 is the main result of the paper. It shows that the elite will choose a small army, and will not establish a monopoly of violence over its territory, at least for a while, when $p > \hat{p}$ or when $p > p^*$—i.e., when a small army is not too ineffective at fighting the rebels. Note, however, that both thresholds $\hat{p}$ and p^* can be very small or equal to zero, so when a small army is maintained, the civil war can persist for a very long time (in the limit forever as $p \to 0$, if both thresholds are zero). Corollary 1 shows that such an outcome is more likely when r is low relative to $\bar{r}$, that is, when the elite receive significant rents even when the civil war is ongoing (for example because the civil war is in peripheral areas and does not interfere

[10] The argument in the text gives the main idea of the proof of this proposition. A more detailed characterization of the MPE and a formal proof are provided in the Appendix.

with the rents that the elite receive from corruption or natural resources). Small armies and persistent civil wars are also more likely when δ is low relative to ϕ, making the income loss (of the elite and of the citizens) relatively small under civil war, and high under military regimes. Finally, a high $\hat{\tau}$ also makes this configuration more likely because of two distinct channels: first, it makes a military dictatorship more costly to the elite (when these happen along the equilibrium path); second, it makes a military dictatorship more attractive for soldiers, thus making it more expensive for the elite to satisfy the no coup constraint (when they prefer to do so). For reasons related to the second channel, a high level of R (high rents for the military from controlling the government) also makes the elite more likely to choose a small army and a weak state. In all cases, the reason why the elite prefer a small army is that they are afraid of the influence of and a potential coup by a strong army following the end of the civil war.

When the elite decide to fight the rebels more vigorously to end the civil war, they can do so using one of two different strategies. In the first one, they build an intermediate-sized army, but because of their inability to commit to not downsizing the army after the civil war ends, they cannot satisfy the no coup constraint, and there is a positive probability of a coup along the equilibrium path. In the second one, they build an over-sized army as a commitment to not reforming the military in the future. This amounts to making permanent concessions (high wages and other policy concessions) to the military as the price that the elite have to pay for fighting the rebels and establishing some sort of monopoly of violence. Note, however, that in this case this monopoly of violence is mostly in the hands of the military not in the hands of the civilian government.

An interesting implication of the model, again highlighted by Corollary 1, is a novel substitutability between fiscal and political capacity of the state. When $\hat{\tau}$ is high, the fiscal capacity of the state is high. This is generally thought to increase the political capacity of the state (e.g., Besley and Persson, 2009). However, a higher fiscal capacity also puts more economic power in the hands of the military if they decide to attempt a coup. Through this channel, it discourages the civilian government from building a strong military and the monopoly of violence necessary for political capacity.

Finally, it is also useful to observe that the entire analysis is predicated on the possibility that the military, once sufficiently large, can take control of the government. In this sense, the model represents the workings of politics in a weakly-institutionalized polity, which does not place major constraints on the exercise of military power.

5 Concluding Remarks

We presented a simple model where civil wars persist because of the endogenous weakness of the state. The civilian government, assumed to be under the control of an elite, may prefer to forgo the establishment of the monopoly of violence over its territory, allowing an ongoing civil war, because, given the weak institutions, the elite are afraid of building a strong military. This fear is particularly relevant when the civilian government is unable to commit to not reforming the military after the civil war is over, and this commitment problem makes a military coup more likely. One, potentially paradoxical, response of the civilian government, when it needs to prevent the continuation of the civil war, is to build an over-sized army as a commitment to not reforming the military after the threat of the civil war is gone.

We view this paper as part of our broader research on the interaction between civilian segments of the society and the military, and on the ability of society to control the use of force. Our simple model shows how this interaction is affected by an ongoing civil war and at the same time determines the persistence of the civil war. Other interesting directions would be to investigate how international relations (including possibility of international wars and international trade) affect the balance of power between the elite, non-elite elements in the society and the military, and also how the interplay between the military and civilian branches of the government may affect the development of the fiscal capacity of the state.

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Appendix

Additional Details on the Characterization of the MPE

We first state a version of our main result, Proposition 1, more formally.

Proposition 2 *Define the thresholds $\hat{p}$ and p^* as in the text. When (14) and (15) hold, $\hat{p}$ and p^* are interior. When (14) does not hold, set $\hat{p} = 0$, and (15) does not hold, set $p^* = 0$. Suppose that $p \neq \hat{p}$ and $p \neq p^*$. Then, the political game described in the main text has a unique MPE with the following structure.*

1. *Suppose that (a): (16) is satisfied and $p \in (\hat{p}, 1]$, or (b) (16) is not satisfied and $p \in (p^*, 1]$. Then when $s_t = W$, the elite choose a small army, i.e., $x_t = x_\ell$, set $\tau_t = \bar{\tau}_\ell$, and pay military wages given by $w_t = \bar{w} = h/q$. The civil war ends with probability p in each period $t \geq 0$. After (or if) the civil war ends at some date t, the elite choose $x_u = x_\ell$, set $\tau_u = \tau_\ell$, pay military wages $w_u = (1 - \tau_\ell) A$ and $s_u = D$ (i.e., the elite remain in power) for all $u > t$.*

2. *Suppose that (16) does not hold and $p \in [0, p^*)$. Then, at time $t = 0$, the elite choose an intermediate-sized army, i.e., $x_0 = x_m$, set $\tau_0 = \bar{\tau}_m$, pay military wages given by $w_0 = \bar{w} = h/q$ and the civil war ends in the same period. At time $t = 1$, the military can be reformed with probability λ, and in this case the elite choose $x_u = x_\ell$, set $\tau_u = \tau_\ell$, pay military wages $w_u = (1 - \tau_\ell) A$ and $s_u = D$ for all $u \geq 1$. With probability $1 - \lambda$ the army cannot be reformed, the military undertakes a coup, and a permanent military dictatorship is established. The military chooses $x_u = x_m$, sets $\tau_u = \hat{\tau}$, sets military wages given by $w_u = \hat{\tau}(1 - \phi)(a + (n - x_m)A)/x_m$, and $s_u = M$ for all $u \geq 1$.*

3. *Suppose that condition (16) is satisfied and $p \in [0, \hat{p})$. Then, at time $t = 0$, the elite choose an over-sized army, i.e., $x_0 = x_h$, set $\tau_0 = \bar{\tau}_h$, pay military wages $w_0 = \bar{w} = h/q$, and the civil war ends in the same period. The military cannot be reformed in all future periods, i.e., $x_u = x_h$, the elite set $\tau_u = \tau_h^P$ and $w_u = w_h^P$ given respectively by (8) and (7), and $s_u = D$ for all $u \geq 1$.*

When $p = p^*$, configurations in parts 1 and 2 are MPEs, and when $p = \hat{p}$, configurations in parts 1 and 3 are MPEs.

Proof. Condition (16) is obtained from $V^E(W, x_h) > V^E(W, x_m)$ using (13) and (12) and rearranging terms. When this condition is satisfied, the elite prefer an over-sized army (x_h) to an intermediate one (x_m), so that their choice will be between $x = x_h$ and $x = x_\ell$. Then, note that $V^E(W, x_\ell) = V^E(W, x_\ell|p)$ defined in (11) is a strictly increasing function of the probability p that a small army (x_ℓ) will defeat the rebels, while $V^E(W, x_h)$ in (13) is independent on p. This implies that there exists a threshold $\hat{p} \in [0, 1]$ such that $V^E(W, x_h) \gtreqless V^E(W, x_\ell|p = \hat{p})$ whenever $p \lesseqgtr \hat{p}$. Moreover, $\hat{p}$ is always strictly lower than 1 as $V^E(W, x_h) < V^E(W, x_\ell|p = 1)$, and it is strictly positive if and only if $V^E(W, x_h) > V^E(W, x_\ell|p = 0)$, which is equivalent to (14). This establishes parts 1(a) and 3 of the proposition.

When condition (16) does not hold, $V^E(W, x_h) < V^E(W, x_m)$, the elite prefer an intermediate-sized army (x_m) to an over-sized one (x_h) and, therefore, $x = x_h$ is never chosen. Similarly to the previous case, from $V^E(W, x_\ell)$ increasing in p and $V^E(W, x_m)$, defined in (12), independent on p follows that there exists a threshold $p^* \in [0, 1]$ such that $V^E(W, x_m) \gtreqless V^E(W, x_\ell|p = p^*)$ whenever $p \lesseqgtr p^*$. Again, p^* is always strictly lower than 1 as $V^E(W, x_m) < V^E(W, x_\ell|p = 1)$, while it is strictly positive if and only if $V^E(W, x_m) > V^E(W, x_\ell|p = 0)$, which is equivalent to (15). This establishes parts 1(b) and 2 of the proposition.

When $p = \hat{p}$, $V^E(W, x_h) = V^E(W, x_\ell)$, the elite will have two best responses and configurations in parts 1 and 3 are MPEs. Again, when $p = p^*$, $V^E(W, x_m) = V^E(W, x_\ell)$, and configurations in parts 1 and 2 are MPEs. ■

Proof of Corollary 1

The following relation

$$\mathcal{L} \equiv V^E(W, x_\ell|\hat{p}) - V^E(W, x_h) = 0$$

implicitly defines the threshold probability $\hat{p}$ such that $V^E(W, x_h) \gtreqless V^E(W, x_\ell|p = \hat{p})$ whenever $p \lesseqgtr \hat{p}$. Suppose that this threshold is interior (otherwise, small changes in the parameters would have no effect on $\hat{p}$, and the results in the corollary apply directly).

Using the implicit function theorem, we have that

$$\frac{\partial \hat{p}}{\partial \delta} = -\frac{\partial \mathcal{L}/\partial \delta}{\partial \mathcal{L}/\partial \hat{p}},$$

where

$$\frac{\partial \mathcal{L}}{\partial \hat{p}} = \frac{\partial V^E(W, x_\ell|\hat{p})}{\partial \hat{p}} > 0. \tag{17}$$

Then, using (3) and the fact that $\partial\bar{\tau}_\ell/\partial\delta = \bar{\tau}_\ell/(1-\delta)$ and $\partial\bar{\tau}_h/\partial\delta = \bar{\tau}_h/(1-\delta)$, we obtain $\partial V^E(W, x_\ell|\hat{p})/\partial\delta = -a/(1-\beta(1-\hat{p}))$ and $\partial V^E(W, x_h)/\partial\delta = -a$. Therefore, we have

$$\frac{\partial\mathcal{L}}{\partial\delta} = \frac{\partial V^E(W, x_\ell|\hat{p})}{\partial\delta} - \frac{\partial V^E(W, x_h)}{\partial\delta} = -\frac{\beta a(1-\hat{p})}{1-\beta(1-\hat{p})} < 0.$$

This combined with (17) implies that $\partial\hat{p}/\partial\delta > 0$, i.e., that $\hat{p}$ is increasing in δ as stated.

Using the same procedure and taking into account (17), we also obtain:

$\partial\hat{p}/\partial r = -(\partial\mathcal{L}/\partial r)/(\partial\mathcal{L}/\partial\hat{p}) > 0$ as $\partial\mathcal{L}/\partial r = \partial V^E(W, x_\ell|\hat{p})/\partial r - \partial V^E(W, x_h)/\partial r = -\beta\,(1-\hat{p})\,/\,(1-\beta(1-\hat{p})) < 0$.

$\partial\hat{p}/\partial\bar{r} = -(\partial\mathcal{L}/\partial\bar{r})/(\partial\mathcal{L}/\partial\hat{p}) < 0$ as $\partial\mathcal{L}/\partial\bar{r} = \partial V^E(W, x_\ell|\hat{p})/\partial\bar{r} - \partial V^E(W, x_h)/\partial\bar{r} = \beta\,(1-\hat{p})\,/\,(1-\beta(1-\hat{p})) > 0$.

$\partial\hat{p}/\partial x_h = -(\partial\mathcal{L}/\partial x_h)/(\partial\mathcal{L}/\partial\hat{p}) < 0$ as $\partial\mathcal{L}/\partial x_h = -\partial V^E(W, x_h)/\partial x_h = a\,(1-\delta)\,(\partial\bar{\tau}_h/\partial x_h) + \beta a(\partial\tau_h^P/\partial x_h)/(1-\beta) > 0$, where in the last expression we have used the fact that $\partial\bar{\tau}_h/\partial x_h$ and $\partial\tau_h^P/\partial x_h$ are both strictly positive (see (3) and (8)).

$\partial\hat{p}/\partial\hat{\tau} = -(\partial\mathcal{L}/\partial\hat{\tau})/(\partial\mathcal{L}/\partial\hat{p}) < 0$ as $\partial\mathcal{L}/\partial\hat{\tau} = -\partial V^E(W, x_h)/\partial\hat{\tau} = \beta a\,(1-\phi)\,/\,(1-\beta) > 0$. In the last expression, we have taken into account the expression for τ_h^P in (8) (as we will also do for the next two cases).

$\partial\hat{p}/\partial\phi = -(\partial\mathcal{L}/\partial\phi)/(\partial\mathcal{L}/\partial\hat{p}) > 0$ as $\partial\mathcal{L}/\partial\phi = -\partial V^E(W, x_h)/\partial\phi = -\beta a\hat{\tau}/\,(1-\beta) < 0$.

$\partial\hat{p}/\partial R = -(\partial\mathcal{L}/\partial R)/(\partial\mathcal{L}/\partial\hat{p}) < 0$ as $\partial\mathcal{L}/\partial R = -\partial V^E(W, x_h)/\partial R = \beta a x_h/(1-\beta)(a + (n-x_h)\,A) > 0$.

This completes the proof of the first part of the corollary.

For the second part, consider the relation

$$\mathcal{F} \equiv V^E(W, x_\ell|p^*) - V^E(W, x_m) = 0$$

that implicitly defines the threshold probability p^* such that $V^E(W, x_m) \gtreqless V^E\,(W, x_\ell|p = p^*)$ whenever $p \lesseqgtr p^*$. Again we consider the case where this threshold is interior. From the implicit function theorem

$$\frac{\partial p^*}{\partial\delta} = -\frac{\partial\mathcal{F}/\partial\delta}{\partial\mathcal{F}/\partial p^*} > 0,$$

because

$$\frac{\partial\mathcal{F}}{\partial p^*} = \frac{\partial V^E(W, x_\ell|p^*)}{\partial p^*} > 0. \tag{18}$$

Since as in the previous case, $\partial\bar{\tau}_\ell/\partial\delta = \bar{\tau}_\ell/(1-\delta)$ and $\partial\bar{\tau}_m/\partial\delta = \bar{\tau}_m/(1-\delta)$, we have

$\partial V^E(W, x_\ell | p^*)/\partial\delta = -a/(1-\beta(1-p^*))$ and $\partial V^E(W, x_m)/\partial\delta = -a$, and thus

$$\frac{\partial\mathcal{F}}{\partial\delta} = \frac{\partial V^E(W, x_\ell | p^*)}{\partial\delta} - \frac{\partial V^E(W, x_m)}{\partial\delta} = -\frac{\beta a(1-p^*)}{1-\beta(1-p^*)} < 0.$$

In order to show that p^* is increasing in r if λ is high enough and vice versa, note that

$$\frac{\partial\mathcal{F}}{\partial r} = \frac{\partial V^E(W, x_\ell | p^*)}{\partial r} - \frac{\partial V^E(W, x_m)}{\partial r} = \frac{\beta}{1-\beta}\left(\frac{p^*}{1-\beta(1-p^*)} - \lambda\right)$$

is monotonically decreasing in λ, $\partial\mathcal{F}/\partial r > 0$ when $\lambda = 0$ and $\partial\mathcal{F}/\partial r < 0$ when $\lambda = 1$. These observations and (18) imply that $\partial p^*/\partial r = -(\partial\mathcal{F}/\partial r)/(\partial\mathcal{F}/\partial p^*) > 0$ for high levels of λ and vice versa.

Again, taking into account (18), we have the following results:

$\partial p^*/\partial\bar{r} = -(\partial\mathcal{F}/\partial\bar{r})/(\partial\mathcal{F}/\partial p^*) < 0$ as $\partial\mathcal{F}/\partial\bar{r} = \partial V^E(W, x_\ell|p^*)/\partial\bar{r} - \partial V^E(W, x_m)/\partial\bar{r} = \beta(1-p^*)/(1-\beta\,(1-p^*)) > 0$.

$\partial p^*/\partial x_m = -(\partial\mathcal{F}/\partial x_m)/(\partial\mathcal{F}/\partial p^*) < 0$ as $\partial\mathcal{F}/\partial x_m = -\partial V^E(W, x_m)/\partial x_m = a(1-\delta)(\partial\bar{\tau}_m/\partial x_m) > 0$ because $\partial\bar{\tau}_m/\partial x_m > 0$ (see (3)).

$\partial p^*/\partial\hat{\tau} = -(\partial\mathcal{F}/\partial\hat{\tau})/(\partial\mathcal{F}/\partial p^*) < 0$ as $\partial\mathcal{F}/\partial\hat{\tau} = -\partial V^E(W, x_m)/\partial\hat{\tau} = \beta a(1-\lambda)(1-\phi)/(1-\beta) > 0$.

$\partial p^*/\partial\phi = -(\partial\mathcal{F}/\partial\phi)/(\partial\mathcal{F}/\partial p^*) < 0$ as $\partial\mathcal{F}/\partial\phi = -\partial V^E(W, x_m)/\partial\phi = \beta a(1-\hat{\tau})/(1-\beta) > 0$.

$\partial p^*/\partial R = 0$ as $V^E(W, x_\ell|p^*)$ and $V^E(W, x_m)$ are both independent on R.

This completes the second part of the corollary. ■