Principles of Optimal Taxation

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This lecture

- Principles of optimal taxes
- Focus on linear taxes (VAT, sales, corporate, labor in some countries)
- (Almost) no heterogeneity across consumers
  - highlight the key driving forces behind taxes and distortions associate with them
  - sidestep questions of the optimal taxation of redistribution:
    - large topic in itself
    - many insights do not depend on it

Plan

1. Optimal commodity taxation
2. Optimal intermediate goods taxation
3. Taxation of capital income
4. Tax smoothing

Optimal commodity taxation

- Static economy
- General equilibrium
- 4 main elements:
    - consumers
    - firms
    - government
    - market clearing

Consumers

- A representative consumer supplies labor l and consumes n different consumption goods.
- Normalizing his wage rate to 1, the representative consumer solves consumer's problem (1)

$$\max_{c,l} U(c_1, ..., c_n, l)$$

$$s.t. \qquad \sum_i p_i (1 + \tau_i) c_i = l$$

Firms

- A large number of firms operate identical, constant returns to scale technology to produce consumption goods.
- The firm solves firm's problem (2)

$$\max_{x,l} \sum_i p_i x_i - l$$

$$s.t \qquad F(x_1, ..., x_n, l) = 0$$

Government

- The government has to rely on commodity taxes to finance exogenous expenditures $\{g_i\}$.
- Government's budget constraint (3) is

$$\sum_i p_i g_i = \sum_i p_i \tau_i c_i$$

Market clearing

Market clearing condition (4) is

$$c_i + g_i = x_i \qquad \forall \quad i$$

Definition of Competitive Equilibrium

With taxes $\{\tau_i\}$ and government purchases $\{g_i\}$, allocations $\{c_i, l\}$ and prices $\{p_i\}$ are CE if and only if the following conditions are satisfied.

- consumers take $\{p_i\}$ as given and solve consumer's problem (1).
- firms take $\{p_i\}$ as given, solve firm's problem (2) and make 0 profit in equilibrium.
- government's budget constraint (3) is satisfied.
- market clearing condition (4) is satisfied.

Question: How to find $\{\tau_i\}$ to finance government expenditure $\{g_i\}$ so that welfare is maximized?

2 approaches:

1. Express everything as a function of τ and maximize w.r.t. τ directly;
2. Use "primal"/"Ramsey" approach.

We will take the second approach.

Idea: find necessary and sufficient conditions on $\{c_i, l\}$ that should be true in any CE, and then find the $\{c_i, l\}$ that satisfy these conditions and maximize the welfare.

Consumer's FOCs:

$$U_{ci} = \lambda p_i (1 + \tau_i)$$

$$U_l = -\lambda$$

which implies that

$$p_i (1 + \tau_i) = -\frac{U_{ci}}{U_l}$$

Substitute back into consumer's budget constraint to get

$$\sum U_{ci} c_i + U_l l = 0$$

Theorem

For any exogenous stream $(g_1, ..., g_n)$ *consider* $(c_1^*, ..., c_n^*, l^*)$ *that satisfy*

$$\sum U_{ci} c_i + U_l l = 0$$

$$F(c_1 + g_1, ..., c_n + g_n, l) = 0$$

Then there exists a competitive equilibrium with taxes for which $(c_1^*, ..., c_n^*, l^*)$ *are equilibrium allocations*

This may seem a little surprising since we have $n + 1$ variables and only two constraints. This means that there exist many solutions to this system of equations. For any solution that satisfies these conditions, we can find some taxes that would implement them.

Re-constructing equilibrium from allocations

- Pick any $(c_1^*, ..., c_n^*, l^*)$ that satisfies the conditions above.
- **Construct prices**: from firm's problem we have

$$p_i = \lambda F_i$$

$$-1 = \lambda F_l$$

Therefore,

$$p_i^* = -\frac{F_i(c_1^* + g_1, ..., c_n^* + g_n, l^*)}{F_l(c_1^* + g_1, ..., c_n^* + g_n, l^*)}$$

- **Construct taxes**: from consumer's FOCs

$$1 + \tau_i^* = -\frac{\frac{U_{ci}(c^*, l^*)}{U_l(c^*, l^*)}}{p_i^*} = \frac{U_{ci}(c^*, l^*)}{U_l(c^*, l^*)} \frac{F_l(c_1^* + g_1, ..., c_n^* + g_n, l^*)}{F_i(c_1^* + g_1, ..., c_n^* + g_n, l^*)}$$

Remaining sufficiency conditions

- Are firms making zero profit? Yes, since F is CRS.

$$\sum_i F_i c_i^* + F_l l^* = 0$$

- Does it raise enough money to finance the government?

$$\sum p_i g_i = \sum p_i \tau_i c_i$$

  - substitute definition of prices, taxes and consumer budget constraint to verify that it holds
  - also follows from Walras law

How to find something that maximizes social surplus?

$$\max_{c,l} U(c_1, ..., c_n, l)$$

$$s.t. \quad \sum U_{ci} c_i + U_l l = 0$$

$$F(c_1 + g_1, ..., c_n + g_n, l) = 0$$

- For simplicity that $U(c_1, ..., c_n, l) = u_1(c_1) + ... + u_n(c_n) + v(l)$
- Consider the FOCs

$$(1+\lambda)u_i'(c_i) + \lambda u_i''(c_i)c_i = \gamma F_i$$

$$(1+\lambda)v'(l) + \lambda v''(l)l = \gamma F_l$$

- Let $H_i = -u_i'' c_i / u_i'$, and $H_l = -v'' l / v'$. Then

$$\frac{(1+\lambda) - \lambda H_i}{(1+\lambda) - \lambda H_l} \frac{U_i}{U_l} = \frac{F_i}{F_l}$$

We know that

$$1 + \tau_i^* = \frac{U_i}{U_l}\frac{F_l}{F_i}$$

Therefore,

$$1 + \tau_i^* = \frac{(1+\lambda) - \lambda H_l}{(1+\lambda) - \lambda H_i}$$

$$\frac{\tau_i^*}{1+\tau_i^*} = \frac{\lambda(H_i - H_l)}{(1+\lambda) - \lambda H_l}$$

Combining with the same condition for good j, we get

$$\frac{\frac{\tau_i^*}{1+\tau_i^*}}{\frac{\tau_j^*}{1+\tau_j^*}} = \frac{H_i - H_l}{H_j - H_l}$$

$H_i > H_j$ implies that $\tau_i > \tau_j$.

What is Hi?

- Consumer theory: consume solves

$$\max \sum u_i(c_i)$$

s.t.

$$\sum p_i c_i \leq m$$

- The FOC of the consumer's maximization problem becomes

$$U_i(c_i(p,m)) = \lambda(p,m)p_i$$

Differentiate this with respect to non-labor income

$$U_{ii}\frac{\partial c_i}{\partial m} = p_i\frac{\partial \lambda}{\partial m} = \frac{U_i}{\lambda}\frac{\partial \lambda}{\partial m}$$

- This implies that

$$H_i \equiv -\frac{U_{ii}c_i}{U_i} = -\frac{c_i}{\lambda}\frac{\frac{\partial\lambda}{\partial m}}{\frac{\partial c_i}{\partial m}}$$

- Income elasticity of demand:

$$\eta_i = \frac{\partial c_i}{\partial m}\frac{m}{c_i}$$

We have

$$H_i = -\frac{\frac{\partial \lambda}{\partial m}\frac{m}{\lambda}}{\frac{\partial c_i}{\partial m}\frac{m}{c_i}} = \frac{-\frac{\partial \lambda}{\partial m}\frac{m}{\lambda}}{\eta_i}.$$

Thus,

$$\frac{H_i}{H_j} = \frac{\eta_j}{\eta_i}$$

where η_i is income elasticity of demand.
From

$$\frac{\frac{\tau_i^*}{1+\tau_i^*}}{\frac{\tau_j^*}{1+\tau_j^*}} = \frac{H_i - H_l}{H_j - H_l}$$

this implies that if a good has a higher income elasticity, it should be taxes at a lower rate. So it is optimal to tax necessities at a higher rate than luxury goods.

Lesson 1

- Spread out tax distortions across all goods
- Tax more heavily the goods for which demand is inelastic
- Higher taxes distort inelastic goods less $\longrightarrow$ deadweight burden is smaller
- Remark: the result that necessities should be taxed at a higher rate than luxuries is not very robust
    - derived under assumption that all agents are identical
    - if we allow for heterogeneity and income taxation, often obtain a *uniform commodity taxation* result: if consumption is weakly separable from labor, tax all goods at the same rate, do all the redistribution through labor income taxation.

Intermediate goods

- How would we tax goods that consumers do not consume directly such as intermediate goods?
- A general result (Diamond and Mirrlees (1971)) is that economy should always be on the production possibility frontier with optimal taxes.
- This implies that intermediate goods should not be taxed.

Two sectors:
Final goods sector has technology

$$f(x, z, l_1) = 0$$

where z is an intermediate good.

Intermediate goods sector has technology

$$h(z, l_2) = 0$$

Consumers maximize their utility subject to budget constraint.

$$\max U(c, l_1 + l_2)$$

$$s.t. \qquad p(1+\tau)c \leq w(l_1 + l_2)$$

Final goods sector maximizes its profit subject to feasibility constraint.

$$\max px - wl_1 - q(1+\tau_z)z$$

$$s.t. \qquad f(x, z, l_1) = 0$$

FOCs

$$-w = \gamma f_l$$
$$-q(1+\tau_z) = \gamma f_z$$

so that

$$\frac{f_l}{f_z} = \frac{w}{q(1+\tau_z)}$$

Intermediate goods sector maximizes its profit subject to feasibility constraint

$$\max qz - wl_2$$

$$s.t. \qquad h(z, l_2) = 0$$

FOCs

$$q = \gamma h_z$$

$$-w = \gamma h_l$$

so that

$$\frac{h_l}{h_z} = -\frac{w}{q}$$

$$\frac{h_l}{h_z} = -(1 + \tau_z)\frac{f_l}{f_z} \tag{2.1}$$

- Government budget constraint

$$\tau pc + \tau_z qz = pg$$

- Market clearing

$$c + g = x$$

- Following steps similar to those we did before, we can derive the implementability constraint

$$U_c c + U_l (l_1 + l_2) = 0$$

The social planner's problem is

$$\max U(c, l_1 + l_2)$$

$$s.t. \qquad U_c c + U_l(l_1 + l_2) = 0$$

$$f(c + g, z, l_1) = 0$$

$$h(z, l_2) = 0$$

FOC w.r.t z:

$$f_z\gamma_f + h_z\gamma_h = 0$$

or

$$\frac{f_z}{h_z} = -\frac{\gamma_h}{\gamma_f}$$

FOC w.r.t l_1

$$[l_1] : U_l(1+\lambda) + \lambda(U_{ll}(l_1 + l_2) + U_{cl}c) = f_l\gamma_f$$

FOC w.r.t l_2

$$[l_2] : U_l(1+\lambda) + \lambda(U_{ll}(l_1 + l_2) + U_{cl}c) = h_l\gamma_h$$

which implies that

$$\frac{f_l}{h_l} = \frac{\gamma_h}{\gamma_f}$$

or

$$\frac{f_l}{f_z} = -\frac{h_l}{h_z}$$

This suggests that when taxes are set optimally, the marginal rate of transformation should be undistorted across goods.
Comparing with the condition for CE (2.1) we see that in the optimum $\tau_z = 0$

Lesson 2

- Tax consumption goods but not intermediate goods
- The same final bundle of consumption can be achieved with either consumption or intermidate taxes
- ... but intermediate taxes distort more by misallocating intermediate inputs

Limitations

- externalities (obvious)
- intermidiate goods are not used as a consumption good
  - if can, tax final consumption but not intermidiate consumption, but that may not be feasible.
  - if cannot, the results need not apply, similar to what we show below.
- perfect competitition
  - may need to tax them if cannot tax monopoly's pure profits

Optimal capital taxation

- Dynamic economy
- Government: finances a stream of government purchases g_t.
- Assume that government can use only linear taxes.
- No lump sum taxes.
- No taxation of capital in the first period (equivalent to lump sum tax)

Environment

Representative infinitely lived agent with utility $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$.

Government:

- Needs to finance g_t.
- Chooses
  - taxes to finance g_t
  - government debt b_t to smooth out the distortions
- Representative agent with taxes.

Consumer's problem (1)

$$\max_{c,l,k} \sum \beta^t u(c_t, l_t)$$

s.t.

$$(1+\tau_{ct})c_t + k_{t+1} + b_{t+1} \leq (1-\tau_{kt})(1+(r_t-\delta))k_t + (1-\tau_{lt})w_t l_t + R_t b_t$$

$$k_0 = \bar{k}_0$$

Firm's problem (2)

$$\max_{k,l} F(k_t, l_t) - w_t l_t - r_t k_t$$

Government budget constraint (3)

$$g_t + R_t b_t \leq \tau_{lt} w_t l_t + \tau_{kt} \left(1 + (r_t - \delta)\right) k_t + \tau_{ct} c_t + b_{t+1}$$

Market clearing (4)

$$c_t + g_t + k_{t+1} \leq F(k_t, l_t) + (1 - \delta) k_t$$

Definition: CE with taxes $\{\tau_{lt}, \tau_{kt}\}$ and government purchases $\{g_t\}$ is allocations $\{c_t, l_t, k_k, b_t)$ and prices $\{w_t, r_t\}$ s.t.

- consumers take $\{w_t, r_t\}$ as given and solve consumer's problem (1)
- firms take $\{w_t, r_t\}$ as given, solve producer's problem and make 0 profit in equilibrium (2)
- government's budget constraint is satisfied (3)
- markets clear (4)

Some observations

Observation 1: Irrelevance of some taxes
FOCs:

$$\frac{u_l(t)}{u_c(t)} = -\frac{(1-\tau_{lt})w_t}{1+\tau_{ct}}$$

$$\frac{\beta u_c(t+1)}{u_c(t)} = \frac{p_{t+1}}{p_t} = \frac{(1+\tau_{ct+1})}{(1+\tau_{ct})\,(1-\tau_{kt+1})(1+r_{t+1}-\delta)}$$

$$(1+r_{t+1}-\delta) = R_{t+1}$$

Too many taxes, can get rid of some.
We will assume that $\tau_{ct} = 0$ for al t
Equivalently, we could assume that $\tau_{kt} = 0$ and have

$$\frac{(1+\hat{\tau}_{ct})}{(1+\hat{\tau}_{ct+1})} = (1-\tau_{kt+1})$$

Positive tax on capital and constant tax on consumption is *equivalent* to zero tax on capital and increasing tax on consumption.

Observation 2: Nothing fancy about dynamics
Instead of thinking about period t consumption, think about period 0 consumption of a good with label "t": equivalent to the static commodity taxation problem with infinitely many goods.

Observation 3: Non-distortionary taxation of capital in period 0

- Note that taxes on capital in period 0 does not distort any decisions: equivalent to a lump sum tax.
- If government could use this tax, it would set it at a very high level to get enought revenues to finance all future g_t.
- Assume (without any justification) that this tax is unavailable to make the problem interesting

$$\tau_{k0} = 0$$

Observation 4: (as before) Many ways to ensure that distortions hold
Here: tax gross return on capital $1 + r - \delta$
Could instead (as usually done in practice) tax net return $r - \delta$: nothing changes in the analysis.

Finding necessary conditions

Proceed as before: substitute FOCs into budget constraint:

$$u_c(t)c_t + u_l(t)l_t + u_c(t)\left[k_{t+1} + b_{t+1}\right] \leq \beta^{-1}u_c(t-1)\left[k_t + b_t\right]$$

Feasibility

$$c_t + k_{t+1} + g_t \leq F(k_t, l_t) + (1-\delta)k_t$$

These conditions necessary. Depends on 4 variables: c, l, k, b. Equivalently can re-write in terms of c, l, k, a where

$$a(t+1) \equiv u_c(t)\left[k_{t+1} + b_{t+1}\right]$$

so that we get

$$u_c(t)c_t + u_l(t)l_t + a(t+1) \leq \beta^{-1}a(t)$$

Sum over all the periods to get

$$\sum \beta^t \left[u_c(t)c_t + u_l(t)l_t\right] = u_c(0)k_0 \qquad \text{(ImC)}$$

Optimal taxes

Solve for the optimal allocations

$$\max \sum \beta^t u(c_t, l_t)$$

s.t. (F), (ImC).

FOCs:

$$\beta^t u_c(t) + \eta[\beta^t u_{cc}(t)c_t + \beta^t u_c(t) - \beta^t u_{cl}(t)l_t] = \lambda_t$$

$$\lambda_t = [F_k(t+1) + (1-\delta)]\lambda_{t+1}$$

Therefore

$$\begin{aligned} & \frac{u_c(t) + \eta[u_{cc}(t)c_t + u_c(t) - u_{cl}(t)l_t]}{u_c(t+1) + \eta[u_{cc}(t+1)c_{t+1} + u_c(t+1) - u_{cl}(t+1)l_{t+1}]} \qquad (*) \\ = \quad & \beta(F_k(t+1) + (1-\delta)) \end{aligned}$$

Theorem

No capital taxes in the steady state

Proof.

Suppose $c_t \longrightarrow c$, $k_t \longrightarrow k$, $l_t \longrightarrow l$. Then the equation above says

$$\beta(F_k(t+1) + (1-\delta)) = 1$$

Consumer (Euler) in the steady state

$$(1-\tau_k)\beta(1 + (r-\delta)) = 1$$

and $r = F_k$. These equations give

$$(1-\tau_k) = 1$$

so that $\tau_k = 0$ □

Theorem

Suppose $u(c, l) = \frac{1}{1-\sigma} c^{1-\sigma} + v(l)$ *(actually need much weaker conditions). Then* $\tau_t = 0$ *for all* $t > 1$.

Proof.

In this case $u_{cl} = 0$ and $u_{cc} c = -\sigma u_c$ and

$$\begin{aligned} & u_c + \eta [u_{cc} c + u_c - u_{cl} l] \\ = \; & u_c \left(1 + \eta \left[\frac{-\sigma u_c}{u_c} + 1\right]\right) \\ = \; & u_c \left(1 + \eta \left[1 - \sigma\right]\right) \end{aligned}$$

so that (*) becomes

$$\frac{u_c(t)}{u_c(t+1)} = \beta(F_k(t+1) + (1-\delta))$$

Definition of taxes on capital immidiately implies that $\tau_{kt} = 0$ for all $t > 1$ □

Discussion

General results:

- high tax on capital in the beginning, goes to zero.
- labor taxes typically positive
- government revenues high in the beginning, decrease over time: budget surplus in the beginning, deficit later on

Judd (JPubE 1987)
Add heterogeneity. Two types of agents, capitalists (who do not work and own capital) and workers (who work but cannot save). Capital and labor taxes not only create distortions but also redistributed from capitalists to workers. Showed a start result that even if the planner cares only about workers, still taxes are zero on capitalists in the long run.

Lesson 3

- Tax labor/consumption, not capital
- Capital distortions quickly accumulate due to compounding
- Contrast with a naive view that want to distribute distortions across all sources of income

Time consistency

- Compute the optimal policy from $t = 0$ perspective
  - high capital taxes early, go to zero (say, by $t = 100$)
- Compute the optimal policy from $t = 100$ perspective
- The two will not be the same
  - government has strong incentives to deviate from the optimal policy and choose different taxes later on

Time consistency II

- If government cannot commit, agents will take that into account when taxes are announced
- Invest less, even though they are promised low taxes tomorrow
  - know that tomorrow government cannot keep its promise and will revert to high taxes
- Welfare losses can be large without commitment
- Kydland and Prescott's 2004 Nobel prize
- **Lesson 4**: Optimal policy is not time consistent. It is important to be able to commit and avoid temptation ex-post

Taxation over business cycle

- Big topic, sketch one general idea
- Suppose preferences are $u(c_t, l_t) = c_t - \frac{1}{1+\gamma} l_t^{1+\gamma}$

$$u_l(c_t, l_t) = (1 - \tau_{lt}) w_t$$

- Suppose g_t follows some stochastic process
- Ignore capital (or set taxes on capital to zero)

Implementability constraint

- As before

$$u_c(t)c_t + u_l(t)l_t + a(t+1) \leq \beta^{-1}a(t)$$

- But now $u_c(t) = 1$
- Cannot sum, due to uncertainty
- SP solves

$$\max E \sum \beta^t \left[c_t - v(l_t)\right]$$

Taxes as random walk

- FOCs imply that
$$u_l(t) = E_t u_l(t+1)$$
if $v(l)$ is quadratic
$$\tau_t = E_t \{\tau_{t+1}\} \quad (*)$$
- Taxes follow random walk
  - independent of the stochastic process for g_t
- Implication:
  - consider a positive shock for g_t
  - government revenues must go up
  - for (*) to be satisfied, they must go up in all future periods by the same (small) amount
  - from government b.c., debt must go up a lot, be repaid over time

Lesson 5

- **Tax smoothing**: smooth tax distortions in response to shocks, use debt to help doing that

Summary

- Lesson 1: tax elastic goods less than less elastic
- Lesson 2: do not tax intermediate goods
- Lesson 3: do not tax capital
- Lesson 4: commitment is important
- Lesson 5: smooth taxes in response to shocks